Multiple Parton Scattering: Symmetrising PYTHIA's Model of nPDFs

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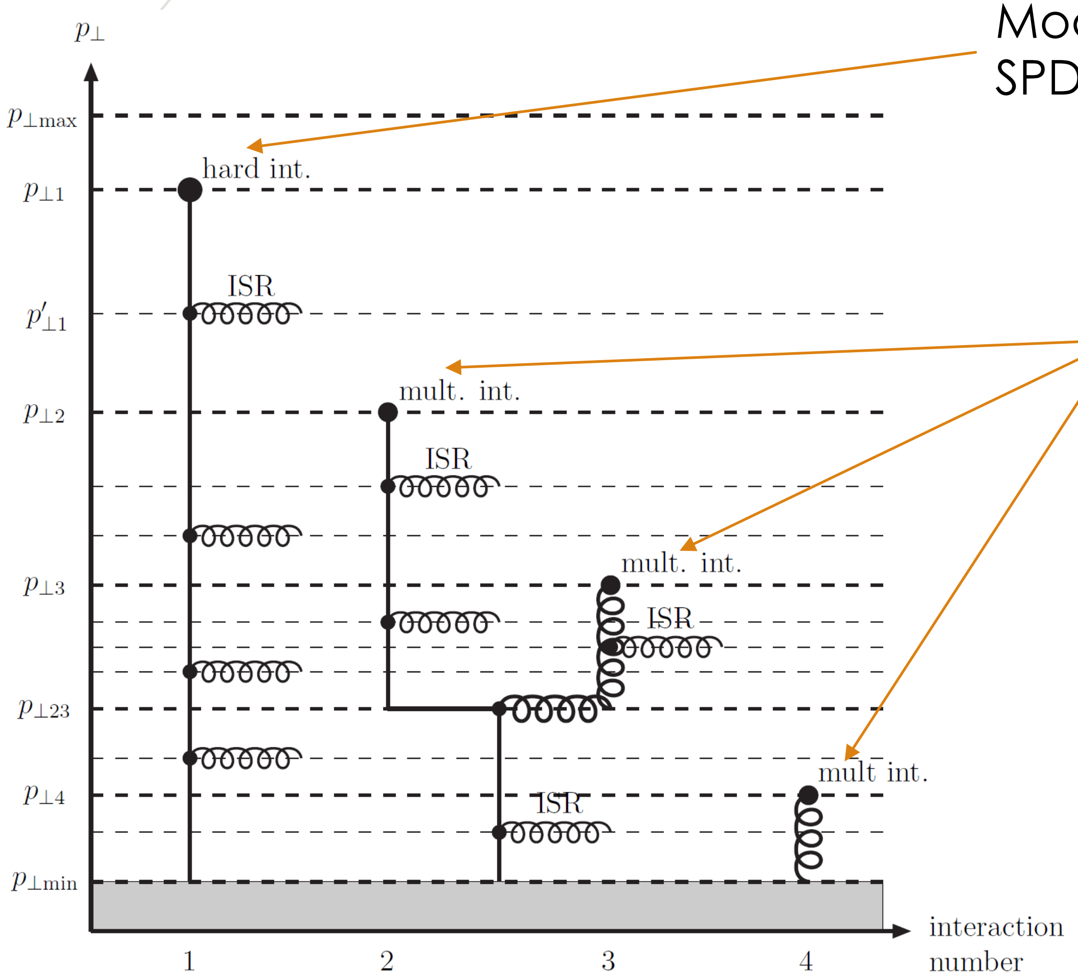
PYTHIA Model of MPI

How Does PYTHIA approach MPI?



