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Parton showers and matching for top physics

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Top Quark Physics at the Precision Frontier

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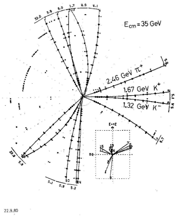
Light-quark simulations timeline

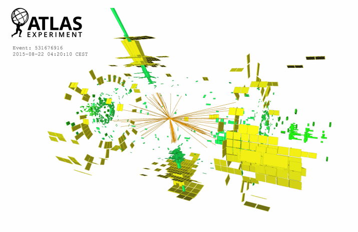
Accuracy → Precision

- Color coherent evolution
  Backward evolution
- Color dipole formalism
  Spin correlations
- Color dipole showers
- LO merging
  NLO matching
- Dipole-like showers
  Antenna showers
  NLO merging
- Color ME corrections
  NNLO matching
- Threshold effects
  NLO splitting kernels
  Higher-order PS framework
- Accuracy quantification
  Amplitude evolution
- NLL & NGLs in one framework
  Efficient $1/N_c$ corrections
- NNLL soft-only evolution

1985 — 1990 — 1995 — 2000 — 2005 — 2010 — 2015 — 2018 — 2022 →



$\sqrt{s} \times 500$ →
e^+e^- vs. pp



Top-quark simulations timeline

Accuracy → Precision

Coherent heavy quark evolution

Production / decay correlations

LO merging

NLO matching

Dipole-like showers

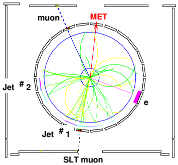
Antenna showers

NLO merging

NNLO matching

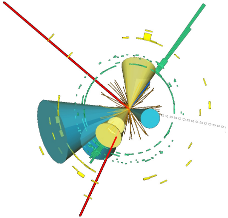
NLL parton showers

1990 — 1995 — 2000 — 2005 — 2010 — 2015 — 2018 — 2022 →



$\sqrt{s} \times 7$ →

$p\bar{p}$ vs. pp



Simulation of QCD radiation

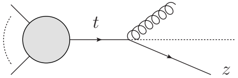
Approaches, problems & solutions

Radiative corrections as a branching process

[Marchesini,Webber] NPB238(1984)1
[Sjöstrand] PLB157(1985)321

- Probability for parton splitting in collinear limit

$$\lambda \to \frac{1}{\sigma_n}\int_t^{Q^2} \mathrm{d}\bar{t}\,\frac{\mathrm{d}\sigma_{n+1}}{\mathrm{d}\bar{t}} \approx \sum_{\text{jets}}\int_t^{Q^2}\frac{\mathrm{d}\bar{t}}{\bar{t}}\int \mathrm{d}z\,\frac{\alpha_s}{2\pi}P(z)$$



- Perturbative unitarity leads to a Markov process
  - Assume bosonic final state → naive probability for n emissions

$$P_{\text{naive}}(n,\lambda) = \frac{\lambda^n}{n!}$$

  - Probability conservation implies no-emission probability

$$P(n,\lambda) = \frac{\lambda^n}{n!}\exp\{-\lambda\} \qquad \longrightarrow \qquad \sum_{n=0}^{\infty} P(n,\lambda) = 1$$

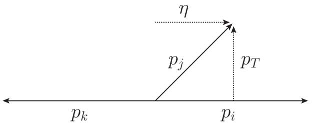
$\Delta(t,Q^2) := \exp\{-\lambda\}$ → Sudakov factor

- Practical challenges
  - Color coherence
  - Four-momentum conservation
  - Matching to soft limit

Dual description and the Lund plane

[Gustafson] PLB175(1986)453

- Compute everything in center-of-mass frame of fast partons



- Simple expressions for transverse momentum and rapidity

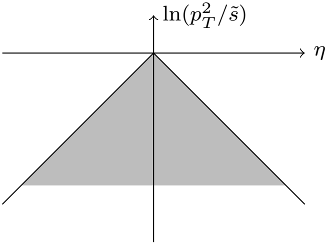
$$p_T^2 = \frac{2(p_i p_j)(p_k p_j)}{p_i p_k}\;, \qquad \eta = \frac{1}{2}\ln\frac{p_i p_j}{p_k p_j}$$

