

# Constraining Fragmentation Functions through photon-hadron production

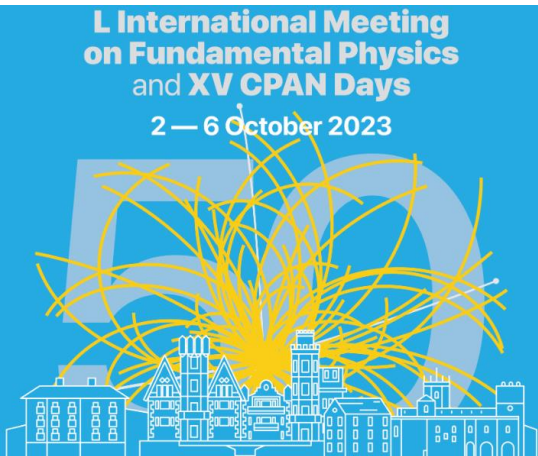


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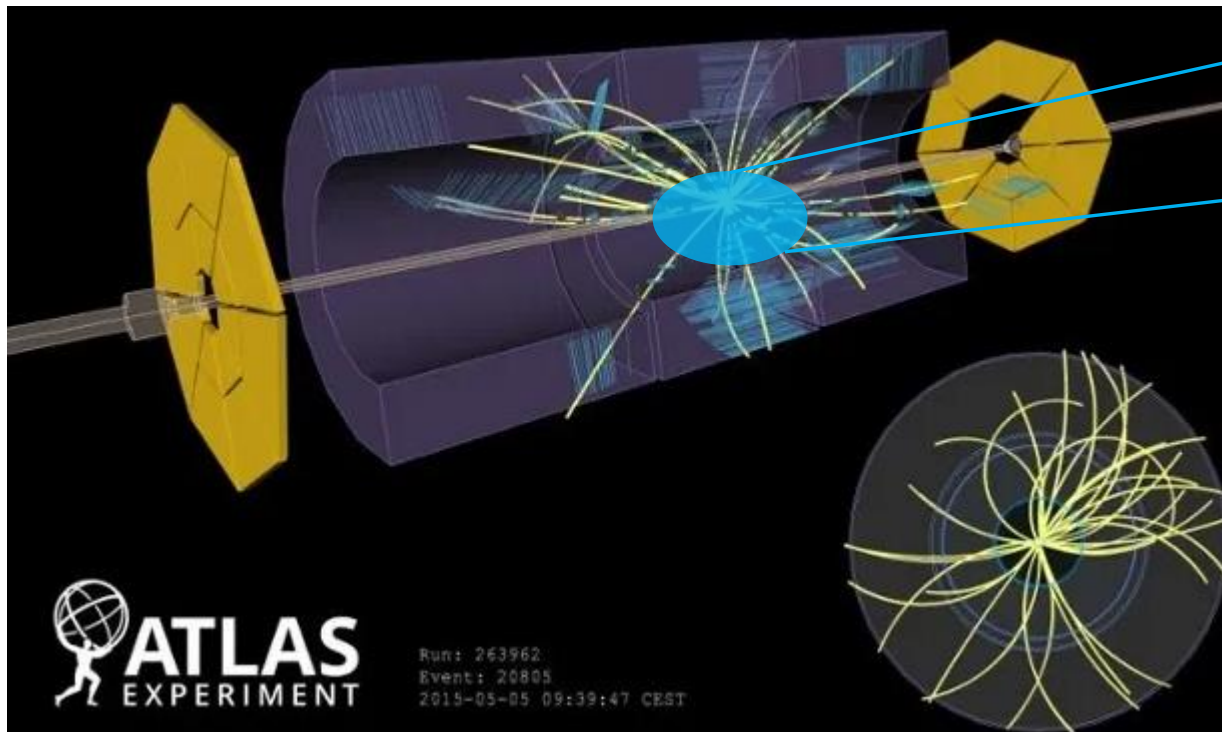
1. Motivation
2. Computational details
3. Reconstructing the parton kinematics
  - A. Reconstruction @ (N)LO
  - B. Error estimation
4. Constraining FFs (main result)
  - A. Enhancing different partonic channels
5. Conclusions

**Based on the following  
published articles:**

- **1011.0486 (inspiration)**
- **2104.14663**
- **2112.05043**
- **2303.04965 (new)**




- Experiments are collecting more and more data
- **More data available**  **More accurate results!!**
- Hadrons are composite states  **Non-perturbative description**
- Several aspects must be explored, but we must **enter into their internal structure**



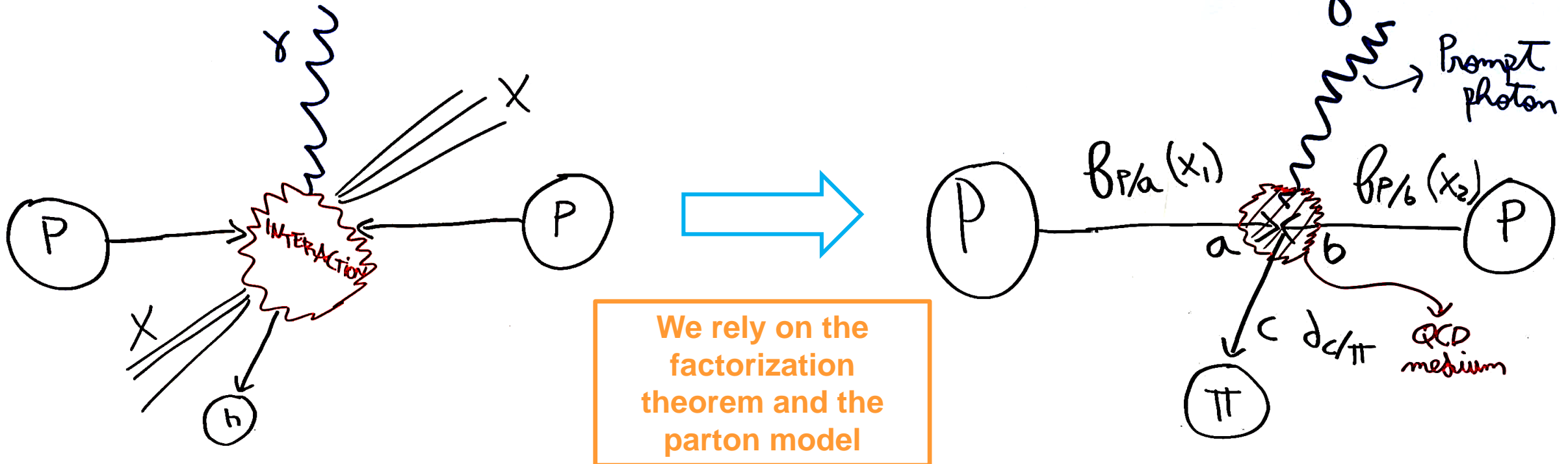
**We must reconstruct  
what's going on  
inside the collisions**



- **Process of interest:** photon-hadron production at colliders
- **Why?**  Photon acts as a clean probe of the parton collision
- We can access information regarding PDFs and FFs
- **Hadron = Pion** (higher-production rates, more statistics)
- **Use to impose restrictions on heavier hadron FF!**



