A Bayesian analysis with Machine Learning of EFT Operators in Direct Dark Matter Detection

Instituto de Física Teórica UAM-CSIC

Universidad Autónoma de Madrid

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Santander

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In collaboration with David Cerdeño and Martín de los Rios

Outline

DM-nucleon interaction with NR-EFT

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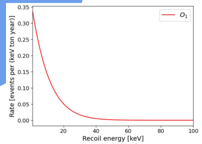
XENONnT 20ty

DM-nucleon interaction with NR-EFT

→

DM differential rate for a DD experiment



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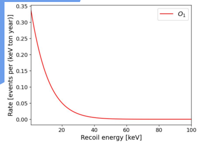

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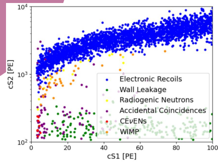
Simulate the signal reported by the experiment



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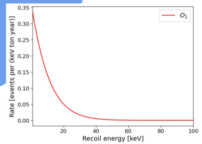

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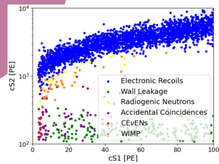
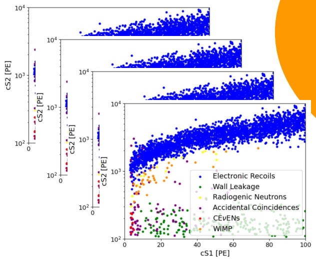
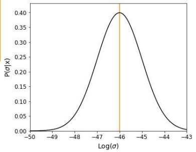




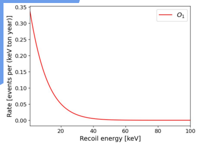
Data analysis with ML to obtain posteriors

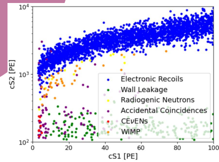


Outline

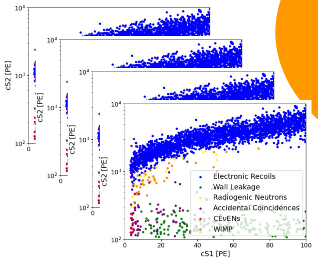
XENONnT 20ty

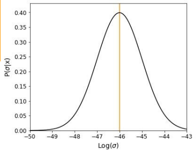
DM-nucleon interaction with NR-EFT → DM differential rate for a DD experiment → Simulate the signal reported by the experiment

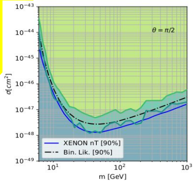




→ Data analysis with ML to obtain posteriors → Parameter space that can be reconstructed







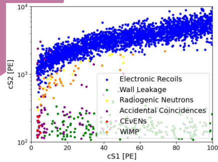
Outline

XENONnT 20ty

DM-nucleon interaction with NR-EFT

DM diff

Simulate the signal reported by the experiment



preliminary results

Data ana
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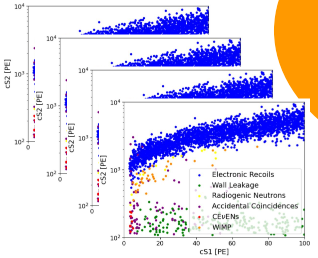
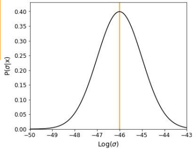
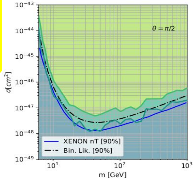




Parameter space that can be reconstructed



Non-relativistic effective field theory (NR-EFT)

DM-nucleon non-relativistic effective field theory (NR-EFT)

For DM particles with spin up to ½ , the effective DM-nucleon scattering interaction Lagrangian

$$\mathcal{L}_{\rm EFT} = \sum_{\tau} \sum_{i} c_i^{\tau} \mathcal{O}_i \bar{\chi} \chi \bar{\tau} \tau$$

i=14 possible interactions

$$c_i^0 \mathbb{1}_{2\times 2} + c_i^1 \tau_3$$

isospin basis
c^0: isoscalar
c^1: isovector

$$c_i^0 = \frac{1}{2}(c_i^p + c_i^n)$$

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nucleon basis
c^p: proton
c^n: neutron

O1: spin-independent (SI)
O4: spin-dependent (SD)

usually shown assuming isoscalar interactions

$c^p = c^n$ $c^0 = 1$ and $c^1 = 0$

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Change to polar coordinates:

$$c_i^0 = \frac{1}{2}(c_i^p + c_i^n) \ = A_i \sin(\theta_i)$$

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Natural choice for the EFT parameter space because the interaction cross section:

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For SI (O1)

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For each operator

2 parameters

amplitude (cross-section)

phase

also DM mass

(σ_i, θ_i, m_{DM})

Data sample generation

DM Differential rate

From NR-EFT operators to differential rate with WimPyDD

Inputs:

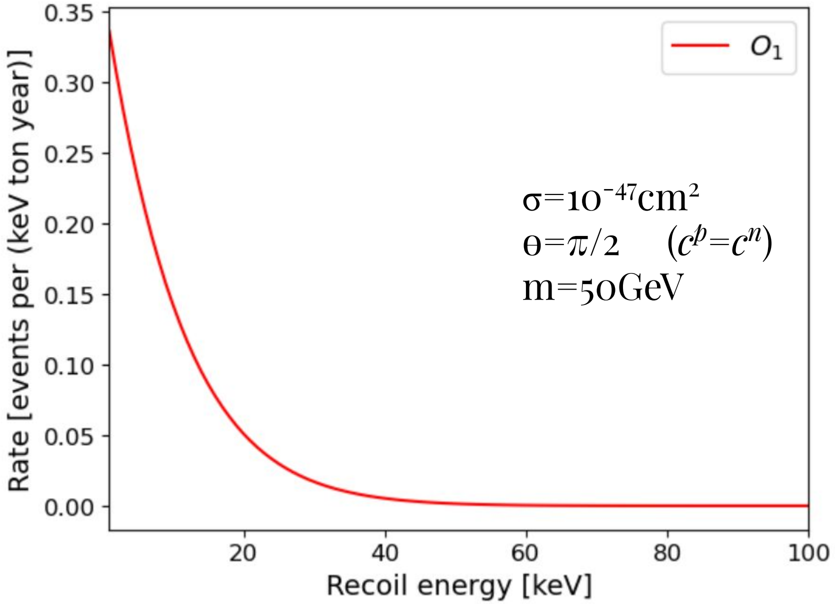
- Operator

- Parameters $\longrightarrow$ amplitude (cross-section)
  phase
  DM mass

- DM halo model

- DD experiment (XENONnT)

Output:



