Quantum entanglement: from foundations to precision

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Top quark physics at the precision frontier, October 2nd 2023

Project PID2019-110058GB-C21 funded by
MINISTERIO DE CIENCIA E INNOVACIÓN
UNIÓN EUROPEA
FONDO EUROPEO DE DESARROLLO REGIONAL
"Una manera de hacer Europa"
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Outline

Not covering other types of measurements: discord and steering. Bell inequalities in extra slides.

1. Quick introduction to quantum entanglement
2. Quick introduction to spin correlations
3. Qubits at the energy frontier: *t t*-bar
4. Qutrits at the energy frontier: *VV* from *H* decays
5. Novel tests of QM: post-decay *tW* entanglement
6. Looking for new physics via entanglement

QM is one fundamental pillar of modern physics. We have to test it at all possible situations

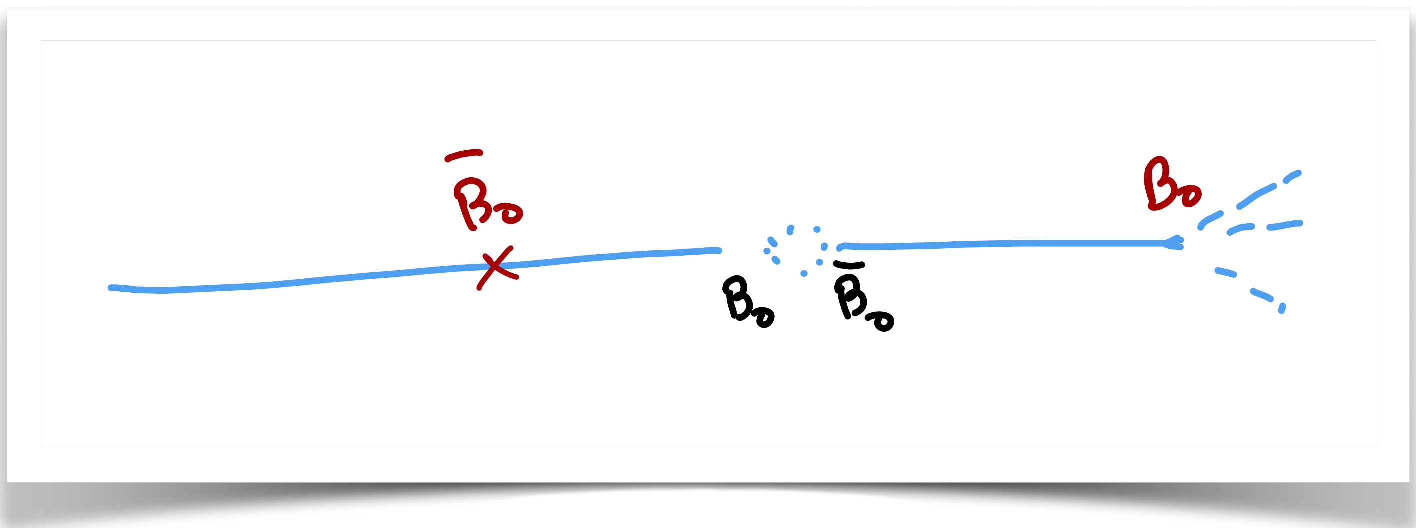
Particle physics offers new ingredients [decay and time evolution] that allow for novel tests not possible in previous experiments with e^- and γ

No formal summary! - wrap-up slides after each block

Quantum Entanglement: basics

Quantum entanglement: basics

Entanglement is routinely used for the measurement of time-dependent CP asymmetries in *B* decays, at the LHCb experiment, *B* factories, etc.



At the exact time one meson decays as B_0, the other one is anti-B_0

Entanglement is a genuinely quantum property of the systems.

Quantum entanglement: basics

The state of a system composed by two sub-systems A and B is separable if it can be written as

$$|\psi\rangle = |a\rangle_A \otimes |b\rangle_B$$

Otherwise, it is entangled, e.g. something like

$$|\psi\rangle = |a_1\rangle_A \otimes |b_1\rangle_B + |a_2\rangle_A \otimes |b_2\rangle_B$$

A typical example of entanglement is the combination of two spin-1/2 systems in the spin-0 configuration

$$|\psi\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B$$

Quantum entanglement implies that measurements on one subsystem affect the other instantaneously, even if there is a large spatial separation.

Quantum entanglement: basics

Example: top pair production

q_L q_L[-bar] → *t t*-bar gives a spin configuration $|\leftarrow\rangle \otimes |\leftarrow\rangle$[in the q_L direction]

This is obviously not entangled.

q_R q_R[-bar] → *t t*-bar gives a spin configuration $|\rightarrow\rangle \otimes |\rightarrow\rangle$

Not entangled either.

g g → *t t*-bar at threshold gives $$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle\right)$$

This one is entangled.

[Dropping -bar in light quarks from now on…]

Mixed states in top pair production

qq → t t-bar is 50% of the time q_L q_L and 50% of the time q_R q_R

Then, we have 50% of the time $|\leftarrow\rangle \otimes |\leftarrow\rangle$ and 50% $|\rightarrow\rangle \otimes |\rightarrow\rangle$

Obviously, in qq → t t-bar we do have t t-bar spin correlations. But not entanglement!

This example illustrates the need of the density operator formalism. Otherwise, we could not describe qq → t t-bar !

Quantum entanglement: basics

Pure states are those that are described by a vector $|\psi\rangle$ in Hilbert space, up to a phase.

Mixed states correspond to states with classical probabilities $p_1, p_2, \ldots p_n$ for the system to be in pure states $|\psi_1\rangle$, $|\psi_2\rangle$, $\ldots$ $|\psi_n\rangle$

They are conveniently represented by a density operator

$$\rho = p_1|\psi_1\rangle\langle\psi_1| + \cdots + p_n|\psi_n\rangle\langle\psi_n|$$

Of course, this is different from the pure state

$$p_1|\psi_1\rangle + \cdots + p_n|\psi_n\rangle$$

Quantum entanglement: basics 6/8

Any operator cannot be a density operator. A valid density operator has several characteristics:

- Unit trace
- Hermitian
- Positive semidefinite: eigenvalues ≥ 0

A density operator describing a composite system is separable if it can be written as

$$\rho_{\text{sep}} = \sum_n p_n \rho_n^A \otimes \rho_n^B$$

Note: in general, one has something like

$$\rho = \sum_{ijkl} p_{ij}^{kl} |\psi_i\rangle\langle\psi_j| \otimes |\psi_k\rangle\langle\psi_l|$$

Quantum entanglement: basics

Necessary criterion for separability:

Peres, quant-ph/9604005
Horodecki, quant-ph/9703004

taking the transpose in subspace of B [for example] the resulting density operator is valid.

Example: composite system A ⊗ B with dim $\mathcal{H}_A$ = n, dim $\mathcal{H}_B$ = m

P_{ij} are m x m matrices, $(P_{ij})^{kl} = p_{ij}^{kl}$

$$\rho = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & & \\ \vdots & & \ddots & \\ P_{n1} & & & P_{nn} \end{pmatrix} \quad \longrightarrow \quad \rho^{T_2} = \begin{pmatrix} P_{11}^T & P_{12}^T & \cdots & P_{1n}^T \\ P_{21}^T & P_{22}^T & & \\ \vdots & & \ddots & \\ P_{n1}^T & & & P_{nn}^T \end{pmatrix}$$

(n∗m) × (n∗m) matrix

Not easily tractable!

Quantum entanglement: basics

To take away:

- It is quite complicated to prove [analytically] that a composite system is in a separable state.
- Numerically, it can be done but there may be a bias [see later]
- However, we are interested in showing that the system is entangled.
- To prove that, in some systems there are simple sufficient conditions that do the work

ρ^{T2} non-positive $\Rightarrow$ ρ^{T2} not valid $\Rightarrow$ system entangled

Showing this for a single vector is enough → simple conditions

Spin measurements: basics

Spin measurements: basics

Top quarks have spin 1/2, as it is well known.

This corresponds to a Hilbert space $\mathcal{H}$ of dimension 2

I have mentioned that a valid density operator is Hermitian and with unit trace. Therefore, I can `expand´ it in terms of Pauli matrices as

$$\frac{1}{2}\left(1_{2\times 2} + \sum_i B_i^+ \sigma_i\right) \qquad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

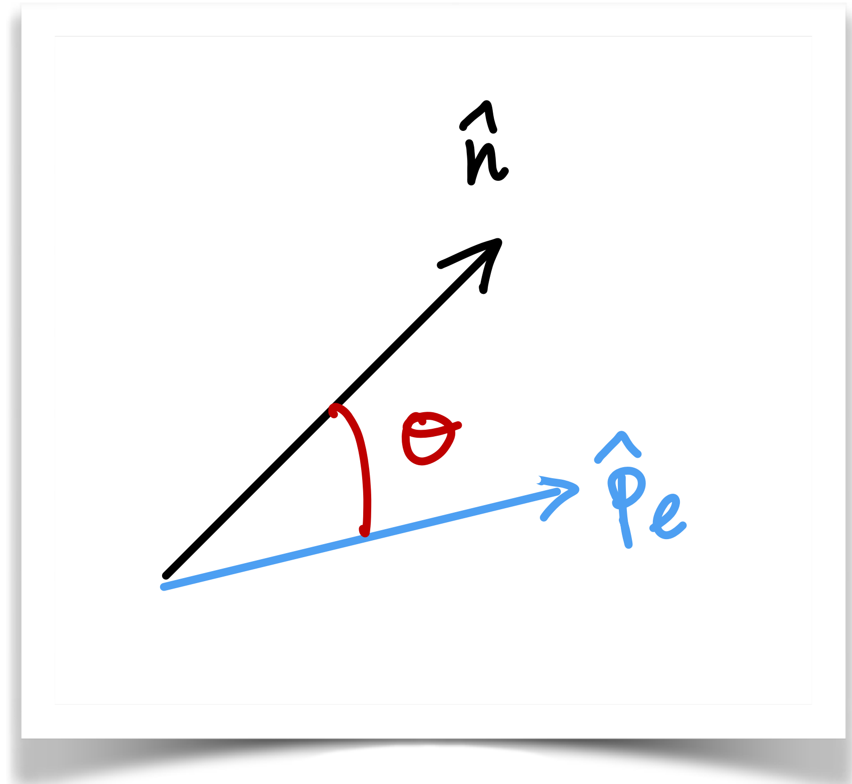
The B_i are constants and correspond to the top polarisation . There are additional degrees of freedom [momentum] that we can integrate out, or consider a specific region in phase space.

The spin of the top quark cannot be directly measured, but statistically the spin state can be determined from angular distributions.

Spin measurements: basics

The charged lepton distribution in the top quark rest frame, with respect to any axis n, is

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta} = \frac{1}{2}(1 + \alpha P_{\hat{n}} \cos\theta)$$



constant that depends on decay product: α = 1 for charged lepton

Polarisation: 2 × <S>

> The charged lepton distribution allows to measure expected value of spin operators for the top quark / antiquark

Spin measurements: basics

When we have a top-antitop pair, we have a composite system of two spin-1/2 particles.

The `spin space´ is $\mathcal{H}_A \otimes \mathcal{H}_B$, of dimension 2 × 2.

The density operator for the top-antitop pair can be written as

$$\rho = \frac{1}{4}\left(1 \otimes 1 + \sum_i B_i^+ \sigma_i \otimes 1 + \sum_i B_i^- 1 \otimes \sigma_i + \sum_{ij} C_{ik} \sigma_i \otimes \sigma_j \right)$$

polarisation of top

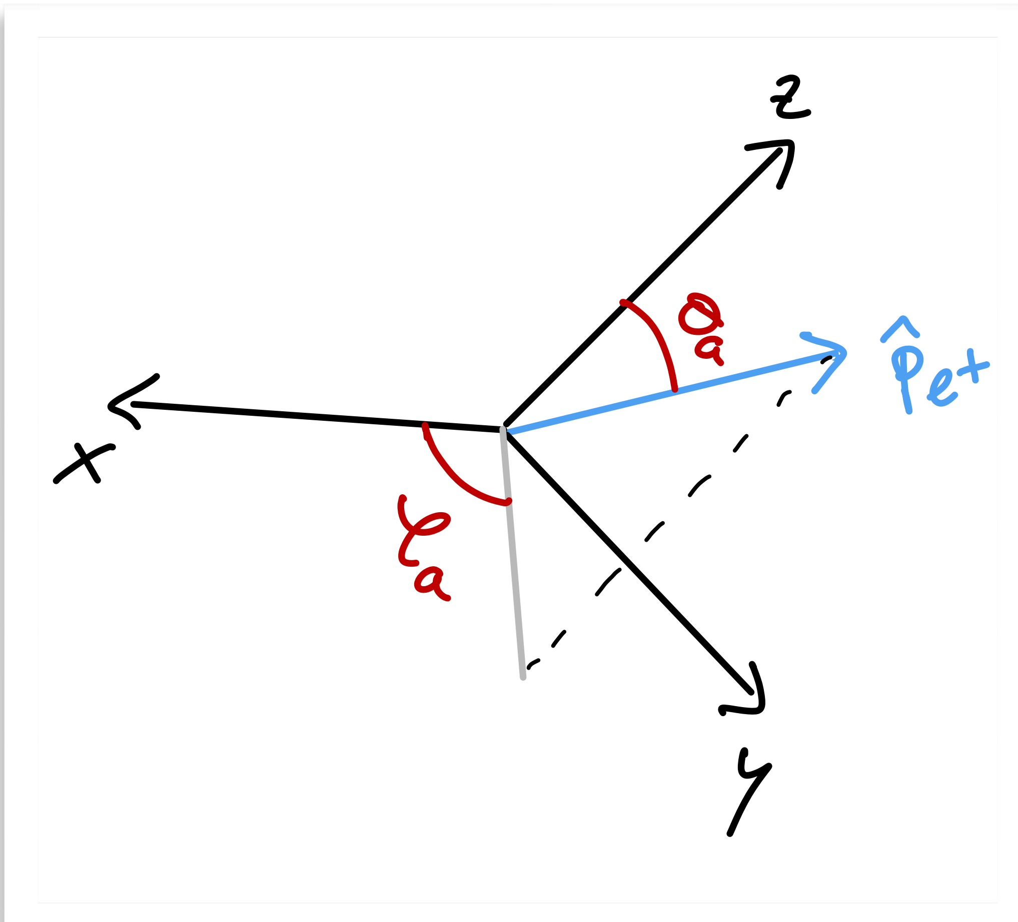
polarisation of anti-top

spin correlations

The identification of the coefficients with polarisations, etc. can be done by calculating expected values of spin operators

Spin measurements: basics

Again, the *B* and *C* coefficients characterising the spin state of top pair production can be measured from the charged lepton distributions, fixing a reference system



ℓ^+ from top: θ_a ϕ_a

ℓ^- from anti-top: θ_b ϕ_b

The corresponding 4-dimensional distribution for the charged leptons is

$$\hat{n}_a = (\sin\theta_a \cos\varphi_a, \sin\theta_a \sin\varphi_a, \cos\theta_a)$$
$$\hat{n}_b = (\sin\theta_b \cos\varphi_b, \sin\theta_b \sin\varphi_b, \cos\theta_b)$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} = \frac{1}{(4\pi)^2}\left[1 + \alpha_a \vec{B}^{+} \cdot \hat{n}_a + \alpha_b \vec{B}^{-} \cdot \hat{n}_b + \alpha_a \alpha_b \hat{n}_a^T C \hat{n}_b\right]$$

normalisation

3 coefficients corresponding to top polarisation

3 coefficients corresponding to anti-top polarisation

9 spin correlations

With suitable integrations the coefficients in red can be extracted from LHC data. And they have been.

Spin measurements: basics

For a weak boson, the 3×3 density matrix can be written as a linear combination of the identity [L = 0] plus irreducible tensors $T^L{}_M$ [L = 1,2]

$$T_1^1 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad T_0^1 = \sqrt{\frac{3}{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$T_2^2 = \sqrt{3} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad T_2^1 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T_0^2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T^1{}_{-1} = -(T_1^1)^\dagger$$
$$T^2{}_{-2} = -(T_2^2)^\dagger$$
$$T^2{}_{-1} = -(T_1^2)^\dagger$$

Alternative: Gell-Mann matrices

Spin measurements: basics

7/8

For a V_1V_2 pair, the density matrix is

$$\rho = \frac{1}{9}\left(1_{9\times 9} + A^1_{LM} T^L_M \otimes 1_{3\times 3} + A^2_{LM} 1_{3\times 3} \otimes T^L_M + C_{L_1 M_1 L_2 M_2} T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right)$$

8 coefficients corresponding to V_1 polarisation

8 coefficients corresponding to V_2 polarisation

64 spin correlation coefficients

… and the 4-d angular distribution of the V_1V_2 decay products [we will use the charged leptons in leptonic decays] has a remarkably compact form

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\Big[1 + B^1_{L_1} A^1_{L_1 M_1} Y^{M_1}_{L_1}(\Omega_1) + B^2_{L_2} A^2_{L_2 M_2} Y^{M_2}_{L_2}(\Omega_2) + B^1_{L_1} B^2_{L_2} C_{L_1 M_1 L_2 M_2} Y^{M_1}_{L_1}(\Omega_1) Y^{M_2}_{L_2}(\Omega_2)\Big]$$

$$\Omega_1 = (\theta_1, \varphi_1)$$
$$\Omega_2 = (\theta_2, \varphi_2)$$

using Gell-Mann matrices the expansion is lengthier

$$B_1 = -\sqrt{2\pi}\eta_\ell\,, \quad B_2 = \sqrt{\frac{2\pi}{5}}$$

$$\eta_\ell = \begin{cases} \dfrac{g_L^2 - g_R^2}{g_L^2 + g_R^2} & Z \\ 1 & W^- \\ -1 & W^+ \end{cases}$$

Spin measurements: basics

8/8

In $H \to ZZ \to 4\ell$ and $H \to WW \to 2\ell\, 2q$ (with charm tagging) the decay can be fully reconstructed, and the As and Cs measured.

