Theoretical predictions for top-pair and $t + X$ production

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- **Higher-order soft-gluon corrections**
- $t\bar{t}$ **and** $t\bar{t}\gamma$ **production**
- tW **production**
- $tq\gamma$ **and** tqZ **production**
- tqH **production**

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TPF23 Workshop

Soft-gluon corrections

They are important for top-quark processes and they approximate known exact results at NLO and NNLO very well

partonic processes $a(p_a) + b(p_b) \to t(p_t) + X$

define $s = (p_a + p_b)^2$, $t = (p_a - p_t)^2$, $u = (p_b - p_t)^2$

For a $2 \to n$ **process with** $p_a + p_b \to p_t + p_2 + \cdots + p_n$
we define the threshold variable $s_4 = s + t + u - m_t^2 - (p_2 + \cdots + p_n)^2$

Also $s_4 = (p_2 + \cdots + p_n + p_g)^2 - (p_2 + \cdots + p_n)^2$ **where extra gluon with** p_g **emitted**

At partonic threshold $p_g \to 0$ **and thus** $s_4 \to 0$

Soft corrections $\left[\frac{\ln^k(s_4/m_t^2)}{s_4}\right]_+$ **with** $k \le 2n-1$ **for the order** α_s^n **corrections**

Resum these soft corrections for the double-differential cross section

Finite-order expansions $\to$ **no prescription needed or used**
(this avoids underestimating the size of the corrections)

Approximate NNLO (aNNLO) and/or approximate N^{3}LO (aN^{3}LO) predictions

for cross sections and differential distributions

Soft-gluon Resummation

$$d\sigma_{pp\to tX} = \sum_{a,b} \int dx_a \, dx_b \, \phi_{a/p}(x_a,\mu_F) \, \phi_{b/p}(x_b,\mu_F) \, d\hat{\sigma}_{ab\to tX}(s_4,\mu_F)$$

take Laplace transforms $d\hat{\sigma}_{ab\to tX}(N) = \int (ds_4/s) \, e^{-Ns_4/s} d\hat{\sigma}_{ab\to tX}(s_4)$

and $\tilde{\phi}(N) = \int_0^1 e^{-N(1-x)} \, \phi(x) \, dx$ **with transform variable** N

Then

$$d\tilde{\sigma}_{ab\to tX}(N) = \tilde{\phi}_{a/a}(N_a,\mu_F) \, \tilde{\phi}_{b/b}(N_b,\mu_F) \, d\tilde{\hat{\sigma}}_{ab\to tX}(N,\mu_F)$$

Refactorization for the cross section

$$d\sigma_{ab\to tX}(N) = \tilde{\psi}_a(N_a,\mu_F) \, \tilde{\psi}_b(N_b,\mu_F) \, \tilde{J}_q(N,\mu_F) \, \mathrm{tr} \left\{ H_{ab\to tX}\left(\alpha_s(\mu_R)\right) \tilde{S}_{ab\to tX}\left(\frac{\sqrt{s}}{N\mu_F}\right) \right\}$$

ψ_a, ψ_b $\to$ **collinear emission from incoming partons**

J_q $\to$ **collinear emission from final-state massless quarks (if any)**

$H_{ab\to tX}$ **is hard function** $\to$ **short distance**

$S_{ab\to tX}$ **is soft function** $\to$ **noncollinear soft gluons**

Thus

$$d\tilde{\hat{\sigma}}_{ab\to tX}(N) = \frac{\tilde{\psi}_{a/a}(N_a,\mu_F) \, \tilde{\psi}_{b/b}(N_b,\mu_F)}{\tilde{\phi}_{a/a}(N_a,\mu_F) \, \tilde{\phi}_{b/b}(N_b,\mu_F)} \, \tilde{J}_q(N,\mu_F) \; \mathrm{tr} \left\{ H_{ab\to tX}\left(\alpha_s(\mu_R)\right) \tilde{S}_{ab\to tX}\left(\frac{\sqrt{s}}{N\mu_F}\right) \right\}$$

$S_{ab\to tX}$ **satisfies the renormalization group equation**

$$\left(\mu_R \frac{\partial}{\partial \mu_R} + \beta(g_s)\frac{\partial}{\partial g_s}\right) S_{ab\to tX} = -\Gamma^\dagger_{S\,ab\to tX}\, S_{ab\to tX} - S_{ab\to tX}\, \Gamma_{S\,ab\to tX}$$

Soft anomalous dimension $\Gamma_{S\,ab\to tX}$ **controls the evolution of the soft function which gives the exponentiation of logarithms of** N

Renormalization group evolution → resummation

$$d\tilde{\hat{\sigma}}^{\rm resum}_{ab\to tX}(N) = \exp\left[\sum_{i=a,b} E_i(N_i)\right] \exp\left[\sum_{i=a,b} 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\gamma_{i/i}(N_i)\right] \exp\left[E'_q(N)\right]$$

$$\times {\rm tr}\left\{H_{ab\to tX}\left(\alpha_s(\sqrt{s})\right) \bar{P}\exp\left[\int_{\sqrt{s}}^{\sqrt{s}/N} \frac{d\mu}{\mu}\,\Gamma^\dagger_{S\,ab\to tX}(\alpha_s(\mu))\right] \tilde{S}_{ab\to tX}\left(\alpha_s\left(\frac{\sqrt{s}}{N}\right)\right) P\exp\left[\int_{\sqrt{s}}^{\sqrt{s}/N} \frac{d\mu}{\mu}\,\Gamma_{S\,ab\to tX}(\alpha_s(\mu))\right]\right\}$$

The soft anomalous dimensions Γ_S **and the hard and soft functions are in general matrices in the space of color exchanges in the hard scattering**

Top processes studied - total and differential cross sections

Top pair

$t\bar{t}$ **aN^{3}LO (total; top p_T, y, and double-differential; also A_{FB})**
$t\bar{t}$ **aN^{3}LO + EW (total; top p_T, y)**
$t\bar{t}$ **SMEFT aNNLO (total; top p_T)**

Top-pair+X

$t\bar{t}\gamma$ **aNNLO + EW (total; top p_T, y)**
$t\bar{t}W$ **aNNLO + EW (total; top p_T, y)**

Single top

t**- and** s**-channel aNNLO (total; top p_T) and aN^{3}LO (total)**
tW **aN^{3}LO (total; p_T, y for top and W)**

Single-top+X

tqH **aNNLO (total; top p_T, y)**
$tq\gamma$ **aNNLO (total; top p_T, y)**
tqZ **aNNLO (total; top y)**

Single-top BSM

$t\gamma$, tZ, tZ' **aNNLO (total; top p_T, y)**
tg **aNNLO (total)**
tH^- **aNNLO (total; top p_T, y) and aN^{3}LO (total)**

$t\bar{t}$ production

Soft anomalous dimension matrix is 2×2 for $q\bar{q} \to t\bar{t}$ channel and 3×3 for $gg \to t\bar{t}$ channel

I calculated them at one loop in the mid-90's and at two loops over a dozen years ago

more recent partial results at three loops

Four-loop massive cusp anomalous dimension from asymptotics [NK, PRD 107, 054006 (2023)] contributes to 4-loop result

NLO expansions agree with exact NLO results very well

NNLO expansions (aNNLO) predicted the exact NNLO results to high accuracy (percent or per mille) for total cross sections and top-quark p_T and rapidity distributions

aN^{3}LO is the state of the art

electroweak corrections also included

$t\bar{t}$ production at aN^{3}LO QCD + NLO EW at LHC energies

(with Marco Guzzi and Alberto Tonero [PRD 108, 054012 (2023)])

$t\bar{t}$ total cross sections at LHC energies with MSHT20 aN^{3}LO pdf						
σ in pb	5.02 TeV	7 TeV	8 TeV	13 TeV	13.6 TeV	14 TeV
LO QCD	$40.0^{+14.9+1.1}_{-10.1-1.2}$	103^{+35+3}_{-24-3}	146^{+48+3}_{-34-4}	$469^{+133+9}_{-97\ -10}$	$518^{+145+10}_{-106-11}$	$553^{+153+11}_{-113-11}$
NLO QCD	$58.1^{+6.8+1.8}_{-7.8-2.0}$	151^{+17+4}_{-20-5}	215^{+25+5}_{-27-6}	700^{+80+15}_{-80-15}	775^{+89+16}_{-88-16}	828^{+94+16}_{-94-18}
NLO QCD+EW	$58.1^{+6.6+1.8}_{-7.8-2.0}$	150^{+17+4}_{-19-4}	214^{+25+6}_{-26-6}	698^{+78+14}_{-80-16}	772^{+88+16}_{-87-16}	825^{+92+16}_{-93-18}
NNLO QCD	$65.3^{+2.8+2.0}_{-4.4-2.2}$	$169^{+7\ +5}_{-11-5}$	$240^{+9\ +6}_{-15-7}$	781^{+27+16}_{-43-17}	864^{+30+18}_{-47-19}	922^{+32+18}_{-49-20}
NNLO QCD+EW	$65.3^{+2.8+2.0}_{-4.4-2.2}$	$168^{+7\ +5}_{-11-5}$	$239^{+9\ +6}_{-15-7}$	779^{+27+16}_{-43-17}	861^{+30+18}_{-47-19}	919^{+32+18}_{-49-20}
aN^{3}LO QCD	$68.2^{+2.1+2.1}_{-3.2-2.3}$	175^{+5+5}_{-7-5}	249^{+7+6}_{-9-7}	804^{+22+16}_{-17-17}	889^{+24+18}_{-19-20}	948^{+26+19}_{-21-21}
aN^{3}LO QCD+EW	$68.2^{+2.1+2.1}_{-3.2-2.3}$	174^{+5+5}_{-7-5}	248^{+7+6}_{-9-7}	802^{+22+16}_{-17-17}	886^{+24+18}_{-19-20}	945^{+26+19}_{-21-21}