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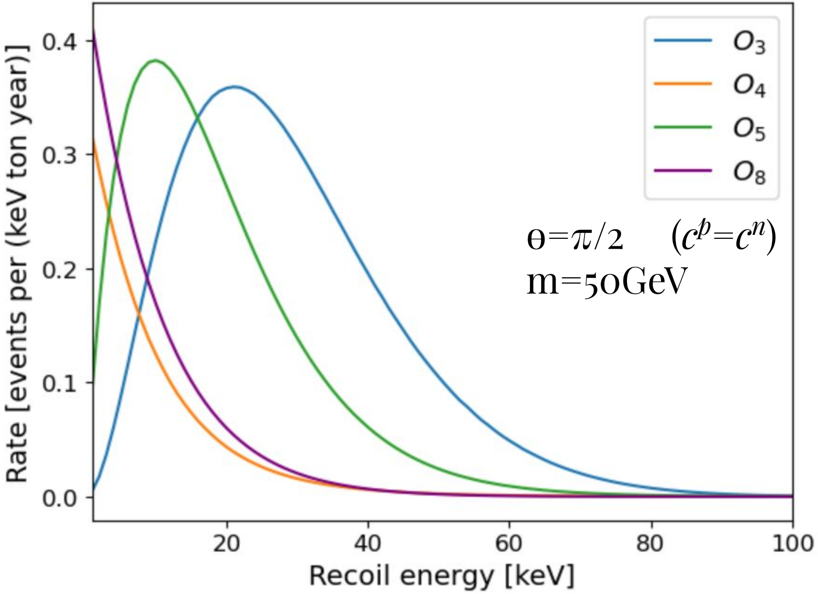
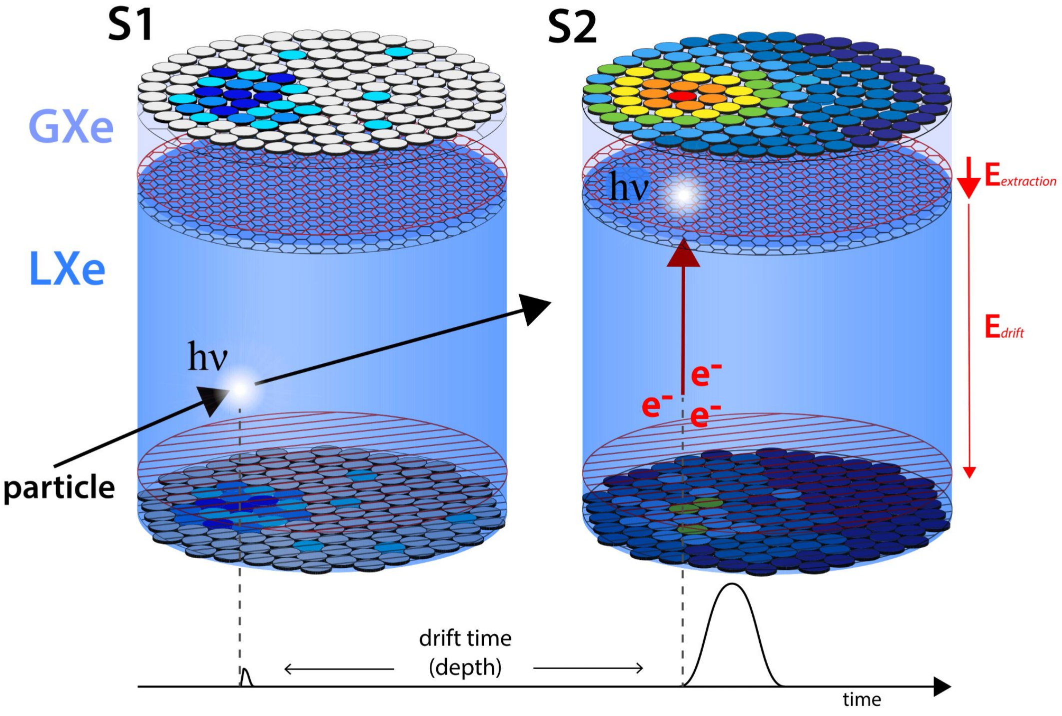
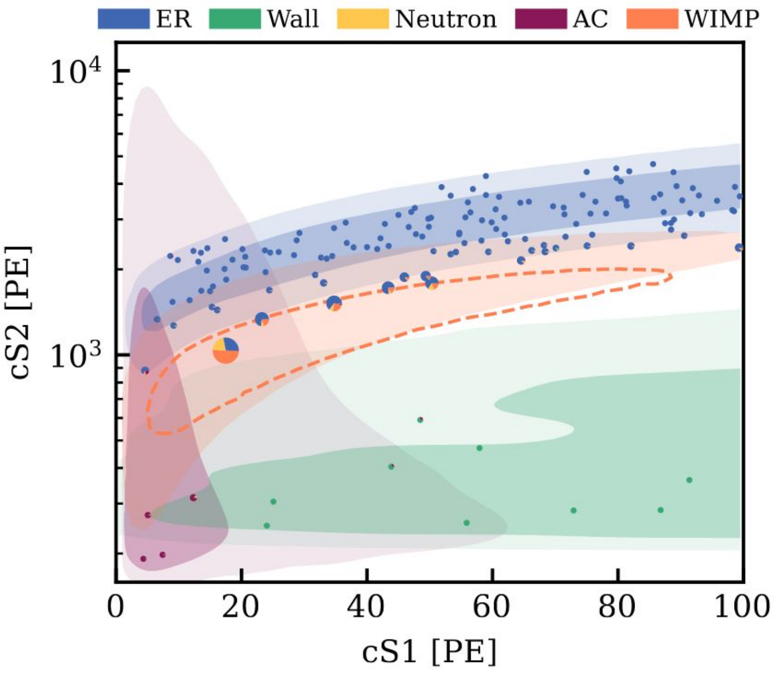