Because spherical harmonics are orthogonal functions, to pick selected terms in the distribution one just has to take averages

constants you can calculate

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y_{L_1}^{M_1}(\Omega_1)^* Y_{L_2}^{M_2}(\Omega_2)^* = \frac{1}{(4\pi)^2} B_{L_1} B_{L_2} C_{L_1 M_1 L_2 M_2}$$

data follow this distribution

calculate the average of this quantity on your data

the quantity you want

In $H \to WW \to 2\ell\, 2\nu$ the system is underconstrained and the kinematics cannot be uniquely reconstructed.

Top pair production

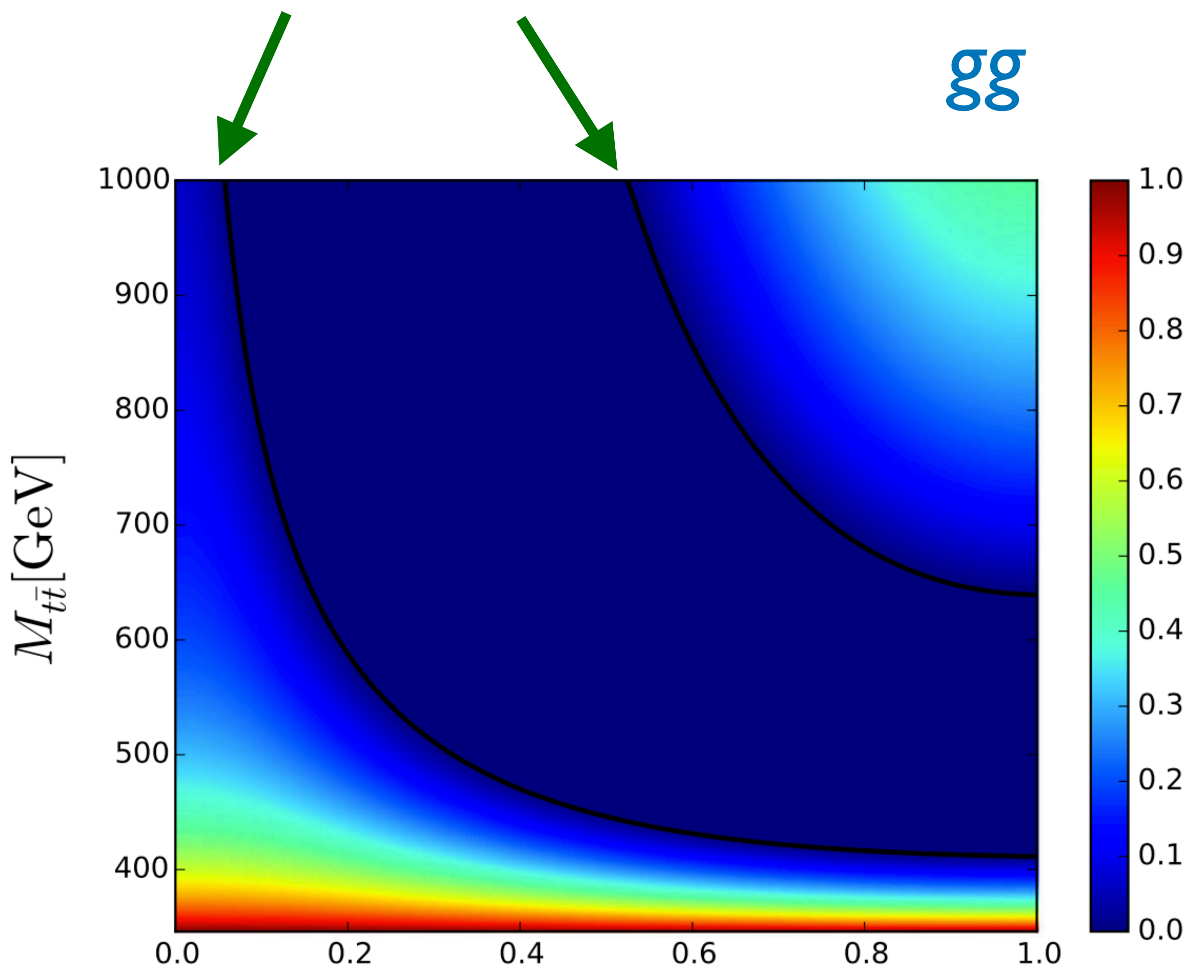
Top pair production

Both *gg* → *t t*-bar and *qq* → *t t*-bar exhibit entanglement in wide regions of phase space — but not in the same state!

Afik & de Nova, 2003.02280
Afik & de Nova, 2203.05582

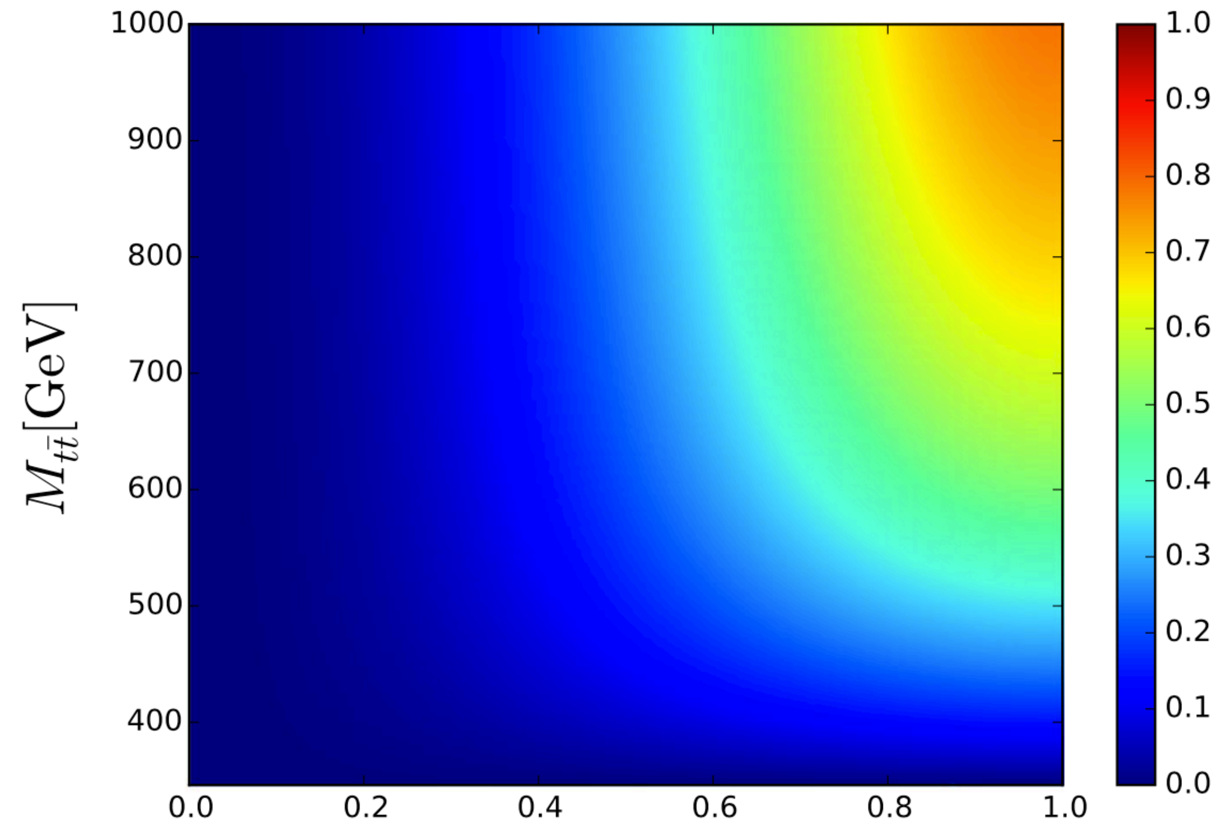
entanglement boundary

gg



entanglement everywhere?

qq



However, *gg* entanglement + *qq* entanglement ≠ more entanglement (!!!)

Let us look at the details.

I have mentioned that simple sufficient conditions for entanglement can be written.

For the case of the top quark, some of these conditions are

$$|C_{11} + C_{22}| > 1 + C_{33}$$
$$|C_{11} - C_{22}| > 1 - C_{33}$$

These remarkably simple conditions result from requiring $\langle a|\rho^{T_2} a\rangle < 0$ for strategically-chosen vectors a

The coefficients C_{ij} are just the ones ATLAS and CMS have measured

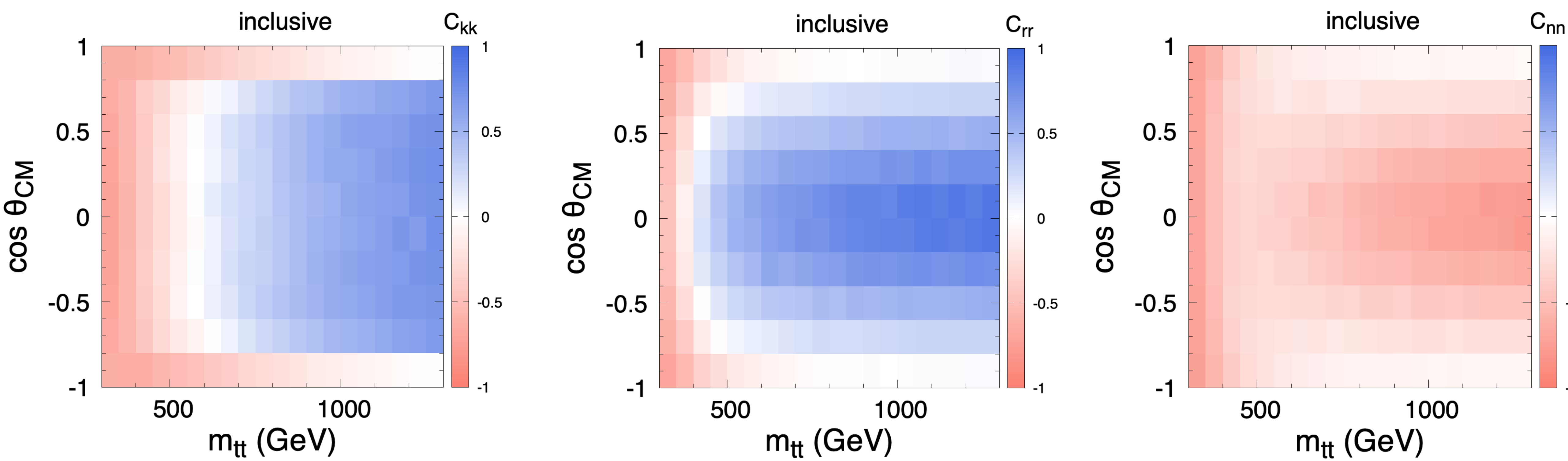
Observables measured long ago by ATLAS and CMS allow to test the entanglement of the top pair

Top pair production

There is a dependence of the C_{ij} coefficients on the kinematics.

In the helicity basis:

Notice: $C_{nn} < 0$



K: top helicity

$$\hat{k} = \hat{p}_t$$

R: ⊥ in production plane

$$\hat{r} \propto [\hat{p}_p - (\hat{p}_p \cdot \hat{p}_t)\hat{k}]$$

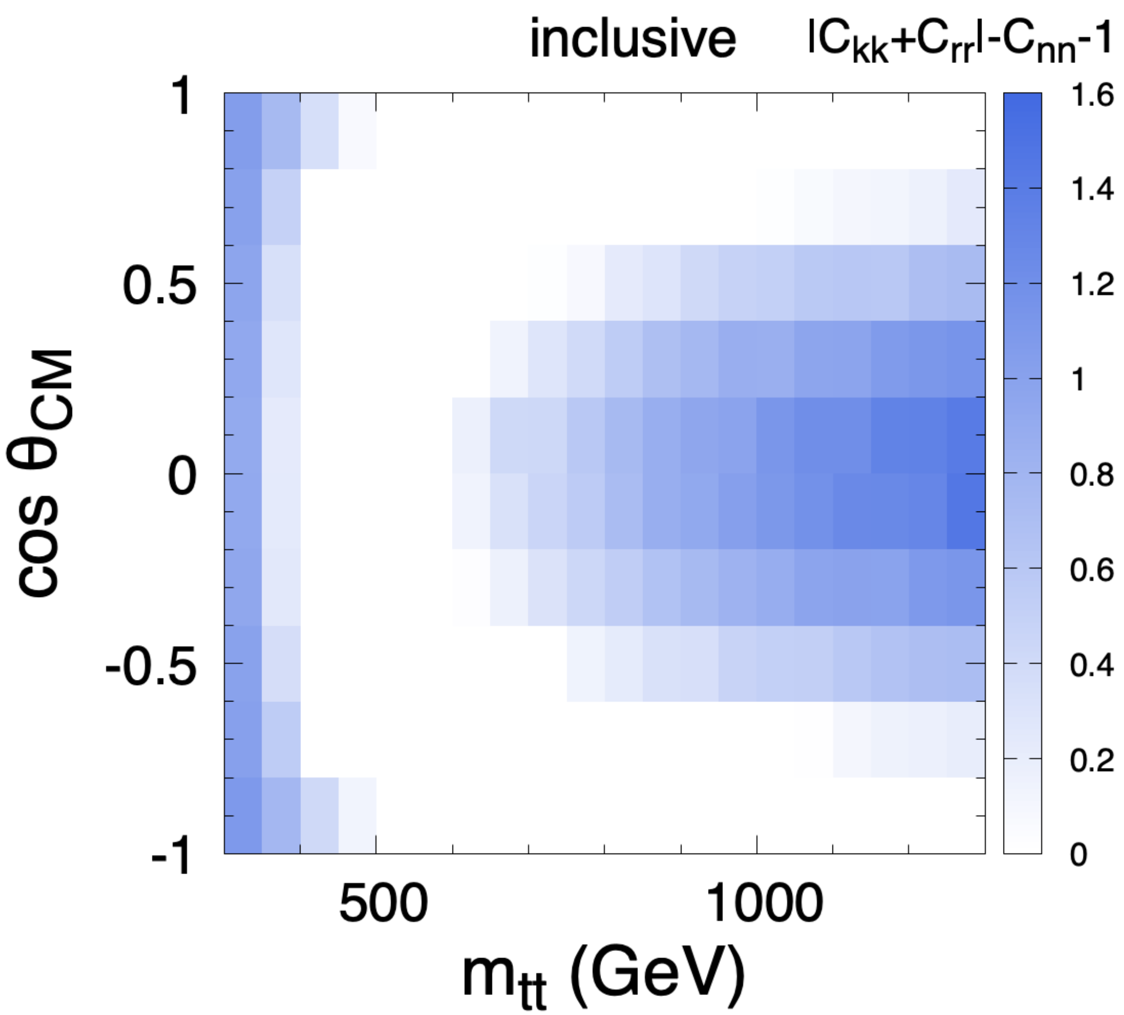
N: ⊥ to K and N

$$\hat{n} = \hat{k} \times \hat{r}$$

1 → K axis; 2 → R axis; 3 → N axis

Since $C_{nn} < 0$, one of the sufficient conditions is stronger:

$$|C_{kk} + C_{rr}| - C_{nn} > 1$$



Near threshold:

$C_{kk} + C_{rr} < 0$

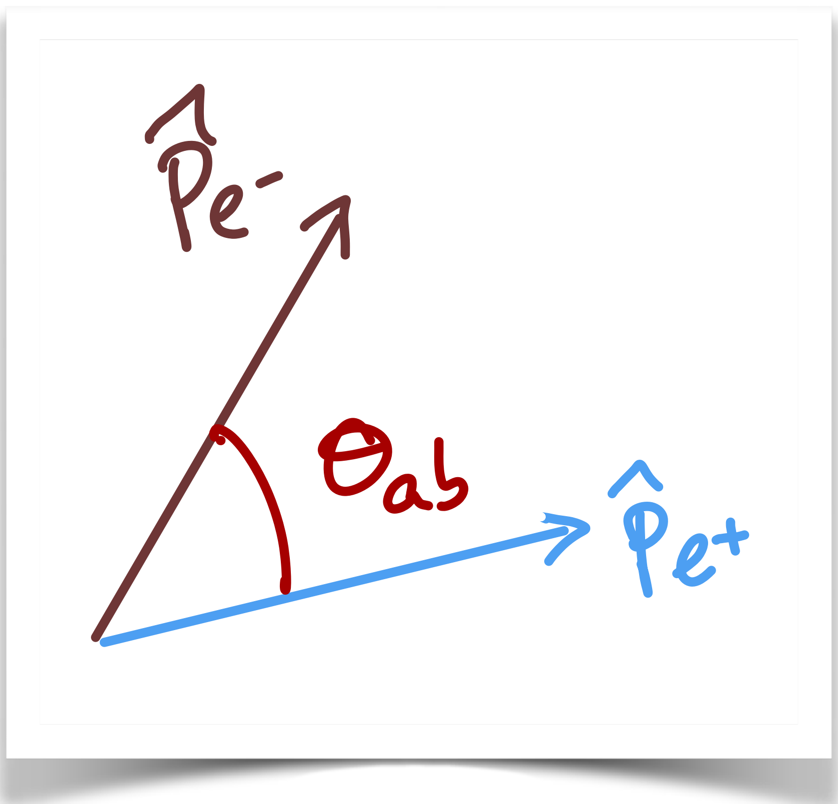
Measure $-C_{kk} - C_{rr} - C_{nn}$

Boosted, central:

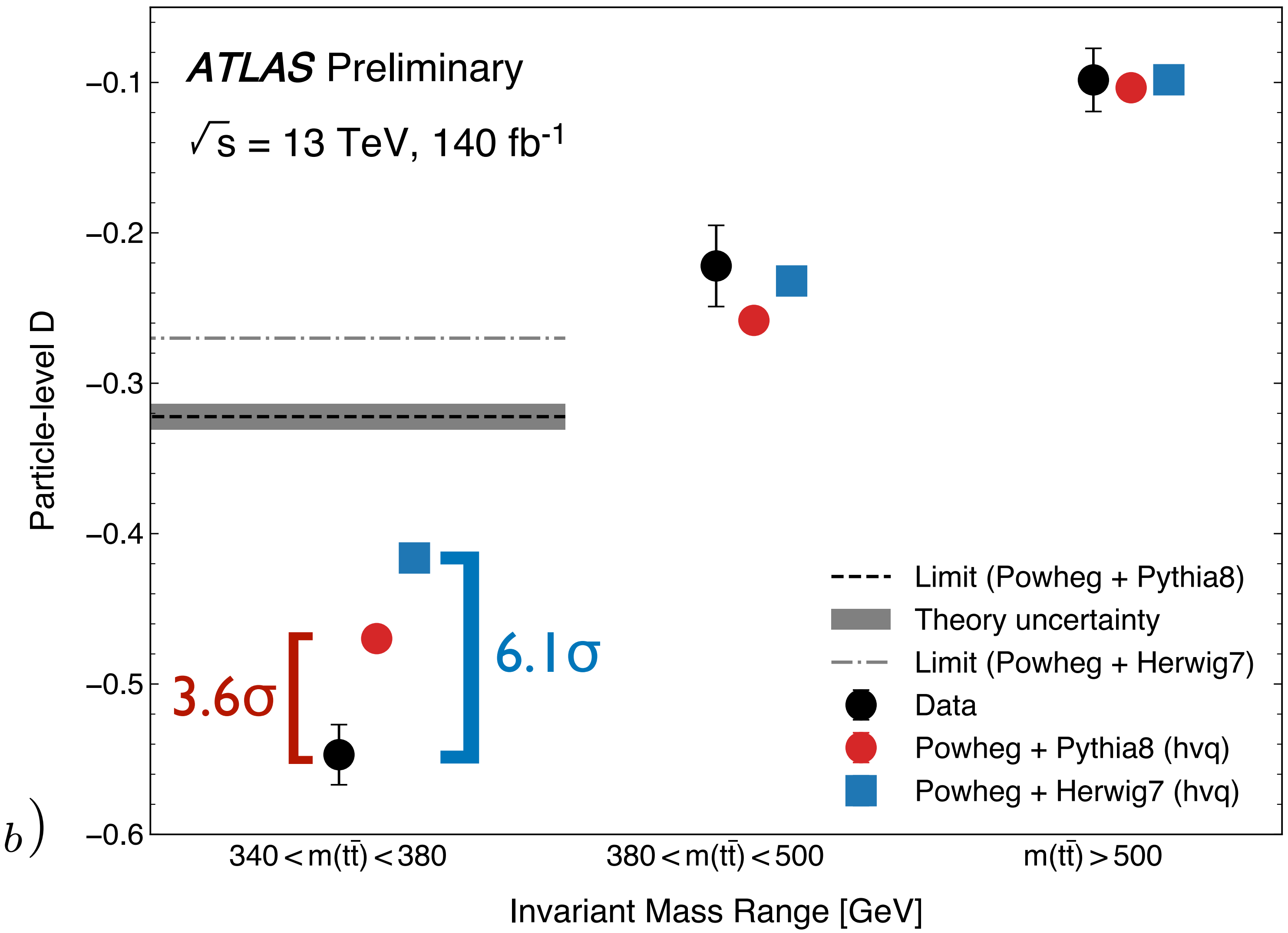
$C_{kk} + C_{rr} > 0$

Measure $C_{kk} + C_{rr} - C_{nn}$

ATLAS has performed [and CMS is pursuing] a measurement at threshold using the *D* observable, related to the angle between the two leptons



$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_{ab}} = \frac{1}{2}\left(1 + \alpha_a\alpha_b D\cos\theta_{ab}\right)$$



$$D = \frac{1}{3}(C_{11} + C_{22} + C_{33})$$

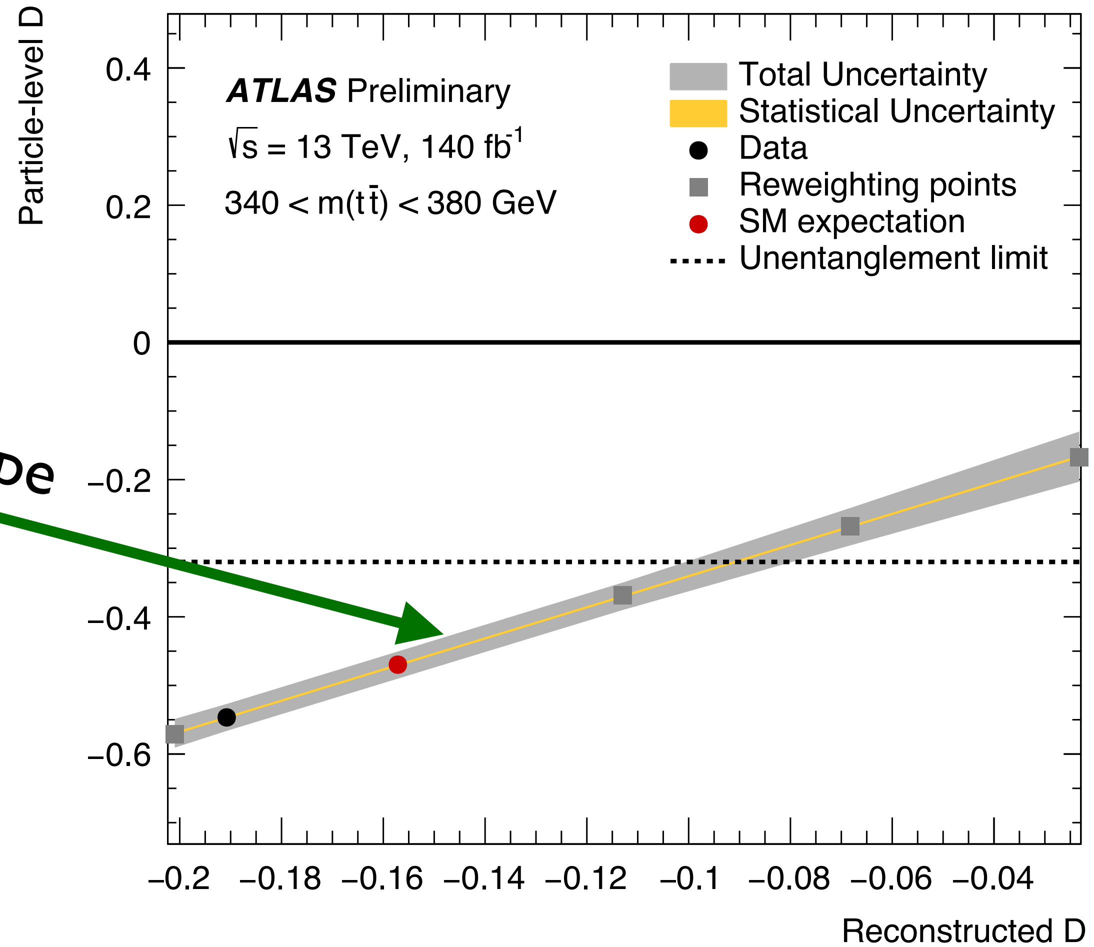
Entanglement test near threshold: $-3D - 1 > 0$

Caveat in measurement: calibration of D from reconstructed to particle level [3x correction]

ATLAS: unphysical reweighting of cos ϕ distribution

$$w = \frac{1 - D_\Omega(m_{t\bar{t}}) \cdot \mathcal{X} \cdot \cos\varphi}{1 - D_\Omega(m_{t\bar{t}}) \cdot \cos\varphi}$$

$\mathcal{X} \in [0.4, 1.2]$ 👉 slope



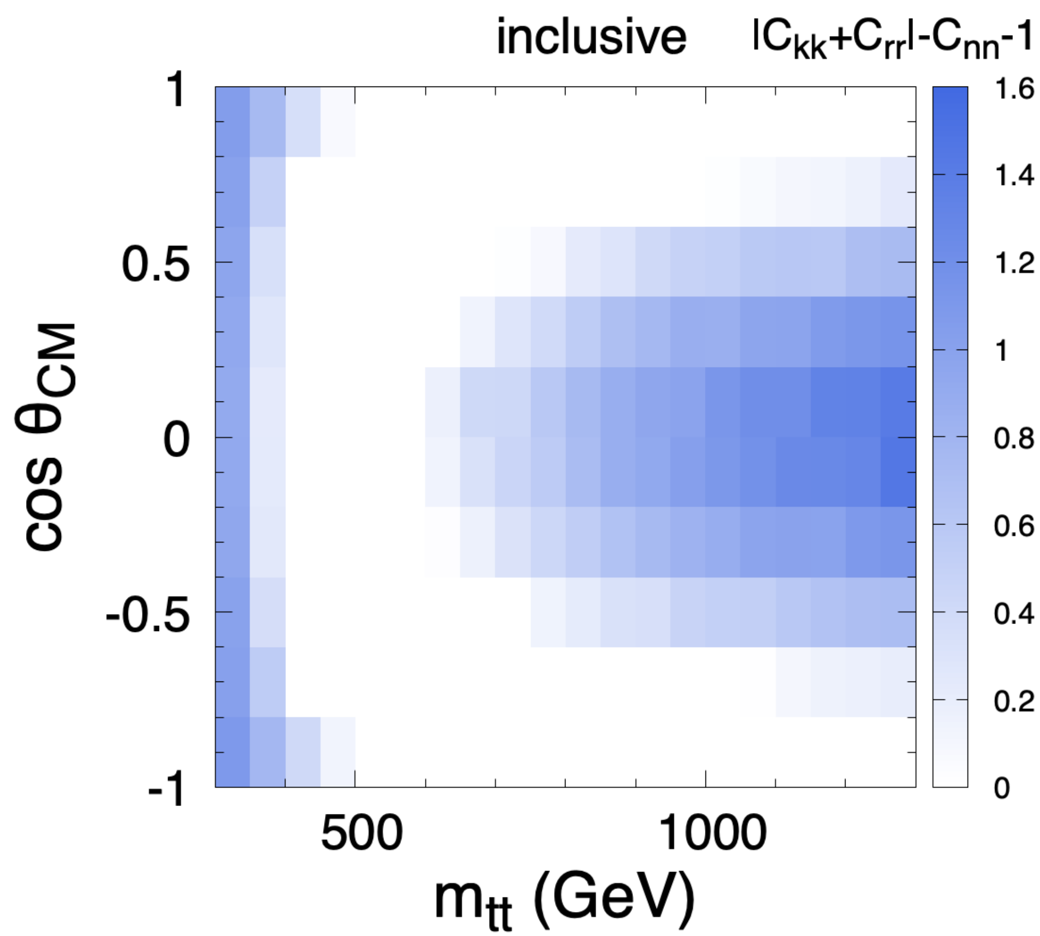
Physical alternative: mixture samples, e.g.

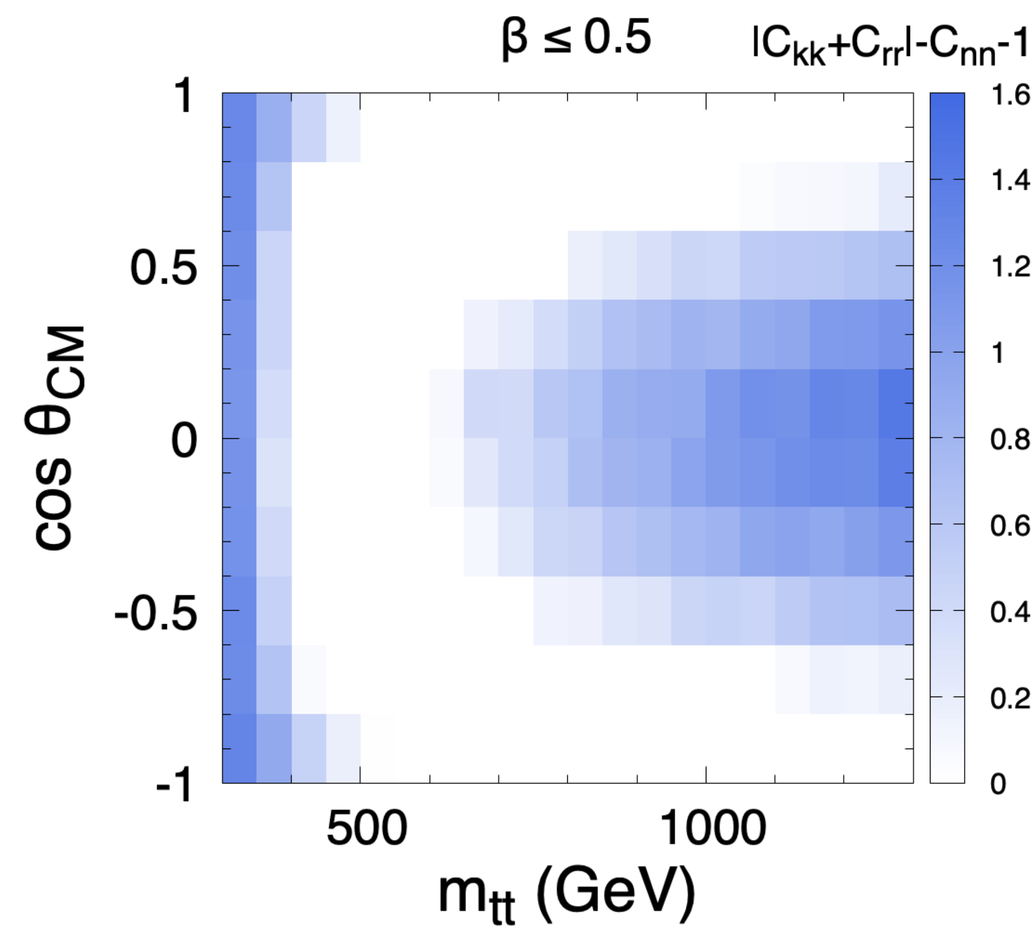
- gg sample ~ spin singlet
- separable sample

	D = -0.73 (LO)		*D* = 0.33 (LO)
f	gg → *t* *t*-bar	+ (1-f)	qq → *t* *t*-bar
f	gg → *t* *t*-bar	+ (1-f)	pp → t_R t_L-bar
f	gg → *t* *t*-bar	+ (1-f)	pp → t_L t_R-bar

Is calibration model dependent?
Do other choices give the same slope?
👉 3.6−6.1σ deviation from SM may well be due to this

Possible improvement: consider events that are more central: upper cut on $t\,\bar{t}$ velocity β in LAB frame JAAS, Casas,2205.00542





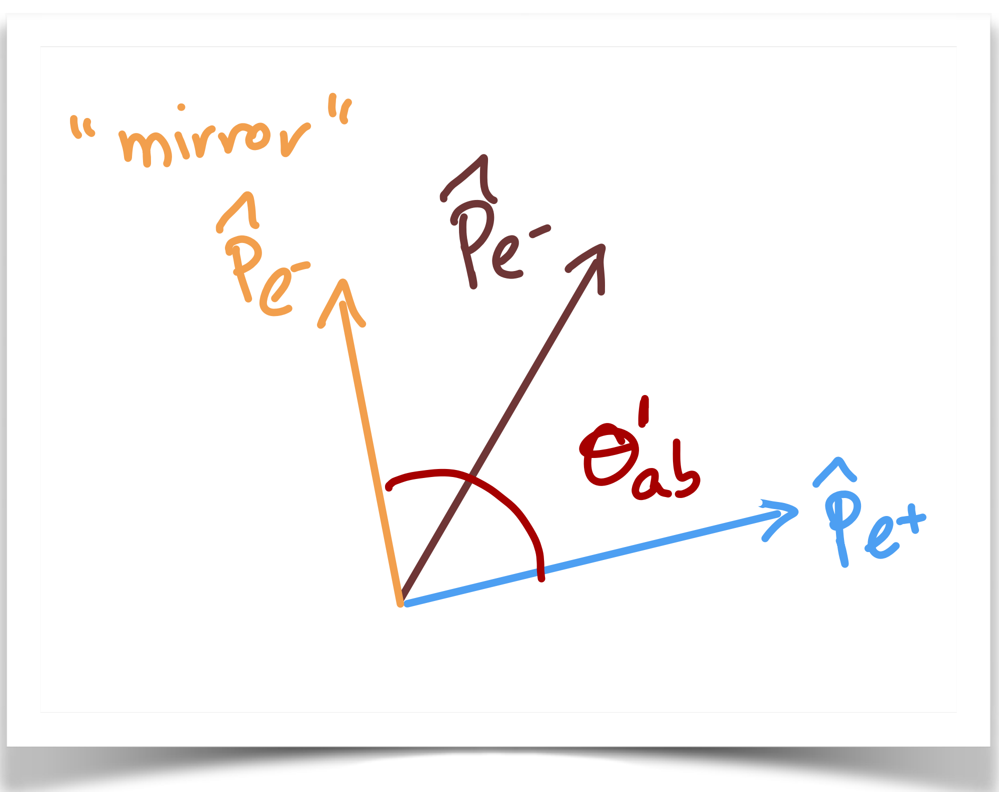
- opposite contributions from *qq* and *gg* sub-processes
- the upper cut reduces the *qq* fraction
- can relax upper cut on m_{tt}, reducing systematics

What about the boosted central region?

The relevant quantity to test is $C_{kk} + C_{rr} - C_{nn}$ and there was no specific observable for this combination [one can however measure C's and sum]

Let's build a new one!

JAAS, Casas,2205.00542



Use the mirror image of ℓ^- momentum, reflected in the K-R plane

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta'_{ab}} = \frac{1}{2}(1 + \alpha_a\alpha_b D_3 \cos\theta'_{ab})$$

$$D_3 = \frac{1}{3}(C_{11} + C_{22} - C_{33})$$

Entanglement test for boosted region: $3D_3 - 1 > 0$

Top pair production

What can we learn from *tt* entanglement?