aN^{3}LO QCD + NLO EW cross section with scale and pdf uncertainties is

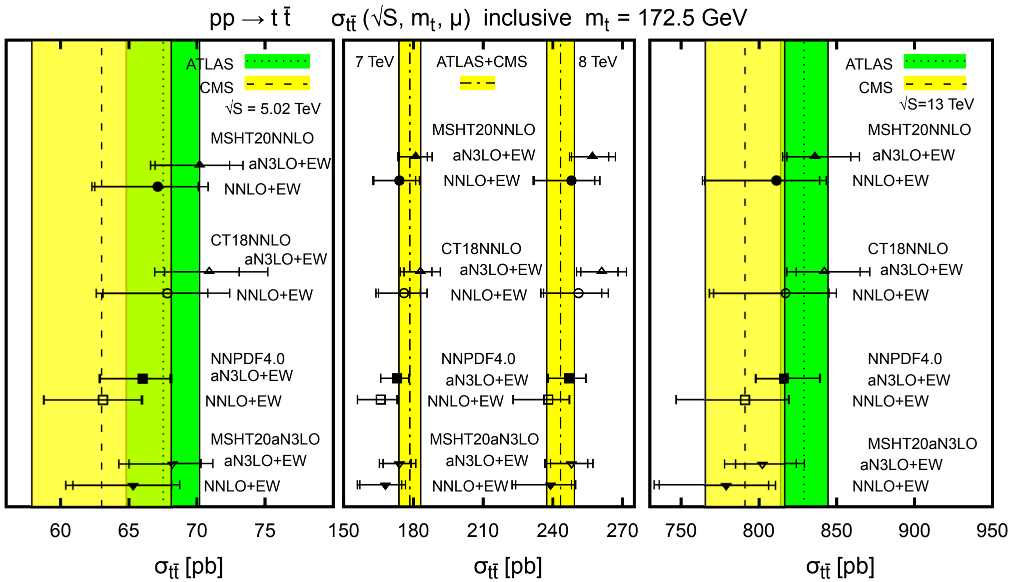
with MSHT20 NNLO pdf at 13 TeV: 836^{+23+17}_{-18-11} **pb ; at 13.6 TeV:** 925^{+25+18}_{-20-12} **pb**

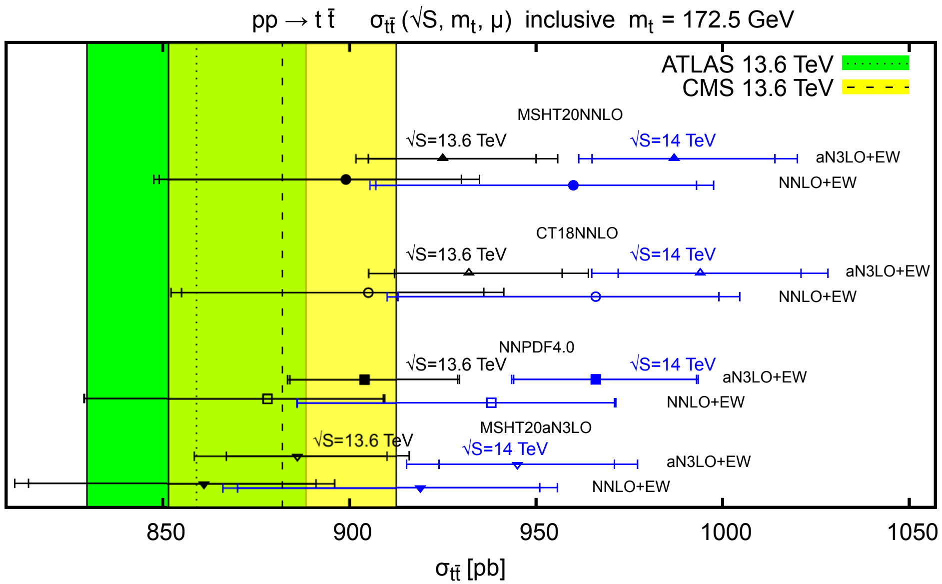
with CT18 NNLO pdf at 13 TeV: 842^{+23+18}_{-18-16} **pb ; at 13.6 TeV:** 932^{+25+20}_{-20-18} **pb**

with NNPDF4.0 NNLO pdf at 13 TeV: 816^{+23+5}_{-18-4} **pb ; at 13.6 TeV:** 904^{+25+5}_{-20-5} **pb**

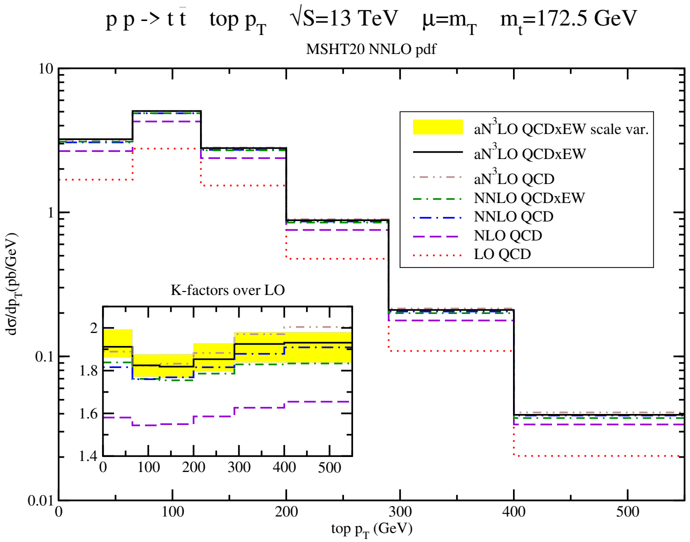
with PDF4LHC21 NNLO pdf at 13 TeV: 837^{+23+20}_{-18-16} **pb ; at 13.6 TeV:** 926^{+25+22}_{-20-17} **pb**

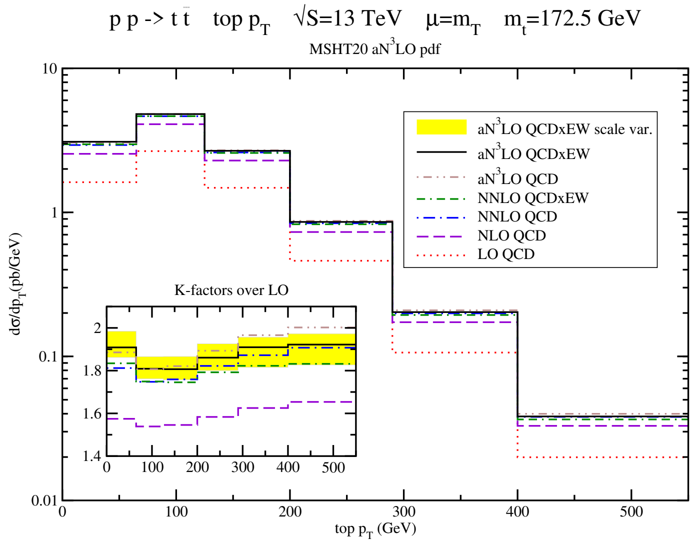
$t\bar{t}$ production at aN^{3}LO QCD + NLO EW at LHC energies

