Cosmic Acceleration from First Principles

Based on:
JGB, Espinosa, arXiv:2106.16012 , 2106.16014
Arjona, Espinosa, JGB, Nesseris, arXiv:2111.13083
and JGB, arXiv:2302.08537, **2306.10593**
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Forces in Physics

- **Fundamental Forces**

  Gravitation, Strong, Weak, E.M.

- **Residual Forces**

  Molecular, Nuclear, Surface Tension

- **Collective Forces**

  Brownian motion, Entropic Forces

$$F dx = dW = -dU + T dS \;\Rightarrow\; F = -\frac{dU}{dx} + T\frac{dS}{dx}$$

Our proposal

- Entropic Forces are responsible for present cosmic acceleration and many other LSS phenomena.
- Use a covariant formalism of out-of-equilibrium phenomena in GR.
- Just Quantum Mechanics (QFT), (Non eq.) Thermodynamics and GR.

Entropic forces in mechanics

General mechanical system with two components:

- **Slow** d.o.f. described with **canonical coordinates** (q, p)
- **Fast** d.o.f. **coarsegrained** as a thermodynamical system with macroscopic quantities (S, T)
- The interaction between the slow and fast d.o.f. are described by the Thermodynamical constraint: the First Law of Thermodynamics

Entropic forces in GR

JGB, Espinosa (2021)

$$\mathcal{S} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\,R + \int d^4x\sqrt{-g}\,\mathcal{L}_m(g_{\mu\nu},\,S)$$

Entropy

$$\delta\mathcal{S} = \int d^4x\left(\frac{1}{2\kappa}\frac{\delta(\sqrt{-g}\,R)}{\delta g^{\mu\nu}} + \frac{\delta(\sqrt{-g}\,\mathcal{L}_m)}{\delta g^{\mu\nu}}\right)\delta g^{\mu\nu} + \int d^4x\sqrt{-g}\,\frac{\partial\mathcal{L}_m}{\partial S}\delta S$$

Variational constraint: **First law thermodynamics**

$$\frac{\partial\mathcal{L}_m}{\partial s}\delta s = \frac{1}{2}\,f_{\mu\nu}\delta g^{\mu\nu}$$

Non-equilibrium Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa\,(T_{\mu\nu} - f_{\mu\nu})$$

Entropic force

Entropy (anti)gravitates !

GREA = General Relativistic Entropic Acceleration

Gravitational Collapse

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa \left(T_{\mu\nu} - f_{\mu\nu} \right) \equiv \kappa \, \mathcal{T}_{\mu\nu}$$

Variational constraint: **First law thermodynamics**

$$-dW = -\vec{F} \cdot d\vec{x} \; = \; dU + \left(P - T \frac{dS}{dV} \right) dV$$

Effective Pressure

$$\equiv \; dU + \tilde{P} \, dV$$

Coeff. viscosity

$$f_{\mu\nu} = \zeta \, D_\lambda u^\lambda \left(g_{\mu\nu} + u_\mu u_\nu \right) = \zeta \, \Theta \, h_{\mu\nu}$$

$$\begin{aligned} \mathcal{T}^{\mu\nu} \; &= \; P \, g^{\mu\nu} + (\epsilon + P) u^\mu u^\nu - \zeta \, \Theta \, h^{\mu\nu} \\ &= \; \tilde{P} \, g^{\mu\nu} + (\epsilon + \tilde{P}) u^\mu u^\nu \, , \end{aligned}$$

$$\zeta = \frac{T}{\Theta} \frac{dS}{dV}$$

Maintains the perfect fluid form

Gravitational Collapse

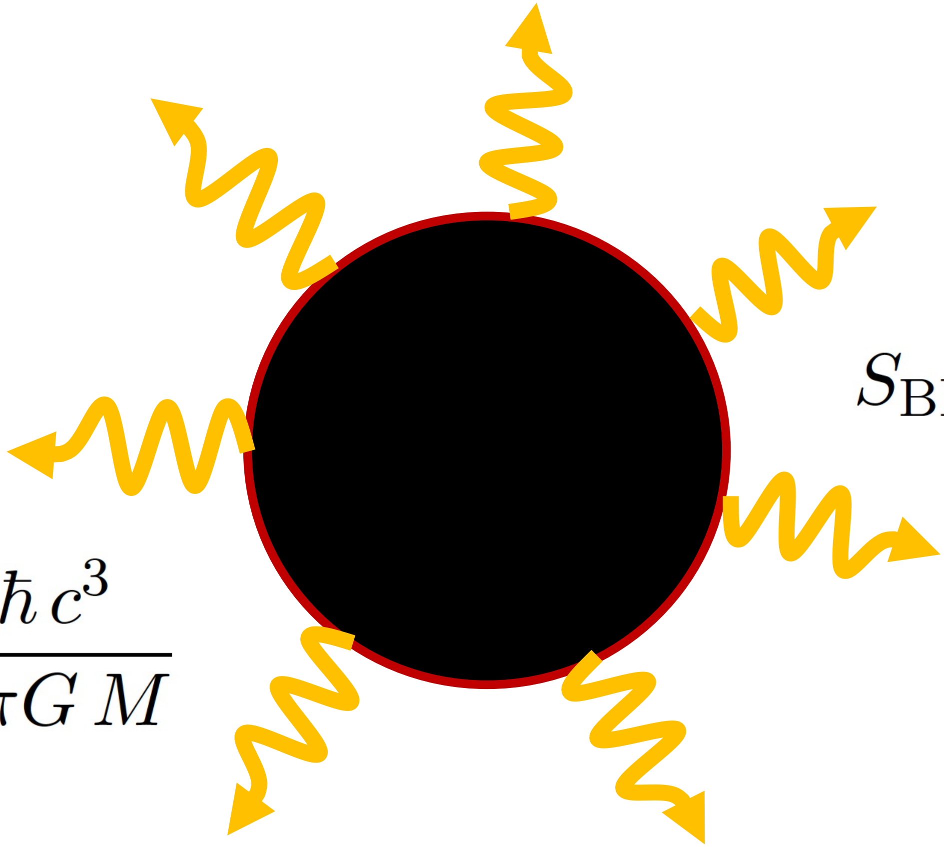
Raychaudhuri equation for geodesic motion

$$\begin{aligned}\frac{D}{d\tau}\Theta + \frac{1}{3}\Theta^2 &= -\sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}u^\mu u^\nu \\ &= -\kappa\left(T_{\mu\nu}u^\mu u^\nu + \frac{1}{2}T^\lambda_{\ \lambda} - \frac{3}{2}\zeta\Theta\right) \\ &= -\frac{\kappa}{2}(\epsilon + 3\tilde{P}) = -\frac{\kappa}{2}\left(\epsilon + 3P - 3T\frac{dS}{dV}\right).\end{aligned}$$

Due to the extra entropic term in the effective pressure, even for matter that satisfies the strong energy condition, $\epsilon+3P>0$, it's possible to prevent gravitat. collapse, $\dot{\Theta} + \Theta^2/3 > 0$, as long as entropy production is significant, i.e. $3TdS/dV > (\epsilon+3P) > 0$

Hawking Radiation

Temperature & Entropy of a black hole horizon



$$S_{\rm BH} = k_{\rm B} \frac{4\pi G\, M^2}{\hbar\, c}$$

$$k_{\rm B} T_{\rm BH} = \frac{\hbar\, c^3}{8\pi G\, M}$$

$$(\hbar,\ c,\ G,\ k_{\rm B})$$

Bekenstein (1973)

Hawking (1974)

Entropic forces in GR

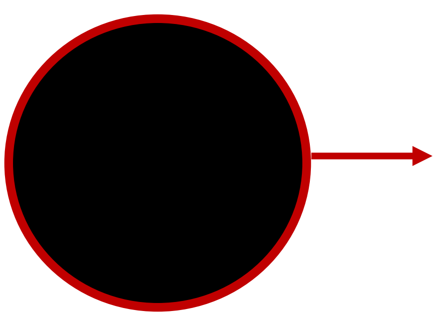
Temperature and Entropy from the gravity sector

- Horizon H with induced metric h

$$\mathcal{S}_{\mathrm{GHY}} = \frac{1}{8\pi G}\int_H d^3y\,\sqrt{h}\,K = \frac{1}{8\pi G}\int_H dt\,\sin\theta d\theta\,d\phi\,\sqrt{h}\,K$$

- Schwarzschild black hole

$$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2 .$$



$$n = -\sqrt{1-\frac{2GM}{r}}\partial_r$$

normal vector to S_2 of radius r

Entropic forces in GR

Gibbons & Hawking (1977), York (1973) JGB, Espinosa (2021)

$$\mathcal{S}_{\rm GHY} = \frac{1}{8\pi G}\int_H d^3y\,\sqrt{h}\,K = \frac{1}{8\pi G}\int_H dt\,\sin\theta d\theta\,d\phi\,\sqrt{h}\,K$$

$$\sqrt{h}K = (3GM - 2r)\sin\theta$$ event horizon @ $r = 2GM$

$$\mathcal{S}_{\rm GHY} = -\frac{1}{2}\int dt\,Mc^2 = -\int dt\,T_{\rm BH}S_{\rm BH}$$

Classical (emergent)

$$k_{\rm B}T_{\rm BH} = \frac{\hbar c^3}{8\pi G\,M}$$

Quantum origin

$$S_{\rm BH} = k_{\rm B}\frac{4\pi G\,M^2}{\hbar c}$$

Cosmology

Homogeneous and isotropic universe

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - k\, r^2} + r^2 d\Omega_2^2 \right)$$

Filled with a perfect (ideal) fluid

$$T_{\mu\nu} = p\, g_{\mu\nu} + (\rho + p)\, u_\mu u_\nu$$

Friedmann (Einstein) equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \qquad \text{(F1)}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\,(\rho + 3p) \qquad \text{(F2)}$$

Cosmology

Covariant Energy-Momentum Tensor Conservation

$$D_\mu T^{\mu\nu} = 0 \qquad \Rightarrow \qquad \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

Also derived from First Law Thermodynamics (in equil.)

$$T\frac{dS}{dt} = \frac{d}{dt}\left(\rho\, a^3\right) + p\frac{d}{dt}\left(a^3\right) = 0$$

On a few occasions (e.g. Big Bang)

$$T\frac{dS}{dt} \geq 0$$

entropy production

Cosmology

Beyond adiabatic cosmology

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = \frac{T\dot{S}}{a^3}$$

(continuity eq.)

