Shape coexistence and superdeformation in ^{28}Si

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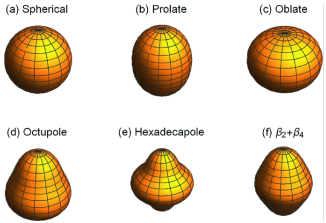

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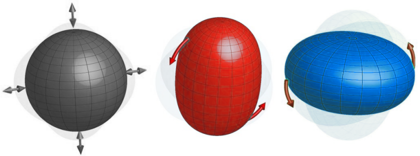
Shape coexistence in ^{28}Si

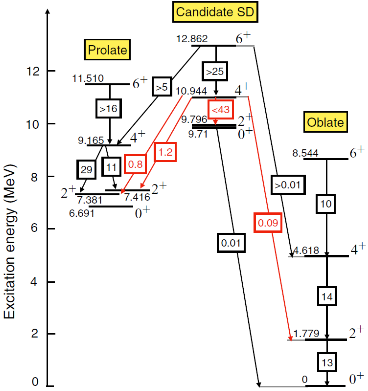
- **Different shapes** among states of the same nucleus within few MeV.
- Motivation for ^{28}Si (N=Z=14):
  1. Oblate ground state.
  2. Prolate structure ($\sim$6 MeV).
  3. Superdeformed structure? ($E \gtrsim 10$ MeV)

Taniguchi, Y., et al. Physical Review C **80**, 044316 (2009).



Spherical Prolate Oblate

Takeuchi, S., et al. Physical review letters, 109. 182501 (2012).



Morris, L. et al. Physical Review C **104**, 054323 (2021).

Spherical mean field

Schrödinger equation

$$\mathcal{H}|\Psi\rangle = E|\Psi\rangle$$

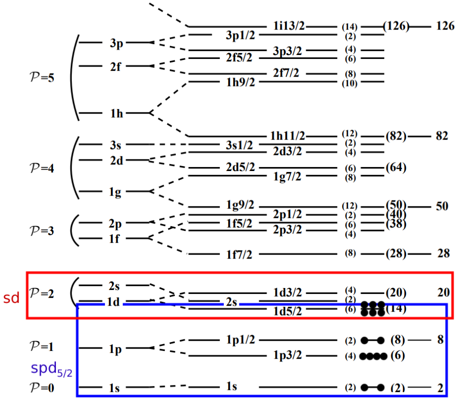
- Mean field approach:

$$\mathcal{H} = \sum_i h_i = \sum_i (t_i + v_i)$$

- ^{28}Si: Z=N=14 Slater determinant (**spherical!**)

$$|\Psi\rangle = \frac{1}{\sqrt{A!}} \det \left\{ \prod_i^A |\phi_i\rangle \right\}$$

Mayer, M. G. Physical Review, 1949. 75



Z=N=14

Poves, A., Nowacki, F. Springer, 2001. 70-101.

Interacting shell model

- Interacting shell model:

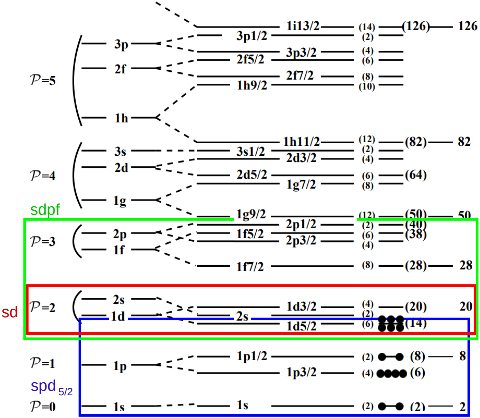
$$\mathcal{H}_{\text{eff}} = \mathcal{H}_0 + \mathcal{H}_{\text{res}}$$

- **Diagonalization** of $\mathcal{H}_{\text{eff}}$:

$$\mathcal{H}_{\text{eff}} |\Psi_{\text{eff}}\rangle = E |\Psi_{\text{eff}}\rangle$$

- Configuration mixing:

$$|\Psi_{\text{NSM}}\rangle = \sum_{\alpha} C_{\alpha} |\Phi_{\alpha}\rangle$$



Poves, A. and Nowacki F. Springer, Berlin, Heidelberg, 2001. 70-101.

