eHIJING for final-state interactions in *e*-*A*

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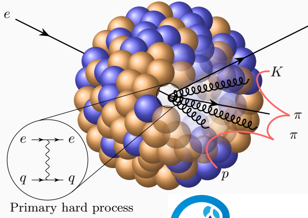
[1] WK, Y. Zhang, H. Xing, and X.-N. Wang 2304.10779



Nuclear effects in deep inelastic scattering

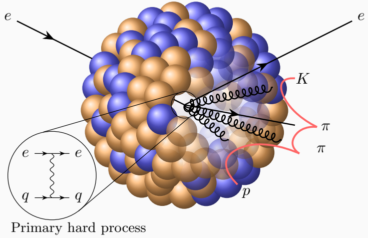
Nuclear non-perturbative input: Nuclear structures, nuclear parton distribution [e.g. extract transverse momentum dependent PDF & FF, Alrashed, Anderle, Kang, Terry, Xing PRL129(2022)242001])

Dynamical effects:

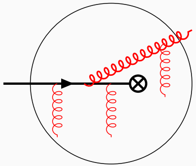
- Dynamical shadowing, in-medium parton shower, hadronic interactions, target dynamics, ...
- Dynamical medium effects can be process dependent.

Understanding dynamical effects is critical to define what are the NP inputs.

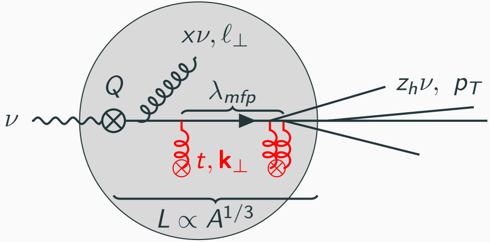
DIS in eA



Drell-Yan in pA



The DIS limit of e-A collisions



- DIS: asymptotically large Q^2 and fixed x_B.
- Localized hard production in the nucleus

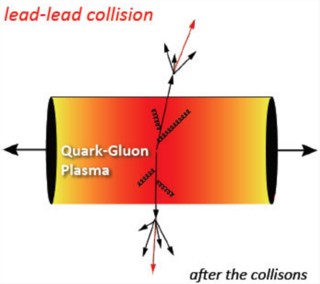
$$\Delta r_\perp \sim \frac{1}{Q} \ll r_0 A^{1/3},$$

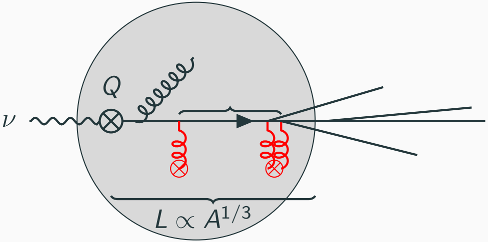
$$\Delta r^+ \sim \frac{\nu}{Q^2} = \frac{1}{2x_B m_p} \approx \frac{0.1}{x_B}\ \mathrm{fm} < r_0 A^{1/3}.$$

- Meaningful to talk about final-state interactions after the hard process with a given path length L: $\frac{d\sigma_{eA\to h}}{dx_B dQ^2 dz_h} = \frac{2\pi\alpha_e^2}{Q^4} \sum_{i,j} f_{i/A} \otimes e_q^2 H_{ij} \otimes (d_{h/j} + \Delta d_{h/j})$.

- Partonic final-state interactions mediated by Glauber gluons $k^+k^- \ll k_\perp^2 \Leftrightarrow$ transport properties ($\hat{q} = \frac{d\langle p_T^2 \rangle}{dL}$) of cold nuclear matter (CNM).

Compare eA and AA

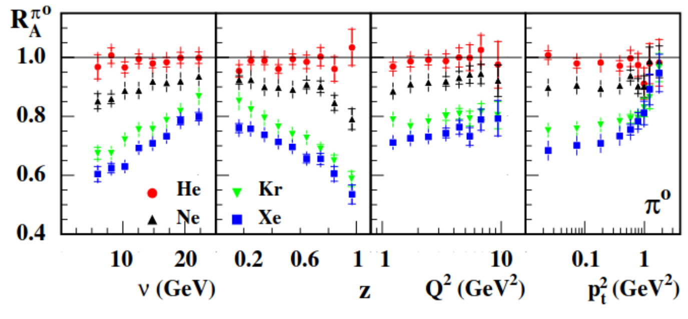




- Near $\eta = 0$, medium comes to a complete stop in z. $\Rightarrow \tau_{\text{hard}} \ll \tau_{\text{int.}}$
- High-temperature medium with real-time dynamics (fast expansion).
- $\hat{q}_F \approx 1$ GeV^2/fm for $T = 0.4$ GeV.

- In eA, $\tau_{\text{hard}} \ll \tau_{\text{int.}}$ is only satisfied at large x_B.
- The medium is the ground state of the nucleus, $T = 0$, and static.
- $\hat{q}_F = 0.02 \sim 0.05$ GeV^2/fm at $x_B \approx 0.1$.