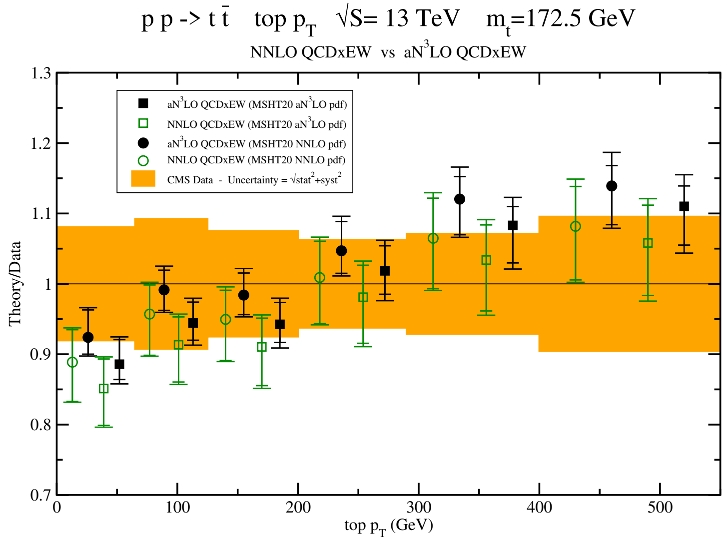


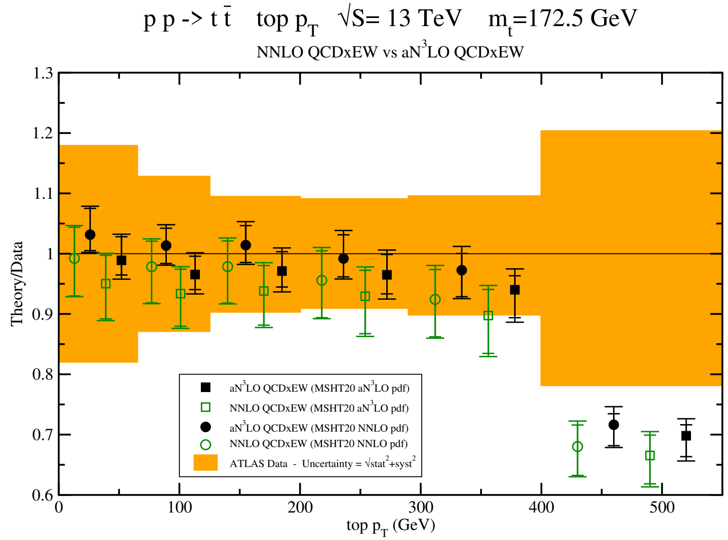


Top p_T distributions in $t\bar{t}$ production at 13 TeV

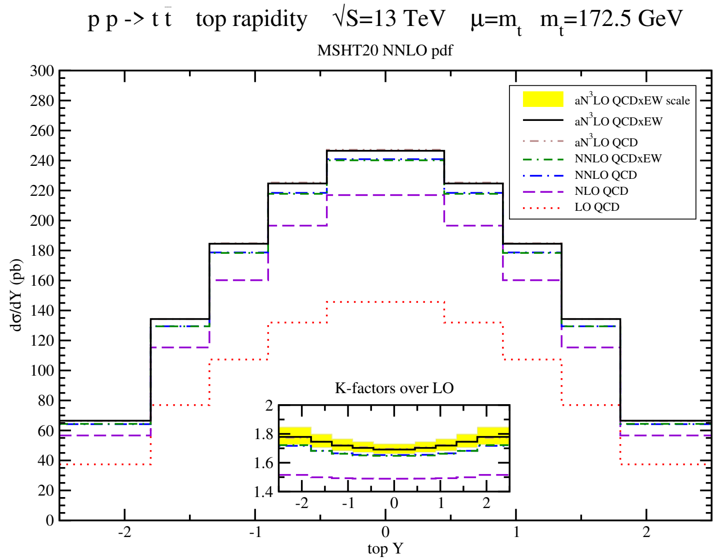


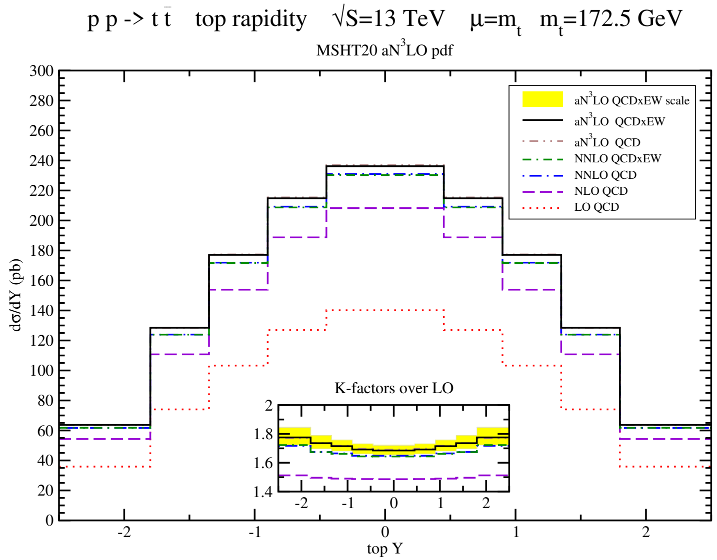


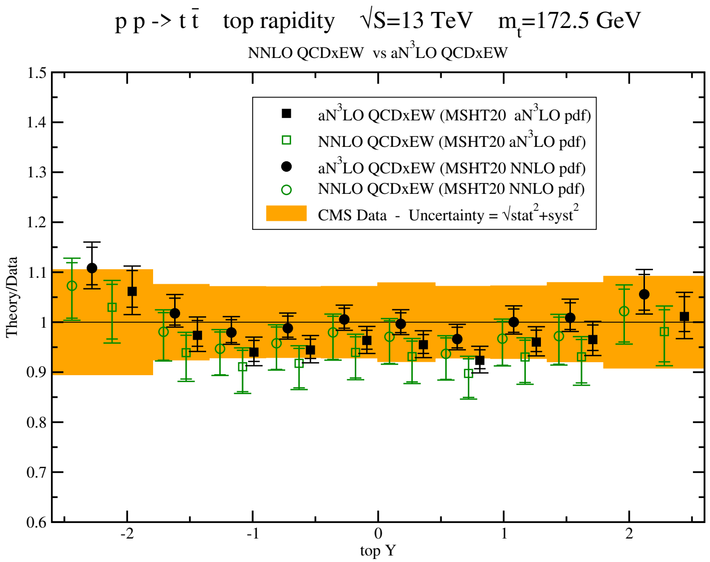


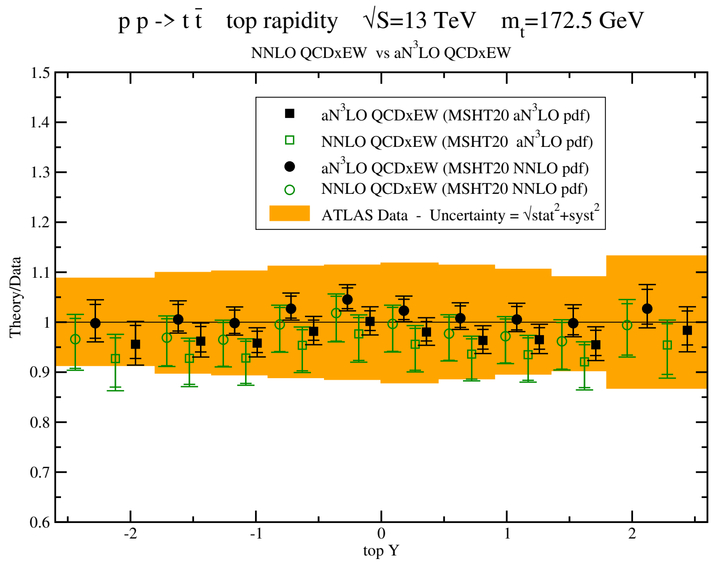


Top rapidity distributions in $t\bar{t}$ production at 13 TeV









$t\bar{t}$ production in SMEFT

(with Alberto Tonero (2023))

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i}{\Lambda^2} \mathcal{O}_i + {\rm h.c.}$$

chromomagnetic operator $\frac{c_{tG}}{\Lambda^2} \mathcal{O}_{tG} + {\rm h.c.}$ **with** $\mathcal{O}_{tG} = g_S \bar{q}_{3L} \sigma_{\mu\nu} T^A t_R \tilde{\varphi} G_A^{\mu\nu}$

cross section $$\sigma(c_{tG}) = \beta_0 + \frac{c_{tG}}{(\Lambda/1{\rm TeV})^2} \beta_1 + \frac{c_{tG}^2}{(\Lambda/1{\rm TeV})^4} \beta_2$$

SM and SMEFT contributions to $t\bar{t}$ cross sections at 13 TeV pp collisions (MSHT20 NNLO pdf)			
in pb	β_0	β_1	β_2
LO QCD	$487^{+142}_{-103}{}^{+10}_{-6}$	$155^{+46}_{-32}{}^{+4}_{-2}$	$28.1^{+8.6}_{-6.1}{}^{+0.7}_{-0.4}$
NLO QCD	$730^{+85}_{-86}{}^{+14}_{-10}$	$232^{+27}_{-27}{}^{+5}_{-3}$	$41.8^{+4.8}_{-5.0}{}^{+1.0}_{-0.6}$
aNNLO QCD	$814^{+28}_{-46}{}^{+16}_{-11}$	$259^{+9}_{-15}{}^{+6}_{-3}$	$46.6^{+1.6}_{-2.6}{}^{+1.1}_{-0.7}$

NLO/LO and aNNLO/LO ratios essentailly the same for β_0**,** β_1**,** β_2

allowed region for c_{tG} **is reduced at aNNLO**

$t\bar{t}\gamma$ production

(with Alberto Tonero, PRD 107, 034013 (2023))

observation of $t\bar{t}\gamma$ events at 7, 8, 13 TeV collisions at the LHC

the cross section for $t\bar{t}\gamma$ is sensitive to the top-quark charge and any modifications of the t-γ interaction vertex

QCD corrections at NLO are large and similar to $t\bar{t}$ production $\sim 50\%$

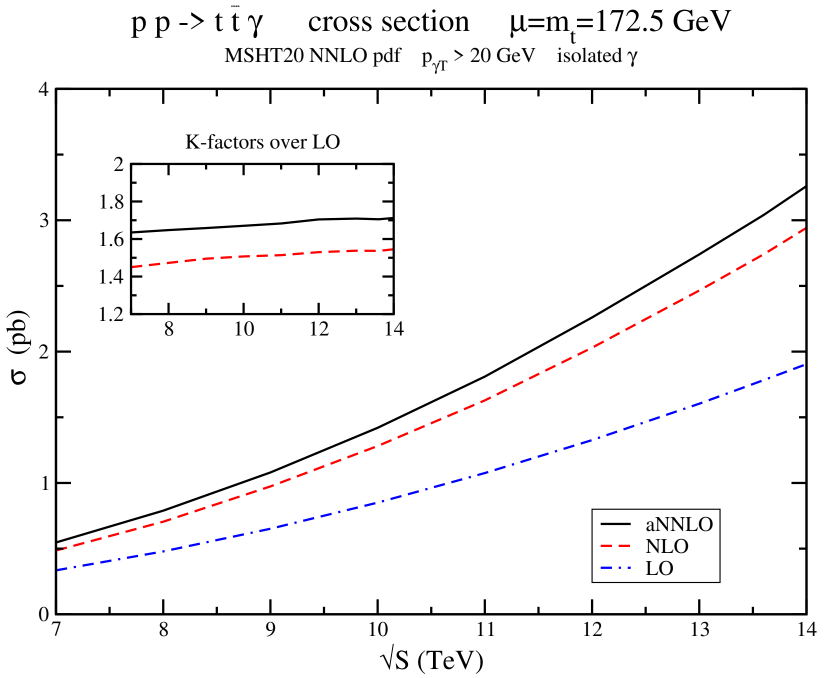
electroweak corrections are smaller than 1%

