Evolution of heavy quarks in the QGP using JETSCAPE

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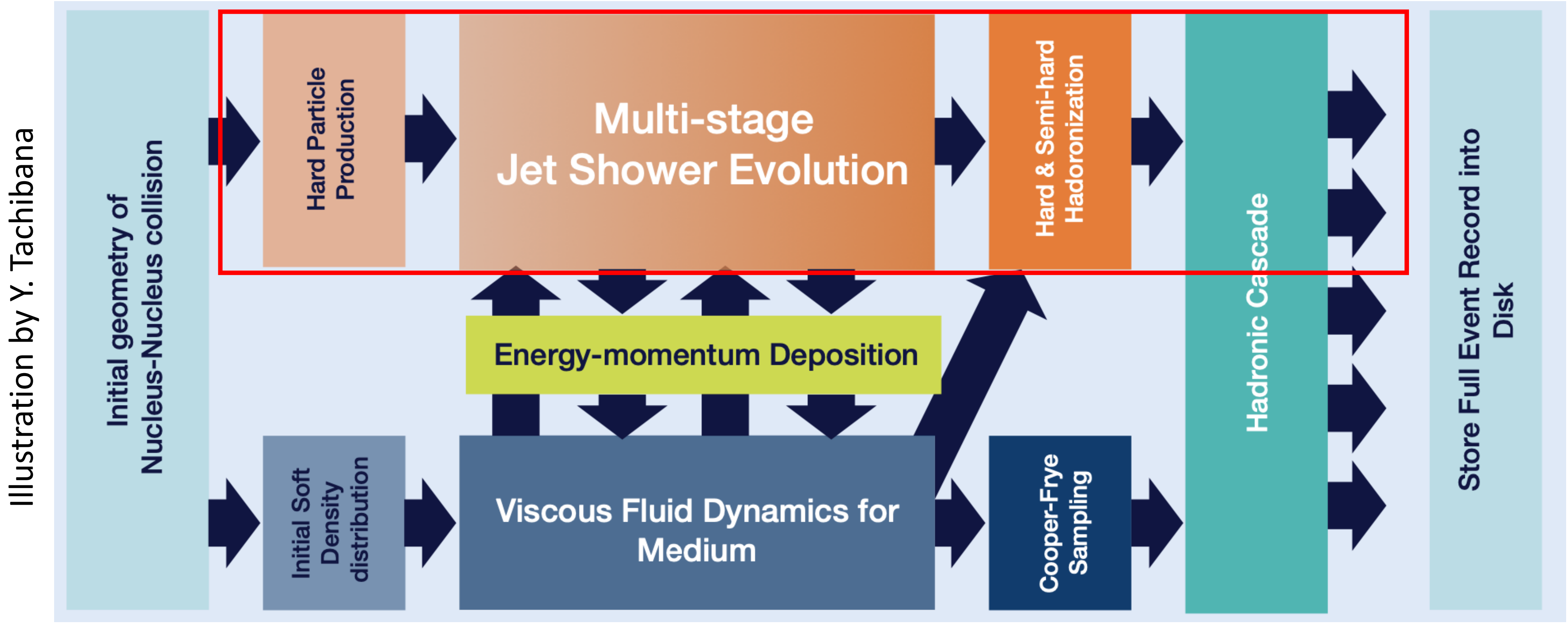
The JETSCAPE Framework



- JETSCAPE framework allows :
  - Multiple energy loss formalisms to be present simultaneously, each applied in its region of validity.
  - Provides a set of Bayesian tools to characterize the interaction of hard probes with the QGP (see tomorrow's Bayesian session).

Outline

- Overview of physics and JETSCAPE modules
- MATTER and the high-virtuality evolution
- LBT and low-virtuality evolution
- Results with heavy flavors and future developments
- Conclusion & Outlook

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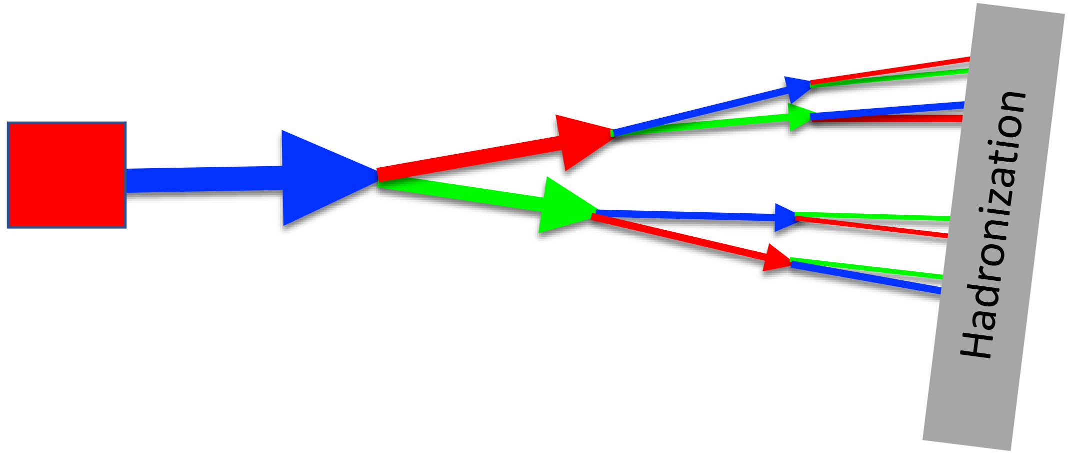
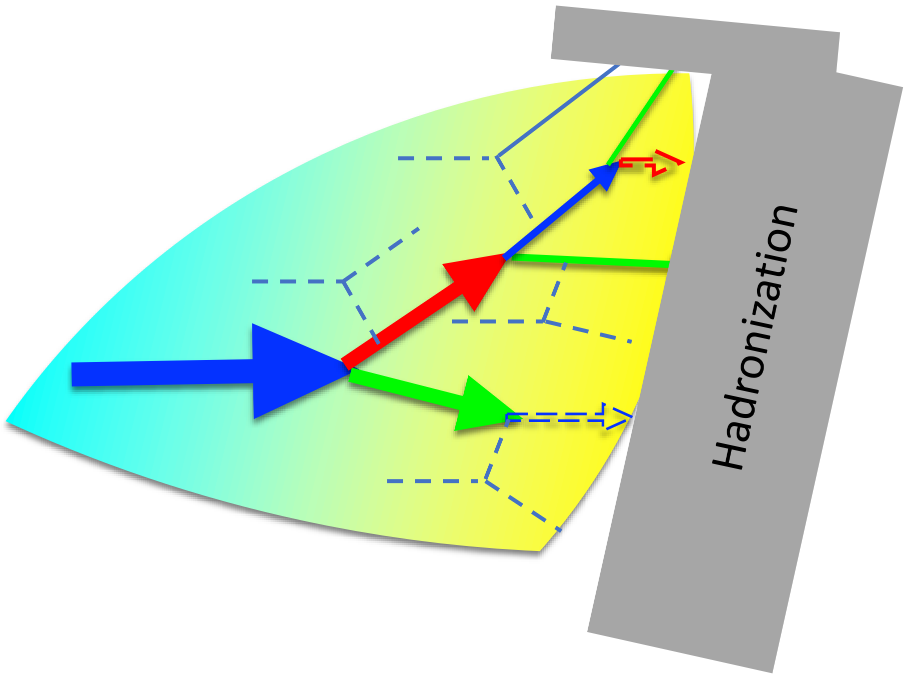
Monte Carlo jet shower simulation in vacuum



- In p-p collisions, the initial highly virtual quark or gluon produces a jet shower solely through radiation.
- In PYTHIA, a Monte Carlo event generator, this shower is angular ordered, which generate a jet in a narrow cone.

Modified splitting inside the QGP



- In the nuclear medium, the particles in a jet after hadronization occupy a wider cone.
- This widening is given by scatterings, which modify the radiation pattern:

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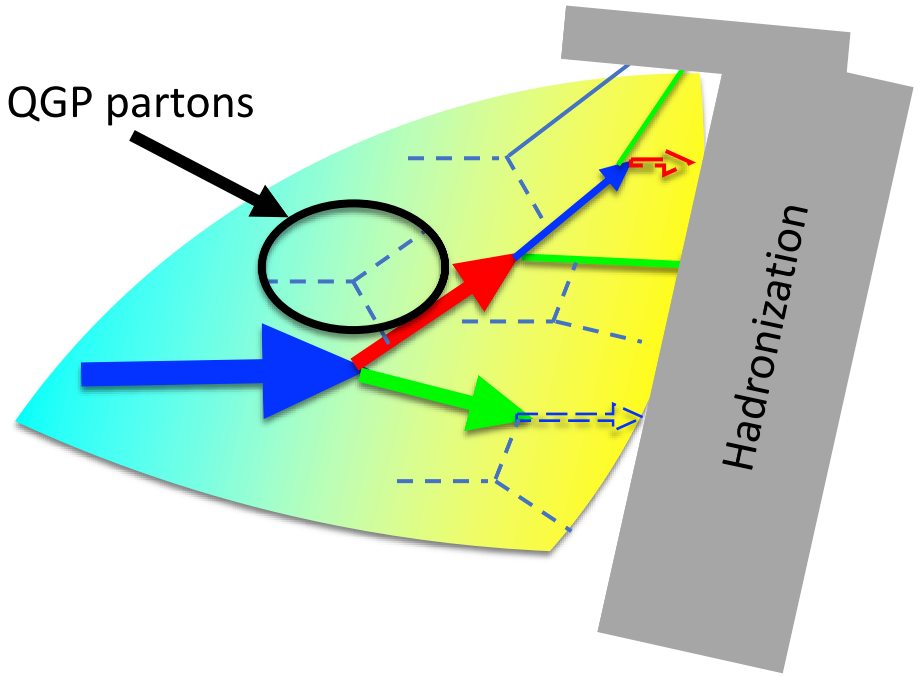

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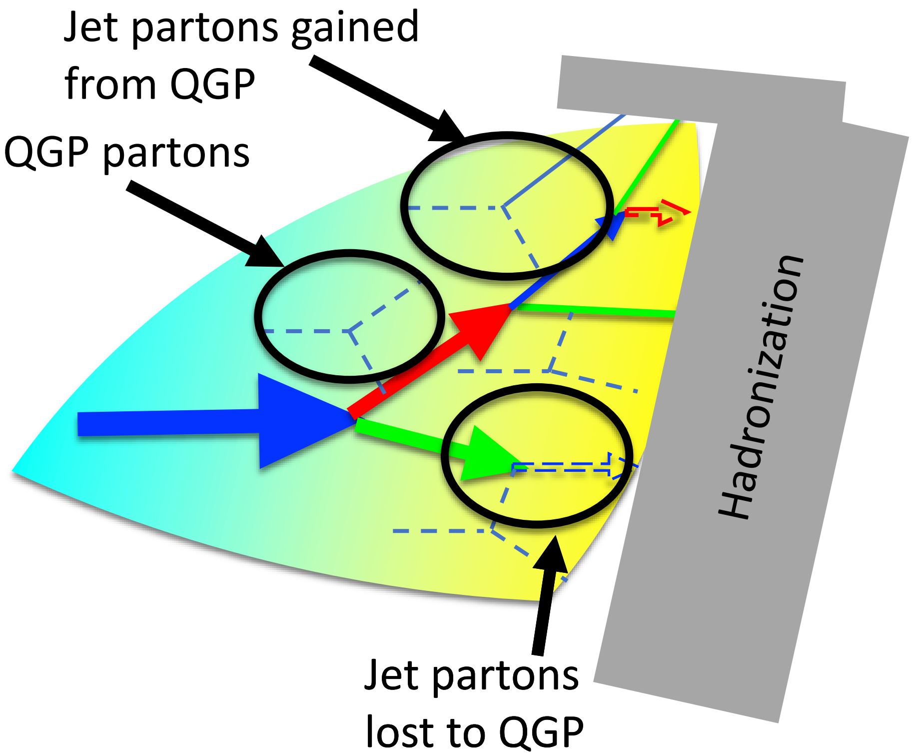

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  - Scattering processes can pick-up (or deposit) partons into the QGP (see dashed to solid lines).

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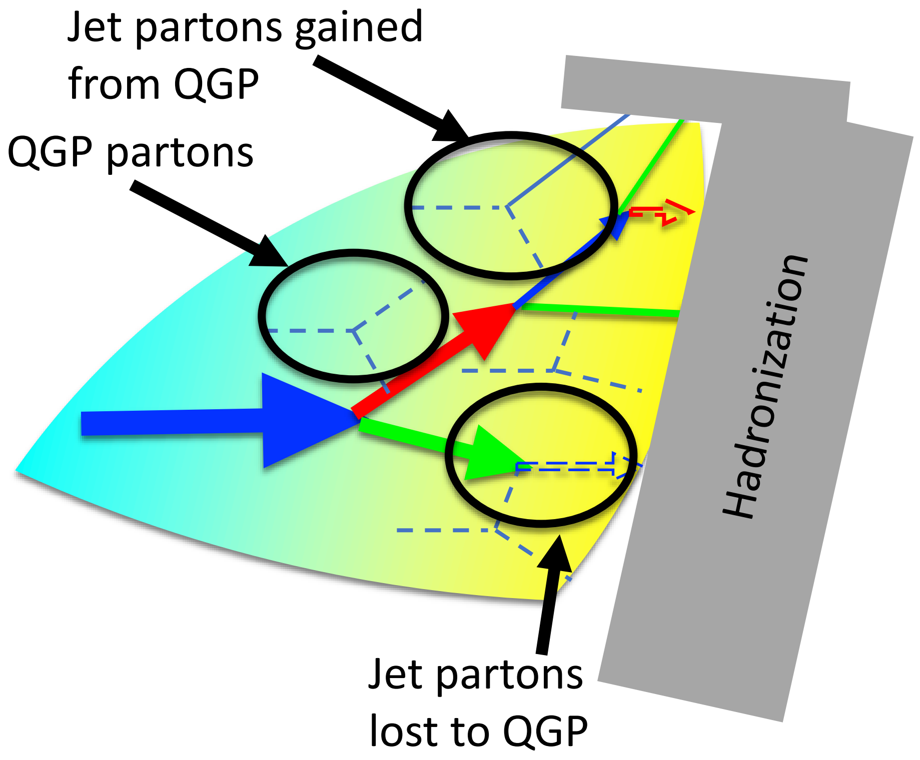

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  - Medium-induced radiation/absorption also change parton chemistry of light-flavored jets. [arxiv:2211.15553]

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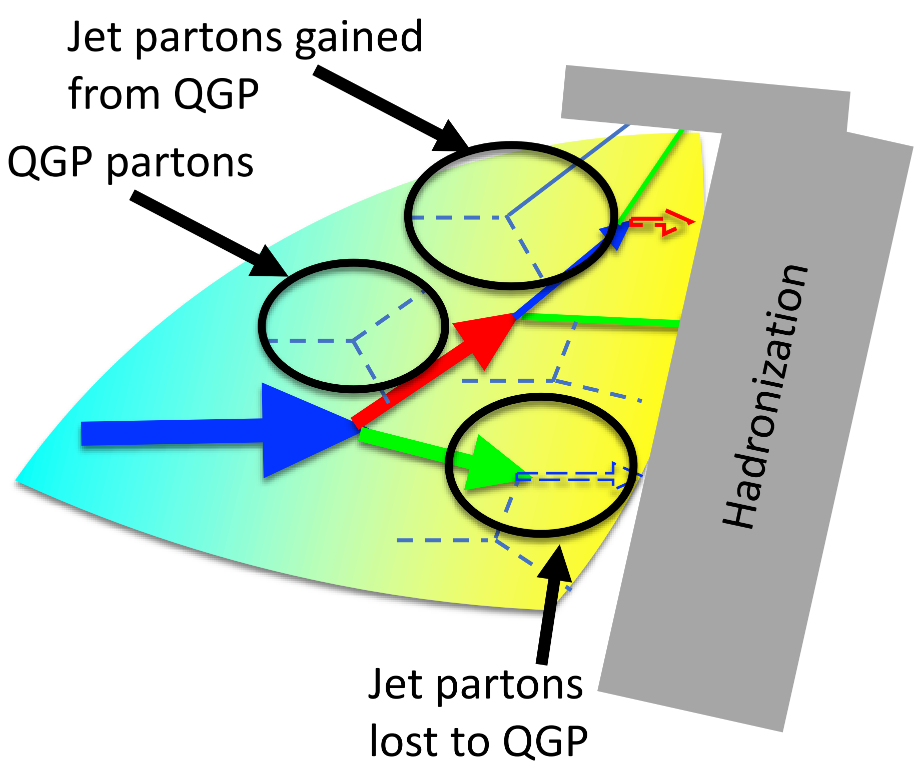

