The smooth out of shape coexistence around Z=40

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What is shape coexistence?

Shape coexistence: It appears in quantum systems where eigenstates with very different density distribution coexist.

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Shape of the nucleus

(Implicit geometric interpretation)

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- **Stabilizing effect:** closed shell
- **Deformed tendency**: pairing and quadrupole force

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Regions around closed shells with spherical shapes and near mid-shell are well deformed

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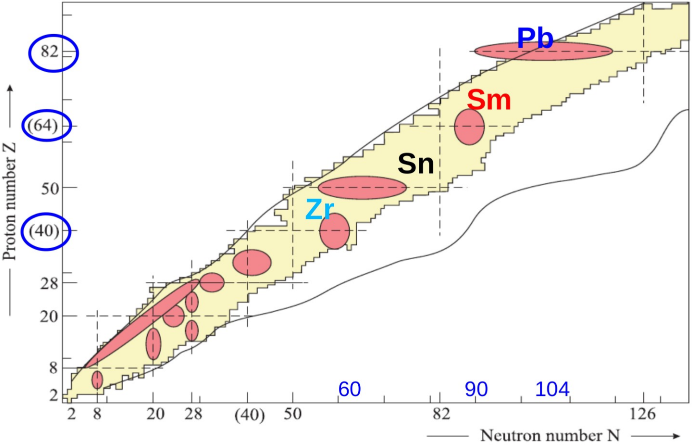
Regions of interest



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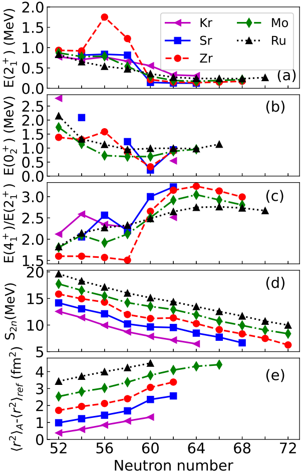
Experimental data around A=100 region

Experimental values for key quantum phase transition and shape coexistence observables for Kr, Sr, Zr, Mo, and Ru isotopes as a function of neutron number.



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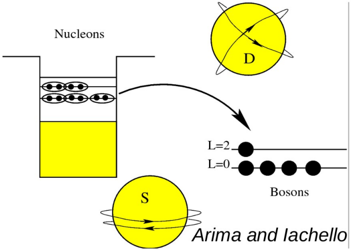
Interacting boson Model. IBM

Nucleons couple preferably in pairs with angular momentum either equal to 0 (S) or equal to 2 (D).

$$s^{\dagger}, d^{\dagger}_{m} (m = 0, \pm 1, \pm 2)$$
$$s, d_{m} (m = 0, \pm 1, \pm 2)$$

$$\boxed{\hat{H}_{ECQF} = \epsilon \hat{n}_d + \kappa \hat{Q}\cdot\hat{Q} + \kappa' \hat{L}\cdot\hat{L}}$$



- Model based on a u(6) spectrum generator algebra. It is especially suited for medium and heavy-mass nuclei.
- The number of bosons, N, corresponds the number of nucleons pairs, regardeless its proton, neutron, particle or hole nature.

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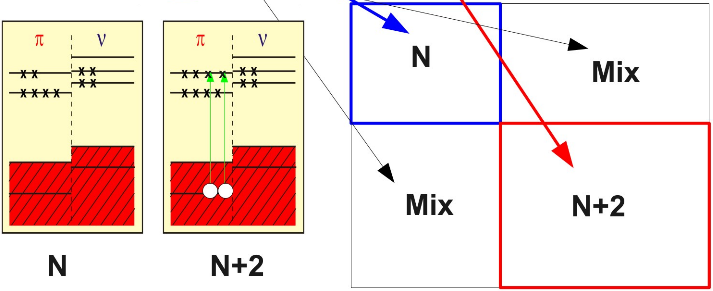
IBM with configuration mixing

$$\hat{H} = \hat{P}_N^{\dagger} \hat{H}^{N}_{\mathrm{ECQF}} \hat{P}_N + \hat{P}^{\dagger}_{N+2} \left(\hat{H}^{N+2}_{\mathrm{ECQF}} + \Delta^{N+2} \right) \hat{P}_{N+2} + \hat{V}^{N,N+2}_{\mathrm{mix}},$$



A different Hamiltonian, $\hat{H}^{N}_{ECQF}$ and $\hat{H}^{N+2}_{ECQF}$, acts on the regular [N] and intruder [N+2] sectors, separately

