Synchrotron Radiation from Muons in a Muon Collider

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Abstract

At TeV beam energies and in small radius collider rings, muons will radiate synchrotron photons. A Muon Collider will scan a certain mass range in $\sqrt{s}$ as the beam muons lose energy turn-by-turn, in contrast to an e^+e^- or pp machine. This gradual decrease will have few consequential effects, other than the precisely known energy decrease per turn, but could be beneficial in searches for narrow high-mass states. It is like an "energy vernier" measuring small precise energy changes as the beams circulate.

Introduction: mean radiation

A 1.5 TeV muon in a 1 km radius ring will radiate about 0.245 MeV on the first turn, and in 1500 turns it will radiate an accumulated 0.735 GeV. This is not much energy loss, and it may play a small role in machine optics, but it can be employed to allow more efficient mass searches as the $\sqrt{s}$ of the colliding muons scans down turn-by-turn. One argument against electron-positron colliders (pre-Nov. 1974) was that a mass scan would be very time-consuming since the beam energy spread was so narrow, whereas the whole mass range is scanned at once in pN collisions. This is actually a correct argument: The J/ψ was found in pN collisions months earlier than in e^+e^- collisions, as was the Υ.

A muon collider has a beam energy spread[1] of about 0.1%, and the central beam energy will be reduced as the muons gradually lose energy turn-by-turn by synchrotron radiation during a single fill. In the context of synchrotron radiation, it might be useful to think about extreme undulators or wigglers in the straight section of a RLA.

Derivation of useful formulas

Relativistic synchrotron radiation power is often simply written as the non-relativistic Larmor radiated power formula times the Lorentz boost ($\gamma = E/m$) to the fourth power, or

$$\text{Power radiated,} \quad P = \left[\frac{1}{6\pi\epsilon_0}\frac{e^2a^2}{c^3}\right]\ \gamma^4,$$

where e^2, the acceleration a, and the time for one revolution T are given by

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \qquad a = \frac{v^2}{R} \qquad \text{and} \quad T = \frac{2\pi R}{v}.$$

The energy loss per turn (substituting and multiply by T and taking $v = c$) is

$$\Delta E \text{ (energy loss/turn)} = \frac{4\pi}{3}(\alpha\hbar c)\frac{\gamma^4}{R} = \frac{4\pi}{3}\left(\frac{\alpha\hbar c}{m^4}\right)\frac{E^4}{R},$$

Choosing the units of $\hbar c$ sets the energy unit: $\hbar c \approx 0.197$ GeV·F $\approx 0.197 \times 10^{-18}$ GeV·km. For $\gamma = E/m$, inserting masses in TeV units for the electron ($m_e = 0.511 \times 10^{-6}$ TeV), the muon (0.10566×10^{-3} TeV), and the proton ($m_p = 0.938256 \times 10^{-3}$ TeV) yields these expressions for radiative energy loss in GeV, for beam energy E in TeV and bending radius R in km:

$$e^{\pm} \qquad \Delta E \text{ (GeV/turn)} = (8.85 \times 10^{4}) \ \frac{E^4}{R} \qquad (E \text{ in TeV}, \ R \text{ in km})$$

$$\mu^{\pm} \qquad \Delta E \text{ (GeV/turn)} = (4.84 \times 10^{-5}) \ \frac{E^4}{R} \qquad \text{''}$$

$$p, \bar{p} \qquad \Delta E \text{ (GeV/turn)} = (7.78 \times 10^{-9}) \ \frac{E^4}{R} \qquad \text{''}$$

Numerical checks: **LEP II:** $\Delta E = (8.85 \times 10^4)(0.10)^4/3.026 = 2.92$ GeV/turn.
LHC: $\Delta E = (7.78 \times 10^{-9})(7.0)^4/3.026 = 6.65 \times 10^{-6}$ GeV/turn = 6.65 keV/turn. Both of these agree with the numbers in Prat [2].

Beam radiative energy spread during each fill (Γ)

The total number of useful turns of the beams in one fill is approximately equal to 300B (for B the bending field in Tesla)[1].

A muon beam will lose ΔE(GeV/turn) on the first turn, and on the second turn E(TeV) is smaller by ΔE, and E^4 is smaller by $4E^3\Delta E$, so the subsequent energy losses rapidly grow smaller.