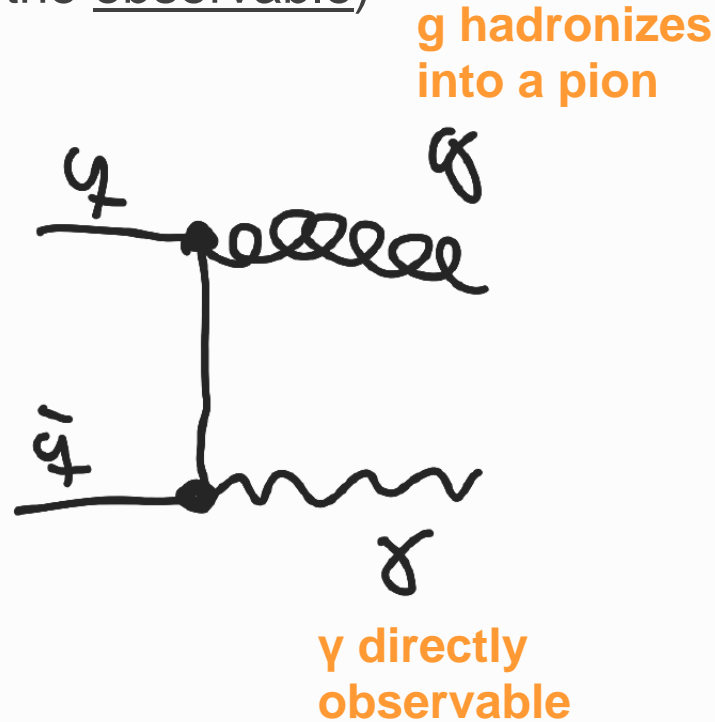
The photon is our "magnifying glass"



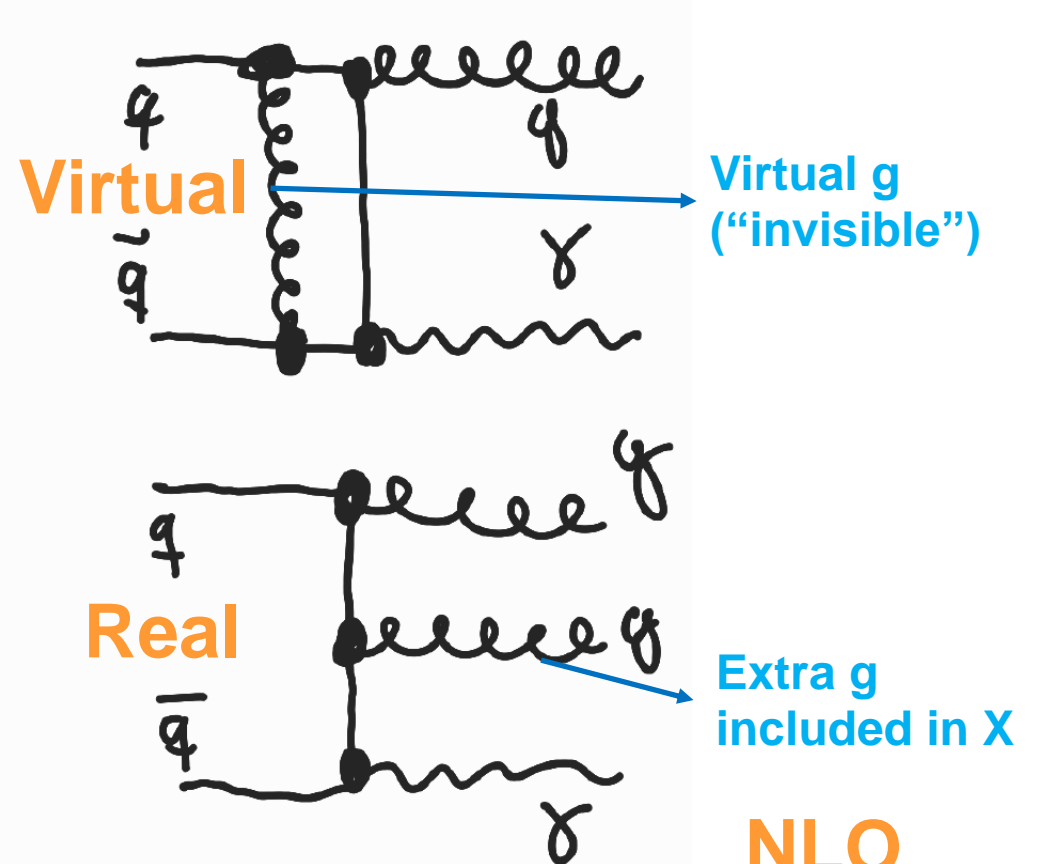


- **Idea:** LO vs NLO
- Higher-orders contain unresolved (not measurable) particles
- Loops are quantum fluctuations of vacuum; real emission includes extra-particles (not considered as part of the observable)

$$P+P \rightarrow \gamma + \pi + X$$



**LO**



**NLO**



- **Framework:** Higher-order contributions implemented in **FKS** (virtual + real + UV counter-terms + ISR counter-terms)  $\Rightarrow$  **Separated integrations!**

## Hadronic cross-section

## Partonic cross-section

$$d\sigma_{H_1 H_2 \rightarrow h \gamma} \approx \sum_{a_1 a_2 a_3} \int dx_1 dx_2 dz f_{H_1/a_1}(x_1, \mu_I) f_{H_2/a_2}(x_2, \mu_I) D_{a_3/h}(z, \mu_F) \times d\hat{\sigma}_{a_1 a_2 \rightarrow a_3 \gamma}^{(ISO)}(x_1 P_1, x_2 P_2, P^h/z, P^\gamma; \mu_I, \mu_F, \mu_R)$$

$$d\hat{\sigma}_{a_1 a_2 \rightarrow a_3 \gamma}^{\text{ISO,(1),finite}} = d\hat{\sigma}_{a_1 a_2 \rightarrow a_3 \gamma}^{\text{ISO,(1),ren.}} - \frac{C_{a_1 a_2 \rightarrow a_3 \gamma}^{\text{UV}}}{\epsilon} \times d\hat{\sigma}_{a_1 a_2 \rightarrow a_3 \gamma}^{\text{ISO,(0)}} - d\hat{\sigma}_{a_1 a_2 \rightarrow a_3 \gamma}^{\text{ISO,cnt,(I)}} - d\hat{\sigma}_{a_1 a_2 \rightarrow a_3 \gamma}^{\text{ISO,cnt,(F)}}$$

$f_{H_1/a_1}$  NNPDF 3&4

$D_{a_3/h}$  DSS2014

- **Isolation:** smooth isolation criteria  $\xi(r) = E_T^\gamma \left( \frac{1 - \cos(r)}{1 - \cos(r_0)} \right)^4$

**More details:  
2112.05043 [hep-ph]**

- **Cuts** (used by STAR/PHENIX @ RHIC):

$$|\eta^h| \leq 0.35, \quad |\eta^\gamma| \leq 0.35, \quad p_T^h \geq 2 \text{ GeV}, \quad 5 \text{ GeV} \leq p_T^\gamma \leq 15 \text{ GeV} \quad + \quad |\phi^h - \phi^\gamma| > 2$$

# Reconstructing the partonic kinematics



- Momentum fractions are not “physical” (consequence of parton model)
- **Still, they allow to understand what’s going on inside hadrons**
- **Experimentally accessible quantities:**

$$\mathcal{V}_{\text{Exp}} = \{p_T^\gamma, p_T^\pi, \eta^\gamma, \eta^\pi, \cos(\phi^\pi - \phi^\gamma)\}$$

- **Analytic formulae (exact only at LO – Born kinematics)**

$$X_{1,\text{REC}} = \frac{p_T^\gamma \exp(\eta^\pi) + p_T^\gamma \exp(\eta^\gamma)}{\sqrt{S_{CM}}}$$