Models the Hardest Interaction first, with SPDFs $f_j^r(x, Q)$ as in single parton scattering

"Subsequent" interactions modelled with modified SPDFs $f_{j_n}^{m \leftarrow j_1 x_1, \ldots, j_{n-1} x_{n-1}}(x, Q)$

ISR handled for each of these interactions as it would be done in SPS

Sjostrand, Skands, hep-ph/0402078, hep-ph/0408302

PYTHIA Model of MPI

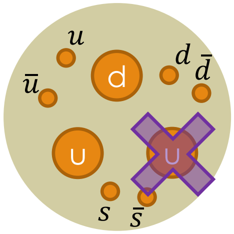
Four Principal Modifications:

(I) MOMENTUM "SQUEEZING": $f_{j_n}^{m \leftarrow j_1, x_1 \ldots j_{n-1}, x_{n-1}}(x, Q) = \frac{1}{X} f_i^r \left(\frac{x}{X}, Q \right)$ $\quad [X = 1 - \sum_{j=1}^{n-1} x_j]$

Ensures $\sum_{jn} \int dx\, x\ f_{j_n}^{m \leftarrow j_1, x_1 \ldots j_{n-1}, x_{n-1}}(x, Q) = X$.

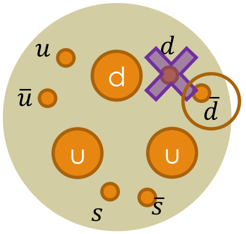
(II) VALENCE NUMBER SUBTRACTION:

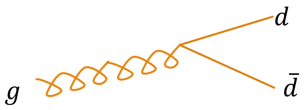
$$f_{i_v}^{m \leftarrow j_1, x_1 \ldots j_{n-1}, x_{n-1}}(x, Q) = \frac{N_{i_v n}}{N_{i_v 0}} \frac{1}{X} f_{i_v}^r \left(\frac{x}{X}, Q \right)$$



(III) COMPANION QUARK ADDITION:

$$q_c(x, x_s) = C(x_s) P_{g \to q\bar{q}} \left(\frac{x_s}{x_s + x} \right) \frac{g(x_s + x)}{x_s + x}$$





(IV) SEA QUARK AND GLUON RESCALING: Steps (II) and (III) break (I). To fix, rescale all sea quark and gluon distributions by a factor "a".

Sjostrand, Skands, hep-ph/0402078, hep-ph/0408302

PYTHIA Model of MPI

Full expression for for multi-parton PDFs in PYTHIA:

$$F_{j_1 \dots j_n}(x_1 \dots x_n, Q_1 \dots Q_n) = f^{r}_{j_1}(x_1, Q_1) f^{m \leftarrow j_1, x_1}_{j_2}(x_2, Q_2) \dots f^{m \leftarrow j_1, x_1 \dots j_{n-1}, x_{n-1}}_{j_n}(x_n, Q_n)$$

PYTHIA thus uses the nPDFs, instead of y_i-dependent nPDs

$$F_{j_1 \dots j_n}(x_1 \dots x_n, Q_1 \dots Q_n) \equiv \int d^2 y_1 \dots d^2 y_{n-1} \Gamma(x_1 \dots x_n, \boldsymbol{y_1} \dots \boldsymbol{y_{n-1}}, Q_1 \dots Q_n)$$

Can these objects be constrained by theory?

The Sum Rules

For DPS ($n = 2$), we know for $Q_1 = Q_2 = Q$ the **sum rules**:

Gaunt, Stirling, 0910.4347
Blok, Dokshitzer, Frankfurt, Strikman, 1306.3763
Diehl, Plößl, Schäfer, 1811.00289

Momentum rule: $\sum_{j_2} \int dx_2\, x_2\; D_{j_1 j_2}(x_1, x_2, Q) = (1 - x_1)\, f_{j_1}(x_1, Q)$

Available momentum after "taking out" j_1

Number rule: $\int dx_2\; D_{j_1 j_{2v}}(x_1, x_2, Q) = \left(N_{j_{2v}} - \delta_{j_1 j_2} + \delta_{j_1 \bar{j}_2}\right) f_{j_1}(x_1, Q)$

Number of j_2 quarks - number of $\bar{j}_2$ quarks after "taking out" j_1

In TPS ($n = 3$), it has been shown that similar rules hold:

Gaunt, Fedkevych, 2208.08197

Momentum rule: $\sum_{j_3} \int dx_3\, x_3\; T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) = (1 - x_1 - x_2)\, D_{j_1 j_2}(x_1, x_2, Q)$

Number rule:

$$\int dx_3\; T_{j_1 j_2 j_{3v}}(x_1, x_2, x_3, Q) = \left(N_{j_{3v}} - \delta_{j_1 j_3} - \delta_{j_2 j_3} + \delta_{j_1 \bar{j}_3} + \delta_{j_2 \bar{j}_3}\right) D_{j_1 j_2}(x_1, x_2, Q)$$

Similar rules have been proven for the QPDS (n=4) case.