Together with Friedmann equation (F1)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

entropic force

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p - \frac{T\dot{S}}{a^2\dot{a}}\right)$$

Entropic forces in FLRW

Non-equilibrium thermodynamics in expanding universe

$$ds^2 = -N(t)^2 dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\right)$$

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu + p g^{\mu\nu} \qquad D^\mu T_{\mu\nu} = D^\mu f_{\mu\nu}$$

First law thermodynamics

$$T dS = d(\rho a^3) + p\, d(a^3) \quad \Rightarrow \quad \dot\rho + 3H(\rho + p) = \frac{T\dot S}{a^3}$$

Hamiltonian constraint $\dot a^2 + k = \frac{8\pi G}{3}\rho\, a^2$

Friedmann/Raychaudhuri equation

Entropic Force

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{4\pi G}{3}\frac{T\dot S}{a^3 H}$$

Entropic forces in FLRW

- Causal Cosmological Horizon H

$$\sqrt{h}K = 2N(t)\, r\, a\sqrt{1-kr^2}\,\sin\theta$$ Trace extrinsic curvature

$$d_H \;=\; a\,\eta$$ Causal horizon distance

$$r_H = \sinh(\eta\sqrt{-k})/\sqrt{-k}$$ Conformal time η

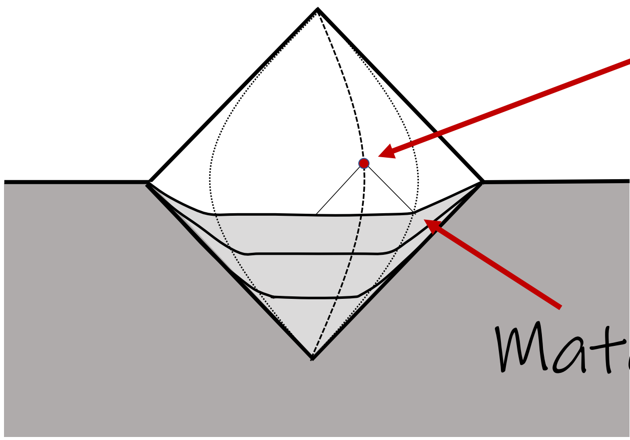
$$S_{GHY} \;=\; -\frac{1}{2G}\int dt\, N(t)\,\frac{a}{\sqrt{-k}}\sinh(2\eta\sqrt{-k})$$

$$= \;-\int dt\, N(t)\, T_H S_H = -\int dt\, N\, a^3 \rho_H$$

$$T_H = \frac{\hbar c}{2\pi}\frac{\sinh(2\eta\sqrt{-k})}{a\, r_H^2\sqrt{-k}}\,, \qquad S_H = \frac{c^3}{\hbar}\frac{\pi a^2 r_H^2}{G}$$

Emergent

Cosmic Acceleration



Observer's causal horizon

$$\rho_H\, a^2 = \frac{T_H S_H}{a} = \frac{1}{2G}\,\frac{\sinh(2a_0 H_0 \eta)}{a_0 H_0}$$

Matching: $H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \implies \sqrt{-k} = a_0 H_0$

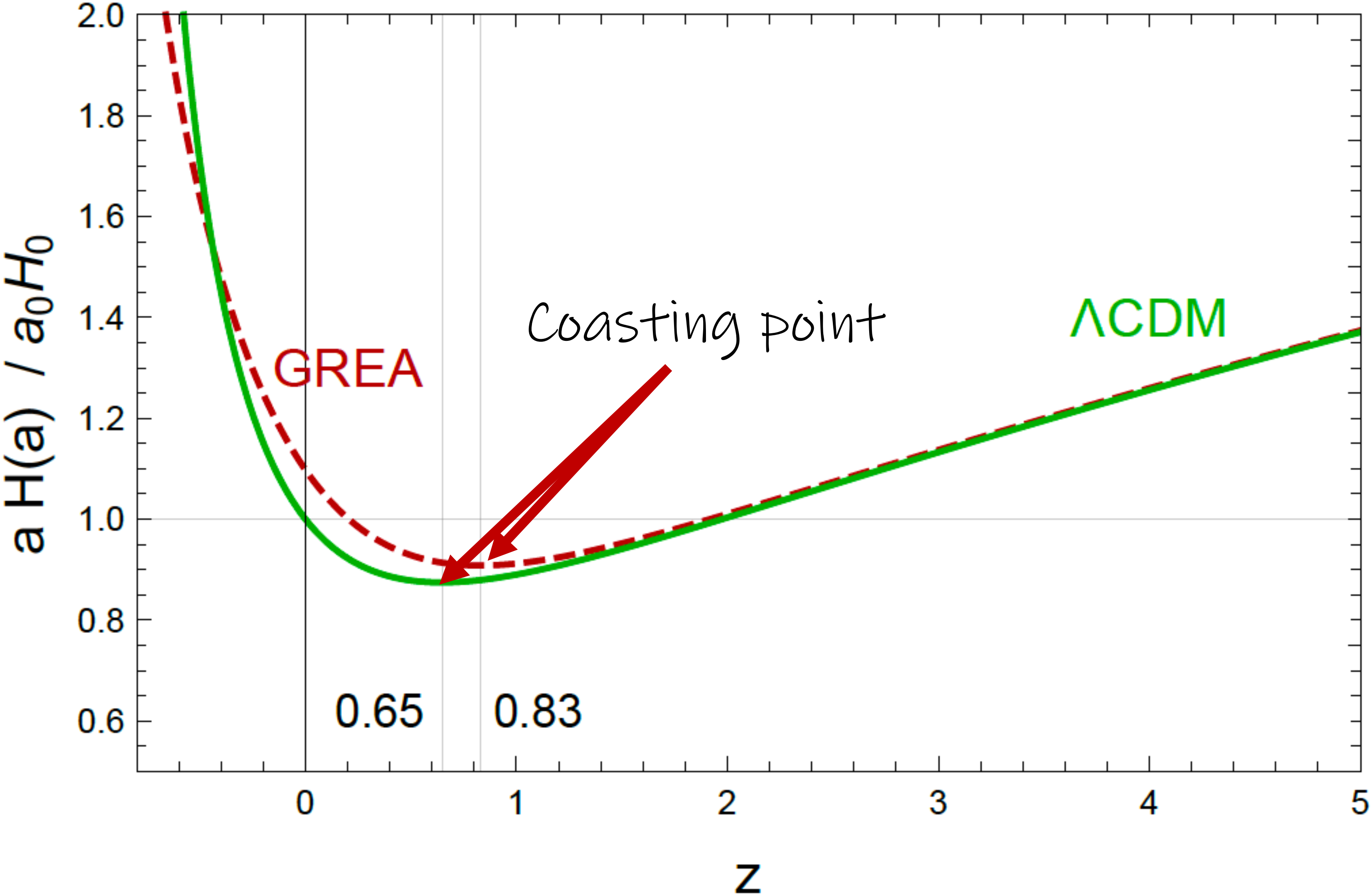
Hamiltonian constraint in conformal time
(primes denote derivatives w.r.t. $\tau = a_0 H_0 \eta$)

$$\left(\frac{a'}{a}\right)^2 = \Omega_{\rm M}\left(\frac{a_0}{a}\right) + \Omega_{\rm K} + \frac{4\pi}{3}\Omega_{\rm K}^{3/2}\sinh(2\tau)$$