- In momentum conserving parton branching $(\tilde{p}_i, \tilde{p}_k) \to (p_i, p_k, p_j)$

$$-\ln \tilde{s}_{ik}/p_T^2 \le 2\eta \le \ln \tilde{s}_{ik}/p_T^2$$

Differential phase-space element $\propto \mathrm{d}p_T^2\,\mathrm{d}\eta$

- Visualized best in Lund plane
  - Gluon emission probability is constant
  - QCD evolution creates fractal structure
  - Recent revival in experimental analyses



Angular ordered parton showers

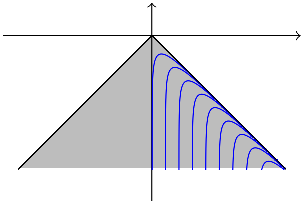
[Marchesini,Webber] NPB238(1984)1, ...

- Differential radiation probability

$$\mathrm{d}\mathcal{P} = \mathrm{d}\Phi_{+1}|M|^2 \approx \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2}\,\mathrm{d}z\,\frac{\alpha_s}{2\pi}\,P_{\widetilde{ij}i}(z)$$

  - Ordering parameter $\tilde{q}^2 = \dfrac{2p_ip_j}{z(1-z)} \approx 4E_{\widetilde{ij}}^2\sin^2\dfrac{\theta_{ij}}{2}$

- Lund plane filled from center to edges
  - Random walk in p_T^2
  - Color factors correct for observables insensitive to azimuthal correlations
  - Small dead zone at $\ln(p_T^2/\tilde{s}) \approx 0$



- Usually disfavored due to dead zones
  Not suitable to resum non-global logartihms

Dipole & antenna showers

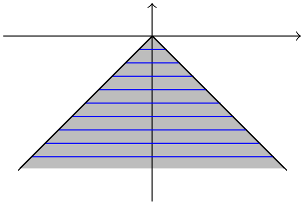
[Gustafson,Pettersson] NPB306(1988)746, ...

- Differential radiation probability for the dipole

$$\mathrm{d}\mathcal{P} = \mathrm{d}\Phi_{+1}|M|^2 \approx \frac{\mathrm{d}p_T^2}{p_T^2}\,\mathrm{d}\eta\,\frac{\alpha_s}{2\pi}\,\tilde{P}_{\widetilde{ij}}(z)$$

  - Ordering parameter p_T^2

- Lund plane filled from top to bottom
  - Random walk in η
  - Color factors in CFFE approximation
  - Pairs of partons evolve simultaneously
  - No dead zones

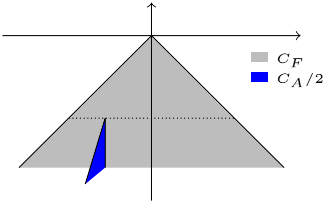


- Solves problem of dead zones
  Known issues with color coherence

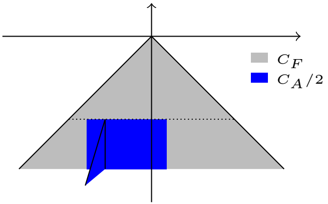
Problems with color coherence

[Gustafsson] NPB392(1993)251

- In angular ordered showers angles are measured in the event center-of-mass frame $\rightarrow$ coherence effects modeled by angular ordering variable agree on average with matrix element



- In dipole-like showers angles effectively measured in center-of-mass frame of emitting color dipole $\rightarrow$ angular coherence not reflected by setting average QCD charge

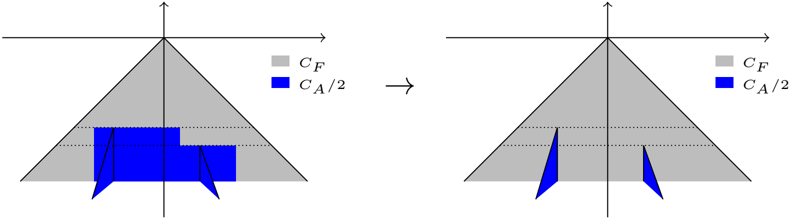


- Emission off “back plane” in Lund diagram should be associated with C_F, but is partly associated with $C_A/2$ in dipole showers

Emulating color coherence on average

[Gustafsson] NPB392(1993)251

- Analyze rapidity of gluon emission in event center-of-mass frame
- Sectorize phase space, use color charge of parton closest to soft gluon