How well do the Pythia double and triple PDFs satisfy these?

Pythia nPDFs and the sum rules

Sum rules satisfied by construction when integrating over final parton

$$F_{j_1 \dots j_n}(x_1 \dots x_n, Q) = f_{j_1}^{r}(x_1, Q) f_{j_2}^{m \leftarrow j_1, x_1}(x_2, Q) \dots f_{j_n}^{m \leftarrow j_1, x_1 \dots j_{n-1}, x_{n-1}}(x_n, Q)$$

Rules violated when integration conducted over any other x_i

In an nPDF we expect $\{x_i, j_i\} \leftrightarrow \{x_k, j_k\}$ symmetry

$$F_{j_1 \dots j_i \dots j_k \dots j_n}(x_1 \dots x_i \dots x_k \dots x_n, Q) = F_{j_1 \dots j_k \dots j_i \dots j_n}(x_1 \dots x_k \dots x_i \dots x_n, Q)$$

A symmetry not maintained by the PYTHIA model

Pythia nPDFs and the sum rules

Can construct 'naïve' symmetrisation for arbitrary nPDF:

$$F^{symm}_{j_1 \dots j_n}(x_1 \dots x_n, Q) = \frac{1}{n!} \sum_{\{1,\dots,n\}} F_{j_1 \dots j_n}(x_1 \dots x_n, Q)$$

Gaunt, Fedkevych, 2208.08197

DPDF (n=2)

$$D^{sym}_{j_1 j_2}(x_1, x_2, Q) = \frac{1}{2!} \sum_{\{1,2\}} D_{j_1 j_2}(x_1, x_2, Q)$$

TPDF (n=3)

$$T^{sym}_{j_1 j_2 j_3}(x_1, x_2, x_3, Q) = \frac{1}{3!} \sum_{\{1,2,3\}} T_{j_1 j_2 j_3}(x_1, x_2, x_3, Q)$$

Pythia nPDFs and the sum rules

Symmetrised DPDF : satisfies sum rules to ~10-25% level across most x_1, but very large deviations elsewhere

x_1	Momentum sum rule (MSR) ($j_1 = u$). Should = 1.	$u\bar{u}$ number sum rule (NSR). Should = -1.	$\bar{u}u$ NSR. Should = 3.
10^{-6}	0.979	-1.227	2.961
10^{-3}	0.980	-0.847	3.351
0.1	1.014	-0.925	3.491
0.2	1.047	-0.928	3.580
0.4	1.133	-0.884	3.858
0.8	1.679	-0.740	7.048

Connected to companion quark mechanism when both quarks have large x

Symmetrised TPDF: broadly similar trends, extreme values of x_1 problematic

$x_2 = 10^{-4}$

x_1	$j_1 = j_2 = u$ MSR. Should = 1.	uuu NSR. Should = 0.	$u\bar{u}u$ NSR. Should = 2.
10^{-6}	0.965	0.108	2.542
10^{-3}	0.967	-0.276	2.154
0.1	0.998	-0.232	2.188
0.2	1.029	-0.242	2.189
0.4	1.117	-0.317	2.161
0.8	1.719	-0.589	2.079

Gaunt, Fedkevych, 2208.08197

Improving Symmetrisation

Companion quark asymmetry

$$D^{comp}_{j_1 j_2}(x_1, x_2, Q) = f^s_{j_1}(x_1, Q) q_c(x_2, x_1) = f^s_{j_1}(x_1, Q) C(x_1) P_{g\to q\bar{q}}\left(\frac{x_1}{x_1 + x_2}\right)\frac{g(x_1 + x_2)}{x_1 + x_2}$$