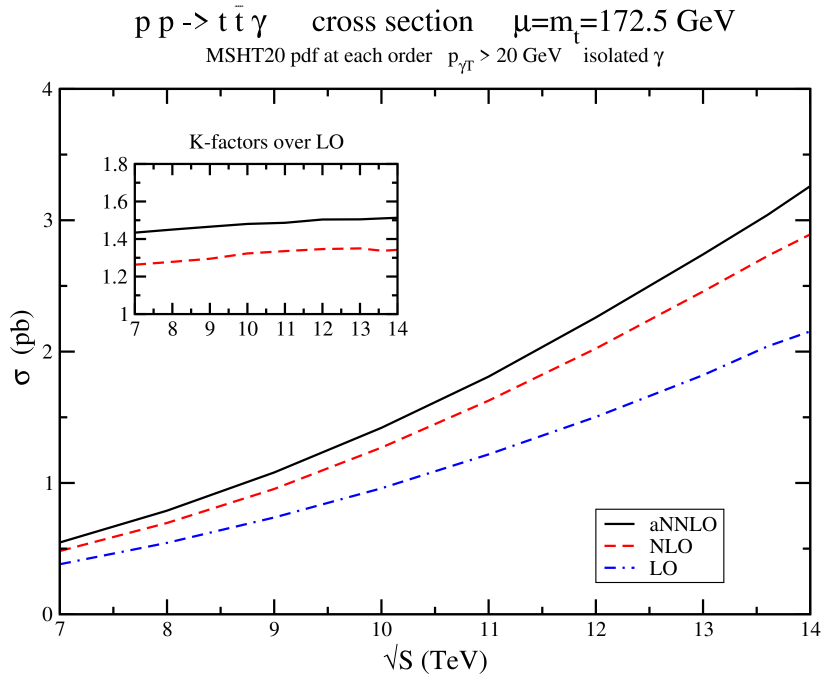
the QCD corrections are dominated by soft-gluon emission

further improvement in theoretical accuracy by the inclusion of higher-order soft-gluon corrections

$\rightarrow$ approximate NNLO (aNNLO) predictions

aNNLO cross sections for $t\bar{t}\gamma$ production





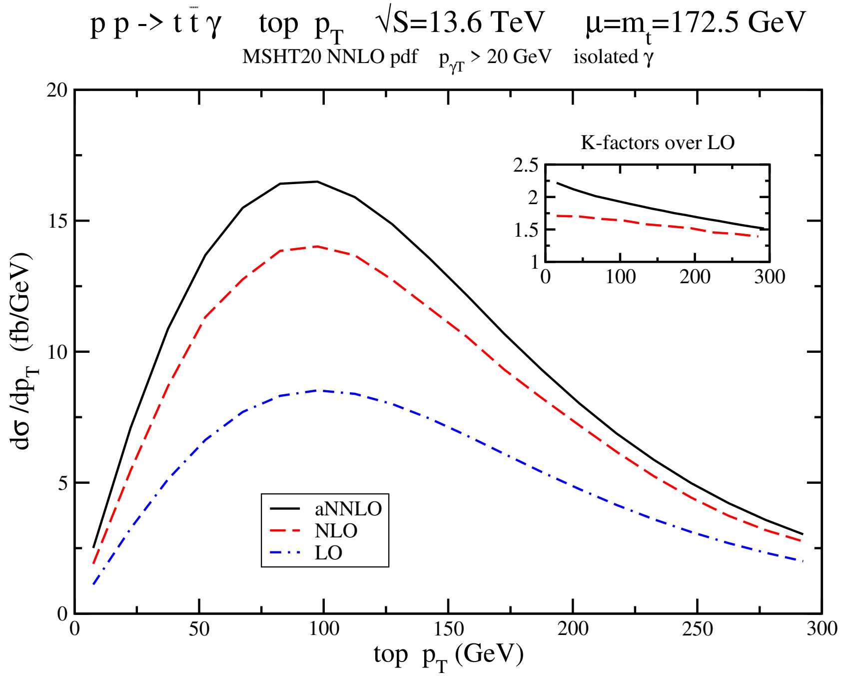
Comparison with 13 TeV CMS data

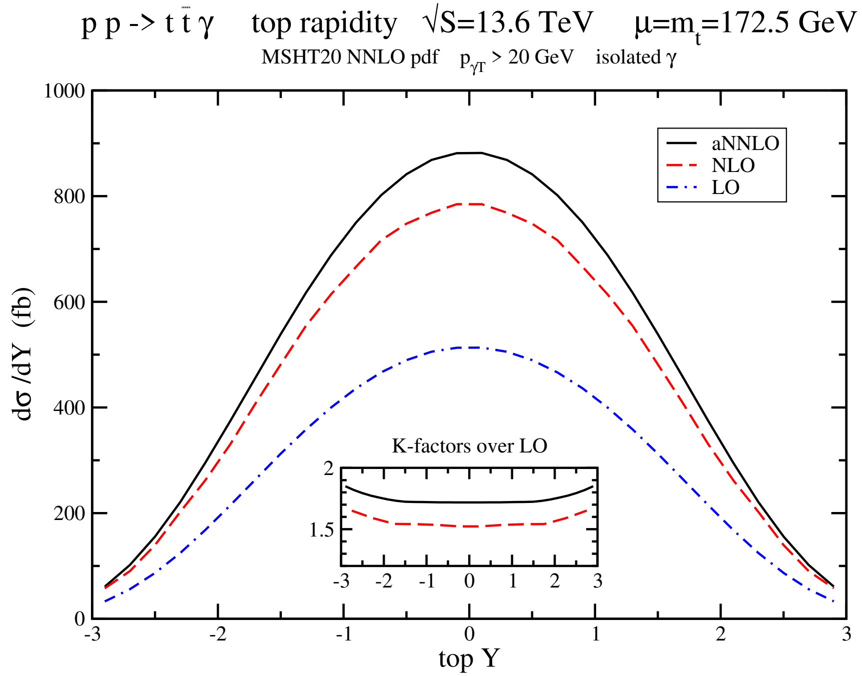
CMS measure a cross section in dilepton decay channel of 175.2 ± 2.5(stat) ± 6.3(syst) fb

which is compared to an NLO prediction (with scale + pdf uncertainty) of 155 ± 27 fb

Our aNNLO result is $173^{+11\,+3}_{-10\,-2}$ fb which is much closer to the data

Top-quark p_T and rapidity distributions in $t\bar{t}\gamma$ production





K-factors decrease at larger top p_T

K-factors are relatively flat at central and small top rapidities but increase at larger rapidities

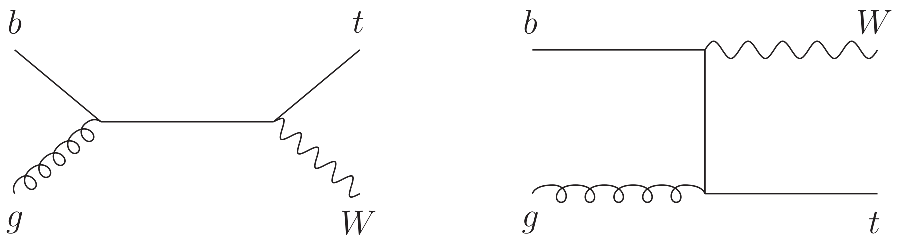
scale uncertainties in most of p_T and rapidity range are similar to those of the total cross section

a little smaller at large p_T

bigger at large rapidities

tW production

leading-order diagrams



At one loop $\Gamma_S^{(1)\,bg\to tW} = C_F \left[\ln\left(\frac{m_t^2-t}{m_t\sqrt{s}}\right) - \frac{1}{2}\right] + \frac{C_A}{2}\ln\left(\frac{u-m_t^2}{t-m_t^2}\right)$

At two loops $\Gamma_S^{(2)\,bg\to tW} = K_2\,\Gamma_S^{(1)\,bg\to tW} + \frac{1}{4}C_F C_A(1-\zeta_3)$

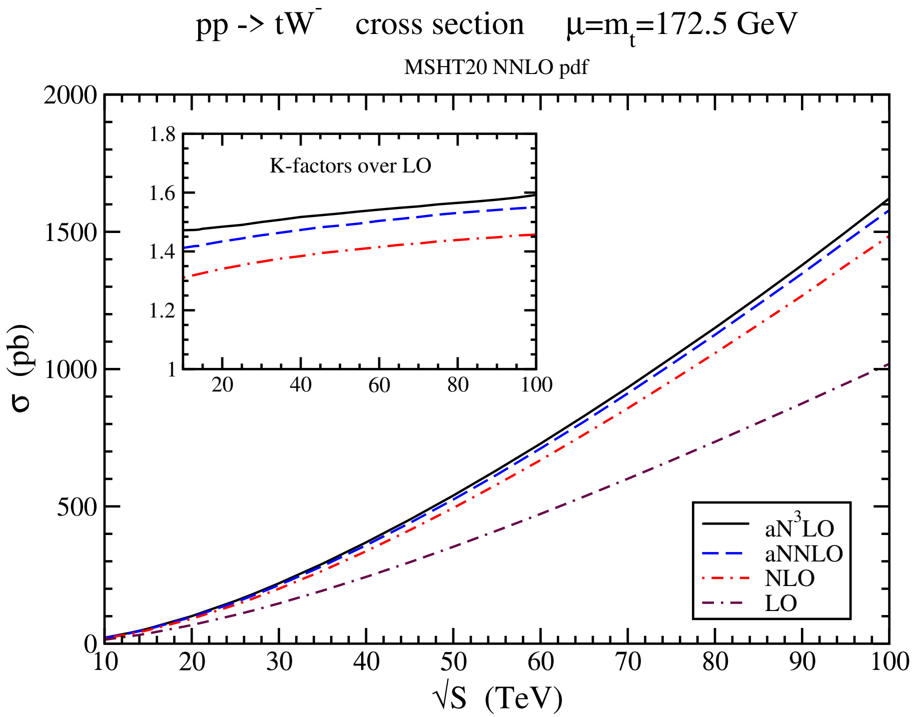
where $K_2 = C_A\left(\frac{67}{36}-\frac{\zeta_2}{2}\right) - \frac{5}{18}n_f$