- Alternatively reweight to double-soft ME [Giele,Kosower,Skands] arXiv:1102.2126
  Algorithm scales as N^2 but can be simplified while retaining accuracy
  $\rightarrow$ Nested double-soft corrections in rapidity segments of parent dipole
  [Hamilton,Medves,Salam,Scyboz,Soyez] arXiv:2011.10054
- Starting with 4 emissions, there be "color monsters"
  [Dokshitzer,Troian,Khoze] SJNP47(1988)881, YF47(1988)1384
  - Quartic Casimir operators (easy)
  - Non-factorizable contributions (hard)

The problem of on-shell momentum mapping

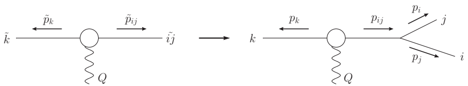
[Dasgupta,Dreyer,Hamilton,Monni,Salam] arXiv:1805.09327

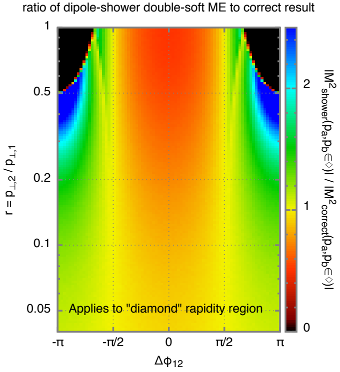
- Subtle problems in standard dipole, dipole-like and antenna mapping

$$p_k^\mu = \left(1 - \frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\right)\tilde{p}_k^\mu$$

$$p_i^\mu = \tilde{z}\,\tilde{p}_{ij}^\mu + (1-\tilde{z})\frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\tilde{p}_k^\mu + k_\perp^\mu$$

$$p_j^\mu = (1-\tilde{z})\,\tilde{p}_{ij}^\mu + \tilde{z}\frac{p_{ij}^2}{2\tilde{p}_{ij}\tilde{p}_k}\tilde{p}_k^\mu - k_\perp^\mu$$





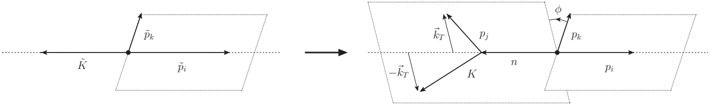
- Induces accidental angular correlations
  Spoils agreement w/ analytic resummation
- Good recoil schemes preserve logarithmic accuracy, but also impact phase-space coverage, especially for angular ordered evolution
  [Bewick,Ferrario-Ravasio,Richardson,Seymour] arXiv:1904.11866

NLL compatible on-shell mappings for light quarks

- Partitioning of antenna radiation pattern paired with suitable choice of evolution variable [Dasgupta,Dreyer,Hamilton,Monni,Salam,Soyez] arXiv:2002.11114

$$k_T = \rho v e^{\beta|\bar{\eta}|} \qquad \rho = \left(\frac{s_i s_j}{Q^2 s_{ij}}\right)^{\beta/2}$$

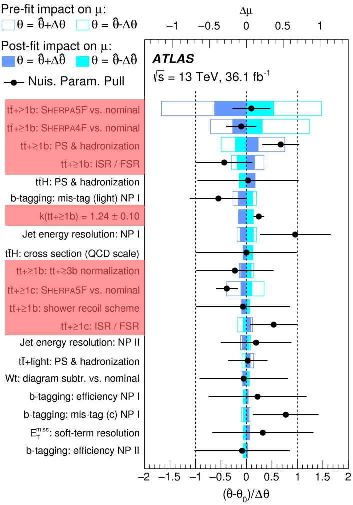
- Global transverse recoil, global longitudinal recoil gives analytic proof of NLL correctness for dedicated observables (thrust, multiplicity) [Forshaw,Holguin,Plätzer] arXiv:2003.06400
- Local transverse recoil, global longitudinal recoil allows analytic proof of NLL correctness, based on kinematics in $s \to \infty$ limit [Nagy,Soper] arXiv:2011.04773
- Keeping emitter along original direction & recoil vector arbitrary allows to match analytic resummation and prove NLL precision analytically [Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057



