Gluon TMD fragmentation function into quarkonium

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L International Meeting on Fundamental Physics and XV CPAN days

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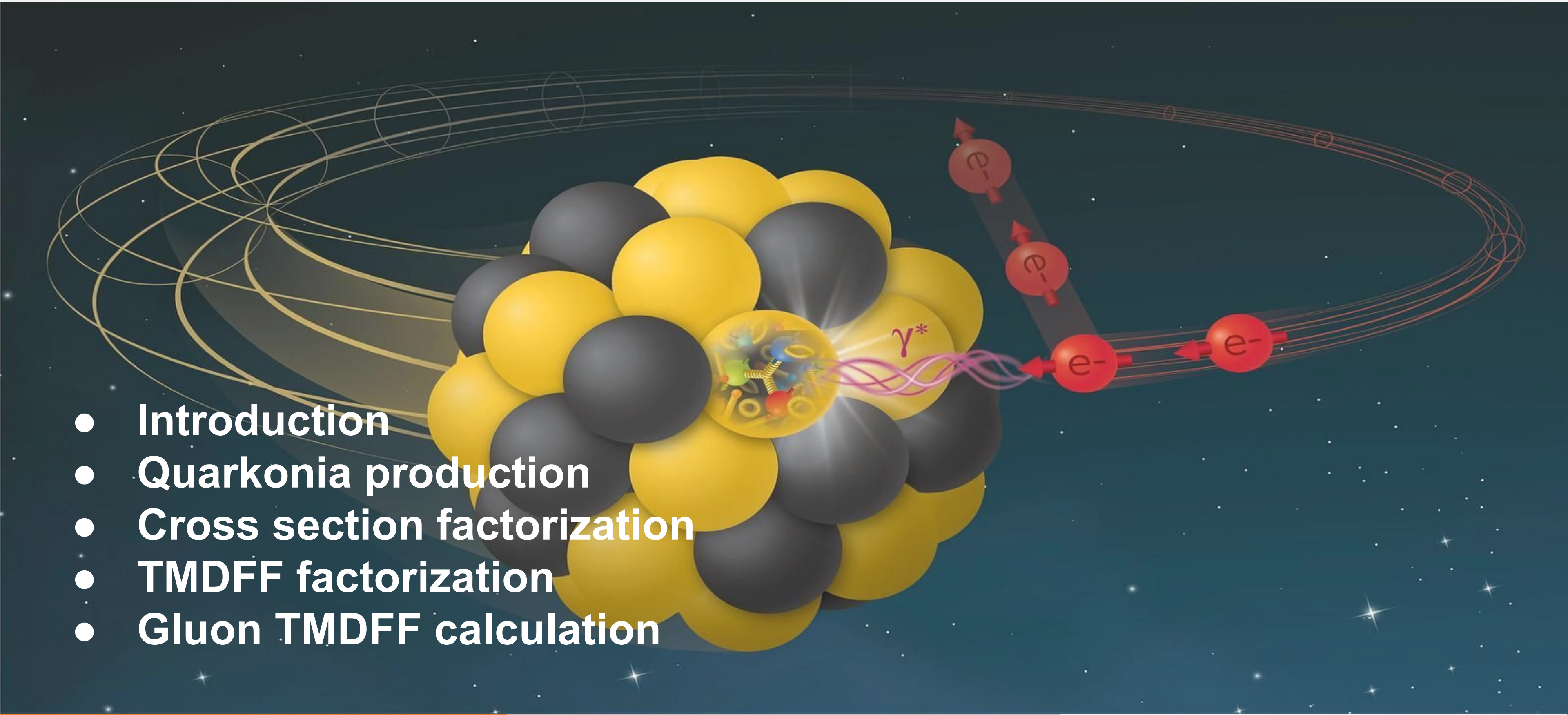
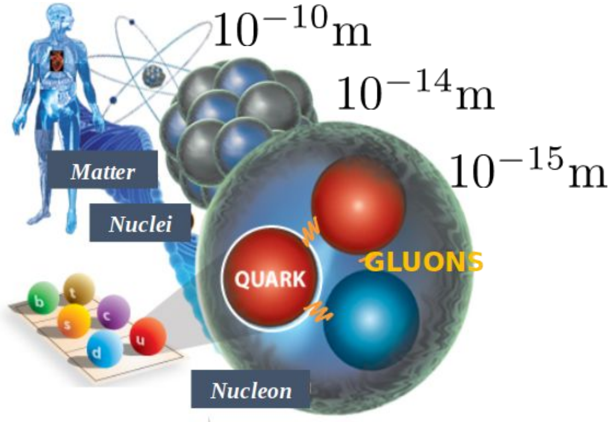
UPV EHU

Outline



- **Introduction**
- **Quarkonia production**
- **Cross section factorization**
- **TMDFF factorization**
- **Gluon TMDFF calculation**

Current status

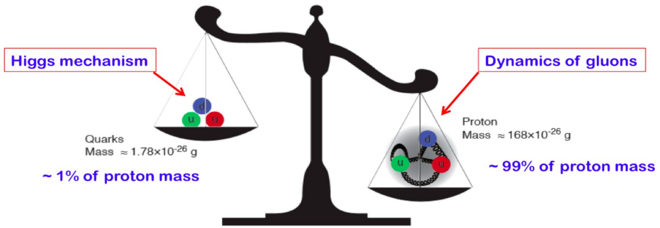


What do we know about the interior of the proton?

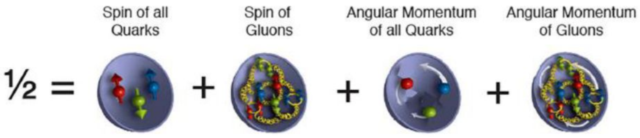
2 up quarks + 1 down quark
Sea of quarks and antiquarks
Gluons

Fundamental questions:

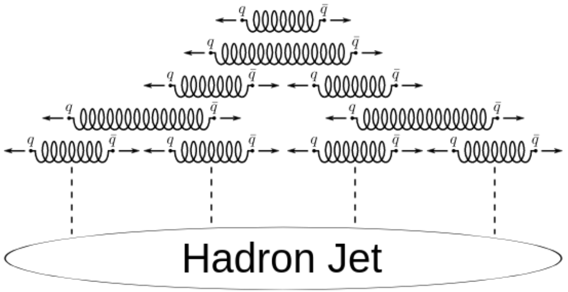
Proton mass?



Proton spin?



Color confinement?



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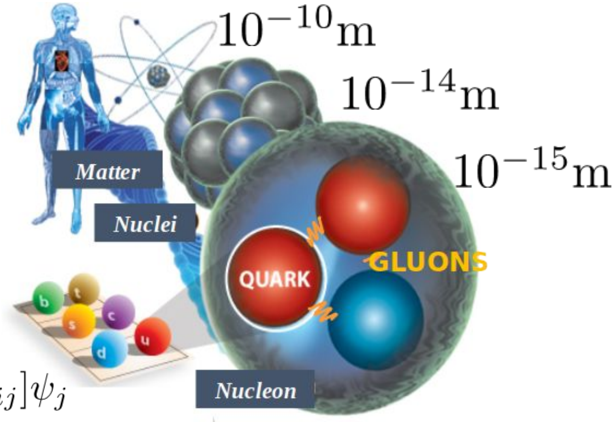

What do we know about the interior of the proton?

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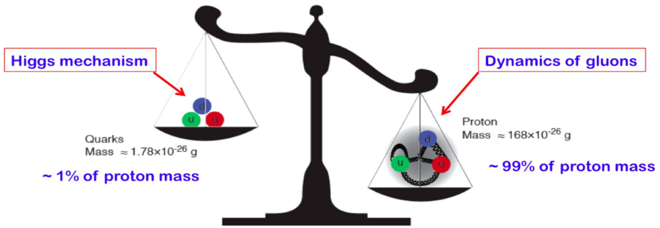
QCD AT WORK!

