## A study of entanglement in $e^{+} e^{-} \rightarrow B \bar{B}$ events at Belle

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Top Quark Physics at the Precision Frontier, Purdue, 2nd October 2023


## Outline

(1) Einstein, Podolsky, and Rosen, via Bohm
(2) Belle and the flavor singlet state
(3) $B \bar{B}$, measurement, and conspiracy
(4) QM versus specific local realistic models
(5) Adapting an existing analysis measuring $\Delta m_{d}$
(6) Summary and reflections

## Einstein, Podolsky, and Rosen, via Bohm

spin-singlet state of photons or particles: $\frac{1}{\sqrt{2}}\left[|\Uparrow\rangle_{1}|\Downarrow\rangle_{2}-|\Downarrow\rangle_{1}|\Uparrow\rangle_{2}\right]$


- measurements on $1(2)$ indeterminate, but $\Longrightarrow$ full knowledge of 2 (1)
- Bell's Theorem (via Clauser, Horne, Shimony, and Holt):
- correlation coeff: $E(\vec{a}, \vec{b})=\frac{R_{++}(\vec{a}, \vec{b})+R_{--}(\vec{a}, \vec{b})-R_{+-}(\vec{a}, \vec{b})-R_{-+}(\vec{a}, \vec{b})}{R_{++}(\vec{a}, \vec{b})+R_{--}(\vec{a}, \vec{b})-R_{+-}(\vec{a}, \vec{b})+R_{-+}(\vec{a}, \vec{b})}$
- $S=E(\vec{a}, \vec{b})-E\left(\vec{a}, \overrightarrow{b^{\prime}}\right)+E\left(\vec{a}^{\prime}, \vec{b}\right)+E\left(\vec{a}^{\prime}, \vec{b}^{\prime}\right)$
- $|S| \leq 2$ for any local realistic model; $S_{Q M}= \pm 2 \sqrt{2}$ for optimal settings
- QM-like results rule out LR, even if we eventually "get behind" QM


## Einstein, Podolsky, and Rosen, via Bohm: Aspect

 Aspect et al., Phys. Rev. Lett. 92, 91 (1982)
## source: 2-photon cascade decay

 $\nu_{1}, \nu_{2}$ polarizations are correlated

FIG. 2. Experimental setup. Two polarimeters I and II, in orientations $\vec{a}$ and $\vec{b}$, perform true dichotomic measurements of linear polarization on photons $\nu_{1}$ and $\nu_{2}$. Each polarimeter is rotatable around the axis of the incident beam. The counting electronics monitors the singles and the coincidences.
[two-channel polarimeters used]

## correlation coeffs in data vs QM

 optimum relative angles $22.5^{\circ}$ and $67.5^{\circ}$

FIG. 3. Correlation of polarizations as a function of the relative angle of the polarimeters. The indicated errors are $\pm 2$ standard deviations. The dotted curve is not a fit to the data, but quantum mechanical predictions for the actual experiment. For ideal polarizers, the curve would reach the values $\pm 1$.

$$
S=2.697 \pm 0.015 ; c f . S_{Q M}=2.70 \pm 0.05
$$

## Einstein, Podolsky, and Rosen, via Bohm: Franson

## J.D. Franson, Phys. Rev. Lett. 62, 2205-2208 (1989)

some more recent experiments are based on a different design with alternative paths setting up a position-time correlation:

here the polarizer orientations are fixed, and variable phase delays $\Phi_{1,2}$ (Pockels cells or similar) are introduced

## the KEKB/Belle facility



superconducting solenoid (1.5 T) [tracking; calorimetry; $K / \pi, e^{-}, \mu \mathrm{ID}$ ] 772 million $B \bar{B}$ pairs on tape

## CP: violated in the neutral $B$-meson system?

instead of $K^{0} \equiv \bar{s} d$, try $B^{0} \equiv \bar{b} d$, using $b \bar{b}$ resonance as a source $\ldots$


## $e^{+} e^{-} \rightarrow \Upsilon(4 S) \rightarrow$ [flavor singlet state of] $B^{0} B^{0}$

the $B$-pair has the same property, substituting flavor for spin/polarization:

- the $\Upsilon(4 S)$ is C-odd

$\mathrm{Y}(4 \mathrm{~S})$ resonance


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|\Psi(t)\rangle=\frac{e^{-t / \tau} B^{0}}{\sqrt{2}}\left[\left|B^{0}(\vec{p}) \bar{B}^{0}(-\vec{p})\right\rangle-\left|\bar{B}^{0}(\vec{p}) B^{0}(-\vec{p})\right\rangle\right]
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$\begin{cases}B_{T A G}^{0} & \text { definite flavor state } \\ B_{C P}^{0} & \text { definite CP state }\end{cases}$


$$
\Gamma_{C P}(\Delta t)=\frac{e^{-|\Delta t| / \tau_{B^{0}}}}{4 \tau_{B^{0}}}\left[1 \pm\left\{S_{C P} \sin (\Delta m \Delta t)+A_{C P} \cos (\Delta m \Delta t)\right\}\right]
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- and another rate for $\bar{B}_{\text {TAG }}^{0}$ : CPV


$$
\Gamma_{C P}(\Delta t)=\frac{e^{-|\Delta t| / \tau_{B^{0}}}}{4 \tau_{B^{0}}}\left[1-\left\{S_{C P} \sin (\Delta m \Delta t)+A_{C P} \cos (\Delta m \Delta t)\right\}\right]
$$

## Measuring a time-dependent CPV asymmetry

$$
\Delta t \sim 10^{-12} \mathrm{~s} \text { unmeasurable, so use } \Delta z \tilde{\propto} \Delta t
$$



## Measuring a time-dependent CPV asymmetry

asymmetric c.m. system $\longrightarrow$ an asymmetric detector


## What one of those events actually "looks" like ...

vertexing: SVD
silicon vertex detector

tracking: CDC
central drift chamber


## CP: violated in the neutral $B$-meson system!!

Belle: I. Adachi et al., Phys. Rev. Lett. 108, 171802 (2012)

- measured by 2000, observed by 2001; 2012 final measurement shown $\longrightarrow$
- clear offset of $\bar{B}^{0}$ and $B^{0}$ tags
- decent fit to the expected sinusoidal modulation in $\Delta t$ in the rate asymmetry
- opposite shift (with poorer precision) seen for $B^{0} \rightarrow J / \psi K_{L}^{0}$
- opposite $\eta_{C P}$ to other modes
- $\Delta t$ measurement is $\approx$ the same
- (for validation, not extra precision)
- huge project at Belle \& BaBar:

- confirm expected SM results
- find deviations - NP signals [cf. top quark first "seen" in loops]


## $K^{0} K^{0} \& B^{0} \bar{B}^{0}$ systems: what can be measured

there is a beautiful optical analogy called quasi-spin due to Lee and Wu (1966) and Lipkin (1968):

| $K$ meson | spin- $\frac{1}{2}$ | photon |
| :---: | :---: | :--- |
| $\left\|K^{0}\right\rangle$ | $\|\Uparrow\rangle_{z}$ | $\|V\rangle$ |
| $\left\|\bar{K}^{0}\right\rangle$ | $\|\Downarrow\rangle_{z}$ | $\|H\rangle$ |
| $\left\|K_{S}^{0}\right\rangle$ | $\|\Rightarrow\rangle_{z}$ | $\|L\rangle=\frac{1}{\sqrt{2}}(\|V\rangle-i\|H\rangle)$ |
| $\left\|K_{L}^{0}\right\rangle$ | $\|\Leftarrow\rangle_{z}$ | $\|R\rangle=\frac{1}{\sqrt{2}}(\|V\rangle+i\|H\rangle)$ |

- we are limited in the "polarization axes" we can choose:
- can't measure along arbitrary $\alpha\left|K^{0}\right\rangle+\beta\left|\bar{K}^{0}\right\rangle=\alpha|\Uparrow\rangle+\beta|\Downarrow\rangle$
- even more restricted for $B$-mesons: only $B^{0}, \bar{B}^{0}$ are practical
- but $\left|B^{0}\right\rangle \xrightarrow{t} \frac{1}{2}\left[\left\{1+\cos \left(\Delta m_{d} t\right)\right\}\left|B^{0}\right\rangle+\left\{1-\cos \left(\Delta m_{d} t\right)\right\}\left|\bar{B}^{0}\right\rangle\right]$, so time difference $\Delta m_{d} \Delta t$ plays the role of phase difference $\Delta \phi$


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- however, there is a catch ...