- ☑ Test of qubit entanglement at energy frontier 🥳
- ☑ The test is non-trivial and motivates effort to reduce experimental systematics
- ☑ New observables available for new physics searches

$H \rightarrow VV$

This is a decay $0 \to 1 + 1$. Angular momentum conservation implies that many A and C coefficients are zero. The non-zero ones are

$$A^1_{10} = -A^2_{10}\,, \quad A^1_{20} = A^2_{20}$$
$$C_{1010}\,, \quad C_{2020}\,, \quad C_{1020}\,, \quad C_{2010}$$
$$C_{111-1} = C^*_{1-111}\,, \quad C_{222-2} = C^*_{2-222}\,, \quad C_{212-1} = C^*_{2-121}\,,$$
$$C_{112-1} = C^*_{1-121}\,, \quad C_{211-1} = C^*_{2-111}$$

and the 9×9 ρ matrix is sparse [relations among coefficients used below]

$$\rho = \frac{1}{3}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 - C_{2020} & 0 & C_{212-1} & 0 & C_{222-2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C^*_{212-1} & 0 & -1 + 2C_{2020} & 0 & C_{212-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C^*_{222-2} & 0 & C^*_{212-1} & 0 & 2 - C_{2020} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Separability —Peres-Horodecki→ $C_{212-1} = 0\,, \quad C_{222-2} = 0$

←$H \to VV$ special case—

Prospects for $H \to ZZ \to 4\ell$ JAAS, Bernal, Casas, Moreno, 2209.13441

- Parton level, no detector simulation, approximate eff [0.25] injected
- Background not included [1/4 size of signal]
- Only statistical uncertainties, estimated with pseudo-experiments

	$C_{212\text{-}1}$	$C_{222\text{-}2}$	Significance
Run 2 + 3 : 300 fb^{-1}	-0.98 ± 0.31	0.60 ± 0.37	3σ
HL-LHC : 3 ab^{-1}	-0.95 ± 0.10	0.60 ± 0.12	many σ

$H \to VV$

What about $H \to WW$, which has larger statistics?

Full reconstruction of $H \to WW \to \ell\nu qq$ possible by using c-tagging to distinguish jets

Fabbri, Howarth, Maurin, 2307.13783

Penalties of full reconstruction:

to reduce bkg

- 1/2 BR because $W \to ud$ is not usable
- 1/2 BR because $W \to cs$ is assumed on shell, $W \to \ell\nu$ off shell
- 0.4 efficiency for charm tagging

With BR 12x larger than $WW \to 2\ell\, 2\nu$, still 20% more statistics

$H \to VV$

Prospects:

- Detector simulation and unfolding
- Background included
- Only statistical uncertainties

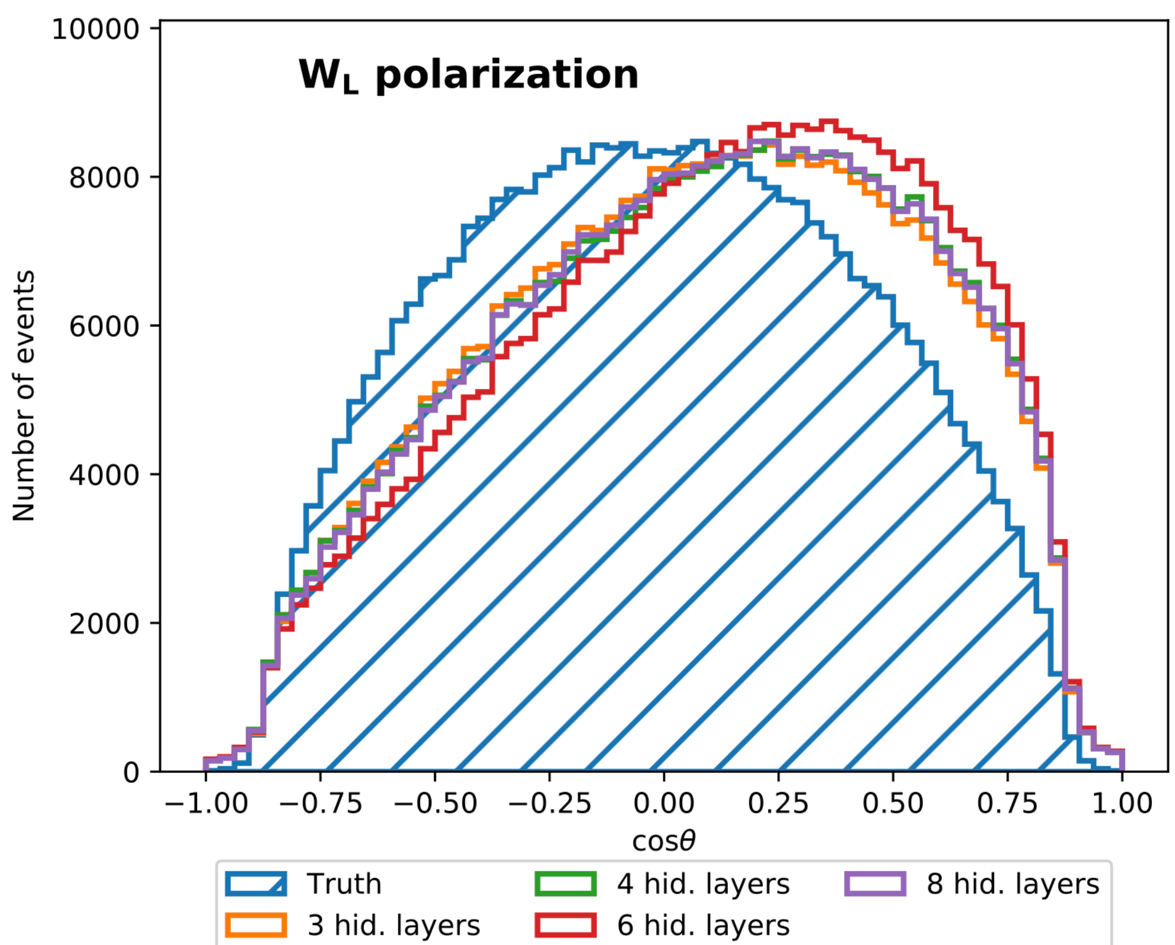
typically, higher significance than for Bell inequalities

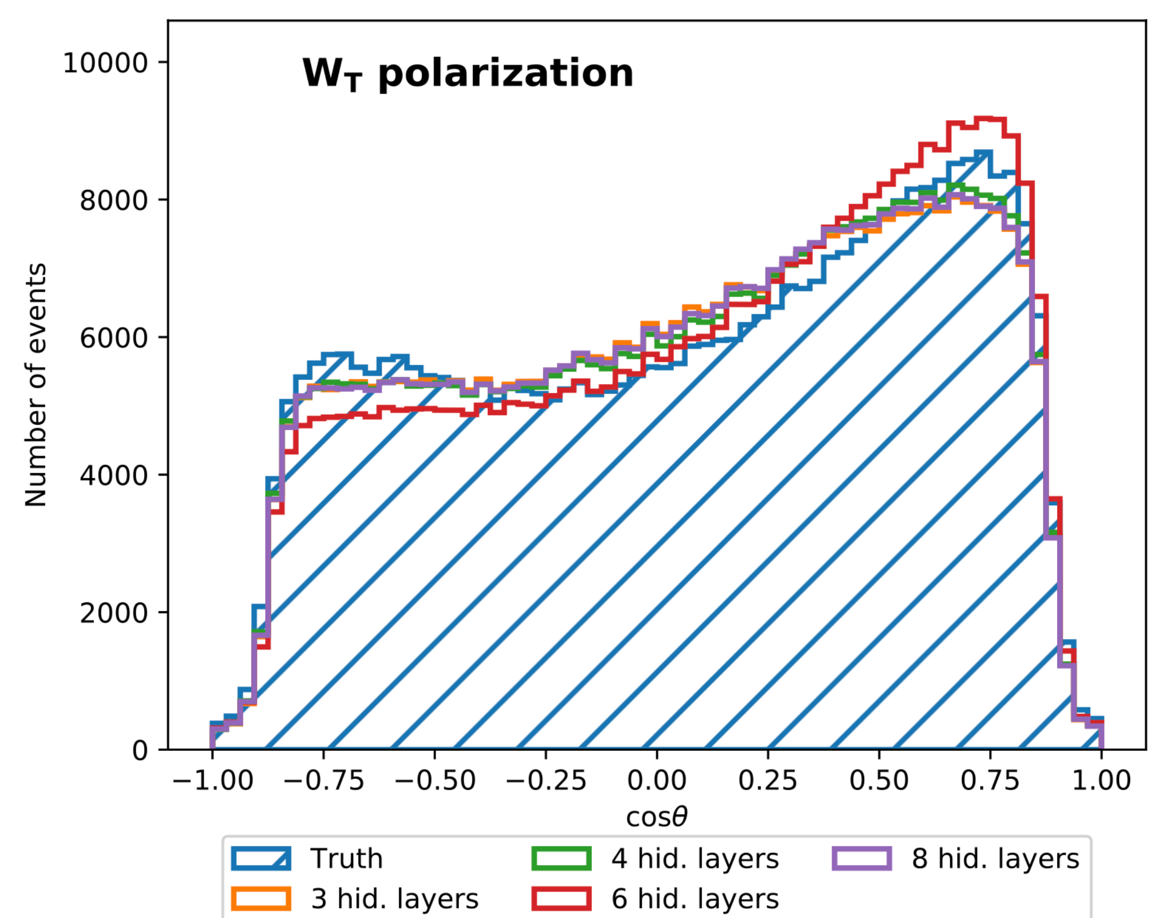
standard operator [could be better]

	Entanglement	Bell inequalities
Run 2 : 139 fb^{-1}	?	1.8σ
Run 2 + 3 : 300 fb^{-1}	??	2.7σ
HL-LHC : 3 ab^{-1}	???	many σ

The decay $H \to WW \to 2\ell\, 2\nu$ cannot be uniquely reconstructed because of the two neutrinos: the system is underconstrained.

Promising attempts in VBF $WW \to 2\ell\, 2\nu$ Grossi et al, 2008.5316

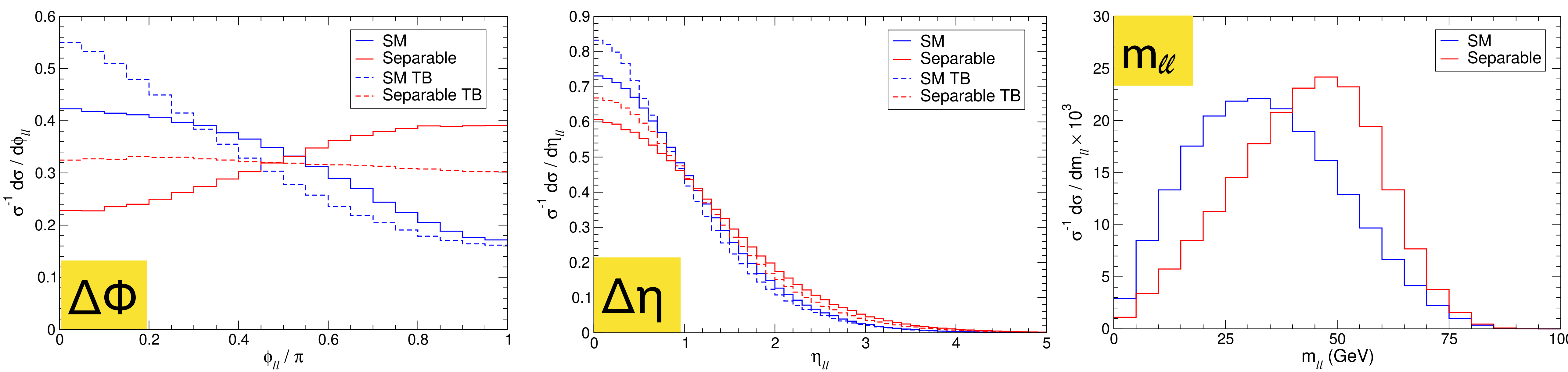




No studies for (θ,φ) reconstruction nor for general polarisations.
Entanglement measurements are quite demanding!

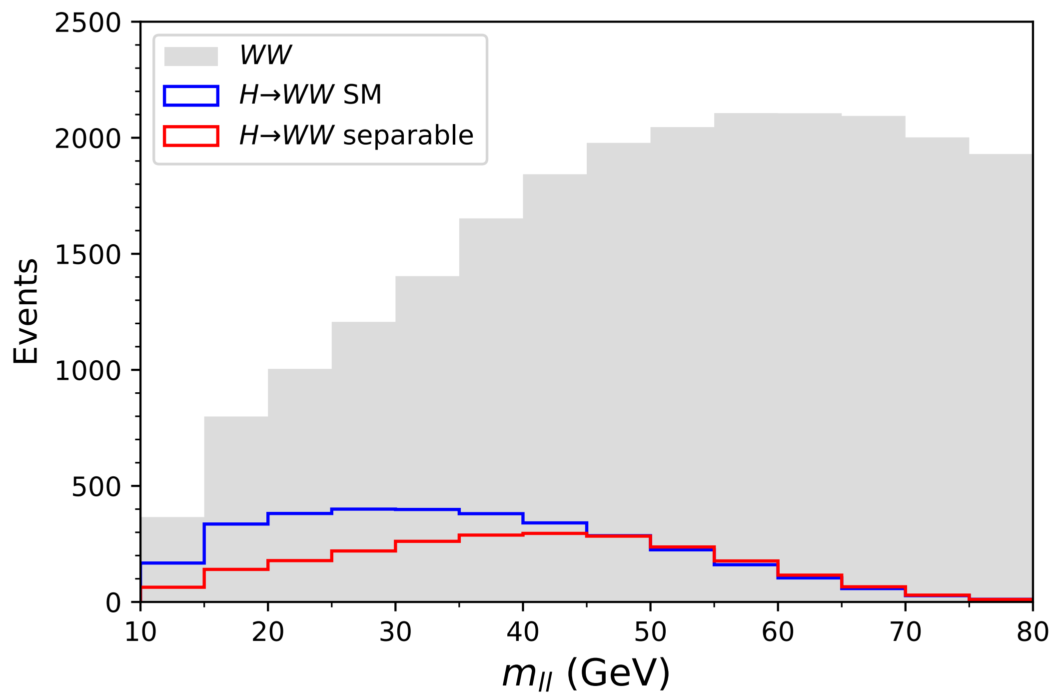
However, for entanglement a binary test can be made in lab frame only using dilepton kinematical distributions. JAAS, 2209.14033

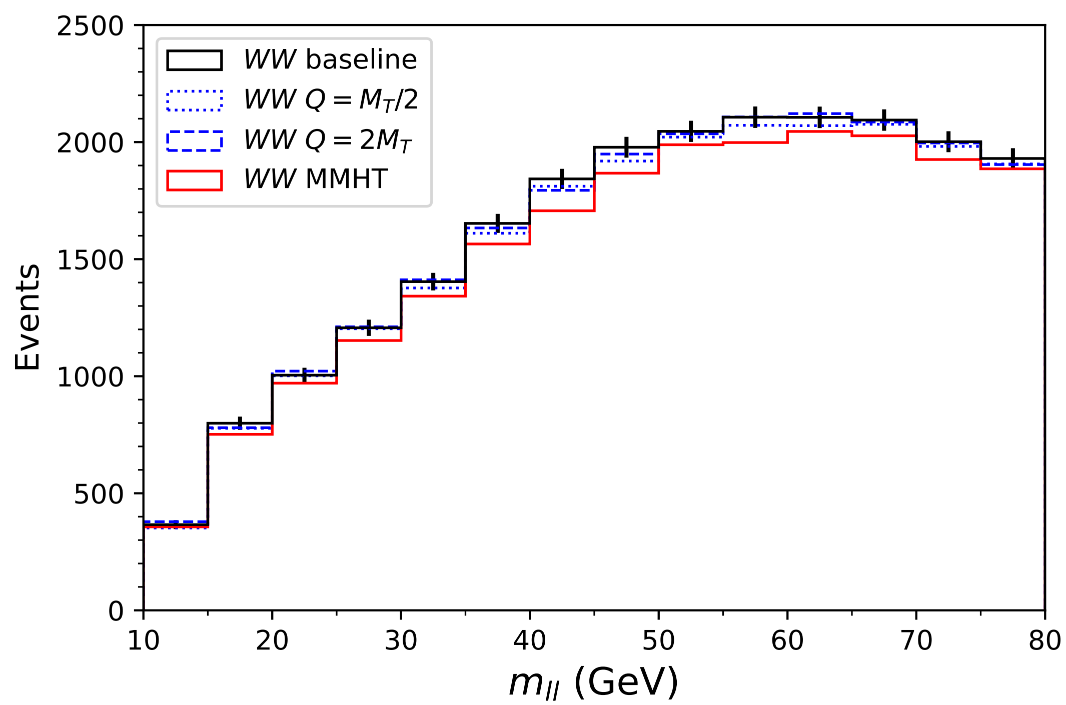
SM vs separability hypothesis $C_{212-1} = 0$ $C_{222-2} = 0$



Note: such a trick is not possible to test Bell inequalities

Results after Delphes simulation, eμ channel, L = 138 fb[-1]





The differences between the SM and separable hypotheses arise in the region with smaller bkg

The bkg systematics are small provided we normalise it with a sideband

	Significance
stat only	7.1σ
stat + modeling syst	6.1σ

likely, observation possible already for Run 2

What can we learn from $H \to VV$ entanglement?

- ☑ First-ever measurement of elementary qutrit entanglement
- ☑ Test done at energy frontier
- ☑ Improved reconstruction of $H \to WW \to \ell\nu qq$
- ☐ Improved reconstruction of $H \to WW \to 2\ell\, 2\nu$

Post-decay entanglement in top pair production

Post-decay tW entanglement

Question:

Assume t t-bar are entangled, then t-bar decays into $W^- b$,

is the top entangled with the W^- from t-bar?

Problem:

When we have several entangled particles and trace over [unobserved] degrees of freedom, entanglement may be lost.