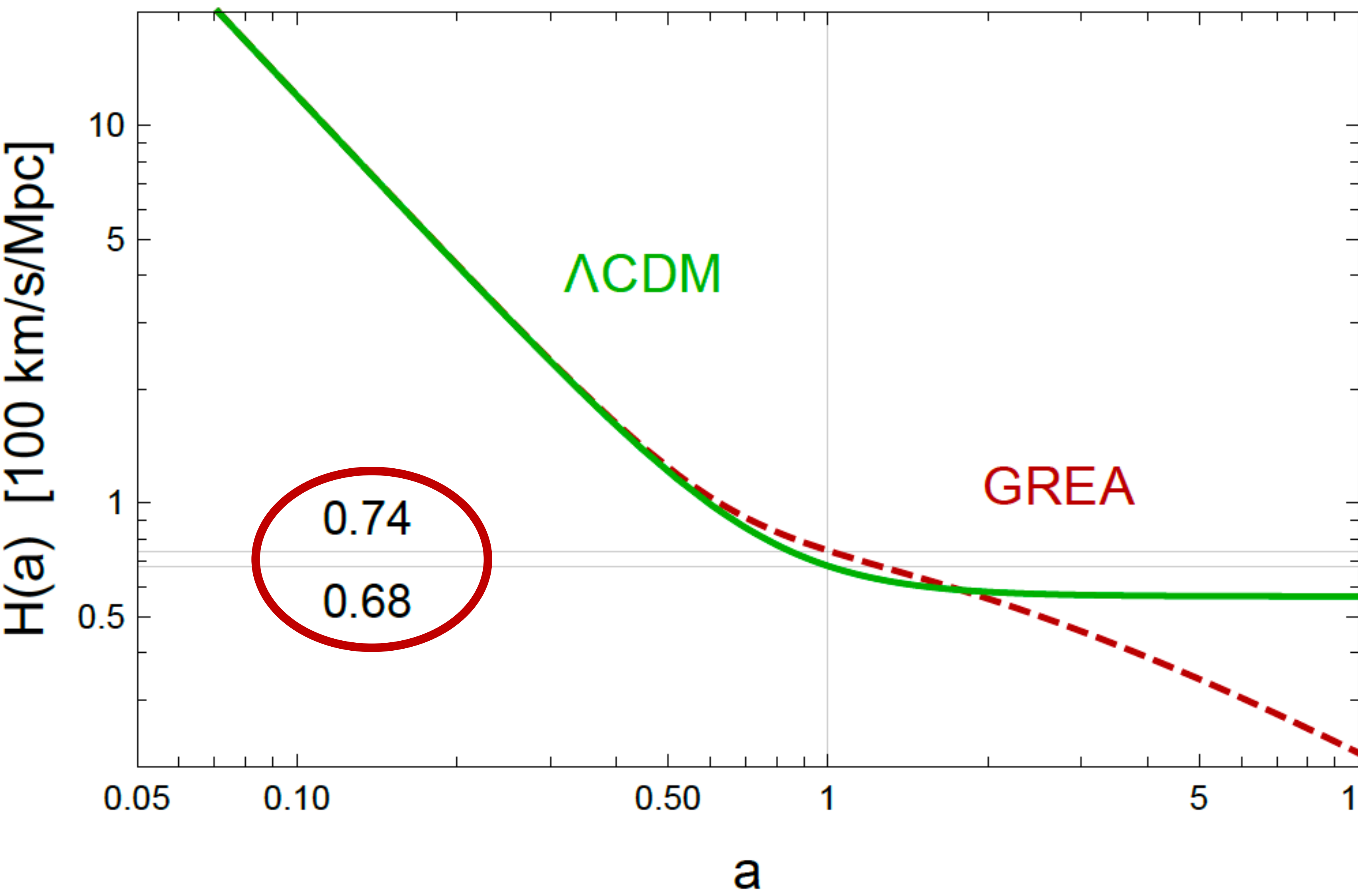
Entropic force term

Note: $\Lambda = 0$

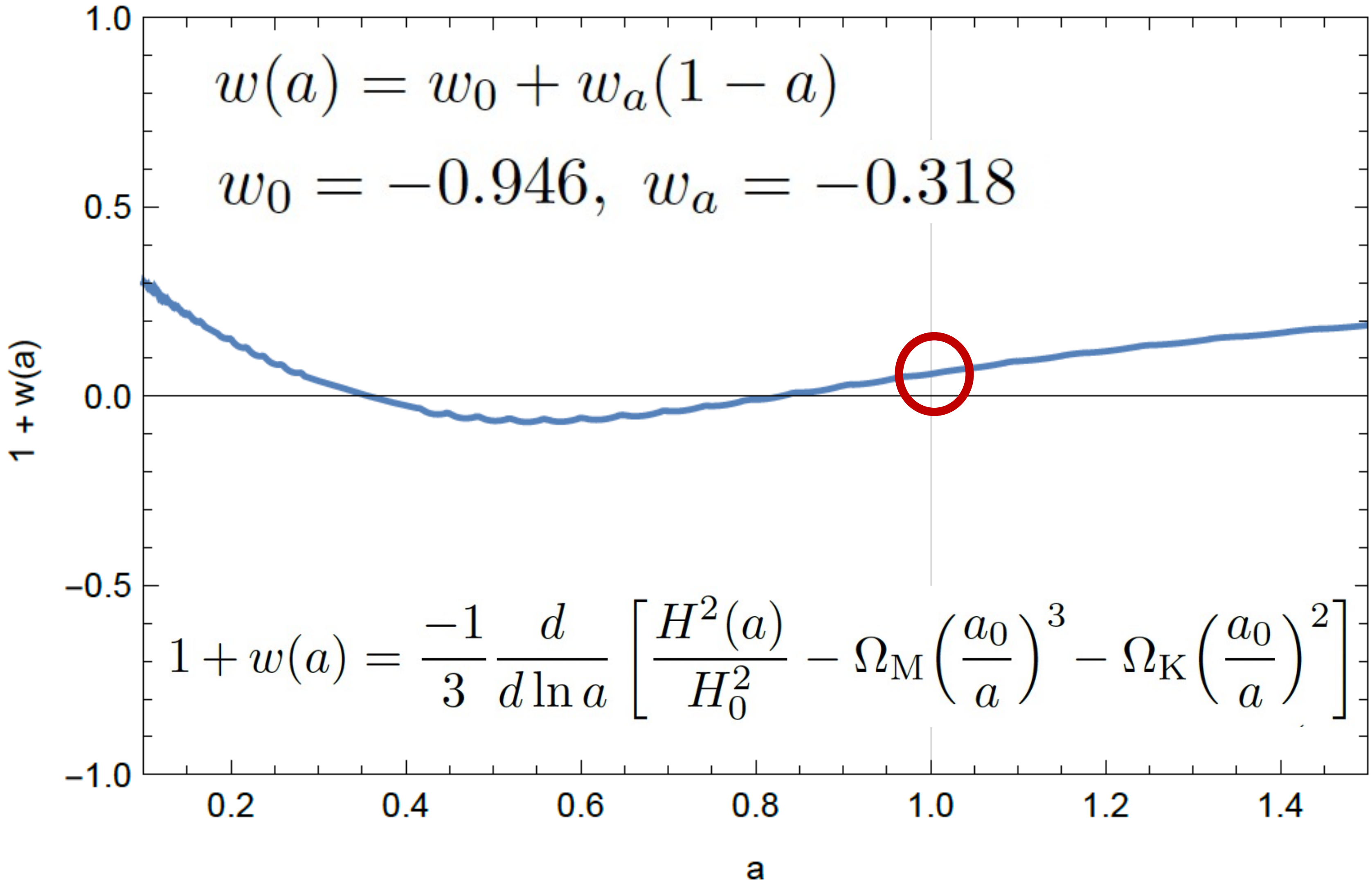
Cosmic Acceleration



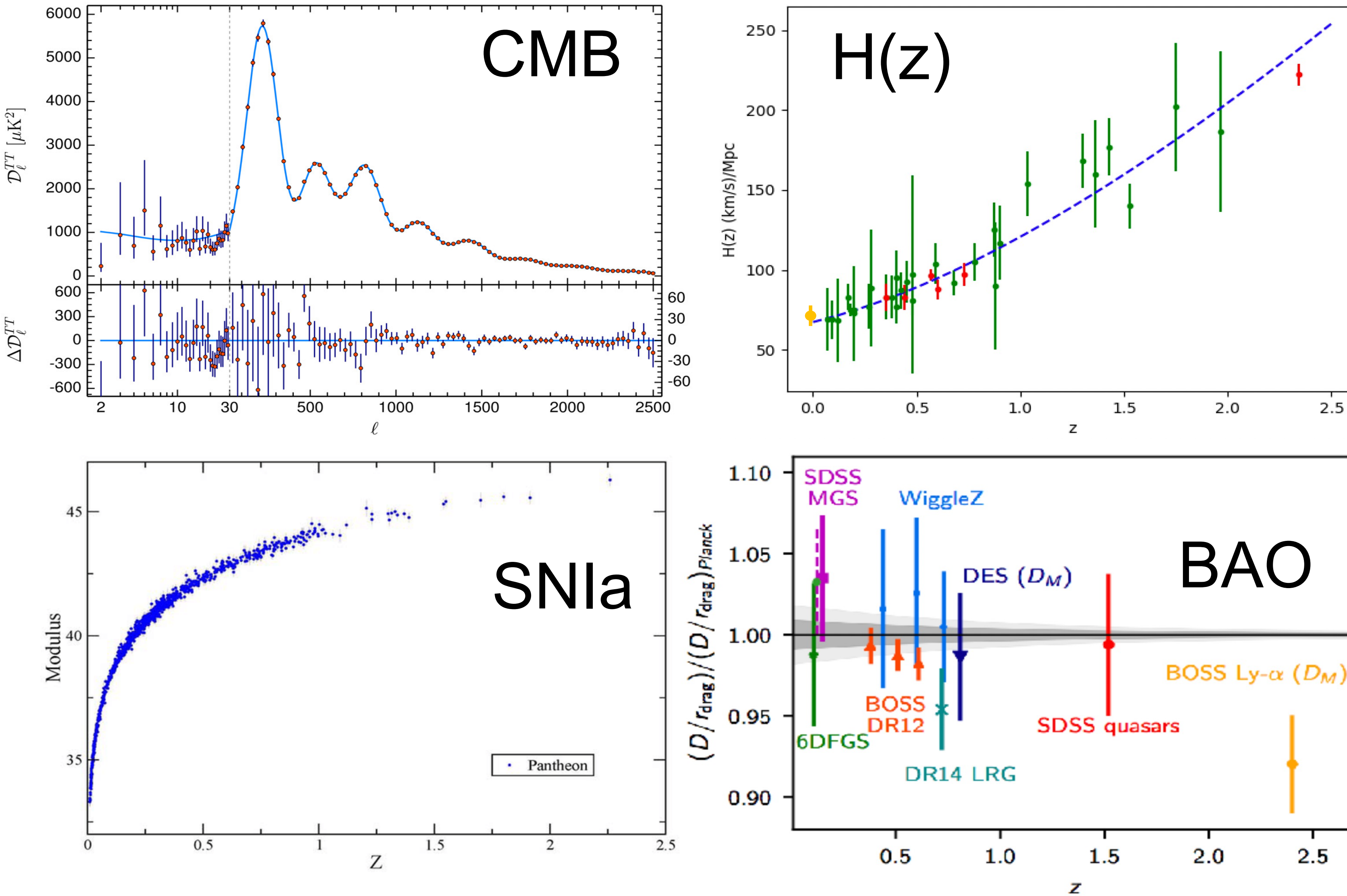
Cosmic Acceleration



Cosmic Acceleration

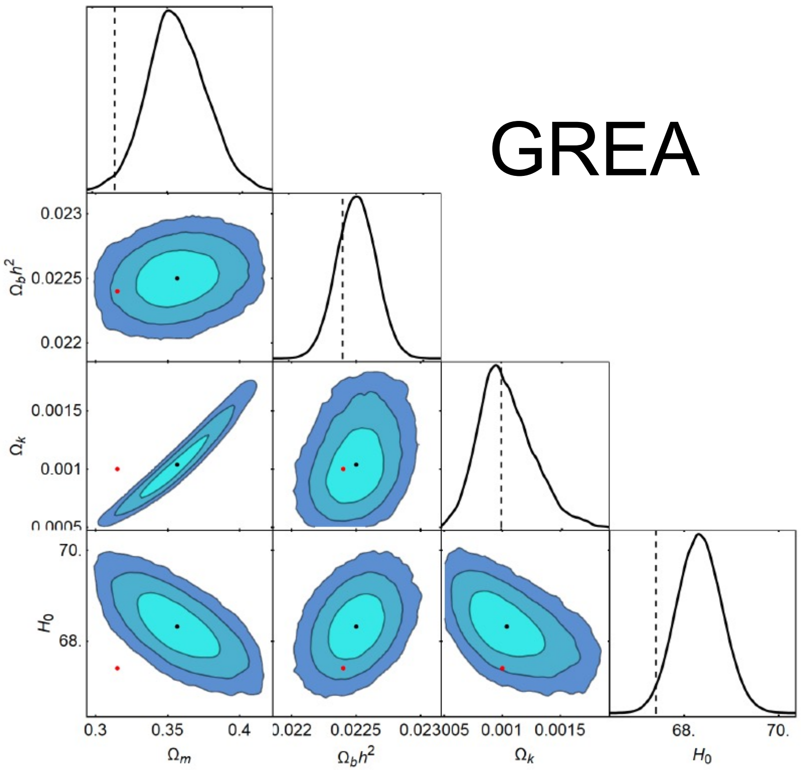


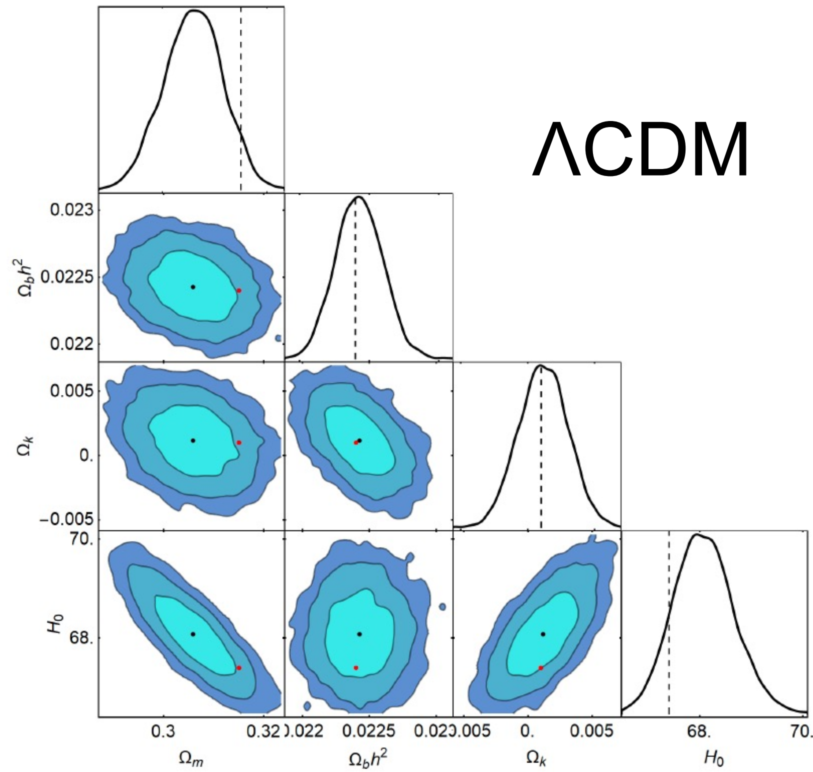
Cosmo Observations



Cosmic Constraints

Arjona, Espinosa, JGB & Nesseris (2021)