Exact diagonalization (NSM)

- Configuration mixing:

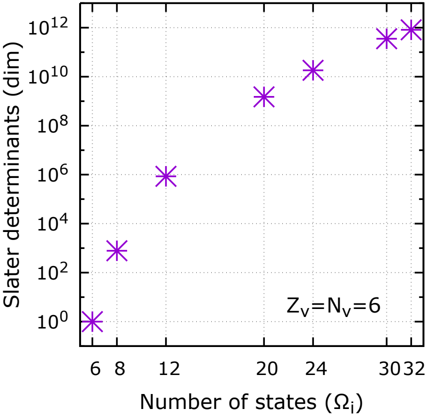
$$|\Psi_{\text{NSM}}\rangle = \sum_{\alpha} C_{\alpha} |\Phi_{\alpha}\rangle$$

- **Slater** determinant $|\Phi_{\alpha}\rangle$:

$$\text{dim} = \binom{\Omega_Z}{Z_v}\binom{\Omega_N}{N_v}$$

- Diagonalization of $\mathcal{H}_{\text{eff}}$: $\mathcal{H}_{\text{eff}}|\Psi_{\text{eff}}\rangle = E|\Psi_{\text{eff}}\rangle$
- Lanczos method

Caurier, E. et al. Shell model code ANTOINE. IReS, Strasbourg, 2002. 1989



Variational method (PGCM)

Exact diagonalization

- Most accurate solution of $\mathcal{H}_{\text{eff}}|\Psi_{\text{eff}}\rangle = E|\Psi_{\text{eff}}\rangle$
- Maximal dimensions of the many-body basis!

Variational method

- **Approximate** solution to diagonalization (Ritz principle):

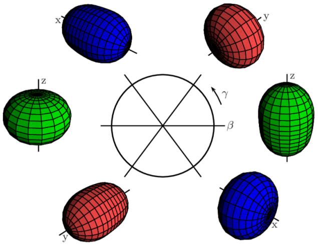
$$E\left[|\phi_0\rangle\right] = \frac{\langle\phi_0|\mathcal{H}_{\text{eff}}|\phi_0\rangle}{\langle\phi_0|\phi_0\rangle} \geq E_0$$

- Smaller set of more **complex** wavefunctions
- Alternative for **large valence spaces**
- Exploration of relevant degrees of freedom $(Q_{\lambda\mu})$
- Allows **visualization** of wavefunctions

Ring, P. & Schuck, P. (Springer Science & Business Media, 2004).

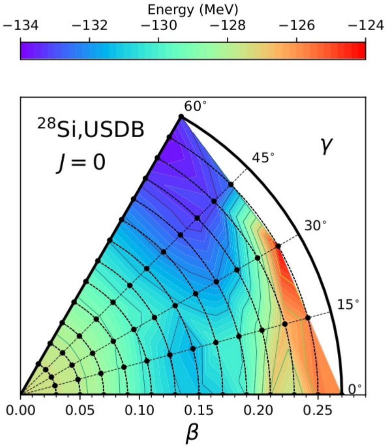
Ground state

- Quadrupole-constrained HFB wavefunctions
- (β, γ) parameters:



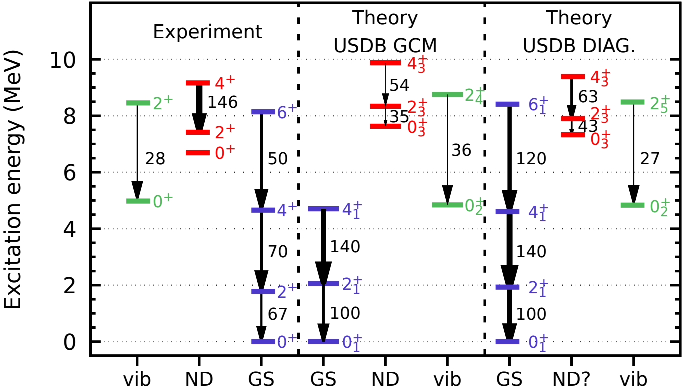
Fortunato, L. The European Physical Journal A, 2005. 1-30