DM signal

XENONnT



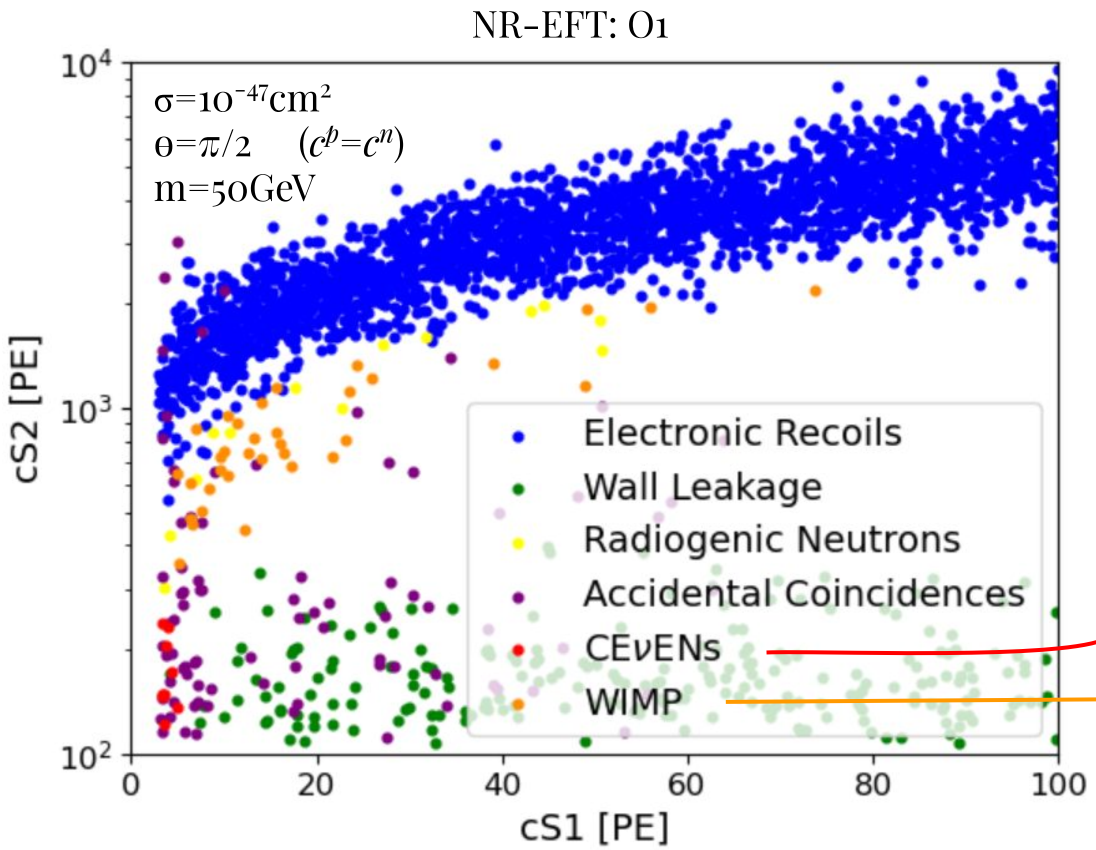


Universe **2021**, *7*(8), 313

Phys. Rev. Lett. 131, 041003 (2023)

DM signal

XENONnT 20ty



XENONnT simulator

We specify background and signal characteristics

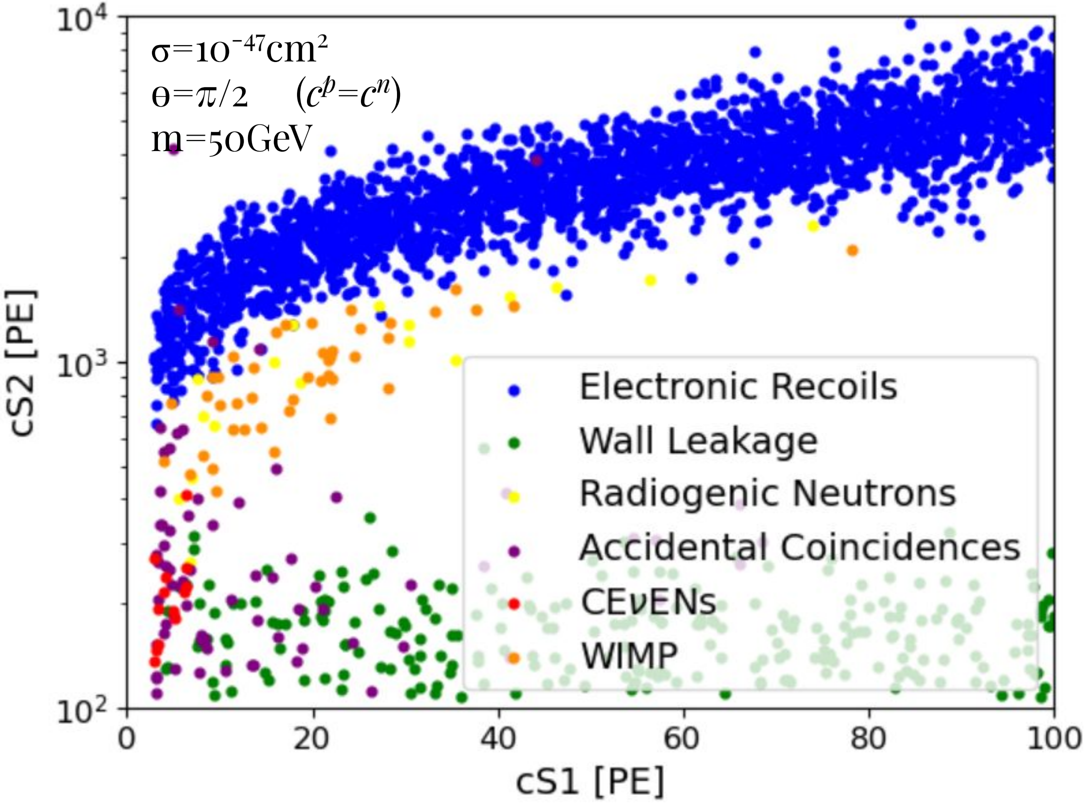
differential rate compute with SnuDD

differential rate compute with WimPyDD for a particular operator, amplitud, phase and DM mass.

DM signal

XENONnT 20ty

NR-EFT: O1



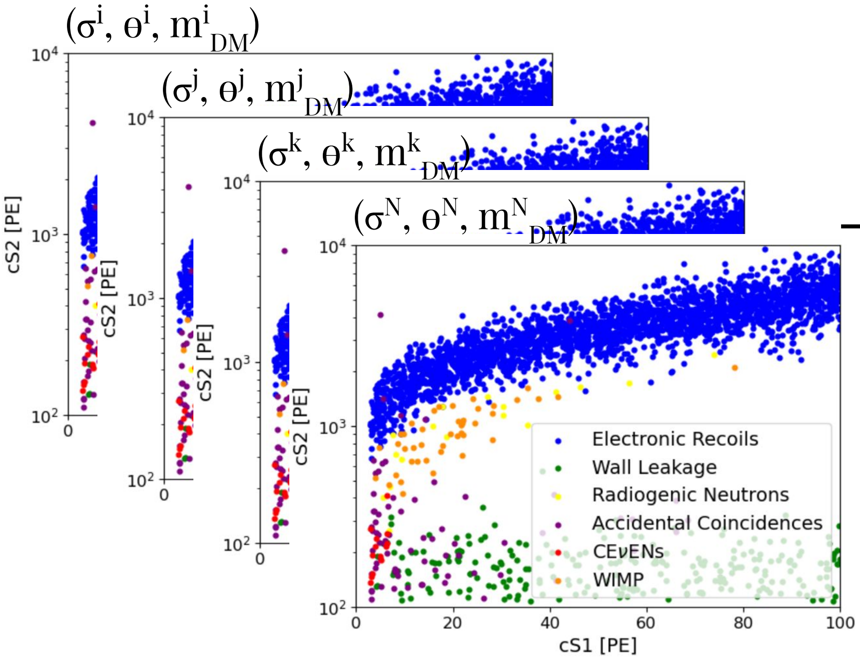
We generate a 10k pseudo experiments per operator varying σ, θ, and m_{DM}

	name	events
0	er	2459
1	radiogenics	17
2	ac	71
3	wall	246
4	WIMP	43
5	CEVNS-SM	13

Analysis with SWYFT

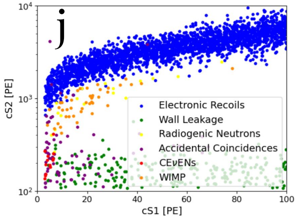
Data analysis to obtain posteriors

SWYFT → Sampling-based inference tool that estimates likelihood to evidence ratio with ML algorithms to obtain marginal and joint posteriors



Matching (parameter, data) → **label 1**

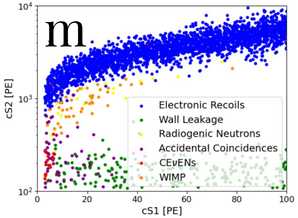
$\left((\sigma^j, \theta^j, m^j_{DM}) \right.$,