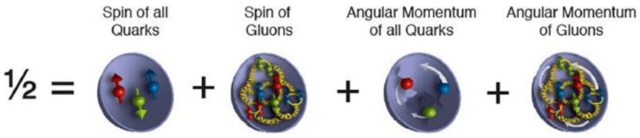
$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \bar{\psi}_i[i(\gamma^\mu D_\mu)_{ij} - m\delta_{ij}]\psi_j$$

Fundamental questions:

Proton mass?



Proton spin?



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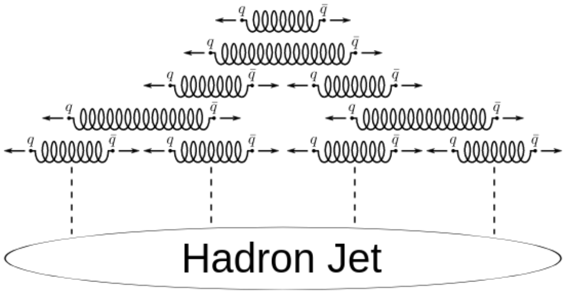
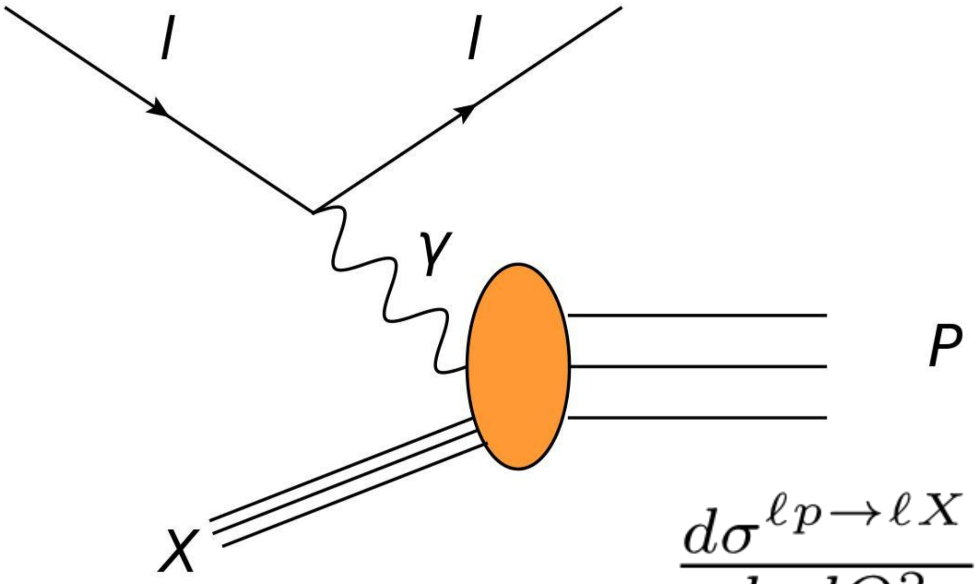


Probing the structure of the proton

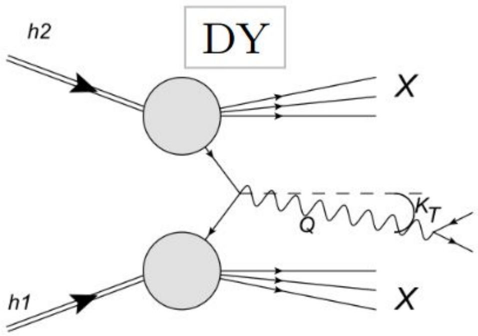


Deep Inelastic Scattering (DIS)

- Scattering through a virtual photon
- Interacts with the quarks in the nucleon
- Parton Distributions (PDFs)

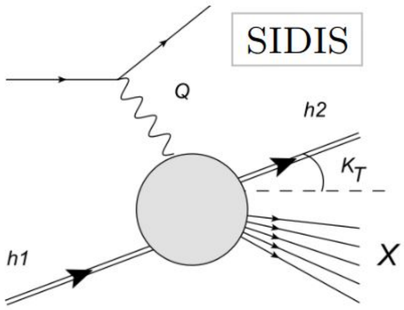
$$\frac{d\sigma^{\ell p \to \ell X}}{dx dQ^2} = \sum_q q(x) \frac{d\hat{\sigma}^{\ell q \to \ell q}}{dQ^2}$$

Probing the structure of the proton



$$h_1(P_1) + h_2(P_2) \to \ell(l) + \ell'(l') + X$$

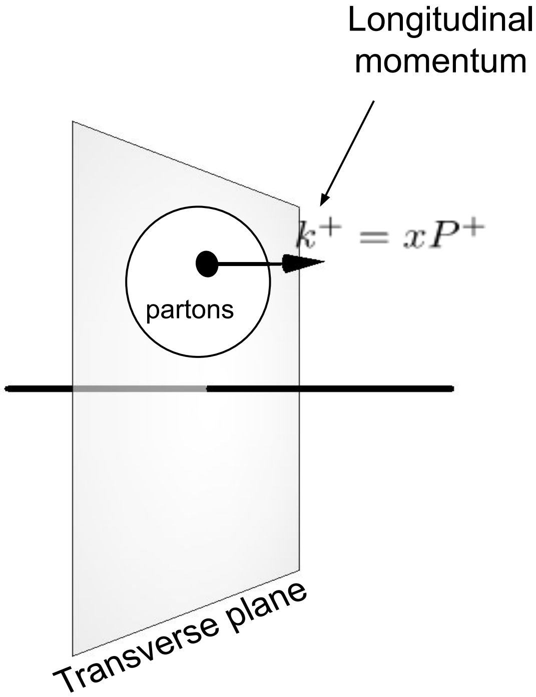
- PDF x PDF
- LHC, Tevatron, Fermilab, RHIC
- Best studied



$$\ell(l) + h_1(P_1) \to \ell(l') + h_2(P_2) + X$$

- PDF x FF
- JLab, COMPASS, HERMES
- Some studies

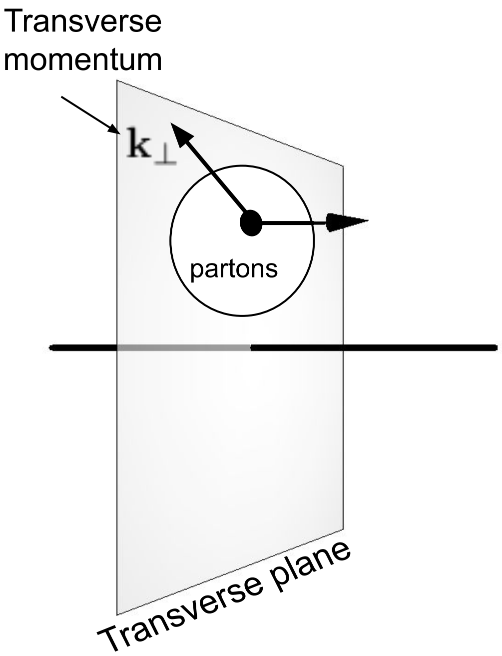
Multi-dimensional mapping of the nucleon



Parton Distribution Functions (PDFs)

- Motion of quarks in the transverse plane is ignored
- They are extracted by fitting data
- Scale evolution of PDFs can be calculated
- Independent of process

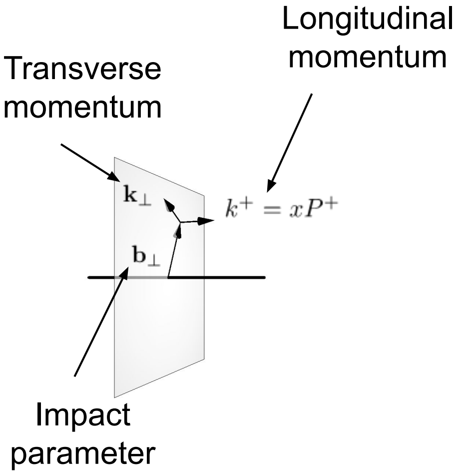
Multi-dimensional mapping of the nucleon