Companion term asymmetric

Proposed Modification

Change companion function:

Gaunt, Stirling, 0910.4347

$$q_c(x, x_s) = \frac{C(x_s + x)}{f^s_{j_1}(x_s)}\left\langle P_{g\to q\bar{q}}\left(\frac{x_s}{x_s + x}\right)\right\rangle\frac{g(x_s + x)}{x_s + x} = \frac{-[\partial_y f^s_{j_1}(y)]\big|_{y=x+x_s}}{f^s_{j_1}(x_s)}$$

Gives a symmetric companion contribution:

$$D^{comp}_{j_1 j_2}(x_1, x_2, Q) = f^s_{j_1}(x_1, Q) q_c(x_2, x_1) = -[\partial_y f^s_{j_1}(y)]\big|_{y=x_2+x_1}$$

Improving Symmetrisation

High-x_1 Deviation

x_1	$j_1 = u$ MSR Should = 1.	$u\bar{u}$ NSR. Should = -1.	$\bar{u}u$ NSR. Should = 3.
0.8	1.679	-0.740	7.048

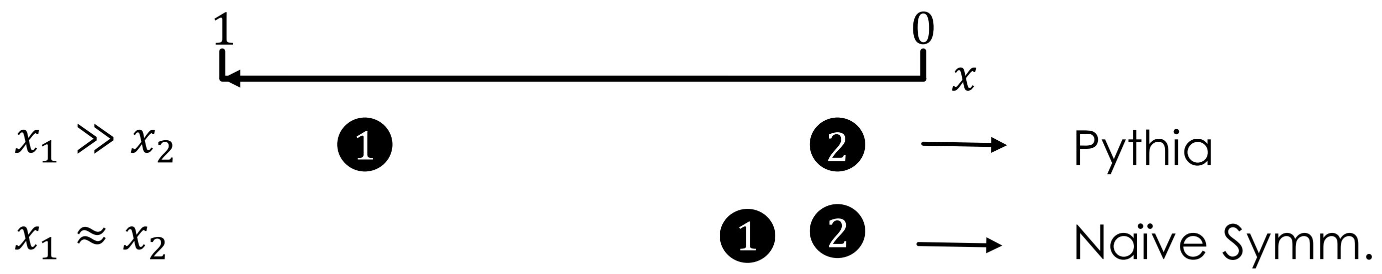
Large deviation from expectation when x_1 large

Implies $D^{sym}_{j_1 j_2}(x_1, x_2, Q)$ behaving unlike an accurate PDF

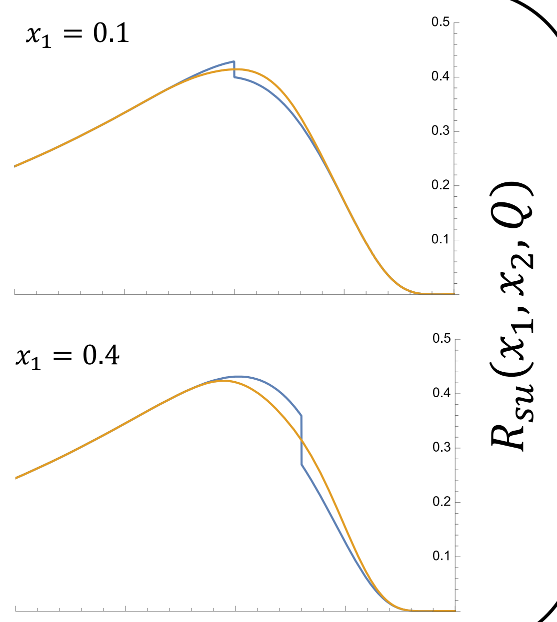
Introduce 'X-Ordering'

Step 1) Force the largest x 'first'

Step 2) Smoothly interpolate between the PYTHIA and naïvely symmetrised terms away from problem areas



Smooth vs. Strict



Modified nPDFs and the sum rules

Modified DPDF : satisfies sum rules to <10% level across most x_1, introduced >10% deviations at low x_1.