$\left. \right)$

Scrambled (parameter, data) → **label 0**

$\left((\sigma^k, \theta^k, m^k_{DM}) \right.$,



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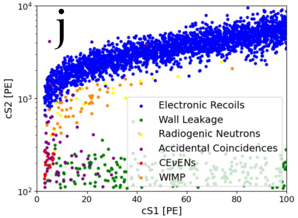
$\left((\sigma^j, \theta^j, m^j_{DM}) , \right.$  $\left. \right)$

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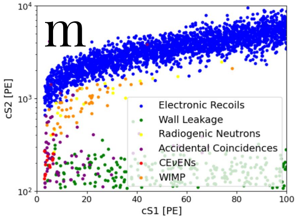
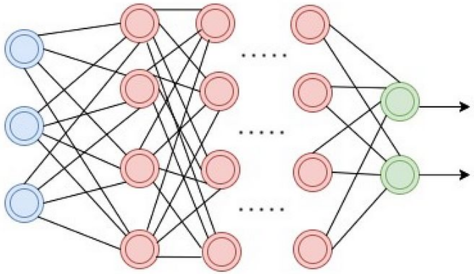
$\left((\sigma^k, \theta^k, m^k_{DM}) , \right.$  $\left. \right)$

Binary classifier (DNN, CNN, ...)



Estimates the density ratio

(likelihood ratio trick)

$$r(\mathbf{x};\mathbf{z}) \equiv \frac{p(\mathbf{x}\,|\,\mathbf{z})}{p(\mathbf{x})} = \frac{p(\mathbf{z}\,|\,\mathbf{x})}{p(\mathbf{z})} = \frac{p(\mathbf{x},\mathbf{z})}{p(\mathbf{x})p(\mathbf{z})}$$

Data analysis to obtain posteriors

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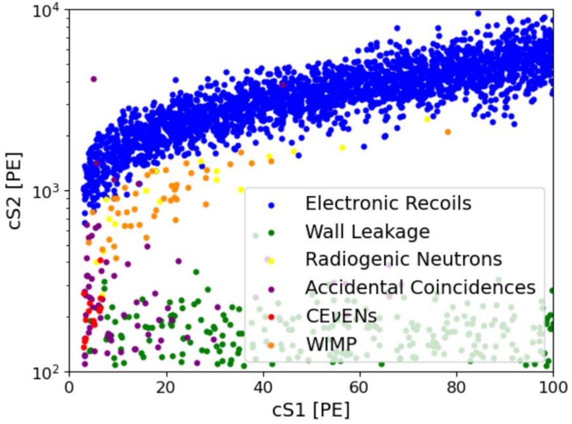
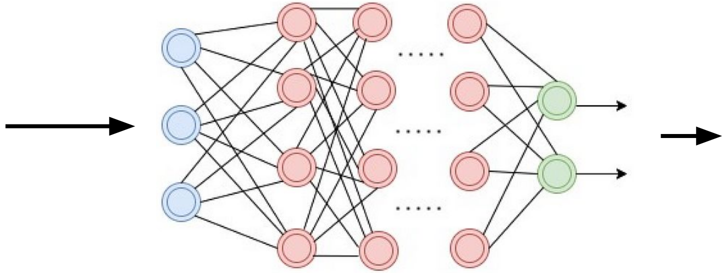
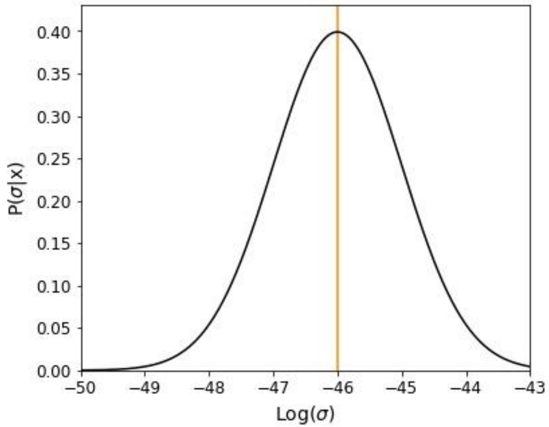
New data sample x^{new}



Trained binary classifier



Posterior $P(\sigma|x^{new})$



For another data sample → we do **not** need to train everything again, use the same classifier

Results

Posteriors

Once we trained SWYFT we can compute the posterior for any new pseudo experiment

For example $P(\sigma|x)$

x: a data generated with

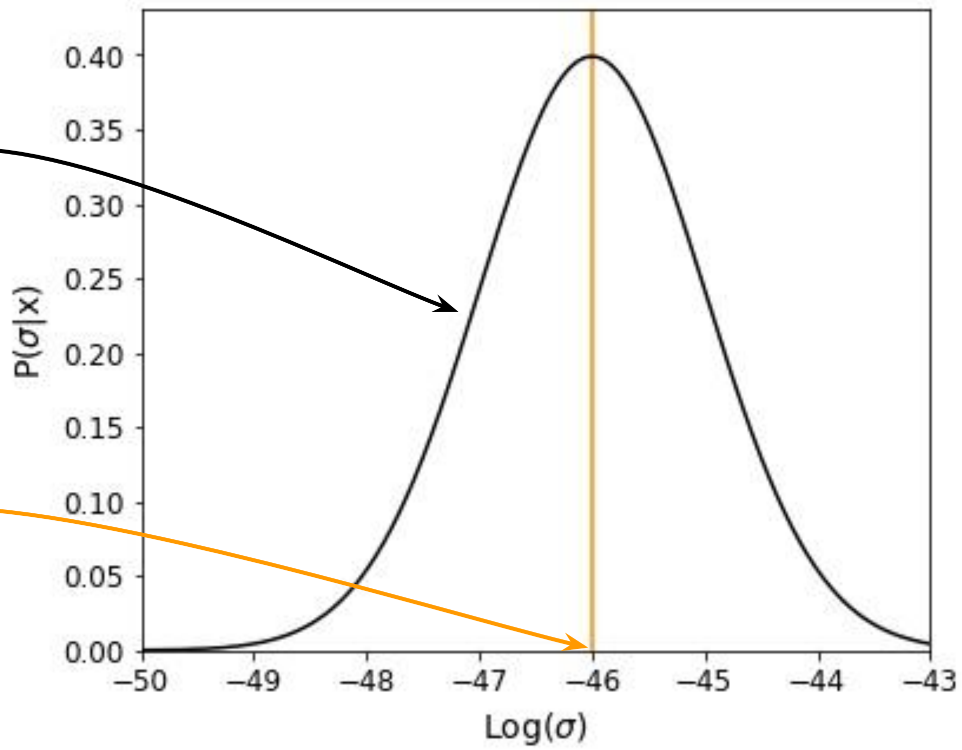
O1 (SI)

m_{DM}=85GeV

$\theta=\pi/2$

$\sigma=10^{-46}cm^2$

this is a gaussian as an example, not the actual posterior!



Reconstruction of parameters

Once we trained SWYFT we can compute the posterior for any new pseudo experiment

We define a σ_{th} threshold:

$\sigma_{th}=10^{-49}cm^2 \longrightarrow$ NO SIGNAL!

