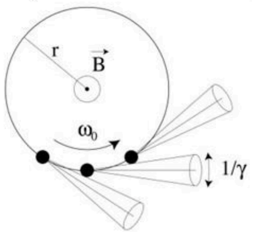
International MUON Collider Collaboration

Synchrotron Radiation from Muons in a Muon Collider



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This is a simple simulation study of synchrotron radiation of muons in several imaginary muon colliders that are simple circles, of radii 1 and 2 km, with uniform bending fields.

The main point is that muons will lose energy turn-by-turn and thereby scan downwards in $\sqrt{s}$ during each fill of the machine. This can be used to search for narrow high-mass states.

First, derive simple expressions for the energy loss per turn, then cycle over colliders from $\sqrt{s} = 0.125$ to 30 TeV, at two radii R=1.0 and R=2.0 km. Calculate the full energy spread due to radiation loss (2Γ) and the fractional energy loss to radiation ($2\Gamma/\sqrt{s}$).

The relativistic synchrotron radiation energy loss can be written as the non-relativistic Larmor power, times the Lorentz boost, $\gamma = E/m$, to the fourth power. This is

$$P = \left[\frac{1}{6\pi\epsilon_0}\frac{e^2a^2}{c^3}\right]\gamma^4$$

and simplifying with

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}, \qquad a = \frac{v^2}{R}, \qquad \text{and} \qquad T = \frac{2\pi R}{v},$$

substitute, multiply by T, and take $v = c$, the energy loss per turn is

$$\Delta E \text{ (energy loss per turn)} = \frac{4\pi}{3}\,(\alpha\hbar c)\,\frac{\gamma^4}{R} \quad = \quad \frac{4\pi}{3}\left[\frac{\alpha\hbar c}{m^4}\right]\frac{E^4}{R}$$

The units are chosen by the choice of $\hbar c$: $\hbar c \approx 0.197\ \mathrm{GeV} \cdot \mathrm{F} = 0.197 \times 10^{-18}\ \mathrm{GeV} \cdot \mathrm{km}$.

$$\Delta E\ (\text{energy loss per turn}) = \frac{4\pi}{3}\,(\alpha \hbar c)\,\frac{\gamma^4}{R} \quad = \quad \frac{4\pi}{3}\left[\frac{\alpha \hbar c}{m^4}\right]\frac{E^4}{R}$$

For $e^{\pm}$, $\mu^{\pm}$ and $p, \bar{p}$ these are:

$$e^{\pm} \qquad \Delta E\ (\mathrm{GeV/turn}) = (8.85 \times 10^{4})\,\frac{E^4}{R} \qquad (E \text{ in TeV}, R \text{ in km})$$

$$\mu^{\pm} \qquad \Delta E\ (\mathrm{GeV/turn}) = (4.84 \times 10^{-5})\,\frac{E^4}{R} \qquad \text{''}$$

$$p, \bar{p} \qquad \Delta E\ (\mathrm{GeV/turn}) = (7.78 \times 10^{-9})\,\frac{E^4}{R} \qquad \text{''}$$

The simulation integrates the radiative energy loss in steps of 0.1 turn, and the useful number of turns is $N_{\text{turns}} \sim 300\ B(\text{T})$, at which the luminosity is down to $1/e^2 \approx 13.5\,\%$. This is calculated from $p = 0.3BR$, $\tau = \gamma\tau_\mu$, and $T = 2\pi R/c$.

Define Γ(GeV) to be the energy loss in one fill, so 2Γ(GeV) is the reduction in $\sqrt{s}$.