Let's define Γ to be the "full width beam radiative energy spread" during one fill, or

$$\Gamma = \Delta E \text{ (GeV/fill)}$$

and since both beams lose energy the same way, the center-of-mass energy $\sqrt{s}$ spread is 2Γ. At low energy, E is essentially constant and

$$\Gamma \approx \left[(4.84 \times 10^{-5}) \ \frac{E^4}{R}\right] \text{ (GeV/turn)} \times [300B \text{ (turns)}]$$

or

$$\Gamma(\text{GeV}) = [1.45 \times 10^{-2}] \ \frac{B}{R} \ E^4.$$

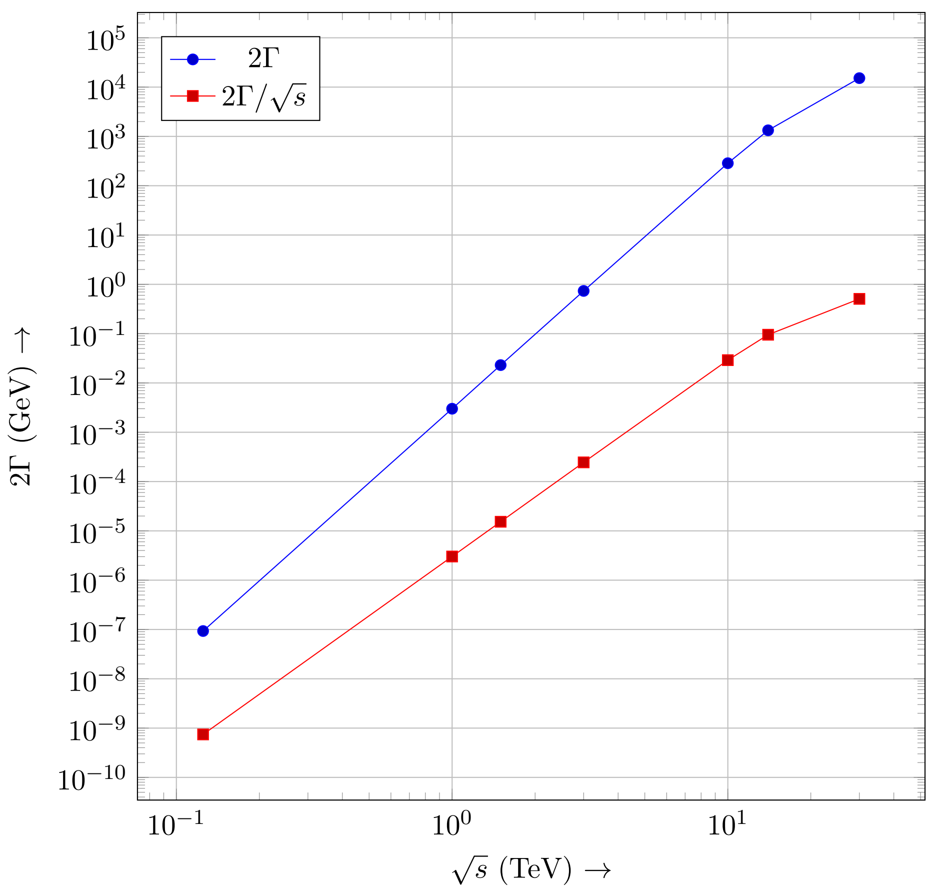
The integration to get the total energy spread in the table below was done is steps of 1/10 of the circumference over which E is assumed constant. A table of values of 2Γ for several cm energies, beam energies, radii and bending fields are shown in the table. The last column is the fraction of the original $\sqrt{s}$ that is scanned, $2\Gamma/\sqrt{s}$. For lower energy muon colliders, below 3 TeV, synchrotron radiation plays a small role and a muon collider mass scan will resemble that of an electron collider, *viz.*, one $\sqrt{s}$-run at a time and the natural energy spread of the beams, 0.1%, is the mass reach. For an $R = 1$ km ring, the cross-over point (between electron machine mass scan and muon machine mass scan, where $2\Gamma/\sqrt{s} \approx 0.1\%$) is a little below 10 TeV, and for an $R = 2$ km ring, it is a little below 14 TeV.

[1] This can be derived from $p = 0.3BR$, the mean beam lifetime, $\tau = \gamma\tau_\mu$ and the time per turn, $T = 2\pi R/c$. After $300B$ turns, each beam is down by $1/e$ and the luminosity is down to $1/e^2 \approx 13.5\%$.