- Oblate minimum ($\beta \approx 0.25$)



Total Energy surface (USDB)

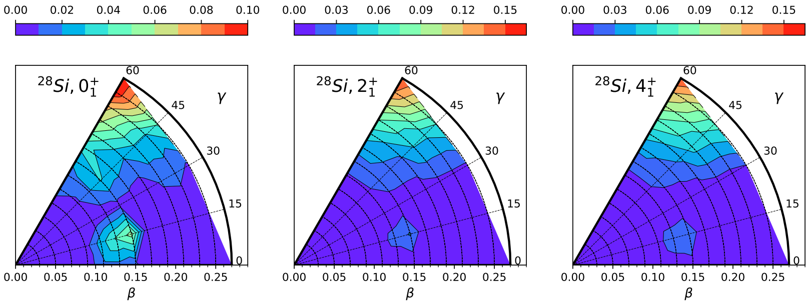
Spectrum of ^{28}Si (USDB)



Agreement with **experimental** data and **exact** diagonalization

Oblate rotational band

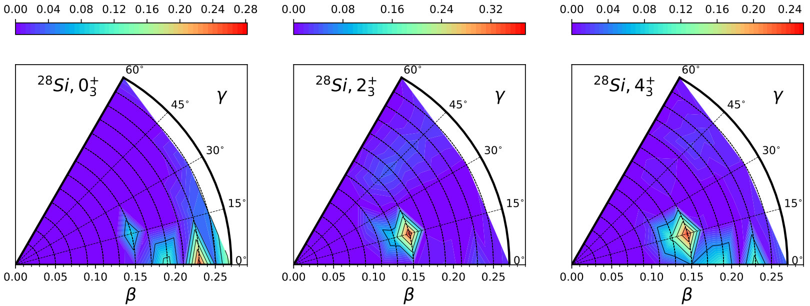
Collective wavefunctions (USDB): $|F^{\Gamma}_{\sigma}(q)|^2 = |\sum_{K\lambda} G^{\Gamma}_{\sigma;\lambda} u^{\Gamma}_{\lambda;qK}|^2$



- **Shared** intrinsic deformation ($\beta \approx -0.27$)
- Rotational band behaviour:
  - $E \sim J(J+1)$ and **strong** $B(E2)$ transition strengths

Prolate rotational band

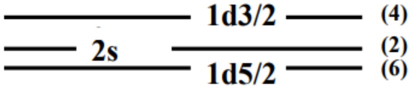
Collective wavefunctions (USDB): $|F^{\Gamma}_{\sigma}(q)|^2 = |\sum_{K\lambda} G^{\Gamma}_{\sigma;\lambda} u^{\Gamma}_{\lambda;qK}|^2$



- Normal deformation for $J = 0$
- **Diminished** deformation for $J = 2$ and $J = 4$
- **Weak** $B(E2)$ transition strengths

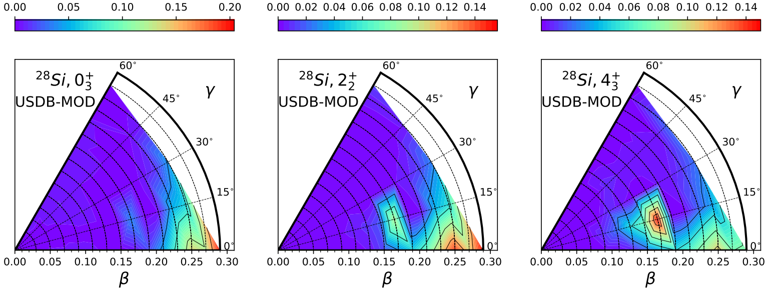
Modification of the interaction

- Prolate: excitations from the $d_{5/2} + s_{1/2}$ to the $d_{3/2}$ orbit.