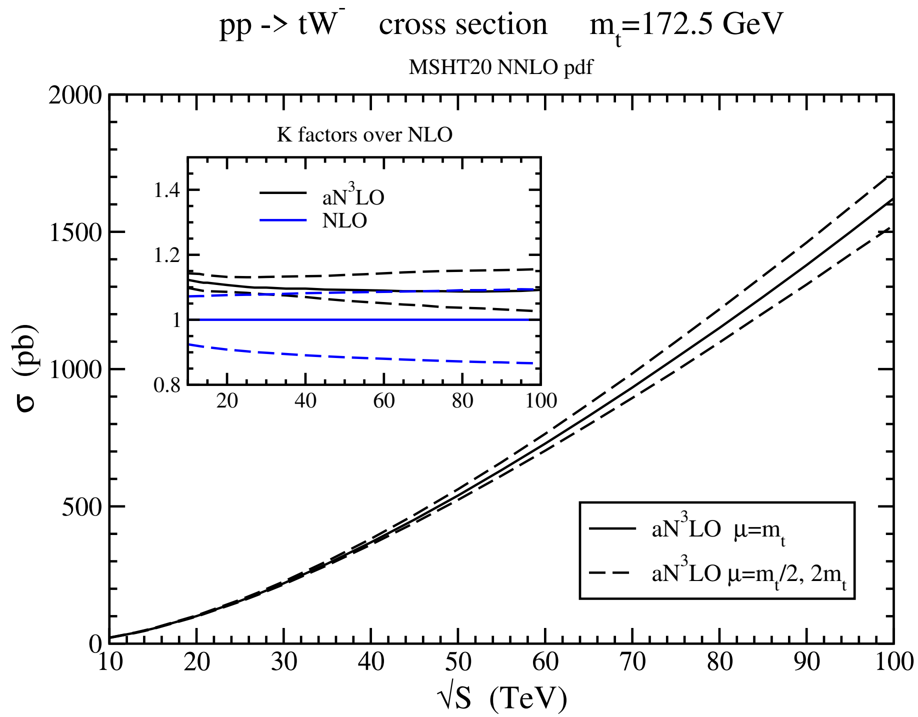
At three loops

$$\Gamma_S^{(3)\,bg\to tW} = K_3\,\Gamma_S^{(1)\,bg\to tW} + \frac{1}{2}K_2 C_F C_A(1-\zeta_3) + C_F C_A^2\left[-\frac{1}{4}+\frac{3}{8}\zeta_2-\frac{\zeta_3}{8}-\frac{3}{8}\zeta_2\zeta_3+\frac{9}{16}\zeta_5\right]$$

tW production in high-energy collisions

(with Nodoka Yamanaka, JHEP 05, 278 (2021) & updates)

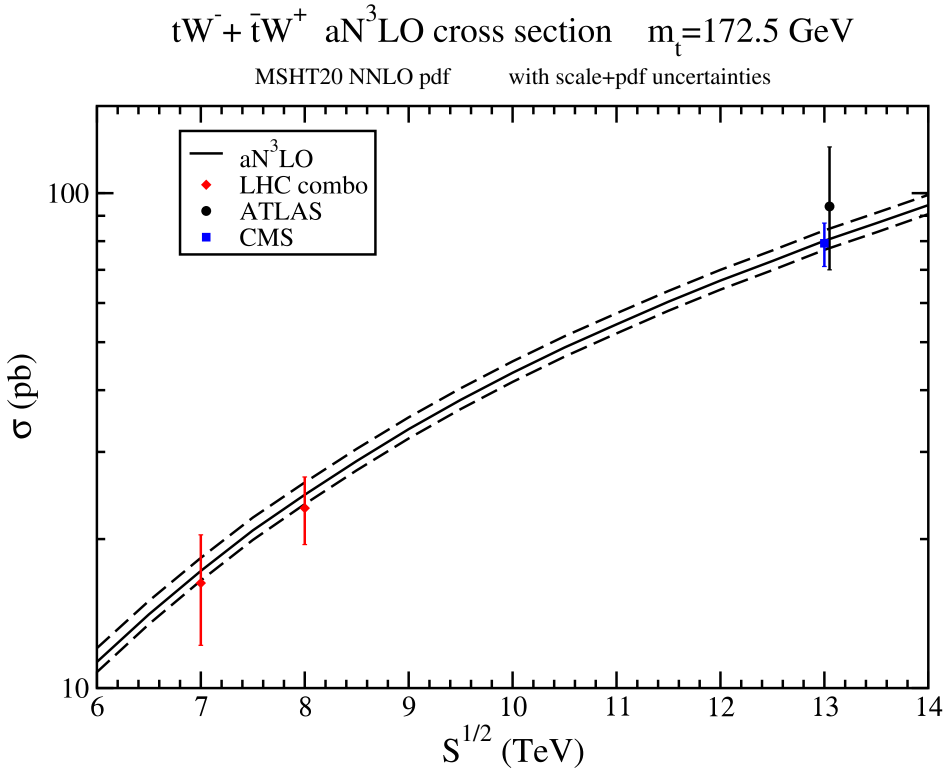




The aNLO cross section is a very good approximation to the complete NLO result for all foreseeable collider energies → the soft-gluon corrections are dominant

The aNNLO and $\mathrm{aN^3LO}$ corrections (at NNLL) are also significant

The scale dependence at $\mathrm{aN^3LO}$ is reduced relative to NLO



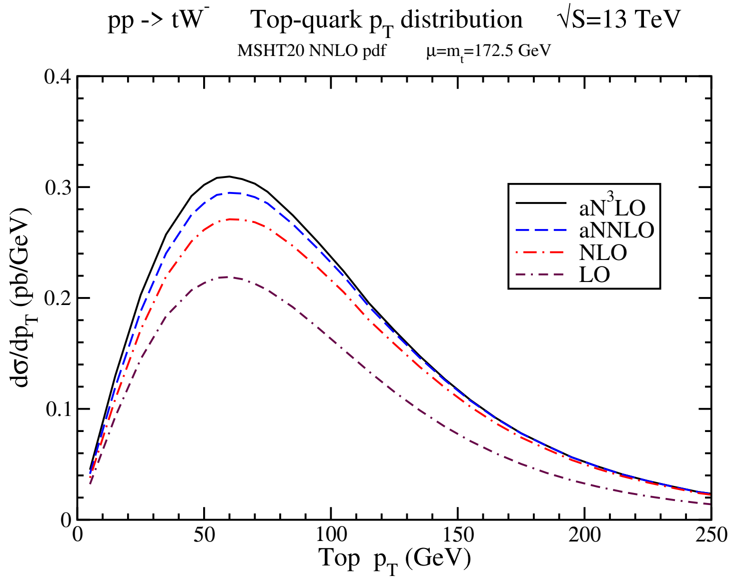
The aN^{3}LO cross section for $tW^- + \bar{t}W^+$ with scale and pdf uncertainties is

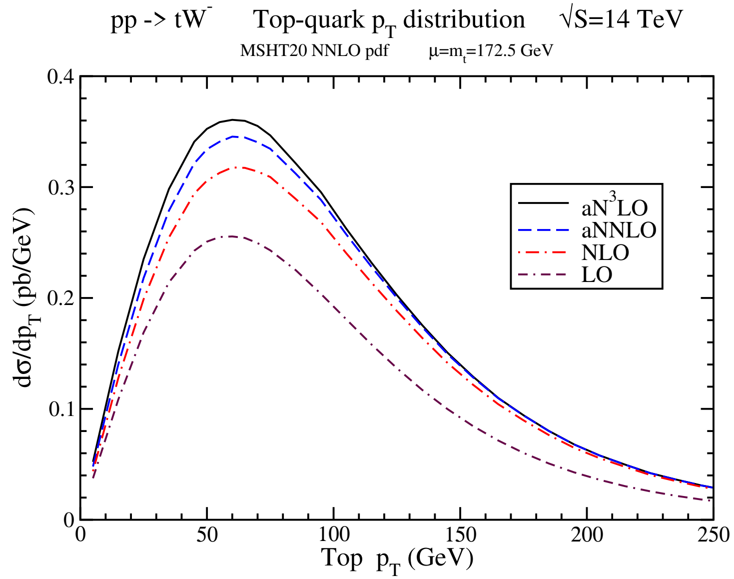
with MSHT20 NNLO pdf at 13 TeV: $79.5^{+1.9+2.0}_{-1.8-1.4}$ **pb ; at 13.6 TeV:** $87.6^{+2.0+2.1}_{-1.9-1.5}$ **pb**

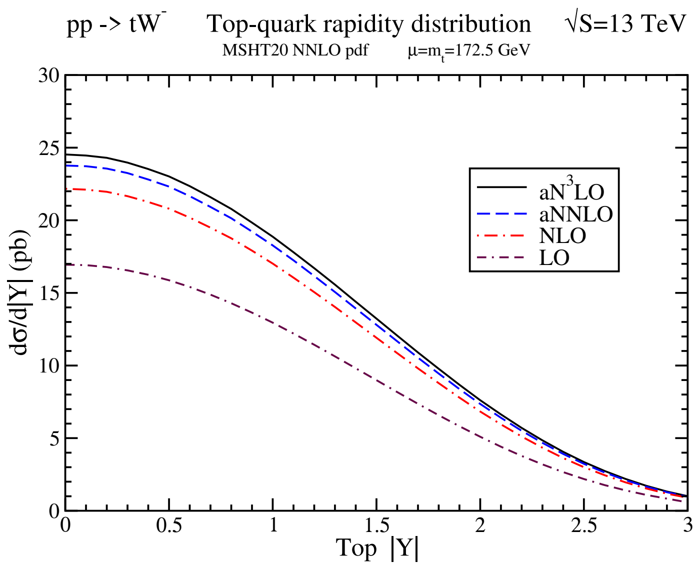
with MSHT20 aN^{3}LO pdf at 13 TeV: $77.3^{+1.9+2.0}_{-1.8-2.1}$ **pb ; at 13.6 TeV:** $85.6^{+2.0+2.2}_{-1.9-2.3}$ **pb**