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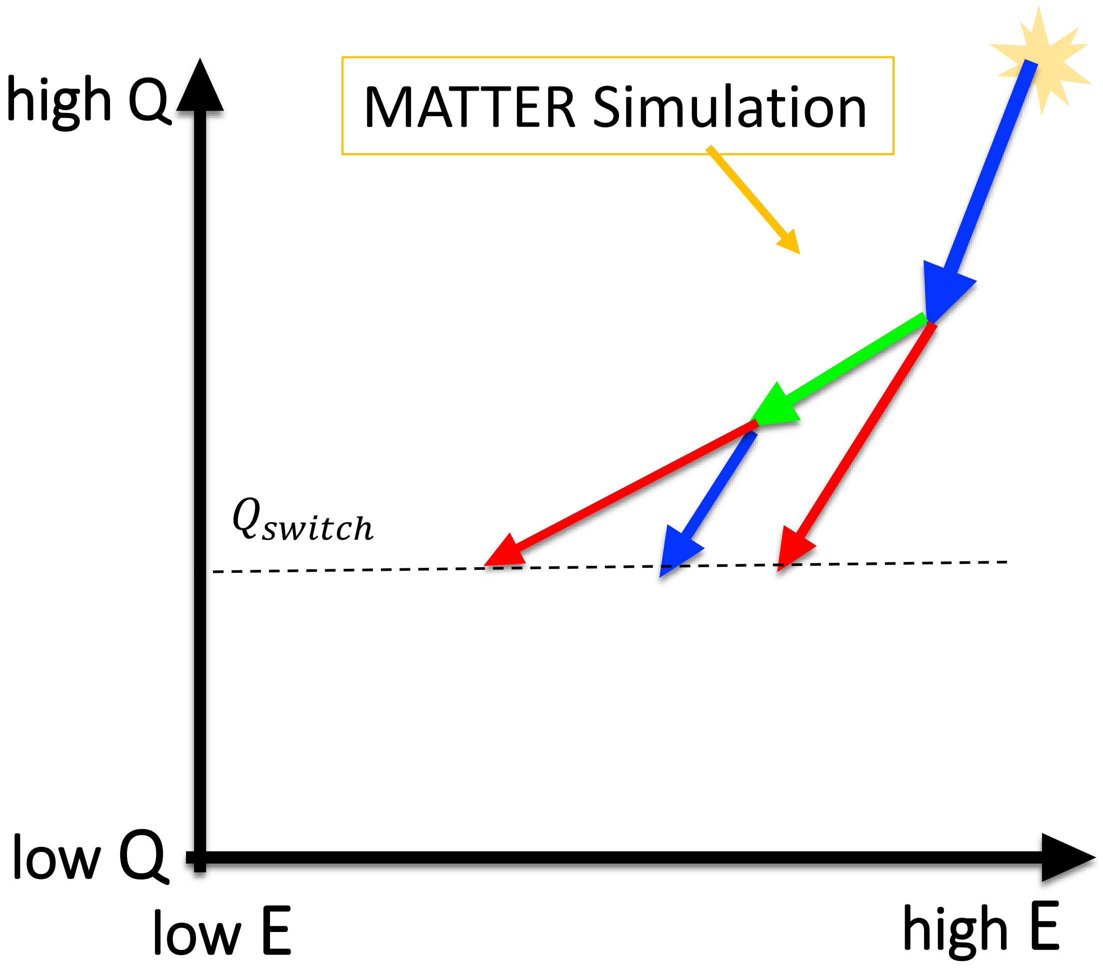
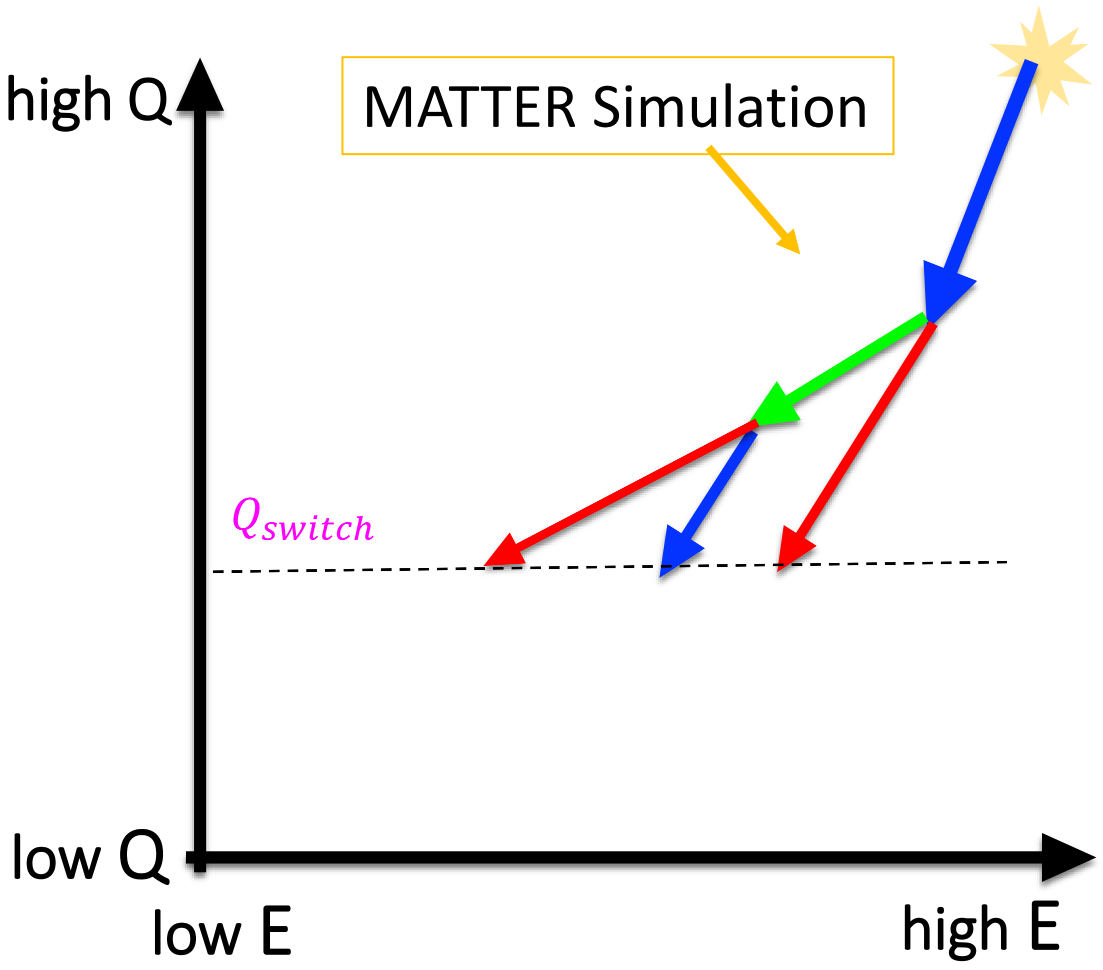
- Scatterings affect differently highly virtual particles compared to near-on-shell particles

Multistage parton evolution in JETSCAPE



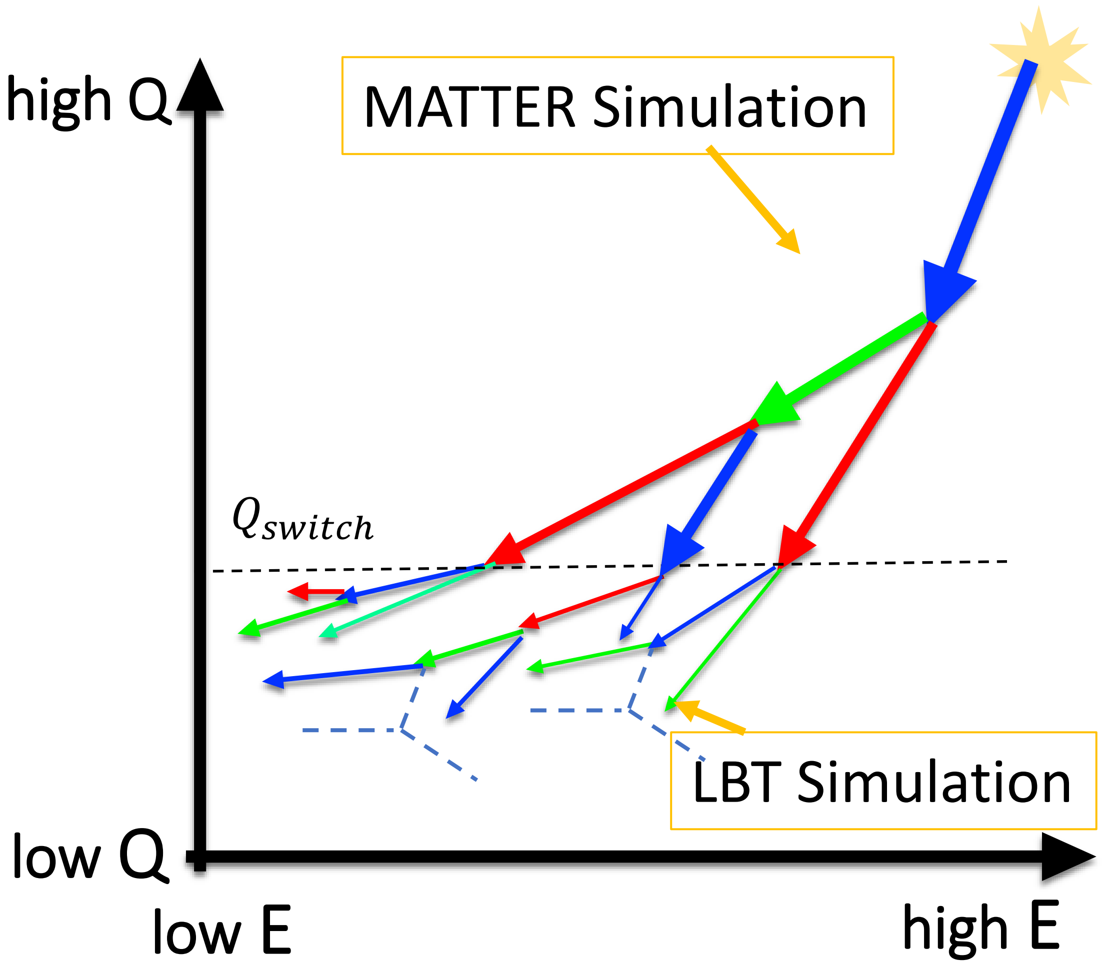
- High→Lower Q, High E: Rapid virtuality loss through radiation. MATTER (via Higher Twist) generates scattering-modified radiation.

Multistage parton evolution in JETSCAPE



- High→Lower Q, High E: Rapid virtuality loss through radiation. MATTER (via Higher Twist) generates scattering-modified radiation. MATTER is evolved until a switching virtuality (Q_{switch}) is reached.

Multistage parton evolution in JETSCAPE



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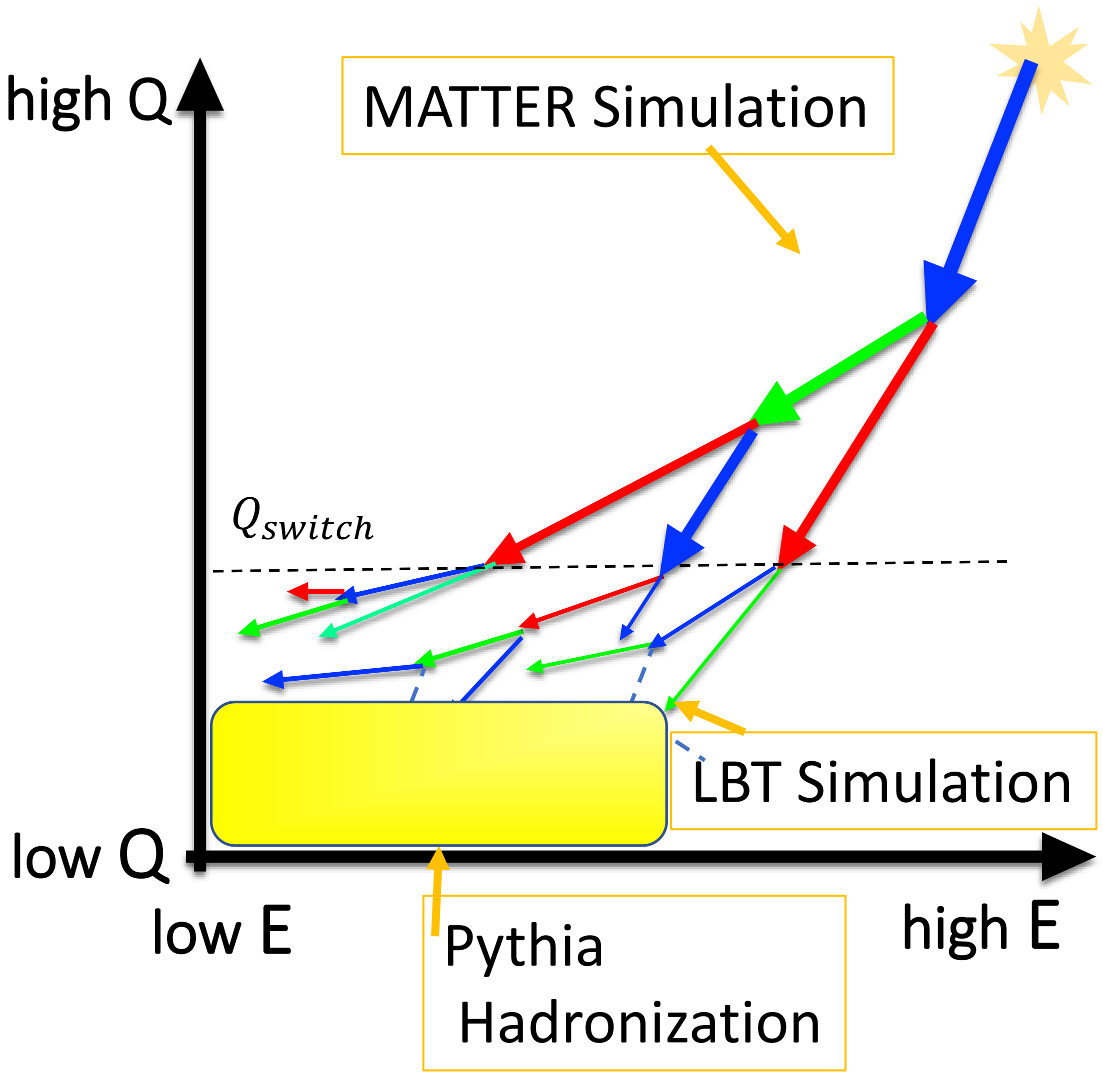

- High→Lower Q, High E: Rapid virtuality loss through radiation. MATTER (via Higher Twist) generates scattering-modified radiation.
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- Low Q, Low E: Hadronization physics important (partons→Pythia for hadronization)

