ANALYTICAL SOLUTIONS OF THE BOHR HAMILTONIAN WITH THE SEXTIC OSCILLATOR: PT-OS ISOTOPES.

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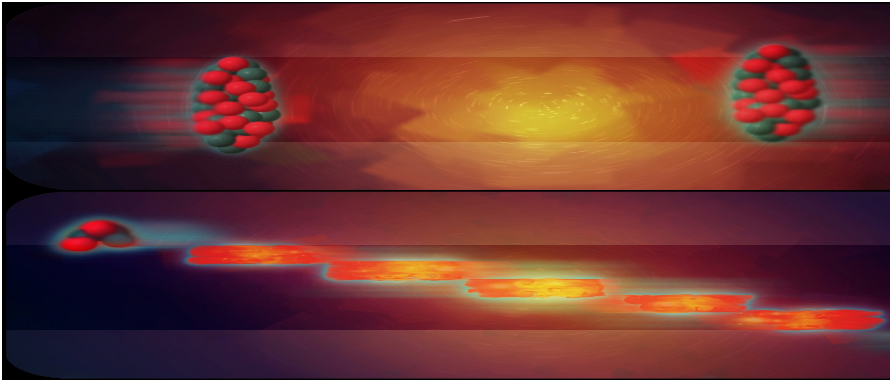
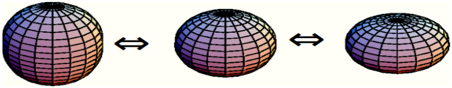


INTRODUCTION

NUCLEAR STRUCTURE

- The nucleus is a system of many strongly interacting fermions.
- A variety of models have been introduced for the theoretical description of this system, each focusing on different aspects.
- Bohr-Mottelson Collective Model:
- One of the most significant characteristics of nuclei is their shape. .

$$R(\theta,\phi) = R_o\left(1 + \sum_{\lambda} \sum_{\mu=-\lambda}^{+\lambda} \alpha_{\lambda\mu} Y^{*}_{\lambda\mu}(\theta,\phi)\right)$$



BOHR HAMILTONIAN

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$$H_B = -\frac{\hbar^2}{4B}[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2 \sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_k \frac{Q_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)}] + V(\beta,\gamma),$$

where:
$\Longrightarrow$ β being a deformation coordinate measuring departure from spherical shape, and γ being an angle measuring departure from axial symmetry.
$\Longrightarrow$ $Q_k (k = 1, 2, 3)$ are the components of angular momentum.
$\Longrightarrow$ B is the mass parameter.

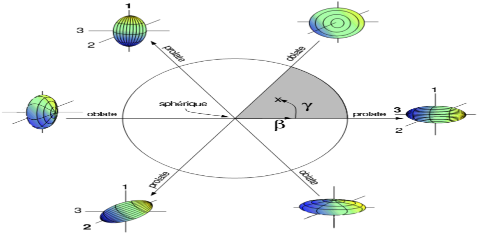

Figure: Nuclear deformations in the β, γ plane for quadrupolar nuclei.

BOHR HAMILTONIAN

$V(\beta,\gamma)$ Potentials of frequent use

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- $\Longrightarrow V(\beta,\gamma)=u(\beta)$: γ-unstable nuclei.

BOHR HAMILTONIAN

$V(\beta)$

- For these potentials the β-dependence can be separated into an equation similar to the radial Shrödinger equation by using the substitution

$$\Psi(\beta,\gamma,\theta_i) = \beta^{-2}\phi(\beta)\Phi(\gamma,\theta_i), \tag{1}$$

The $\beta-$differential equation is then

$$-\frac{d^2\phi(\beta)}{d\beta^2} + \left(\frac{(\tau+1)(\tau+2)}{\beta^2} + u(\beta)\right)\phi(\beta) = \epsilon\phi(\beta). \tag{2}$$

where τ is the seniority quantum number, and the reduced energies and potentials are defined as $\epsilon = \frac{2B}{\hbar^2}E$ and $u(\beta) = \frac{2B}{\hbar^2}V(\beta)$, respectively.

SEXTIC OSCILLATOR POTENTIAL

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- The form of the sextic potential used in the Bohr Hamiltonian is:

$$u(\beta) = (b^2 - 4ac^{\pi})\beta^2 + 2ab\beta^4 + a^2\beta^6 + u_0^{\pi}, \quad (3)$$

where a and b are real parameters.