Then, we can ***reconstruct*** σ if:

$$\int_{\sigma_{th}}^{inf} P(\sigma|x) > 0.90$$

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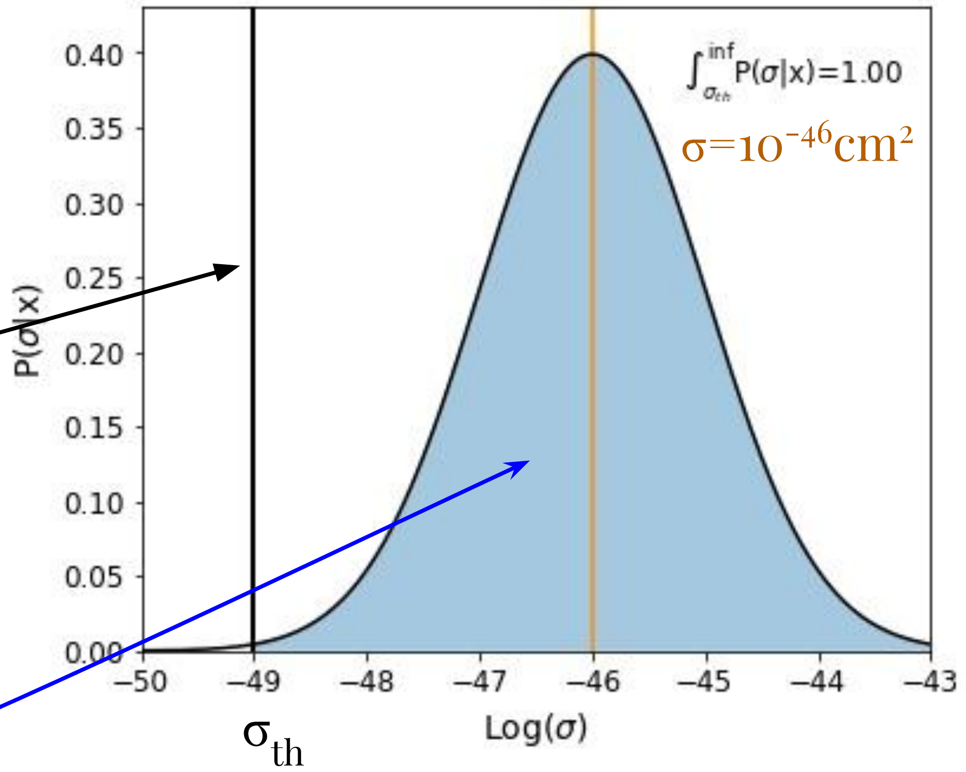

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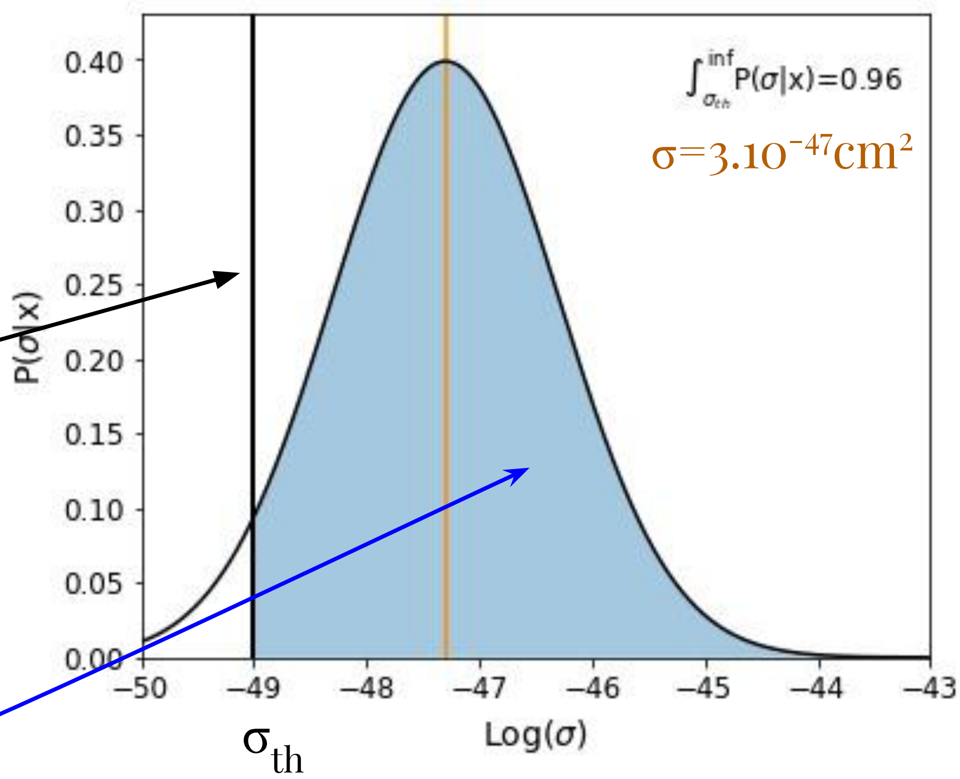