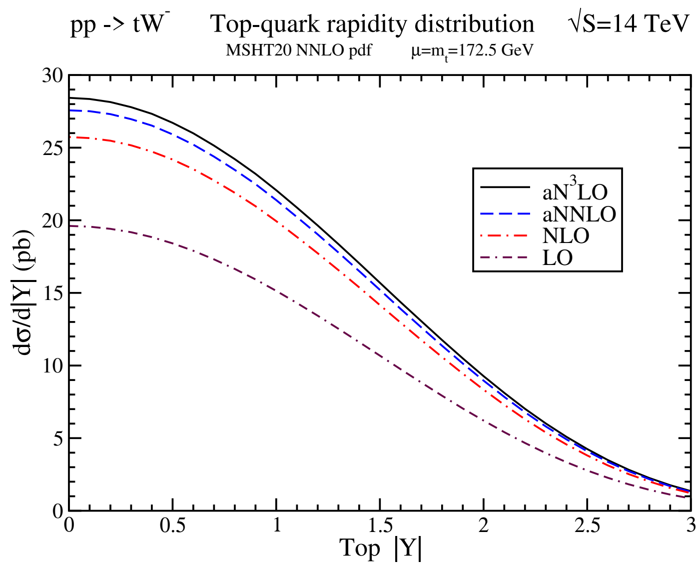
with PDF4LHC21 pdf at 13 TeV: $79.3^{+1.9+2.2}_{-1.8-2.2}$ **pb ; at 13.6 TeV:** $87.9^{+2.0+2.4}_{-1.9-2.4}$ **pb**

Top-quark p_T and rapidity distributions in tW production









$tq\gamma$ production

(with Nodoka Yamanaka, EPJC 82, 670 (2022))

evidence for $pp \to tq\gamma$ at 13 TeV collisions at the LHC

the cross section for $tq\gamma$ is sensitive to the top-quark charge and any anomalous electric and magnetic dipole moments

also sensitive to any anomalous t-q-γ couplings with FCNC

QCD corrections are significant at NLO and they are needed for good theoretical predictions

further improvement in theoretical accuracy by inclusion of soft-gluon corrections

$\to$ approximate NNLO predictions

aNNLO cross sections for $tq\gamma$ production

$tq\gamma$ cross sections at aNNLO for LHC energies with $p_{\gamma T} > 10$ GeV, $\lvert\eta_\gamma\rvert < 2.6$, and $\Delta R > 0.05$				
σ in pb	8 TeV	13 TeV	13.6 TeV	14 TeV
MSHT20 NNLO pdf	$0.857^{+0.019}_{-0.020}{}^{+0.011}_{-0.007}$	$2.30^{+0.05}_{-0.07} \pm 0.02$	$2.50^{+0.05}_{-0.08} \pm 0.02$	$2.65^{+0.06}_{-0.08} \pm 0.02$
CT18 NNLO pdf	$0.864^{+0.018}_{-0.019}{}^{+0.022}_{-0.020}$	$2.29^{+0.05}_{-0.07}{}^{+0.04}_{-0.05}$	$2.51^{+0.06}_{-0.08}{}^{+0.04}_{-0.05}$	$2.65^{+0.06}_{-0.08}{}^{+0.04}_{-0.05}$
NNPDF4.0 NNLO pdf	$0.829^{+0.017}_{-0.018} \pm 0.003$	$2.27^{+0.05}_{-0.07} \pm 0.01$	$2.45^{+0.06}_{-0.08} \pm 0.01$	$2.58^{+0.06}_{-0.08} \pm 0.01$

For $\bar{t}q\gamma$ production with the same cuts and MSHT20 NNLO pdf, the aNNLO cross section is $1.61^{+0.04}_{-0.05}{}^{+0.02}_{-0.01}$ pb at 13 TeV, $1.77^{+0.05}_{-0.06}{}^{+0.02}_{-0.01}$ pb at 13.6 TeV, and $1.89^{+0.05}_{-0.07} \pm 0.02$ pb at 14 TeV

Comparison with 13 TeV CMS $tq\gamma$ data

For the CMS cuts of $p_{\gamma T} > 25$ GeV, $\lvert\eta_\gamma\rvert < 1.44$, $\Delta R > 0.5$ at 13 TeV:

the aNNLO $tq\gamma$ cross section with MSHT20 NNLO pdf is $0.584^{+0.011}_{-0.015}{}^{+0.007}_{-0.005}$ pb
the aNNLO $\bar{t}q\gamma$ cross section with MSHT20 NNLO pdf is $0.406^{+0.010}_{-0.012}{}^{+0.005}_{-0.004}$ pb

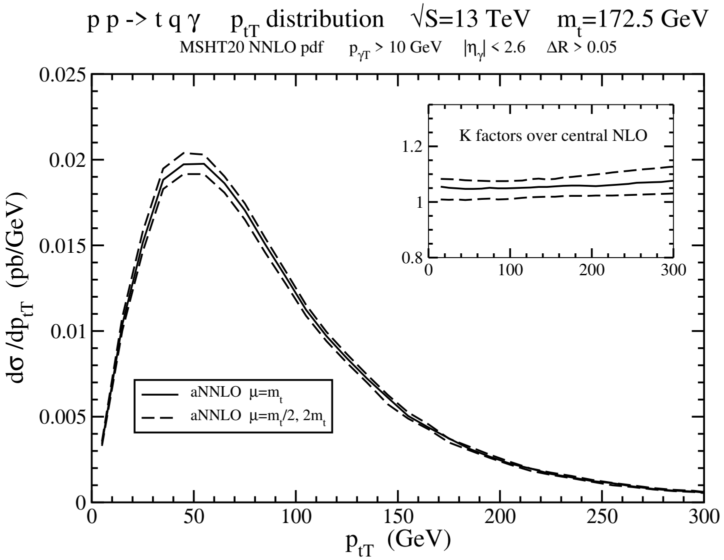
Total $tq\gamma$+$\bar{t}q\gamma$ aNNLO cross section is $0.990^{+0.021}_{-0.027}{}^{+0.012}_{-0.008}$ pb

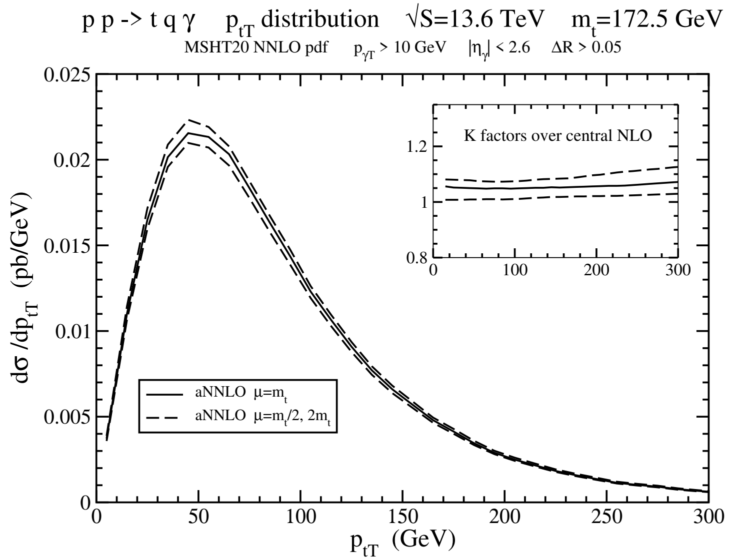
Multiplying by the branching fraction for $t \to \mu\nu b$ ($11.40 \pm 0.20\%$), we find $(113 \pm 2)^{+2}_{-3} \pm 1$ fb

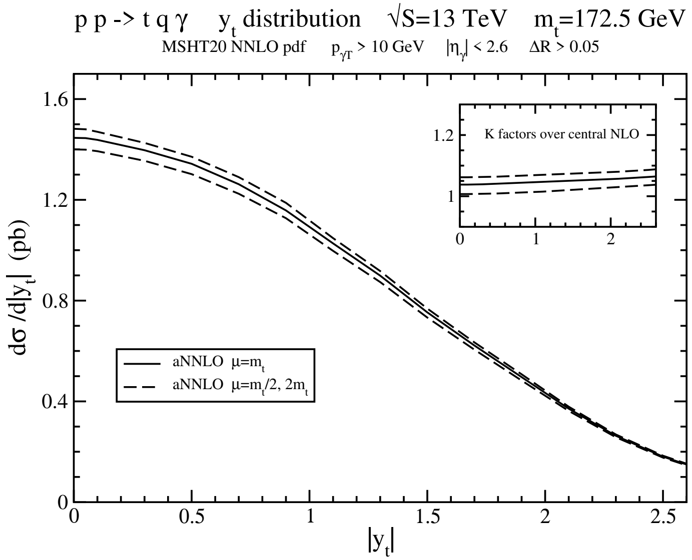
CMS measured value of 115 ± 17 (stat) ± 30 (syst) fb includes further cuts and is compared to NLO theory 81 ± 4 fb

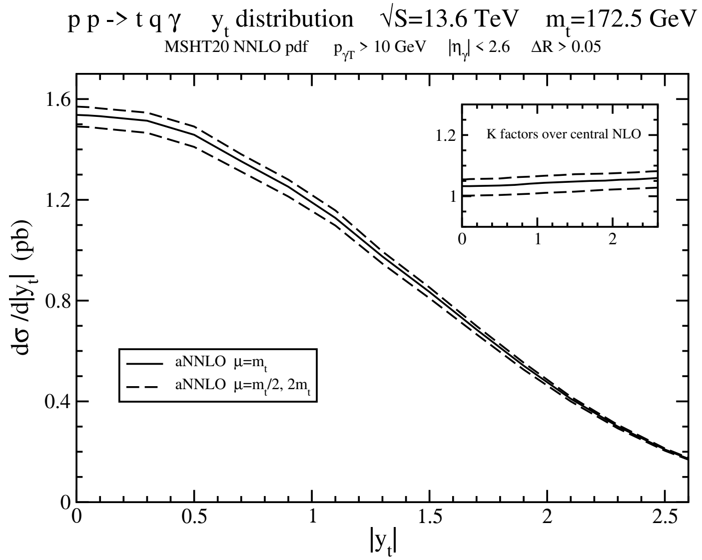
We note that aNNLO corrections increase the NLO cross section by 5.8%