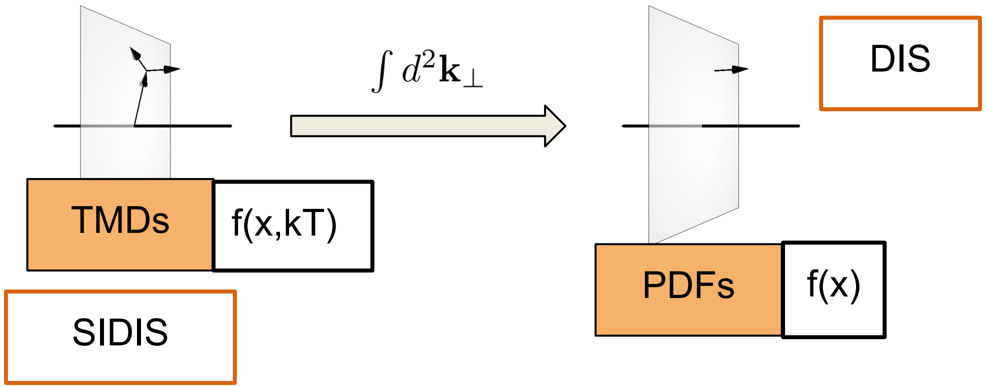


Transverse Momentum dependent parton distributions (TMDs)

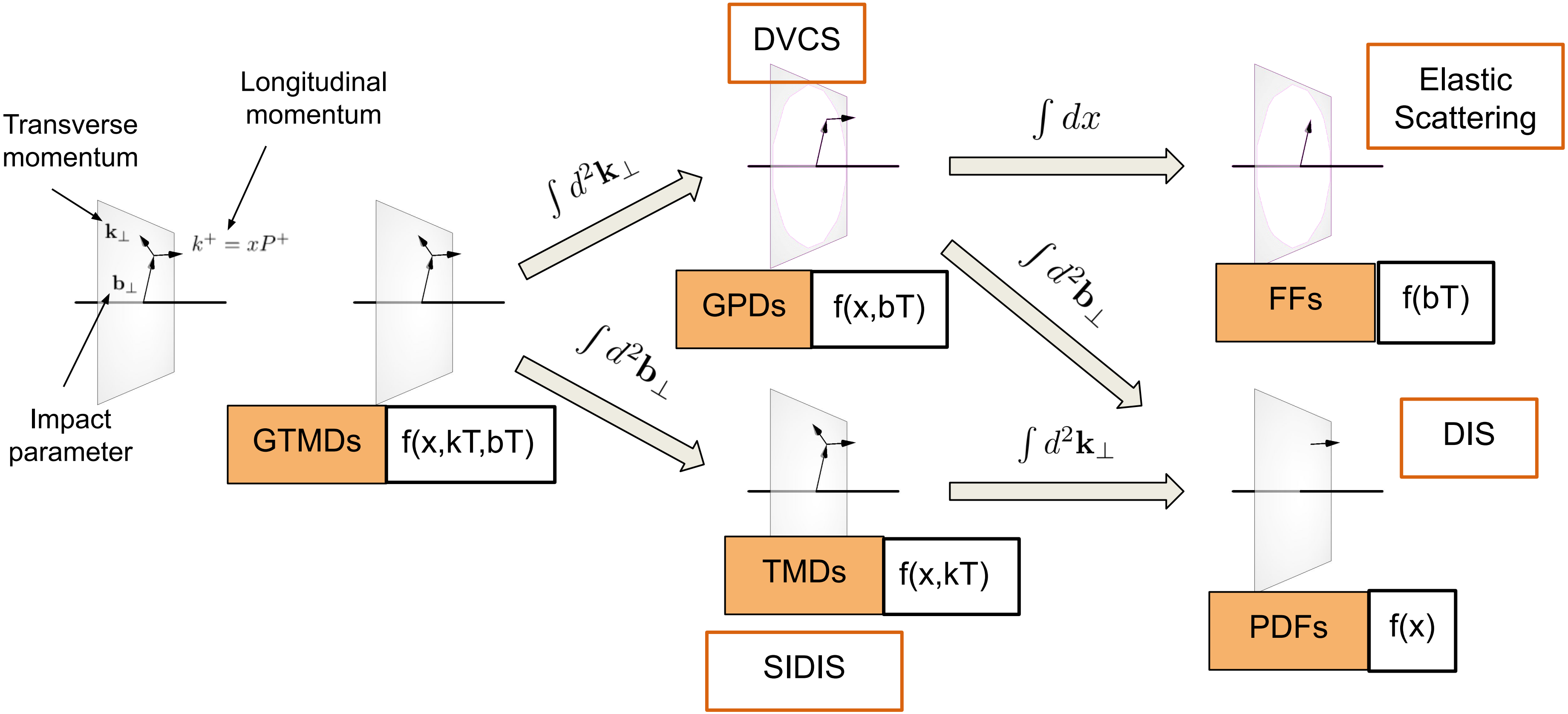
- Single spin asymmetries
- Motion of quarks in the transverse plane is considered
- They are extracted by fitting data
- Gives a 3D picture of the nucleon in momentum space

Multi-dimensional mapping of the nucleon



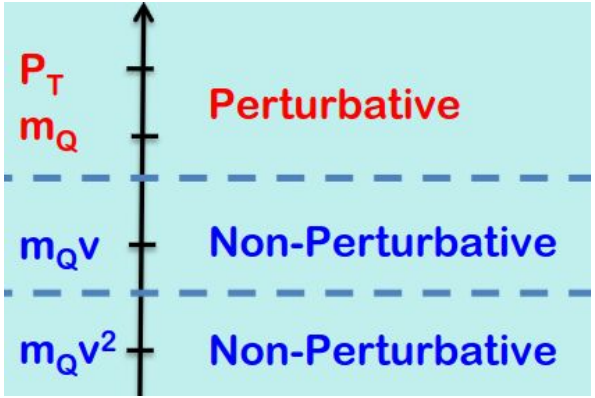


Multi-dimensional mapping of the nucleon



Why quarkonia?

- Many heavy quarkonia, such as J/psi, psi(2S) and upsilon(nS), have clean decay channels that enable precision measurements in colliders.
- Production of heavy quark pair could be perturbative
- One of the simplest QCD bound states
- Well-separated momentum scale → effective theory