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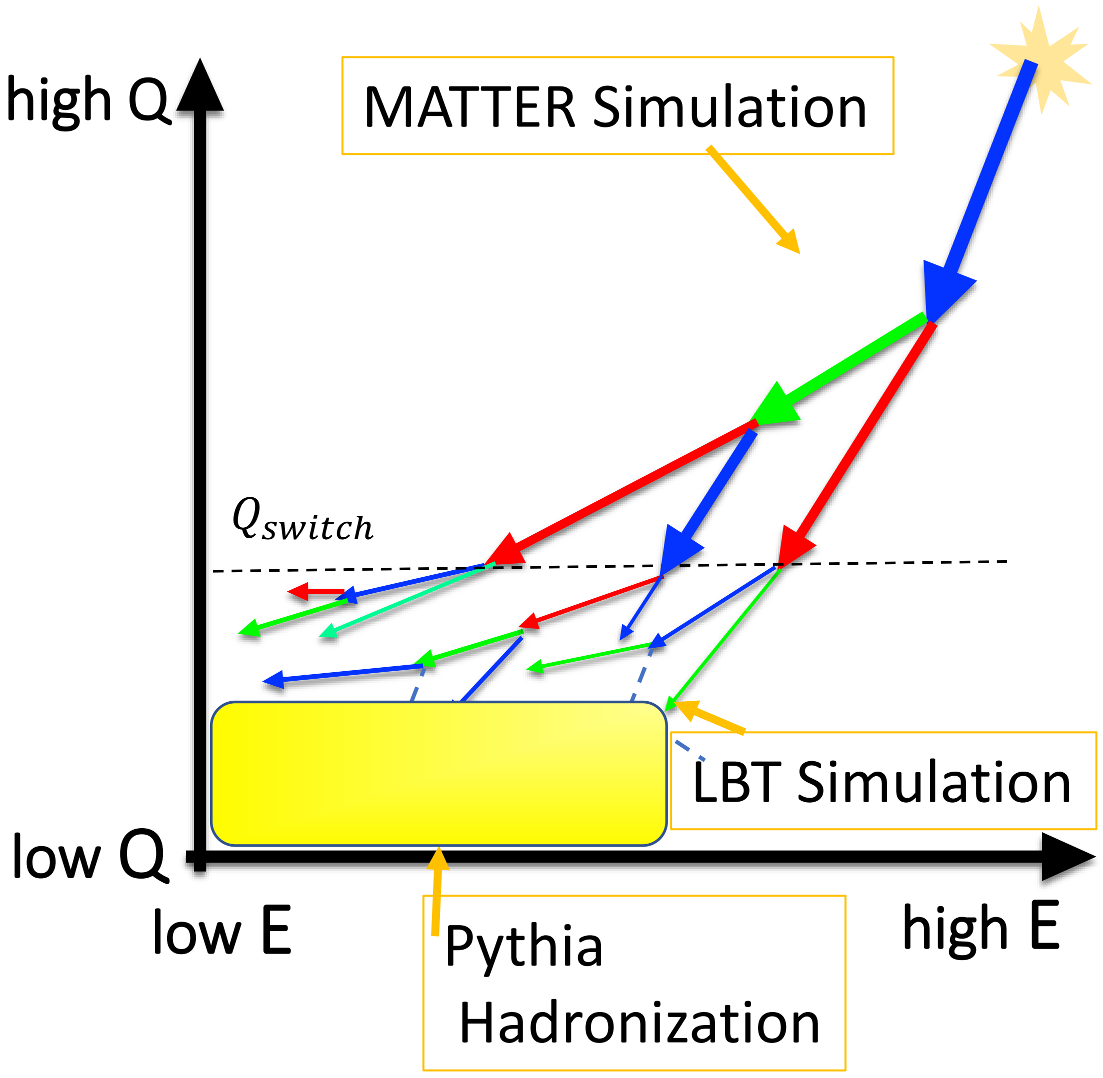

- High→Lower Q, High E: Rapid virtuality loss through radiation. MATTER (via Higher Twist) generates scattering-modified radiation.
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- Low Q, Low E: Hadronization physics important (partons→Pythia for hadronization)

The JETSCAPE framework combines these multiple stages for an improved description of parton energy loss.

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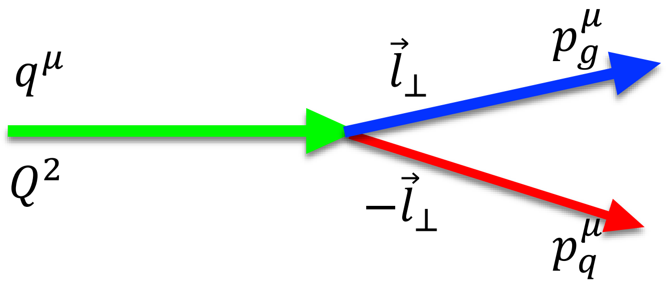
The splitting function for a quark



- Splitting function for a quark in vacuum (Peskin & Schroeder):

$$P_{g\leftarrow q}(y) = P(y) = C_F \frac{1+(1-y)^2}{y}$$

w/ the Casimir for $SU(N_c = 3)$ $C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$

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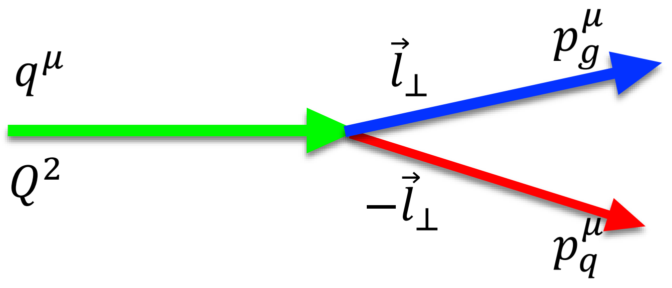

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$$P(y) = \frac{4}{3}\frac{1+(1-y)^2}{y}$$

- A few definitions:

$$q^\mu = (q^+, q^-, \vec{q}_\perp) = \left(q^+, \frac{Q^2+M^2}{2q^+}, \vec{0}_\perp\right)$$

$$q^+ = \frac{q^0+q^z}{\sqrt{2}}; \quad q^- = \frac{q^0-q^z}{\sqrt{2}}$$

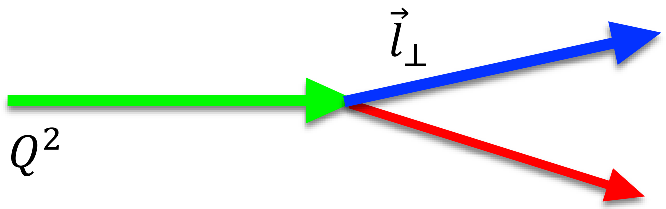
$$\Rightarrow q^2 - M^2 = 2q^+q^- - \vec{q}_\perp^{\,2} - M^2 = Q^2$$

$$q^\mu = p_q^\mu + p_g^\mu$$

$$\Rightarrow p_q^\mu = \left((1-y)q^+, \frac{\vec{l}_\perp^{\,2}+M^2}{2(1-y)q^+}, -\vec{l}_\perp\right)$$

$$\Rightarrow p_g^\mu = \left(yq^+, \frac{\vec{l}_\perp^{\,2}}{2yq^+}, \vec{l}_\perp\right)$$

The splitting function for a quark



- Splitting function for a quark in vacuum (Peskin & Schroeder):

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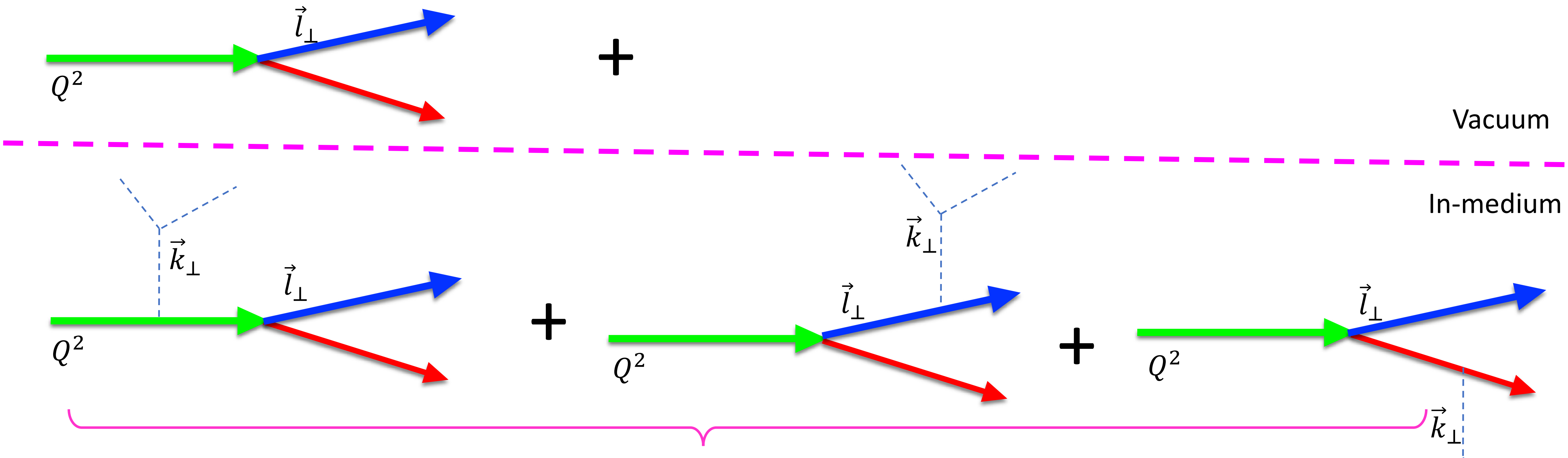
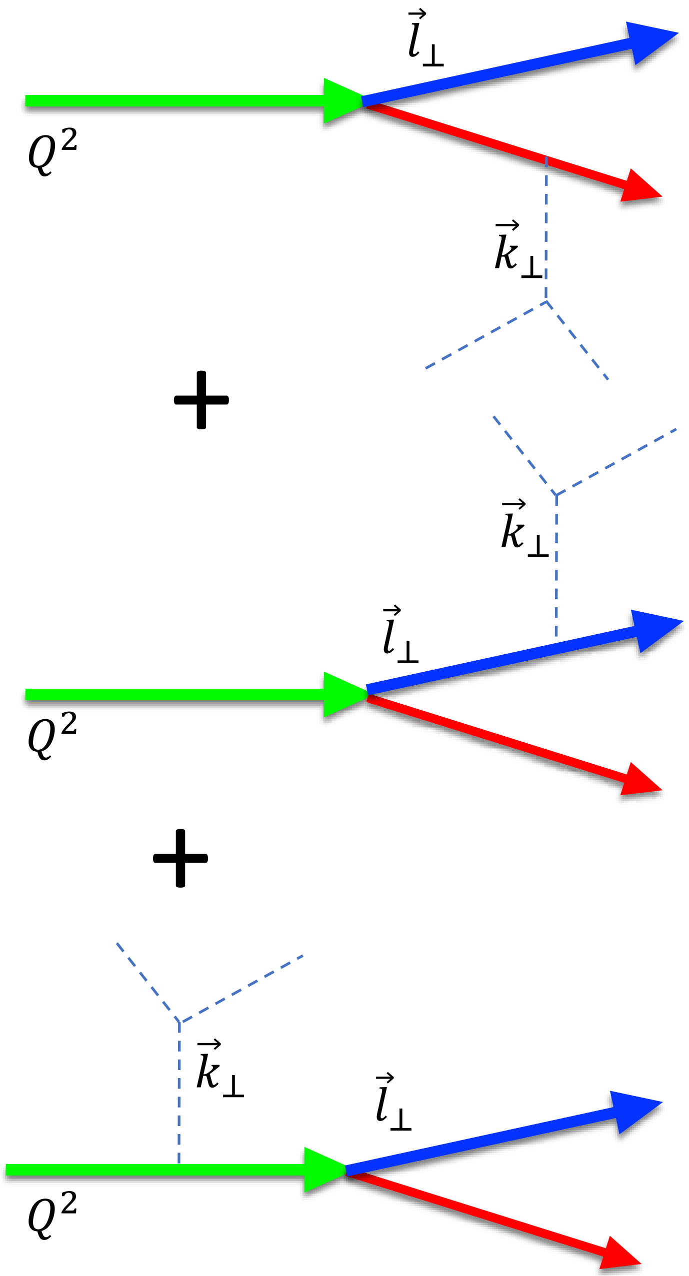
- The in-medium splitting function is affected by scattering. At high Q^2, only a single scattering is allowed, thus the full splitting function is :



There 3 diagrams interfere in the medium!

Coherent scatterings at high virtuality



- If $l_\perp^2 \sim k_\perp^2 \Rightarrow$ medium can resolve the two daughter partons

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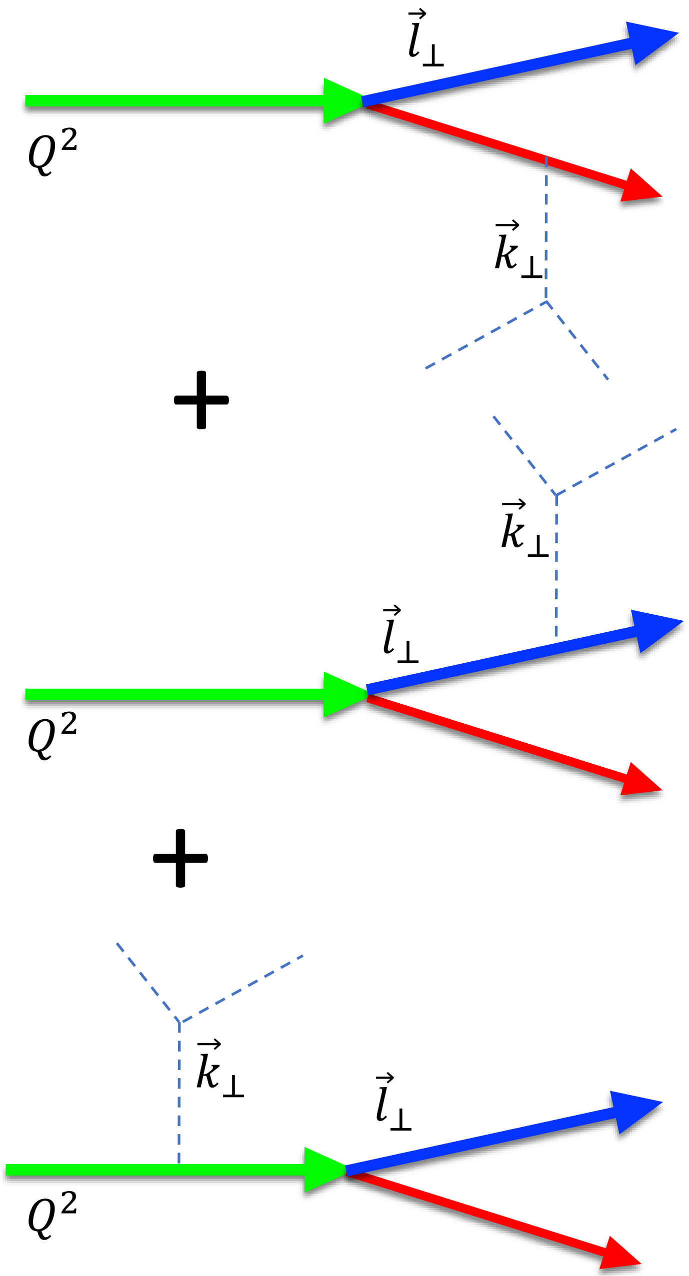
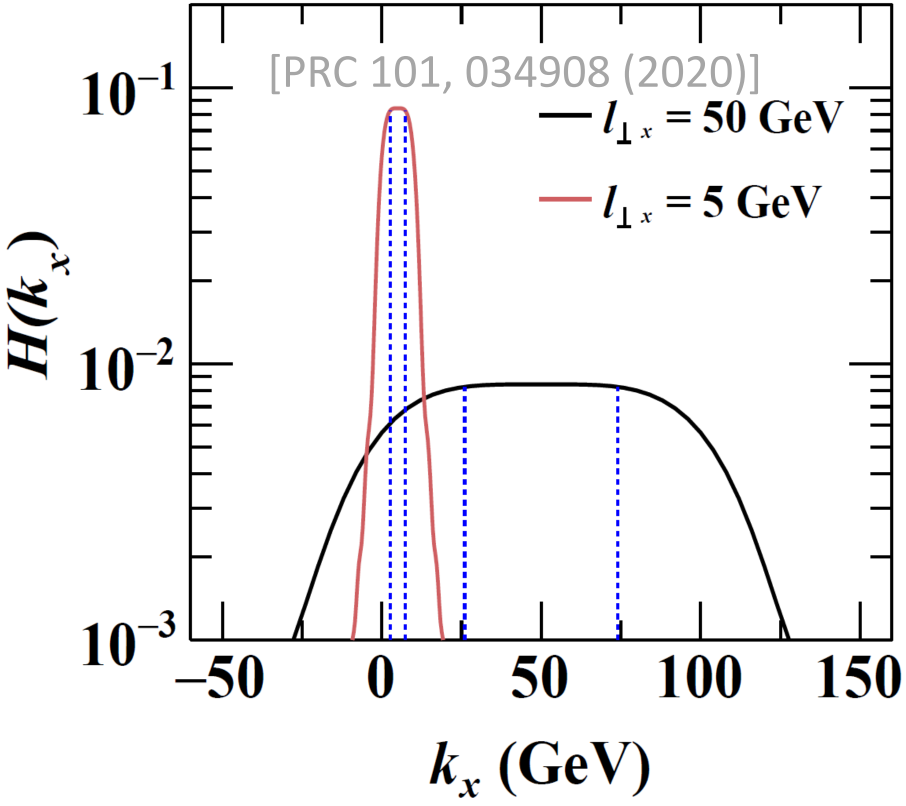