Top-quark p_T and rapidity distributions in $tq\gamma$ production









tqZ production

(with Nodoka Yamanaka, PLB 838, 137708 (2023))

observation of $pp \to tqZ$ at 13 TeV collisions at the LHC

recent data is well above NLO theoretical prediction

the cross section for tqZ allows study of t-Z and W-W-Z couplings

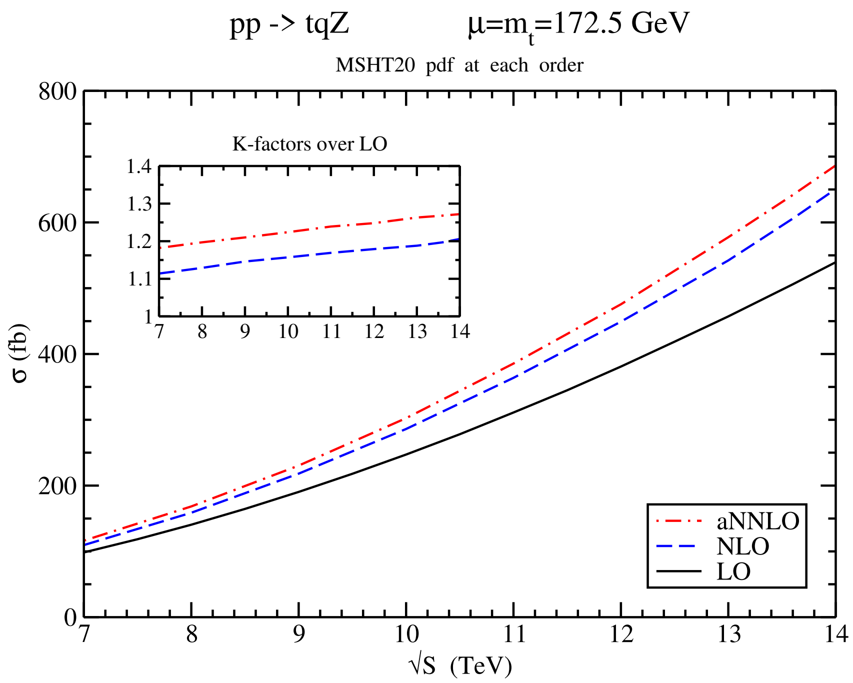
and is sensitive to any anomalous top-quark couplings and moments

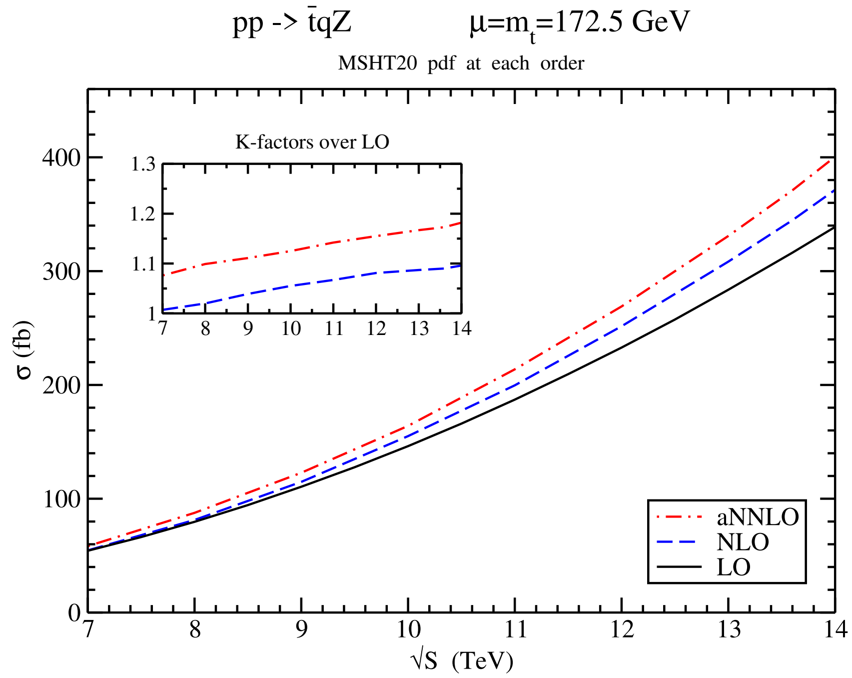
QCD corrections are significant at NLO and they are needed for good theoretical predictions

further improvement in theoretical accuracy by inclusion of soft-gluon corrections

$\to$ approximate NNLO (aNNLO) predictions

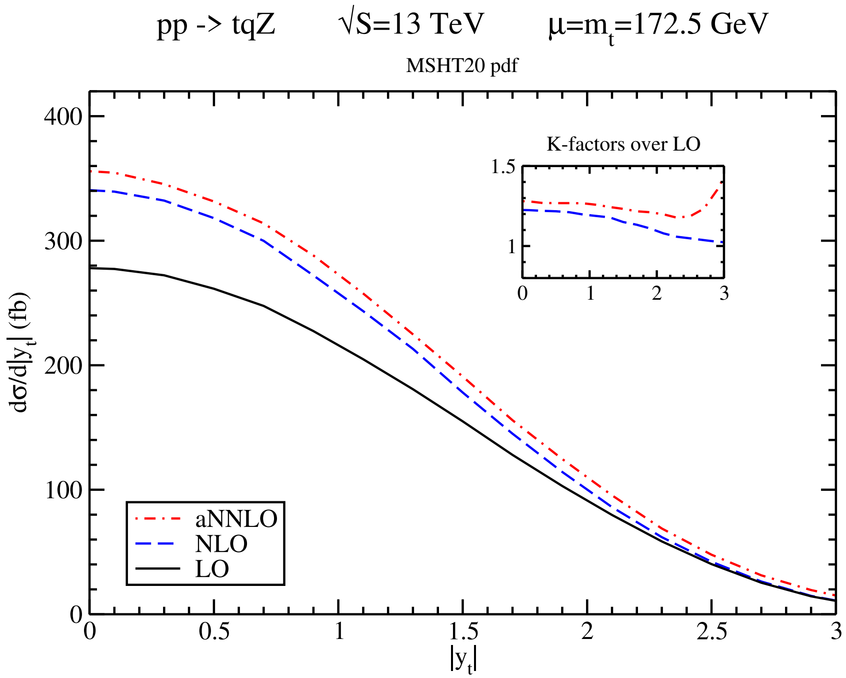
aNNLO cross sections for tqZ production

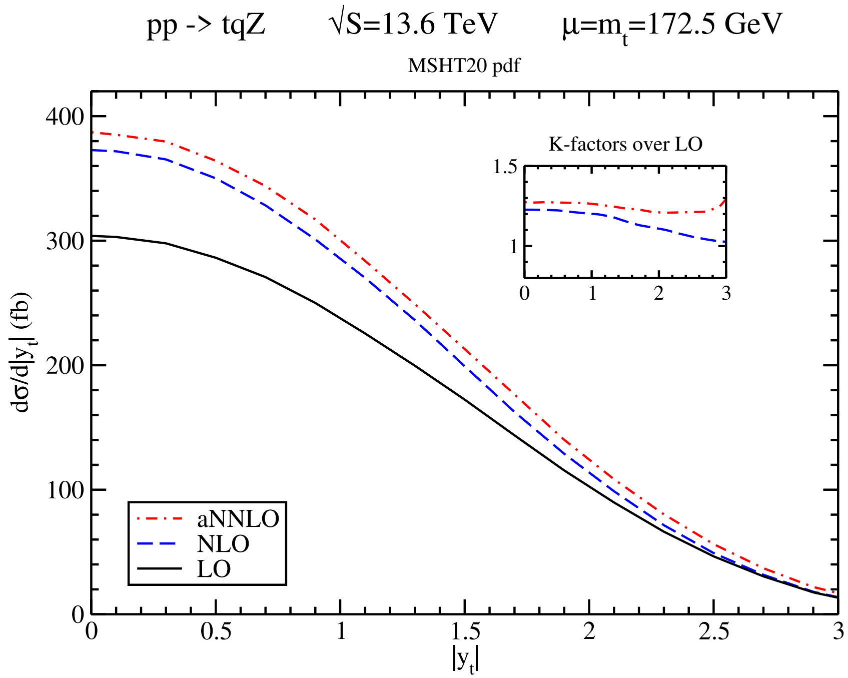




Sum of tqZ and $\bar{t}qZ$ cross sections in pp collisions at the LHC					
σ in fb	7 TeV	8 TeV	13 TeV	13.6 TeV	14 TeV
LO	153^{+2+3}_{-6-2}	$221^{+5\ +3}_{-11-4}$	741^{+34+9}_{-52-8}	$822^{+39}_{-59} \pm 9$	$879^{+43}_{-65} \pm 9$
NLO	$165 \pm 3^{+2}_{-3}$	240^{+5+4}_{-3-3}	850^{+19+11}_{-18-9}	$951^{+19}_{-21} \pm 11$	1022^{+24+12}_{-25-10}
aNNLO	$174^{+1}_{-3} \pm 3$	256^{+2+5}_{-3-3}	$908^{+6\ +10}_{-15-9}$	$1012^{+6\ +10}_{-18-9}$	$1087^{+7\ +12}_{-21-9}$