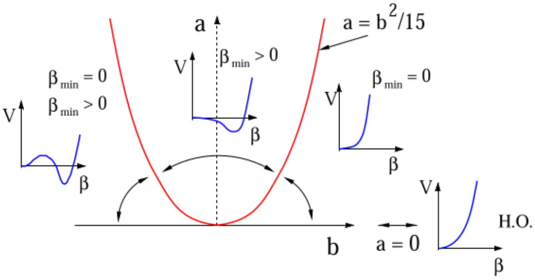

Figure: Different possible shapes for the energy surface u(β) in the parameter (a, b) model space.

Excitation energies and B(E2)transitions probabilities

The energy spectrum and B(E2)

- The rescaled energy eigenvalues will be denoted as

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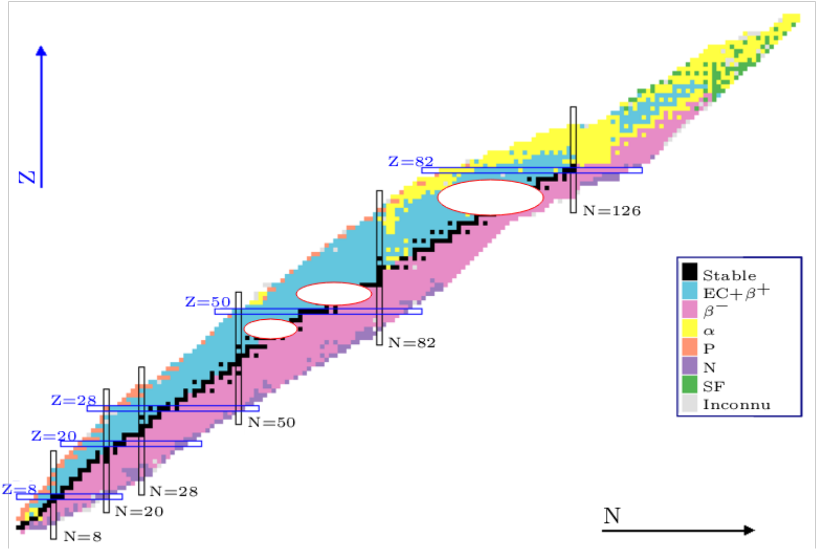
- The electric quadrupole transitions are calculated using the first-order transition operator

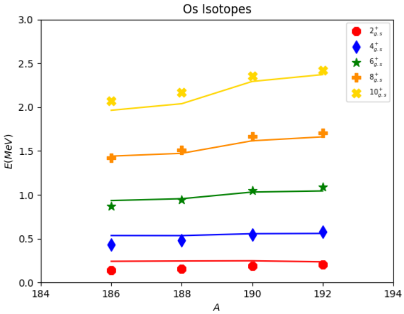
$$T^{(\mathrm{E2})} = t\beta \left[D^{(2)}_{\mu,0}(\theta_i)\cos\gamma + \frac{1}{\sqrt{2}}\left(D^{(2)}_{\mu,2}(\theta_i) + D^{(2)}_{\mu,-2}(\theta_i)\right)\sin\gamma \right] . \tag{5}$$

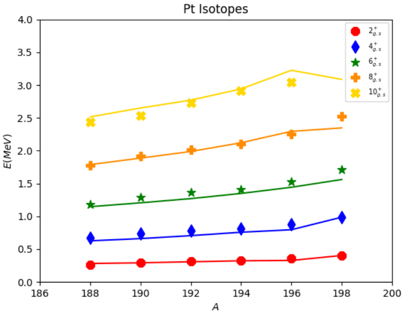
APPLICATION



APPLICATION FOR Pt AND Os ISOTOPES

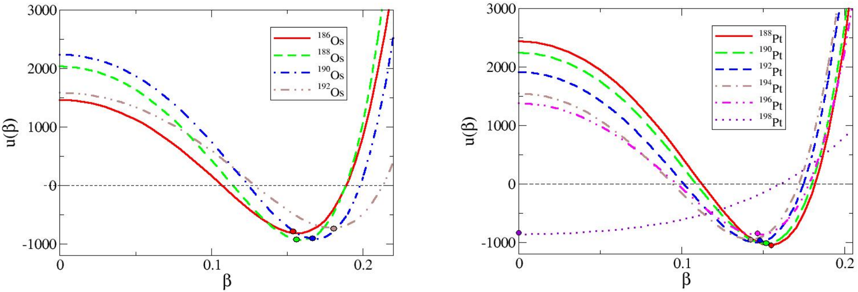
Excitation energies





APPLICATION FOR Pt AND Os ISOTOPES

The potentials u(β)