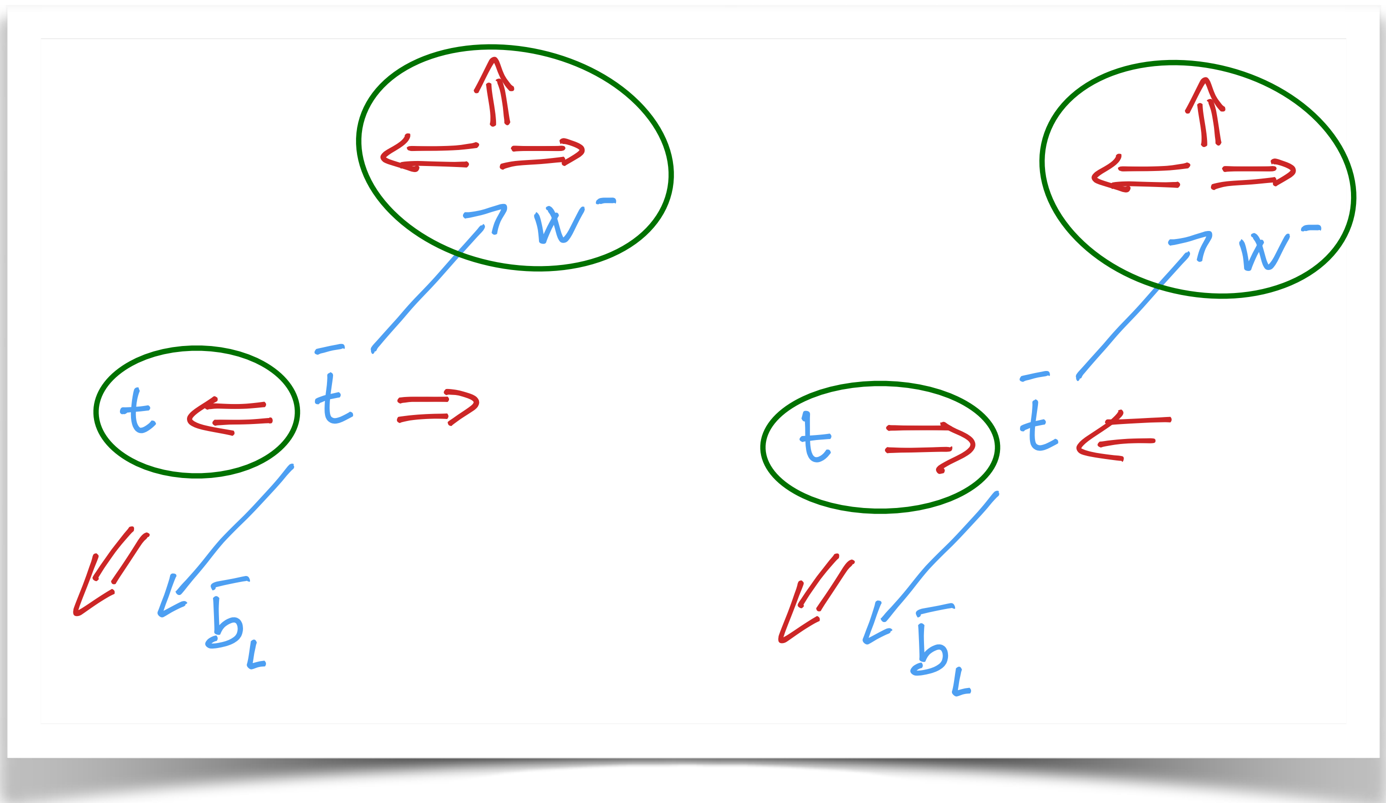
The b-bar spin is not measured, and summing over it destroys entanglement.

Solution:

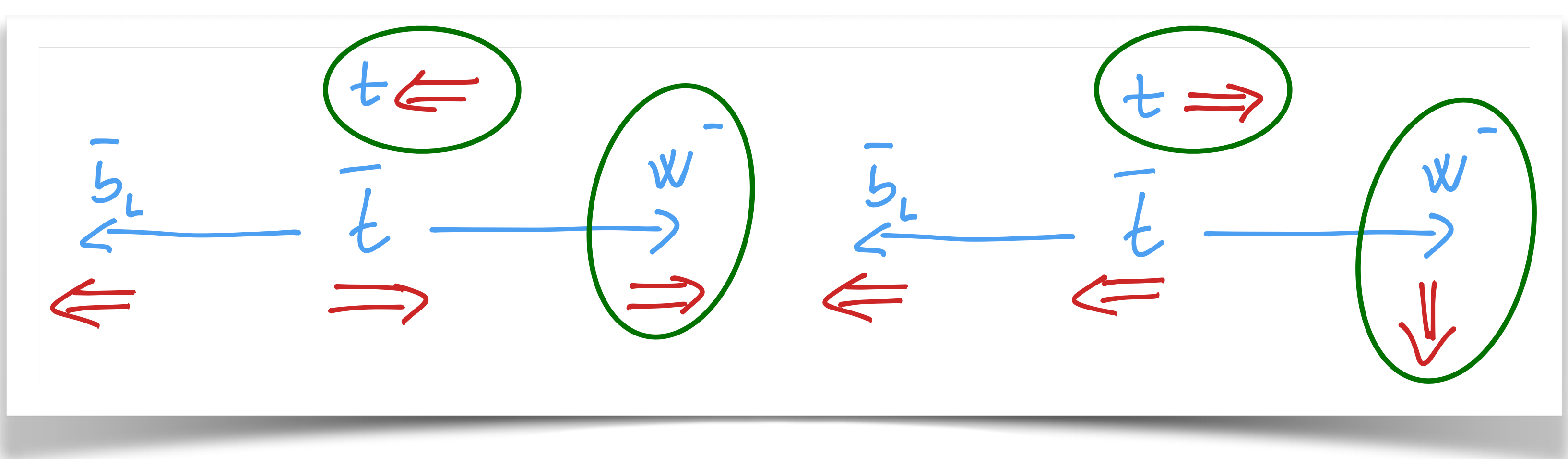
Consider a kinematical region where the b-bar spin aligns with the t-bar one (!)

Post-decay *tW* entanglement

Assume *t* *t*-bar are in a spin-triplet state $\frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |\downarrow\rangle + |\downarrow\rangle \otimes |\uparrow\rangle)$



t W^- not entangled in general



but in some kinematical configurations they are!

Post-decay *tW* entanglement

Threshold, beamline basis z = (0,0,1)

θ_W 👉 angle between W^- momentum in t-bar rest frame and z axis

$$m_{t\bar{t}} \leq 390 \text{ GeV}\,, \beta \leq 0.9\,, \cos\theta_W \geq 0.3$$

Looser cut
less entanglement
more statistics

$$\rho_{tW} \simeq 0.49|\Psi\rangle\langle\Psi| + \ldots \qquad |\Psi\rangle \simeq \frac{1}{\sqrt{2}}\left[|\tfrac{1}{2}\,0\rangle - |-\tfrac{1}{2}\,1\rangle\right]$$

$$m_{t\bar{t}} \leq 390 \text{ GeV}\,, \beta \leq 0.9\,, \cos\theta_W \geq 0.9$$

Tighter cut
more entanglement
less statistics

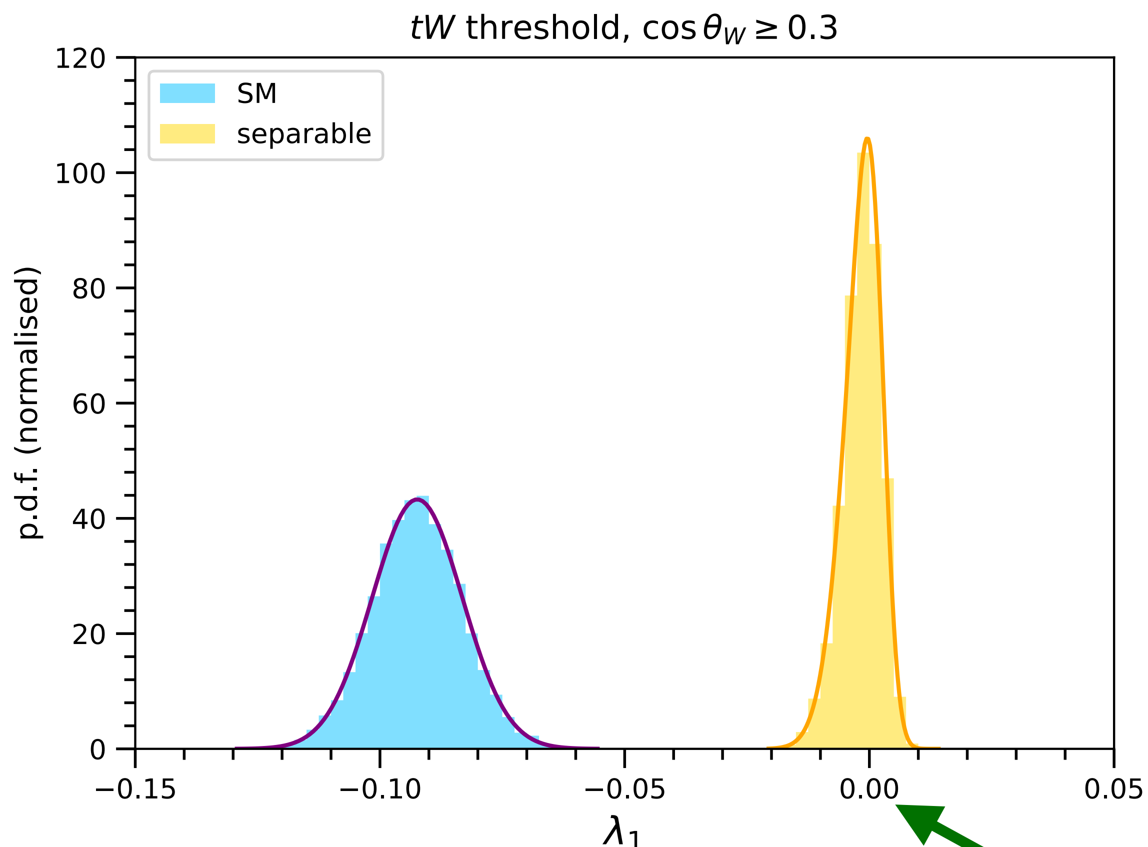
$$\rho_{tW} \simeq 0.62|\Psi\rangle\langle\Psi| + \ldots \qquad |\Psi\rangle \simeq 0.82\,|\tfrac{1}{2}\,0\rangle - 0.57\,|-\tfrac{1}{2}\,1\rangle$$

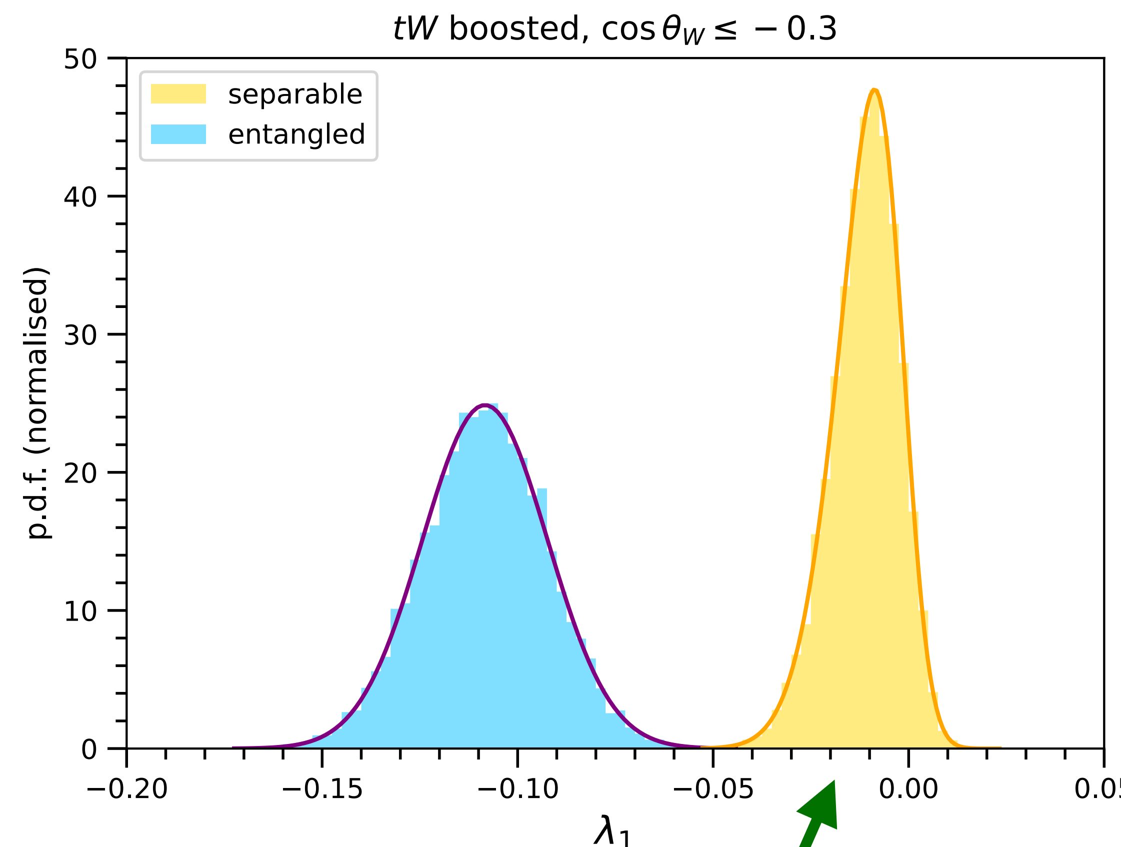
Post-decay tW entanglement

Entanglement indicator:

lowest eigenvalue λ_l of the ρ^{T2} matrix for tW

$\lambda_l < 0 \Leftrightarrow$ Entanglement





Run 2	Significance [stat + 10% sys + bias]
Threshold	7.0 σ
Boosted	5.0 σ

Bias: even if $\lambda_l > 0$, in a small sample we may find it negative

What can we learn from *tW* entanglement?

- ☑ Post-decay entanglement: unique test of QM not possible in experiments with e^- and γ
- ☑ Boson-fermion spin entanglement tests are quite rare, too!

Probing new physics

Probing new physics 1/5

Entanglement observables involve spin correlations, which are sensitive to new physics.

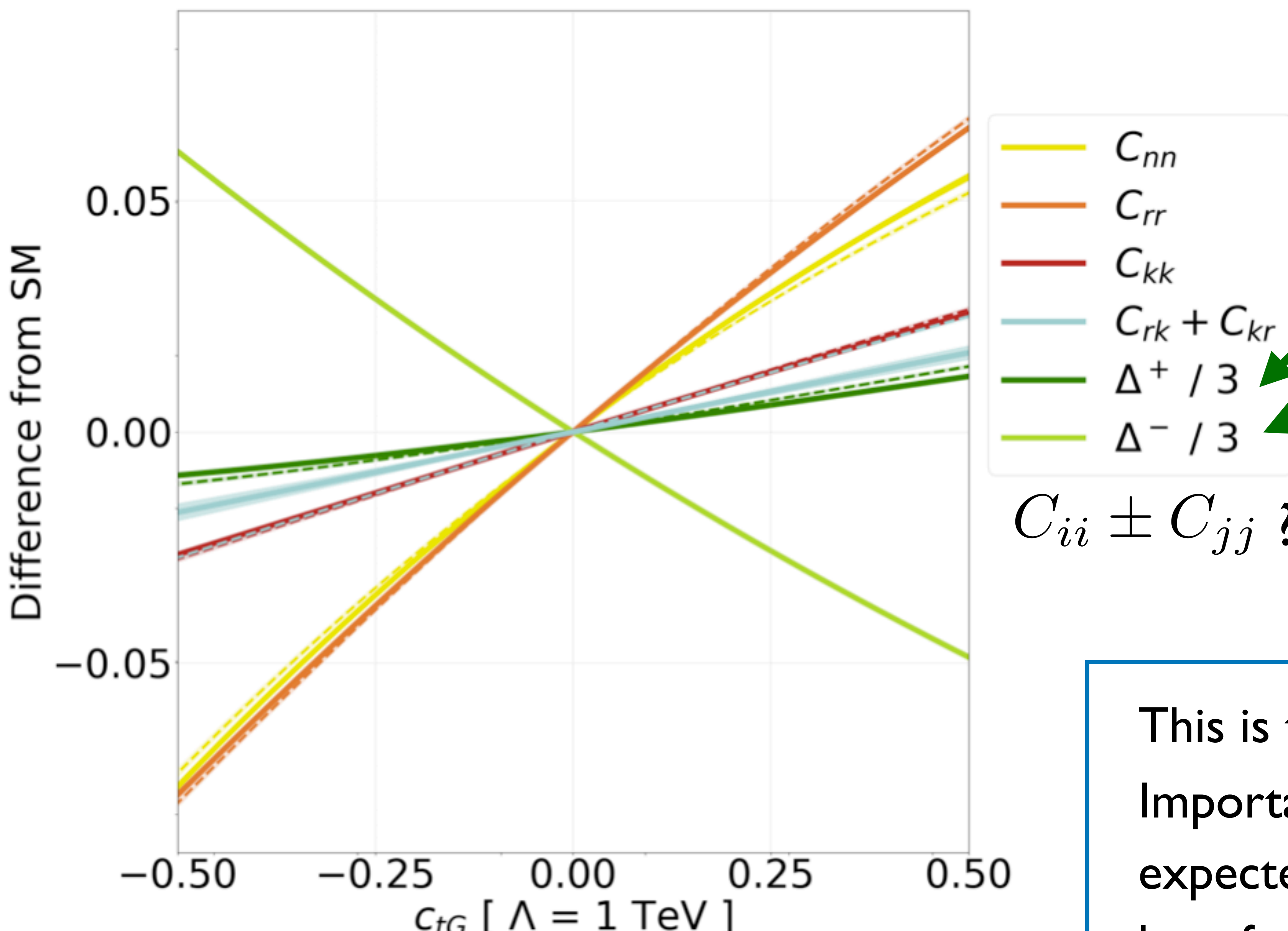
→ we can parameterise deviations from SM in terms of dim-6 operators, which provide a definite framework for comparisons

Spin correlations are measured with angular distributions, with a relation that may be modified by new physics

→ we can also introduce dim-6 operators for the decay of top, *W*, *Z*, but typically there are better ways to constrain them