Model	$\Omega_{m,0}$	$\Omega_{b,0}h^2$	$\Omega_{k,0}$	H_0	χ^2_{min}	$\log Z(1)$
ΛCDM	0.3057 ± 0.0056	0.0224 ± 0.0002	0.0012 ± 0.0018	68.08 ± 0.58	1075.63	-557.515
GREAT	0.3522 ± 0.0190	0.0225 ± 0.0001	0.0010 ± 0.0002	68.38 ± 0.48	1071.35	-548.509

Cosmic Constraints

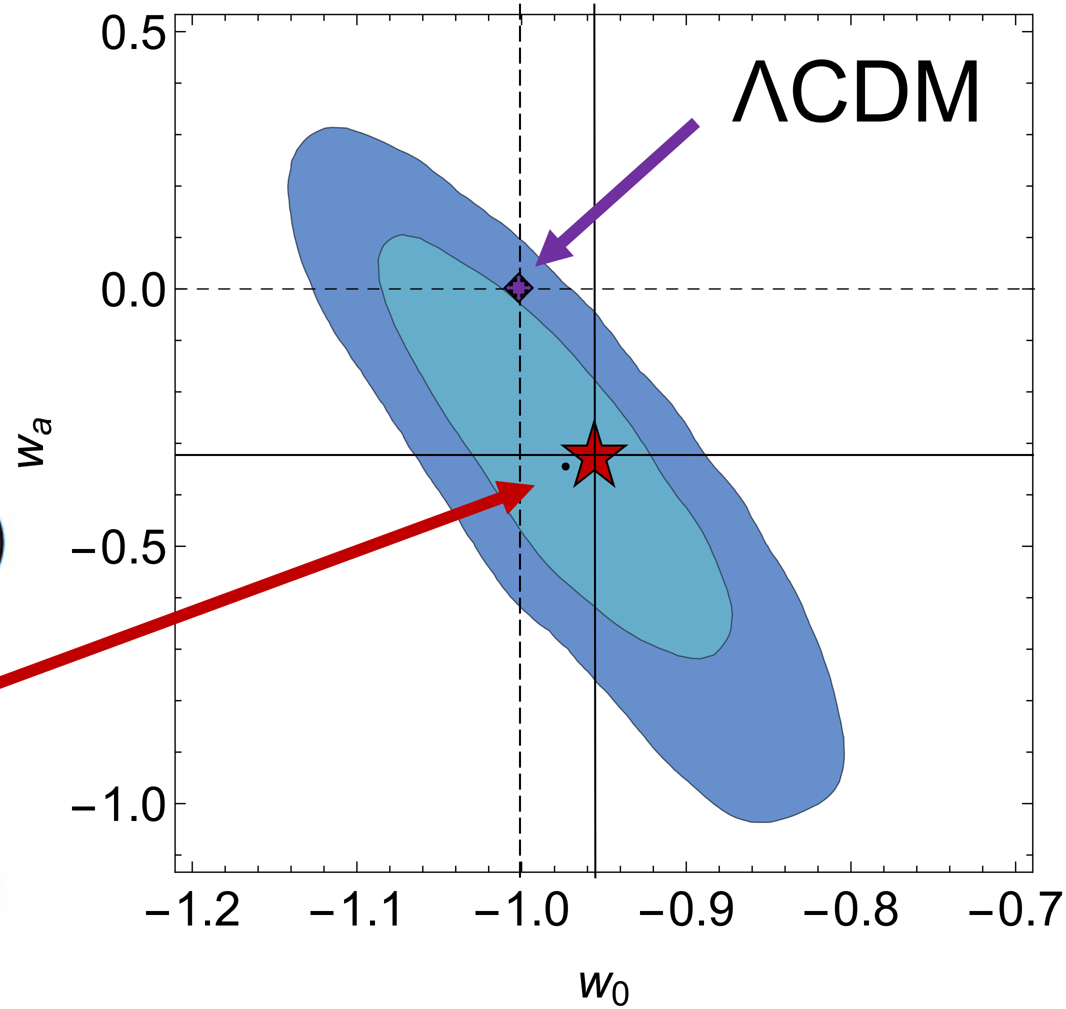
Arjona, Espinosa, JGB & Nesseris (2021)

Same data but with $(w_0,\, w_a)$ free:

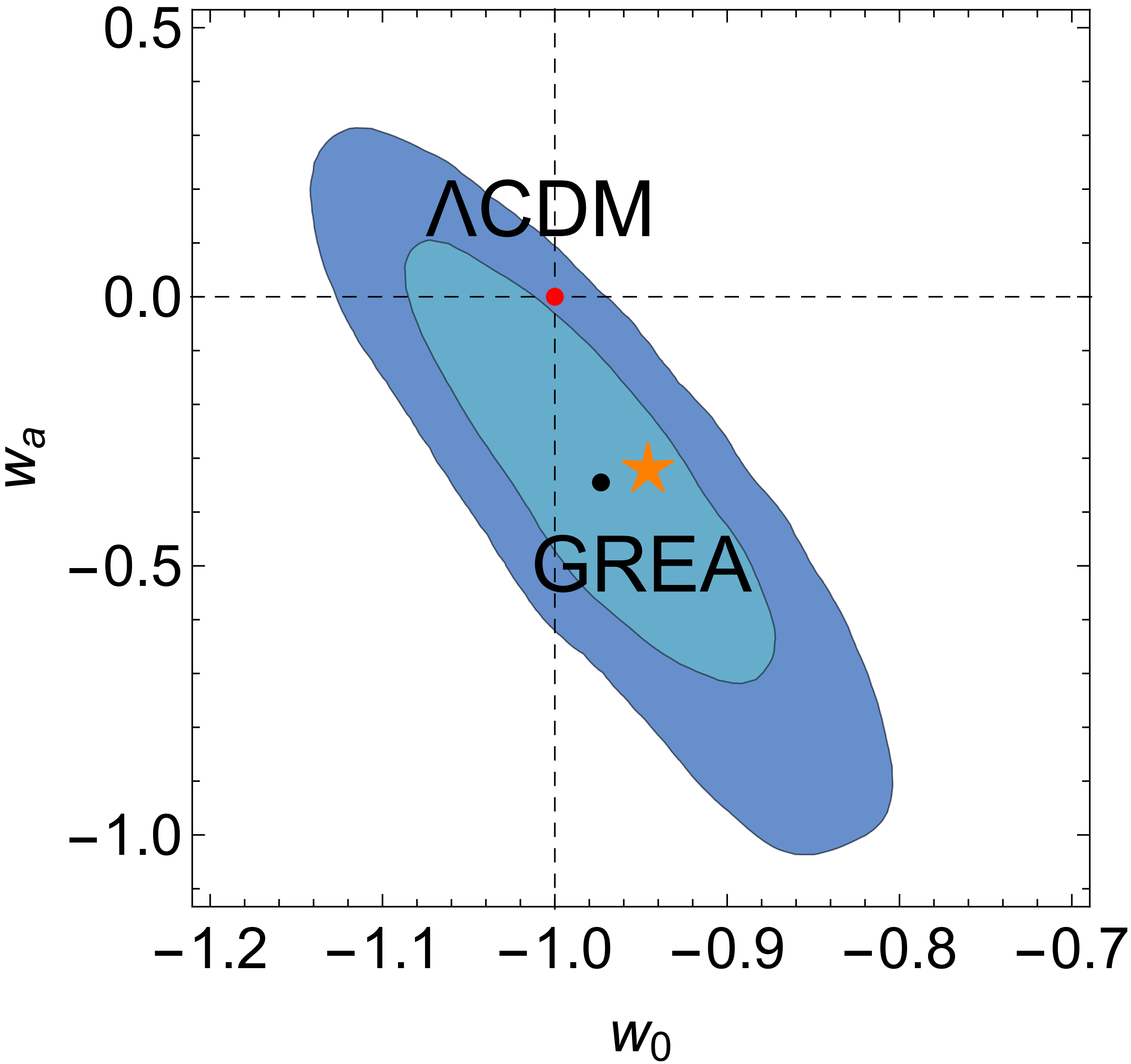
$$w(a) = w_0 + w_a(1-a)$$

GREA

$$w_0 = -0.946,\ w_a = -0.318$$

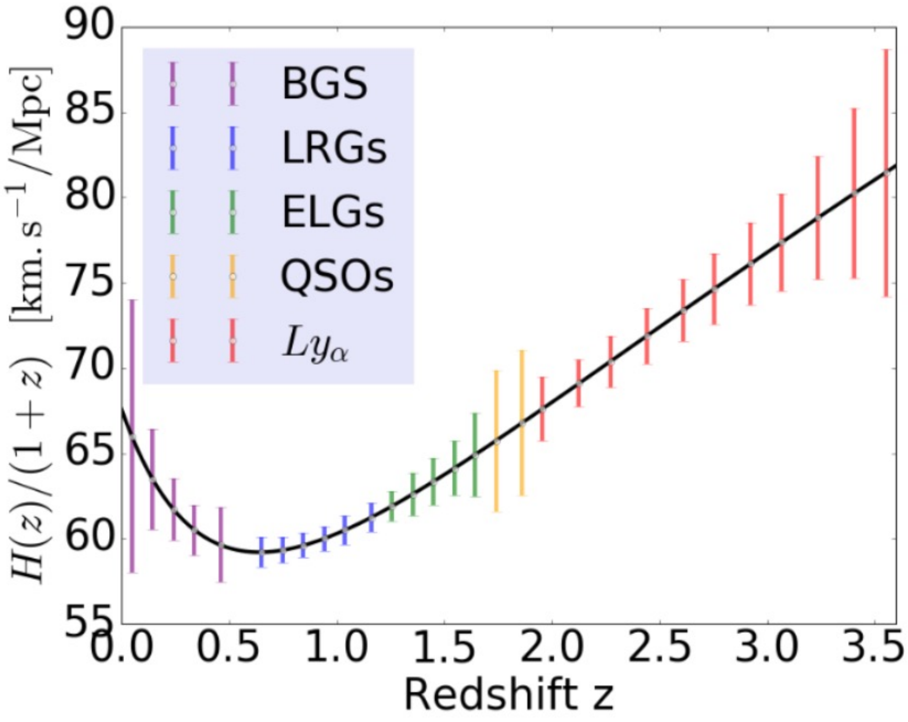


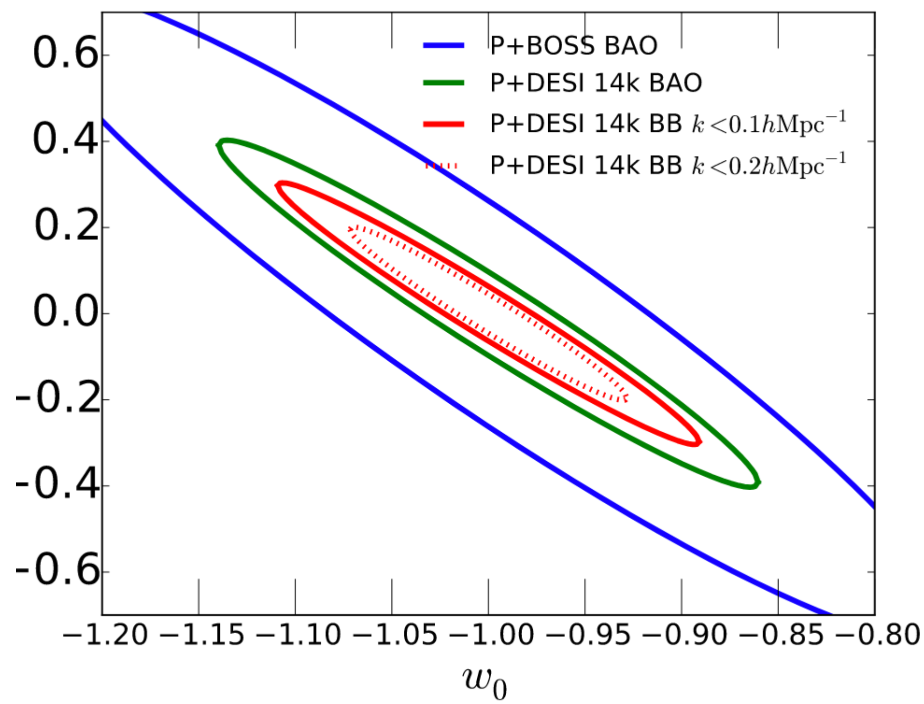
Present Constraints



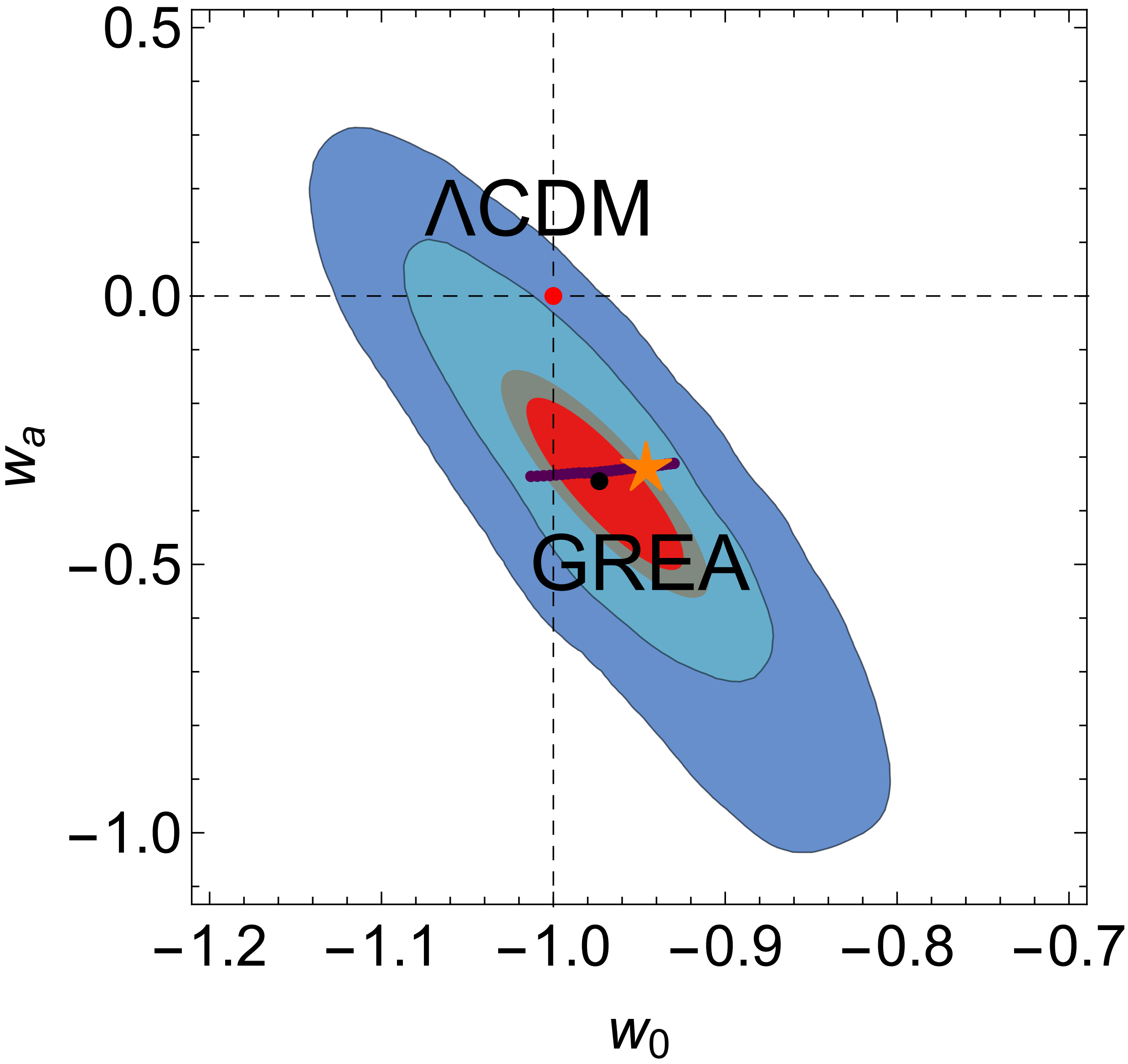
Future Constraints

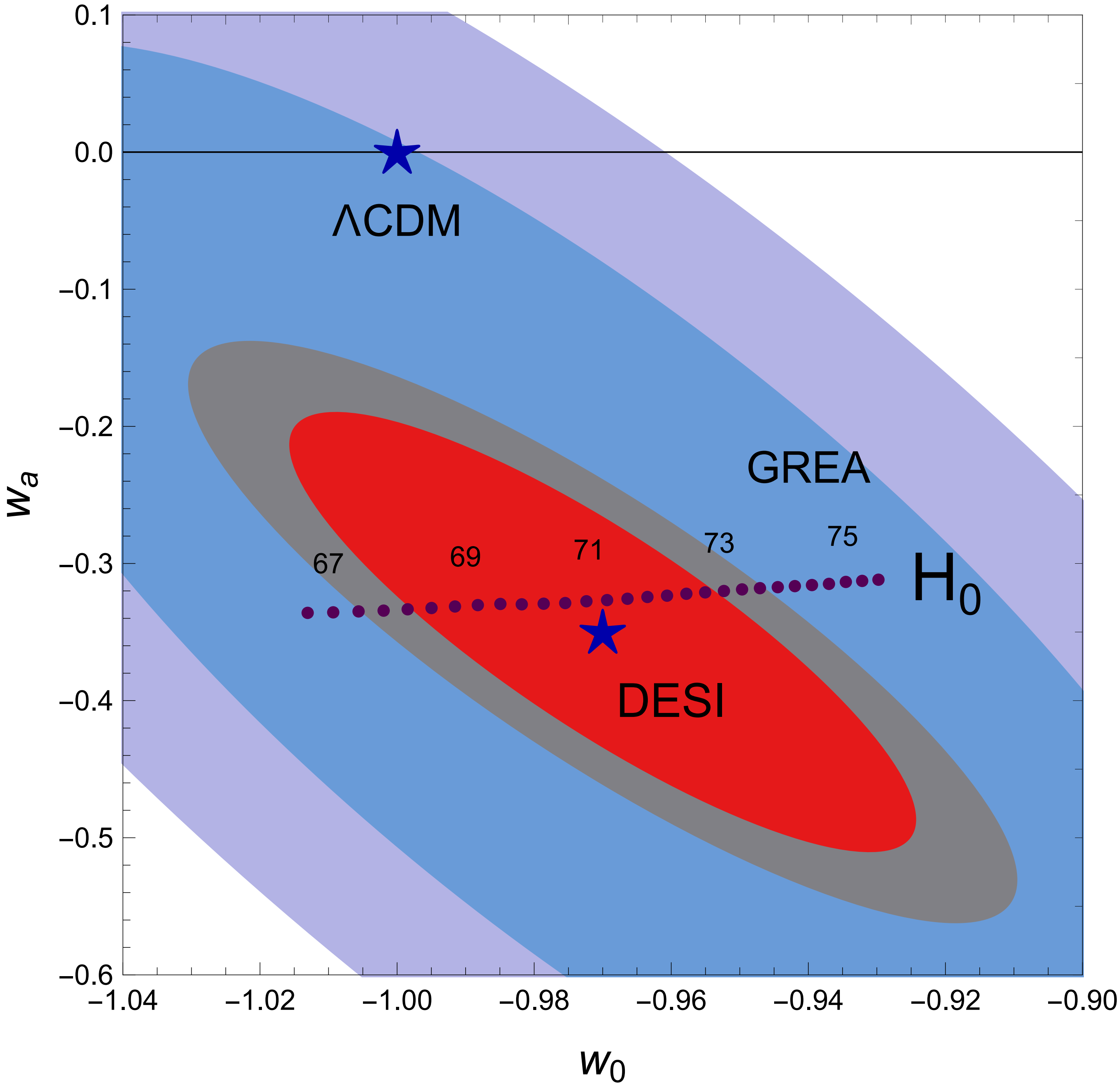
DESI Coll. (2016)





Future Constraints





0.1
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
w_a
−1.04
−1.02
−1.00
−0.98
−0.96
−0.94
−0.92
−0.90
w_0
ΛCDM
GREA
DESI
67
69
71
73
75
H_0

Entropic forces in SMBH

Accretion onto black holes from the gas around them will change their mass and therefore their entropy, inducing an entropic force on space-time around them, according to Raychaudhuri equations.