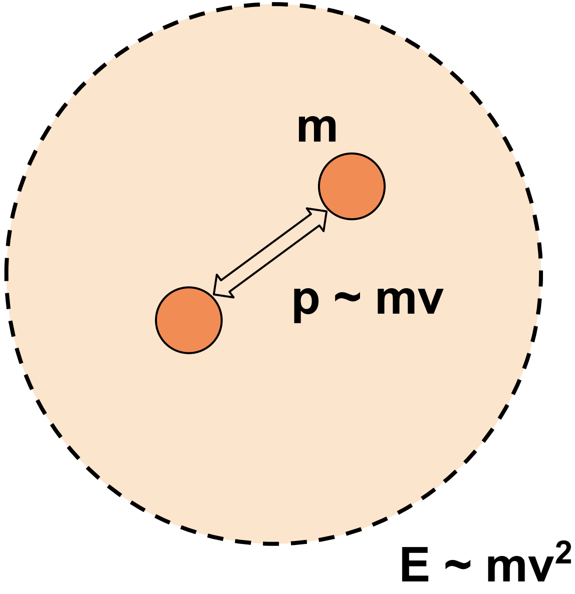


HARD

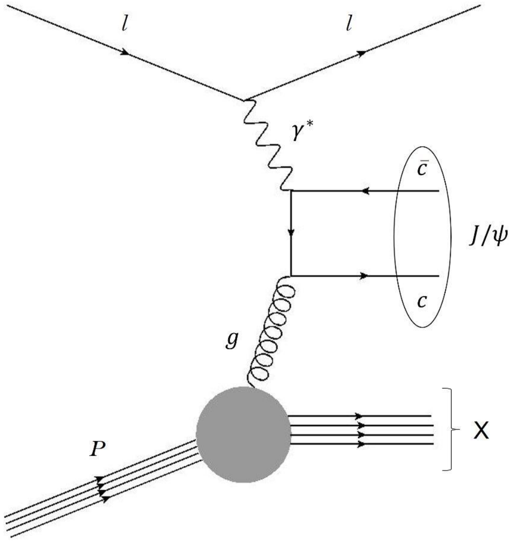
SOFT

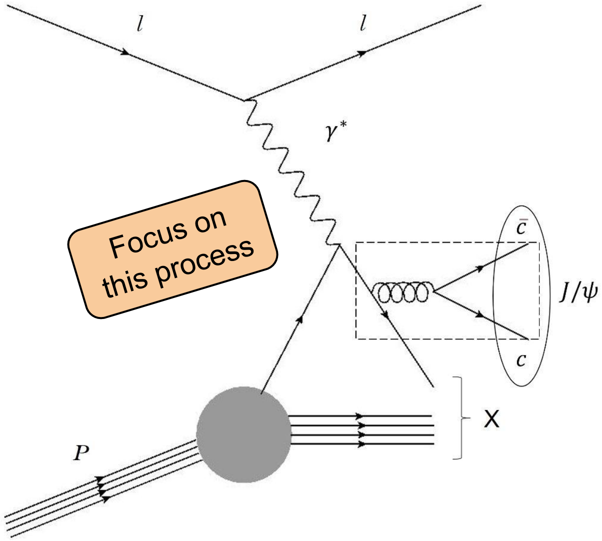
USOFT

$m \gg p \gg E$

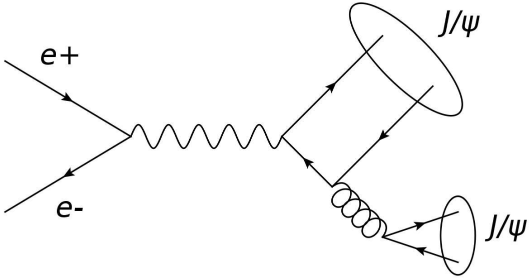


Goal



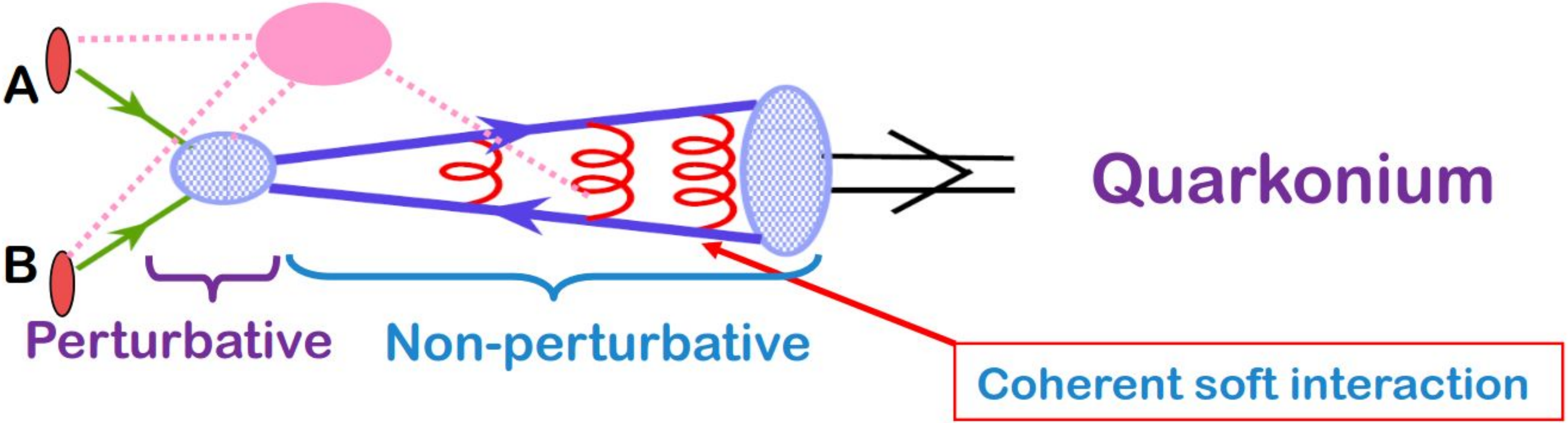


- The main motivation is complete the J/ψ production in ep scattering
- The contribution by the light-quark fragmentation have been done in arXiv:2007.05547
- Typical process carried by EIC: J/ψ production one of the most important



But we can also find the contribution of gluon TMDFF in other processes!

Overview


A
B
Quarkonium
Perturbative
Non-perturbative
Coherent soft interaction

Mechanisms

How the heavy-quark pair becomes a quarkonium?

Mechanisms

How the heavy-quark pair becomes a quarkonium?

- Color Evaporation Model (CEM)

The probability of for the heavy-quark pair to evolve into specific quarkonium state is given by a constant which is independent of momentum and process.

$$\sigma_H = F_H \int_{2m_Q}^{2m_D} \frac{d\sigma_{Q\bar{Q}}}{dm_{Q\bar{Q}}} dm_{Q\bar{Q}}$$

Mechanisms

How the heavy-quark pair becomes a quarkonium?

- Color-Singlet Model (CSM)

It is assumed that the heavy quark pair has the same color, spin and orbital angular momentum quantum numbers as the heavy quarkonium.

Mechanisms

How the heavy-quark pair becomes a quarkonium?

- Non-relativistic QCD Model (NRQCD Model)

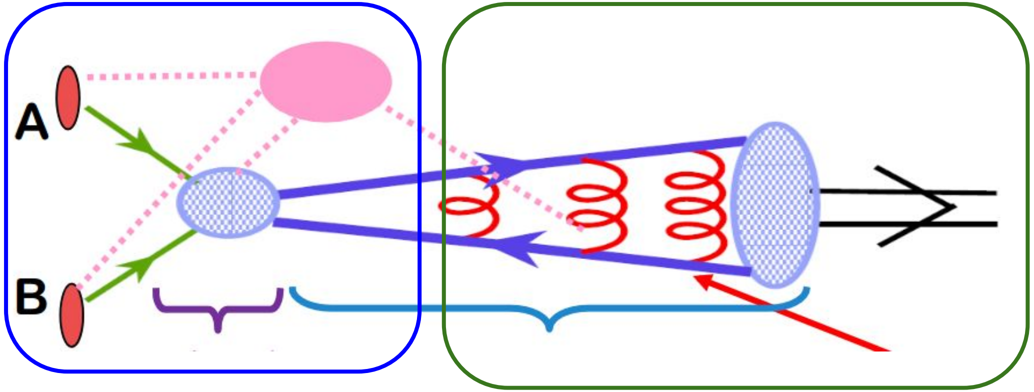
EFT of QCD which reproduces full QCD dynamics at momentum scales of order mv and smaller. NRQCD has a bag convergence in velocity expansion. Best model in agreement with the experimental data.

Mechanisms

How the heavy-quark pair becomes a quarkonium?