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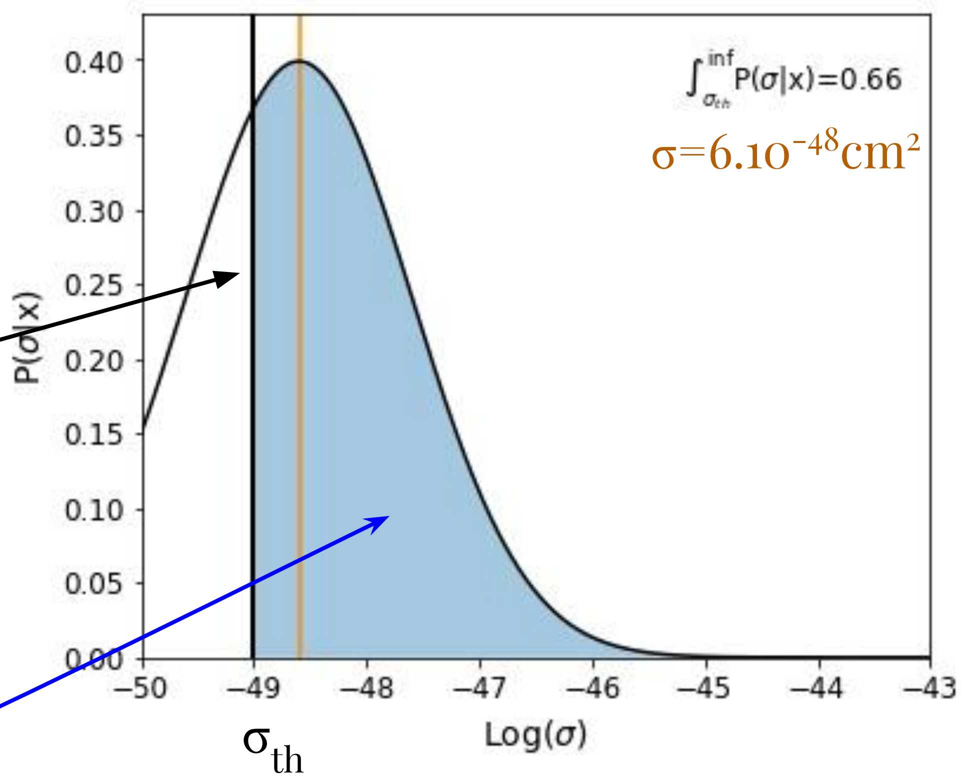
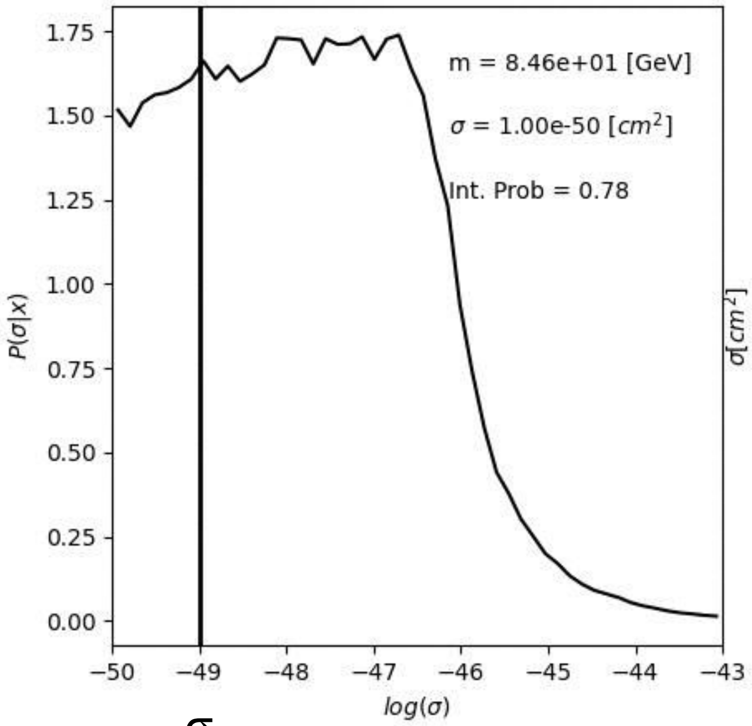
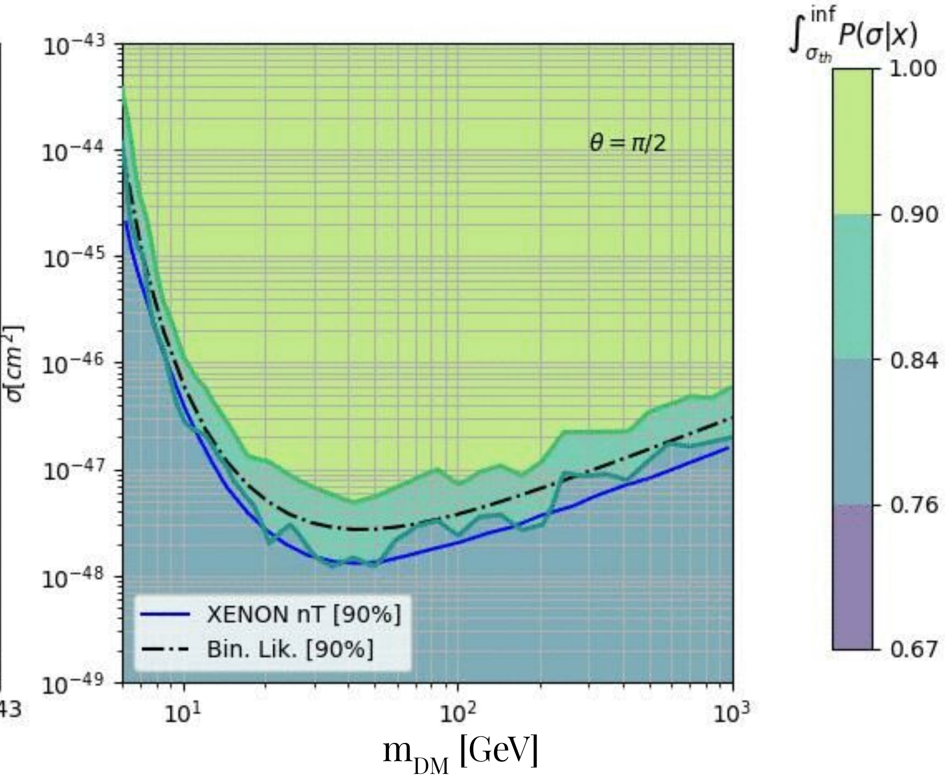