At the Eddington limit, the mass of SMBH grows like

$$\dot{M} = \frac{4\pi G\, m_p}{0.1 c\, \sigma_T}\, M \simeq \frac{M}{40\,\mathrm{Myr}} = \frac{2}{t(z_*)}\, M \qquad (z_* \simeq 35)$$

<u>**Assumption:**</u> SMBH continue to accrete mass at Eddington limit with a rate that decreases with the available gas over cosmological timescales,
at least since 80 Myr

$$M \propto t^2 \propto a^3 = V$$

Entropic forces in SMBH

Growth of BH entropy associated with this mass growth

$$S \propto M^2 \propto V^2 \quad \Rightarrow \quad \frac{dS}{S} = 2\frac{dV}{V}$$

Contributes with a **constant & negative entropic pressure**

$$p_S = -T\frac{dS}{dV} = -2\frac{TS}{V} = -\frac{N_{\rm SMBH} M_{\rm SMBH}}{V} = -\rho_{\rm SMBH}$$

where the total entropy is $S = \sum_i S_{\rm SMBH}^{(i)} = N_{\rm SMBH} S_{\rm SMBH}$

$N_{\rm SMBH}$ is the total number of SMBH in the Universe, assumed constant (i.e. without SMBH mergers)

Acceleration from SMBH

The Raychaudhuri equation in this case becomes

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p + \rho_{\rm SMBH} + 3p_S)$$
$$= -\frac{4\pi G}{3}(\rho + 3p) + \frac{8\pi G}{3}\rho_{\rm SMBH}\,.$$

The entropic force term can be interpreted as an **effective cosmological constant** term $\Lambda = 8\pi G\,\rho_{\rm SMBH}$

Consequence: Primordial seeds of SMBH, rather than contributing as DM, they behave as DE, due to their rapid growth, until accretion stops.

DE from SMBH

Only a small fraction of DM in the form of PBH constitute the seeds of SMBH at the centers of galaxies, and their rapid growth induces GREA that we interpret as DE.

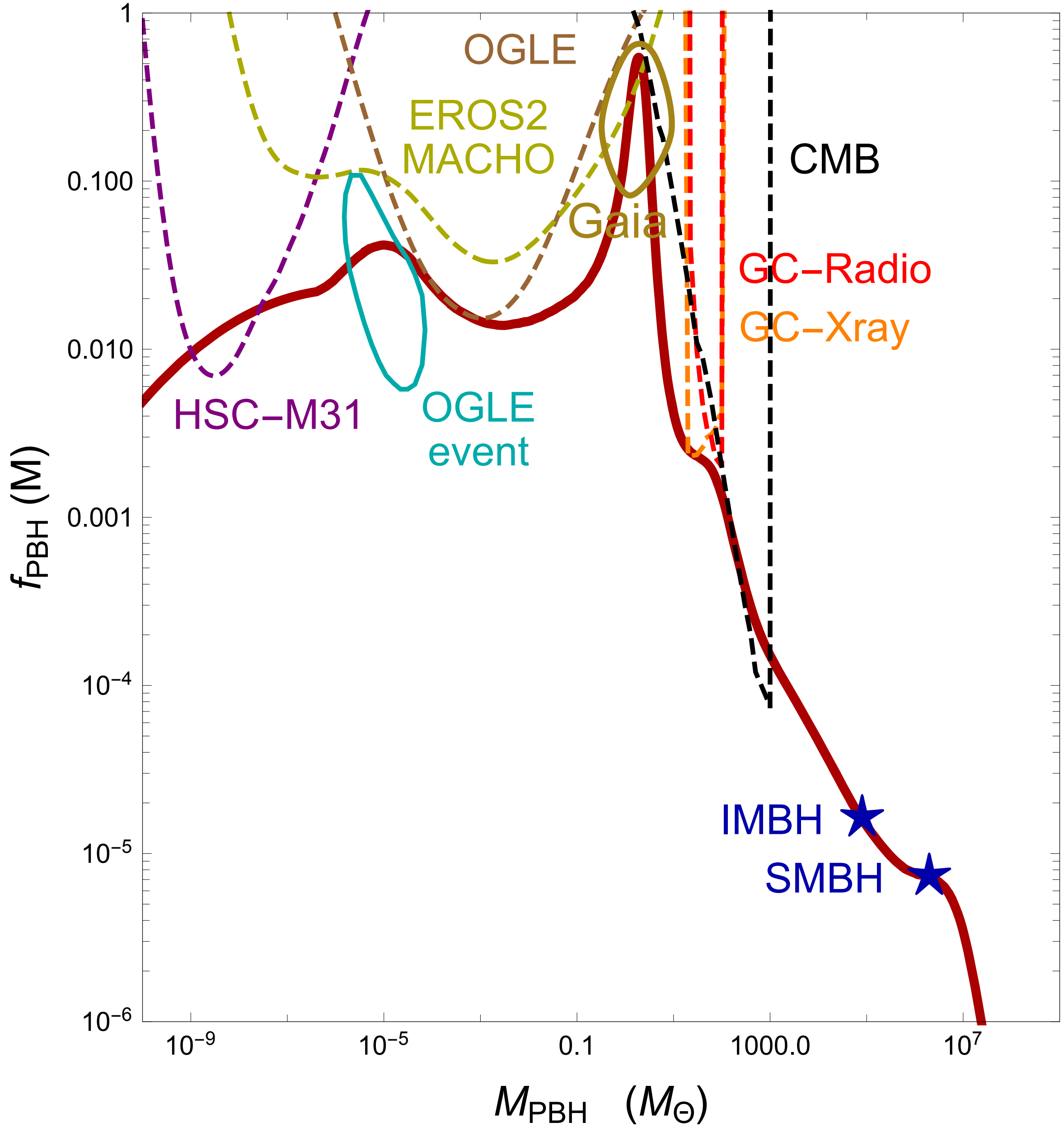
$$\Omega_{\rm DE} = f_{\rm SMBH}\,\Omega_{\rm DM}\,(1+z_*)^3 = 0.69\,,$$

$$f_{\rm SMBH} = 5\times 10^{-5} \qquad \Omega_{\rm DM} = 0.26.$$

A more sophisticated computation is needed for the case of a broad mass distribution f(M) of PBH, and possibly different rates of accretion, M(z)

$$\Omega_{\rm DE} = \Omega_{\rm DM}\int \frac{f(M)}{M}\,\frac{dM}{dz}dz$$

PBH could be all the DM

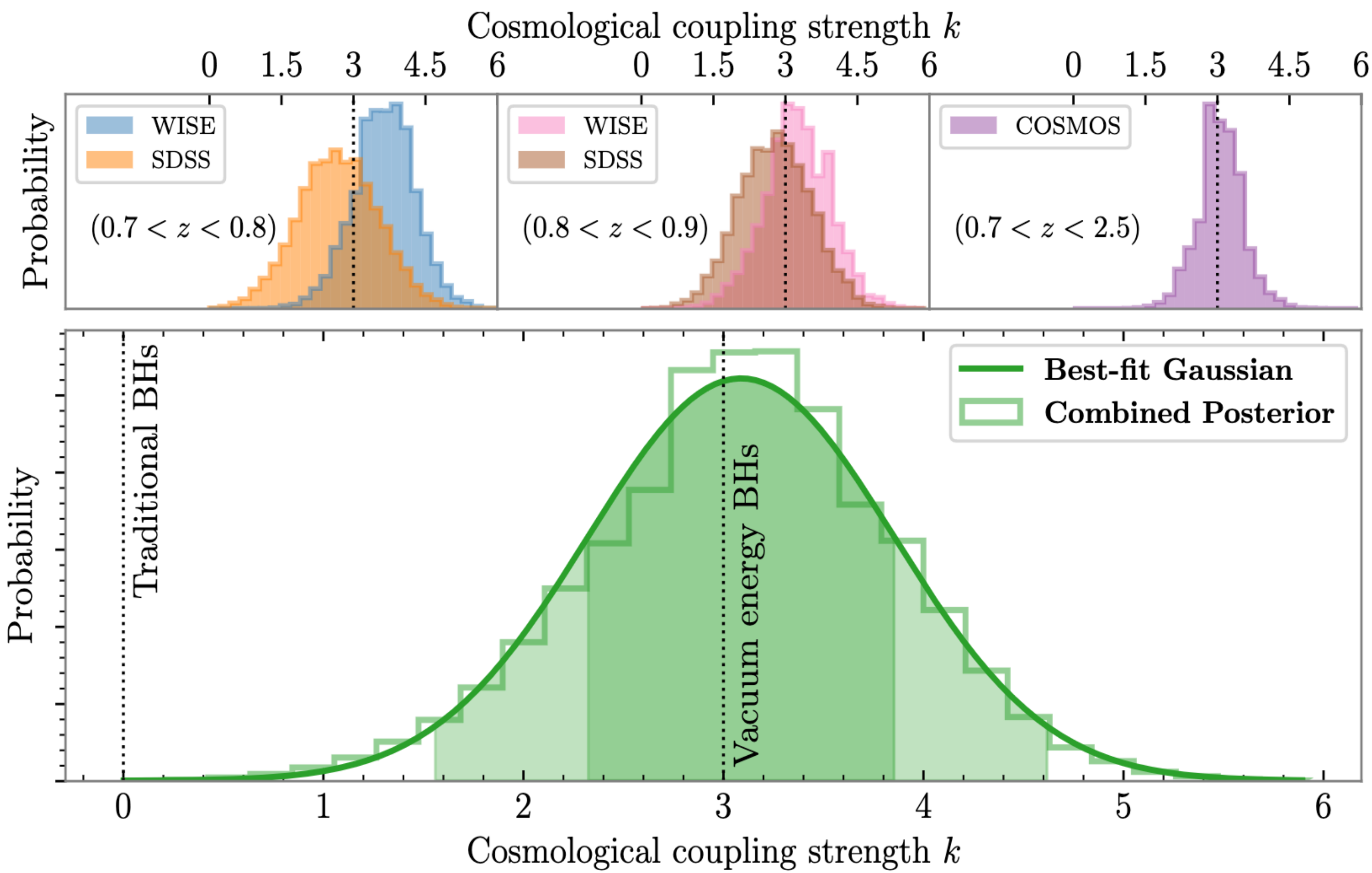


Cappelluti
Hasinger
Natarajan
(2022)

Based on
JGB (2021)

SMBH growth

Farrah+ (2023)



Conclusions

- Non-equilibrium phenomena in GR: entropic forces
- Extra term in Raychaudhuri eq. of grav. Collapse
- FLRW: Cosmic acceleration from first principles
- SMBH growth through Eddington accretion
- BH entropy production generates GREA
- No need for a Cosmological Constant
- Precise knowledge of $M(z)$ & $f(M)$ will give (w_0, w_a)
- Multiple consequences for Large Scale Structure
- Possible solution to the H_0 tension
- DESI able to distinguish GREA from ΛCDM

Backup slides

Real fluids

Relativistic dynamics of real fluids (viscosity & heat)

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} + \tau_{\mu\nu}$$

Is a particular case of the variational formalism if:

$\tau_{\mu\nu} = -f_{\mu\nu}$ with orthogonality cond. $u^\mu \tau_{\mu\nu} = 0$

The 1st law thermodynamics of real fluid takes the form

$$T D_\mu(\sigma u^\mu) = \tau^\nu_\mu D_\nu u^\mu$$

Where T is the **temperature** of the fluid
And σ is the **local entropy density** of the fluid