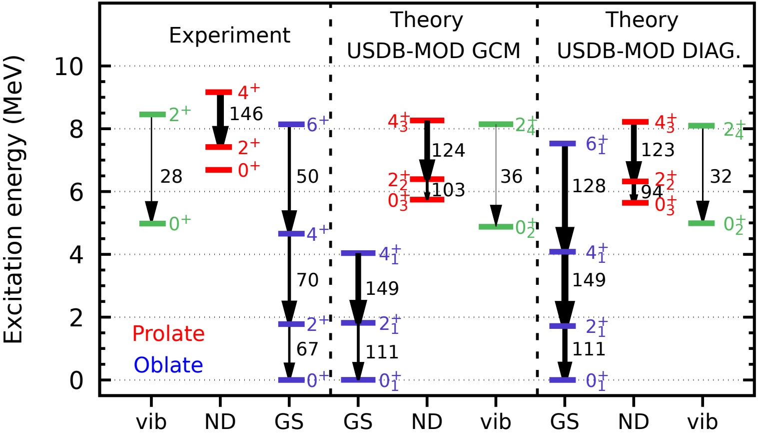


Too high single-particle energies

Gap between $d_{5/2}$+$s_{1/2}$ and $d_{3/2}$: **5 MeV → 3.6 MeV**

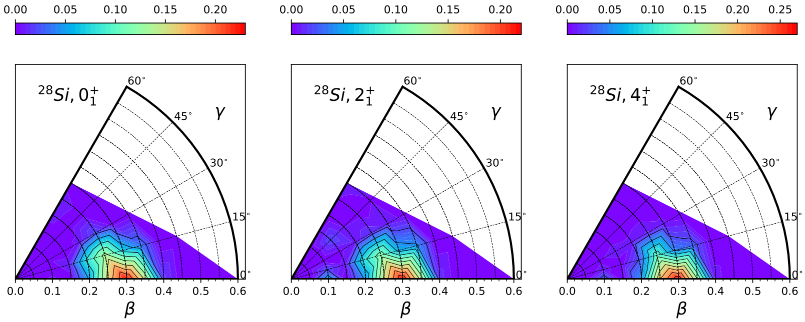


Spectrum of ^{28}Si (USDB-MOD)



pf-shell inclusion

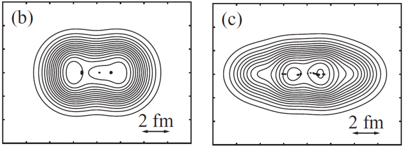
- The SDPF-NR interaction ($sd + pf$ shells) **naturally** reproduces the prolate rotational band
- Only **0.32 particles** in *pf* shell on average



Previous studies

Predicted **superdeformed** band

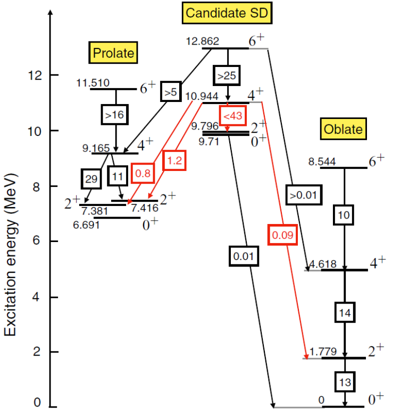
- Deformation: $\beta \approx 0.6$
- **4 particles** into *pf* shell
- $\sim$ 13 MeV bandhead



Taniguchi, Y., et al. Physical Review C, 2009. **80**, 044316

Experimental attempts

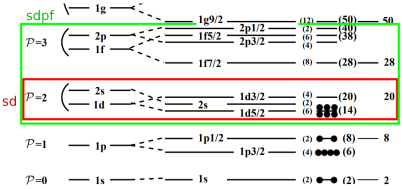
- $B(E2, 4^+ \to 2^+) \leq 217 \text{efm}^2$
- Not found: $\beta \leq 0.35$

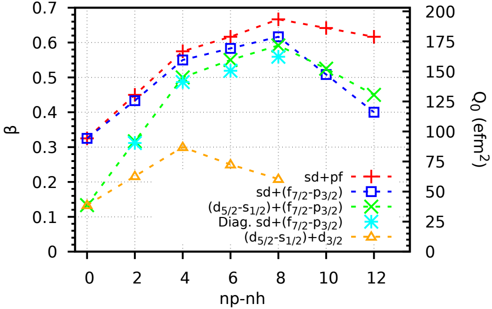


Morris, L., et al. Physical Review C, 2021. **104**, 054323.