- Non-relativistic QCD Model (NRQCD Model)
- Soft-Collinear Effective Theory (SCET) + NRQCD

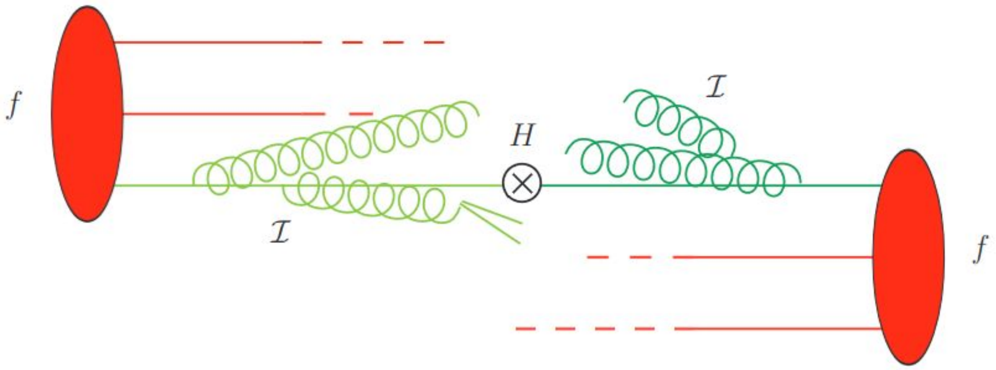
Let's look



Soft-Collinear Effective Theory (SCET)

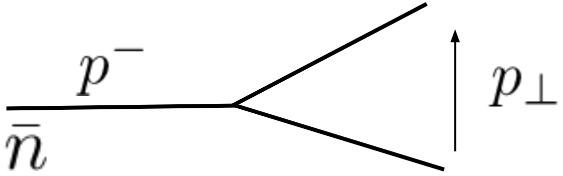
SCET is an EFT for hard interactions in QCD which produce energetic and soft particles. In other words, is a theoretical framework for doing calculations that involve interacting particles carrying different energies.

- Quarkonia production
- Higgs production
- Jets
- Nuclear Physics
- B decays



Momentum regions

$$p^\mu = (n \cdot p, \bar{n} \cdot p, p_\perp) \qquad n^2 = \bar{n}^2 = 0 \quad n \cdot \bar{n} = 2$$



$$\lambda = \frac{p_\perp}{p^-} \ll 1 \Longrightarrow p^\mu \sim (\lambda^2, 1, \lambda)$$

Momentum regions

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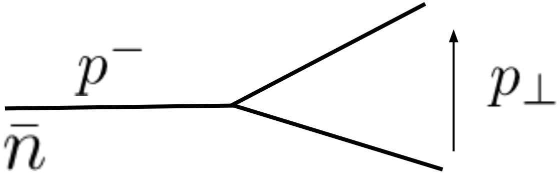
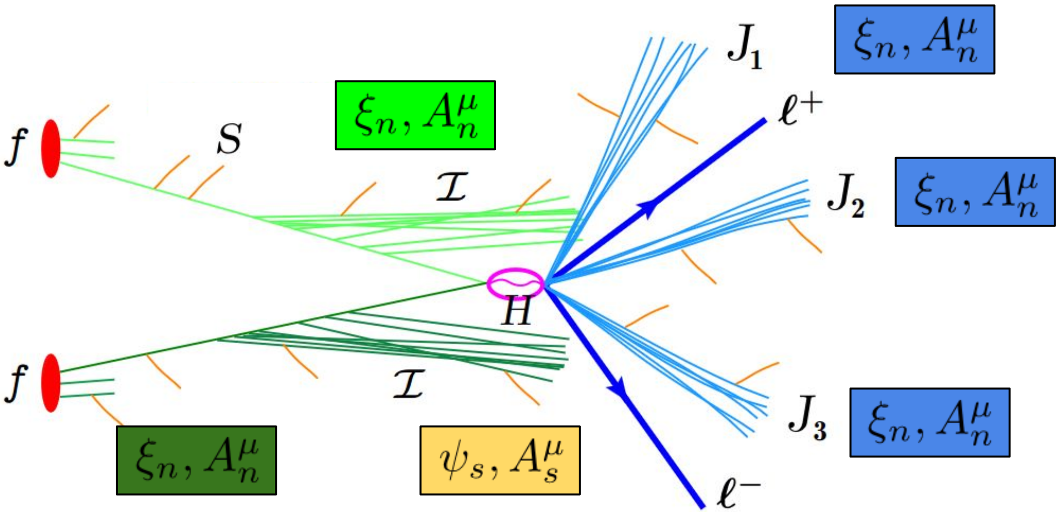


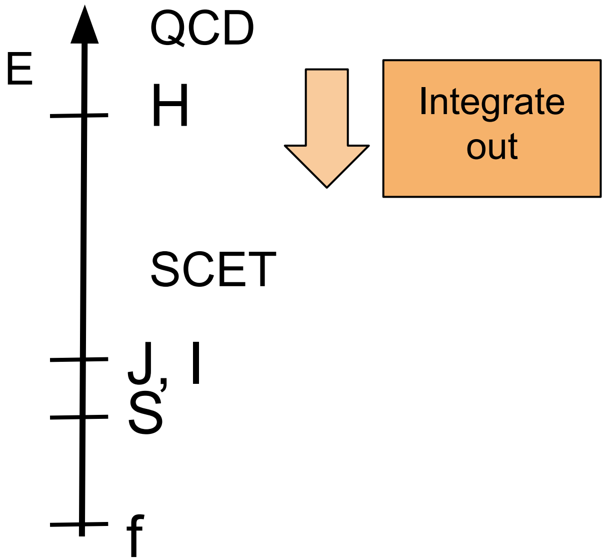
$$\lambda = \frac{p_\perp}{p^-} \ll 1 \quad \Rightarrow \quad p^\mu \sim (\lambda^2, 1, \lambda)$$

$\lambda \ll 1$ Q large

mode	fields	momentum scaling
collinear	ξ_n, A_n^μ	$(p^+, p^-, p_\perp) \sim Q(\lambda^2, 1, \lambda)$
soft	ψ_s, A_s^μ	$\sim Q(\lambda, \lambda, \lambda)$
usoft	ψ_{us}, A_{us}^μ	$\sim Q(\lambda^2, \lambda^2, \lambda^2)$
hard	–	$p^2 \gtrsim Q^2$

Cross section factorization





Non-perturbative: $$d\sigma = f_1 \otimes f_2 \otimes \hat{\sigma}$$

Perturbative: $$\hat{\sigma} = I_1 \otimes I_2 \otimes H \otimes J_1 \otimes J_2 \otimes J_3 \otimes S$$

Unpolarized TMDFF

How can we describe the process when a quark is produced in a hard interaction and then fragments into a detected hadron? → TMD fragmentation function

$$\Delta_{g\to J/\psi}(z,\mathbf{b}_T) \sim \sum_X \int \frac{db^-}{4\pi} e^{-iP^+b^-/z} < 0|T\left[\mathcal{B}^{\mu}_{n\perp}\right](b)\,|X, J/\psi >< X, J/\psi|\bar{T}\left[\mathcal{B}_{n\perp\mu}\right](0)\,|0>$$

Gluon field

$$\mathcal{B}^{\mu}_{n\perp} = \frac{1}{g}\left[W^{\dagger}_{n}\, iD^{\mu}_{n\perp}\, W_n\right]$$

Renormalization!

$$D_{g\to J/\psi} \sim Z_g \times R_g \times \Delta_{g\to J/\psi}$$

Rapidity divergences

Our theory is boost invariant in the light-cone directions

$k^+ \to ak^+, k^- \to k^-/a, k^+k^-$

$a \to \infty$ ⟹ Divergence

$$\frac{1}{k^-} \to \frac{1}{k^- - i\delta}$$

Soft Function

Non-Relativistic QCD (NRQCD)

The relevant scales of the non-relativistic bound state dynamics are

$$E \sim \frac{\mathbf{p}^2}{2m} \sim V \sim mv^2 \qquad p \sim 1/r \sim mv$$

If the velocity is v << 1, then $m \gg mv \gg mv^2$

Quarkonia, bound states of heavy quarks: massive quarks → perturbative QCD. The heavy quarks are not relativistics: v<<1 → QCD turns non-perturbative

→ We have scales strongly separated describing the quarkonium production.