Sizeable modifications to fragmentation observed at EMC, HERMES, CLAS



$$R_A^h = \frac{N_{eA\to\pi^0}(z_h, p_T^2; \nu, Q^2)}{N_{ed\to\pi^0}(z_h, p_T^2; \nu, Q^2)}$$

$$N_{eX\to h} = \frac{d\sigma_{eX\to h}}{d\nu dQ^2 dz_h dp_T^2} \Big/ \frac{d\sigma_{eX}}{d\nu dQ^2}$$

◁ HERMES NPB780(2007)1-27

EMC ZPC52(1991)1–11

CLAS PRC105(2022)015201

eHIJING (electron-Heavy-Ion-Jet-INteraction-Generator) aims to model the development of parton shower in the cold nuclear matter.

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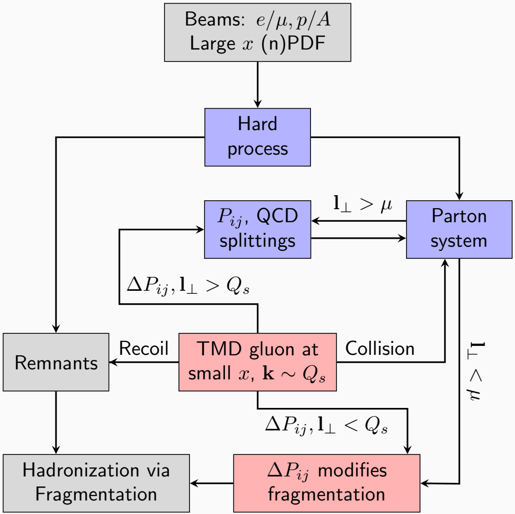


A Monte Carlo model for jet shower in *e*-*A* (DIS limit):

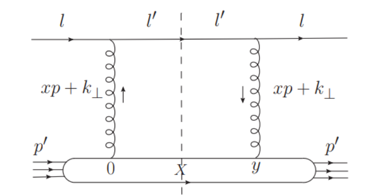
- *e*-*p* (and hard event in *e*-*A*) generation from Pythia8 [T. Sjöstrand et al, Comput.Phys.Commun.191(2015)159].
- *e*-*A* (including changes to Pythia8):
  1) multiple scatterings
  2) modified splitting functions
  3) parton shower in CNM
  4) hadronization

Modelling jet-medium interactions

- Glauber gluon carries a small nucleon momentum fraction $x_g = \frac{\mathbf{k}_\perp^2}{Q^2} x_B$.

Relate jet-medium cross section to the unintegrated gluon distribution $\phi_g(x, \mathbf{k}_\perp^2)$ (or TMD distribution $\phi_g/(4\pi x)$) [J. Casalderrey-Solana, X.-N. Wang, PRC77(2008)024902]

$$\frac{d\sigma_G}{d\mathbf{k}_\perp^2} = \frac{C_R}{d_A} \frac{\alpha_s \phi_g(x, \mathbf{k}_\perp^2)}{\mathbf{k}_\perp^2}$$



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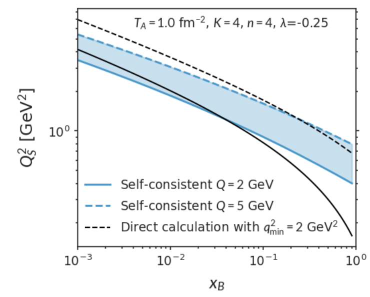
- A saturation-motivated parametrization for $\phi_g(x, \mathbf{k}_\perp^2)$

$$\alpha_s \phi_g(x, \mathbf{k}_\perp) = K \frac{(1-x)^n x^\lambda}{\mathbf{k}_\perp^2 + Q_s^2(x_B, Q^2)}$$

Q_s calculated self-consistently [Y-Y Zhang, X-N Wang PRD105(2022)034015; A. Mueller NPB558(1999)285-303]

$$Q_s^2(x_B, Q^2) = \rho_N L \frac{C_A}{d_A} \int_{\Lambda^2}^{Q^2/x_B} \alpha_s \phi_g\left(\frac{x_B \mathbf{k}_\perp^2}{Q^2}, \mathbf{k}_\perp^2\right) d^2\mathbf{k}_\perp$$



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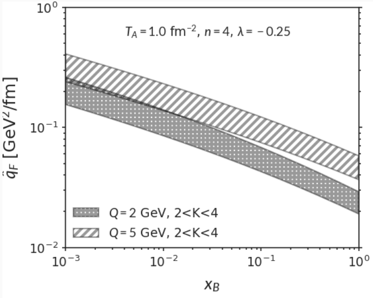
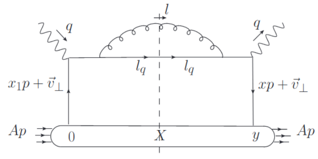


The resulting transport parameter in CNM $\hat{q}_R = \frac{C_R}{C_A} \frac{Q_s^2}{L}$

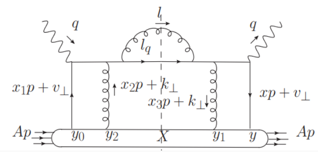
Medium-modified QCD splitting functions: $q \to q$

Theory inputs: (generalized) twist-four calculation of medium-induced radiative correction [Y-Y Zhang, X-N Wang PRD105(2022)034015].

