# Transfer reactions to the continuum involving exotic nuclei 

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## Introduction

- Exotic nuclei

Different properties

- Shell closure
- Halo nature
- Transfer reactions with exotic nuclei Advances in radioactive beam facilities
- Few-body models

Significant effect of deformation


- ${ }^{11} \mathrm{Be}$ and ${ }^{17} \mathrm{C}$
deformed core + neutron structure


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## Structure models

- Deformed 2-body models (neutron+core)
- Nilsson model

$$
H=-\frac{\hbar^{2}}{2 \mu} \nabla^{2}+V_{l s}(r)(\vec{l} \cdot \vec{s})+V_{c}(r)-r \beta \frac{d V_{c}(r)}{d r} Y_{20}\left(\theta^{\prime}\right)+\frac{\hbar^{2}}{2 g} \vec{I}^{2}
$$

- PAMD [PRC89 (2014) 014333]

$$
H=-\frac{\hbar^{2}}{2 \mu} \nabla^{2}+V_{l s}(r)(\vec{l} \cdot \vec{s})+V_{v c}^{A M D}(\vec{r}, \xi)+h_{\text {core }}(\xi)
$$

- Eigenstates from diagonalization in THO basis

$$
\phi_{n l}^{T H O}(r)=\sqrt{\frac{d s}{d r}} \phi_{n l}^{H O}[s(r)]
$$

## Neutron transfer reactions

- $(p, d)$ and $(d, p)$ reactions are studied applying the Adiabatic Distorted Wave Approximation (ADWA).
${ }^{11} \mathrm{Be}(p, d)^{10} \mathrm{Be}$
$\left.{ }^{16} \mathrm{C}(d, p)\right)^{17} \mathrm{C}$



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- These calculations require the $\left\langle{ }^{10} \mathrm{Be} \mid{ }^{11} \mathrm{Be}\right\rangle$ and $\left\langle{ }^{17} \mathrm{C} \mid{ }^{16} \mathrm{C}\right\rangle$ overlaps.
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- Calculations are compared with recent experimental data.

$$
{ }^{11} \mathrm{Be}(p, d)^{10} \mathrm{Be}
$$

$$
{ }^{16} \mathrm{C}(d, p)^{17} \mathrm{C}
$$

- GANIL, 35.3 MeV/nucleon beam [NPA683 (2001) 48]
- GANIL, 17.2 MeV/nucleon beam [PLB811 (2020) 135939]
- RCNP, 26.9 MeV/nucleon beam [Chinese P. L. 35 (2018) 082501]


## Application to ${ }^{17} \mathrm{C}$ and ${ }^{11} \mathrm{Be}$

${ }^{11 B e}$

P. Punta, et al. PRC108 (2023) 024613

## Application to ${ }^{11} \mathrm{Be}(\mathrm{p}, \mathrm{d})^{10} \mathrm{Be}$




## Application to ${ }^{16} \mathrm{C}(\mathrm{d}, \mathrm{p})^{17} \mathrm{C}$




Comparison with data suggest that:

- $\left\langle\left.{ }^{17} C\left(1 / 2_{1}^{+}\right)\right|^{16} C\left(0^{+}\right)\right\rangle$overlaps are realistic in our models.
- $\left\langle\left.{ }^{17} C\left(5 / 2_{1}^{+}\right)\right|^{16} C\left(0^{+}\right)\right\rangle$too small, suggesting a larger spectroscopic factor.


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## Pauli blocking

## N=16 Shell-Gap



Single Particle Leves


## Pseudo-states discretization method

$$
\begin{aligned}
& { }^{16} \mathrm{C}(d, p)^{17} \mathrm{C} \\
& \mathscr{F}(k, \theta)=\sqrt{\frac{\mu_{i} \mu_{f} k_{f}}{\left(2 \pi \hbar^{2}\right)^{2} k_{i}}}\left\langle\chi_{f}^{(-)} \varphi_{p} \Psi^{17} C\right.
\end{aligned}
$$