→ Non-relativistic QCD (NRQCD)

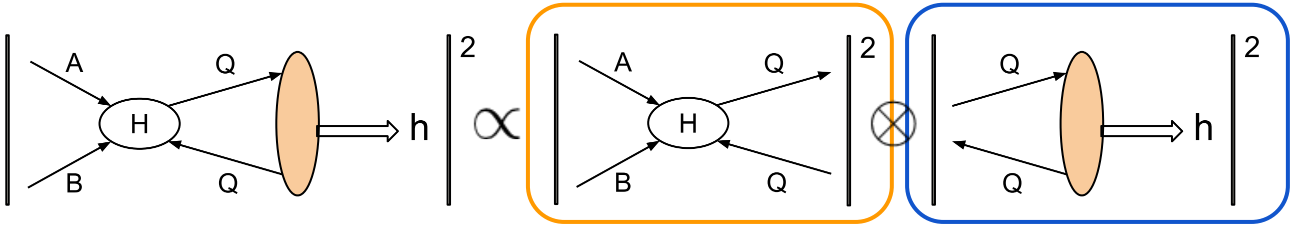
Factorization

$$n = {}^{2S+1}L_J^{[col.]}$$

NRQCD

$$\sigma^H = \sum_n \sigma_{Q\bar{Q}(n)} < 0|\mathcal{O}^H(n)|0 >$$

Can be organized in powers of v



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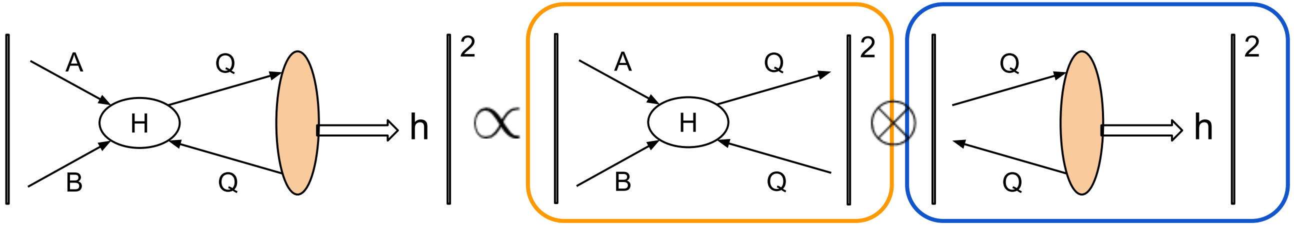


gTMDFF in NRQCD

$$D_{g\to J/\psi}(z, \mathbf{b}_T) = \sum_n d_{g\to Q\bar{Q}(n)}(z, \mathbf{b}_T) < 0|\mathcal{O}^{J/\psi}(n)|0 >$$

SDC

LDME → Universals

$$< \mathcal{O}_n^{J/\psi} > = < 0|\chi^\dagger \mathcal{K}_n \psi a^\dagger_{J/\psi} a_{J/\psi} \psi^\dagger \mathcal{K}'_n \chi|0 >$$

Defined by NRQCD

A few LDMEs…

$$O_8^H({}^3S_1) = \chi^\dagger \sigma_i T^a \psi (a_H^\dagger a_H) \psi^\dagger \sigma_i T^a \chi$$

$$O_1^H({}^3P_0) = \frac{1}{3} \chi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \cdot \sigma \right) \psi (a_H^\dagger a_H) \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \cdot \sigma \right) \chi$$

$$O_1^H({}^3P_1) = \frac{1}{2} \chi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \times \sigma \right)_i \psi (a_H^\dagger a_H) \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \times \sigma \right)_i \chi$$

$$O_1^H({}^3P_2) = \chi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D}_{\{i} \sigma_{j\}} \right) \psi (a_H^\dagger a_H) \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D}_{\{i} \sigma_{j\}} \right) \chi$$

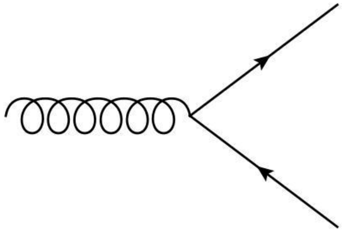
Threshold expansion method

$$D_{g\to J/\psi}(z,\mathbf{b}_T) = \sum_n d_{g\to Q\bar{Q}(n)}(z,\mathbf{b}_T) < 0|\mathcal{O}^{J/\psi}(n)|0 >$$

Threshold expansion method

$$D_{g\to J/\psi}(z,\mathbf{b}_T) = \sum_n d_{g\to Q\bar{Q}(n)}(z,\mathbf{b}_T) < 0|\mathcal{O}^{J/\psi}(n)|0 >$$

- QCD calculation at next to leading order in α_s
- Expansion around the threshold (q = 0)



$$\bar{u}(p)(ig\gamma^\mu T^a)v(\bar{p})$$

$$p = P/2 + \mathbf{q}, \quad \bar{p} = P/2 - \mathbf{q}$$

$$u(p)\,|_{\mathrm{CM}} = \frac{1}{\sqrt{E_q+m_c}}\left((E_q+m_C)\,\xi,\ \ \mathbf{q}\cdot\sigma\,\xi\right)^T \qquad v(\bar{p})\,|_{\mathrm{CM}} = \frac{1}{\sqrt{E_q+m_c}}\left(-\mathbf{q}\cdot\sigma\,\eta,\ \ (E_q+m_C)\,\eta\right)^T$$

$$\bar{u}(p)\gamma^\mu v(\bar{p}) = L^\mu_j\left(2E_q\xi^\dagger\sigma^j\eta - \frac{2}{E_q+m_c}q^j\xi^\dagger(\mathbf{q}\cdot\sigma)\eta\right)$$

Threshold expansion method

$$D_{g\to J/\psi}(z,\mathbf{b}_T) = \sum_n d_{g\to Q\bar{Q}(n)}(z,\mathbf{b}_T) < 0|\mathcal{O}^{J/\psi}(n)|0 >$$

- QCD calculation at next to leading order in α_s
- Expansion around the threshold (q = 0)

$$\bar{u}(p)v(\bar{p}) = -2\,\xi^\dagger(\mathbf{q}\cdot\boldsymbol{\sigma})\eta,$$

$$\bar{u}(p)\gamma^\mu v(\bar{p}) = L^\mu{}_j\left(\boxed{2E_q\,\xi^\dagger\sigma^j\eta} - \frac{2}{E_q+m_c}q^j\,\xi^\dagger(\mathbf{q}\cdot\boldsymbol{\sigma})\eta\right),$$

$$\bar{u}(p)(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)v(\bar{p}) = (P^\mu L^\nu{}_j - P^\nu L^\mu{}_j)\left(\frac{2m_c}{E_q}\,\xi^\dagger\sigma^j\eta + \frac{2}{E_q(E_q+m_c)}q^j\,\xi^\dagger(\mathbf{q}\cdot\boldsymbol{\sigma})\eta\right) + L^\mu{}_j\,L^\nu{}_k\,\xi^\dagger\{[\sigma^j,\sigma^k],\mathbf{q}\cdot\boldsymbol{\sigma}\}\eta,$$