Heavy flavor production & evolution

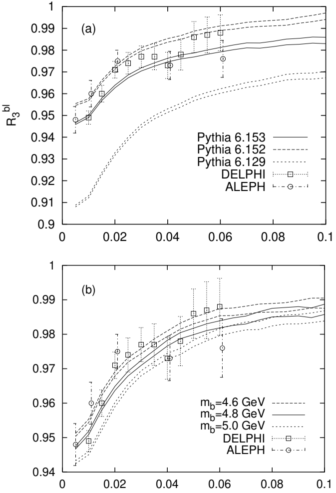
- Example $t\bar{t}b\bar{b}$: MC single largest source of uncertainty on signal strength
- Despite intense study of HF production
  - Fixed order, NLL, FONLL
    [Cacciari,Frixione,Houdeau,Mangano,Nason,Ridolfi,. . .] arXiv:1205.6344, hep-ph/0312132, hep-ph/9801375, NPB373(1992)295
  - In context of particle-level Monte Carlo
    [Marchesini,Webber] NPB330(1990)261,
    [Norrbin,Sjöstrand] hep-ph/0010012,
    [Gieseke,Stephens,Webber] hep-ph/0310083,
    [Schumann,Krauss] arXiv:0709.1027,
    [Gehrmann-deRidder,Ritzmann,Skands] arXiv:1108.6172,
    [Assi,SH] arXiv:2307.00728
- Recurring themes, not special to $t\bar{t}b\bar{b}$
  - PS uncertainties hard to judge and reduce
    [Cascioli,Maierhöfer,Moretti,Pozzorini,Siegert] arXiv:1309.591
  - Matching needed for inclusive predictions
    [Krause,Siegert,SH] arXiv:1904.09382,
    [Ferencz,Katzy,Krause,Pollard,Siegert,SH]

[ATLAS] arXiv:1712.08895



Heavy flavor production & evolution

- Both high-energy limit and threshold region should be described well, but
- Infrared finite prediction for $g \to Q\bar{Q}$ leaves splitting functions somewhat arbitrary
- Soft gluon emission off light/heavy quarks associated with $\alpha_s(k_T^2)$, i.e. "correct" scale is k_T^2 [Amati et al.] NPB173(1980)429, but no such argument to set scale for $g \to Q\bar{Q}$
  $\to$ HQ production rate not very stable w.r.t. parton shower variations
- A number of different prescriptions, e.g.
  [Marchesini,Webber] NPB330(1990)261,
  [Norrbin,Sjöstrand] hep-ph/0010012,
  [Gieseke,Stephens,Webber] hep-ph/0310083,
  [Schumann,Krauss] arXiv:0709.1027,
  [Gehrmann-deRidder,Ritzmann,Skands] arXiv:1108.6172,
  [Assi,SH] arXiv:2307.00728
  varying success in describing expt. data



[Norrbin,Sjöstrand] hep-ph/0010021

Soft radiation and matching to collinear limit

[Marchesini,Webber] NPB238(1984)1, NPB310(1988)461

- Eikonal can be written in terms of energies and angular "radiator" function

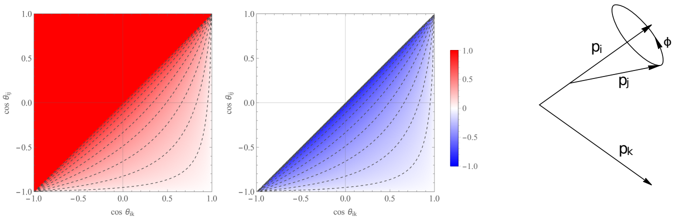
$$J_\mu J^\mu \to \frac{2p_ip_k}{(p_ip_j)(p_jp_k)} = \frac{W_{ik,j}}{E_j^2}\,, \qquad W_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{kj})}$$

- Collinearly divergent as $\theta_{ij} \to 0$ and as $\theta_{kj} \to 0$

  $\to$ Expose individual singularities via $W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j}$

$$\tilde{W}^i_{ik,j} = \frac{1}{2(1-\cos\theta_{ij})}\left[\frac{1-\cos\theta_{ik}}{1-\cos\theta_{kj}} + 1 - \frac{1-\cos\theta_{ij}}{1-\cos\theta_{kj}}\right]$$

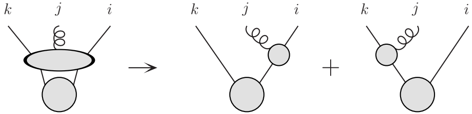
- Azimuthal averaging yields famous angular ordering
  - Differential radiation pattern outside parent dipole more intricate
    Positive & negative contributions sum to zero