Leading twist, e.g.



Twist four, e.g.



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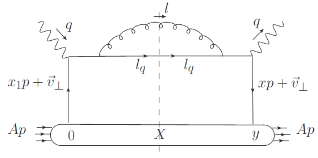

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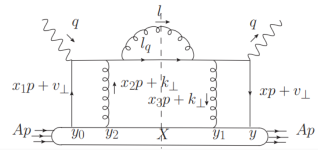


$$\frac{d\sigma^D_{eA}}{dx_B dQ^2 dz d^2l_\perp d^2l_{q\perp}} = \frac{2\pi\alpha^2_{em}}{Q^4}\sum_q e_q^2[1+(1-\frac{Q^2}{x_B s})^2]\frac{\alpha_s}{2\pi}\frac{1+z^2}{1-z}\frac{2\pi\alpha_s}{N_c}\int\frac{d^2k_\perp}{(2\pi)^2}\int d^2b_\perp dy_0^- dy_1^-$$
$$\times\rho_A(y_0^-,b_\perp)\rho_A(y_1^-,b_\perp)q_N(x_B,\vec{v}_\perp,b_\perp)\frac{\phi_N(x_G,\vec{k}_\perp)}{k_\perp^2}[\mathcal{N}_g^{\rm qLPM}+\mathcal{N}_g^{\rm gLPM}+\mathcal{N}_g^{\rm nonLPM}]\,. \qquad (19)$$

$$\mathcal{N}_g^{\rm qLPM} = \frac{1}{N_c}\left(\frac{[\vec{l}_\perp-(1-z)\vec{v}_\perp]\cdot[\vec{l}_\perp-(1-z)(\vec{l}_\perp+\vec{l}_{q\perp})]}{[\vec{l}_\perp-(1-z)\vec{v}_\perp]^2[\vec{l}_\perp-(1-z)(\vec{l}_\perp+\vec{l}_{q\perp})]^2}-\frac{1}{[\vec{l}_\perp-(1-z)\vec{v}_\perp]^2}\right)$$
$$\times\left(1-\cos[(x_L+x_E-x_F)p^+(y_1^--y_0^-)]\right), \qquad (20)$$

Suppressed by $1/N_c^2$

$$\mathcal{N}_g^{\rm gLPM} = C_A\left(\frac{2}{[\vec{l}_\perp-(1-z)\vec{v}_\perp-\vec{k}_\perp]^2}-\frac{[\vec{l}_\perp-(1-z)\vec{v}_\perp-\vec{k}_\perp]\cdot[\vec{l}_\perp-(1-z)(\vec{l}_\perp+\vec{l}_{q\perp})]}{[\vec{l}_\perp-(1-z)\vec{v}_\perp-\vec{k}_\perp]^2[\vec{l}_\perp-(1-z)(\vec{l}_\perp+\vec{l}_{q\perp})]^2}\right.$$
$$\left.-\frac{[\vec{l}_\perp-(1-z)\vec{v}_\perp]\cdot[\vec{l}_\perp-(1-z)\vec{v}_\perp-\vec{k}_\perp]}{[\vec{l}_\perp-(1-z)\vec{v}_\perp]^2[\vec{l}_\perp-(1-z)\vec{v}_\perp-\vec{k}_\perp]^2}\right)\times\left(1-\cos[(x_L+\frac{z}{1-z}x_D+x_S-x_F)p^+(y_1^--y_0^-)]\right), \qquad (21)$$

Main contributions, medium enhanced

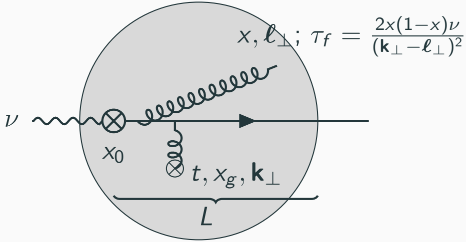
$$\mathcal{N}_g^{\rm nonLPM} = C_F\left(\frac{1}{[\vec{l}_\perp-(1-z)(\vec{l}_\perp+\vec{l}_{q\perp})]^2}-\frac{1}{[\vec{l}_\perp-(1-z)\vec{v}_\perp]^2}\right), \qquad (22)$$

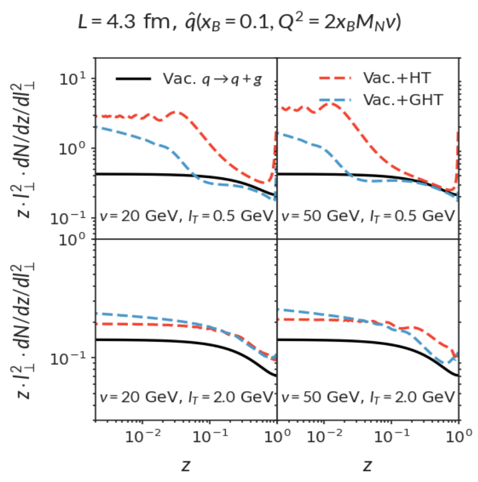
Power suppressed by UV cut off, not enhanced by medium