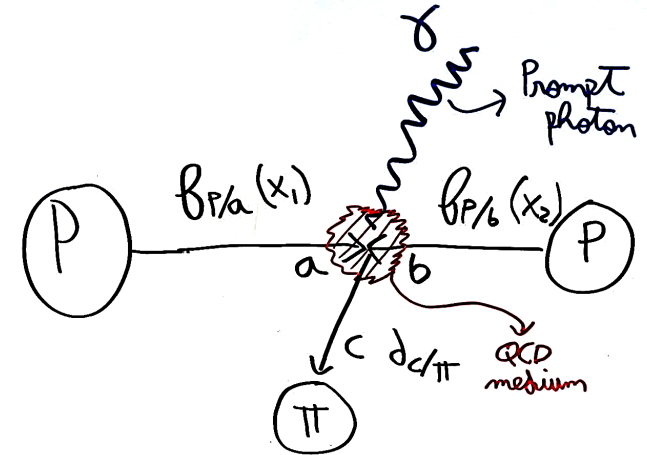
$$X_{2,\text{REC}} = \frac{p_T^\gamma \exp(-\eta^\pi) + p_T^\gamma \exp(-\eta^\gamma)}{\sqrt{S_{CM}}}$$

$$Z_{\text{REC}} = \frac{p_T^\pi}{p_T^\gamma} \quad \text{SET 1}$$

$$X_{1,\text{REC}} = \frac{p_T^\gamma \exp(\eta^\pi) - \cos(\phi^\pi - \phi^\gamma) p_T^\gamma \exp(\eta^\gamma)}{\sqrt{S_{CM}}}$$

$$X_{2,\text{REC}} = \frac{p_T^\gamma \exp(-\eta^\pi) - \cos(\phi^\pi - \phi^\gamma) p_T^\gamma \exp(-\eta^\gamma)}{\sqrt{S_{CM}}}$$

$$Z_{\text{REC}} = -\cos(\phi^\pi - \phi^\gamma) \frac{p_T^\pi}{p_T^\gamma} \quad \text{SET 2}$$

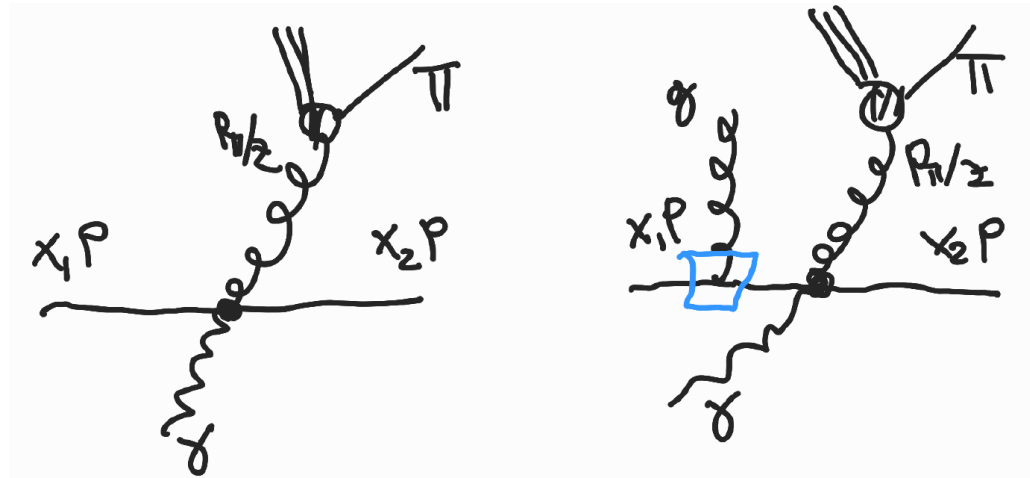


**These expressions are equivalent at LO, but differ at NLO (due to real radiation)**

## Dealing with higher-orders: binning

- NLO corrections involve: real (2-to-3), virtual (2-to-2), counterterms (2-to-2)
- Create “bins” in the external variables and compute the cross-section**
- NLO events don't have a unique momentum fraction  $(x,z)$  ➡ **Weight contributions!**

Hadron & photon momenta fixes  $(x,z)$



Extra-radiation integrated (included in “X”, not resolved)



$(x,z)$  can not be exactly identified

- Goal:** find the maps

$$X_{1,REC} := \bar{\mathcal{V}}_{Exp} \longrightarrow \bar{X}_{1,REAL} = \{(x_1)_j\}$$

$$Z_{REC} := \bar{\mathcal{V}}_{Exp} \longrightarrow \bar{Z}_{REAL} = \{(z)_j\}$$





## Dealing with higher-orders: binning

- NLO corrections involve: real (2-to-3), virtual (2-to-2), counterterms (2-to-2)
- **Create “bins” in the external variables and compute the cross-section**

$$p_j = \{\bar{p}_T^\gamma, \bar{p}_T^\pi, \bar{\eta}^\gamma, \bar{\eta}^\pi, \overline{\cos}(\phi^\pi - \phi^\gamma)\} \in \bar{\mathcal{V}}_{\text{Exp}} \quad \text{Grid}$$

$$\sigma_j(\bar{p}_T^\gamma, \bar{p}_T^\pi, \bar{\eta}^\gamma, \bar{\eta}^\pi, \overline{\cos}(\phi^\pi - \phi^\gamma)) = \int_{(p_T^\gamma)_{j,\text{MIN}}}^{(p_T^\gamma)_{j,\text{MAX}}} dp_T^\gamma \int_{(p_T^\pi)_{j,\text{MIN}}}^{(p_T^\pi)_{j,\text{MAX}}} dp_T^\pi \int dx_1 dx_2 dz d\bar{\sigma}$$

- Weight the MC momentum fractions with the cross-section per bin:

$$(x_1)_j = \sum_i (x_1)_i \frac{d\sigma_j}{dx_1}(p_j; (x_1)_i) \quad (z)_j = \sum_i z_i \frac{d\sigma_j}{dz}(p_j; z_i)$$