x_1	$(j_1 = u)$ MSR. Should = 1.	$u\bar{u}$ NSR. Should = -1.	$\bar{u}u$ NSR. Should = 3.
10^{-6}	0.974	-1.137	3.134
10^{-3}	0.968	-1.085	3.089
0.1	1.023	-1.003	2.928
0.2	1.022	-0.996	2.923
0.4	1.007	-0.994	2.965
0.8	1.000	-0.997	2.934

NB: All numbers hereon out are preliminary!

x_1	$j_1 = s$ MSR. Should = 1.	$s\bar{s}$ NSR. Should = -1.	$\bar{s}s$ NSR. Should = 1.
10^{-6}	0.974	-0.999	0.999
10^{-3}	0.967	-1.000	1.000
0.1	0.957	-1.000	1.000
0.2	0.976	-1.000	1.000
0.4	0.986	-1.000	1.000
0.8	1.014	-0.964	0.964

Improving Symmetrisation

Low-x_1 Deviation Post-Modification

x_1	$u\bar{u}$ NSR. Should = -1.	$\bar{u}u$ NSR. Should = 3.
10^{-6}	-1.137	3.134

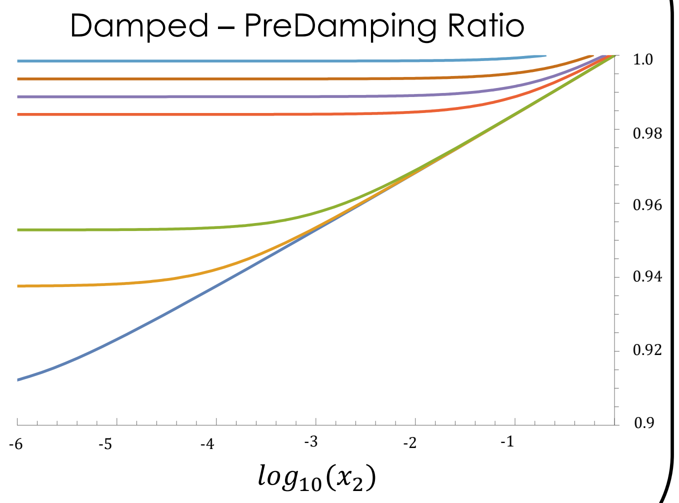
Introduce Damping Factor

Weight the quark components by $(x_1 + x_2)^{\alpha}$, $0 < \alpha \ll 1$, reducing overcontributions when x_1 and x_2 are both low

Modified DPDF, $\alpha = 0.007$: satisfies sum rules to <10% level across all sampled x_1

x_1	$j_1 = u$ MSR. Should = 1.	$u\bar{u}$ NSR. Should = -1.	$\bar{u}u$ NSR. Should = 3.
10^{-6}	0.964	-1.075	3.072
10^{-3}	0.958	-1.033	3.035
0.1	1.018	-0.987	2.902
0.2	1.018	-0.986	2.904
0.4	1.005	-0.989	2.953
0.8	0.999	-0.996	2.931