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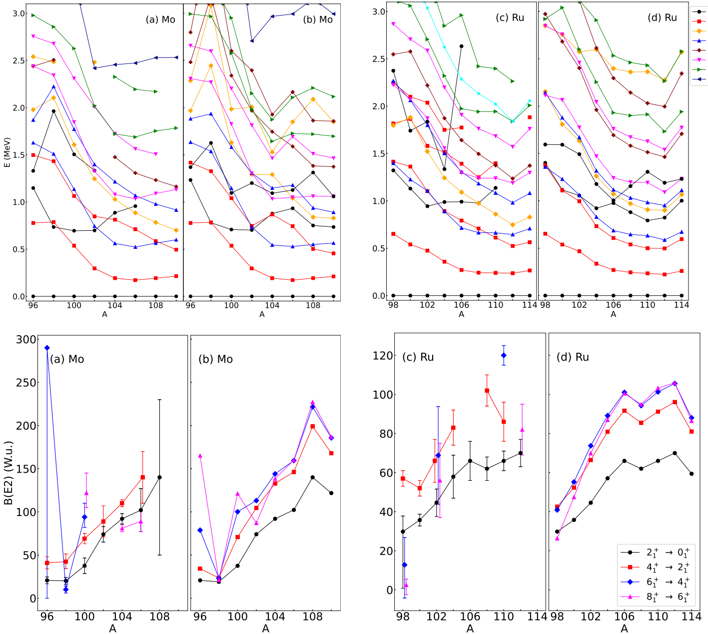
Excitation energies and B(E2)s



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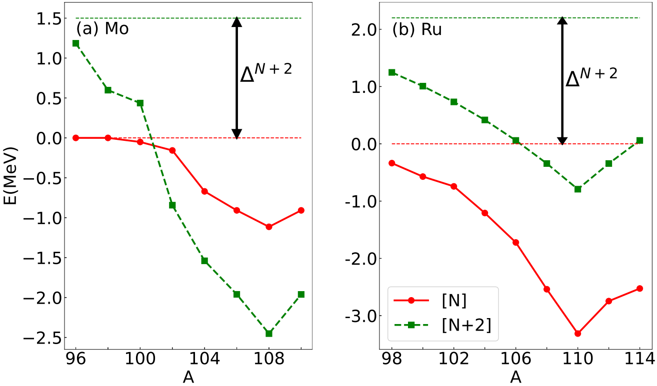
Correlation energy

- Increases with the number of bosons, being larger for the intruder configuration. Although it is corrected by the pairing energy gain resulting from the formation of two extra 0^+ pairs.

$$\boxed{V_{mix} = 0}$$



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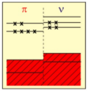
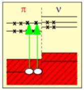
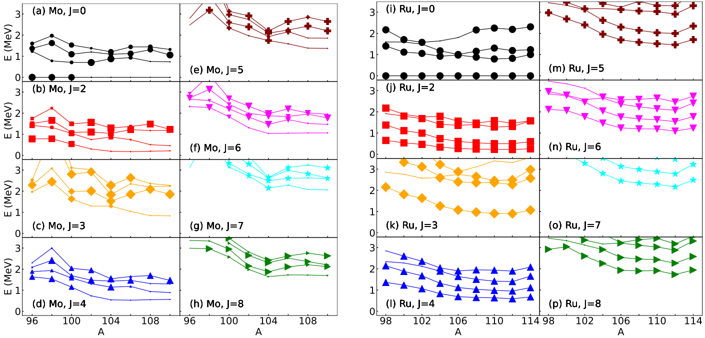
Wave function: Regular component

$\phi(J,M)=a(J,M)$  $+\ b(J,M)$ 

- The size of each dot associated with a state is proportional to the regular content of its wave function.
- **Reference point:** the size of the dot for the 0_1^+ states in ^{96}Ru (panel (l)) corresponds to 100% of regular content.



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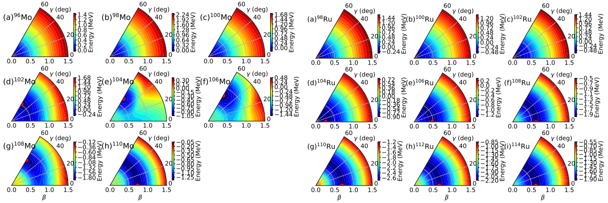
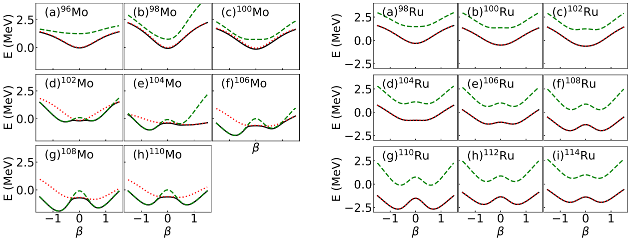
Energy Surfaces

Using the IBM-CM mean-field formalism, we can calculate the energy density functional based on deformation parameters.

Energy surfaces for Mo and Ru



Axial symmetry energy for Mo and Ru



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Quantum Phase Transitions in Ru and Mo

- QPT occurs in systems where the ground state's structure undergoes a sudden change when a control parameter varies slightly around a specific value.
- The presence of a QPT is generally associated with a combination of Hamiltonians possessing different symmetries (A or B).

$$\hat{H} = (1 - x)\hat{H}_A + x\hat{H}_B.$$

Key elements for finding QPTs in Mo and Ru isotopes

- In the case of Mo, a crossing of regular and intruder configurations exists at the phase transition point.
- In Ru isotopes the evolution of the ground state is fully determined by a single configuration and the energy surface of Ru isotopes is initially spherical for the lighter ones, but it starts flattening and becoming fully flat at $A = 104$. From this point onwards, a γ-unstable deformation develops.