- **Goal:** find the maps

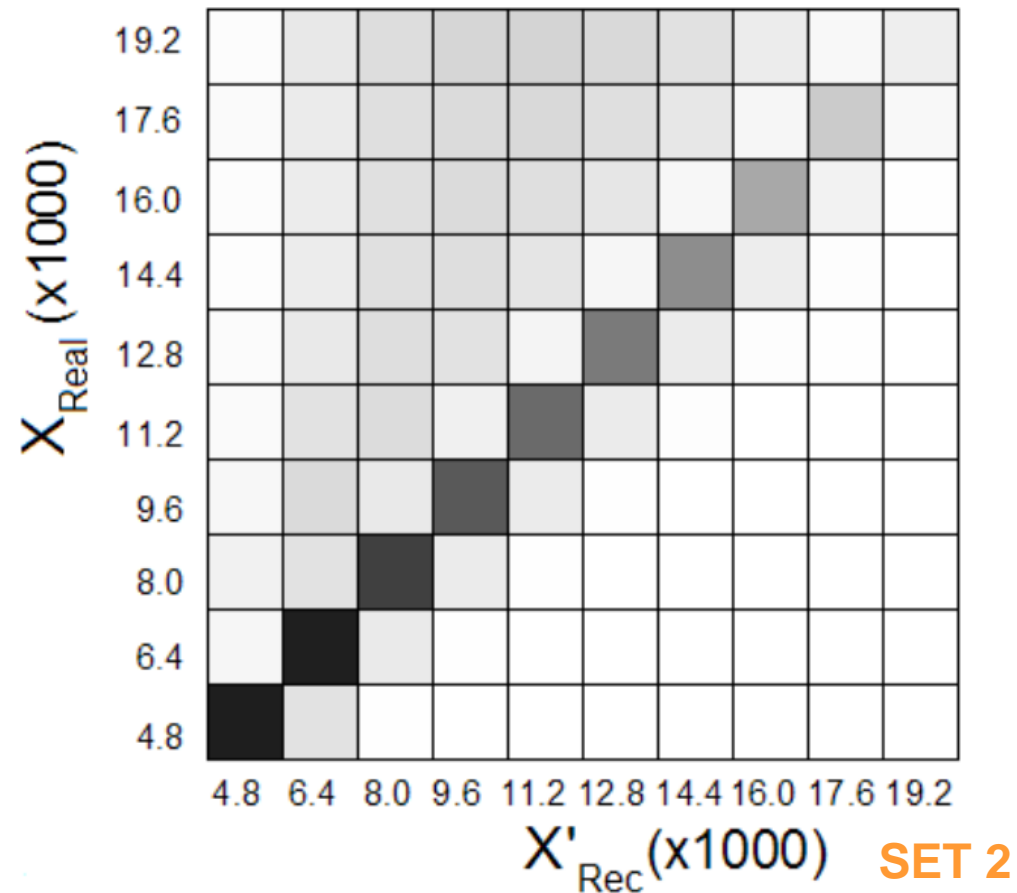
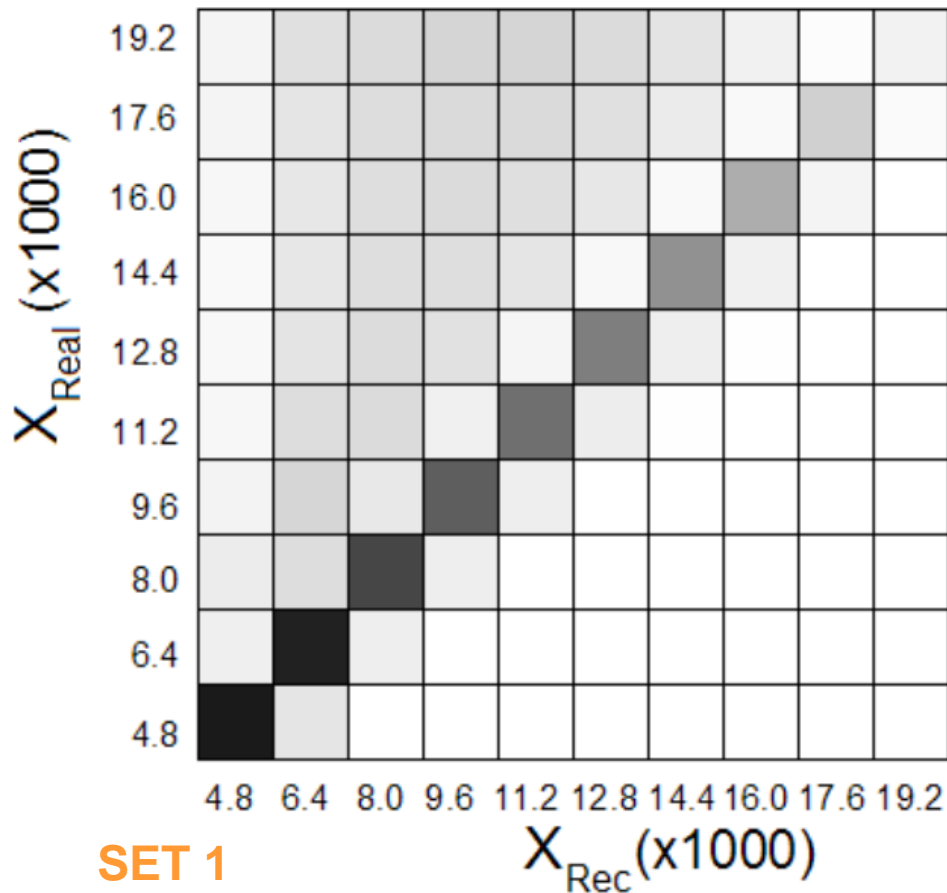
$$X_{1,\text{REC}} := \bar{\mathcal{V}}_{\text{Exp}} \longrightarrow \bar{X}_{1,\text{REAL}} = \{(x_1)_j\}$$

$$Z_{\text{REC}} := \bar{\mathcal{V}}_{\text{Exp}} \longrightarrow \bar{Z}_{\text{REAL}} = \{(z)_j\}$$



## Previous results (from 2011)

- **SET 1** and **SET 2** tested against real Monte-Carlo momentum fractions
- **Good agreement for low x region; dispersion increases at larger values**

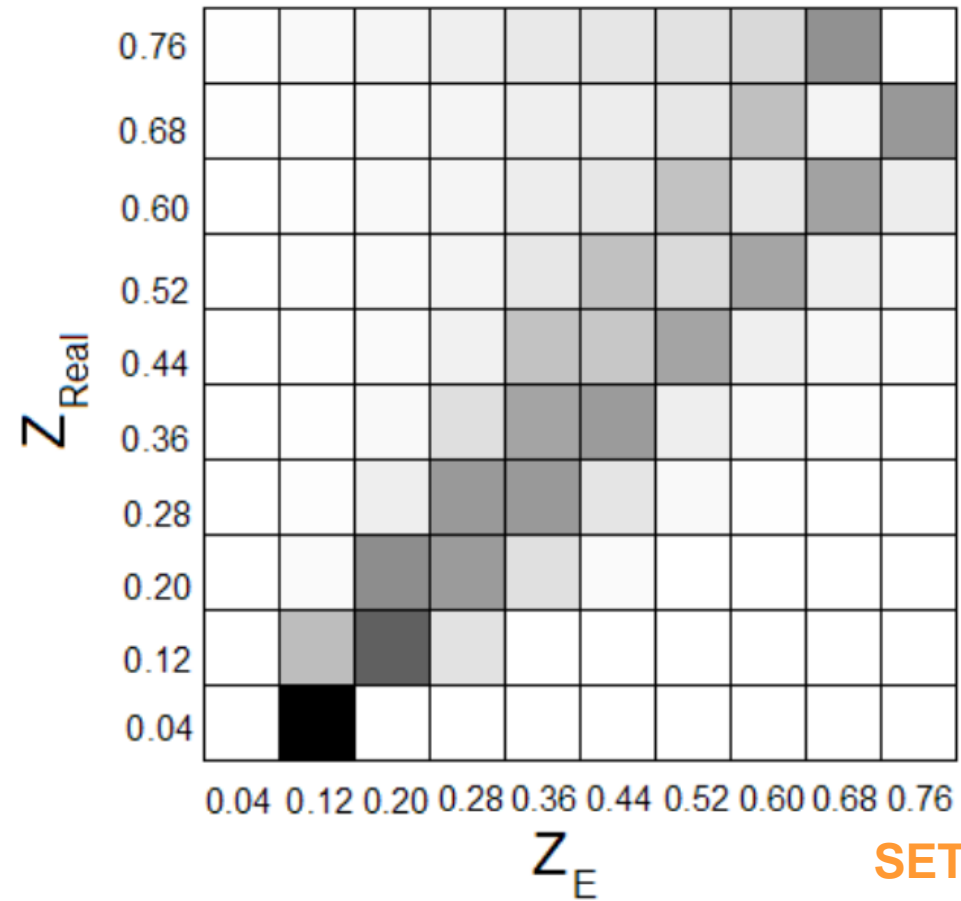
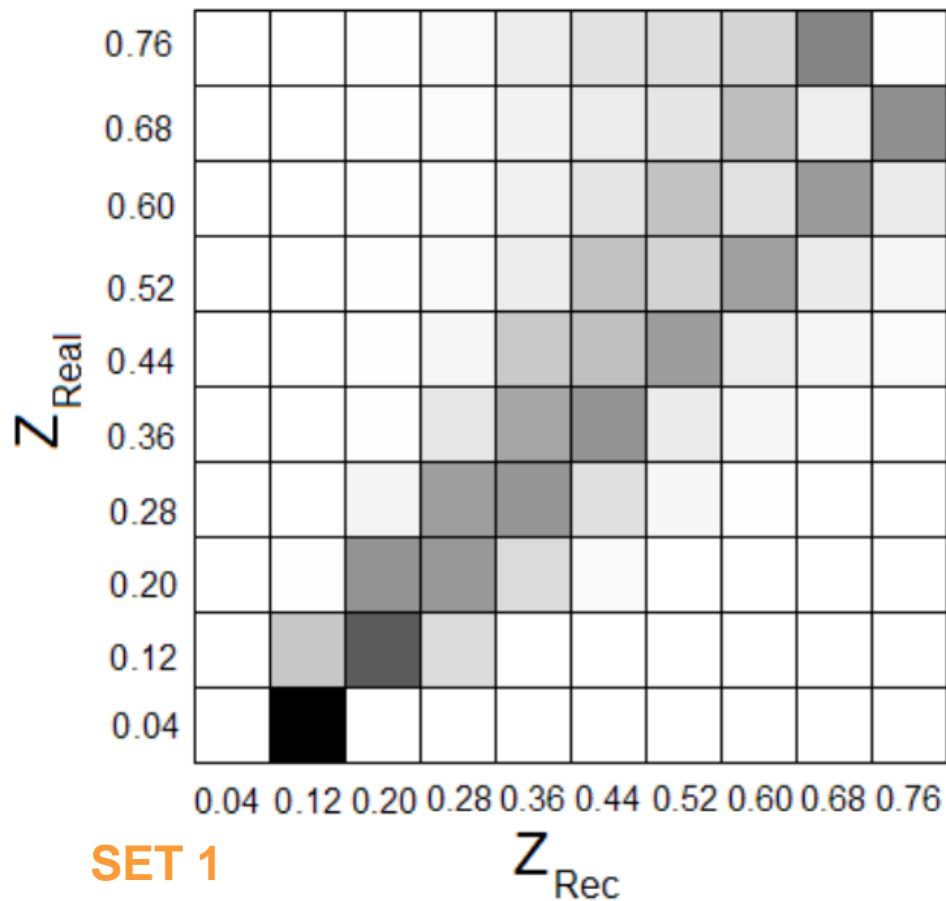