Fixed *np*-*nh* configurations

- **sd-shell** deformations: $\beta \leq 0.3$
- With **pf-shell** orbits: $\beta \geq 0.3$
- Quasi-SU(3): $f_{7/2}$-$p_{3/2}$

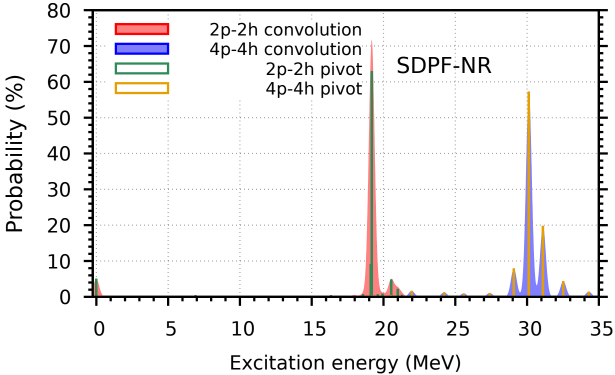




β parameters for SU(3) schemes and nuclear shell model *np*-*nh* configurations

Lanczos strength functions

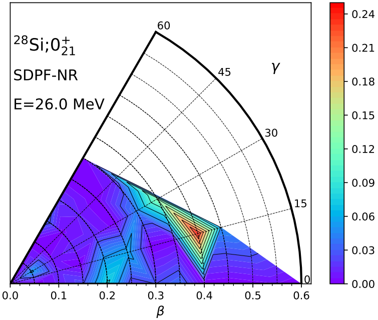
$$|0^+_{np\text{-}nh}\rangle_{sdfp} = \frac{1}{N}\sum_\sigma S(\sigma)|0^+_\sigma\rangle_{sdfp}$$



- 2p-2h at 19 MeV and 4p-4h at **30 MeV**!!!

Full *sd*+*pf* space

- **Full** *sd* + *pf* space
- Almost superdeformed state ($\beta \approx 0.4$)
- $\sim$ **2.5 particles** into the *pf* shell
- Energy (**SDPF-4MeV**): $E = 26$ MeV
- **Not compatible** with SD state $E = 10 - 20$ MeV

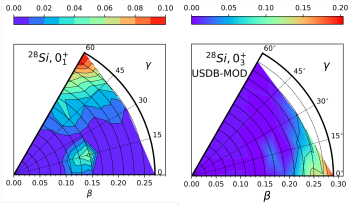


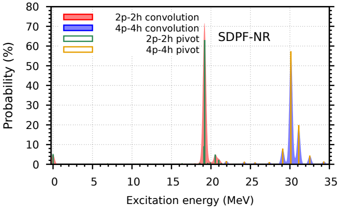
Conclusions

Shape coexistence of **ND structures** within the *sd* shell

- Simple SU(3) model
- Exact **diagonalization** and variational method
- USDB interaction describes oblate band
- USDB-MOD or SDPF is needed for ND prolate band

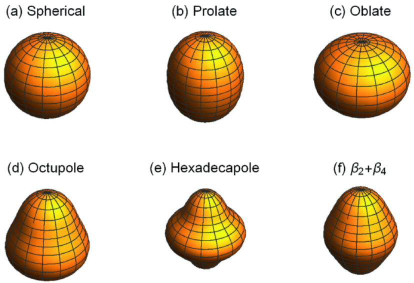
SD structures are **disfavoured** at low energies (10 – 20 MeV)





Outlook

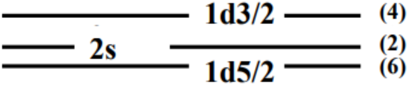
- Monopole modification of USDB
- Multipole deformations
  - $\beta_4(^{28}\mathrm{Si}) = 0.03 \pm 0.01$
- SDPF diagonalization
- Other nuclei: ^{44}Ti, ^{40}Ar, ^{32}S, ^{24}Mg...



Kota, V. K. B. SU (3) symmetry in atomic nuclei. Springer, 2020.