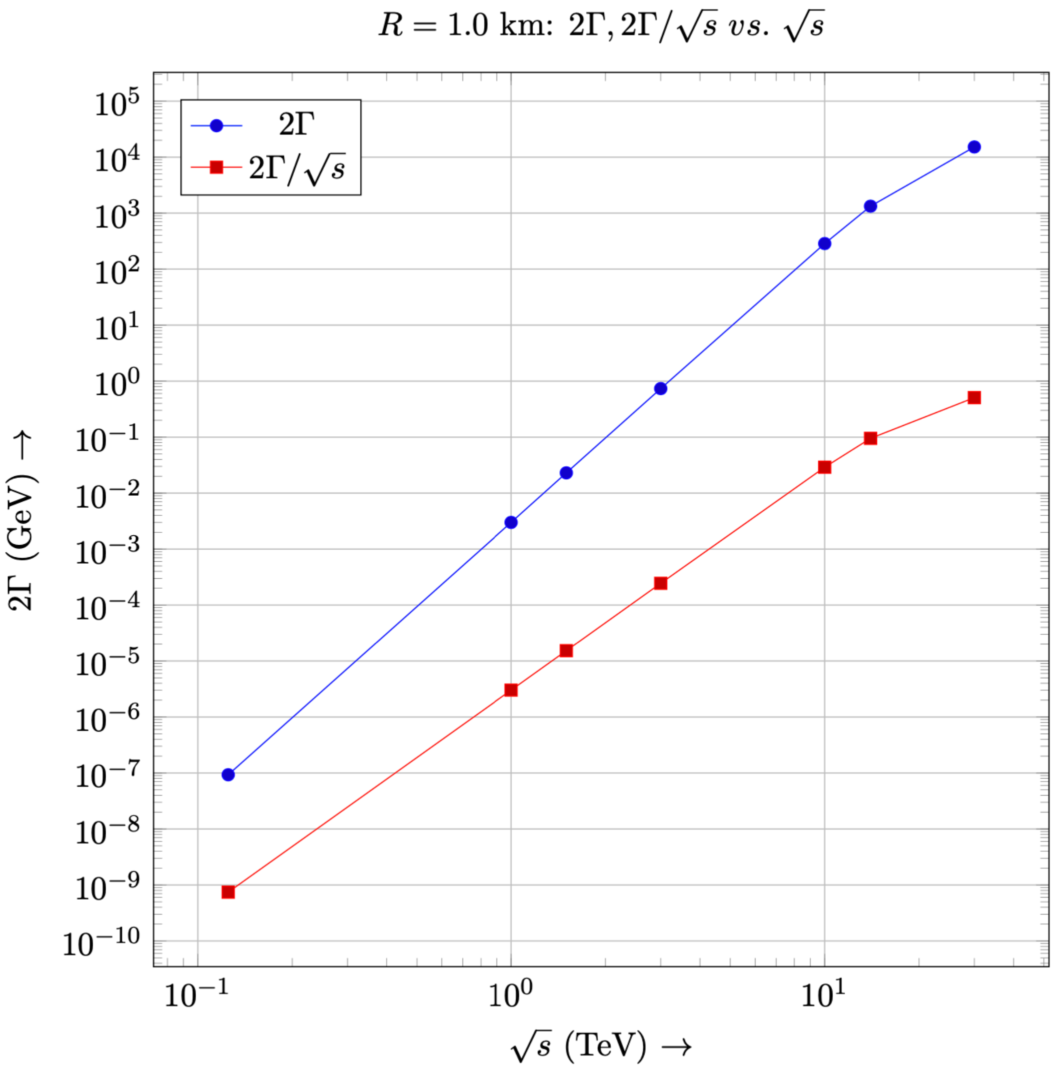
The energy loss starts at some ΔE(GeV) at energy E on the first turn. Crudely, the energy loss on the second turn is less, proportional to $(E - \Delta E)^4$, and so on.

I apologize for the tables, and I will turn these tables into log-log plots after introducing stochastic fluctuations into this simple calculation.