Benchmark example: top chromomagnetic dipole operator

Severi, Vryonidou, 2210.09330



this is a new observable

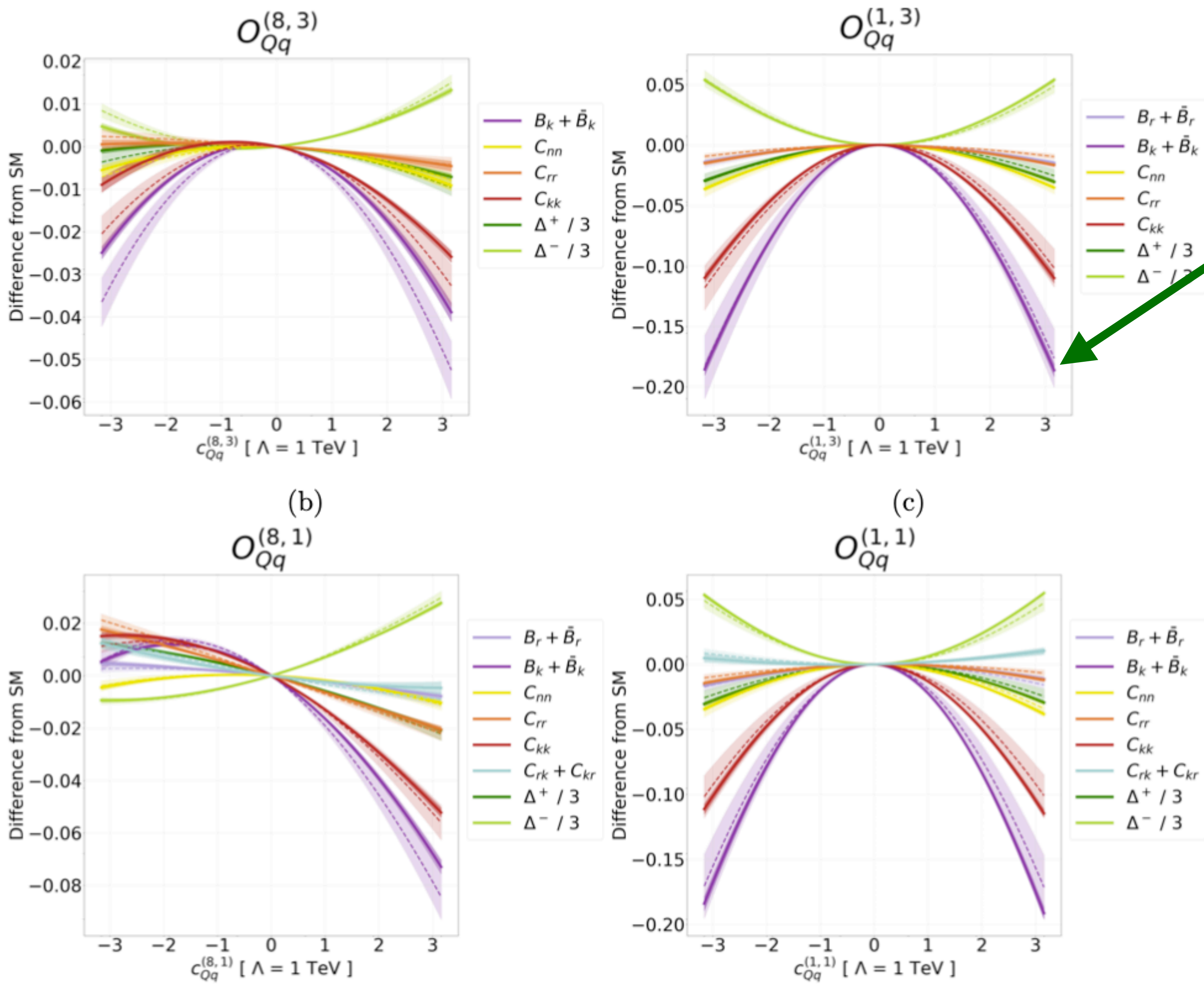
this is essentially the old good D

$C_{ii} \pm C_{jj}$???

This is the first step.
Important missing piece:
expected experimental error bars for these quantities

Probing new physics

Benchmark example: some four-fermion operators



polarisation in helicity direction

Polarisation seems to outperform the rest of observables [note that experimental uncertainties are likely smaller] but this statement is basis-dependent (!)

Probing new physics 4/5

Naming a cancellation as `flat direction´ is misleading because that suggests some fine-tuning.

Well-known examples of operators & models that produce such cancelations.

JAAS, 0811.3842

...

$$\mathcal{L}_{Ztt} = -\frac{g}{2c_W}\bar{t}\gamma^\mu(c_L^t P_L + c_R^t P_R)tZ_\mu \qquad \delta c_L^t = \left[C_{\phi q}^{(3,3+3)} - C_{\phi q}^{(1,3+3)}\right]\frac{v^2}{\Lambda^2}$$

$$\mathcal{L}_{Zbb} = -\frac{g}{2c_W}\bar{b}\gamma^\mu(c_L^b P_L + c_R^b P_R)bZ_\mu \qquad \delta c_L^b = \left[C_{\phi q}^{(3,3+3)} + C_{\phi q}^{(1,3+3)}\right]\frac{v^2}{\Lambda^2}$$

In the one-operator-at-a-time framework, both $C_{\phi q}^{(3,3+3)}$ and $C_{\phi q}^{(1,3+3)}$ are tightly constrained by $Z \to bb$ at LEP.

But a VLQ singlet T precisely generates $C_{\phi q}^{(1,3+3)} = -C_{\phi q}^{(3,3+3)}$ and no tree-level contribution to the *Zbb* vertex (!!!)

What can we learn with respect to new physics?

- [x] New observables motivated by entanglement are sensitive to new physics.
- [ ] More sensitive than old ones? That is yet to be determined.
- [ ] UV matching may help identify interesting observables and scenarios

End

Bell inequalities

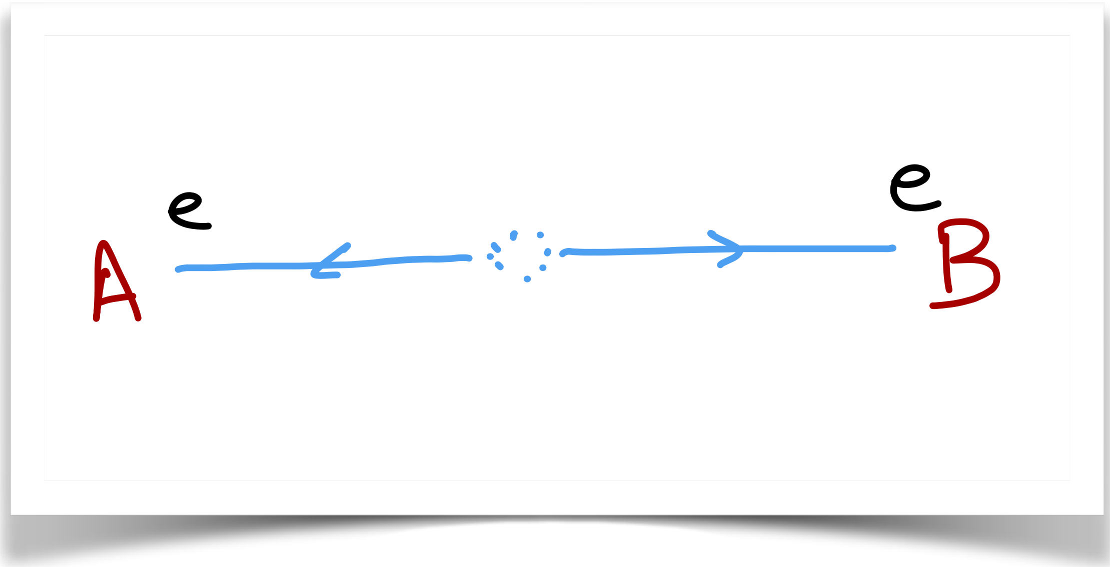
Bell inequalities

Bell-like inequalities hold for classical systems. Their violation implies quantum mechanics.

In particular, the violation implies that the quantum system is not described by hidden variables.

Bell-like inequalities are based on measurements on two separate sub-systems A [Alice] and B [Bob], of photons, electrons, …

Experiments usually performed measuring spins



Bell inequalities

A useful formulation of Bell-like inequalities for spin-1/2 systems is provided by the so-called CHSH inequalities for two systems A (Alice) and B (Bob).

Clauser, Horne, Shimony, Holt, '69

Alice measures two spin observables A, A'. Bob measures two spin observables B, B'. [Both normalised to unity]. Then, clasically:

$$|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle| \leq 2$$

these are spin correlation observables!

One can show violation of CHSH inequalities if one finds spin observables A, A' for Alice and B, B' for Bob such that the inequality is violated.

in a given quantum state!

Bell inequalities

The CHSH inequalities involve spin correlations. Therefore, for a particle of spin 1/2, they involve the C_{ij} spin-correlation coefficients [already measured for top pair production]

It can be shown that the maximum of the l.h.s.

$$|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle|$$

is given by

$$2\sqrt{\lambda_1 + \lambda_2}$$

where λ_1 and λ_2 are the two largest eigenvalues of the positive definite matrix $C^T C$

Horodecki, Horodecki, Horodecki, ‘ 95

Bell inequalities

Simpler but equally effective: Take judicious choice of [non-commuting] spin observables

$$A \to 2S_i \qquad B \to \frac{1}{\sqrt{2}}(2S_i + 2S_j)$$

$$A' \to 2S_j \qquad B' \to \frac{1}{\sqrt{2}}(-2S_i + 2S_j)$$

$$|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle| \to |C_{ii} + C_{jj}|$$

$$|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle| \to |C_{ii} - C_{jj}|$$

$$A \to 2S_i \qquad B \to \frac{1}{\sqrt{2}}(-2S_i - 2S_j)$$

$$A' \to 2S_j \qquad B' \to \frac{1}{\sqrt{2}}(2S_i - 2S_j)$$

CHSH violation is probed by testing if $|C_{ii} \pm C_{jj}| > \sqrt{2}$

These estimators are optimal when off-diagonal C_{ij} vanish

Bell inequalities

For spin-1 systems there is an inequality that is stronger than CHSH. For any observables A_1, A_2 [on system A], B_1, B_2 [on system B] CGLMP PRL '02

$$I_3 = P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) - [P(A_1 = B_1 - 1) + P(B_1 = A_2) + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)] \leq 2$$

if the systems are classical.

There is a well-known choice of A_1, A_2, B_1, B_2 that is believed to maximise I_3 for the spin-singlet state

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left(|+-\rangle - |00\rangle + |-+\rangle\right)$$

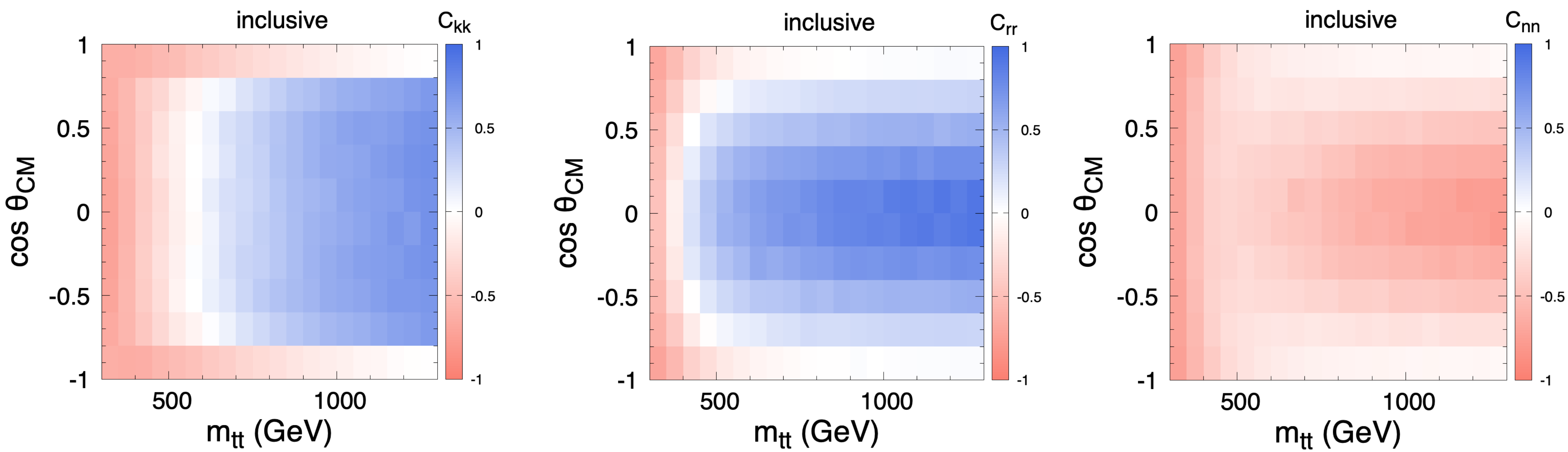
However, it is not optimal for the mixed spin state of the *VV* pair resulting from *H* decay

$$\rho = \int d\beta\ \mathcal{P}(\beta)|\psi_\beta\rangle\langle\psi_\beta| \qquad |\psi_\beta\rangle = \frac{1}{\sqrt{1+\beta^2}}\left(|+-\rangle - \beta|00\rangle + |-+\rangle\right)$$

See more of it later!

Top pair production

Violating CHSH inequalities is much harder than finding entanglement!



Having $|C_{ii} \pm C_{jj}| > \sqrt{2}$ requires two C's of order 0.7, which can only be achieved quite close to threshold, or in the very boosted central region.

low statistics even at HL-LHC

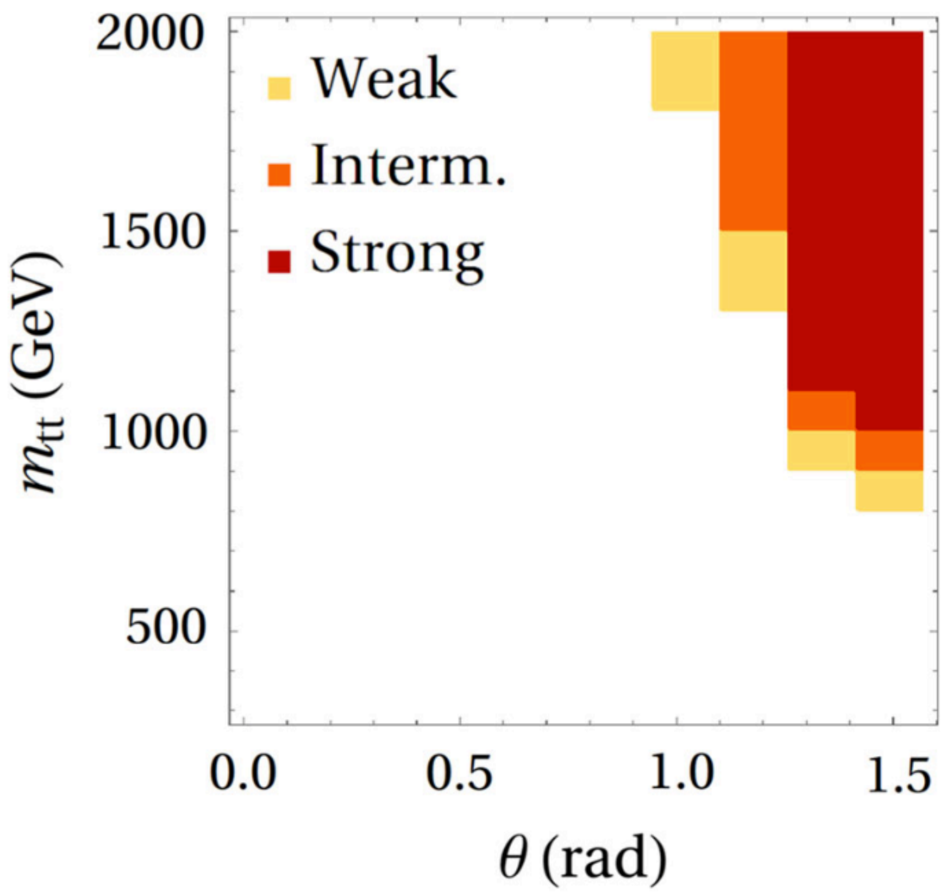
- Threshold: $C_{kk} + C_{nn}$ → but beamline basis slightly better
- Boosted: $C_{rr} - C_{nn}$

Top pair production

Severi et al. 2110.10112

Results with fast simulation

Kinematical selection



Optimal observable

High-p_T Selection	$\lambda + \lambda'$ Parton–level	Significance for > 1 [3 ab^{-1}]
Weak	1.12	1.9 σ
Intermediate	1.20	2.1 σ
Strong	1.30	1.3 σ

Simple observable

High-p_T		CHSH on fixed axes, $\sqrt{2}\lvert - C_{rr} + C_{nn} \rvert$			Significance for > 2
Selection	Cross section	Parton–level	Reconstructed [350 fb^{-1}]	Reconstructed [3 ab^{-1}]	[3 ab^{-1}]
Weak	0.19 pb	2.10	2.12 ± 0.17	± 0.06	1.7 σ
Intermediate	0.10 pb	2.18	2.20 ± 0.30	± 0.10	1.8 σ
Strong	0.06 pb	2.25	2.30 ± 0.76	± 0.26	1.0 σ

Top pair production

Why is the simple estimator so close to the optimal observable?