Real fluids

In the comoving orthogonal gauge, $u^\mu = n^\mu$:

$$D_\mu(\sigma u^\mu) = \pounds_n\sigma + \nabla_\mu(\sigma n^\mu) \quad \text{and} \quad D_\mu u_\nu = \frac{1}{2}\pounds_n h_{\mu\nu}$$

Then the 2nd law thermodynamics of a real fluid can be written in terms of the phenomenological constraint, by making the following identifications:

$$f_{\mu\nu} = -\tau_{\mu\nu} \quad s^{tot} = \sigma \quad j_s^i = -\sigma u^i$$

$$T = -\frac{1}{N\sqrt{h}}\frac{\partial \mathcal{L}}{\partial s}$$

Real fluids in FLRW

Further identification between the entropic force tensor and the viscosity tensor

$$\tau_{\mu\nu} = -\,\eta\left(D_\nu u_\mu + D_\mu u_\nu - u_\nu u^\alpha D_\alpha u_\mu - u_\mu u^\alpha D_\alpha u_\nu\right) - \left(\zeta - \frac{2}{3}\eta\right) D_\alpha u^\alpha \left(g_{\mu\nu} + u_\mu u_\nu\right) ,$$

Where η and ζ are, respectively, the shear and bulk viscosity coefficients of the real fluid

Real fluids in FLRW

Consider a homogeneous and isotropic fluid in FLRW, where covariant derivatives are given by

$$D_\mu u^\nu = \frac{\dot{a}}{a}\delta^\nu_\mu$$

and therefore the viscosity tensor $\tau_{\mu\nu} = -3\zeta\frac{\dot{a}}{a}h_{\mu\nu}$

Comparing with the trace of the entropic force:

$$\frac{T\dot{S}}{a^3 H} = \tilde{f} = \tilde{f}^{ij}h_{ij} = -\tau_{ij}h^{ij}$$

We find the bulk viscosity coefficient: $\zeta = \frac{T\dot{S}}{9H^2a^3} > 0$

Let us elaborate a bit on the results of this section. First of all, the conventional formulation of general relativistic real fluids can be recovered by means of the variational formulation of non-equilibrium thermodynamics in General Relativity. In fact, one does not even need to impose additional terms on the energy momentum tensor. Instead, they are effectively generated by simply assuming the pressure and the energy density of the fluid to have a dependency on the entropy.

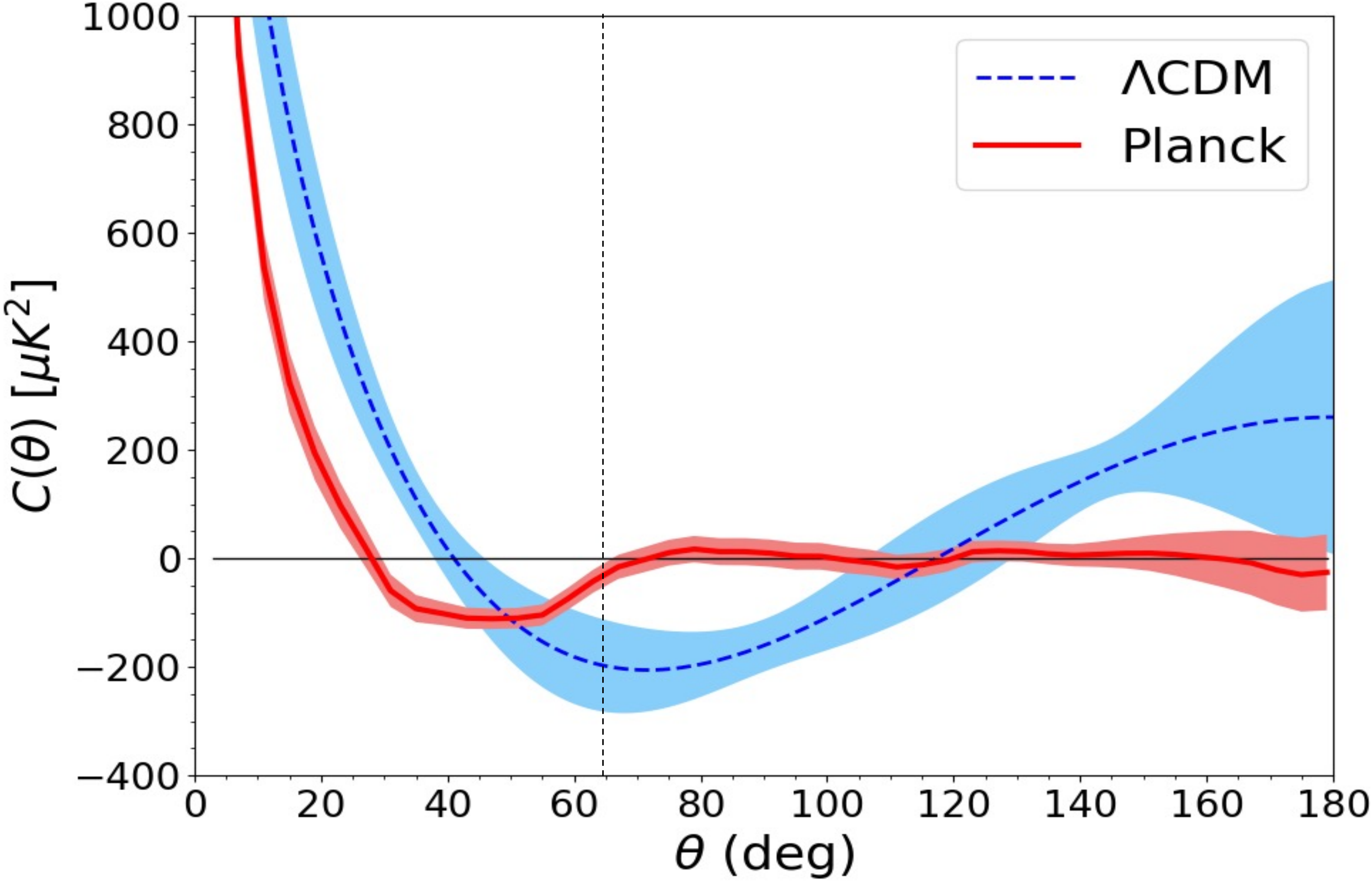
The variational description allows the inclusion of dissipative effects to any matter or gravity content, as long as it has time-dependent entropy. This means that we can interpret non-equilibrium phenomena in General Relativity as an effective viscosity term of a real (i.e. non ideal) fluid. In this sense, our results allow for a variational, first principles formulation of real fluids and the generalization of their dissipative effects to arbitrary matter and gravity contents.

We point out that the variational and phenomenological constraints are imposed before obtaining the equations of motion and must be satisfied at all times. This is a fundamental difference with the theory of real fluids.

Here we considered a vanishing chemical potential, which means that we did not impose particle number conservation. This excludes thermal conduction effects. Nevertheless, one could in principle impose also particle number conservation at the Lagrangian.

In the homogeneous and isotropic limit there is only bulk viscosity, parametrized by ζ. However, shear viscosity, parametrized by η, may play a role in characterizing entropic forces in gravitational collapse and structure formation.

Gaztanaga & Fosalba (2021)



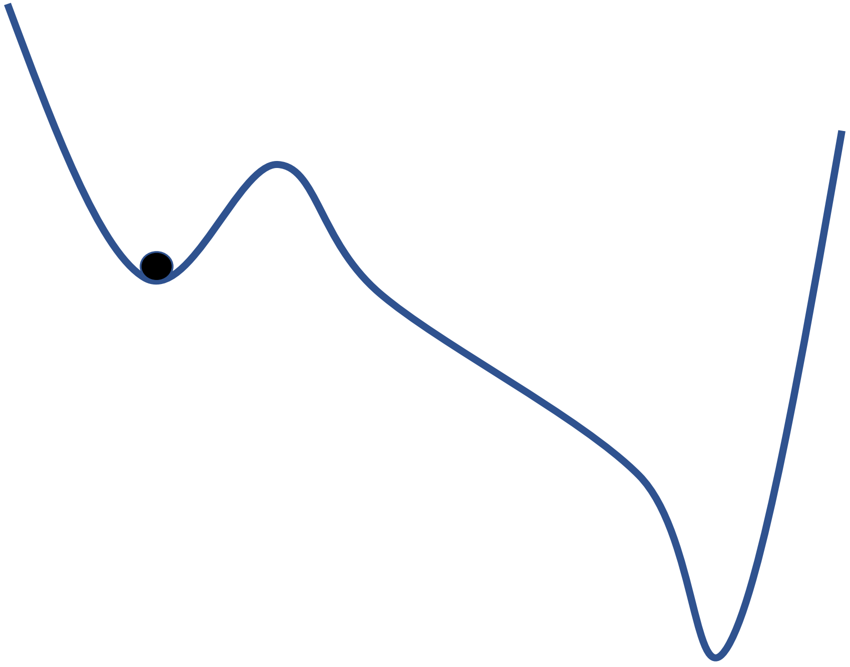
Angular 2-point correlation function of the Planck temperature map (red solid line). For reference we also show the theory prediction for the Planck best-fit ΛCDM cosmology (blue dashed line). Shaded areas display the 68 % Gaussian confidence intervals (see text for details).