Soft radiation and matching to collinear limit

- Alternative to additive matching: partial fraction matrix element & match to collinear sectors [Ellis,Ross,Terrano] NPB178(1981)421, [Catani,Seymour] hep-ph/9605323

$$\frac{W_{ik,j}}{E_j^2} \to \frac{1}{p_i p_j}\frac{p_i p_k}{(p_i+p_k)p_j} + \frac{1}{p_k p_j}\frac{p_i p_k}{(p_i+p_k)p_j}$$

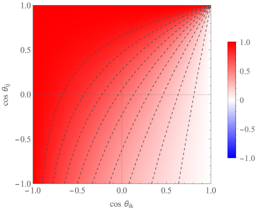


  - Captures matrix element both in angular ordered and unordered region
  - Caveat: Oversampling difficult for certain kinematics maps

- Separate into energy & angle first [Herren,Krauss,Reichelt,Schönherr,SH] arXiv:2208.06057
  Partial fraction angular radiator only: $W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j}$

$$\bar{W}^i_{ik,j} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(2-\cos\theta_{ij}-\cos\theta_{kj})}$$

  - Bounded by $(1-\cos\theta_{ij})\bar{W}^i_{ik,j} < 2$
  - Strictly positive



Soft-collinear matching for heavy quarks

[Marchesini,Webber] NPB330(1990)261

- Singularity in angular radiator screened by velocity $\to$ deadcone $\theta_0 \approx m/E$

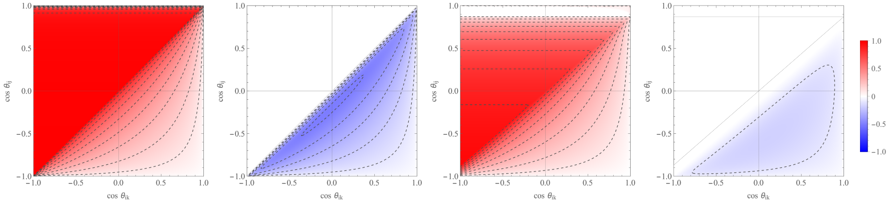
$$W_{ik,j} = \frac{1 - v_i v_k \cos\theta_{ik}}{(1 - v_i \cos\theta_{ij})(1 - v_k \cos\theta_{jk})} - \frac{(1 - v_i^2)/2}{(1 - v_i \cos\theta_{ij})^2} - \frac{(1 - v_k^2)/2}{(1 - v_k \cos\theta_{jk})^2}$$

- Quasi-collinear divergence if $m_Q \propto k_T$ as $k_T \to 0$
  $\to$ Expose individual singularities via $W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j}$

$$\tilde{W}^i_{ik,j} = \frac{1}{2(1 - v_i \cos\theta_{ij})}\left[\left(\frac{1 - v_i v_k \cos\theta_{ik}}{1 - v_k \cos\theta_{kj}} - \frac{1 - v_i^2}{1 - v_i \cos\theta_{ij}}\right) + 1 - \frac{1 - v_i \cos\theta_{ij}}{1 - v_k \cos\theta_{kj}}\right]$$

- Approximate angular ordering after azimuthal averaging

$v^2 = 1 - m_b^2/m_Z^2$ $\qquad$ $v^2 = 1 - m_t^2/(350\ \text{GeV})^2$



Soft-collinear matching for heavy quarks

[Assi,SH] arXiv:2307.00728

- Alternative: separate into energy & angle first
  Partial fraction angular radiator only: $W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j}$

$$\bar{W}^i_{ik,j} = \frac{1 - v_k \cos\theta_{kj}}{2 - v_i \cos\theta_{ij} - v_k \cos\theta_{kj}} W_{ik,j}$$

- Can be written in more intuitive form (n^μ defines reference frame)

$$\bar{W}^i_{ik,j} = \frac{1}{2 l_i l_j}\left(\frac{l_{ik}^2}{l_{ik} l_j} - \frac{l_i^2}{l_i l_j} - \frac{l_k^2}{l_k l_j}\right), \qquad \text{where} \qquad l_i^\mu = \sqrt{n^2}\,\frac{p_i^\mu}{p_i n}$$