Results

Testing with:
m_{DM}=84.6GeV ⟶ fixed
θ=π/2 ⟶ fixed



σ_{th}



Results

These are all the posteriors for

m_{DM}=84.6GeV $\longrightarrow$ fixed

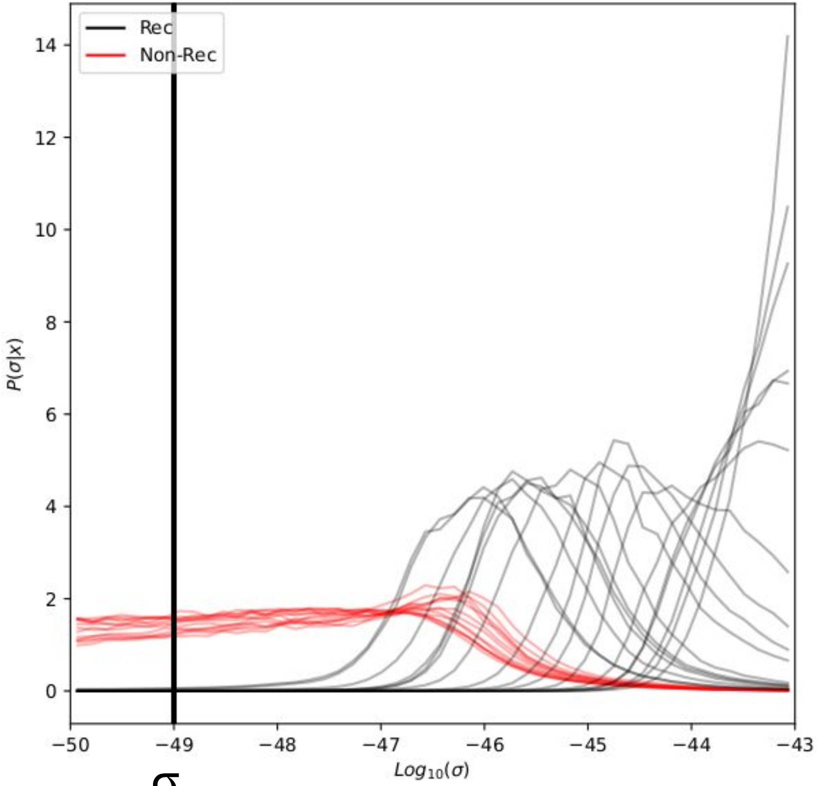
$\theta=\pi/2 \longrightarrow$ fixed

red $\longrightarrow$ σ not reconstructed

~no signal,

~similar posteriors

black $\longrightarrow$ σ reconstructed

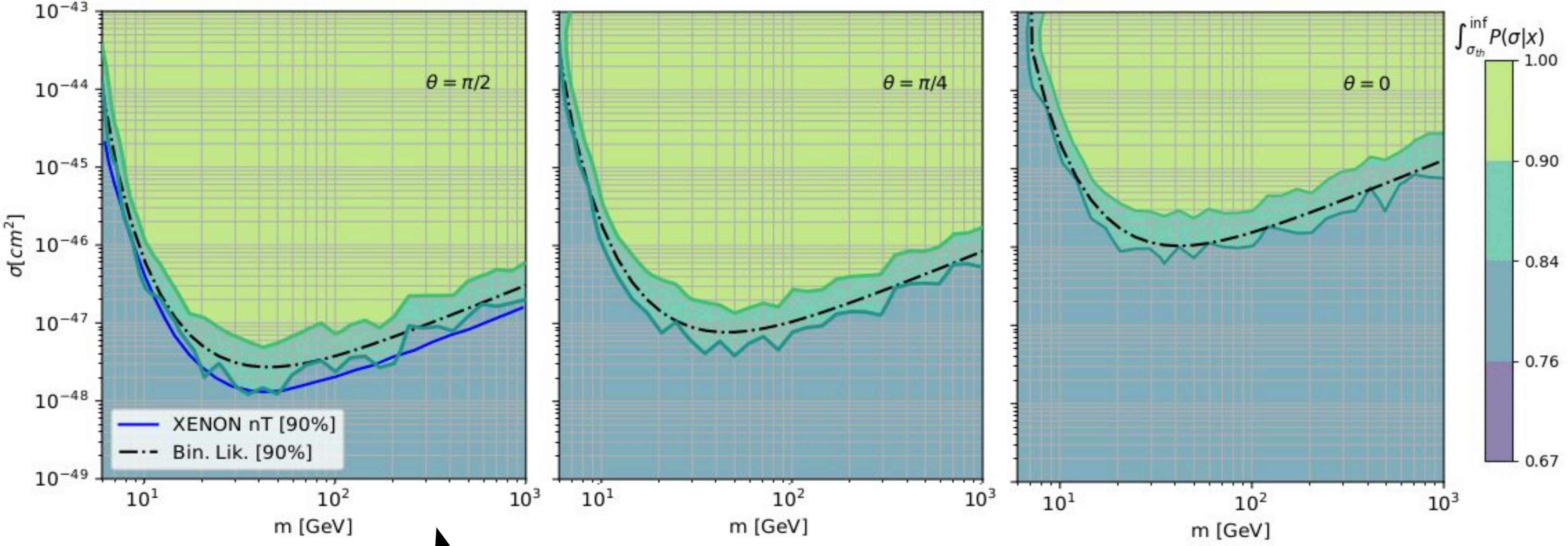


σ_{th}

Results

Data:
the entire cS1 vs cS2 plane

O1 operator
XENONnT

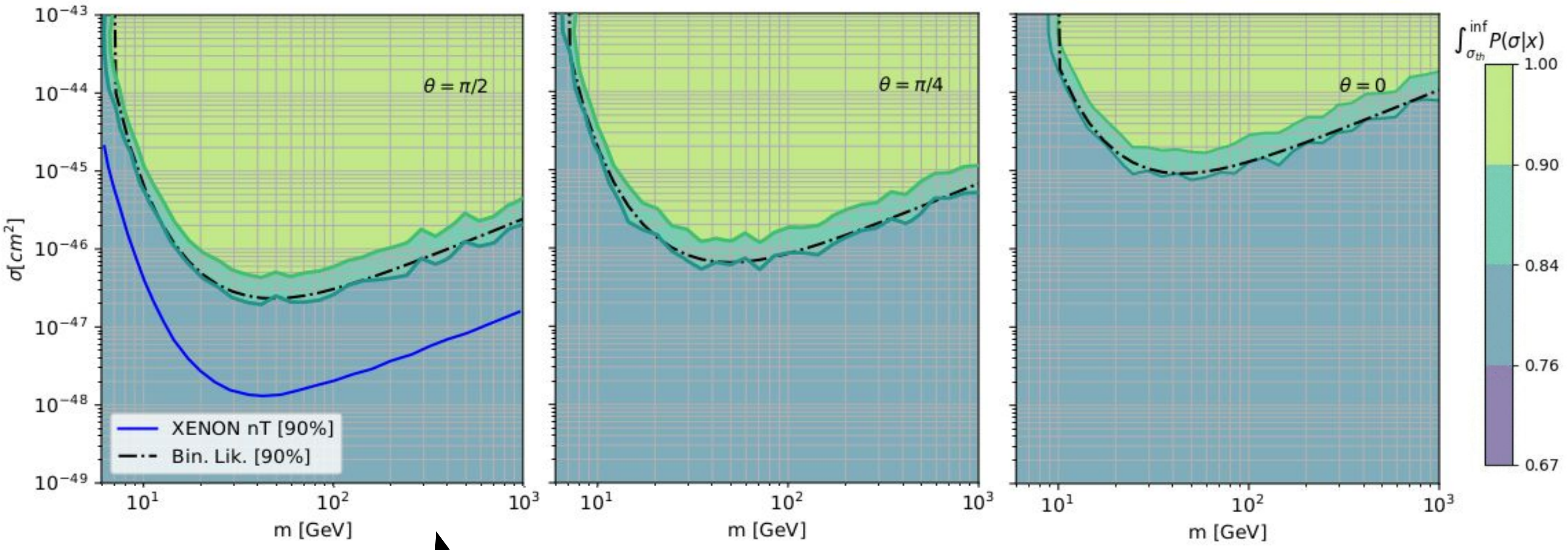


This is the usually shown
SI parameter space

Results

Data: the total number of events

O1 operator XENONnT



This is the usually shown **SI** parameter space

Conclusions

preliminary results

- A Bayesian analysis to explore the reach of direct detection experiments that can be applied to any DM model (translate it into NR-EFT)

  - O1 (SI) presented here as an example,
  - SWYFT, a data driven tool, allows a really fast estimation of posteriors,
  - we computed the parameter space that can be reconstructed,
  - we compared total number of events vs the full cS1,cS2 space.

- **Next:**
  Apply to other NR-EFT operators → combine operators
  Different Direct Detection experiments → combine experiments


Thank you!

Back-up

DM-nucleon non-relativistic effective field theory (NR-EFT)

For DM particles with spin up to ½ , the effective DM-nucleon scattering interaction Lagrangian

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i=14 possible interactions

$$i\frac{\vec{q}}{m_N}, \quad \vec{v}^{\perp}, \quad \vec{S}_{\chi}, \quad \vec{S}_N.$$

$$\vec{v} \equiv \vec{v}_{\chi,\text{in}} - \vec{v}_{N,\text{in}}$$

$$\vec{v}^{\perp} = \vec{v} + \frac{\vec{q}}{2\mu_N}$$

momentum transfer, spin operators, relative velocity

$\mathcal{O}_1 = 1_{\chi} 1_N$	$\mathcal{O}_9 = i\vec{S}_{\chi} \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N})$
$\mathcal{O}_3 = i\vec{S}_N \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^{\perp})$	$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N}$
$\mathcal{O}_4 = \vec{S}_{\chi} \cdot \vec{S}_N$	$\mathcal{O}_{11} = i\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_N}$
$\mathcal{O}_5 = i\vec{S}_{\chi} \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^{\perp})$	$\mathcal{O}_{12} = \vec{S}_{\chi} \cdot (\vec{S}_N \times \vec{v}^{\perp})$
$\mathcal{O}_6 = (\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$	$\mathcal{O}_{13} = i(\vec{S}_{\chi} \cdot \vec{v}^{\perp})(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$
$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^{\perp}$	$\mathcal{O}_{14} = i(\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \vec{v}^{\perp})$
$\mathcal{O}_8 = \vec{S}_{\chi} \cdot \vec{v}^{\perp}$	$\mathcal{O}_{15} = -(\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_N})((\vec{S}_N \times \vec{v}^{\perp}) \cdot \frac{\vec{q}}{m_N})$