**NLO**



## Previous results (from 2011)

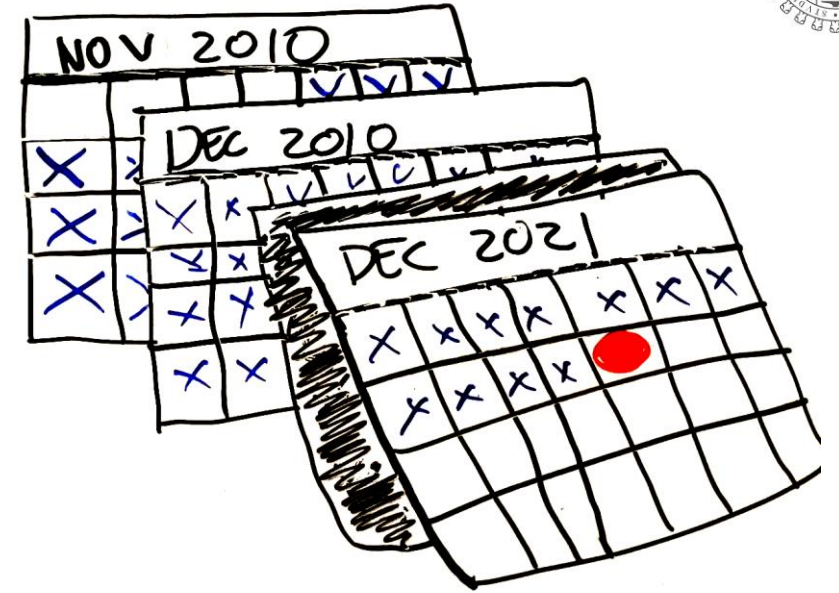
- **SET 1** and **SET 2** tested against real Monte-Carlo momentum fractions
- **Good agreement for low  $z$  region; dispersion increases at larger values**



NLO

## But 11 years after...

- New technologies are easier to use (even for non-experts)
- Computing the NLO corrections is faster (better hardware)
- New high-precision PDFs and FFs are available
- **Machine learning (ML) is becoming a crucial discovery tool for HEP!**

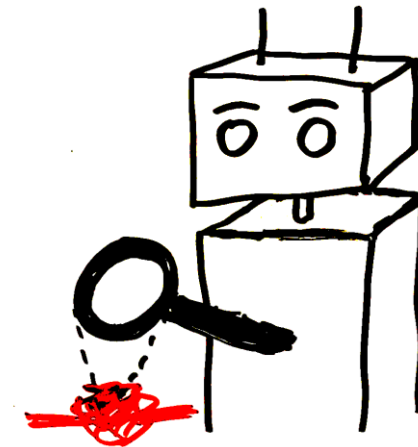


## So...

- We updated our 2011 MC code, including **QED corrections**
- We adapted it to the **LHAPDF framework** (with the latest PDFs)  
*NNPDF3.1QED; NNPDF4.0*

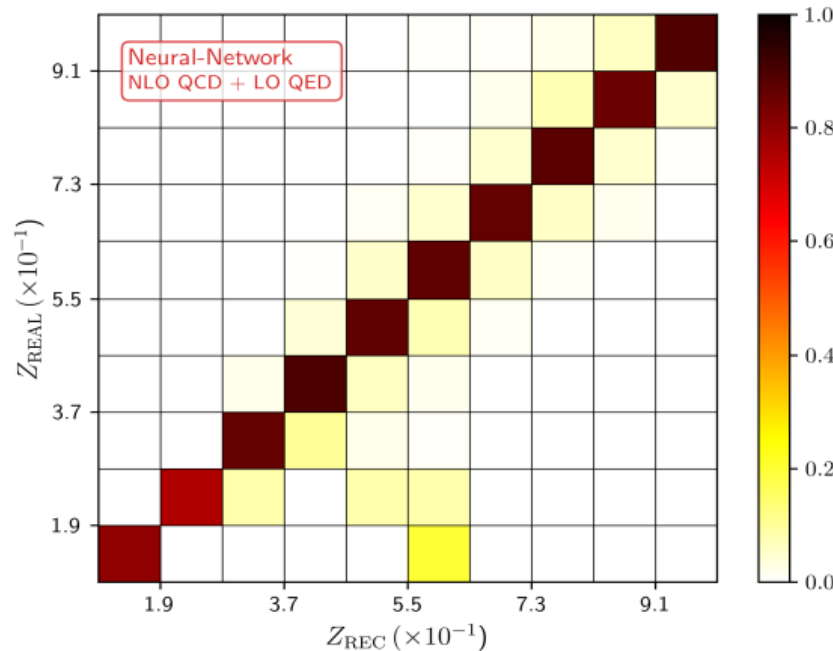
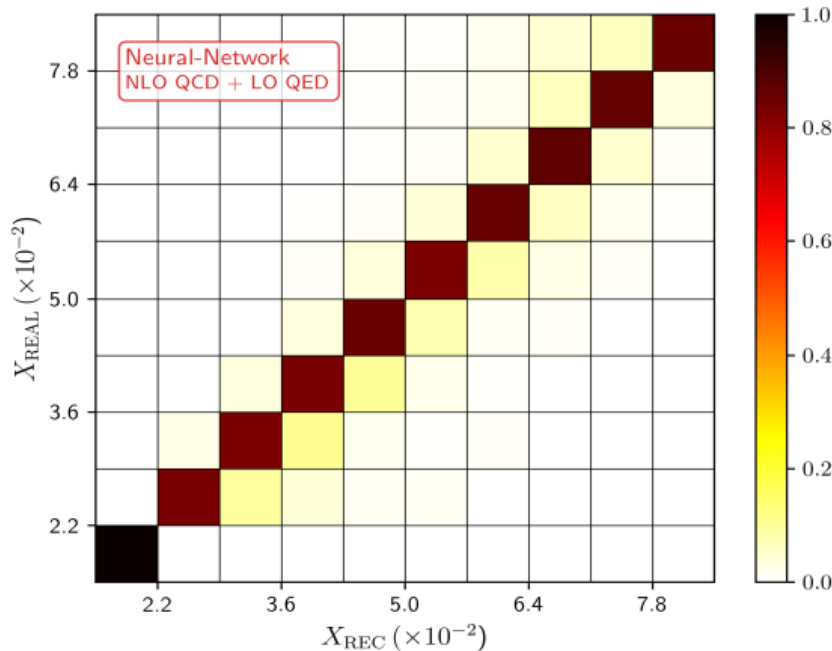
## In order to...