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## Bramon/Escribano/Garbarino, J. Mod. Opt. 52, 1681 (2005) via Chris Carter

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- if $\left(t_{1}, f_{1}, t_{2}, f_{2}\right)$ are chosen randomly according to QM ... the phenomena look like QM!
- because $\Delta m_{d} \Delta t$ plays the role of phase difference $\Delta \phi$, and
 the decays set $\Delta t$, we cannot choose $\Delta \phi$ to defeat the conspiracy
Bruce Yabsley (Sydney) $\quad$ Entanglement in $e^{+} e^{-} \rightarrow B \bar{B}$ at Belle $\quad$ Top Precision 2023-10-02 $\quad 15 / 31$


## The Green Baize Table Conspiracy Model (2)

 G. Weihs et al., Phys. Rev. Lett. 81, 5039-5043 (1998): "Aspect++"changing $\Delta \phi$ in flight...

... based on random numbers


Here $\Delta \phi$ is actively chosen: not subject to the same sorts of conspiracy.

## Beyond The Green Baize Table Conspiracy

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| $D^{0} / \bar{D}^{0}$ | $<0.03$ |
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$c f$. Aspect: free to choose $\Delta \phi$


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- use final states $\left(f_{1}, f_{2}\right)$ to determine flavours at $\left(t_{1}, t_{2}\right)$
- check if this is consistent with a given model


## QM versus specific local realistic models

The QM "model" has distinctive predictions for how B-meson flavours change:

- after $\Upsilon(4 S)$ decay, the two $B$-mesons operate as a unit
- when $B_{1}$ decays ( $50 / 50 \% B^{0} / \bar{B}^{0}$ ), $B_{2}$ is in the opposite flavour state; as (proper) time passes, it oscillates opposite (OF) $\longleftrightarrow$ same flavour (SF)
- find asymmetry in pair decays:

$$
\begin{aligned}
A\left(t_{1}, t_{2}\right) & =\frac{R_{O F}-R_{S F}}{R_{0 F}+R_{S F}} \\
& =\cos \left(\Delta m_{d}\left(t_{2}-t_{1}\right)\right)
\end{aligned}
$$

- depends only on $\Delta t \longrightarrow$ (this is an entanglement thing)
- cf. a ( $t_{1}, t_{2}$ ) plot would look complicated

- easy (in principle) to distinguish QM and other models: apart from $\Delta t$, any dependence on individual $t_{i}$ is non-QM


## QM versus specific local realistic models

LR model \#1: spontaneous disentanglement

- after $\Upsilon(4 S)$ decay, $B^{\prime}$ s immediately separate into $B^{0}$ and $\bar{B}^{0}$
- $A_{S D}=\cos \left(\Delta m_{d} t_{1}\right) \cos \left(\Delta m_{d} t_{2}\right)$
$=\frac{1}{2}\left[\cos \left(\Delta m_{d} \Sigma t\right)+\cos \left(\Delta m_{d} \Delta t\right)\right]$
- start with well-defined flavour
- oscillate independently
- $A_{S D}$ depends on both $t_{1}, t_{2}$
- the variables shown are prejudicial: $\left(t_{1}, t_{2}\right)$ would have looked simpler
- $\left(\Delta t, \Sigma t=\left[t_{1}+t_{2}\right]\right)$ likewise
- $\left(\Delta t, t_{\text {min }}\right)$ chosen to compare with QM and $\ldots$