R = 1.0 km

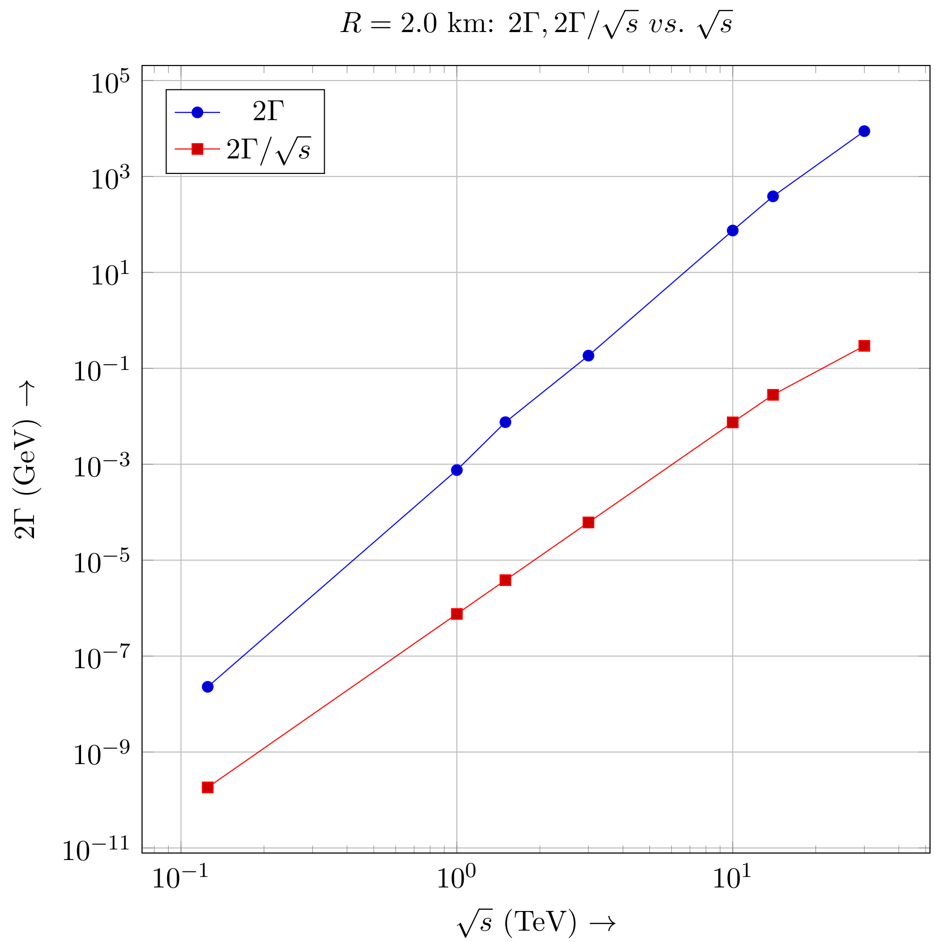
$\sqrt{s}$ (TeV)	E_{beam} (TeV)	R (km)	B (T)	Number turns/fill	ΔE(GeV) (first turn)	2Γ (GeV)	$2\Gamma/\sqrt{s}$
0.125	0.062	1.0	0.208	63	7.39×10^{-10}	9.31×10^{-8}	7.44×10^{-10}
1.000	0.500	1.0	1.667	500	3.03×10^{-6}	3.02×10^{-3}	3.02×10^{-6}
1.500	0.750	1.0	2.500	750	1.53×10^{-5}	0.023	1.53×10^{-5}
3.000	1.500	1.0	5.000	1500	2.45×10^{-4}	0.735	2.45×10^{-4}
							0.1%
10.000	5.000	1.0	16.67	5000	0.030	286.	0.029
14.000	7.000	1.0	23.33	7000	0.116	1330.	0.095
30.000	15.000	1.0	50.00	15000	2.45	1.52×10^{4}	0.507

$R = 1.0$ km: $2\Gamma, 2\Gamma/\sqrt{s}$ *vs.* $\sqrt{s}$



R = 2.0 km

$\sqrt{s}$ (TeV)	E_{beam} (TeV)	R (km)	B (T)	Number turns/fill	ΔE(GeV) (first turn)	2Γ (GeV)	$2\Gamma/\sqrt{s}$
0.125	0.062	2.0	0.104	31	3.69×10^{-10}	2.29×10^{-8}	1.83×10^{-10}
1.000	0.500	2.0	0.833	250	1.51×10^{-6}	7.56×10^{-4}	7.56×10^{-7}
1.500	0.750	2.0	1.250	375	7.66×10^{-6}	5.74×10^{-3}	3.83×10^{-6}
3.000	1.500	2.0	2.500	750	1.23×10^{-4}	0.184	6.12×10^{-5}
10.000	5.000	2.0	8.333	2500	0.015	74.6	7.46×10^{-3} 0.1%
14.000	7.000	2.0	11.67	3500	0.058	385.	0.028
30.000	15.000	2.0	25.00	7500	1.23	8.81×10^{3}	0.294