- **Apply ML to discover new approximations to the real MC momentum fractions (with less assumptions)**



## Dealing with higher-orders: neural networks (NN)

Uses a Multilayer Perceptron (more details in the paper!!!)



NLO

### Advantages

- No need to define an specific basis (**only input variables & architecture**)
- **Minimal human** intervention, with **better reconstruction quality**

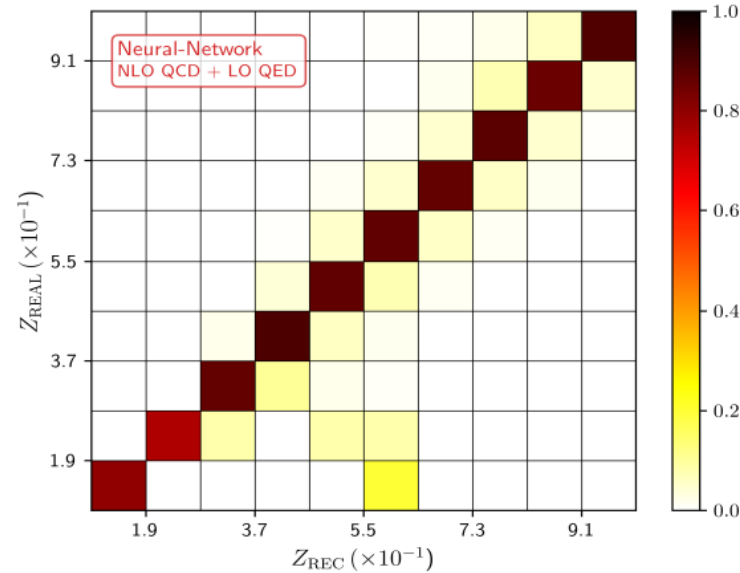
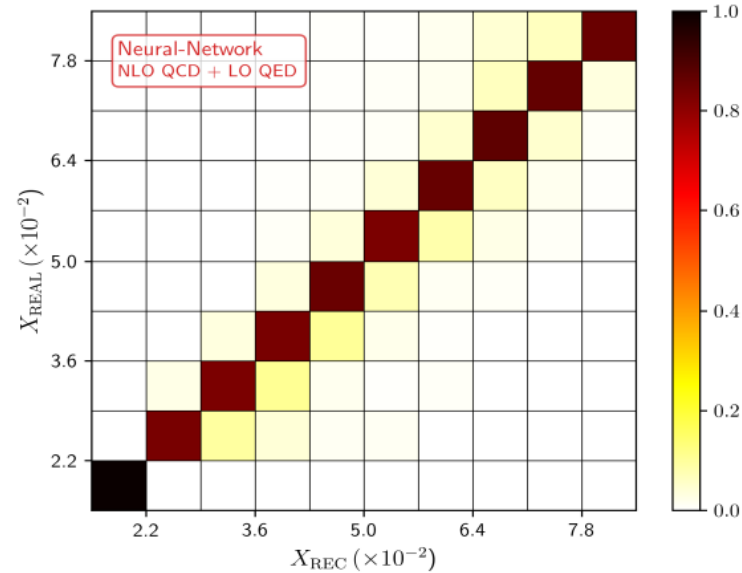
### Disadvantages

- Complicated architectures take **more time** to be trained

# Reconstructing the partonic kinematics

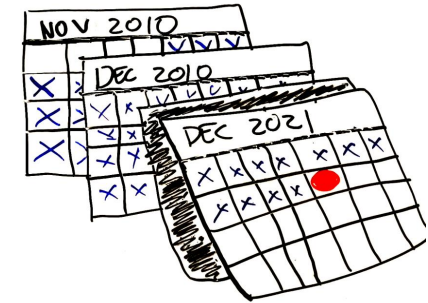
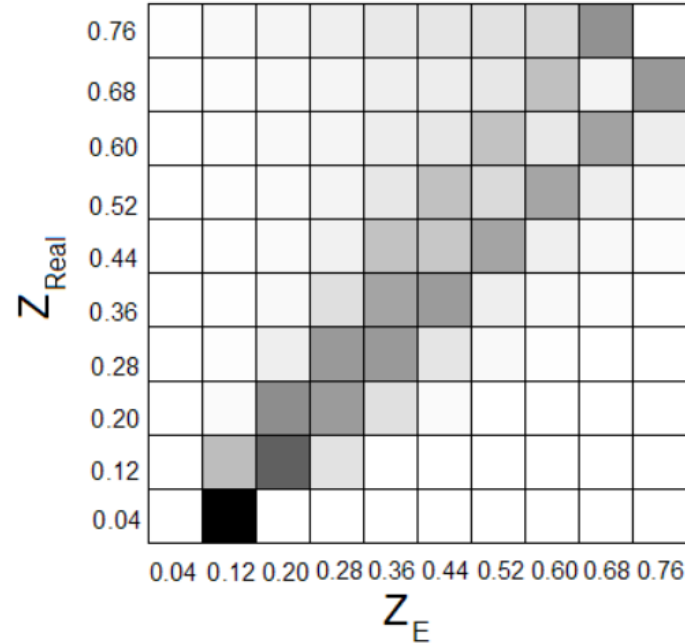
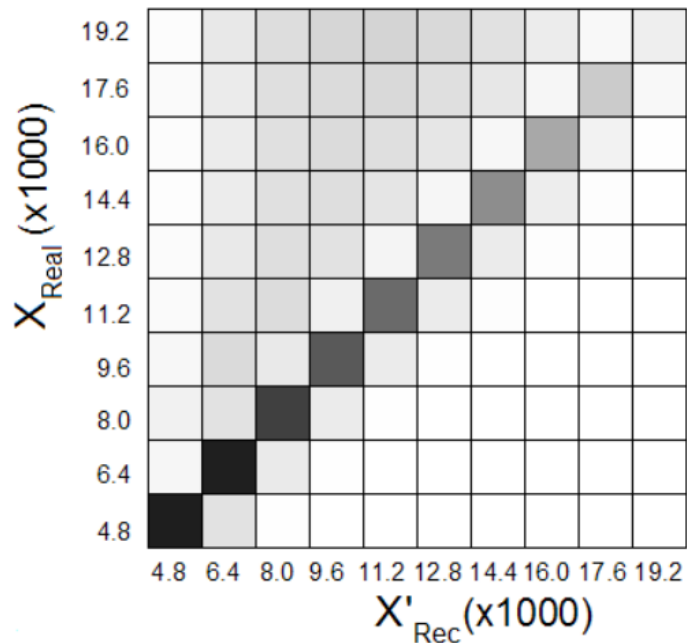


More than 10 years in a single slide....