They obviously coincide when the C spin correlation matrix is diagonal

Example: $m_{t\bar{t}} \geq 1\ \mathrm{TeV}\,, \quad |\cos\theta_{\mathrm{CM}}| \leq 0.2$

σ	C_{kk}	C_{rr}	C_{nn}	C_{kr}	C_{xx}	C_{yy}	C_{zz}
23.3 fb	0.659	0.874	-0.760	0.037	-0.043	-0.043	0.878

largest elements in helicity basis

off-diagonal element in helicity basis

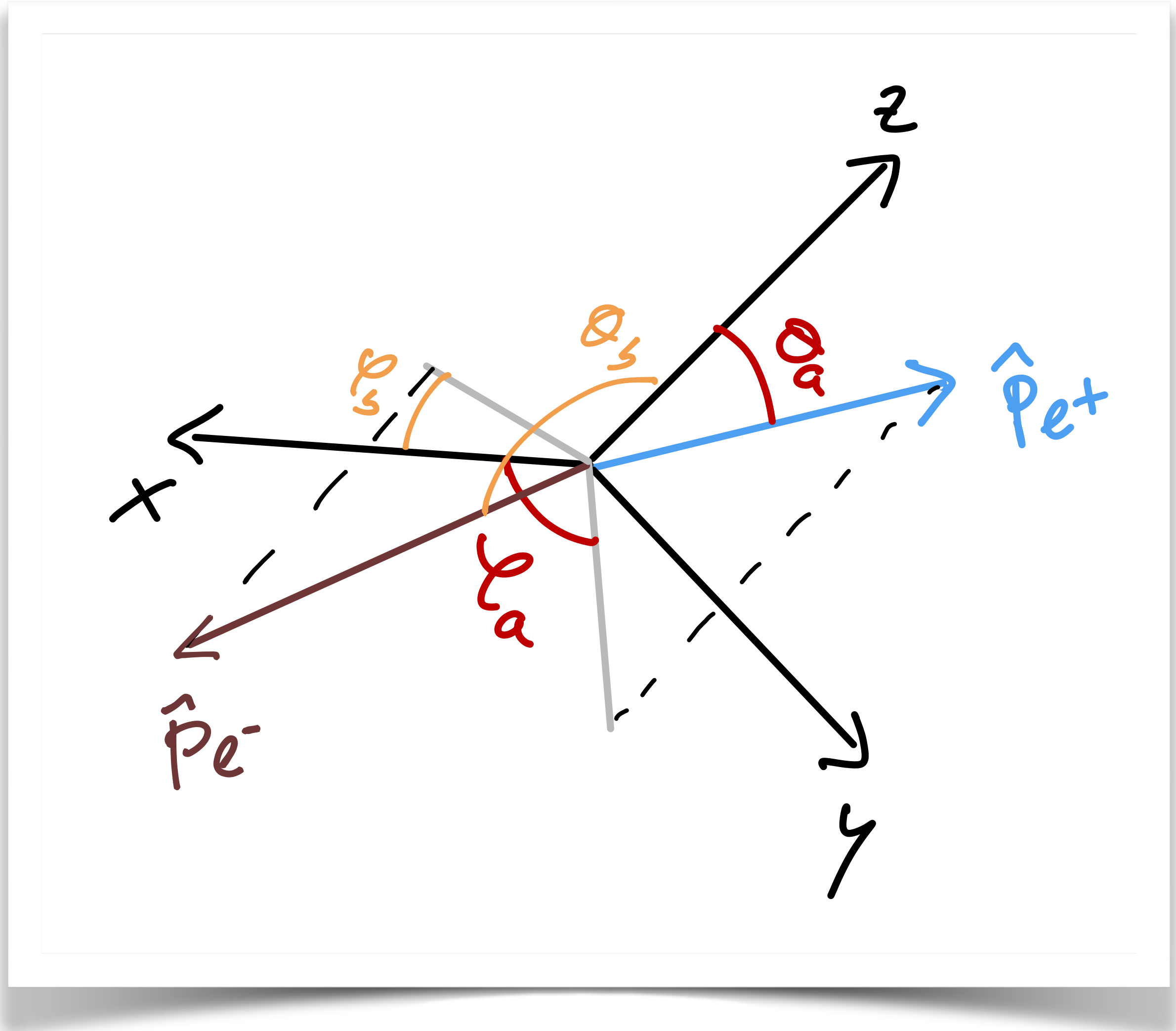
CHSH violation indicator

$$B \equiv |C_{rr} - C_{nn}| - \sqrt{2} = 0.210$$

Top pair production

What about dedicated observables to measure $|C_{ii} \pm C_{jj}|$?

JAAS, Casas,2205.00542



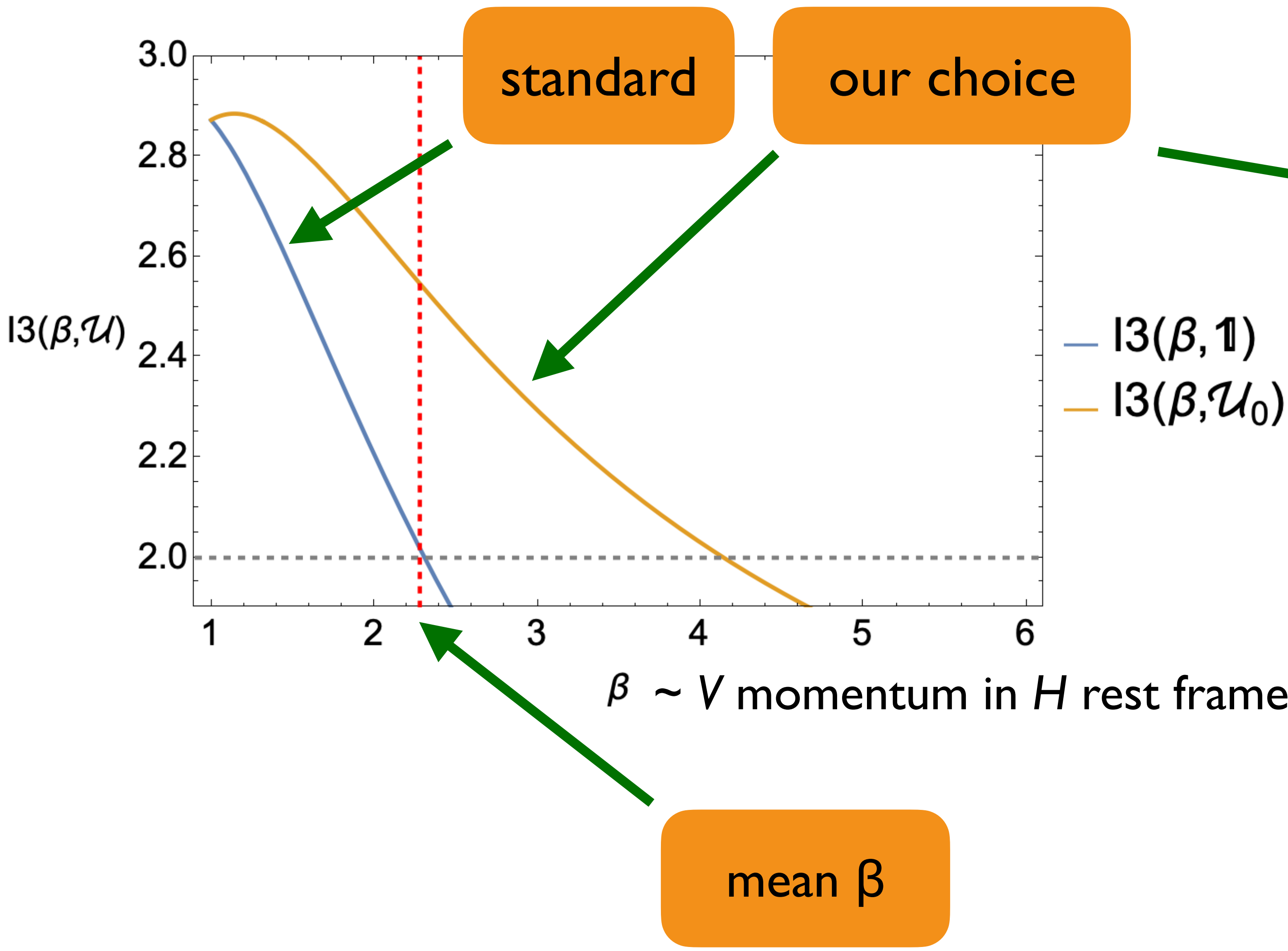
$$A_{\pm} = \frac{N(\cos(\varphi_a \mp \varphi_b) > 0) - N(\cos(\varphi_a \mp \varphi_b) < 0)}{N(\cos(\varphi_a \mp \varphi_b) > 0) + N(\cos(\varphi_a \mp \varphi_b) < 0)}$$

$$= \frac{\pi}{16}\alpha_a \alpha_b (C_{11} \pm C_{22})$$

$H \to VV$

We saw earlier that for a spin singlet there is a `standard´ Bell operator that is believed to be optimal. But this is not the case for $H \to VV$

[*V* not at rest in *H* rest frame]



$$I_3 = \frac{1}{36}\Big[\left(18+16\sqrt{3}\right) - \sqrt{2}\left(9-8\sqrt{3}\right) A^{1}_{2,0} - 8\left(3+2\sqrt{3}\right) C_{2,1,2,-1} + 6\, C_{2,2,2,-2}\Big]$$

	I_3	Significance
Run 2 + 3 : 300 fb^{-1}	2.66 ± 0.46	1.4σ
HL-LHC : 3 ab^{-1}	2.63 ± 0.15	4.2σ

Other

Bell inequalities in top pair production

CHSH violation involves only two coefficients. Near threshold, it pays off to make two of them larger even if the third one is smaller.

Beamline basis: simply $\hat{x} = (1,0,0) \quad \hat{y} = (0,1,0) \quad \hat{z} = (0,0,1)$

$$m_{t\bar{t}} \leq 353 \text{ GeV}$$

really tight cut!

	σ	C_{kk}	C_{rr}	C_{nn}	C_{kr}	C_{xx}	C_{yy}	C_{zz}
no β cut	303 fb	-0.677	-0.562	-0.712	0.067	-0.719	-0.719	-0.506
$\beta \leq 0.8$	181 fb	-0.743	-0.640	-0.761	0.052	-0.767	-0.767	-0.602

estimator nearly optimal

CHSH violation indicator

$$B \equiv |C_{xx} + C_{yy}| - \sqrt{2} = 0.024 \xrightarrow{\beta \leq 0.8} 0.120$$

Bell inequalities in top pair production

Prospects?

Setting systematics aside, one can investigate the statistical significance for non-zero CHSH violation indicators $B = |C_{ii} \pm C_{jj}| - \sqrt{2}$

LHC Run 2+3 300 fb^{-1}

		naive		improved	
threshold [β, A_+]	B :	0.021 ± 0.053	$\longrightarrow$	0.121 ± 0.045	6.8 ×
boosted [A_-]	B :	0.218 ± 0.141	$\longrightarrow$	0.208 ± 0.125	1.13 ×

In all cases the statistics are small, therefore an improvement of the statistical sensitivity is very welcome.

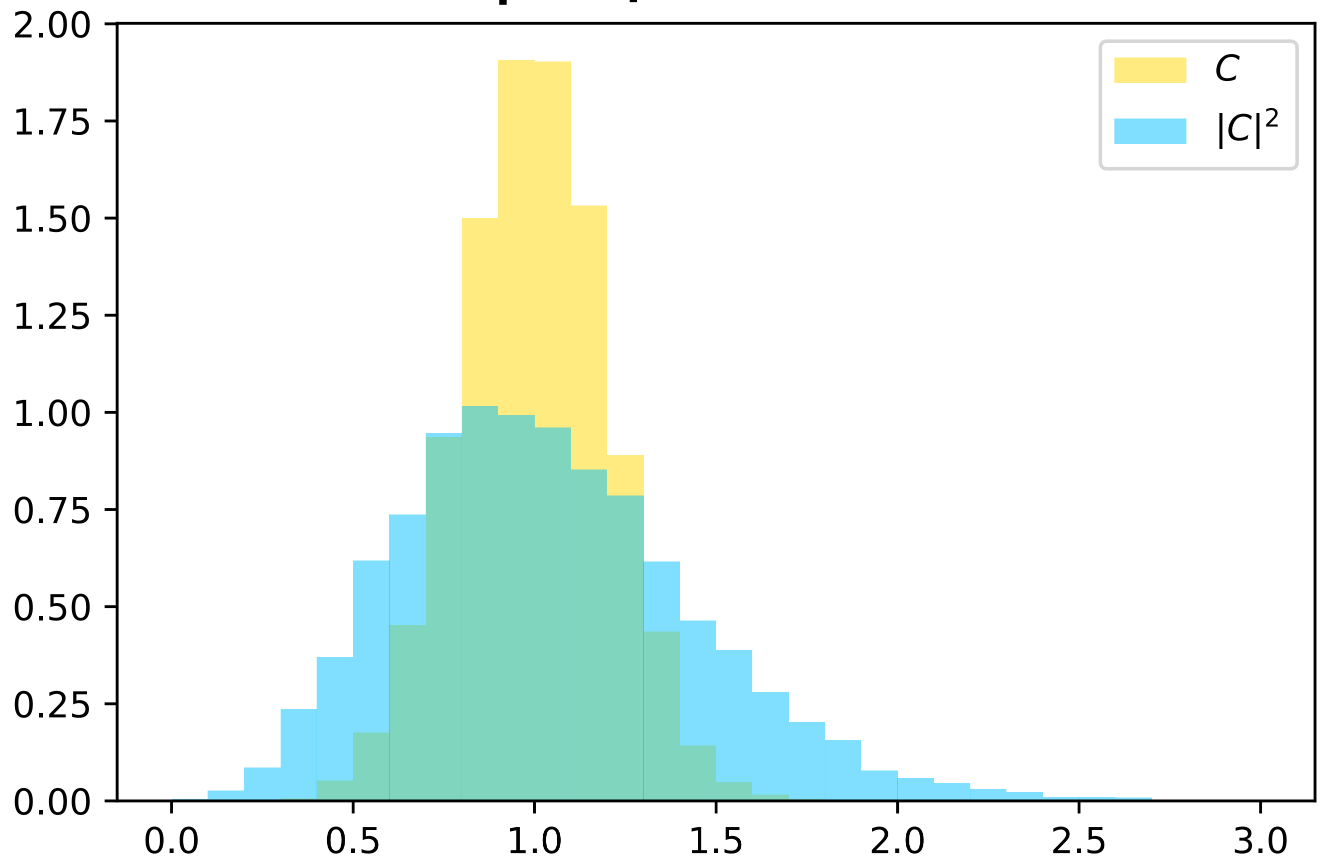
HL-LHC 3 ab^{-1}

threshold [β, A_+]	B :	0.024 ± 0.017	$\longrightarrow$	0.124 ± 0.013	6.8 ×
boosted [A_-]	B :	0.218 ± 0.041	$\longrightarrow$	0.208 ± 0.036	1.13 ×

Bell inequalities

Statistical fluctuations bias the eigenvalues of $C^T C$ towards larger values

Example: $\mu = 1, \sigma = 0.2$



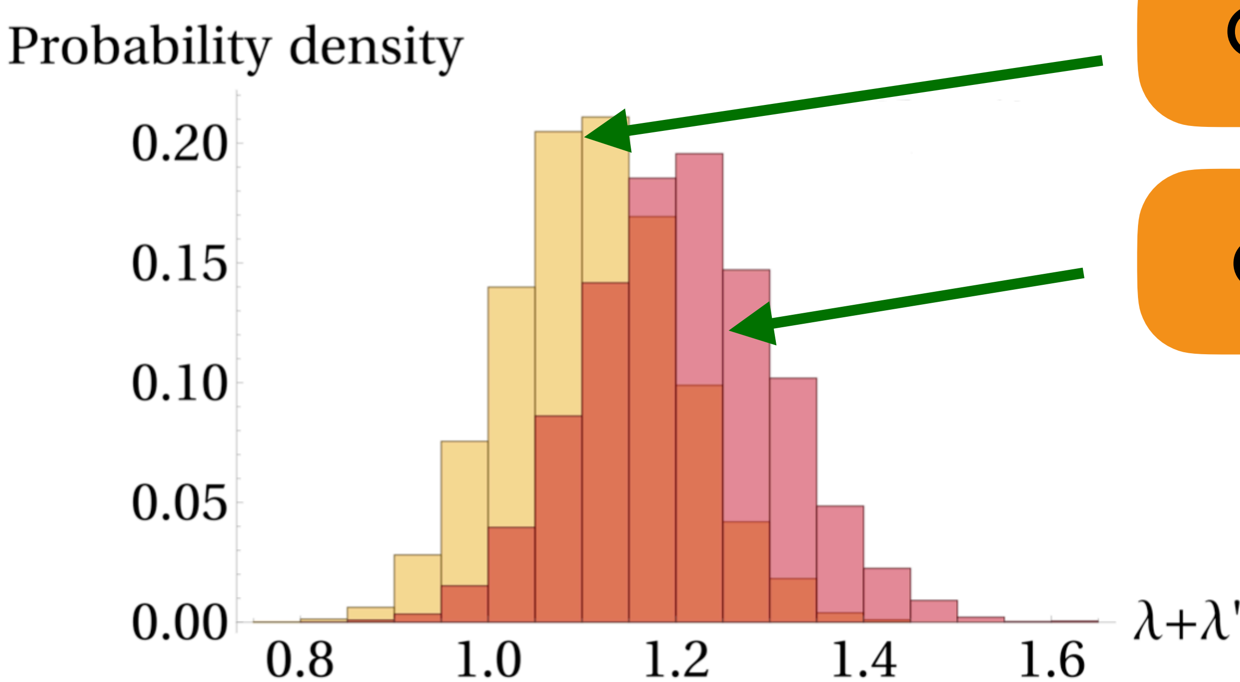
… this `optimal´ procedure is not as good as it seems, and simpler ways are more robust and equally effective!