The soft gluon emission approximation

$$\frac{dN_{gq}^{(1)}}{dx d^2\ell_\perp} = \frac{\alpha_s C_F}{2\pi^2} \frac{P_{qq}(x)}{\ell_\perp^2} \int d^2\mathbf{k}_\perp \frac{C_A}{d_A} \frac{\alpha_s \phi_g(\mathbf{k}_\perp^2)}{\mathbf{k}_\perp^2}$$
$$\frac{2\mathbf{k}_\perp \cdot \ell_\perp}{(\ell_\perp - \mathbf{k}_\perp)^2} \int_0^L dt \rho_N(x_0 + t) \left[1 - \cos\frac{t}{\tau_f}\right]$$

The Landau-Pomeranchuk-Migdal (LPM) effect



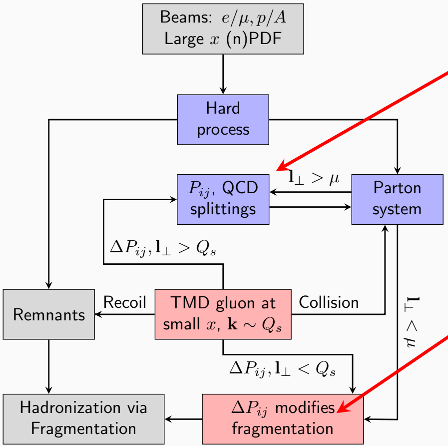


- "Higher-Twist" (HT): in the past, one expands in $1/\ell_\perp^2$ before $\mathbf{k}_\perp$ integration.

⋆ Generalized Higher-Twist (GHT): one performs twist expansion after $\mathbf{k}_\perp$ integration.

Multiple emissions

Multiple emissions follow the modified DGLAP evolution [Chang, Deng, Wang PRC89(2014)034911].



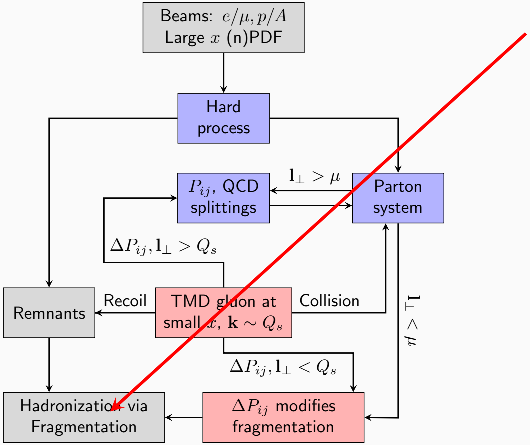
- At high virtuality, eHIJING modifies vacuum splitting functions in Pythia8's p_T-ordered shower, $(\ell_\perp)_{\rm min} < (\ell_\perp)_n < \cdots < (\ell_\perp)_1 < Q/2$

$$\frac{dP}{dx d^2\ell_\perp} = \frac{\alpha_s}{2\pi^2}\frac{1}{\ell_\perp^2}P^{(0)}(x) + \frac{dN^{(1)}}{dx d^2\ell_\perp}\Theta(\ell_\perp^2 - Q_s^2)$$

- At low virtuality, multiple emissions are ordered in formation time $1/Q_s \sim \tau_{f,1} < \ldots < \tau_{f,n} < \frac{p}{\Lambda^2}$

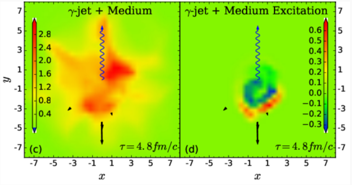
$$\frac{dP}{dx d^2\ell_\perp} = \frac{dN^{(1)}}{dx d^2\ell_\perp}\Theta(Q_s^2 - \ell_\perp^2)$$

Hadronization



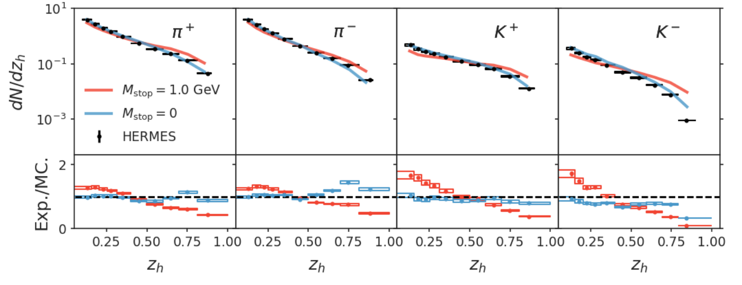
- Shower + remnants hadronization from Lund string model.
- Remnants from multiple collisions: quark + diquark. For future: soft energy-momentum deposition & medium excitation?