- Quasi-collinear limit manifest

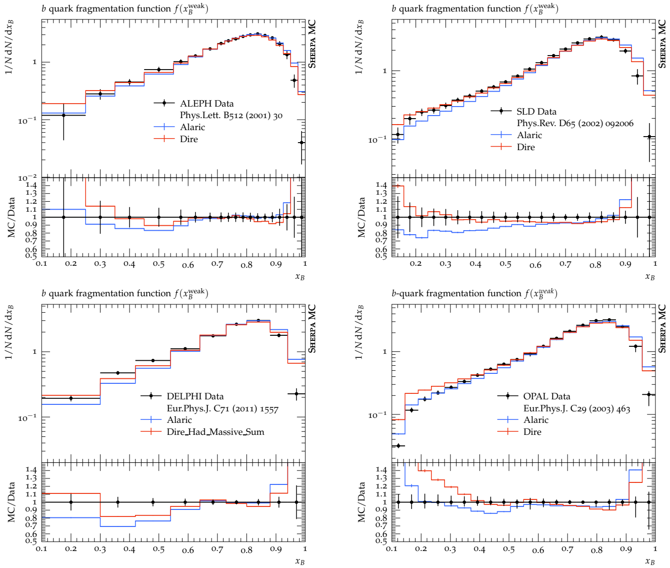
$$\frac{\bar{W}_{ik,j}}{E_j^2} \xrightarrow[m_i \propto p_i p_j]{i||j} w^{(\mathrm{coll})}_{ik,j}(z) := \frac{1}{2 p_i p_j}\left(\frac{2z}{1-z} - \frac{m_i^2}{p_i p_j}\right)$$

- Matching to massive DGLAP splitting functions

$$\frac{P_{(ij)i}(z,\varepsilon)}{(p_i+p_j)^2 - m_{ij}^2} \to \frac{P_{(ij)i}(z,\varepsilon)}{(p_i+p_j)^2 - m_{ij}^2} + \delta_{(ij)i}\,\mathbf{T}_i^2\left[\frac{\bar{W}^i_{ik,j}}{E_j^2} - w^{(\mathrm{coll})}_{ik,j}(z)\right],$$

Implementation in an NLL accurate parton shower

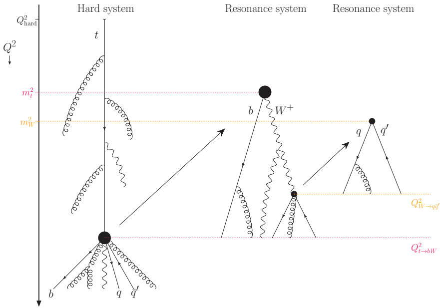
[Assi,SH] arXiv:2307.00728

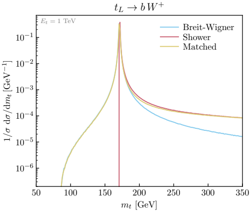


Interleaved MPI+PS evolution for resonance decays

[Brooks,Skands,Verheyen] arXiv:2108.10786

- First antenna shower for resonance decays
- Simultaneous PS & MPI evolution in hard system
- When PS reaches width of resonance, decay occurs and decay PS starts

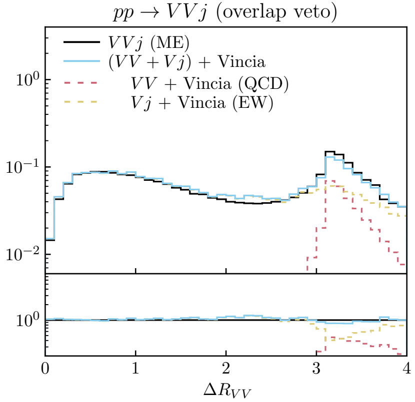


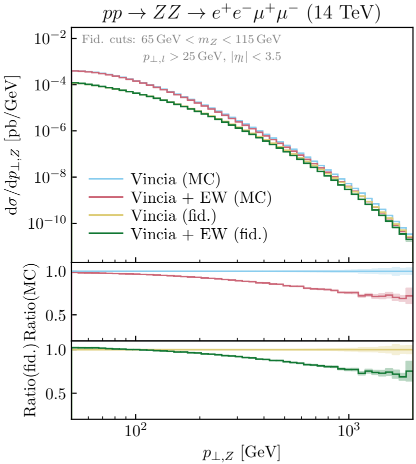


- Novel phase-space mappings, antenna functions & sampling algorithms
- Matching algorithm to reproduce correct Breit-Wigner shapes