## QM versus specific local realistic models

LR model \#2: phenomenological model-family of Pompili \& Selleri

- QM-like states, including $\Delta m$
- individual meson masses are stable
- flavours of the pair are correlated: subject to instantaneous jumps
- require that QM predictions for single $B$-mesons are preserved
- asymmetry for any such model must fall within a range:
- $A_{P S}^{\min }=1-\min (2+\Psi, 2-\Psi)$,

$$
\begin{aligned}
\Psi=\{1 & \left.+\cos \left(\Delta m_{d} \Delta t\right)\right\} \cos \left(\Delta m_{d} t_{\text {min }}\right) \\
& -\sin \left(\Delta m_{d} \Delta t\right) \sin \left(\Delta m_{d} t_{\text {min }}\right)
\end{aligned}
$$



- $A_{P S}^{\text {max }}=1-\mid\left\{1-\cos \left(\Delta m_{d} \Delta t\right)\right\} \cos \left(\Delta m_{d} t_{\text {min }}\right)$

$$
+\sin \left(\Delta m_{d} \Delta t\right) \sin \left(\Delta m_{d} t_{\text {min }}\right)
$$

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$$
+\sin \left(\Delta m_{d} \Delta t\right) \sin \left(\Delta m_{d} t_{\min }\right)
$$

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## $A(\Delta t)$ for QM, SD, and Pompili-Selleri

- at Belle we cannot measure individual decay times [knowledge of the interaction point is poor compared to needed resolution]
- measuring $\Delta t$ is fine for $A_{Q M}(\Delta t) \equiv \frac{R_{O F}-R_{S F}}{R_{O F}+R_{S F}}=\cos \left(\Delta m_{d} \Delta t\right)$
- we must integrate over remaining variable for SD, PS:

SD: $\int_{\Delta t}^{\infty} \mathrm{d}(\Sigma t) R_{O F, S F}(\Sigma t, \Delta t) \longrightarrow$
PS: $\int_{0}^{\infty} \mathrm{d} t_{\text {min }} R_{O F, S F}\left(t_{\min }, \Delta t\right) \longrightarrow$

- these resemble the $\Delta t$ evolution for QM, but differ in the detail: resolve the difference!!
- avoid assuming quantum
 mechanics along the way (which can be difficult)
- $N . B$. event rate at $\Delta t=10 \mathrm{ps}$ is $\sim \frac{1}{700} \times(\Delta t=0)$


## Adapt an existing analysis measuring $\Delta m_{d}$ (1)

Belle: K. Abe et al., Phys. Rev. D 71, 072003 \& 079903 (2005)

Belle's most current $\sin 2 \phi_{1},|\lambda|, \tau_{B}, \Delta m_{d}$ measurement at the time:

- $152 \times 10^{6} B \bar{B}$ pairs
- $5 \times$ the discovery dataset
- $\frac{1}{5} \times$ the eventual dataset
- 5417 CP- and 177368 flavoureigenstate $B$-decay candidates
- sample purities vary 63-98\% depending on the decay mode
- multivariate flavour-tagging of the other $B$ decay; $\epsilon_{\text {eff }}=28.7 \%$
- $\Delta m_{d}=(0.511 \pm 0.005 \pm 0.006) \mathrm{ps}^{-1}$

$$
\text { cf. }(0.5065 \pm 0.0019) \mathrm{ps}^{-1} \text { PDG23 }
$$



We then adapted this in various ways ...

## Adapt an existing analysis measuring $\Delta m_{d}$ (2)

## Belle: A. Go, A. Bay et al., Phys. Rev. Lett. 99, 131802 (2007)

- restrict $177368 \rightarrow 84823$ flavour eigenstates, choosing only $B^{0} \rightarrow D^{*-} \ell^{+} \nu$ where the lepton explicitly determines the $B$-flavour

- restrict $84823 \rightarrow 8565$ by choosing only the best flavour tags of the other $B$ : highest of 7 purity categories; leptons only


## Then: background subtraction (1) fake $D^{*}$

## Belle: A. Go, A. Bay et al., Phys. Rev. Lett. 99, 131802 (2007)

- signal relies on $D^{*-} \rightarrow \bar{D}^{0} \pi^{-}$tag: energy release $Q \ll m_{\pi} \ll m_{D}$
- estimate background under peak using sideband region:


- affects samples differently: we subtract $\left\{\begin{aligned} 126 \pm 6 & \text { OF events } \\ 54 \pm 4 & \text { SF events }\end{aligned}\right.$


## Then: background subtraction (2) bad $D^{*}-\ell$

## Belle: A. Go, A. Bay et al., Phys. Rev. Lett. 99, 131802 (2007)

- true $D^{*}$ mesons; mostly, true leptons, $D^{*}, \ell$ produced by different $B$-decays
- estimated from data using a reversed momentum trick;
Monte-Carlo validated
(a,b) here, unlike the last case, more SF bkgd
(c) $A(\Delta t)$ before \& after correction
(d) residuals: note $\Delta t$ structure






## Then: background subtraction (3) $B^{+} \rightarrow \bar{D}^{* * 0} \ell \nu$

 Belle: A. Go, A. Bay et al., Phys. Rev. Lett. 99, 131802 (2007)- the remaining background is from related decays of charged $B$
- we rely on the different distributions for $D^{* *}$ decays and $D^{*}$ decays
- fit data to get fractions
- rely on MC for details:
- 254 OF vs. 1.5 SF events
- structured in $\Delta t$
- generous systematics
- ( $1.5 \pm 0.1$ ) \% mistag rate of other $B$ corrected using OF and SF distributions; 0.5\% systematic assigned
- effect of background subtraction and mistag correction: $\longrightarrow^{\cos \left(\theta_{\mathrm{am}, \mathrm{O}}\right)}$


## Then: background subtraction (3) $B^{+} \rightarrow \bar{D}^{* * 0} \ell \nu$

## Belle: A. Go, A. Bay et al., Phys. Rev. Lett. 99, 131802 (2007)



## Further adaptation: deconvolution and bias removal

 Belle: A. Go, A. Bay et al., Phys. Rev. Lett. 99, 131802 (2007)- remaining effects are vertex resolution, efficiency losses ... these blur out the distribution in $\Delta t$
- use a deconvolution procedure (DSVD) to remove them:
- due to falling rate with $\Delta t$, events assigned to 11 variable-width bins
- build $11 \times 11$ response matrices in $\Delta t$ using MC
- optimise using toy MC study
- regularisation (rank $11 \longrightarrow 5,6$ )
- MC events themselves produce a bias:
e.g. SM has no SF events at $\Delta t=0$
- replace SF sample with $\mathrm{SF}+0.2 \times \mathrm{OF}$
- replace OF sample with $\mathrm{OF}+0.2 \times \mathrm{SF}$
- measure remaining bias for 3 models: average it \& subtract
- any bias still remaining $\longrightarrow$ systematic error
- check resulting OF \& SF distributions by adding them ...


## Further adaptation: deconvolution and bias removal

## Belle: A. Go, A. Bay et al., Phys. Rev. Lett. 99, 131802 (2007)

$\ldots$ and fitting for the $B^{0}$ lifetime:

finds lifetime $\tau_{B}^{0}=(1.532 \pm 0.017) \mathrm{ps}$, with $\chi^{2} / n_{\text {dof }}=3 / 11$ $c f$. world average $(1.530 \pm 0.009)$ ps from PDG2006

## fitting to the QM, PS, and SD models (1)

world average $\Delta m_{d}$ is dominated by measurements that assume QM!!