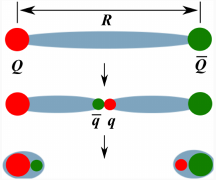
Similar practices in heavy-ion collisions.



[Jet wake in HIC, Wei Chen et al PLB777(2018)86]

The Lund string model in Pythia8: $D(z)$



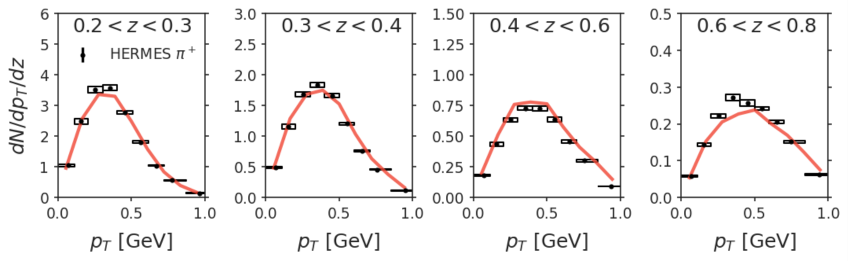


[HERMES, Phys Rev D 87, 074029 (2013)]

- Change a default Pythia8 fragmentation parameter $M_{\rm stop}$ from 1 GeV to 0 to fit π and K spectra in e-d collisions at HERMES energy.
- $M_{\rm stop}$ controls the minimum mass for the string to break $W > m_q + m_{\bar{q}'} + M_{\rm stop}$.

The Lund string model in Pythia8: $d^2N_h/dz_h/dp_T$

$d^2N_h/dz_h/dp_T$



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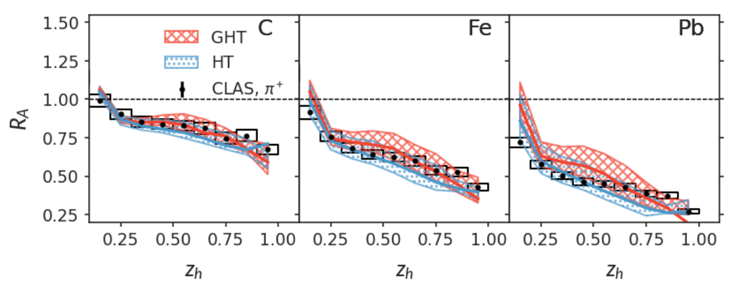
Reasonable agreement with Pythia8's non-perturbative modeling

- The primordial quark k_T, $k_T \sim e^{-k_T^2/2\sigma_1^2}$ [T. Sjöstrand and P.Z. Skands, JHEP 03 (2004) 053].
- k_T from Lund string fragmentation, $k_T \sim e^{-k_T^2/2\sigma_2^2}$ with $\sigma_2 = 0.335$ GeV as default.

Test of medium-modified hadronization at CLAS and HERMES



[CLAS PRC105(2022)015201]

- $R_A = \left[N_h(\nu, Q^2; z_h, p_t)/N_\gamma\right]_{eA} / \left[N_h(\nu, Q^2; z_h, p_t)/N_\gamma\right]_{ed}$.
- Higher Twist (HT, blue) and Generalized Higher Twist (GHT, red) using the same range of $\hat{q}_F(x_B = 0.1, Q^2 = 2.25\ \mathrm{GeV}^2) \in [0.027, 0.06]$ GeV^2/fm.

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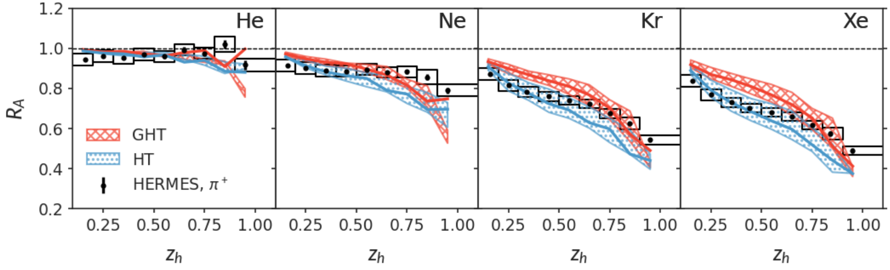

[HERMES, NPB 780, 24 (2007)]

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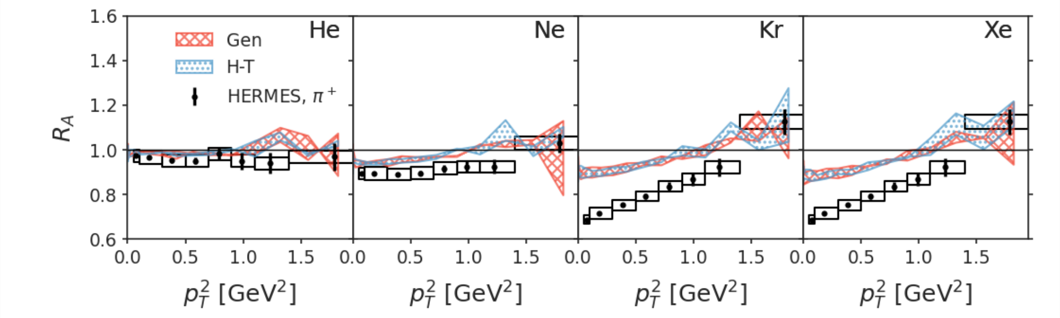
Collisional & radiative contribution to momentum broadening