Damping Effect



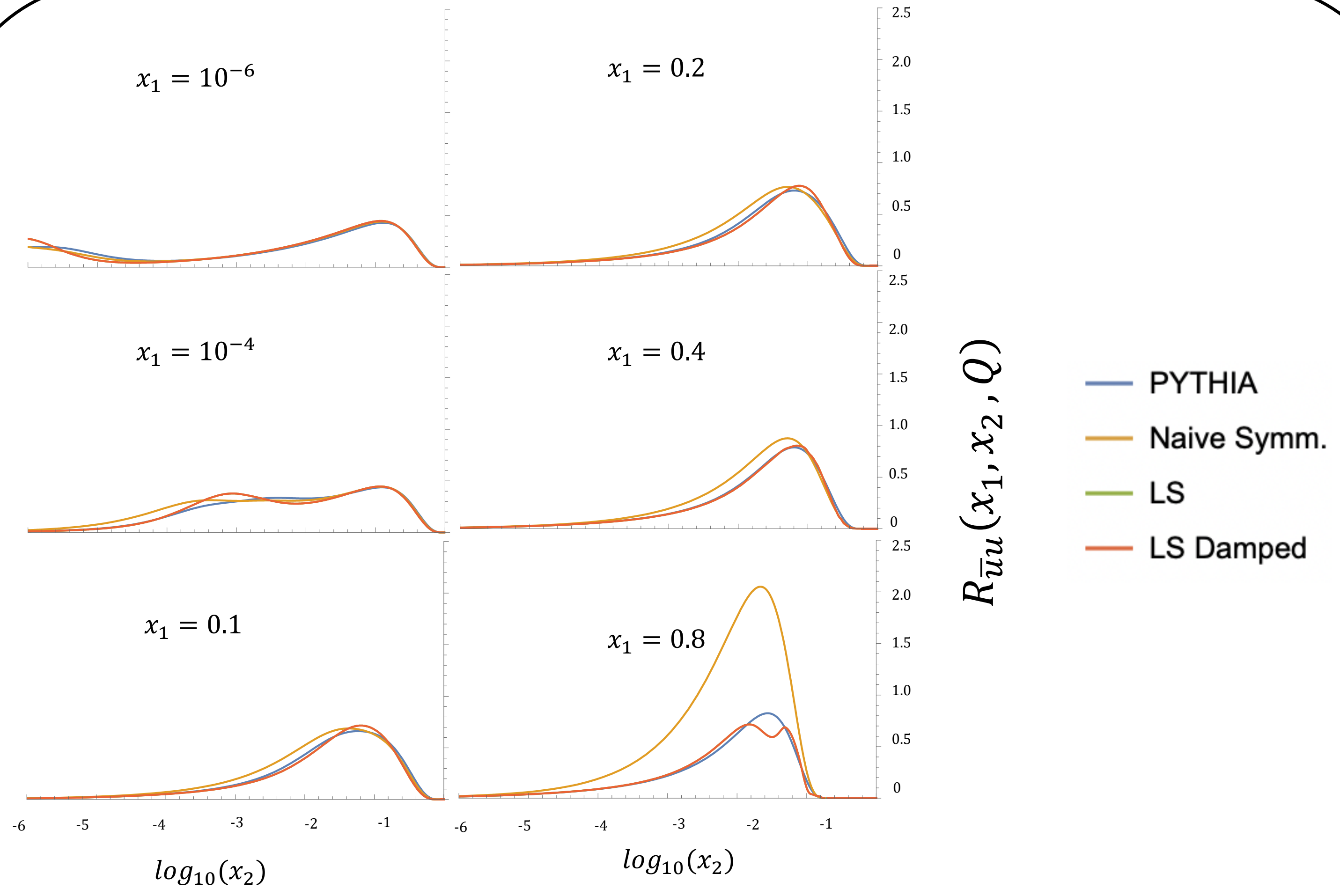
Comparison between schemes

DPDF NSR Response Function

$$R_{j_1 j_2}(x_1, x_2, Q) = \frac{x_2 D_{j_1 j_2}(x_1, x_2, Q)}{f^r_{j_1}(x_1)}$$