- If $l_\perp^2 \sim k_\perp^2 \Rightarrow$ medium can resolve the two daughter partons
- The interference between these diagrams gives (for an incoming light-flavor)



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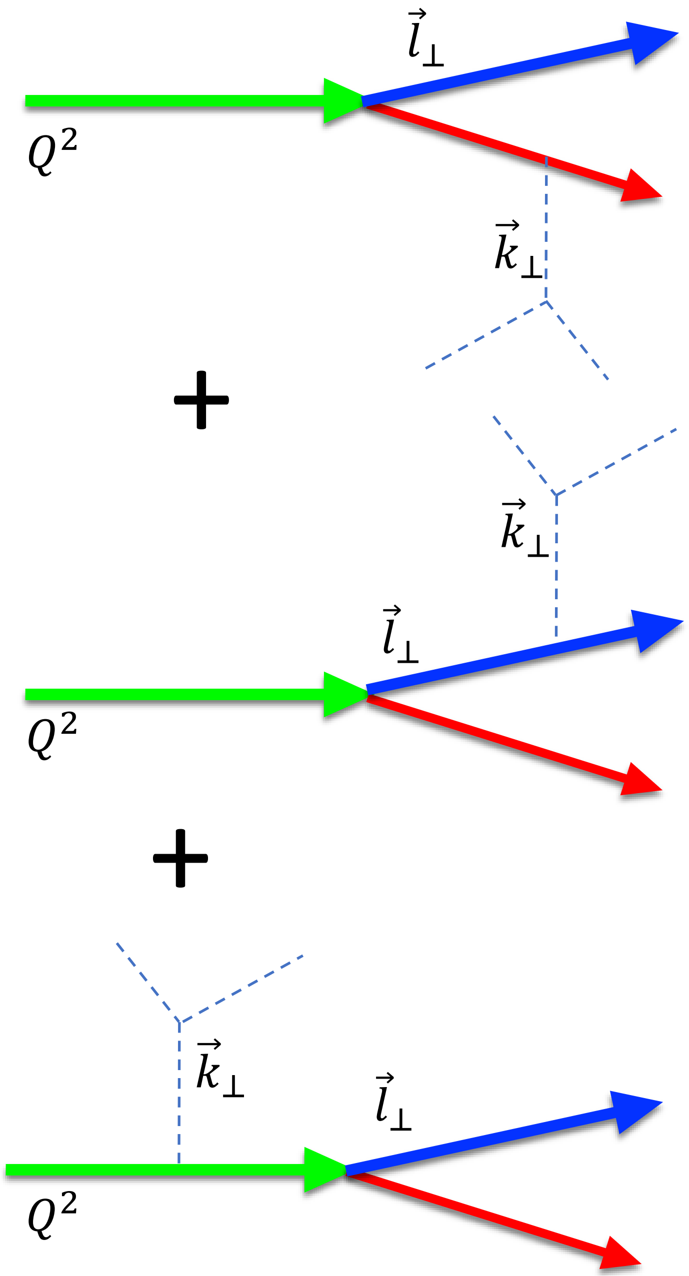

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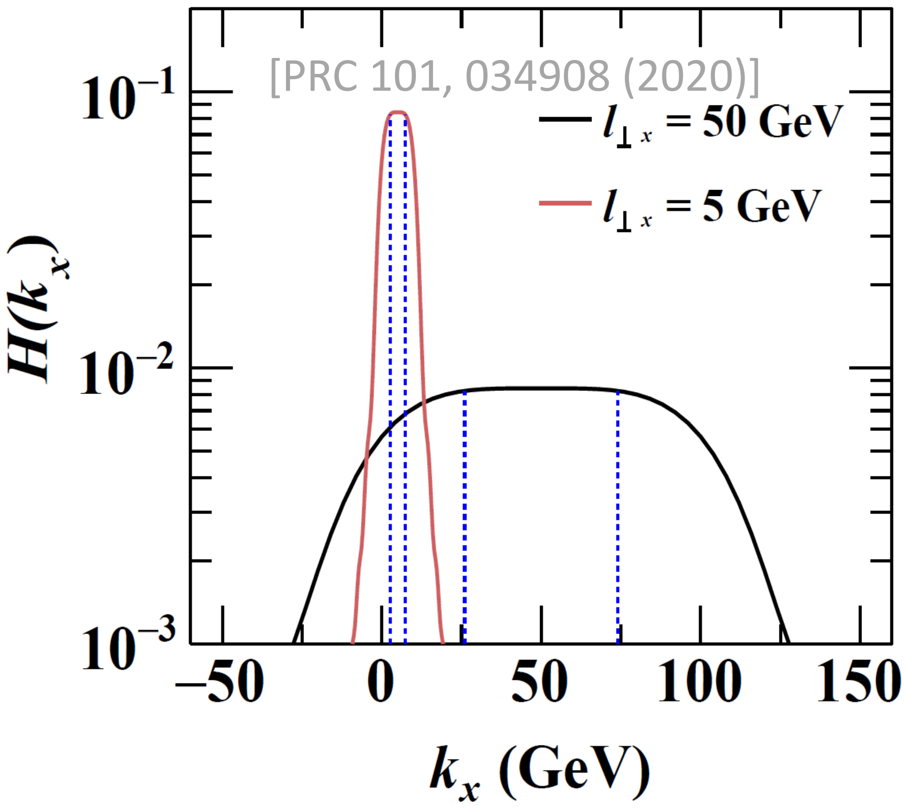
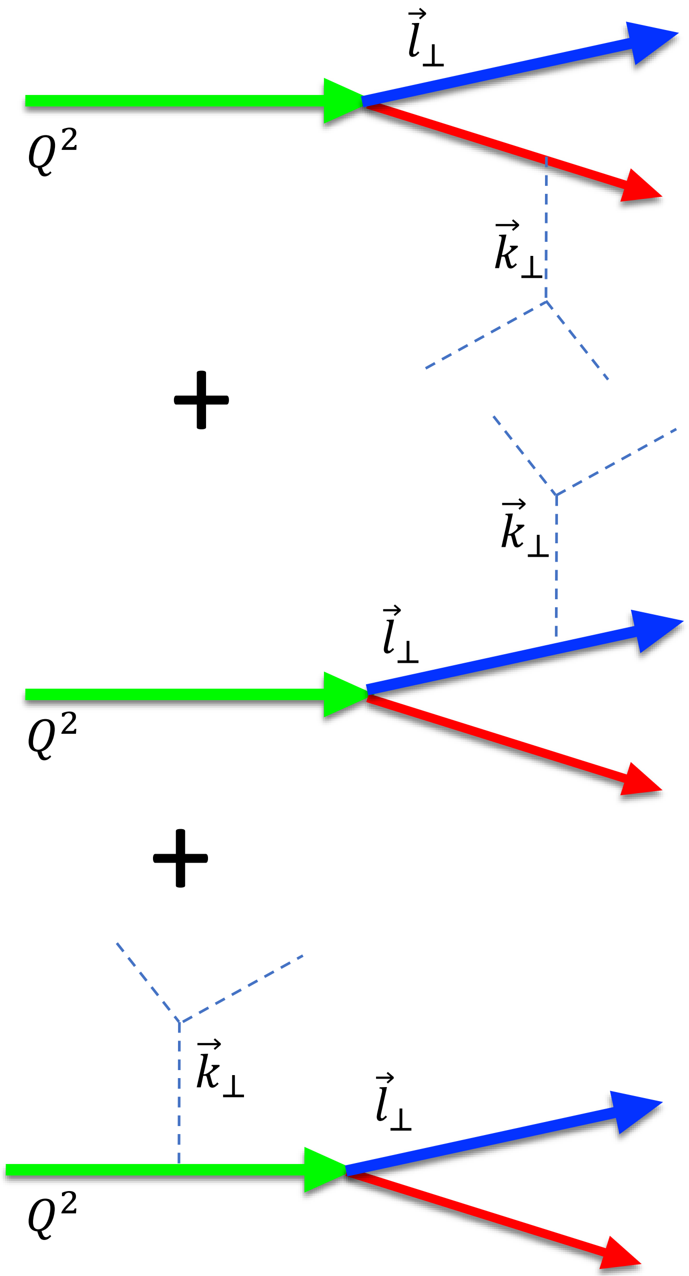


- Inside blue dotted lines: $k_\perp$-range of coherent scattering
- Outside blue dotted lines: scattering and radiation become more and more incoherent

Coherent scatterings at high virtuality



- If $l_\perp^2 \sim k_\perp^2 \Rightarrow$ medium can resolve the two daughter partons
- Taylor-expanding in $k_\perp$ and integrating over $k_\perp$, the medium-modified splitting function (for incoming light quarks) is

$$\mathcal{P}(y) = P(y) + \tilde{\mathcal{P}}(y)$$

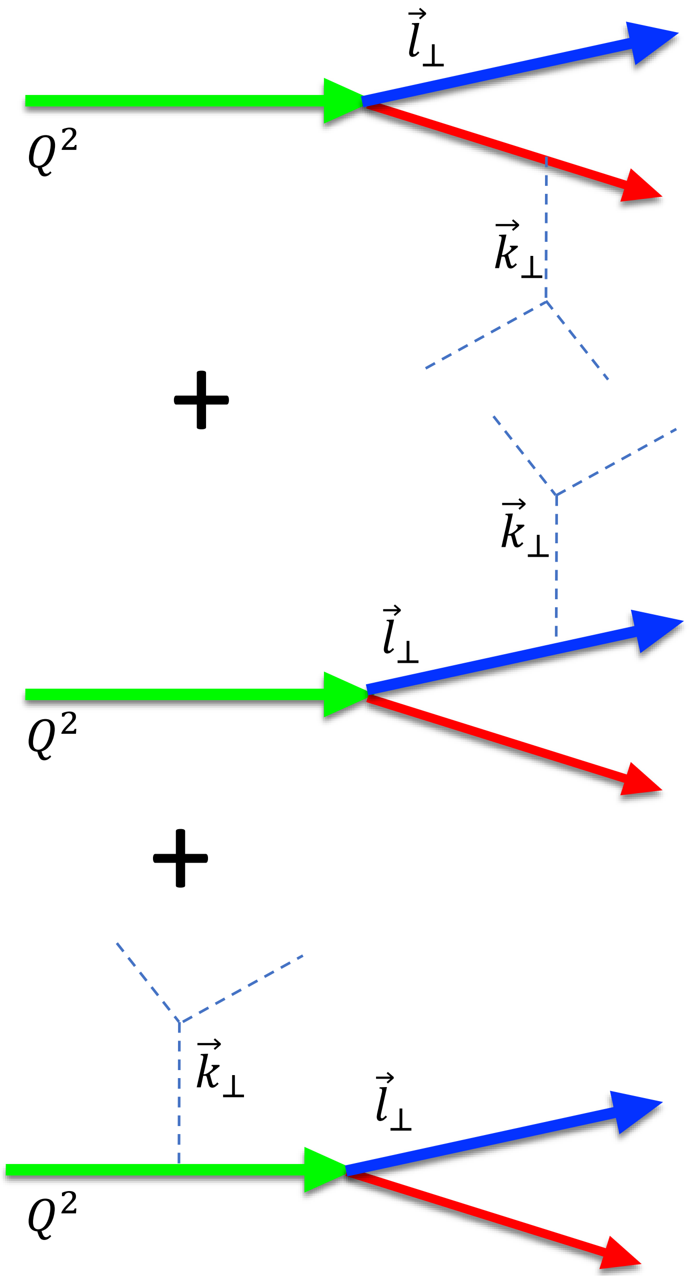
[PRC 101, 034908 (2020)]

$$\tilde{\mathcal{P}}(y) = \frac{P(y) \int_0^{\tau_f^+} d\tau^+ \left[2 - 2\cos\left(\tau^+/\tau_f^+\right)\right] \hat{q}\left(Q^2\right)}{y(1-y)Q^2}$$

$$\tau_f^+ = \frac{2q^+}{Q^2} = \frac{2q^+ y(1-y)}{l_\perp^2}$$

- A part of coherent scattering effects is in $2 - 2\cos\left(\tau/\tau_f\right)$
- The other part coherence is included in $\hat{q}\left(Q^2\right)$

Coherent scatterings at high virtuality



- If $l_\perp^2 \sim k_\perp^2 \Rightarrow$ medium can resolve the two daughter partons
- Applying the light flavor approach on heavy flavors, the in-medium splitting function is:

$$\mathcal{P}(y) = P(y) + \tilde{\mathcal{P}}(y)$$ [PRC 94, 054902 (2016)]

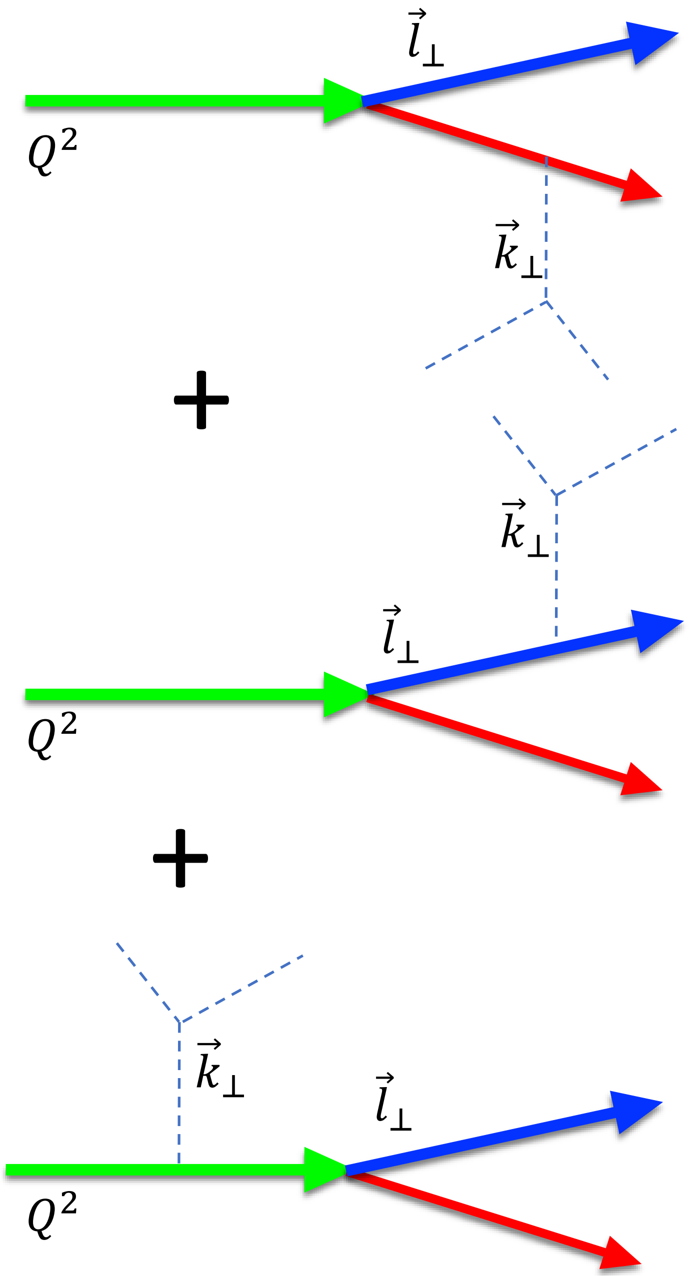
$$\tilde{\mathcal{P}}(y) = \frac{P(y)A(y,\chi)\int_0^{\tau_f^+} d\tau^+\left[2 - 2\cos\left(\tau^+/\tau_f^+\right)\right]\hat{q}}{y(1-y)Q^2(1+\chi)^2}$$

$$A(y,\chi) = \left\{\left(1-\frac{y}{2}\right) - \chi + \left(1-\frac{y}{2}\right)\chi^2\right\}$$