Collisional broadening of the parton shower

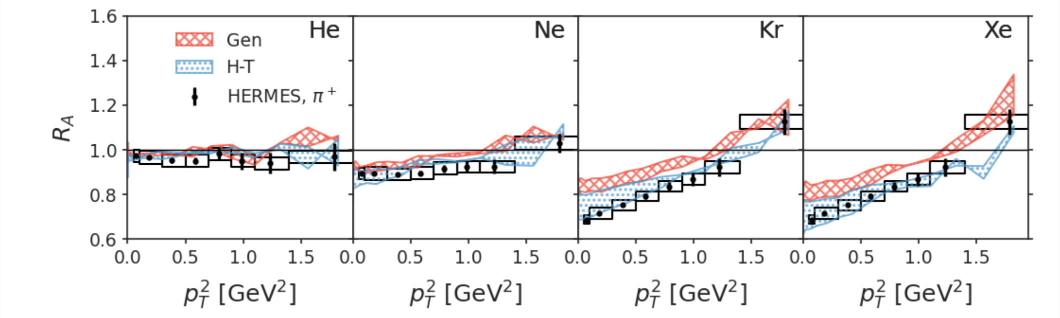


[HERMES, NPB 780, 24 (2007)]

For the same range of $\hat{q}$ that describes $R_A(z_h)$, the pure collisional processes only account for part of the momentum broadening.

Collisional & radiative contribution to momentum broadening

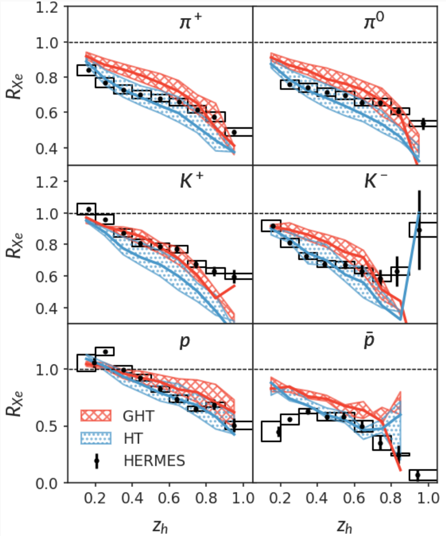
Broadening from both collisions & induced radiations



[HERMES, NPB 780, 24 (2007)]

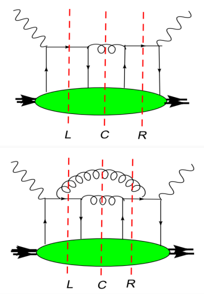
Broadening due to medium-induced radiation is important in large nuclei!

Hadron specie dependence: $\pi^{\pm}$, π^{0}, $K^{\pm}$, p, $\bar{p}$

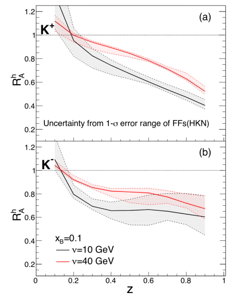


[HERMES, Nuclear Physics B 780, 24 (2007)]

- Notable difference between $R_A(K^+)$ vs $R_A(K^-)$, and $R_A(p)$ vs $R_A(\bar{p})$.
- Importance of medium-induced conversion of $g \to q$ and hadronic transport for future.

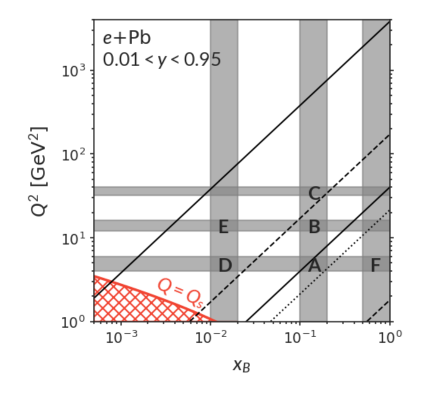


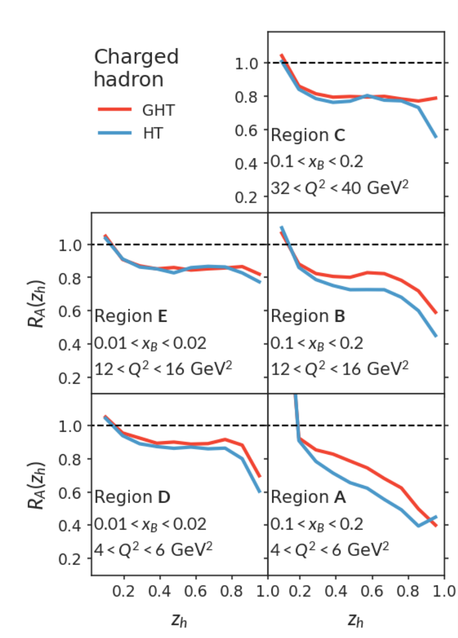
[BW Zhang, XN Wang, A Schaefer]