SU(3) model

- **Quadrupole interactions**: realistic Hamiltonians
- Restriction to a major **shell** (Fermi surface)

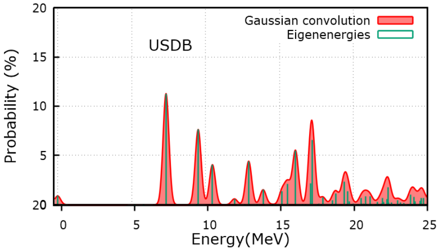


- Energy **competition**: $\mathcal{H} = \mathcal{H}_0 - \kappa Q_0^2$
  - Correlation energy decreases as Q_0^2
  - Single particle energy increases with promoted particles (from $d_{5/2}$ to $s_{1/2}$ or $d_{3/2}$)
- Intrinsic quadrupole moment Q_0:
  - Spherical: $Q_0 = 0$
  - Prolate: $Q_0 > 0$
  - Oblate: $Q_0 < 0$

Elliott, J. P. Proc R Soc Lon Ser-A, 1958. **245**, 128.

Modification of the interaction

- The prolate 4p-4h is **lost** in configuration mixing
- $|{\color{red}4p4h}\rangle = \sum_i c_i |\Psi\rangle_{i,\text{full sd}} \rightarrow$
- USDB: prolate band only has 10% of $|{\color{red}4p4h}\rangle$



Modification of the interaction

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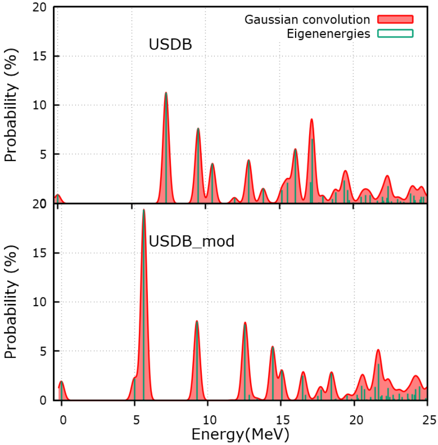
- $|4p4h\rangle = \sum_i c_i |\Psi\rangle_{i,\text{full sd}} \rightarrow$
- USDB: prolate band only has 10% of $|4p4h\rangle$

Too high single-particle energies

Gap between $d_{5/2}$+$s_{1/2}$ and $d_{3/2}$

5 MeV → 3.6 MeV

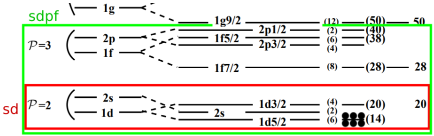
- 4p-4h concentrated in 0_3^+: now goes up to 20%

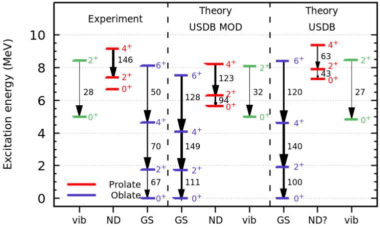


Results

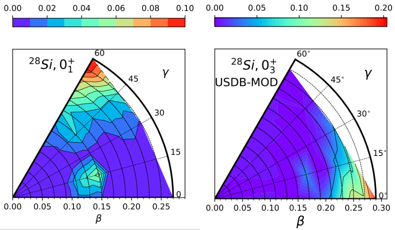
- Shell model techniques:

  Elliott's SU(3): Analytical insight.
  Projected-GCM: Shape of wfs.
  Shell model (ISM): Precise results.





Band structure of the lowest-lying ^{28}Si states.



Problem!

USDB interaction does not reproduce prolate structure.

- **Modified** interaction USDB-MOD.
- Single particle energy adjustment.

Superdeformation (SD)

- Previous studies suggested SD structures at ∼10 MeV.

  Taniguchi, Y., et al. Physical Review C **80**, 044316 (2009).

- Recent experimental attempts **failed** at finding SD structures.

  Morris, L. et al. Physical Review C **104**, 054323 (2021).

Shell model calculations:

- **No SD structures** were found within 10-20 MeV with shell model.
- Possible fragmentation of SD structures?