This same simple code was run for $\sqrt{s} = 10^2$ and 10^3 TeV to check the dependence of $\Gamma/\sqrt{s}$ on $\sqrt{s}$. Below 30 TeV, it is approximately a power-law, and above 30 TeV it saturates: $2\Gamma/\sqrt{s} \approx 0.98$ at 10^3 TeV.

0.1 What's the use, if any, for this?

Here is an interesting case: at $\sqrt{s} = 30$ TeV in an $R = 1$ km ring, the energy loss on the first turn is about $\Delta E = 2.45$ GeV. The beams will continue for 15,000 more turns, at which point $\sqrt{s} \approx 14,800$ GeV, that is, the beam has lost about half its energy and the total energy spread is $2\Gamma \approx 15,200$ GeV. The turn-by-turn step down is $\Delta E \approx 2.45$ GeV at the start (30 TeV) and $\Delta E \approx 0.12$ GeV at the end (near $\sqrt{s} \sim 15$ TeV) of the fill. The natural beam width is 0.1%, or 30 GeV. This is a way to search for a high mass state anywhere between 30 TeV and 15 TeV and with a width larger than 1-10 GeV. It would be interesting to simulate this search, which neither an electron nor a proton machine can do.

The same is possible for $R = 2$ km: The first turn loses 1.23 GeV (at 30 TeV) and after 7500 turns, $2\Gamma \approx 8.81$ TeV, so the last turn loses about 0.08 GeV (at $\sqrt{s} \approx 21$ TeV). Of course, these are both extreme cases.

This type of mass scan will be useful in a muon collider down to about $\sqrt{s} = 10$ TeV. For 10 TeV in an $R = 1$ km ring, 286 GeV in mass is scanned in approximately 0.06 GeV steps. For 10 TeV in a $R = 2$ km ring, 74.5 GeV is scanned in approximately 0.03 GeV steps. These steps are smaller than the beam energy spread of 0.1% at these energies, which is 10 GeV, so the full mass scan range is many times larger than the energy spread of 10 GeV.

As expected, any new or dramatic effects involving synchrotron radiation will be at the higher energies.

0.2 Machine considerations, RF both positive and negative (crazy stuff)

Clearly, the machine is first. A stable design orbit, at a reachable energy, with an acceptable luminosity is the *sine qua non* of a muon collider.

I do not know how critical a decreasing beam energy is to a stable orbit but, simply speaking, it must be OK to vary within the natural beam spread of 0.1%. In any case, as in the LHC and in every electron machine, RF cavities restore the energy lost to synchrotron radiation. For the LHC, this is 6.65 keV/turn, a very modest task for an RF cavity.

For the muon collider, RF cavities could restore the energy loss in all cases in the table. But, RF could be used to "manipulate" $\sqrt{s}$ but retarding the energy loss or, amazingly, but increasing the energy loss. The might be easily done by slipping the phase of the RF, rather than actually raising or lowering the electric field in the cavities.

The only point to be made is that these RF gymnastics and turn-by-turn control of $\sqrt{s}$ for a high-mass scan can only be precisely done at a muon collider.

Work to be done

1. Introduce stochastic fluctuations: beam spread vs. stochastic radiative energy spread
2. Incorporate the machine lattice (bending field that turns on and off): how much will that change things?
3. Understand better the complications with the machine lattice and stable orbits.

4. Solve a problem about beam energy spread damping since $\Delta E/\text{turn} \propto (E+\delta E)^4$ and the δE spread in the muon beam energy in the coasting beam would result in the high energy end damping down faster than the low energy end. This would narrow the beam spread, but would compete with the natural width (determined by the production, cooling and acceleration of the muon beams), and with the stochastic fluctuations introduced by the synchrotron radiation itself.

References

[1] "Muon Colliders," N. Pastrone, et al., DESY, Dec. 17, 2019.

[2] "Synchrotron light sources and X-ray free-electron-lasers," E. Prat,, arXiv:2107.09131v1 [physics.acc-ph] 19 Jul 2021.

[3] William A. Barletta, Director Particle Accelerator School, MIT (lecture notes).