Two-neutron separation energies

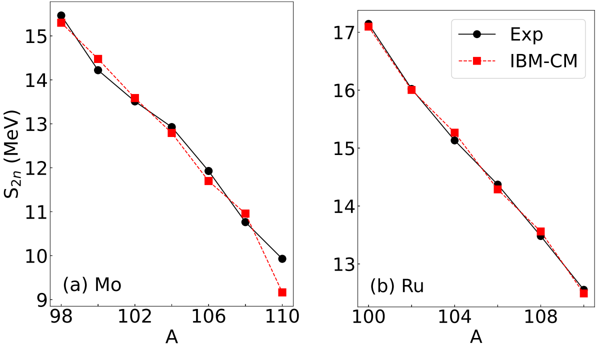
In the framework of the IBM, the definition of the S_{2n} is expressed by,

$$S_{2n}(A) = \mathcal{A} + \mathcal{B}A + BE^{lo}(A) - BE^{lo}(A-2),$$

Where BE^{lo} represents the "local" binding energy and we anticipate that the effective number of bosons will be influenced by the presence of intruder states,

$$S_{2n}(A) = \mathcal{A} + \mathcal{B}(A + 2(1-w)) + BE^{lo}(A) - BE^{lo}(A-2),$$

where $w = w^1(0)$ $(w^k(J) \equiv \sum_i |a_i^k(J)|^2)$.



Conclusions

- Our theoretical results for excitacion energies, B(E2) values, two-neutron separation energies, nuclear radii and isotope shifts show good agreement with experimental data for the entire chain of isotopes.
- Shape coexistence plays a significant role in Mo isotopes, with the crossing of intruder and regular configurations ocurring at neutron number 60 ($A = 102$), which induces a quantum phase transition.
- Ru isotopes present in contrast minimal influence of the intruder states, remaining at higher energies. However at neutron number 60, a quantum phase transition is observed.

THANK YOU

Interacting Boson Model

$$\boxed{\hat{H}^{i}_{\text{ecqf}} = \varepsilon_i \hat{n}_d + \kappa'_i \hat{L} \cdot \hat{L} + \kappa_i \hat{Q}\left(\chi_i\right) \cdot \hat{Q}\left(\chi_i\right)}$$

$$\boxed{\hat{L}_{\mu} = \left[d^{\dagger} \times \tilde{d}\right]^{(1)}_{\mu}}$$

$$\boxed{\hat{Q}_{\mu}\left(\chi_i\right) = \left[s^{\dagger} \times \tilde{d} + d^{\dagger} \times s\right]^{(2)}_{\mu} + \chi_i \left[d^{\dagger} \times \tilde{d}\right]^{(2)}_{\mu}}$$

$$\boxed{\hat{V}^{N,N+2}_{\text{mix}} = \omega^{N,N+2}_{0} \left(s^{\dagger} \times s^{\dagger} + s \times s\right) + \omega^{N,N+2}_{2} \left(d^{\dagger} \times d^{\dagger} + \tilde{d} \times \tilde{d}\right)^{(0)}}$$

$$\boxed{\hat{T}(E2)_{\mu} = \sum_{i=N,N+2} e_i \hat{P}^{\dagger}_{i} \hat{Q}_{\mu}\left(\chi_i\right) \hat{P}_i}$$

Energy Surfaces

We have considered the coherent state:

$$|N,\alpha_m\rangle = \left(s^\dagger + \sum_m \alpha_m d_m^\dagger\right)^N |0\rangle$$

Where the relation with the collective parameters:

$$\alpha_0 = \beta\cos\gamma, \quad \alpha_{\pm1} = 0, \quad \alpha_{\pm2} = \frac{\beta}{\sqrt{2}}\cos\gamma$$

$$|N;\beta,\gamma\rangle = \left\{s^\dagger + \beta\left[\cos\gamma d_0^\dagger + 1/\sqrt{2}\sin\gamma\left(d_{+2}^\dagger + d_{-2}^\dagger\right)\right]\right\}^N |0\rangle$$

$$E(N;\beta,\gamma) = \frac{\langle N;\beta,\gamma|H|N;\beta,\gamma\rangle}{\langle N;\beta,\gamma \mid N;\beta,\gamma\rangle}$$

Weisskopf units

One Weisskopf unit of $B(E\lambda)$ is equal to

$$B(E\lambda) = \frac{(1.2)^{2\lambda}}{4\pi}\left(\frac{3}{\lambda+2}\right)^2 A^{2\lambda/3} \quad \text{in unit of } e^2(fm)^\lambda$$

Transition probability

$$\mathrm{T(E2)} = 1.223 \times 10^9 E_\gamma^5 B(E2)[1/\mathrm{sec}]$$

E_γ is in MeV.
$\mathrm{B(E2)}$ in $e^2(fm)^4$

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