$$\begin{aligned}\bar{u}(p)&(\gamma^\mu\gamma^\nu\gamma^\lambda-\gamma^\lambda\gamma^\nu\gamma^\mu)v(\bar{p})\\ &= L^\mu{}_i\,L^\nu{}_j\,L^\lambda{}_k\Big(-E_q\,\xi^\dagger\{[\sigma^i,\sigma^j],\sigma^k\}\eta + \frac{q^i}{E_q+m_c}\,\xi^\dagger\{[\sigma^j,\sigma^k],\mathbf{q}\cdot\boldsymbol{\sigma}\}\eta\\ &\quad + \frac{q^j}{E_q+m_c}\,\xi^\dagger\{[\sigma^k,\sigma^i],\mathbf{q}\cdot\boldsymbol{\sigma}\}\eta + \frac{q^k}{E_q+m_c}\,\xi^\dagger\{[\sigma^i,\sigma^j],\mathbf{q}\cdot\boldsymbol{\sigma}\}\eta\Big)\\ &\quad - \frac{2}{E_q}\left(P^\mu L^\nu{}_i L^\lambda{}_j + L^\mu{}_i L^\nu{}_j P^\lambda + L^\mu{}_j P^\nu L^\lambda{}_i\right)\left(\xi^\dagger q^i\sigma^j\eta - \xi^\dagger q^j\sigma^i\eta\right).\end{aligned}$$

Threshold expansion method

$$D_{g\to J/\psi}(z,\mathbf{b}_T) = \sum_n d_{g\to Q\bar{Q}(n)}(z,\mathbf{b}_T) < 0|\mathcal{O}^{J/\psi}(n)|0 >$$

NRQCD

$$\langle \chi^\dagger \sigma^j T^a \psi \, \mathcal{P}_{c\bar{c}',c\bar{c}} \, \psi^\dagger \sigma^i T^a \chi \rangle = 4m_c^2 \, \eta'^\dagger \sigma^j T^a \xi' \xi^\dagger \sigma^i T^a \eta$$

$$\langle \chi^\dagger \psi \, \mathcal{P}_{c\bar{c}',c\bar{c}} \, \psi^\dagger \chi \rangle = 4m_c^2 \, \eta'^\dagger \xi' \xi^\dagger \eta,$$

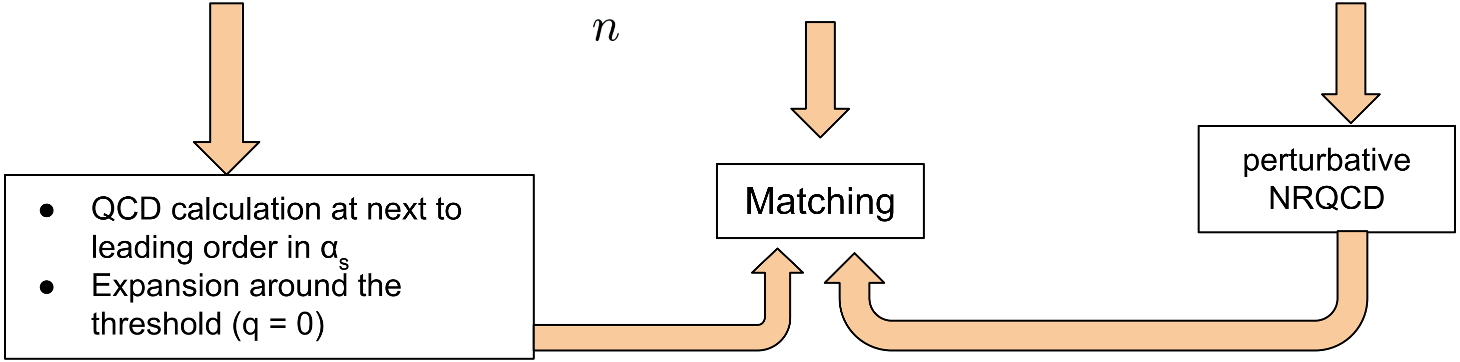
$$\langle \chi^\dagger T^a \psi \, \mathcal{P}_{c\bar{c}',c\bar{c}} \, \psi^\dagger T^a \chi \rangle = 4m_c^2 \, \eta'^\dagger T^a \xi' \xi^\dagger T^a \eta,$$

$$\langle \chi^\dagger (-\tfrac{i}{2}\overleftrightarrow{D}^m) T^a \psi \, \mathcal{P}_{c\bar{c}',c\bar{c}} \, \psi^\dagger (-\tfrac{i}{2}\overleftrightarrow{D}^n) T^a \chi \rangle = 4m_c^2 \, q'^m q^n \, \eta'^\dagger T^a \xi' \xi^\dagger T^a \eta,$$

$$\langle \chi^\dagger (-\tfrac{i}{2}\overleftrightarrow{D}^m) \sigma^i T^a \psi \, \mathcal{P}_{c\bar{c}',c\bar{c}} \, \psi^\dagger (-\tfrac{i}{2}\overleftrightarrow{D}^n) \sigma^j T^a \chi \rangle = 4m_c^2 \, q'^m q^n \, \eta'^\dagger \sigma^i T^a \xi' \xi^\dagger \sigma^j T^a \eta$$

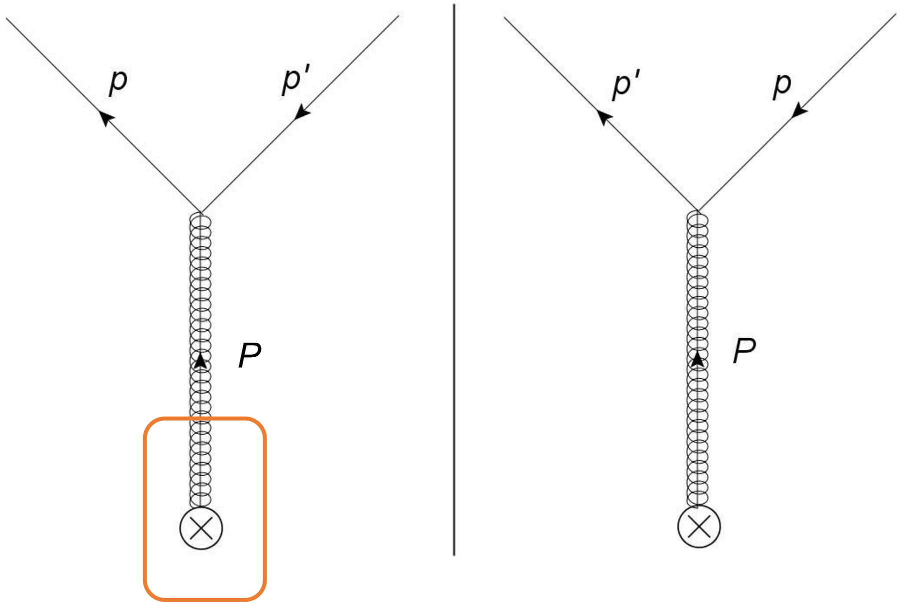
Threshold expansion method

$$D_{g\to J/\psi}(z,\mathbf{b}_T) = \sum_n d_{g\to Q\bar{Q}(n)}(z,\mathbf{b}_T) < 0|\mathcal{O}^{J/\psi}(n)|0 >$$