Top-quark rapidity distributions in tqZ production



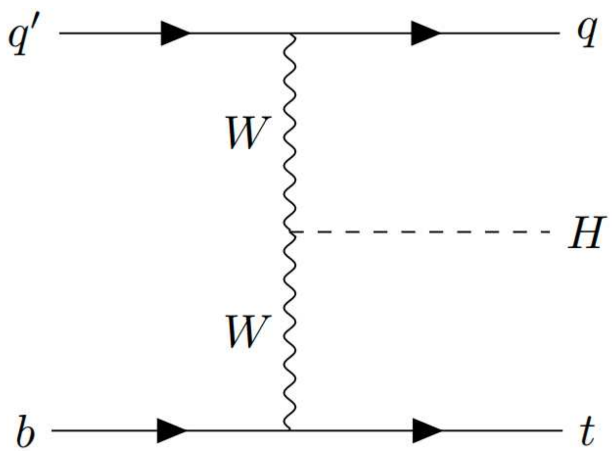


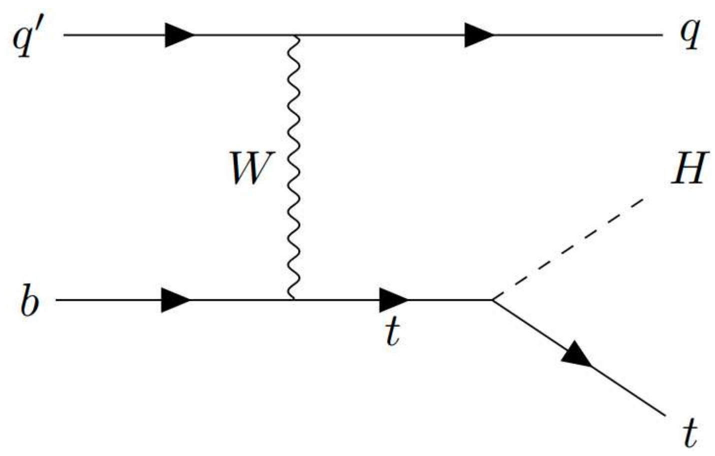
significant enhancements from aNNLO corrections particularly at large rapidities

scale and pdf uncertainties get bigger at larger rapidities, $|y_t| > 2$

tqH production

(with Matthew Forslund, PRD 104, 034024 (2021) & updates)



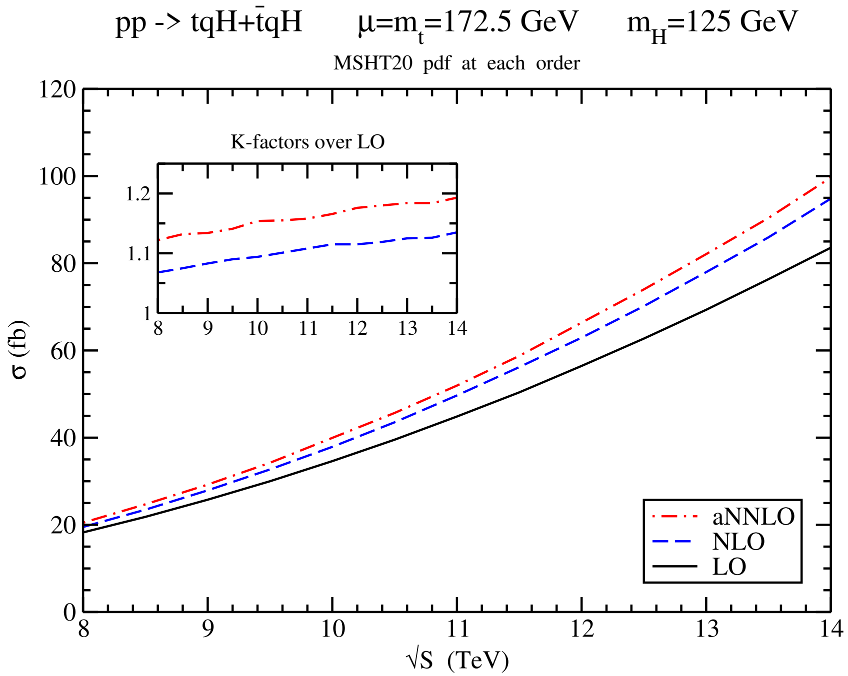


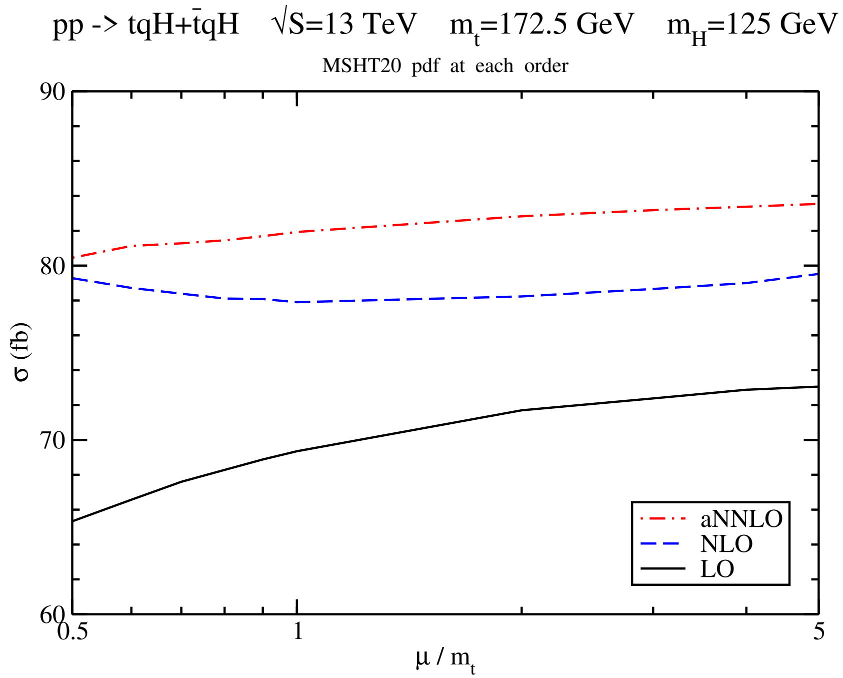
direct probe of the Higgs coupling to the W boson and the top quark

small cross section (100 fb at 14 TeV)

sensitive to new physics

tqH cross sections

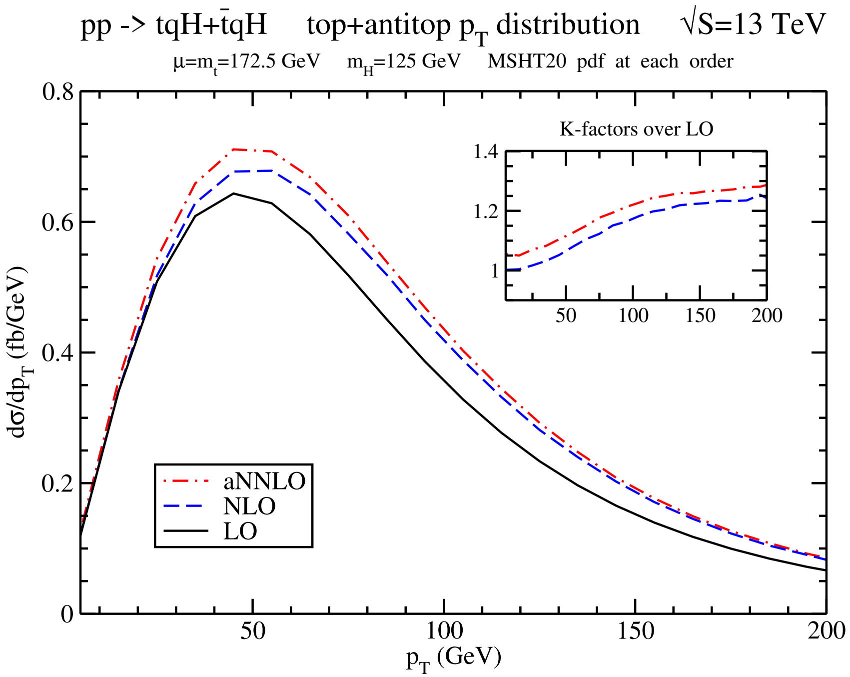


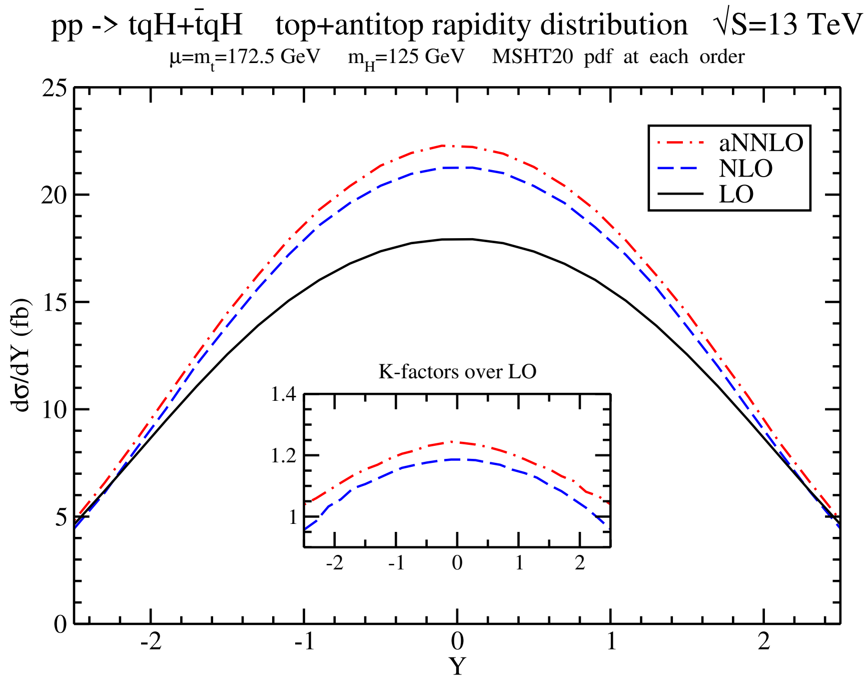


significant enhancement from the higher-order corrections

scale dependence is reduced at higher orders

Top p_T and rapidity distributions in tqH production





large K-factors at high p_T and central rapidity relative to LO

Summary

- $t\bar{t}$ **production**
- $t\bar{t}$ **in SMEFT**
- $t\bar{t}\gamma$ **production**
- tW **production**
- $tq\gamma$**,** tqZ**,** tqH **production**
- **soft-gluon resummation and aNNLO, aN^{3}LO expansions**
- **results for total cross sections and differential distributions**
- **higher-order corrections further enhance and improve the theoretical predictions**
- **good agreement with LHC data**