R = 1.0 km

$\sqrt{s}$ (TeV)	E_{beam} (TeV)	R (km)	B (T)	Number turns/fill	ΔE(GeV) (first turn)	2Γ (GeV)	$2\Gamma/\sqrt{s}$
0.125	0.062	1.0	0.208	63	7.39×10^{-10}	9.31×10^{-8}	7.44×10^{-10}
1.000	0.500	1.0	1.667	500	3.03×10^{-6}	3.02×10^{-3}	3.02×10^{-6}
1.500	0.750	1.0	2.500	750	1.53×10^{-5}	0.023	1.53×10^{-5}
3.000	1.500	1.0	5.000	1500	2.45×10^{-4}	0.735	2.45×10^{-4}
							0.1%
10.000	5.000	1.0	16.67	5000	0.030	286.	0.029
14.000	7.000	1.0	23.33	7000	0.116	1330.	0.095
30.000	15.000	1.0	50.00	15000	2.45	1.52×10^{4}	0.507

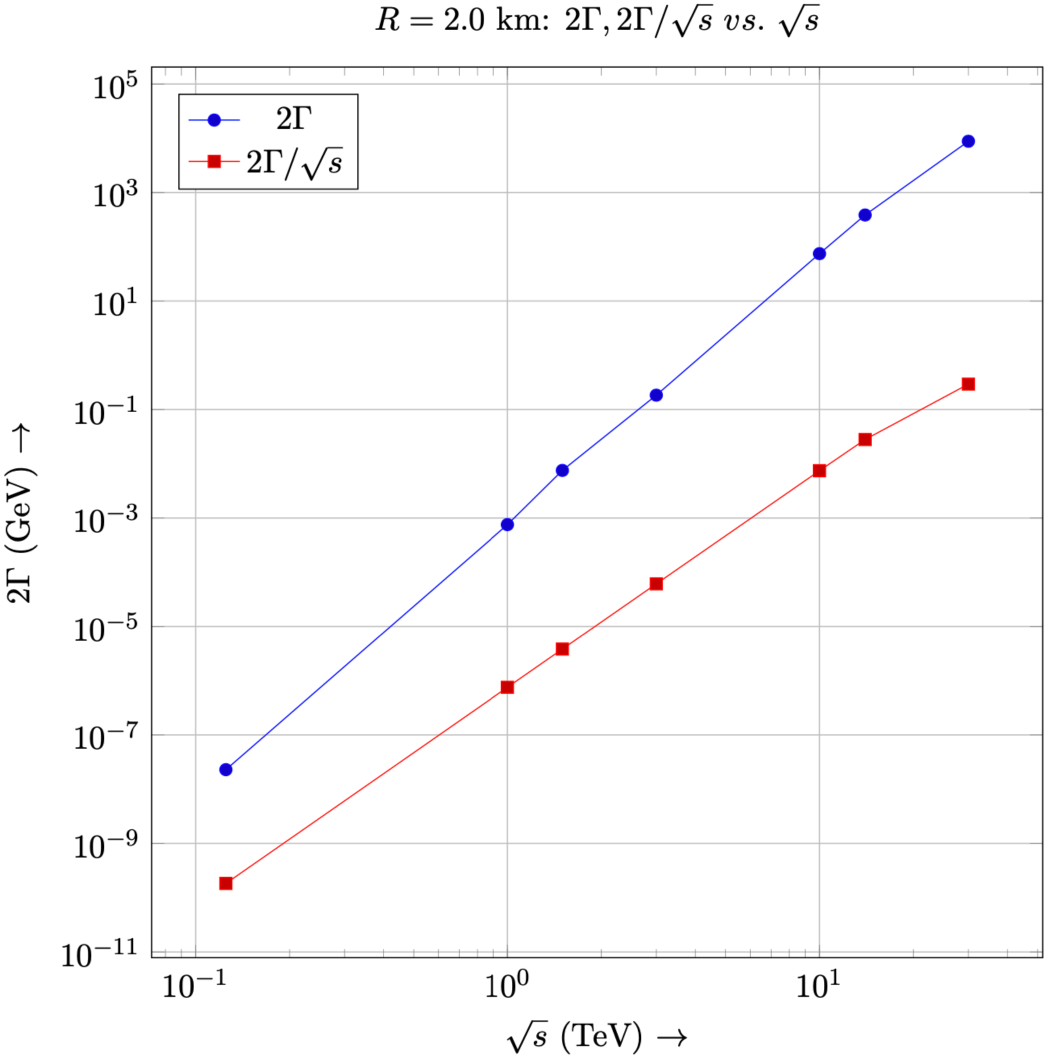
ΔE(GeV) is the energy lost by one muon on the first turn; 2Γ(GeV) is the energy lost by both muons during the entire fill.