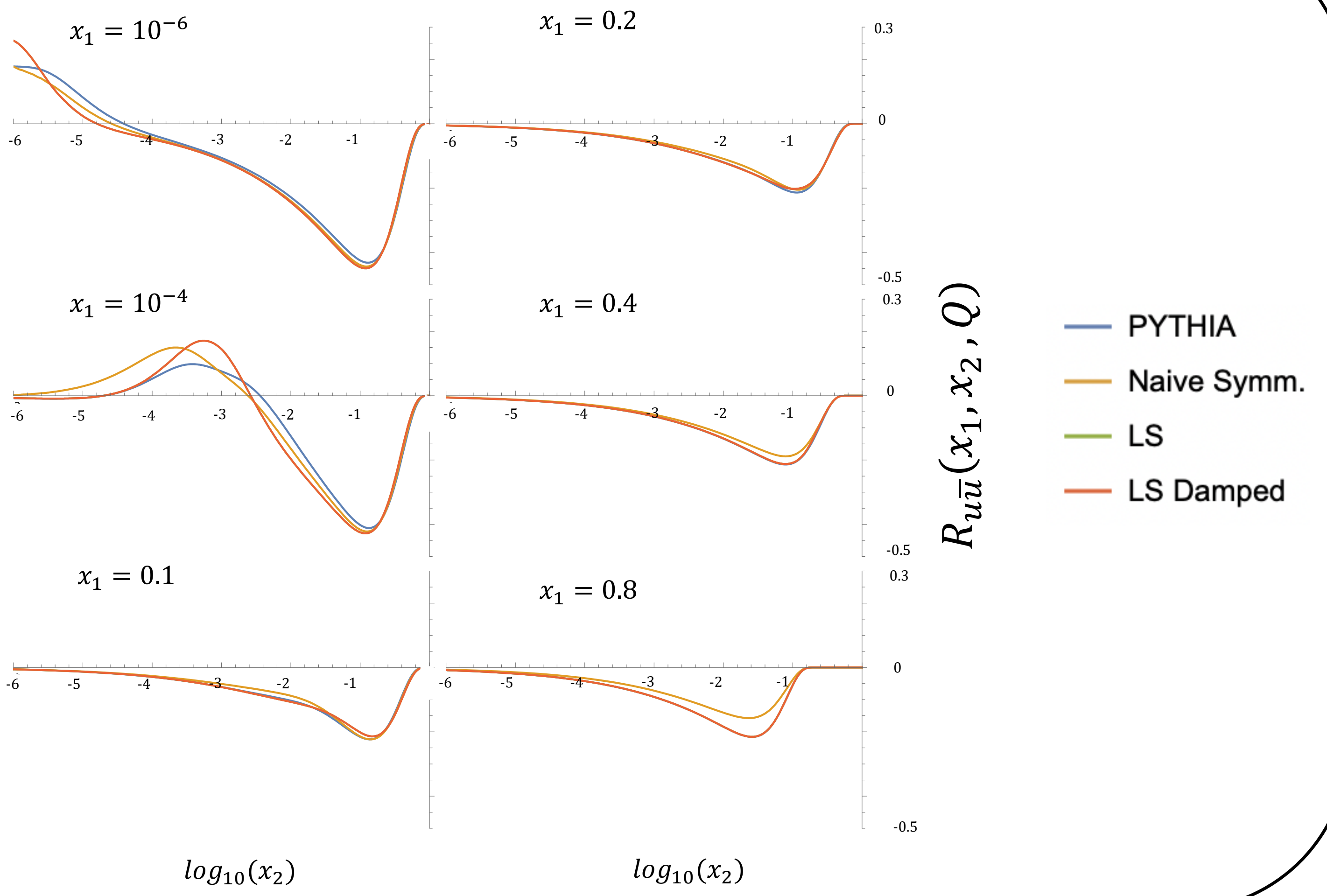
NSR Integrand

Response Functions



Comparison between schemes

Response Functions



DPDF NSR Response Function

$$R_{j_1 j_2}(x_1, x_2, Q) = \frac{x_2 D_{j_1 j_2}(x_1, x_2, Q)}{f^r_{j_1}(x_1)}$$

NSR Integrand

Summary

- Proposed three modifications to the PYTHIA model of MPI that improve adherence to the GS sum rules:
  - Change the companion quark mechanism to one that is manifestly symmetric
  - X-order the PDFs instead of naïvely symmetrise to avoid overcontribution from "incorrect" PDFs
  - Damp out the low x_1, x_2
- These changes are all symmetric in $\{x_i, j_1\} \leftrightarrow \{x_k, j_k\}$, and have improved the GS sum rule adherence of the symmetrised DPDFs to a <10% deviation from theory
- Implementing these changes into PYTHIA to quantify phenomenological effects remains outstanding
  - Similarity of modified response functions and DPDFs to unmodified PYTHIA encouraging