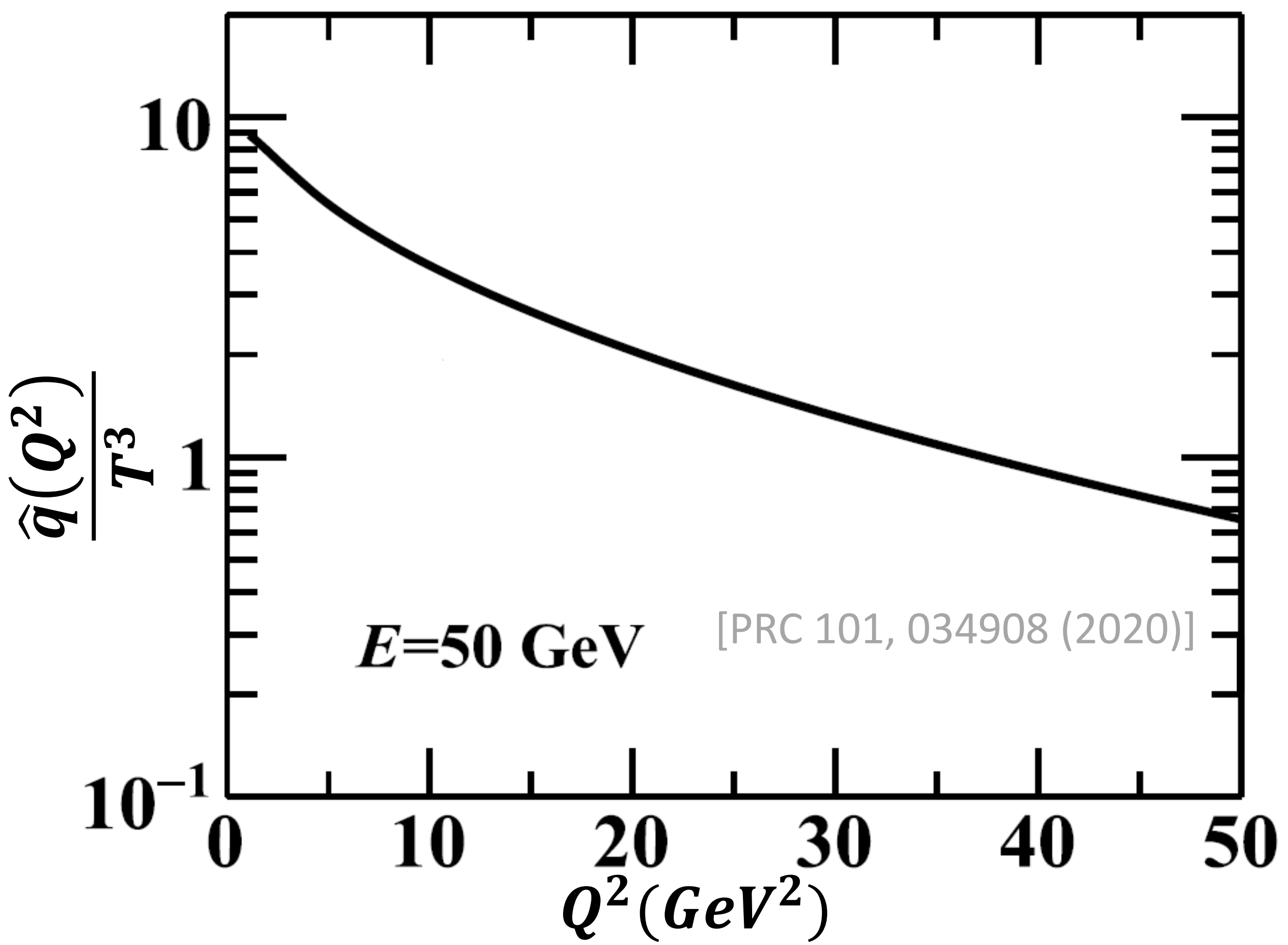
$$\chi = \frac{y^2M^2}{l_\perp^2} = \frac{y^2M^2}{y(1-y)Q^2 - y^2M^2}$$

$$\tau_f^+ = \frac{2q^+y(1-y)}{l_\perp^2 + y^2M^2}$$

Virtuality-dependent $\hat{q}$

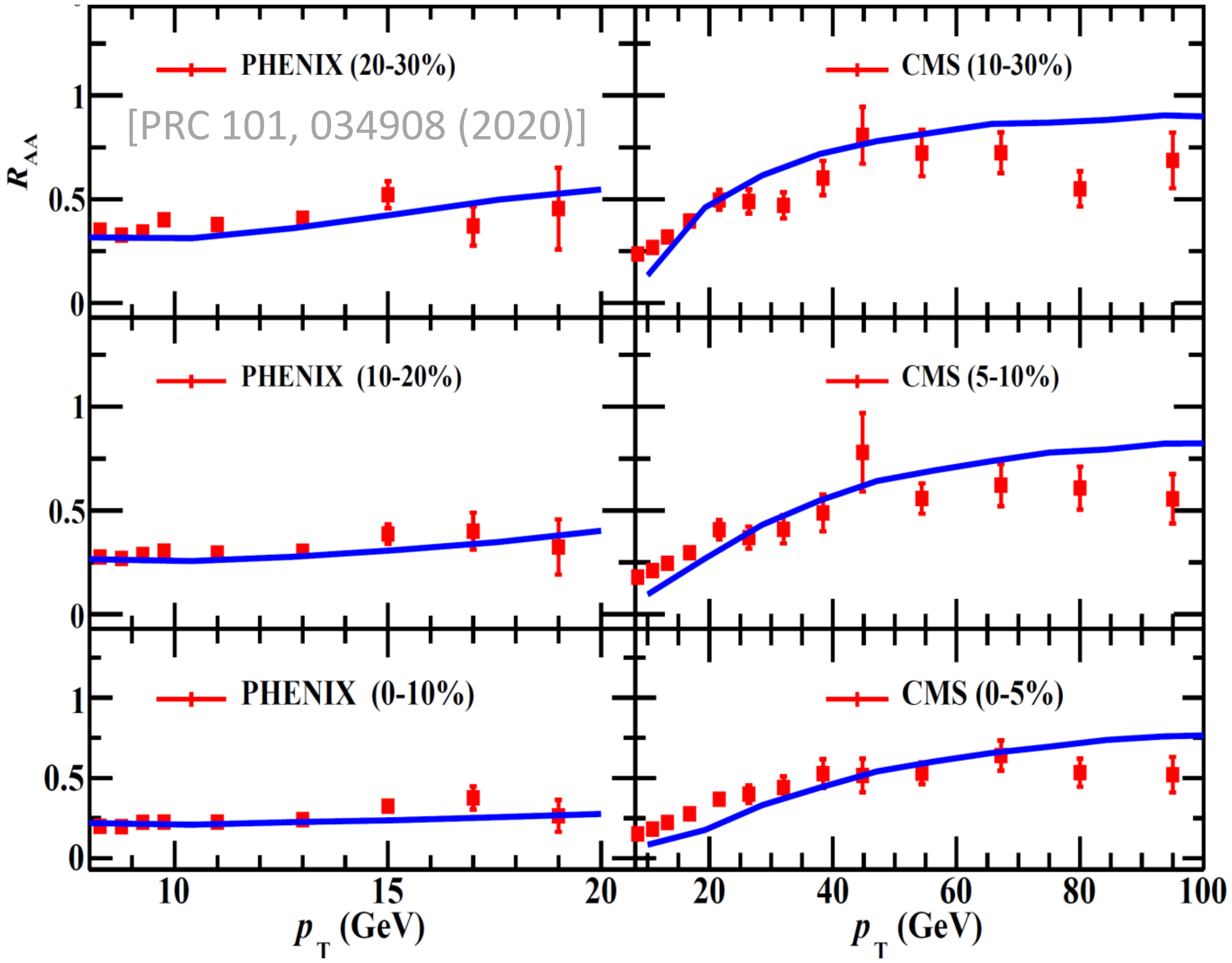


- For light quarks, we get



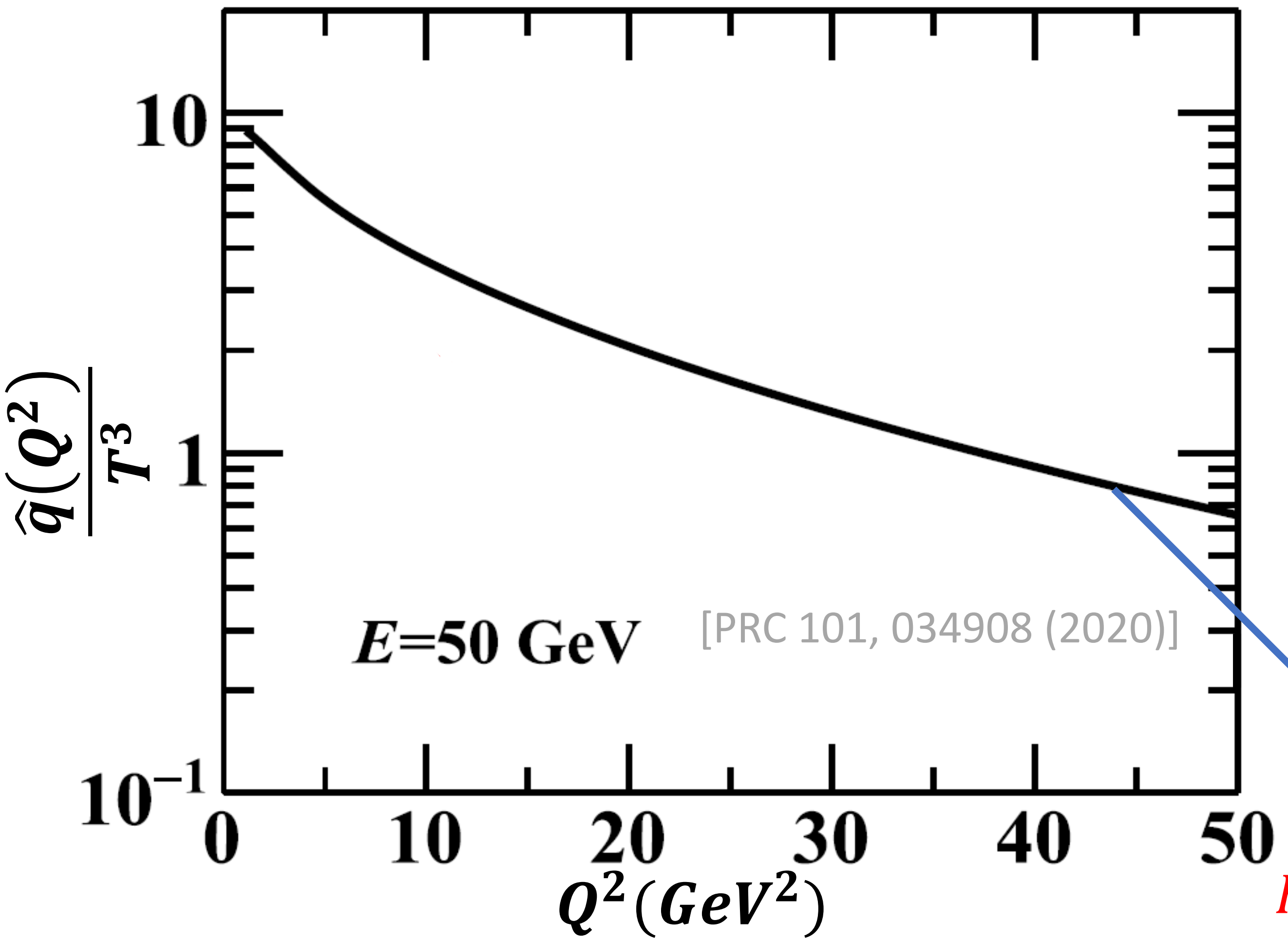
- For heavy quarks $\hat{q}(Q^2, M)$ is yet to be determined using the (SCET) formalism of PRC 94, 054902 (2016)

Why is a virtuality-dependent $\hat{q}$ needed?



- $\hat{q}(Q^2)$ is a key ingredient to simultaneously describe leading hadron R_{AA} at different $\sqrt{s_{NN}}$.

Parametrizing the virtuality dependence of $\hat{q}$



- $\hat{q}(Q^2)$ is a key ingredient to simultaneously describe leading hadron R_{AA} at different $\sqrt{s_{NN}}$.
- Explore how this $\hat{q}(Q^2)$ affects heavy quarks in a multi-scale MATTER+LBT simulation using a parametrization:

$$\hat{q}(Q^2) = \hat{q}_{HTL} H(Q^2)$$

$$\hat{q}_{HTL} \propto \alpha_s^2 T^3 \ln\left[\frac{cE}{\alpha_s T}\right]$$

$$H(Q^2) = \begin{cases} 1 & Q^2 < Q_s^2 \\ \dfrac{1 + c_1 \ln^2(Q_s^2) + c_2 \ln^4(Q_s^2)}{1 + c_1 \ln^2(Q^2) + c_2 \ln^4(Q^2)} & Q^2 \geq Q_s^2 \end{cases}$$

Higher Twist Energy Loss for Heavy Quarks

- **MATTER** (The **M**odular **A**ll **T**wist **T**ransverse-scattering **E**lastic-drag and **R**adiation) valid for High E, High Q^2
  - Virtuality-ordered shower with splittings above $Q^2 \gg Q^2_{switch}$.

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  - Virtuality-ordered shower with splittings above $Q^2 \gg Q^2_{switch}$. This is unlike PYTHIA, since in MATTER the daughter Q_d^2 is found before θ_d (or $l_\perp$) between the daughters is known $\Rightarrow \theta_{d_1} > \cdots > \theta_{d_i} > \cdots > \theta_{d_n}$ is not strictly ensured for a daughter generation $i \in n$.

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  - The Sudakov form factor assigns virtuality to each parton by throwing a random number # and solving for $\# = \Delta(Q)$. [Adv. Ser. Direct. HEP, 573 (1989); NPA 696, 788 (2001)]

$$\Delta(Q_{\max}, Q \geq Q_{\min}) = \exp\left[- \int_{Q^2}^{Q^2_{\max}} \frac{d(Q^2)}{Q^2} \frac{\alpha_s(Q^2)}{2\pi} \int_{y_{\min}}^{y_{\max}} dy\, \mathcal{P}(y, Q^2) \right]$$

$$Q^2_{max} = \frac{E^2}{4} \forall M$$

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- For $Q \to Q + g$

$$Q^2_{\min} = \frac{Q_0^2}{2}\left[1 + \sqrt{1 + \frac{4M^2}{Q_0^2}}\right]$$

$$Q_0^2 = a\ cutoff\ (non-pert.)\ scale\ (typ.\ Q_0 = 1\ \text{GeV})$$

$$y_{max} = 1 - \frac{Q_0^2}{2Q^2}$$

$$y_{\min} = \frac{Q_0^2}{2Q^2} + \frac{M^2}{M^2 + Q^2}$$

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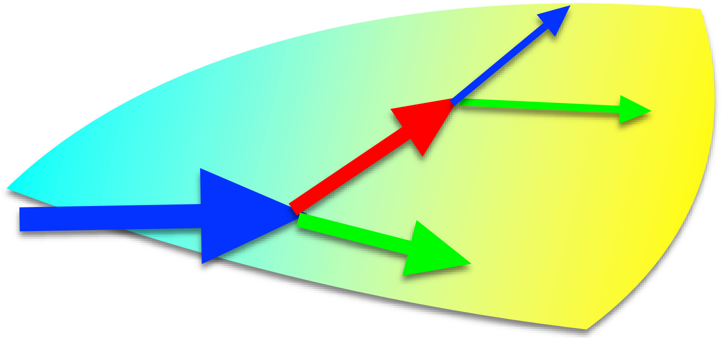
  - Recursive Sudakov application for each daughter (giving Q_d^2) generates a shower.