Human criteria + machine power

Renteria-Estrada et al, arXiv:2112.05043 [hep-ph]



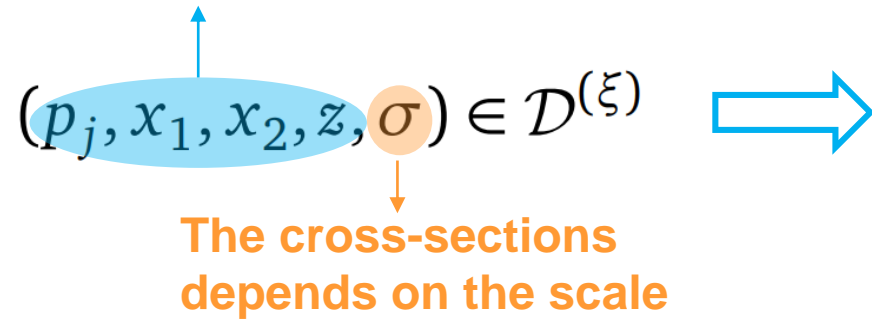
De Florian and Sborlini, arXiv:1011.0486 [hep-ph]

Human intuition + creativity

## How to estimate errors when reconstructing X and Z?

- Several possibilities explored!
- Propagate scale-dependence of partonic cross-section, PDF and FF  $\implies \xi = \{1/2, 1, 2\}$
- For each scale, a training-set is defined and the associated reconstruction functions are obtained

### Kinematical points



$$\begin{aligned}
 X_{1, \text{REC}}^{(\xi)} &:= \bar{\mathcal{V}}_{\text{Exp}}^{(\xi)} \longrightarrow \bar{X}_{1, \text{REAL}}^{(\xi)} = \{(x_1)_j\} \subset \mathcal{D}^{(\xi)} \\
 X_{2, \text{REC}}^{(\xi)} &:= \bar{\mathcal{V}}_{\text{Exp}}^{(\xi)} \longrightarrow \bar{X}_{2, \text{REAL}}^{(\xi)} = \{(x_2)_j\} \subset \mathcal{D}^{(\xi)} \\
 Z_{\text{REC}}^{(\xi)} &:= \bar{\mathcal{V}}_{\text{Exp}}^{(\xi)} \longrightarrow \bar{Z}_{\text{REAL}}^{(\xi)} = \{(z)_j\} \subset \mathcal{D}^{(\xi)}
 \end{aligned}$$

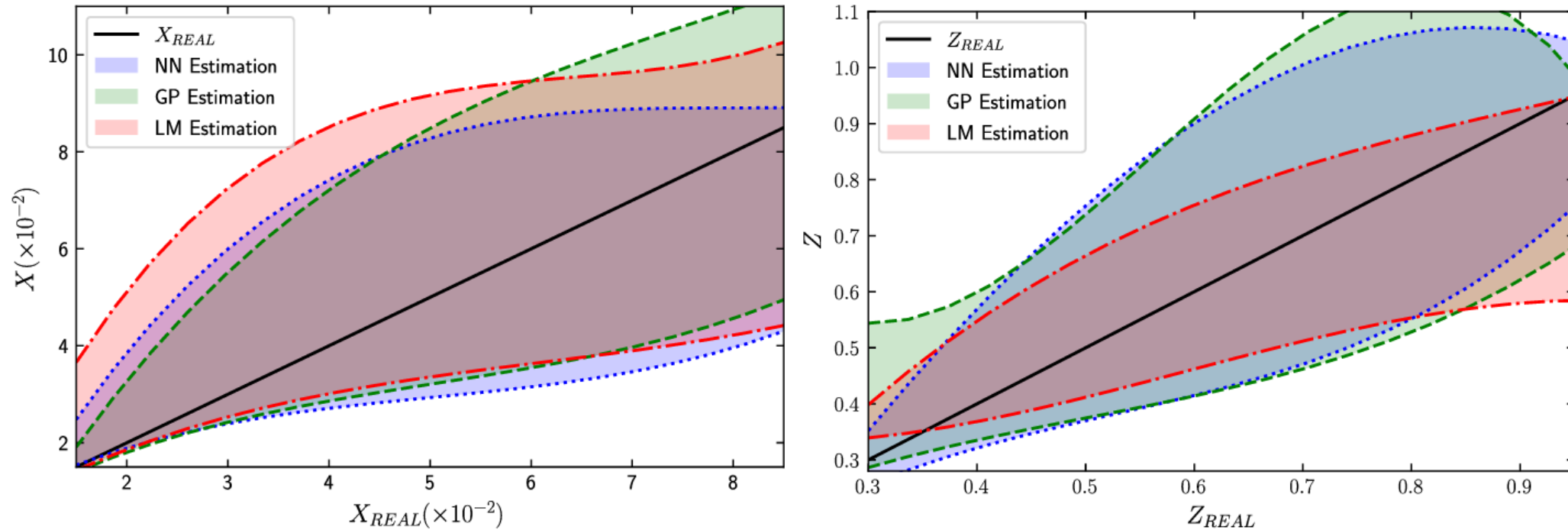
Scale-dependent mappings (reconstructed variables)

- Then, we can provide error estimation!

$$X_{\text{REC}}(p_j) = \overline{X(p_j)} \pm \frac{\max(X(p_j)) - \min(X(p_j))}{2} \equiv \overline{X(p_j)} \pm \Delta X(p_j)$$

Avg. error estimated over the complete dataset!

## How to estimate errors when reconstructing X and Z?



### Error bands

- We evaluate three reconstruction functions over the complete training-set
- Correlation plots are obtained, showing **smaller error for low x/z values**

### Global error

- With our method, **avg. error is 7% and 5% for  $x_1$  and  $z$ , respectively!**





## Imposing constraints on FFs through cross-section ratios

We can rewrite the hadronic cross-section for this process as:

$$\begin{aligned} \frac{d\sigma^{h_i}}{dz} &= \sum_{a_3} d_{a_3}^{h_i}(z) \times \left[ \sum_{a_1, a_2} \int dx_1 dx_2 f_{a_1}^{H_1}(x_1) f_{a_2}^{H_2}(x_2) d\hat{\sigma}_{a_1 a_2 \rightarrow a_3 \gamma} \right] \\ &= \sum_{a_3} d_{a_3}^{h_i}(z) \times g_{a_3}(z) \end{aligned}$$

The function  $g$  is independent of the FS-hadron!

## Kinematics restrictions

- We impose  $|\eta| < 0.5$  to (mainly) retain those events that are closer to the Born-level kinematic.
- We notice that **qg-channel** is  $\sim 10$  times larger than the others (mainly due to PDFs).
- As a consequence of EM-coupling, the U-channels are dominant w.r.t. D-channels. This leads to:

$$|\mathcal{M}_{ug \rightarrow u\gamma}|^2 = 4 |\mathcal{M}_{dg \rightarrow d\gamma}|^2 \quad \text{u-quark dominant w.r.t. c, t quarks (PDF effect)}$$

**Goal:** Relate cross-section and FF ratios, for different hadrons in final state:

$$R^{K/\pi}(d\sigma) = \frac{d\sigma^K/dz_{\text{REC}}}{d\sigma^\pi/dz_{\text{REC}}} \approx \frac{d_u^K(z_{\text{REC}})}{d_u^\pi(z_{\text{REC}})} = R^{K/\pi}(d_u)$$

where  $Z_{\text{REC}} = \frac{p_T^\pi}{p_T^\gamma}$



## Cross-section vs FFs ratios

We present a comparison between the FFs and the cross-section ratios for positive hadron production. In concrete, we restrict to pion and kaon production (higher cross-section).

### Two different approaches:

1) Using the reference energy scale (upper plot):