Cosmic Acceleration

Penrose diagram

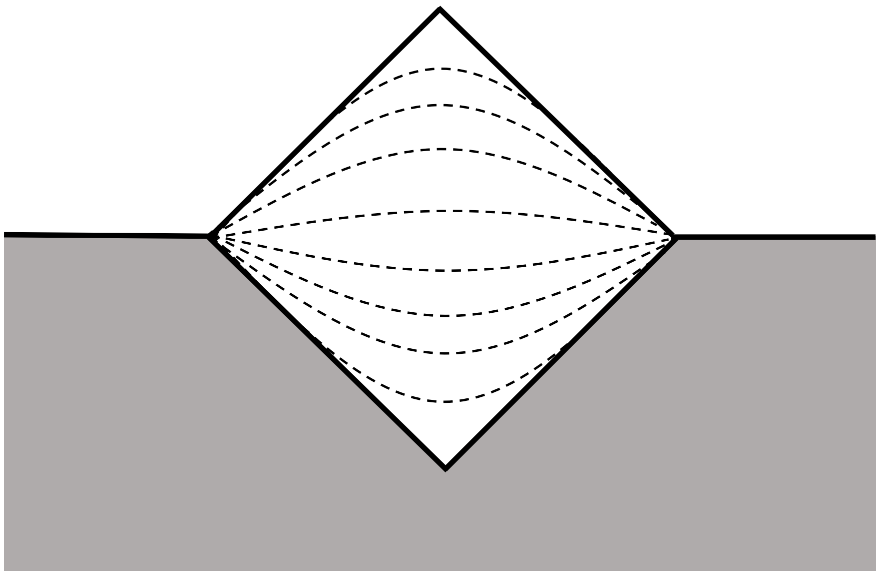
de Sitter

Potential



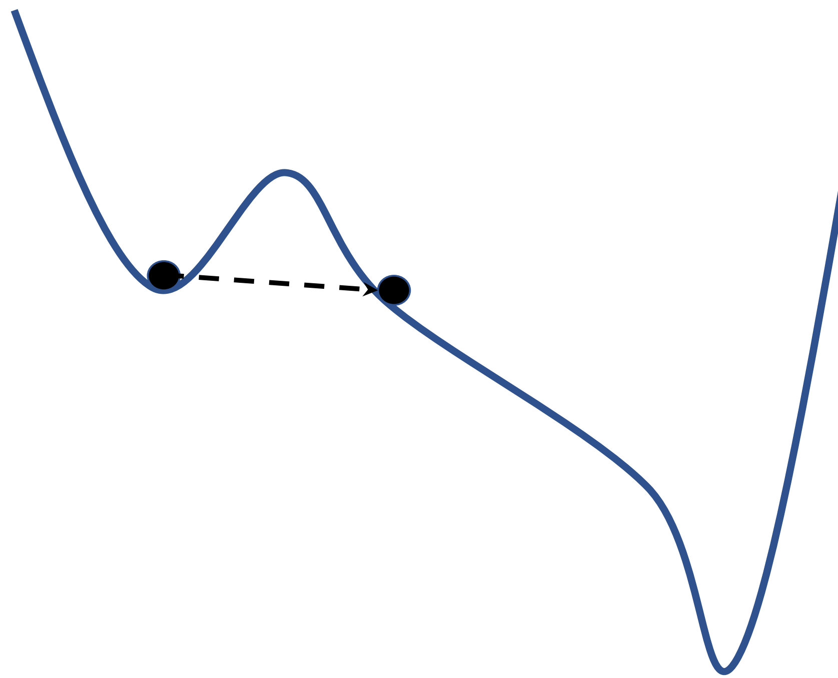
Cosmic Acceleration

Penrose diagram



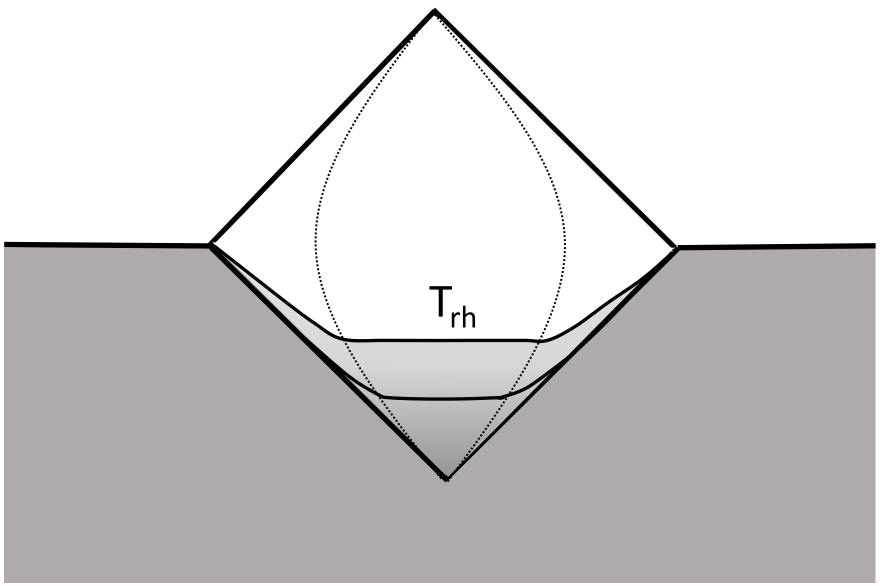
Open **empty** universe

Potential



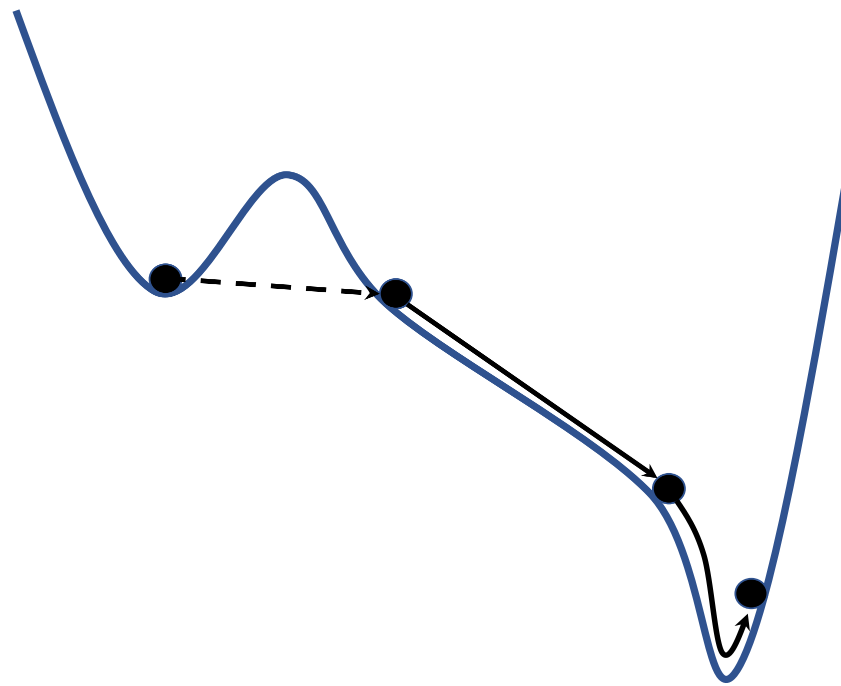
Cosmic Acceleration

Penrose diagram



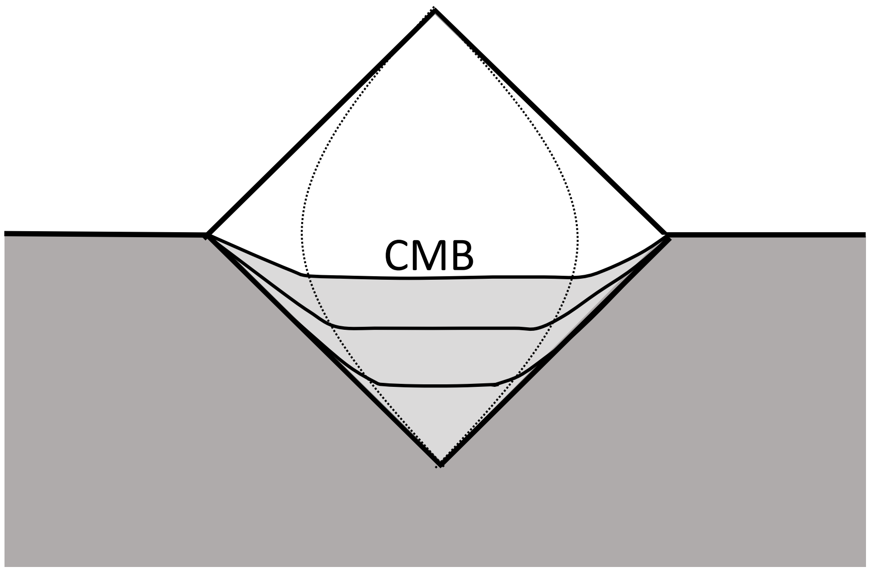
Flat **reheated** universe

Potential



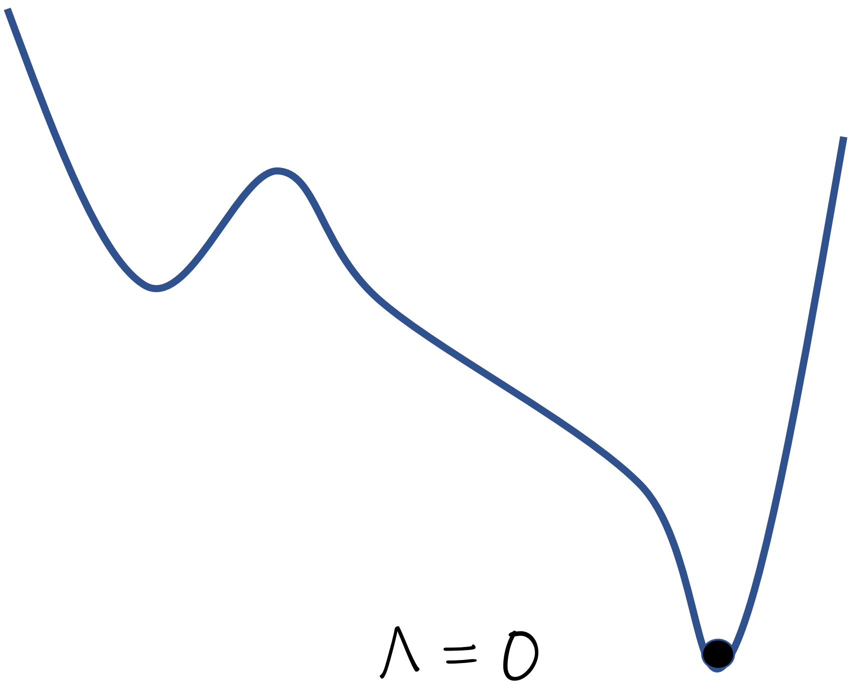
Cosmic Acceleration

Penrose diagram



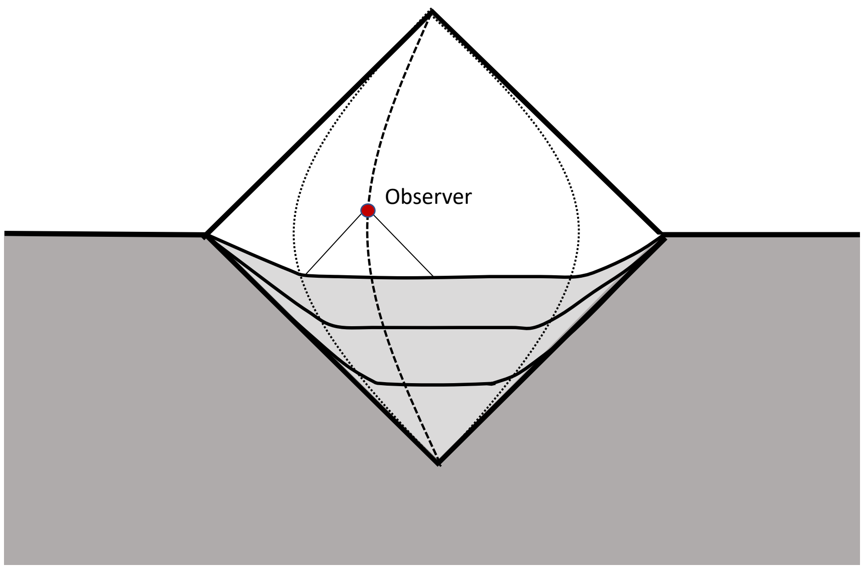
Flat **late** universe

Potential



Cosmic Acceleration

Penrose diagram



Flat **late** universe

Potential