  - Where different decay channels are possible, $\mathcal{P} \to \sum_i \mathcal{P}_i$ where i identifies the processes. For gluons, i labels $g \to gg, g \to q\bar{q}$, and $g \to \bar{Q}Q$.

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  - For $g \rightarrow Q\bar{Q}$**,** the splitting function is phenomenologically estimated using light flavor $g \rightarrow q\bar{q}$, with appropriate kinematic cuts to account for heavy flavor mass (i. e. $y_{\min}, y_{\max}, Q^2_{\min}$).

$$\mathcal{P}(y, \mu^2) = P(y) + \frac{P(y)\int_{\tau_i}^{\tau_f} d\tau \hat{q}(Q^2)\left[2 - 2\cos\left[\frac{\tau}{\tau_f(Q^2)}\right]\right]}{y(1-y)Q^2}$$

$$Q^2_{\min} = Q_0^2 + 2M^2$$

$$y_{max} = 1 - \frac{Q_0^2}{2Q^2}$$

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  - Also, the scale M should also play are role in $\hat{q}\left(Q^2, M\right)$. For now, only $\hat{q}\left(Q^2\right)$ is used.

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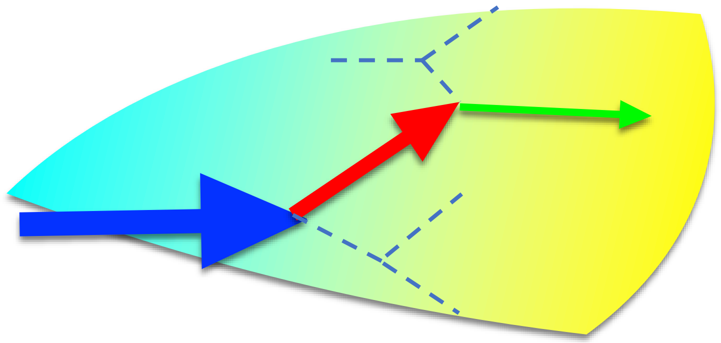
  - Virtuality-ordered shower with splittings above $Q^2 \gg Q^2_{switch}$.
  - The Sudakov form factor assigns virtuality to each parton by throwing a random number # and solving for # $= \Delta(Q)$. [Adv. Ser. Direct. HEP, 573 (1989); NPA 696, 788 (2001)]

$$\Delta(Q_{\max}, Q \geq Q_{\min}) = \exp\left[-\int_{Q^2}^{Q^2_{\max}} \frac{d(Q^2)}{Q^2} \frac{\alpha_s(Q^2)}{2\pi} \int_{y_{\min}}^{y_{\max}} dy\, \mathcal{P}(y, Q^2)\right]$$

  - MATTER also calculates 2 → 2 scatterings, using the LO perturbative QCD (pQCD) formula



  - The details about 2 → 2 scatterings is given in the next section.

Outline

- Overview of physics and JETSCAPE modules
- MATTER and the high-virtuality evolution
- LBT and low-virtuality evolution
- Results with heavy flavors and future developments
- Conclusion & Outlook

Linear Boltzmann Transport for Heavy Quarks

- Valid for high E, assuming particles are (near) on-shell ($\Rightarrow Q \leq Q_{switch}$)
- Solves the *effective* Boltzmann eq. for the phase space distribution function

$$p_1 \cdot \partial f(x, p_1) = \mathcal{C}_{el} + \mathcal{G}_{inel}$$

- The Boltzmann equation is valid in very dilute media where any n-particle correlations $\forall n \geq 2$ are neglected.
    - $2 \rightarrow 2$ scattering is allowed in $\mathcal{C}_{el}$. To calculate $\mathcal{C}_{el}$ one only needs 1-particle distributions.
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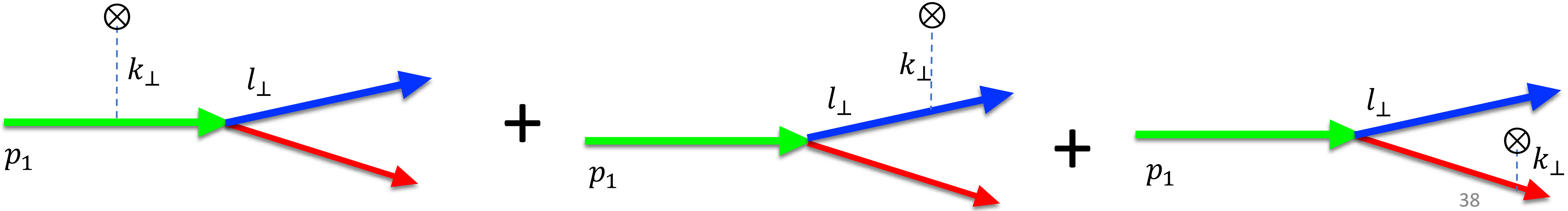
  - $1 \rightarrow 2$ decays $\mathcal{G}_{inel}$ in the vacuum are allowed, only a 1-particle distribution is needed.
  - In-medium $1 \rightarrow 2$ decays would require 2-particle distributions, e.g., $f_2(k_\perp | p_1)$, to capture the correlation/interference between the scattering and the decay.



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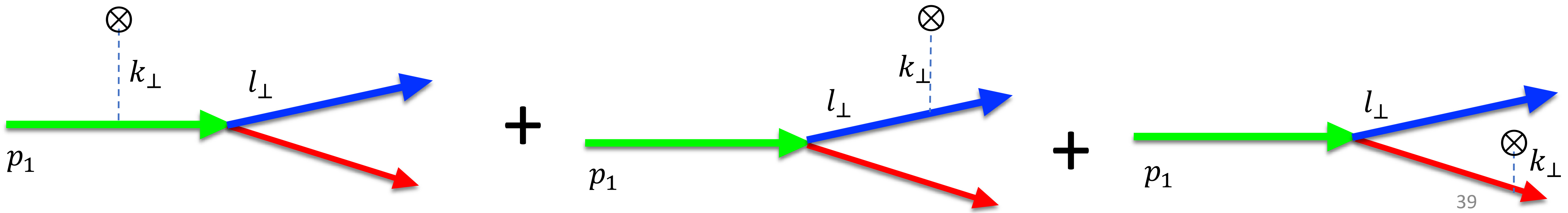
  - $1 \rightarrow 2$ decays $\mathcal{G}_{inel}$ in the vacuum are allowed, only a 1-particle distribution is needed.
- Phenomenologically, keeping track of the n-particle distributions is numerically costly, so an *effective* Boltzmann eq. is used that includes medium-induced decay $\mathcal{G}_{inel}$ on the r.h.s. *w/o* computing the *evolution* of a 2-particle distribution, for example, $f_2(k_\perp | p_1)$.



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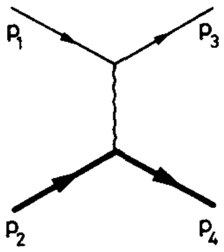
$$\mathcal{C}_{el} \approx -\frac{1}{2p_1^0}\int \frac{d^3p_2}{2p_2^0(2\pi)^3}\int \frac{d^3p_3}{2p_3^0(2\pi)^3}\int \frac{d^3p_4}{2p_4^0(2\pi)^3} f(p_1)f(p_2)\left|\bar{\mathcal{M}}\right|^2[1 \pm f(p_3)][1 \pm f(p_4)](2\pi)^4\delta^{(4)}(p_1 + p_2 - p_3 - p_4)$$

C_{el} gives the rate ***density*** of collisions, i.e.,

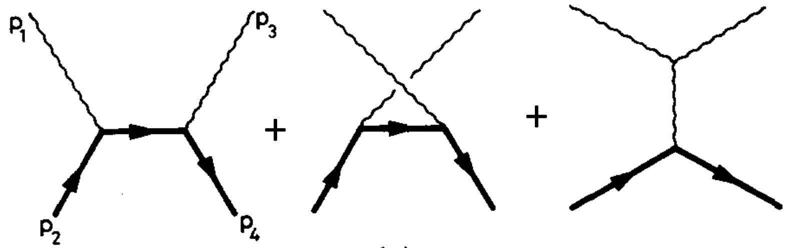
$$C_{el} = (2\pi)^3\frac{d^3R}{d^3p_1} = (2\pi)^3\frac{d^7N}{d^4x d^3p_1}$$

$\Rightarrow$ total number of collisions is:

$$N = \int_{QGP} d^4x \int \frac{d^3p_1}{(2\pi)^3}(2\pi)^3 C_{el}$$



(a)



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- A comparison with vacuum:

Inside the medium:

$$(2\pi)^3\frac{d^3N}{d^3p_1} = \int_{QGP} d^4x\, C_{el} \Rightarrow \text{total number of scatterings N} = \int \frac{d^3p_1}{(2\pi)^3}(2\pi)^3\frac{d^3N}{d^3p_1}$$

Inside vacuum $N = L\sigma$:

$$L \propto \left\{\prod_{i=1,2}\left[\int \frac{d^3p_i}{(2\pi)^3} f(p_i)\right]\right\} 4\sqrt{(p_1 \cdot p_2)^2 - p_1^2 p_2^2}; \qquad \sigma \propto \frac{1}{2p_1^0 2p_2^0}\left\{\prod_{f=3,4}\left[\int \frac{d^3p_f}{2p_f^0(2\pi)^3}\right]\right\}\frac{\left|\bar{\mathcal{M}}\right|^2(2\pi)^4\delta^{(4)}(p_1 + p_2 - p_3 - p_4)}{4\sqrt{(p_1 \cdot p_2)^2 - p_1^2 p_2^2}}$$

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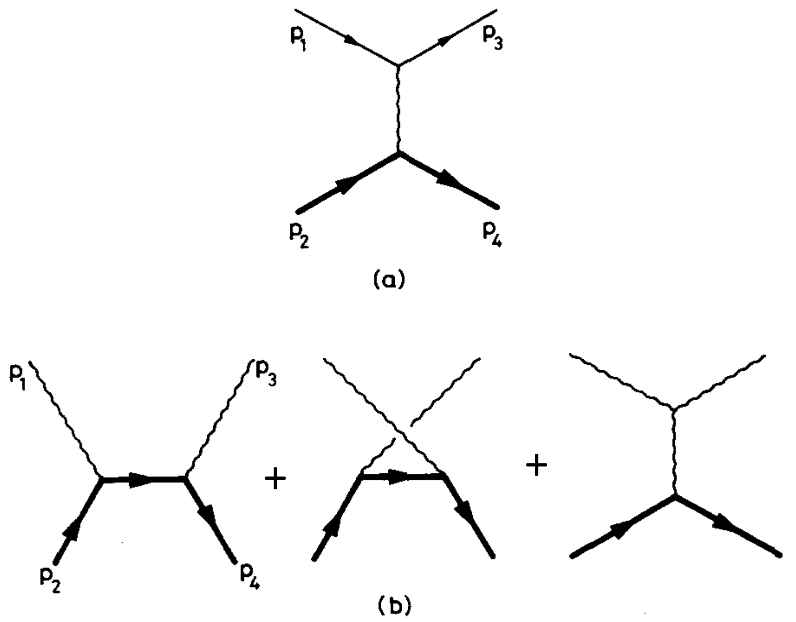
Aside: This is a loss contribution to the $f(p_1)$ evolution on the lhs. Indeed, heavy quarks starts in the p_1 state and is scattered out of p_1 (and into p_3), giving a reduction in $f(p_1)$, hence the minus sign.



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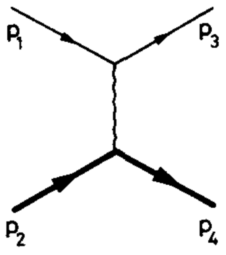
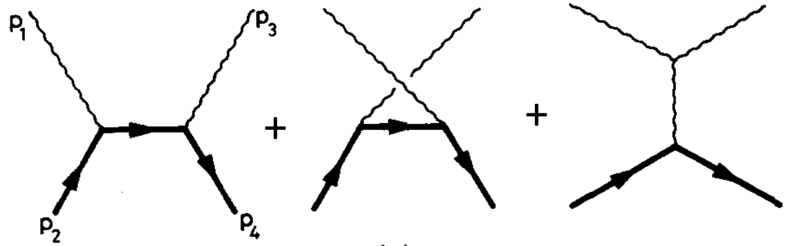


(a)

- When running a Monte Carlo (MC) simulation, $f(x, p_1)$ is replaced by sampled particles in the jet.
- Averaging over many MC simulations can reconstruct $f(x, p_1)$.



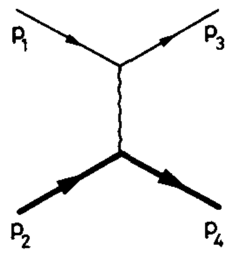
(b)

Linear Boltzmann Transport for Heavy Quarks

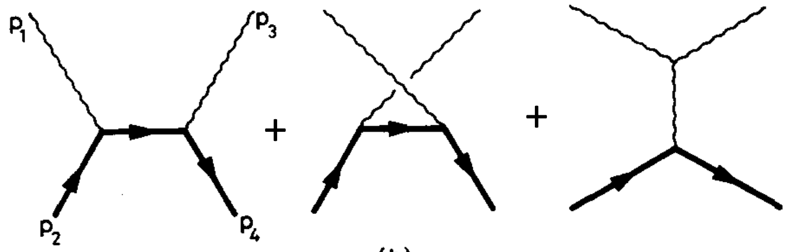
- To calculate the collisional rate density for a single parton in the QGP (at LO pQCD)

$$\mathcal{C}_{el} \approx -\frac{1}{2p_1^0}\int \frac{d^3p_2}{2p_2^0(2\pi)^3}\int \frac{d^3p_3}{2p_3^0(2\pi)^3}\int \frac{d^3p_4}{2p_4^0(2\pi)^3} \cancelto{1}{f(p_1)} f(p_2)\left|\bar{\mathcal{M}}\right|^2 \cancelto{1}{[1\pm f(p_3)]}[1\pm f(p_4)](2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

p_1 and p_3 are not part of the thermal medium: i.e. they are part of the jet, whose *quantum* distribution is MC sampled in LBT. This *unlike* the QGP which uses *classical* hydrodynamics.



(a)



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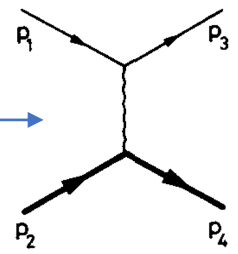
- Leading order QCD $\left|\bar{\mathcal{M}}\right|^2$ for 2 → 2 scatterigns

$$\left|\bar{\mathcal{M}}\right|_a^2 = \frac{64}{9}\pi^2\alpha_s^2\frac{\left(M^2-u\right)^2+\left(s-M^2\right)^2+2M^2t}{t^2}$$

$$\lim_{M\to 0^+}\left|\bar{\mathcal{M}}\right|_a^2 = \frac{64}{9}\pi^2\alpha_s^2\frac{u^2+s^2}{t^2}$$



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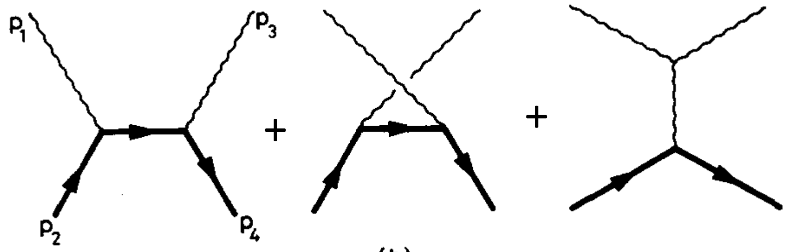

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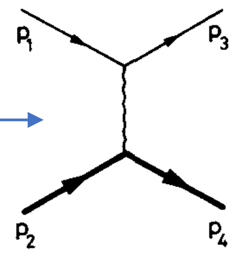
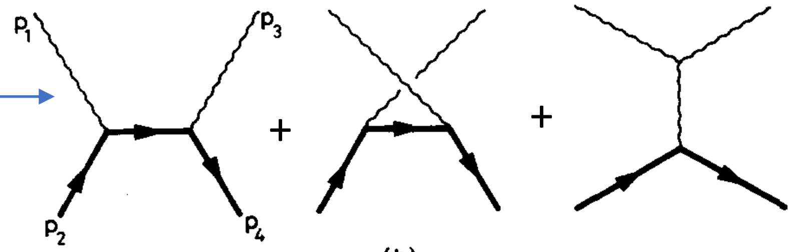


(a)

For $\left|\bar{\mathcal{M}}\right|_b^2$ see Nucl. Phys. B 151, 429 (1979) where the full expression is given.