Severi et al. 2110.10112

CHSH inequalities in top pair production

Because the eigenvalues of $C^T C$ are biased, the violation of CHSH equalities is not signalled by $\lambda_1 + \lambda_2 > 1$

Severi et al. 2110.10112



CHSH-conserving has $\langle \lambda_1 + \lambda_2 \rangle > 1$

CHSH-violating

Significance decreases when taking bias into account

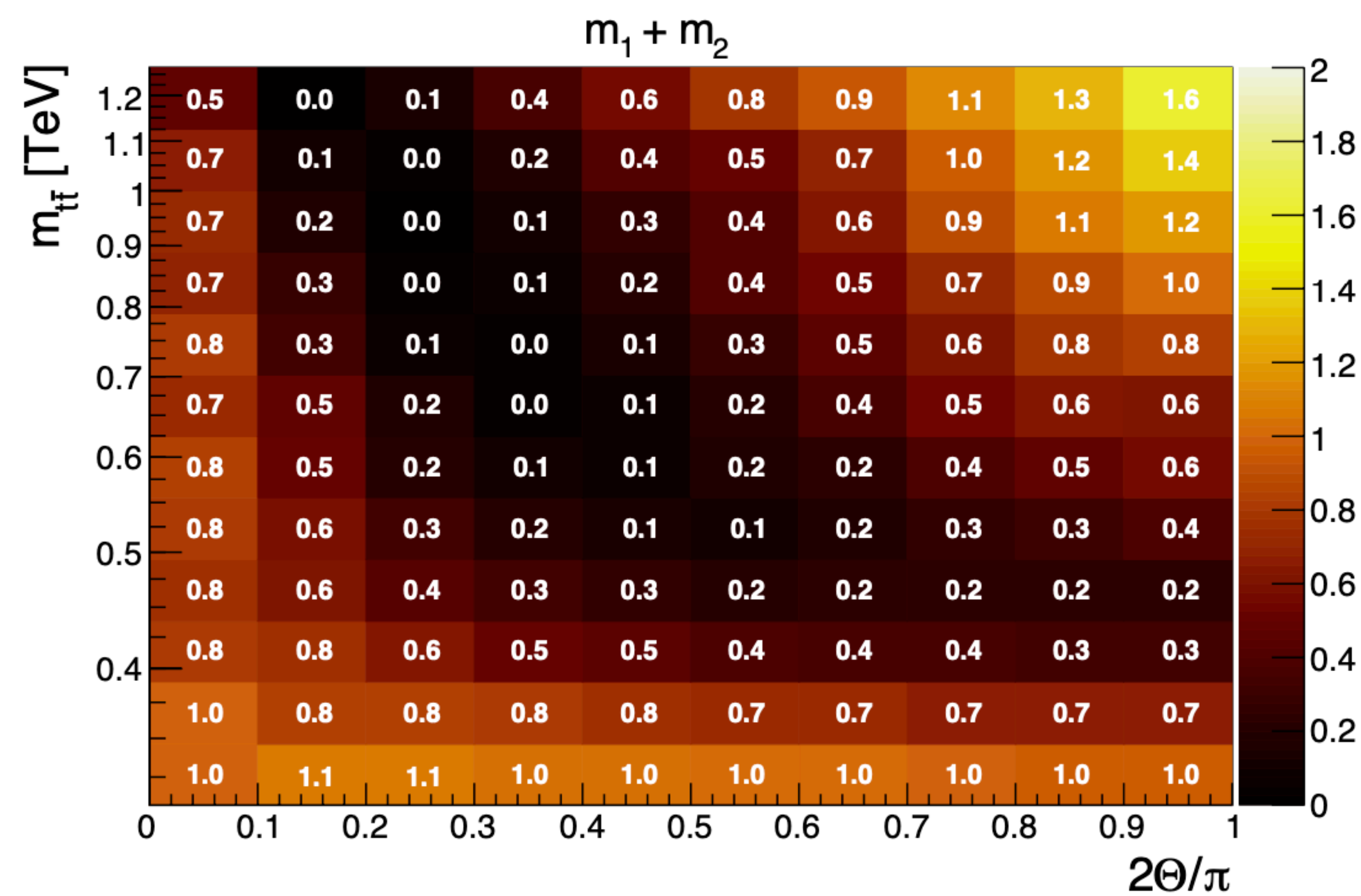
High-p_T Selection	$\lambda + \lambda'$ Parton–level	Significance for > 1 $[3\,\mathrm{ab}^{-1}]$
Weak	1.12	$1.9\,\sigma$
Intermediate	1.20	$2.1\,\sigma$
Strong	1.30	$1.3\,\sigma$

CHSH inequalities in top pair production

CHSH violation tests with $C^T C$ eigenvalues [$\lambda_1 + \lambda_2 > 1$]

- Fast simulation with Delphes
- Only resonant diagrams
- Kinematical reconstruction and unfolding to parton level
- Optimised selection in m_{tt} — θ phase space

Fabbrichesi et al. 2102.11883



Helicity basis used BUT flipping axis for anti-top

CHSH violation at 98% CL [2.3σ] with Run 2 [139 fb^{-1}]

CHSH violation at 4σ with Run 3 [??? fb^{-1}]

CHSH inequalities in top pair production

Why do I worry about the flip sign in the basis definition? It is incorrect.

$$\rho = \begin{pmatrix} 1+C_{33} & 0 & 0 & C_{11}-C_{22}-i2C_{12} \\ 0 & 1-C_{33} & C_{11}+C_{22} & 0 \\ 0 & C_{11}+C_{22} & 1-C_{33} & 0 \\ C_{11}-C_{22}+i2C_{12} & 0 & 0 & 1+C_{33} \end{pmatrix}$$

In some phase space region:

Sign flip

$$C_{11} = 0.743 \quad C_{22} = 0.640$$
$$C_{33} = 0.761 \quad C_{12} = -0.052$$

→ Eigenvalues 1.9, 1.6, 1.6, -1.1

No sign flip

$$C_{11} = -0.743 \quad C_{22} = -0.640$$
$$C_{33} = -0.761 \quad C_{12} = 0.052$$

→ Eigenvalues 3.1, 0.4, 0.4, 0.1

Top pair entanglement

Why dedicated observable?

- If one measures C_{kk}, C_{rr} and C_{nn} independently, and performs the sum, the statistical uncertainty is larger.
- Systematic uncertainties require a detailed study.

however

Measuring C_{kk}, C_{rr} and C_{nn} requires full reconstruction of the K, R and N axes. Measuring the angle θ'_{ab} only requires to reconstruct the N axis.

Top pair entanglement

How much is the improvement?

Setting systematics aside, there is an improvement of the statistical uncertainty of the `entanglement indicator´ $E = |C_{kk} + C_{rr}| - C_{nn} - 1$

LHC Run 2 139 fb^{-1}

threshold [β] $E: \quad 0.559 \pm 0.017 \longrightarrow 0.679 \pm 0.019$ 1.27 ×

boosted [D_3] $E: \quad 0.671 \pm 0.069 \longrightarrow 0.663 \pm 0.056$ 1.23 ×

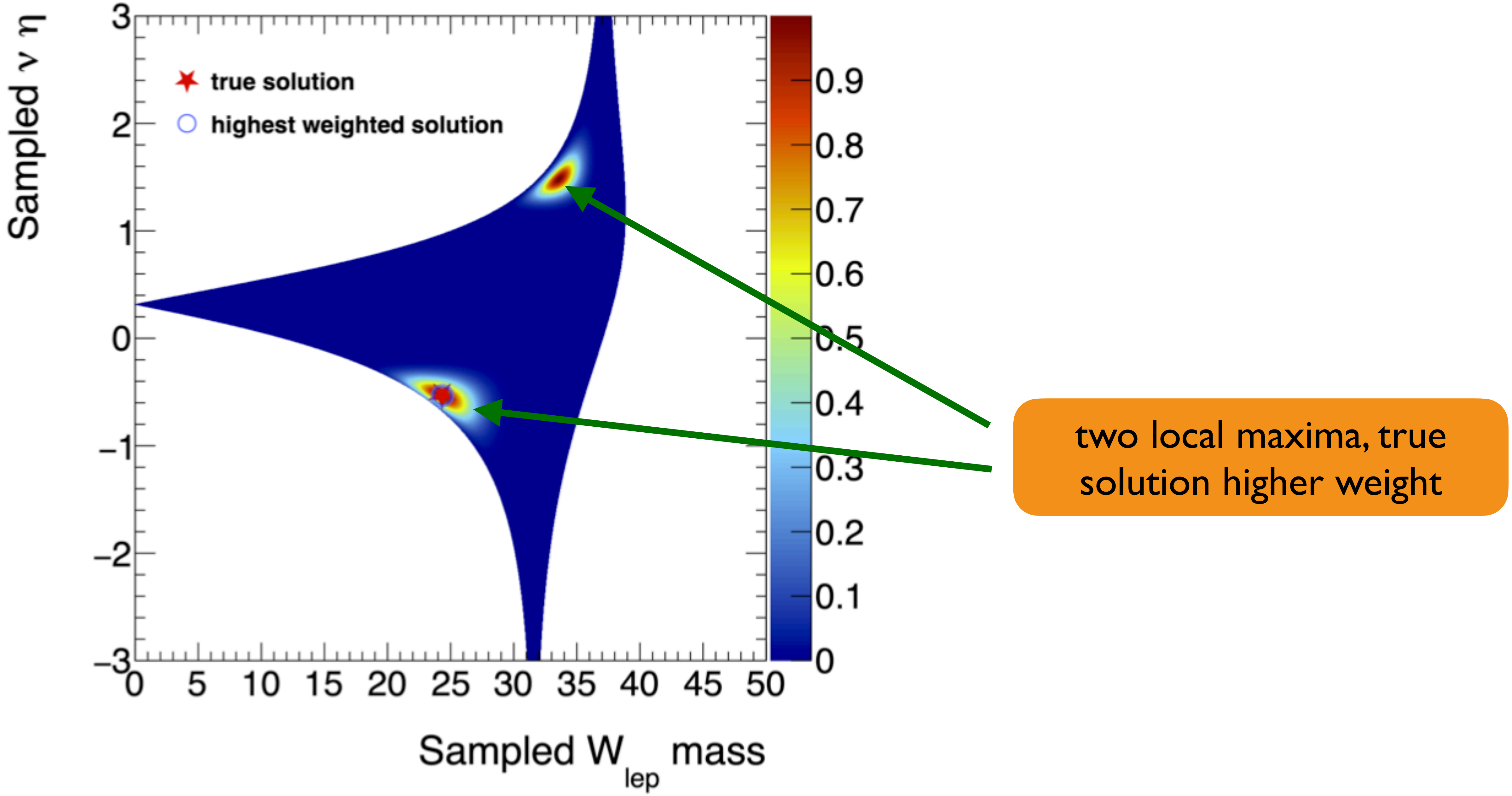
Near threshold there are quite large statistics, but in the boosted central region there are not.

1.23 × improvement in statistical sensitivity for the boosted region is equivalent to 50% more luminosity!

$H \to VV$

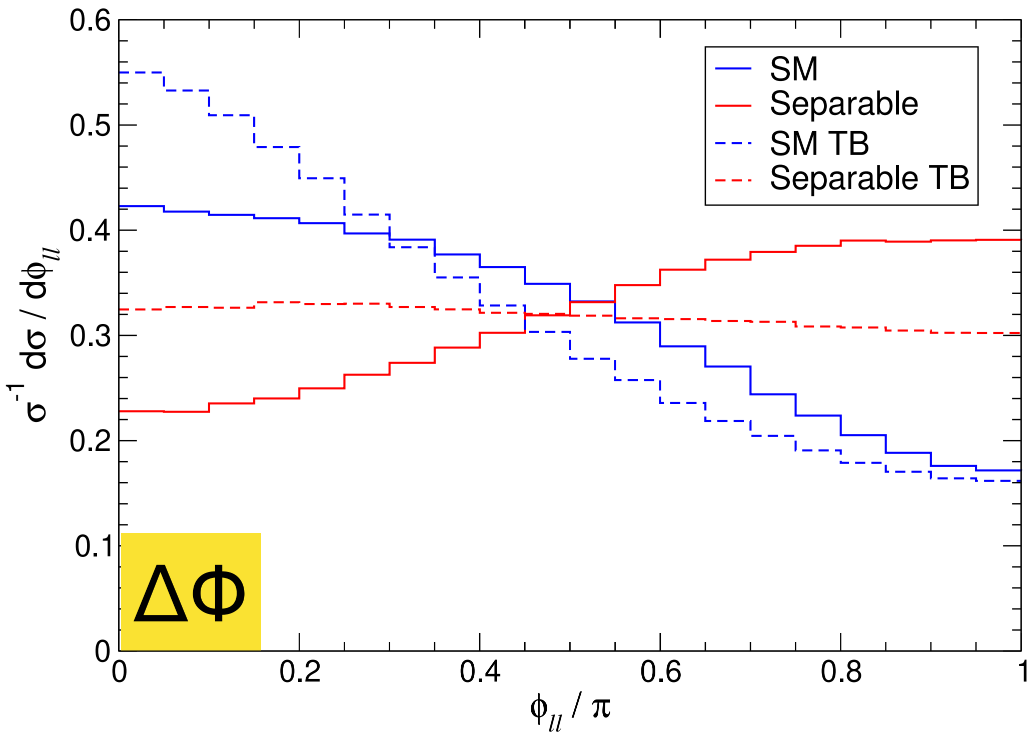
Reconstruction with neutrino weighting works well.

Example of weight distributions for one event

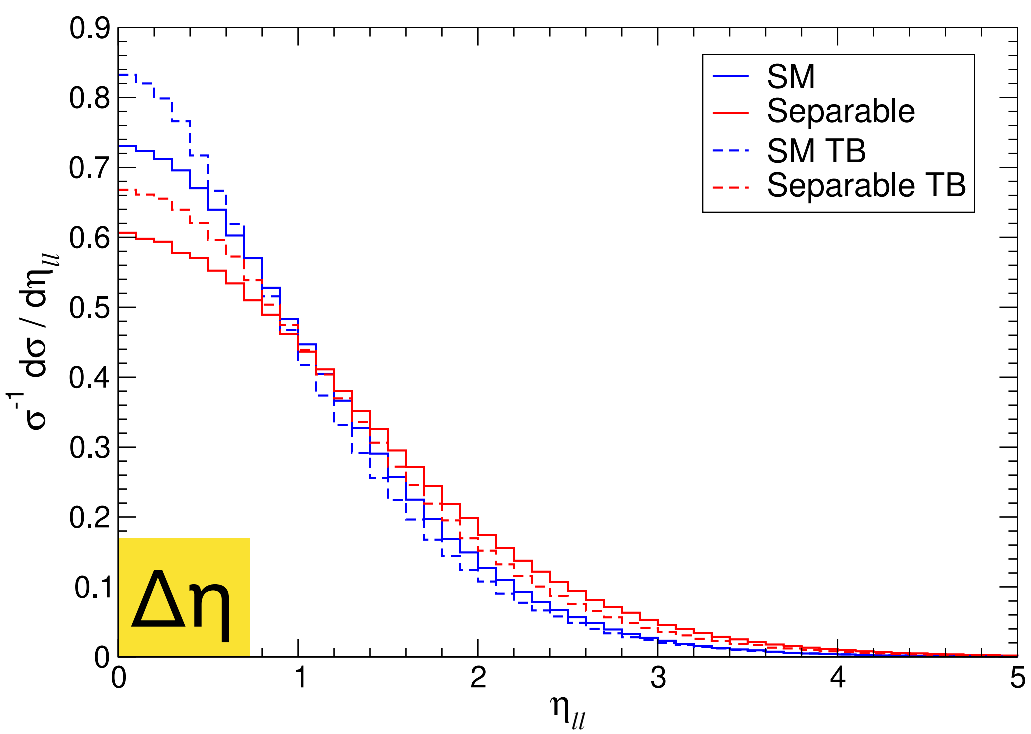


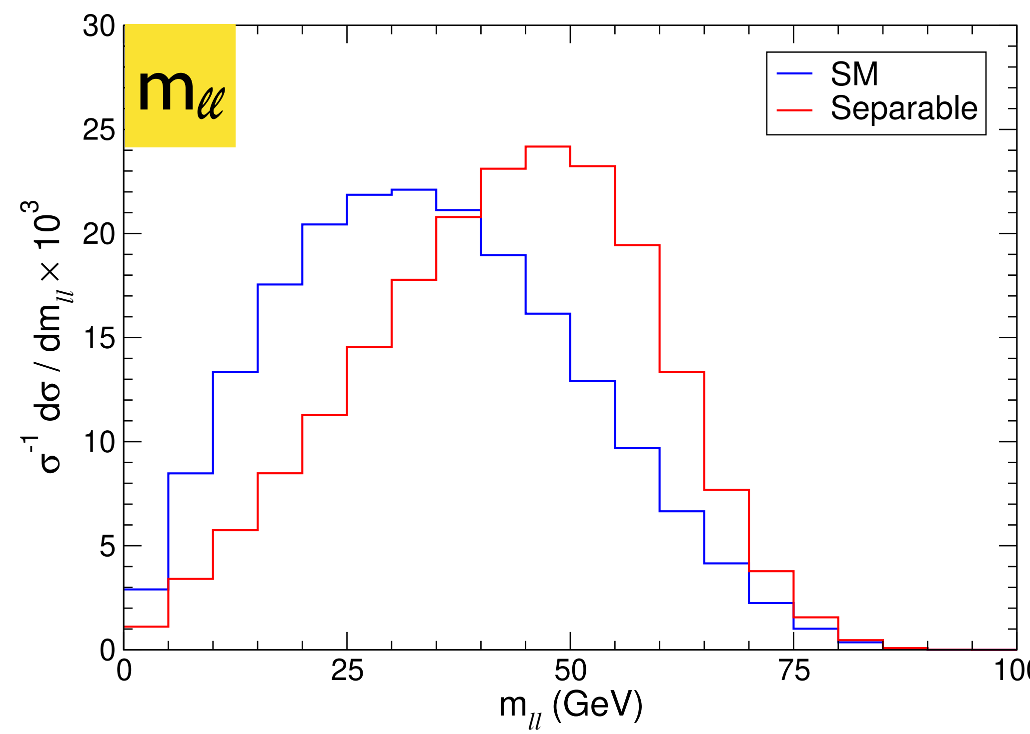
$H \to VV$

Parton-level plots



Though the discrimination from ΔΦ is excellent, transverse boosts from ISR [dashed lines] have a significant impact in the distribution.





Discussion

Loopholes to Bell inequalities? There are many!

- We are assuming quantum mechanics [in the decay of the top quark] to test quantum mechanics [by CHSH inequalities].
- Free-will loophole: we are not measuring spins in the pre-determined directions we want.
- Causal connection: near threshold, the decay of the two top quarks may not be causally disconnected.

… but in any case, these are very nice and demanding measurements!