Feynman Diagrams

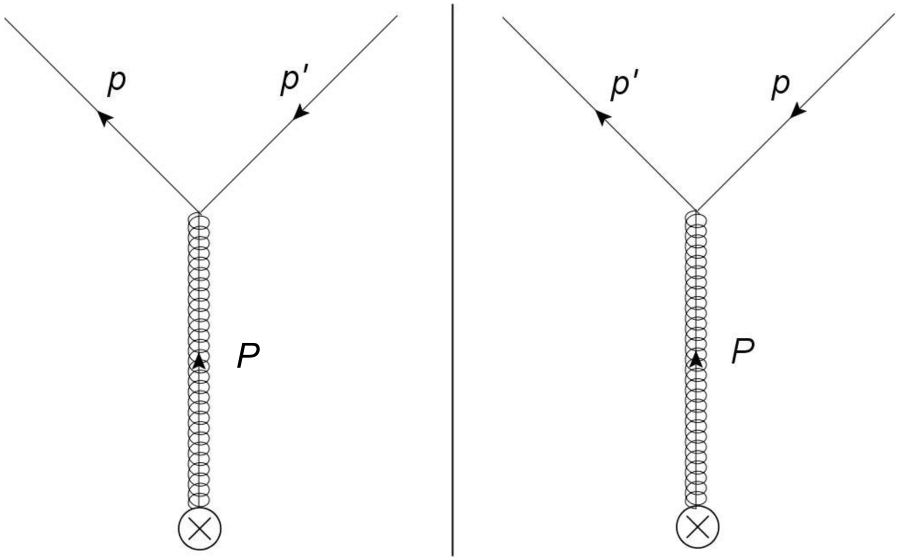
Leading order



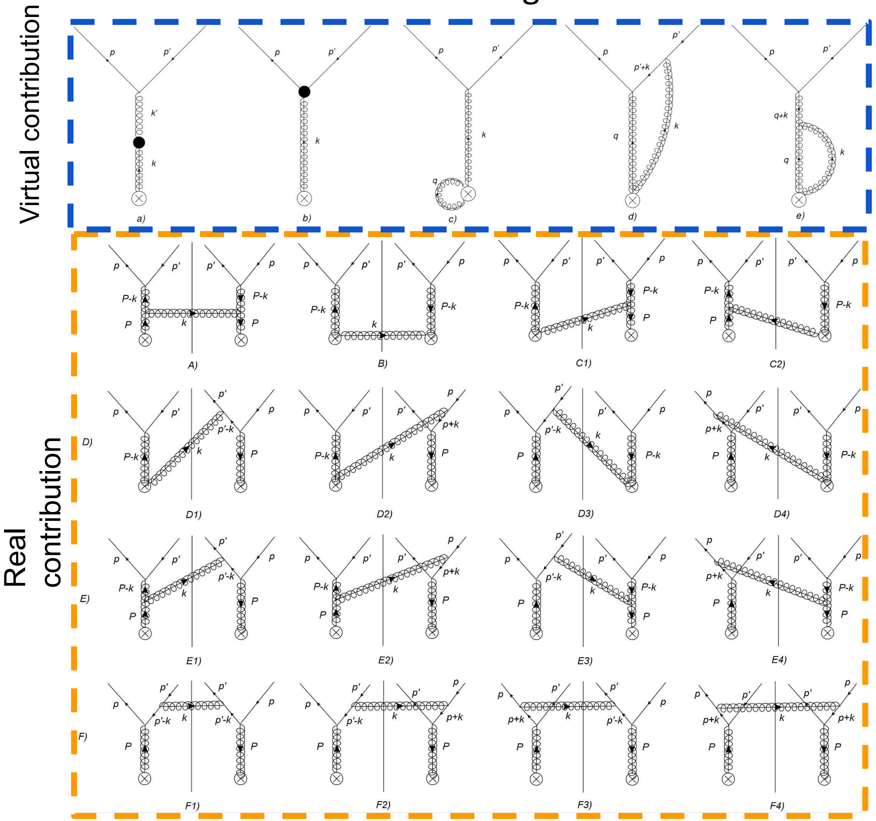
$$\mathcal{B}_{n\perp}^{(0)\alpha\mu}(P)$$

Feynman Diagrams

Leading order

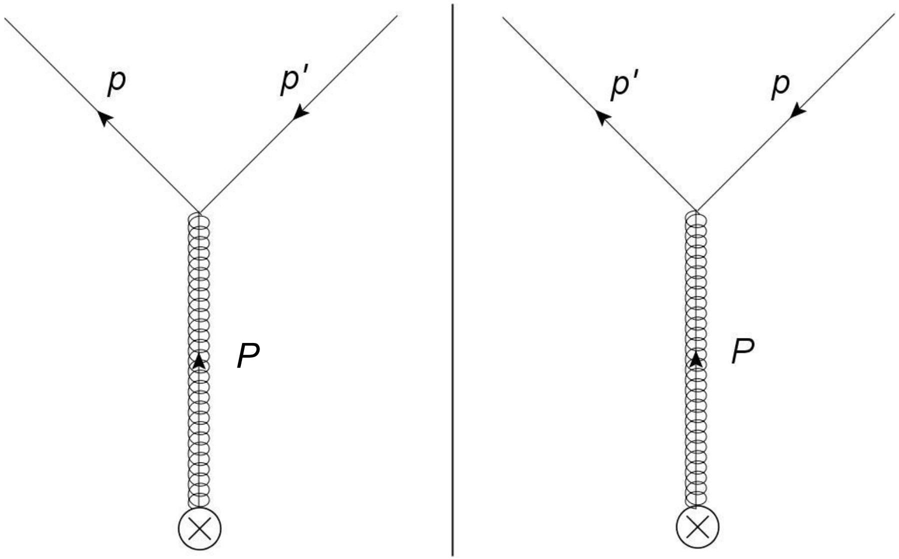


Next to leading order



Feynman Diagrams

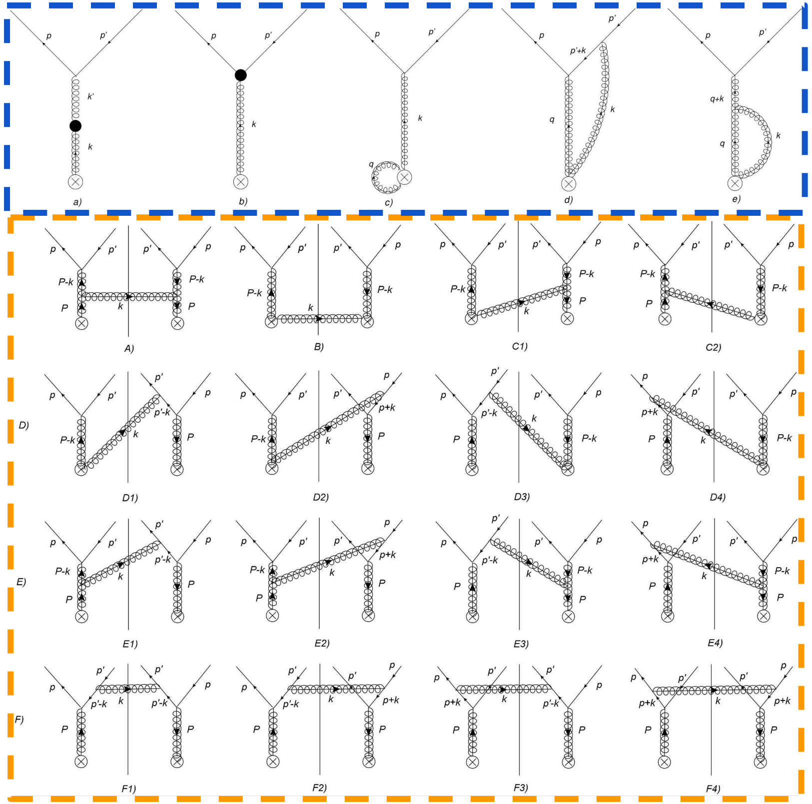
Leading order



Next to leading order

Virtual contribution

Real contribution



arXiv:2308.12356 [hep-ph]

Conclusions

- Quarkonia are very useful tools to study the multidimensional structure of the nucleon
- There are much more physics yet to be understood and much more work yet to be done in quarkonium production mechanisms
- Factorization theorems in SCET allow us to factorize the cross section
- NRQCD factorize the TMD into perturbative and non-perturbative parts $\longrightarrow$ strong tool to study QCD