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- The $\mathcal{G}_{inel}$ calculates medium-induced stimulated $1 \rightarrow 2$ emission at LO in $\left(\alpha_s, \frac{\mathrm{M}^2}{\mathrm{Q}^2}\right)$

[PRC 94, 054902 (2016)]

$$\mathcal{G}_{inel} = \int \frac{d\left(Q^2\right)}{Q^2}\frac{\alpha_s\left(Q^2\right)}{2\pi}\int dy\,\mathcal{P}(y)$$

$$\mathcal{P}(y) = P(y) + \frac{P(y)\left[\left(1-\frac{y}{2}\right)-\chi+\left(1-\frac{y}{2}\right)\chi^2\right]\int_0^{\tau_f^+} d\tau^+\,\hat{q}_{HTL}\left[2-2\cos\left(\tau^+/\tau_f^+\right)\right]}{y(1-y)Q^2(1+\chi)^2}$$

$$\tau_f^+ = \frac{2q^+y(1-y)}{l_\perp^2(1+\chi)}$$

$$\chi = \frac{y^2M^2}{l_\perp^2}$$

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An experimental observable

- To study the nuclear medium's effects on parton shower, one computes nuclear modification factor

$$R_{AA}^{X} = \frac{\frac{d\sigma_{AA}^{X}}{dp_T}}{N_{bin}\frac{d\sigma_{pp}^{X}}{dp_T}}$$

X is the leading (highest energy) hadron in a jet (which can be of an identified species or not)

- If an A-A collisions was the same as p-p collisions, then we can rescale the p-p collision by the N_{bin} binary collisions $\Rightarrow R_{AA}^{X} \to 1$.

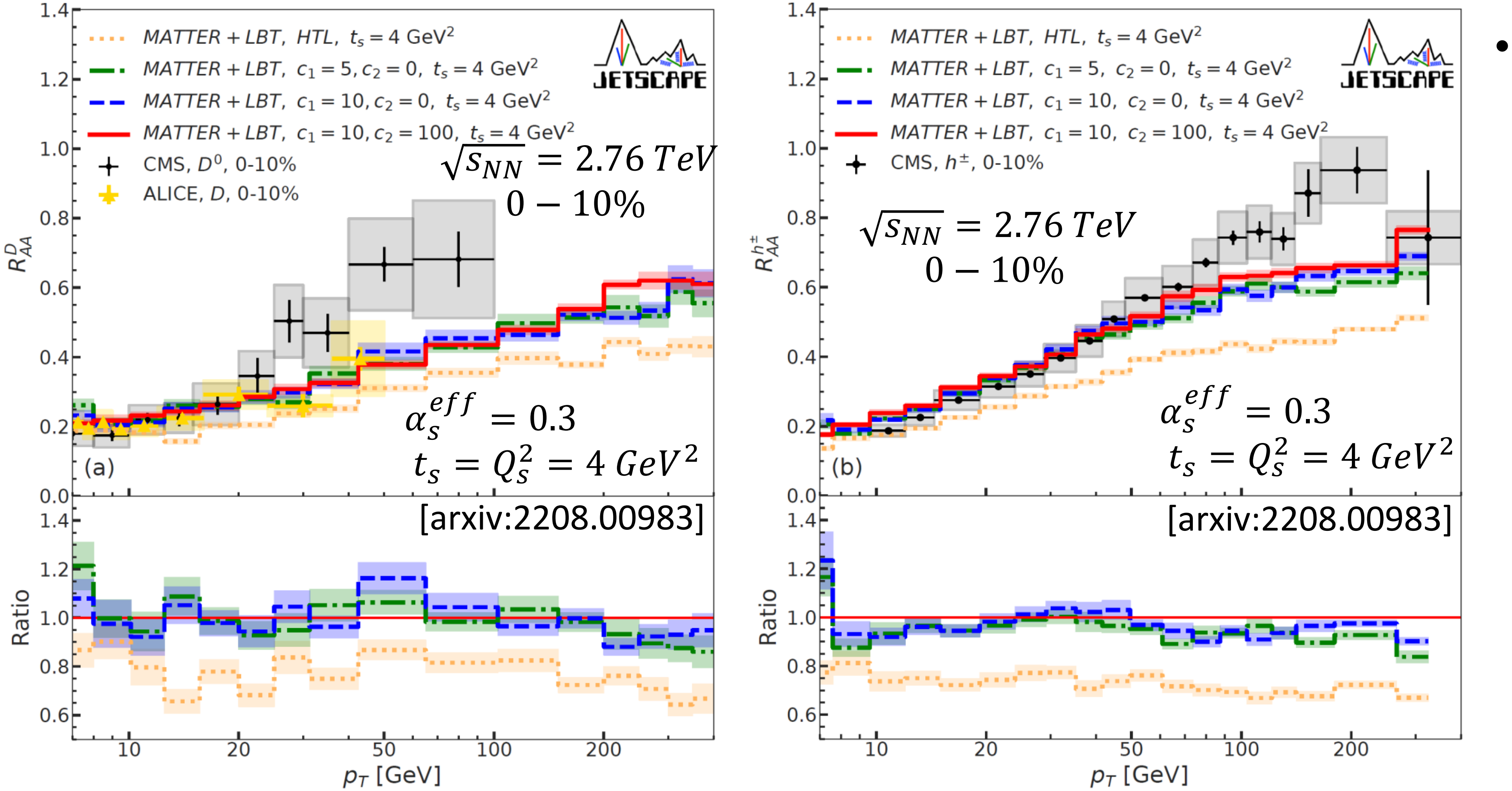
- $R_{AA} < 1$ stems from two different sources:
  - Initial state effects: nuclear modifications to the parton distribution function.
  - Final state effects: creation of the QGP through which partons loose energy and the jet is quenched.

About the QGP medium simulations

- Using *maximum a posteriori* parameters of a Bayesian analysis using soft hadronic observables, QGP evolution profiles were generated for jet energy loss simulations. [**NPA 967 67 (2017); 1804.06469**]

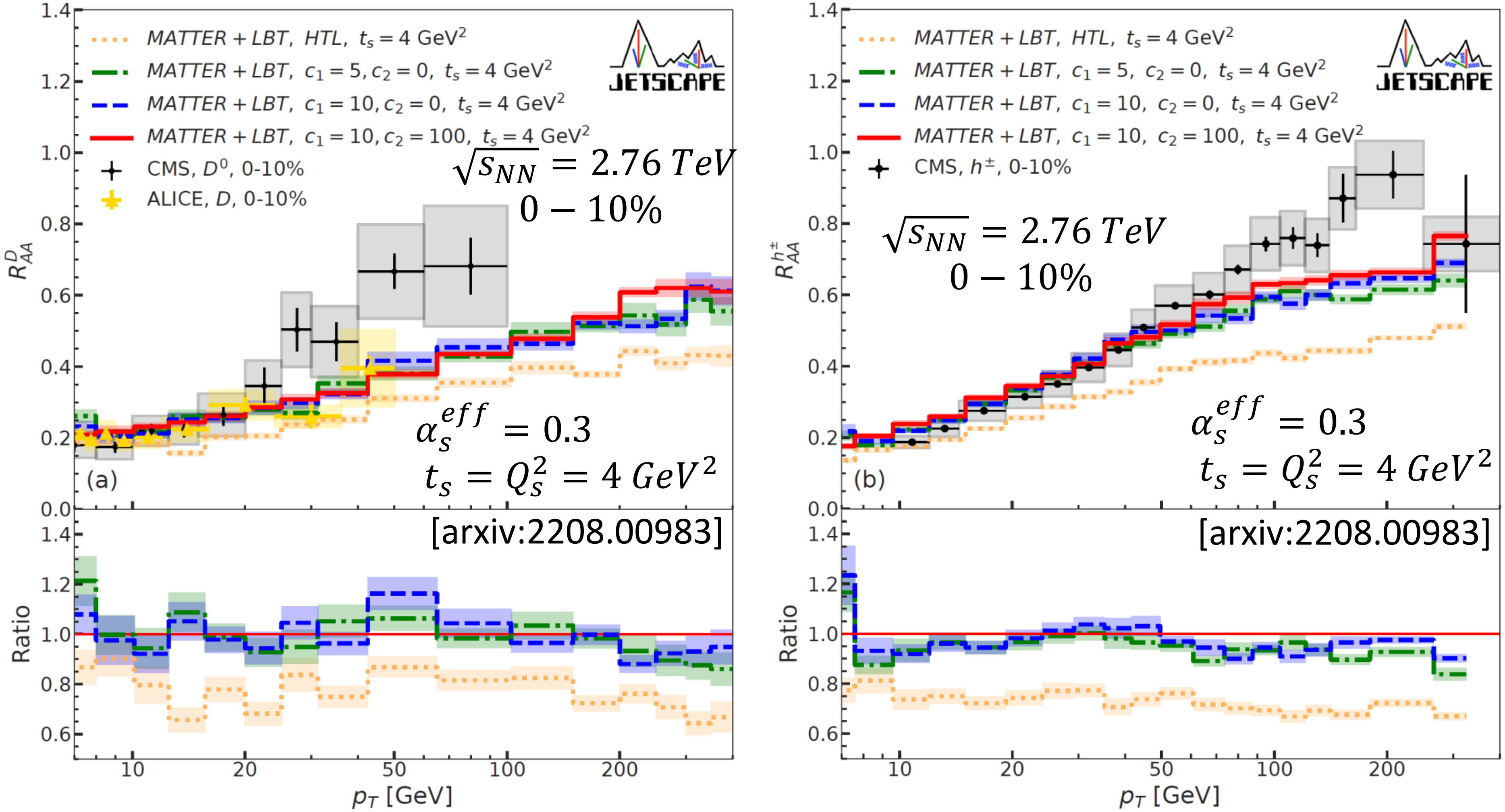
- Event-by-event simulations consist of
  - TRENTO initial conditions
  - 2+1D Pre-equilibrium dynamics (free-streaming)
  - 2+1D 2nd order dissipative hydrodynamics of QGP
  - UrQMD simulation

R_{AA} sensitivity to the presence of $\hat{q}(Q^2)$



- In all cases, parameters were tuned using light flavor jets and charged hadron R_{AA}

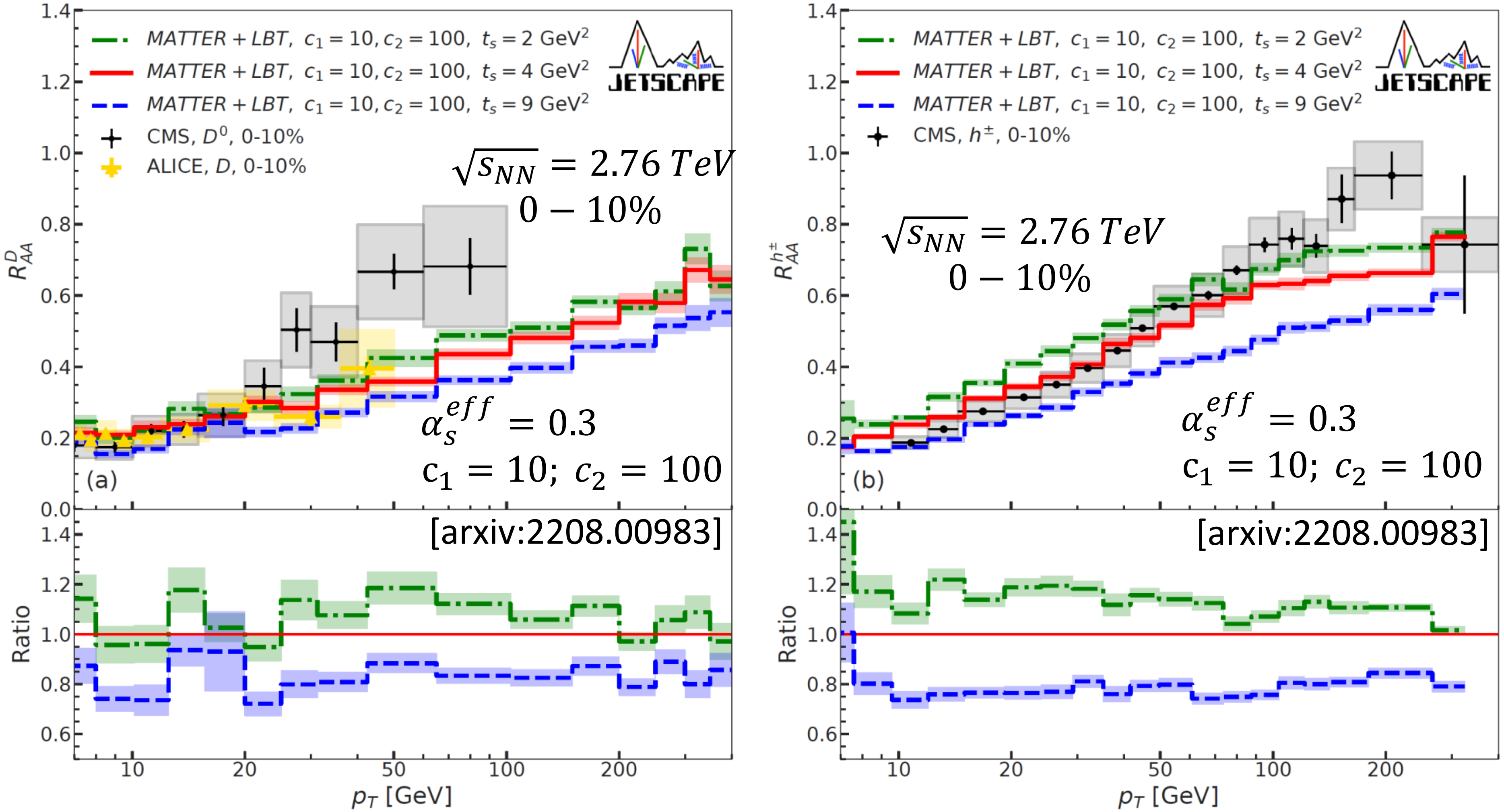
R_{AA} sensitivity to the presence of $\hat{q}(Q^2)$



- The orange curve is for $\hat{q}_{HTL}$ only.
- Red, green, and blue curves use different values of c_1 & c_2 in $\hat{q}(Q^2)$. Same $\hat{q}$ for light and heavy quarks
- Beyond a threshold, ($c_1 = 5$ and $c_2 = 0$) a low sensitivity to c_1 & c_2 is seen.