$R = 1.0$ km: $2\Gamma, 2\Gamma/\sqrt{s}$ $vs.$ $\sqrt{s}$
2Γ
$2\Gamma/\sqrt{s}$
2Γ (GeV) $\rightarrow$
$\sqrt{s}$ (TeV) $\rightarrow$
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-1}
10^{0}
10^{1}

R = 2.0 km

$\sqrt{s}$ (TeV)	E_{beam} (TeV)	R (km)	B (T)	Number turns/fill	ΔE(GeV) (first turn)	2Γ (GeV)	$2\Gamma/\sqrt{s}$
0.125	0.062	2.0	0.104	31	3.69×10^{-10}	2.29×10^{-8}	1.83×10^{-10}
1.000	0.500	2.0	0.833	250	1.51×10^{-6}	7.56×10^{-4}	7.56×10^{-7}
1.500	0.750	2.0	1.250	375	7.66×10^{-6}	5.74×10^{-3}	3.83×10^{-6}
3.000	1.500	2.0	2.500	750	1.23×10^{-4}	0.184	6.12×10^{-5}
10.000	5.000	2.0	8.333	2500	0.015	74.6	7.46×10^{-3}
							0.1%
14.000	7.000	2.0	11.67	3500	0.058	385.	0.028
30.000	15.000	2.0	25.00	7500	1.23	8.81×10^{3}	0.294



$R = 2.0$ km: $2\Gamma, 2\Gamma/\sqrt{s}$ $vs.$ $\sqrt{s}$
2Γ
$2\Gamma/\sqrt{s}$
10^{5}
10^{3}
10^{1}
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
2Γ (GeV) $\rightarrow$
10^{-1}
10^{0}
10^{1}
$\sqrt{s}$ (TeV) $\rightarrow$

Is any of this important, or in any way useful?

From the numbers in the table, it seems that a 1-km radius muon collider at 10 TeV can do a mass scan downwards by 286 GeV from $\sqrt{s}$=10 TeV. At 14 TeV collider, the scan is downwards by 1330 GeV from $\sqrt{s}$=14 TeV. This is nearly 10% of $\sqrt{s}$.

Fact check: It is not conceivable that important decisions on the radius or energy will be made with these slight considerations in mind!

However, this mass scan is only possible in a muon collider, and so we persist.

RF cavities are used to restore energy lost to radiation, which at LEP 2 was 2.92 GeV/turn (and a huge power bill). For more modest RF, suppose ~0.1 GeV/turn, the RF can be used to both *retard* the radiative loss and to *increase* the muon beam energy.

In other words, "RF manipulation of $\sqrt{s}$" for the physics collisions … to my knowledge, this has has never been done (at LEP?).

At RF ~ 0.1 GeV/turn, a $\sqrt{s} = 3.0$ TeV collider with radius R=2.0 km loses only $2\Gamma = 0.184$ GeV in a fill. This is negligible, but the scan range, 2Γ, can be pushed up or down by (750 turns) x (0.1 GeV/turn) ~ 75 GeV, which is 150 GeV out of 3000 GeV, or 5% of $\sqrt{s}$.

With RF ~0.2 GeV/turn, this 5% becomes 10%.

The electric field in the RF cavities can be varied by slipping the phase of the RF, rather than actually changing the electric field.

This stepping down, or stepping up, of the energy of particles in a collider is only possible for a muon collider since muons radiate more than protons, but less than electrons! And it is scientifically useful because the muon is point-like.

All of this may presuppose some theoretical possibility of high-mass narrow states.

Thank you for your attention.