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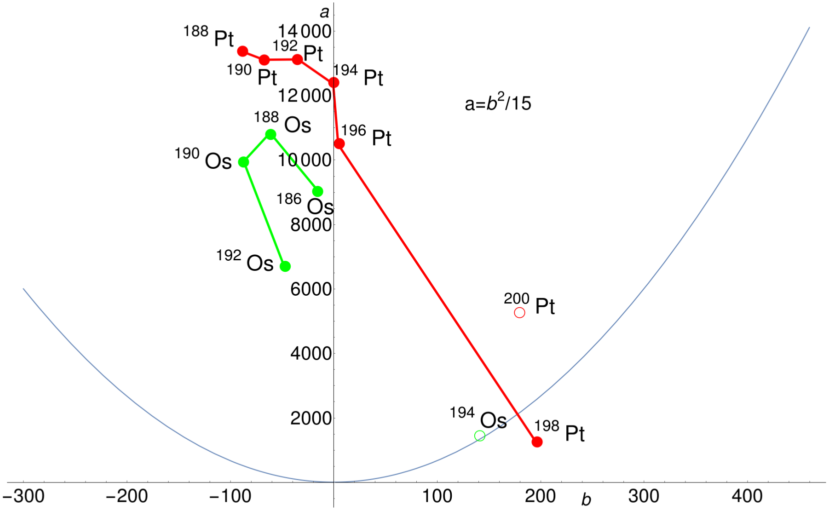
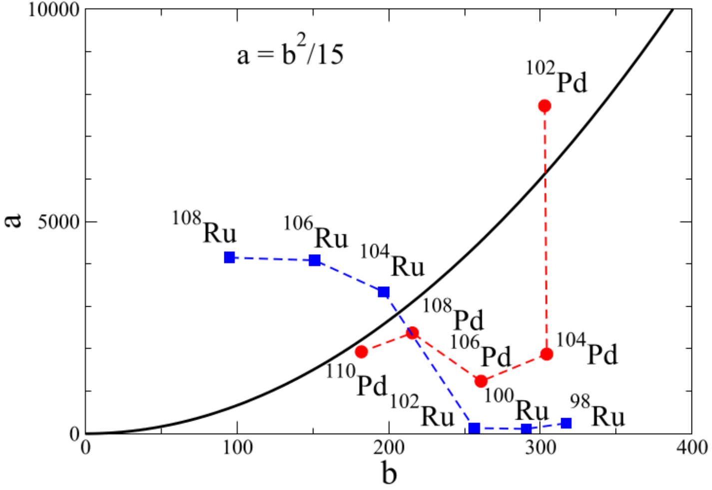
The location of the Pt and Os nuclei in the (a, b) phase space.



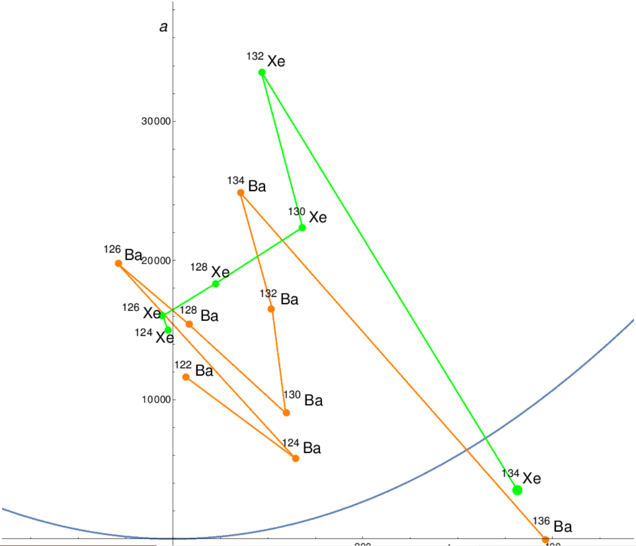
APPLICATION FOR Ru AND Pd ISOTOPES

The location of the Ru and Pd nuclei in the (a, b) phase space.



APPLICATION FOR Xe AND Ba ISOTOPES

The location of the Xe and Ba nuclei in the (a, b) phase space.



CONCLUSION

The γ-unstable BH Sextic Potential

- $\Longrightarrow$The sextic oscillator has been applied in the Bohr Hamiltonian to different isotopic chains of nuclei, with the aim of exploring a possible transition from the γ-unstable phase to the spherical vibrator phase for Pt-Os and Xe-Ba isotopic chains, while for Ru-Pd isotopic chains the reverse occur.

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- $\Longrightarrow$The model gives a reasonable description of the Pt and Os isotopes as γ-unstable nuclei.
- $\Longrightarrow$The Pd-Ru region of the nuclear chart has been studied with the extended model with good results.
- $\Longrightarrow$Based on the results of the last study, the application of the model to other isotopic chains is interesting, with Xe and Ba being prime candidates.


Thank You