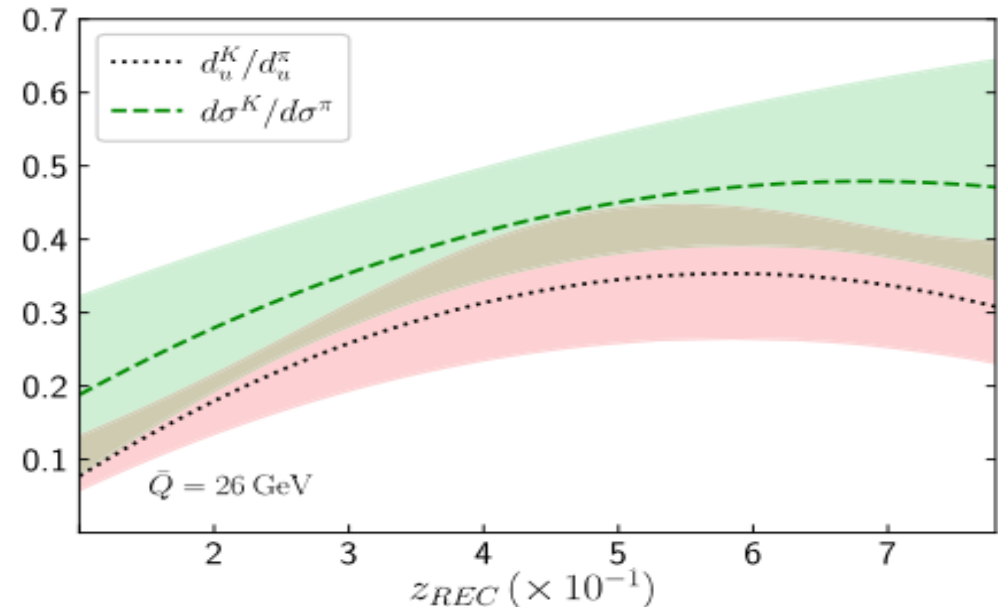
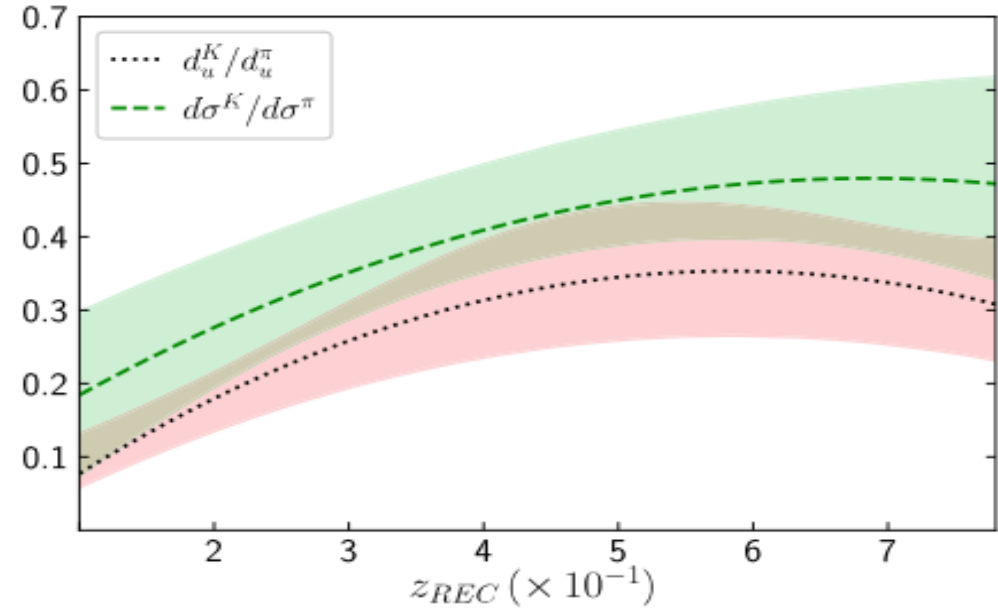
$$\mu = \frac{p_T^\gamma + p_T^h}{2}$$

2) Fixing the reference energy scale (lower plot):

$$\mu = \bar{Q} = 26 \text{ GeV}$$

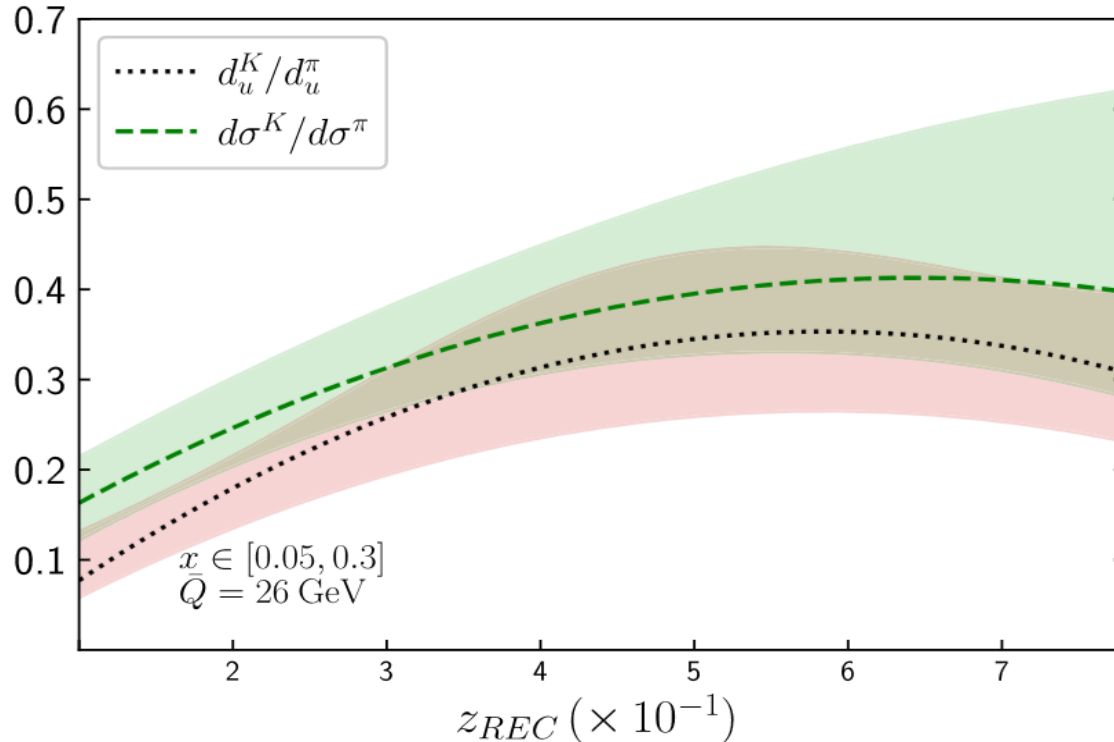
**Motivation:** Reduce the impact of the scale dependence of FFs (improves the validity of the approximation)

$$R^{K/\pi}(d\sigma) = \frac{d\sigma^K/dz_{REC}}{d\sigma^\pi/dz_{REC}} \approx \frac{d_u^K(z_{REC})}{d_u^\pi(z_{REC})} = R^{K/\pi}(d_u)$$





## Improved analysis with physically motivated cuts



- We notice that  $\frac{d\sigma^K/dz_{REC}}{d\sigma^\pi/dz_{REC}}$  and  $\frac{d_u^K(z_{REC})}{d_u^\pi(z_{REC})}$  are much closer than in the previous scenarios, **particularly in the range  $z \in (0.35, 0.65)$ .**
- More stringent constraints can be imposed from the ratio of the cross sections, *paving the road for a more precise determination of heavy-meson FFs from experimental data.*

1) To remove the scale-induced effects (improve the approximation), we fix:

$$\bar{Q} = \frac{\sum_i \mu(p_T^\gamma, p_T^h)_i (\sigma_{\text{BIN}})_i}{\sigma_{\text{TOTAL}}}$$

2) To enhance even more the weight of u-started processes, we look into the region where u-PDF dominates and impose the cut:

$$0.03 \leq \{(x_1)_{\text{REC}}, (x_2)_{\text{REC}}\} \leq 0.5$$

with

$$X_{1,\text{REC}} = \frac{p_T^\gamma \exp(\eta^\pi) + p_T^\gamma \exp(\eta^\gamma)}{\sqrt{S_{CM}}}$$

$$X_{2,\text{REC}} = \frac{p_T^\gamma \exp(-\eta^\pi) + p_T^\gamma \exp(-\eta^\gamma)}{\sqrt{S_{CM}}}$$



- *Photon-hadron production is an interesting process to access to the parton-level kinematics (presence of prompt-photon in final state)*
- **Reconstruction of partonic momentum fractions:** *Validity of analytical approximations confirmed against machine-learning analysis.*
- **Constraining FFs:**
  - Use approximations to relate cross-section and FF ratios for different hadrons.
  - Consider  $z_{\text{REC}}$  spectrum and impose cuts in  $x_{\text{REC}}$ .
- **Outlook:** *Use **ML-assisted optimization** of kinematical cuts (as well as **NN for parton momentum fraction approximations**) to better constrain FFs.*

THANKS!