$$\hat{q}(Q^2) = \hat{q}_{HTL} \frac{c_0}{1 + c_1 \ln^2(Q^2) + c_2 \ln^4(Q^2)}$$

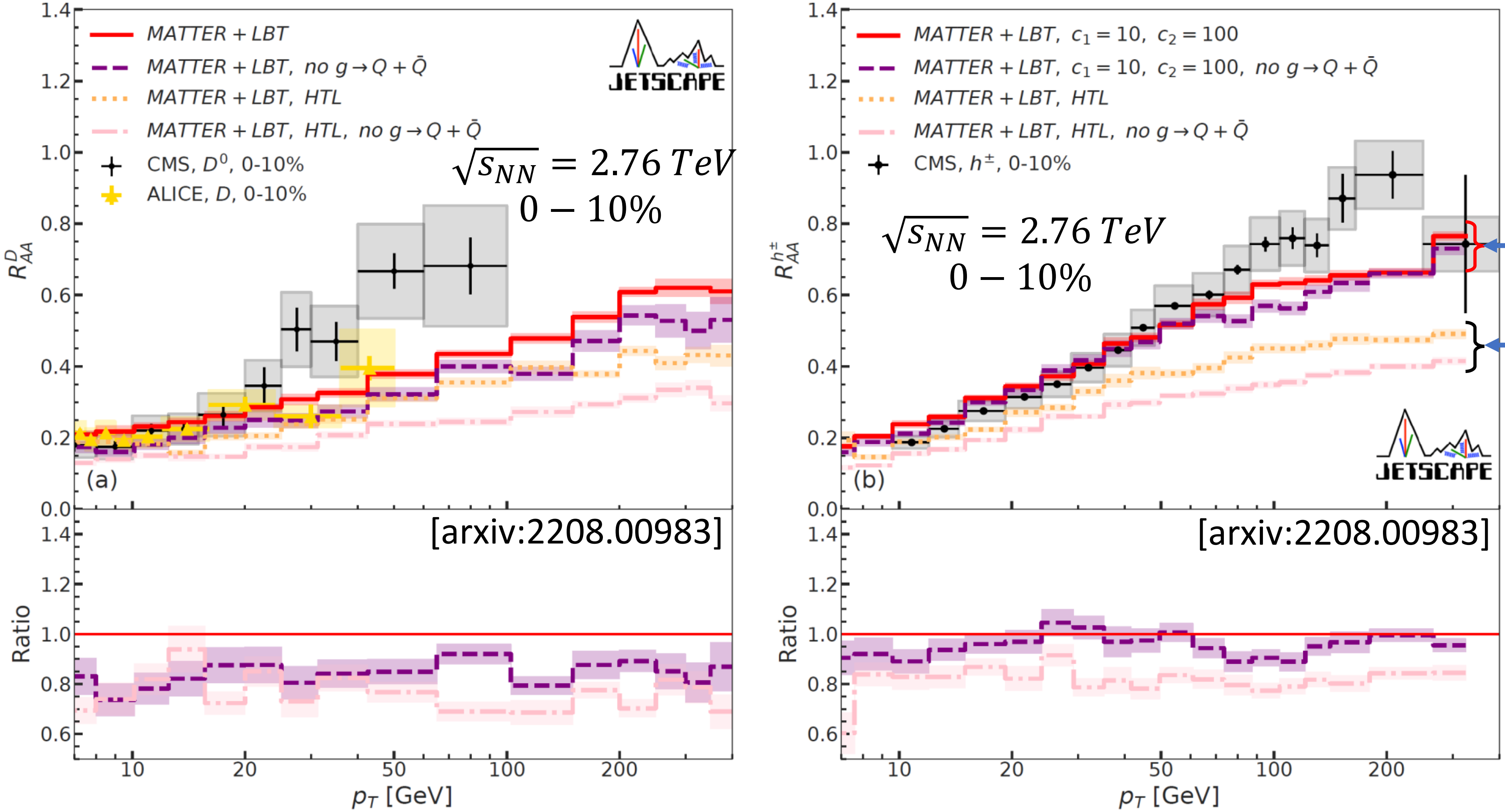
R_{AA} sensitivity to the switching virtuality Q_s^2 between MATTER & LBT



- Green curve: $Q_s^2 = 2\ \mathrm{GeV}^2$
- Red curve: $Q_s^2 = 4\ \mathrm{GeV}^2$
- Blue curve: $Q_s^2 = 9\ \mathrm{GeV}^2$

- The same $\hat{q}(Q^2)$ used for light and heavy flavor ⇒ similar sensitivity to the switching virtuality $t_s = Q_s^2$.
- Will explore how the HF mass *scale* M and virtuality scale Q^2 affects $\hat{q}$ together, i.e. $\hat{q}(Q^2, M)$.

Sensitinity of R_{AA} to $g \to Q + \bar{Q}$



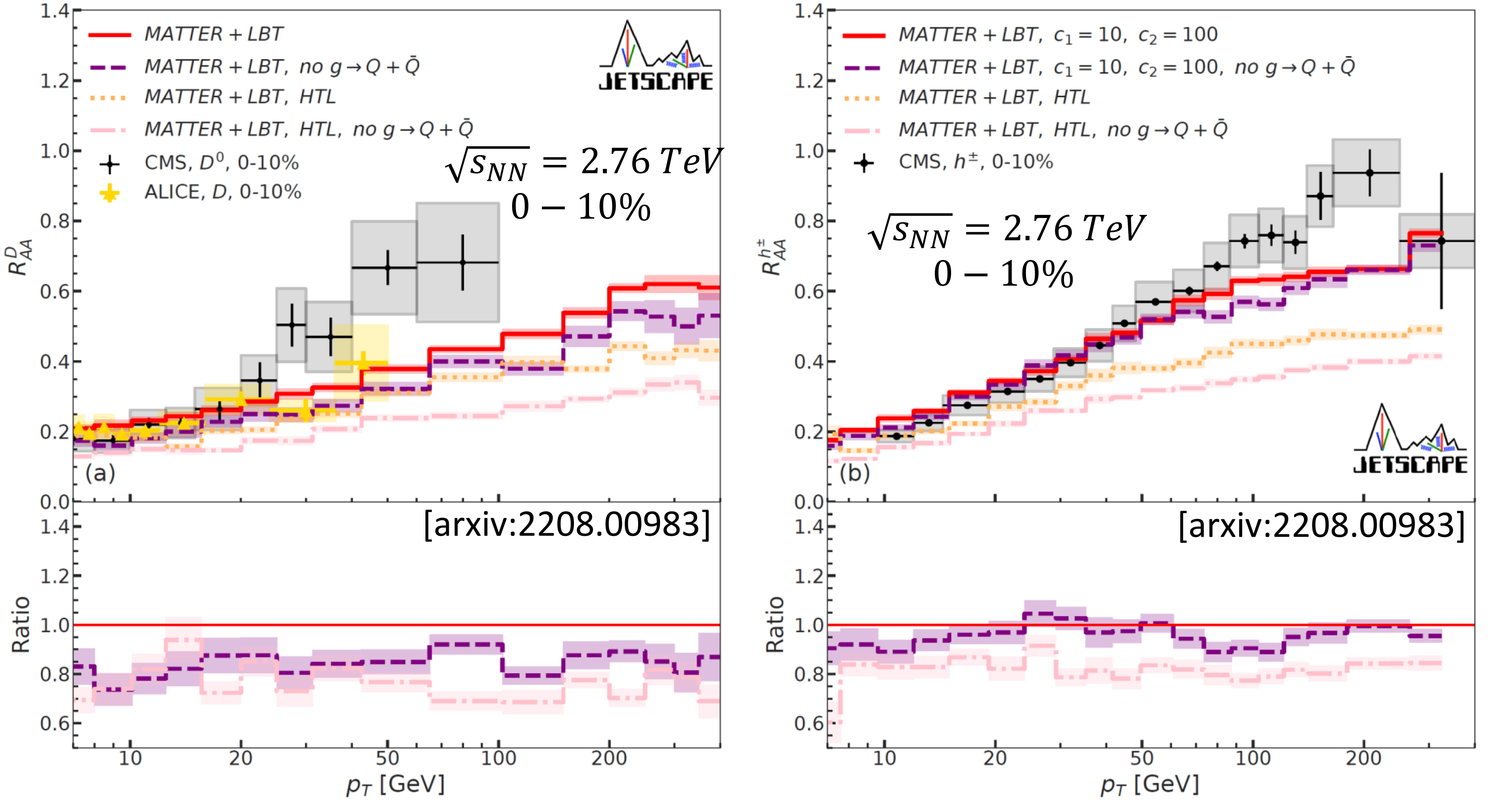
$\hat{q}(Q^2)$: $R_{AA}^{h^\pm}$ is modestly sensitive to $g \to Q + \bar{Q}$

$\hat{q}_{\rm HTL}$: $R_{AA}^{h^\pm}$ is sensitive to $g \to Q + \bar{Q}$

Ratio: relative importance of $g \to Q + \bar{Q}$ for $\hat{q}(Q^2)$ and $\hat{q}_{HTL}$.

- D-meson R_{AA} is sensitive to $g \to Q + \bar{Q}$ at the ~20% level for both parametrizations of $\hat{q}$ (i.e., $\hat{q}(Q^2)$ and $\hat{q}_{HTL}$)

Sensitinity of R_{AA} to $g \to Q + \bar{Q}$



- To explore further: (i) $\hat{q}(Q^2, M)$ and, also,
  (ii) $\mathcal{P}_{g\to Q+\bar{Q}}\left(y, Q^2, M\right)$ beyond the phenomenological approach used here.
- Key message: future simulations of charm energy loss must include $g \to Q + \bar{Q}$!

Conclusion and outlook

- A multi-scale formalism, such as that present inside the JETSCAPE framework, allows for a simultaneous description of light flavor and heavy flavor energy loss inside QGP.
- Realistic simulations of charm energy loss must include dynamical generation of heavy quarks via $g \to Q + \bar{Q}$.
- Future physics improvement for heavy flavors energy loss to include:
  - A multiscale-dependent $\hat{q}\left(Q^2, M\right)$
  - A more realistic splitting function for $g \to Q + \bar{Q}$
  - Including additional energy loss physics, such as longitudinal energy loss $(\hat{e}, \hat{e}_2)$
  - Explore bottom quark energy loss
- A Bayesian analysis including heavy flavors is ongoing…

Thank you

Higher Twist Energy Loss for Heavy Quarks

- **MATTER** (The **M**odular **A**ll **T**wist **T**ransverse-scattering **E**lastic-drag and **R**adiation) valid for High E, High Q^2
  - Virtuality-ordered shower with splittings above $Q^2 \gg Q^2_{switch}$. This is unlike Pythia as the daughter Q_d^2 is found before θ between the daughters is known ⇒ $\theta_{d_1} > \cdots > \theta_{d_n}$ is not strictly ensured.
  - Example: $Q \rightarrow Q + g$

$$q^\mu = p_q^\mu + p_g^\mu$$

$$\Rightarrow p_q^\mu = \left((1-y)q^+, \frac{\vec{l}_\perp^2 + M^2 + Q_q^2}{2(1-y)q^+}, -\vec{l}_\perp \right)$$

$$\Rightarrow p_g^\mu = \left(yq^+, \frac{\vec{l}_\perp^2 + Q_g^2}{2yq^+}, \vec{l}_\perp \right)$$

Conservation of the “–” component of the 4-vector

$$l_\perp^2 = y(1-y)Q^2 - y^2M^2 - (1-y)Q_g^2 - yQ_q^2$$

$$\Rightarrow \theta_g = \arctan\left(\frac{l_\perp}{p^z}\right) = \arctan\left(\frac{\sqrt{2}l_\perp}{yq^+ - \frac{l_\perp^2 + Q_g^2}{2yq^+}}\right)$$

Linear Boltzmann Transport for Heavy Quarks

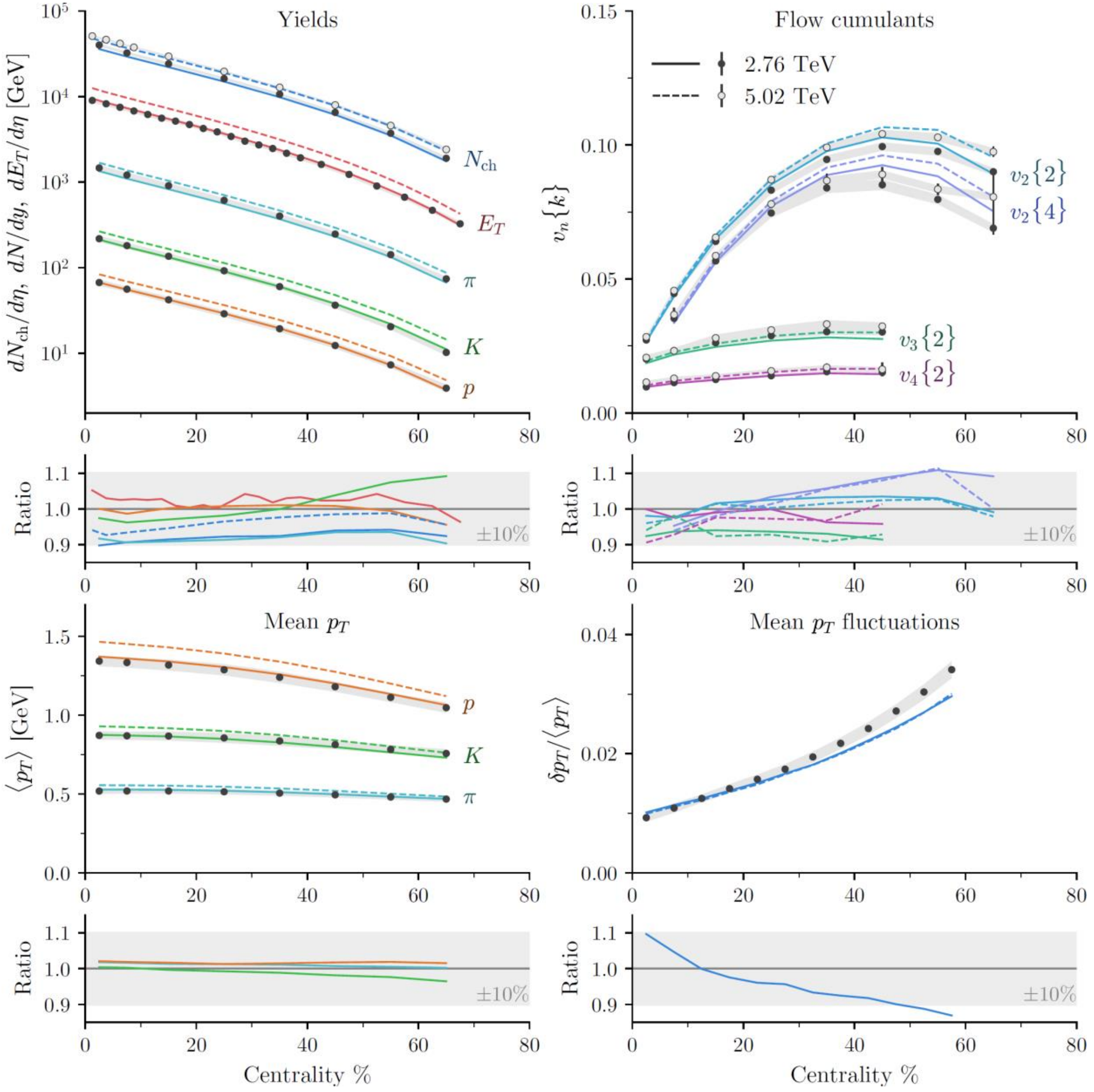
- Valid for high E, assuming particles are on-shell
- Solves the time-ordered evolution for the phase space distribution function

$$p_1 \cdot \partial f(x, p_1) = \mathcal{C}_{el} + \mathcal{G}_{inel}$$

- The LO pQCD $1 + 2 \leftrightarrow 3 + 4$ scattering is included in $\mathcal{C}_{el}$

$$\mathcal{C}_{el} = -\frac{1}{2p_1^0}\int \frac{d^3p_2}{2p_2^0(2\pi)^3}\int \frac{d^3p_3}{2p_3^0(2\pi)^3}\int \frac{d^3p_4}{2p_4^0(2\pi)^3} f(p_1)f(p_2)\left|\bar{\mathcal{M}}\right|^2[1 \pm f(p_3)][1 \pm f(p_4)](2\pi)^4\delta^{(4)}(p+k-l-q)$$
$$+\frac{1}{2p_1^0}\int \frac{d^3p_2}{2p_2^0(2\pi)^3}\int \frac{d^3p_3}{2p_3^0(2\pi)^3}\int \frac{d^3p_4}{2p_4^0(2\pi)^3} f(p_3)f(p_4)\left|\bar{\mathcal{M}}\right|^2[1 \pm f(p_1)][1 \pm f(p_2)](2\pi)^4\delta^{(4)}(p+k-l-q)$$

About the QGP medium simulations



- MAP from Bernhard et al. **NPA 967 67 (2017); 1804.06469** used for QGP evolution profiles
- Event-by-event simulations consist of
  - TRENTO initial conditions
  - 2+1D Pre-equilibrium dynamics (free-streaming)
  - 2+1D 2[nd] order dissipative hydrodynamics of QGP