[NB Chang, WT Deng, XN Wang PRC 92 055207]

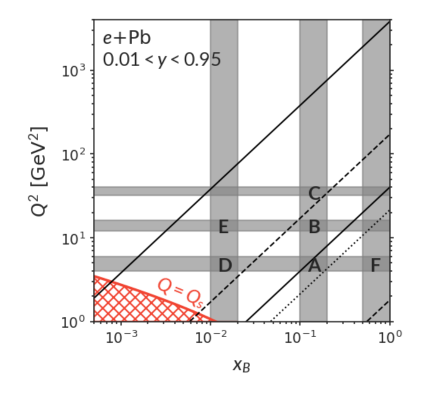
From fixed target to collider energies

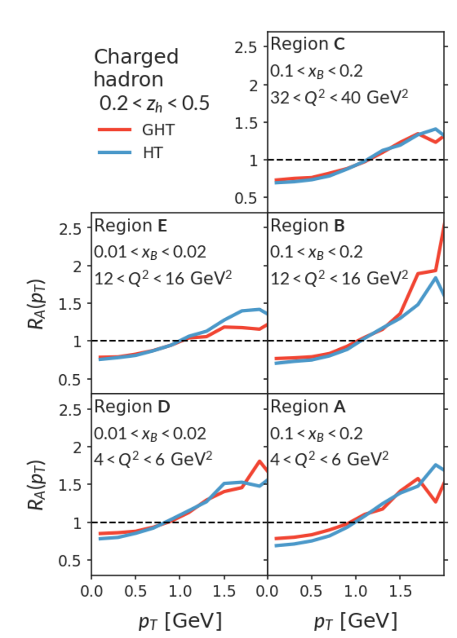




- Coverage from EIC, EicC to CLAS-12.
- Scan regions with $Q^2 \gg Q_s^2$, $x_B > 0.01$.

From fixed target to collider energies





- Coverage from EIC, EicC to CLAS-12.
- Scan regions with $Q^2 \gg Q_s^2$, $x_B > 0.01$.

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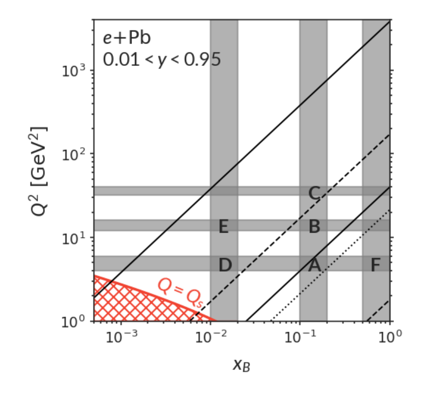

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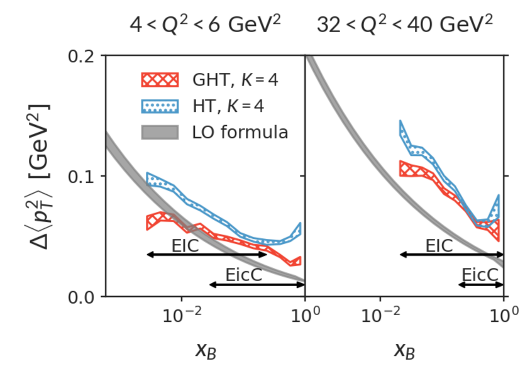


Transverse momentum broadening: simulation v.s. LO expectation

$$\Delta\langle p_T^2\rangle_{\mathrm{LO}} \approx \frac{C_F}{C_A} Q_s^2(x_B, Q^2) \frac{\int_{0.2}^{0.5} z_h^2 \frac{d\sigma}{dz_h dx_B dQ^2} dz_h}{\int_{0.2}^{0.5} \frac{d\sigma}{dz_h dx_B dQ^2} dz_h}.$$

Towards smaller x_B, maintaining large Q^2

✓ Large x_B region $\tau_H/\mathrm{fm} = \frac{0.1}{x_B} \ll 1.2A^{1/3}$. Final-state interaction is completely factorized from hard production.

- For $x_B \sim 0.01$, $\frac{0.1}{x_B} \sim 1.2A^{1/3}$. Multiple collisions become coherent with the hard process $\Rightarrow$ dynamical shadowing [J Qiu, I Vitev, PRL93(2004)262301], or parametrized by nuclear PDF.
- Finally, $\frac{0.1}{x_B} \gg 1.2A^{1/3}$, the dipole regime.

To extend eHIJING to smaller x_B is to consider how to generate shower / excited the nucleus while taking coherence into account.

Summary

A Monte Carlo model for jet shower modifications in e-A:

- e-p event generation from Pythia8.
- e-A (c++17, including modifications to Pythia8):
  Requires large Q^2 and large x_B.

  1) Multiple scatterings with a saturation-motivated TMD gluon density.
  2) Modified splitting functions from generalized higher-twist approach & the HT limit.
  3) In-medium parton shower.
  4) Hadronization (Lund string fragmentation).