- $\left\langle\Delta m_{d}\right\rangle=(0.507 \pm 0.005) \mathrm{ps}^{-1}$
- so we remove Belle
- ... and remove BaBar
- the resulting $\left\langle\Delta m_{d}\right\rangle_{\mathrm{NO}-\mathrm{QM}}$

$$
=(0.496 \pm 0.014) \mathrm{ps}^{-1}
$$

- we add this to the fit as a new datapoint-with-uncertainty
- "Gaussian constraint", in current jargon
- the $\Delta m_{d}$ parameter is then floated in the fits:
 each model chooses its value


## fitting to the QM, PS, and SD models (2)

Belle: A. Go, A. Bay et al., Phys. Rev. Lett. 99, 131802 (2007)
fit: float $\Delta m_{d}$ subject to WA-sans-(Belle+BaBar): $(0.496 \pm 0.014) \mathrm{ps}^{-1}$


QM fits well $\chi^{2} / n_{\text {dof }}=5 / 11$



SD disfavoured: $13 \sigma$
$\chi^{2} / n_{\text {dof }}=174 / 11$

PS disfavoured: $5.1 \sigma$ $\chi^{2} / n_{\text {dof }}=31 / 11$

- "SD fraction": $\left(1-\zeta_{B^{0} \bar{B}^{0}}\right) A_{Q M}+\zeta_{B^{0} \bar{B}^{0}} A_{S D}, \zeta_{B^{0} \bar{B}^{0}}=0.029 \pm 0.057$
- Pompili-Selleri class: QM-like states, stable mass, flavor correlations; QM predictions for single $B$-mesons preserved


## Summary and reflections

- entanglement at $\Upsilon(4 S)$, used many times/second, was tested at Belle
- test of specific models, not a Bell Inequality test ...
- "decoherent fraction" $\zeta_{B^{0} \bar{B}^{0}}=0.029 \pm 0.057$ [modified interf. term]
- Pompili-Selleri class of LR models is ruled out at $5.1 \sigma$
- existing time-dependent $B \bar{B}$ analysis methods were adapted
- this made the measurement feasible
- the adaptation itself was a lot of work
- care was needed to avoid surreptitiously assuming QM at various points
- we benefited enormously from an existing QM foundations study
- excluding decoherence would have been familiar, but uninteresting
- the Pompili-Selleri family was specific to $\Upsilon(4 S) \rightarrow B \bar{B}$, and was something worth ruling out
- importance depends on point of view ...
- future developments?


## BACKUP SLIDES

## The final data for posterity

## Belle: A. Go, A. Bay et al., Phys. Rev. Lett. 99, 131802 (2007)

Systematic errors

| window $[\mathrm{ps}]$ | $A$ and total error | stat. err. | total | event sel. | bkgd sub. | mistags | deconv. |
| ---: | ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| $0.0-0.5$ | $1.013 \pm 0.028$ | 0.020 | 0.019 | 0.005 | 0.006 | 0.010 | 0.014 |
| $0.5-1.0$ | $0.916 \pm 0.022$ | 0.015 | 0.016 | 0.006 | 0.007 | 0.010 | 0.009 |
| $1.0-2.0$ | $0.699 \pm 0.038$ | 0.029 | 0.024 | 0.013 | 0.005 | 0.009 | 0.017 |
| $2.0-3.0$ | $0.339 \pm 0.056$ | 0.047 | 0.031 | 0.008 | 0.005 | 0.007 | 0.029 |
| $3.0-4.0$ | $-0.136 \pm 0.075$ | 0.060 | 0.045 | 0.009 | 0.009 | 0.007 | 0.042 |
| $4.0-5.0$ | $-0.634 \pm 0.084$ | 0.062 | 0.057 | 0.021 | 0.014 | 0.013 | 0.049 |
| $5.0-6.0$ | $-0.961 \pm 0.077$ | 0.060 | 0.048 | 0.020 | 0.017 | 0.012 | 0.038 |
| $6.0-7.0$ | $-0.974 \pm 0.080$ | 0.060 | 0.053 | 0.034 | 0.025 | 0.020 | 0.025 |
| $7.0-9.0$ | $-0.675 \pm 0.109$ | 0.092 | 0.058 | 0.041 | 0.027 | 0.022 | 0.022 |
| $9.0-13.0$ | $0.089 \pm 0.193$ | 0.161 | 0.107 | 0.067 | 0.063 | 0.038 | 0.039 |
| $13.0-20.0$ | $0.243 \pm 0.435$ | 0.240 | 0.363 | 0.145 | 0.226 | 0.080 | 0.231 |