Electroweak antenna showers

[Brooks,Skands,Verheyen] arXiv:2108.10786





- Spin-dependent EW shower, extending QED [Skands,Verheyen] 2002.04939
- Newly computed splitting functions, matched to HELAS
- Consistently combined with QCD shower (overlap veto)

Fixed-order matching for heavy quarks

Recent developments

Matching – Processes with intermediate resonances

[Ježo,Nason] arXiv:1509.09071

- NLO subtraction methods do not preserve virtuality of possible resonances
  IR cancellation takes place highly non-locally $\rightarrow$ efficiency problem
- Problem worsens in POWHEG, as uncontrollable ratios are exponentiated:

$$\Delta(\Phi_B, p_T) = \exp\left\{ -\sum_\alpha \int \mathrm{d}\Phi_1 \frac{\mathrm{R}(\Phi_R^{(\alpha)})}{\mathrm{B}(\Phi_B)}\, \Theta(p_T - k_T) \right\}$$

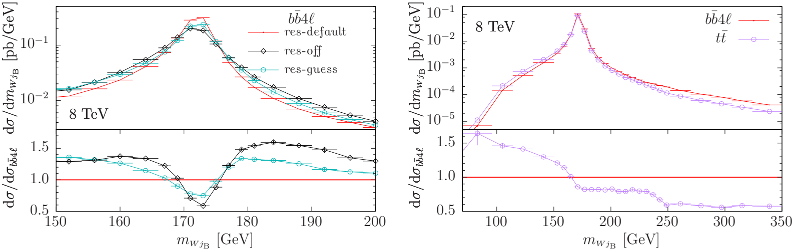
- Proposed solution:
  - Partition phase space such that each region corresponds to a unique resonance history
  - Within each region modify subtraction mappings such that resonance mass is preserved
- Assignment of resonance histories requires algorithm
  $\rightarrow$ Use kinematic proximity to resonance

$$\Pi_{f_b} = \frac{P_{f_b}}{\sum_{f_b' \in \text{res hists}} P_{f_b'}}\,, \quad P_{f_b} = \prod_{i \in \text{ress}} \frac{M_i^4}{(s_i - M_i^2)^2 + \Gamma_i^2 M_i^2}$$

Matching – Wt vs $t\bar{t}$

[Ježo,Lindert,Oleari,Nason,Pozzorini] arXiv:1607.04538

- Wt production in the 5F scheme:
  - NLO corrections swamped by LO $t\bar{t}$ decay
  - Requires ad-hoc subtraction prescription (DR/DS)
- Wt production in the 4F scheme:
  - Unified treatment of Wt and $t\bar{t}$ (identical at LO)
  - Requires off-shell $WWb\bar{b}$ calculation
- Sizable differences compared to resonance-unaware matching and to narrow-width approach [Frixione,Nason,Ridolfi] arXiv:0707.3088



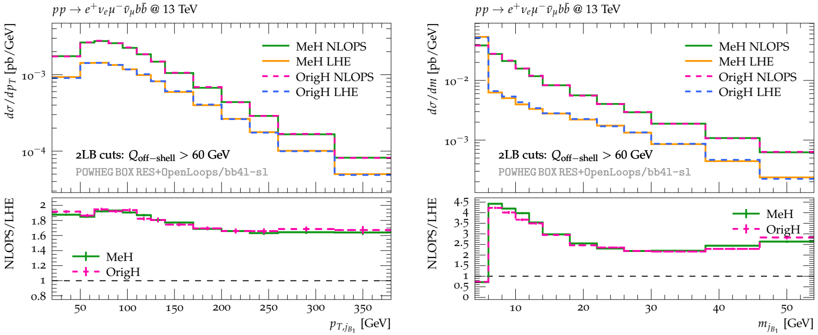
Matching – Wt vs $t\bar{t}$

[Ježo,Lindert,Pozzorini] arXiv:2307.15653

- Various formally sub-leading effects important for physics modeling
  - Treatment of top-quark width at NLO

$$\int_{\mathrm{dec}} \mathrm{d}\sigma^{\mathrm{NLO_{exp}}}_{\mathrm{prod}\times\mathrm{dec}} = \mathrm{d}\sigma_0 \left[\left(1 - \frac{\Gamma_1}{\Gamma_0}\right)\int_{\mathrm{dec}} \frac{\mathrm{d}\Gamma_0}{\Gamma_0} + \int_{\mathrm{dec}} \frac{\mathrm{d}\Gamma_1}{\Gamma_0}\right] + \mathrm{d}\sigma_1 \int_{\mathrm{dec}} \frac{\mathrm{d}\Gamma_0}{\Gamma_0}$$