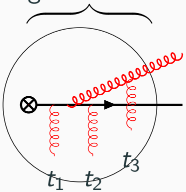
What comes next:

- Hadronic final-state interactions.
- Event generations at smaller x_B (and relatively large Q^2).

Questions?

Multiple collisions in eHIJING

- For each newly generated parton in the shower, compute the in-medium path length L. Currently, only implemented for round nuclei.
- Compute average number of multiple collisions $\langle N \rangle = L/\lambda_G \approx \sigma_G \rho L$
- Number of collisions N follows a Poisson distribution $P(N) = \frac{\langle N \rangle^N e^{-\langle N \rangle}}{N!}$
- Sample the location and kinematics of collisions $(t_i, \mathbf{k}_{\perp i}, x_i)$



Stochastic version of the medium-modified splitting functions

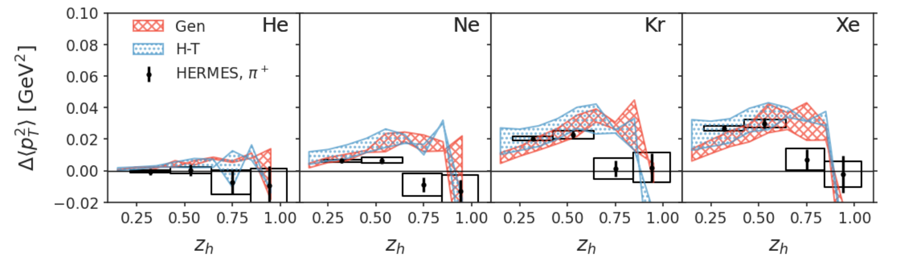
With a large fluctuation in the number of collisions, we constructed fluctuating in-medium splitting functions

$$P_{qq}(x, \boldsymbol{\ell}_\perp) = \frac{\alpha_s C_F}{2\pi} \frac{P_{qq}(x)}{\boldsymbol{\ell}_\perp^2} \left\{ 1 + \int dL \rho(L) \int_{\mathbf{k}_\perp} \frac{C_F}{d_A} \frac{\alpha_s \phi_g(x, \mathbf{k}_\perp^2)}{\mathbf{k}_\perp^2} \frac{C_A}{C_F} \frac{2\mathbf{k}_\perp \cdot \boldsymbol{\ell}_\perp}{(\boldsymbol{\ell}_\perp - \mathbf{k}_\perp)^2} \left[1 - \cos \frac{L}{\tau_f} \right] \right\}$$

$$\Longrightarrow \frac{\alpha_s C_F}{2\pi} \frac{P_{qq}(x)}{\boldsymbol{\ell}_\perp^2} \left\{ 1 + \sum_i \frac{C_A}{C_F} \frac{2(\mathbf{k}_\perp)_i \cdot \boldsymbol{\ell}_\perp}{[\boldsymbol{\ell}_\perp - (\mathbf{k}_\perp)_i]^2} \left[1 - \cos \frac{L_i}{(\tau_f)_i} \right] \right\}$$

The usual average over the medium sources is replaced by the summation over the multiple collisions of the shower parton.

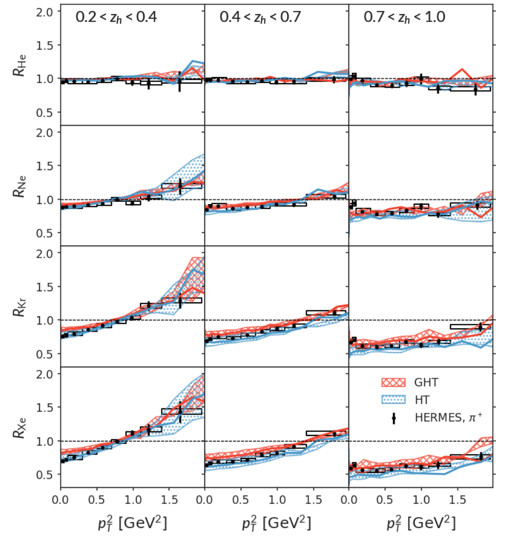
The net broadening $\Delta\langle p_T^2\rangle = \langle p_T^2\rangle_{eA} - \langle p_T^2\rangle_{ed}$



[HERMES, PLB 684 (2010) 114-118]

- Qualitatively similar z-dependence from simulation.
- Data drop more abruptly for $z_h > 0.7$. This region shrinks at higher colliding energies.

p_T-dependent modified fragmentation function $D(z_h, p_T)$



[HERMES, Nuclear Physics B 780, 24 (2007)]

$$R_A = \frac{\left(N_h(\nu, Q^2; z_h, p_t)/N_\gamma\right)_{eA}}{\left(N_h(\nu, Q^2; z_h, p_t)/N_\gamma\right)_{ed}}$$

- Large z: suppression due to parton energy loss of leading particles.
- Intermediate to small z: interplay of $\mathbf{k}_\perp$ broadening and energy loss.