- Scattering amplitudes for a discrete number of pseudo-states

$$
\mathscr{F}_{n}^{T H O}(\theta)=\sqrt{\frac{\mu_{i} \mu_{f} k_{f}}{\left(2 \pi \hbar^{2}\right)^{2} k_{i}}}\left\langle\chi_{f}^{(-)} \varphi_{p} \Psi_{n}^{T H O}\right| U\left|\chi_{i}^{(+)} \psi_{d} \phi_{16}\right\rangle
$$

- Convolution with the exact n+core scattering states

$$
\mathscr{F}(k, \theta) \approx \sum_{n}\left\langle\Psi^{17} C(k, r) \mid \Psi_{n}^{T H O}(r)\right\rangle \mathscr{F}_{n}^{\text {THO }}(\theta)
$$

## Transfer to the continuum: ${ }^{16} \mathrm{C}(\mathrm{d}, \mathrm{p}){ }^{17} \mathrm{C}$



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## Transfer to the Continuum: ${ }^{10} \mathrm{Be}(\mathrm{d}, \mathrm{p}){ }^{11} \mathrm{Be}$



## Conclusions

- We have checked the suitability the PAMD and Nilsson models to describe ${ }^{11} \mathrm{Be}$ and ${ }^{17} \mathrm{C}$ systems.
- ${ }^{11} \mathrm{Be}$ : better with PAMD model (weak-coupling limit)
- ${ }^{17} \mathrm{C}$ : better with Nilsson model (strong-couplig limit)
- These models have been applied to transfer reactions to bound states and to the continuum using pseudo-states.
- A good reproduction of the structure and transfer reactions have been found.
- The reaction ${ }^{16} \mathrm{C}(\mathrm{d}, \mathrm{p}){ }^{17} \mathrm{C}$ to the continuum have been studied in order to confirm the existence of the $\mathrm{N}=16$ shell-gap.
- Extensions to breakup reactions and other weakly bound nuclei are in progress.



## Thanks for your attention



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## Backup

## Nilsson Diagram for ${ }^{17} \mathrm{C}$

$$
H_{s p}=-\frac{\hbar^{2}}{2 \mu} \nabla^{2}+V_{c}(r)+V_{l s}(r)(\vec{l} \cdot \vec{s})-r \beta \frac{d V_{c}(r)}{d r} Y_{20}\left(\theta^{\prime}\right)
$$



I. Hamamoto, PRC76 (2007) 054319

## Occupation of single-particular levels




## Pauli Blocking




## Wave Functions

${ }^{17} \mathrm{C}$ ground state $3 / 2^{+}$


$$
u_{j l}^{\lambda j^{\pi}}(r)=\sqrt{2} \sum_{\Omega}(-1)^{j+\Omega}\langle I 0 \mid j-\Omega J \Omega\rangle u_{j \Omega}^{\lambda j^{j}}(r)
$$

## ${ }^{11} \mathrm{Be}(p, d)^{10} \mathrm{Be}$



## ${ }^{16} \mathrm{C}(\mathrm{d}, \mathrm{p}){ }^{17} \mathrm{C}$



## Transfer to bound states

Sum for the three bound states


## Transfer to the continuum

- The prior form of the ADWA approximation is used
$X\left\langle{ }^{17} \mathrm{C} \mid{ }^{16} \mathrm{C}\right\rangle \rightarrow$ Too much spatial extension
$\left.\left.\checkmark\left\langle{ }^{17} \mathrm{C}\right| V_{n-16}{ }^{16}\right|^{16} \mathrm{C}\right\rangle \rightarrow$ Little spatial extension
- The continuum is discretized with the pseudo-states method
- Scattering amplitudes for a discrete number of pseudo-states
- Convolution with the exact $n+c o r e ~ s c a t t e r i n g ~ s t a t e s ~$

$$
\mathscr{F}(k, \theta) \approx \sum_{n}\left\langle\Psi(k, r) \mid \Psi_{n}^{T H O}(r)\right\rangle \mathscr{F}_{n}^{T H O}(\theta)
$$

## ${ }^{16} \mathrm{C}(\mathrm{d}, \mathrm{p}){ }^{17} \mathrm{C}:$ Prior-Post

Bound states


Transfer to the continuum


## Transfer to the continuum: Decay mode