  - Parton-shower history assignment based on squared partial amplitudes



Matching – $t\bar{t}$ at NNLO

[Mazzitelli,Monni,Nason,Re,Wiesemann] arXiv:2112.12135

- First application of MiNNLO$_{\text{PS}}$ to process with nontrivial color structure
- Based on transverse momentum resummation for a $Q\bar{Q}$ pair

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\vec{p}_T\,\mathrm{d}\Phi_{Q\bar{Q}}} = \sum_{c=q,\bar{q},g} \frac{|M^{(0)}_{c\bar{c}}|^2}{2m^2_{Q\bar{Q}}} \int \frac{\mathrm{d}^2\vec{b}}{(2\pi)^2} e^{i\vec{b}\vec{p}_T} e^{-S_{c\bar{c}}\left(\frac{b_0}{b}\right)} \sum_{i,j} (\mathbf{H}_{c\bar{c}}\mathbf{\Delta})\,(C_{ci}\otimes f_i)\,(C_{\bar{c}j}\otimes f_j)$$

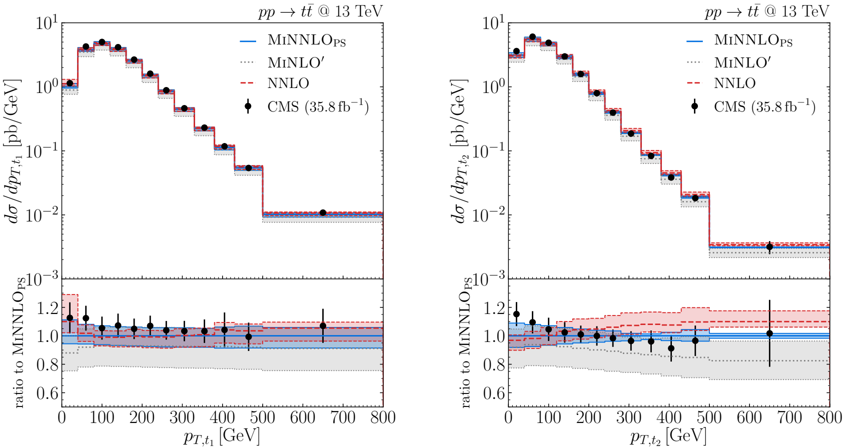
  - Hard function $\mathbf{H}_{c\bar{c}} = |M_{c\bar{c}}\rangle\langle M_{c\bar{c}}|/|M^{(0)}_{c\bar{c}}|^2$
  - Insertion operator given by $\mathbf{\Delta} = \mathbf{V}^\dagger \mathbf{D}\mathbf{V}$
  - Angular correlations encoded in $\mathbf{D} \equiv \mathbf{D}(\Phi_{Q\bar{Q}}, \vec{b}; \alpha_s(b_0/b))$
  - Virtual corrections from exponentiated soft anomalous dimension

$$\mathbf{V} = \mathcal{P}\exp\left\{-\int_{b_0^2/b^2}^{m^2_{Q\bar{Q}}} \frac{\mathrm{d}q^2}{q^2}\mathbf{\Gamma}_t(\Phi_{Q\bar{Q}}; \alpha_s(q))\right\}$$

- Simulation of top-quark decays at LO QCD
  including off-shell effects & spin correlations
  Justified by good physics modeling in fixed-order studies
  [Czakon,Mitov,Poncelet] 2008.11133

Matching – $t\bar{t}$ at NNLO

[Mazzitelli,Monni,Nason,Re,Wiesemann] arXiv:2112.12135



Outlook

Directions of future development

- Parton showers
  - Improved logarithmic precision
  - Higher-order splitting kernels
  - Interplay with analytic resummation
- Matching and merging
  - The role of unitarity constraints
  - Interplay with analytic resummation
  - Fully differential higher-order matching

Apologies for only selecting a small subset of topics
For a comprehensive overview: [Campbell et al.] arXiv:2203.11110