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DM-nucleon non-relativistic effective field theory (NR-EFT)

Contact interaction between a spin ½ DM and nucleon

$$\mathcal{L}^{\rm SI}_{\rm int}(\vec{x}) = c_1 \; \bar{\Psi}_\chi(\vec{x})\Psi_\chi(\vec{x}) \; \bar{\Psi}_N(\vec{x})\Psi_N(\vec{x})$$

$$U_\chi(p) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} \xi_\chi \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m_\chi}\xi_\chi \end{pmatrix} \sim \begin{pmatrix} \xi_\chi \\ \frac{\vec{\sigma}\cdot\vec{p}}{2m_\chi}\xi_\chi \end{pmatrix}$$

at low momenta.

Idem for the nucleon spinor

ξ Pauli spinors

at leading order in p/m $\quad c_1 \; 1_\chi 1_N \; \equiv \; c_1 \; \mathcal{O}_1$

DM-nucleon non-relativistic effective field theory (NR-EFT)

Another interaction

$$\mathcal{L}_{\rm int}^{\rm SD} = c_4\ \bar{\chi}\gamma^\mu\gamma^5\chi\bar{N}\gamma_\mu\gamma^5 N$$

the dominant contribution in the non-relativistic limit comes from the spatial indices

$$\bar{\chi}\gamma^i\gamma^5\chi \sim \xi_\chi^\dagger\sigma^i\xi_\chi$$

Since $\hat{S}^i=\sigma^i/2$

$$-\,4c_4\ \vec{S}_\chi\cdot\vec{S}_N\ \equiv\ -\,4c_4\ \mathcal{O}_4$$

Data analysis to obtain posteriors

SWYFT → Sampling-based inference tool that estimates likelihood to evidence ratio with ML algorithms to obtain marginal and joint posteriors

	MCMC	**SWYFT**
Forward Model	x=f(parameters)	x=f(parameters)
Likelihood	L(x, f(parameters))	Data Driven
Samples	All parameters space > # samples	Only Interesting parameters
Amortization	NO	YES

Motivation

Bayes' Rule: determine a probability distribution over model parameters **θ** given an observation **x**

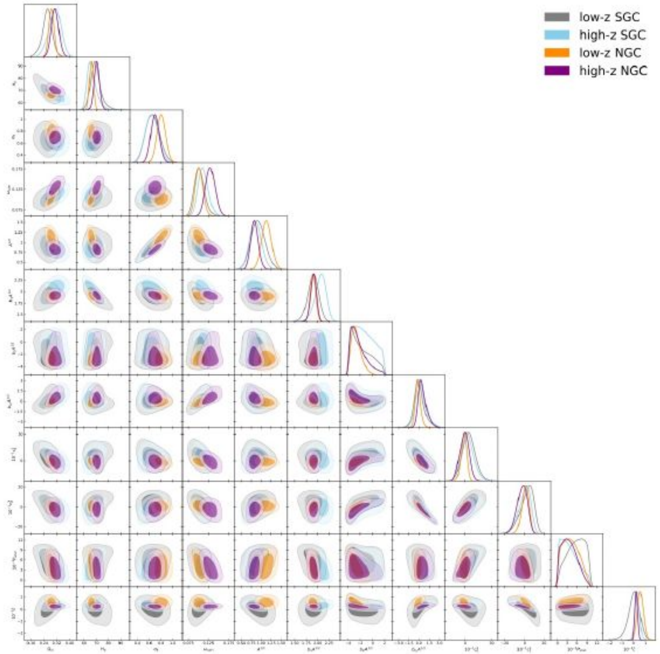
Likelihood of **x** given **θ**

$$p(\boldsymbol{\theta} \mid \boldsymbol{x}) = \frac{p(\boldsymbol{x} \mid \boldsymbol{\theta})}{p(\boldsymbol{x})} p(\boldsymbol{\theta})$$

Posterior

Evidence of the data

Prior



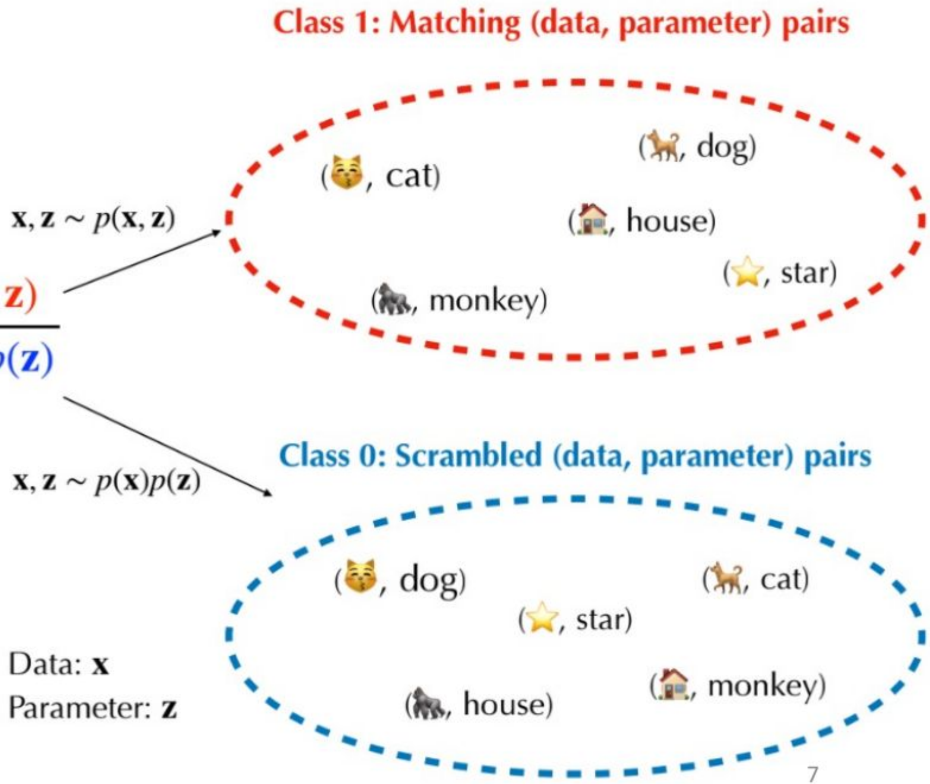
BOSS, Ivanov+ 1909.05277

Samples tipically generated with *Markov Chain Monte Carlo* (MCMC) or *Nested sampling*

Neural Ratio Estimation (NRE)

Approximate density ratios.

$$r(\mathbf{x};\mathbf{z}) \equiv \frac{p(\mathbf{x}\,|\,\mathbf{z})}{p(\mathbf{x})} = \frac{p(\mathbf{z}\,|\,\mathbf{x})}{p(\mathbf{z})} = \frac{p(\mathbf{x},\mathbf{z})}{p(\mathbf{x})p(\mathbf{z})}$$



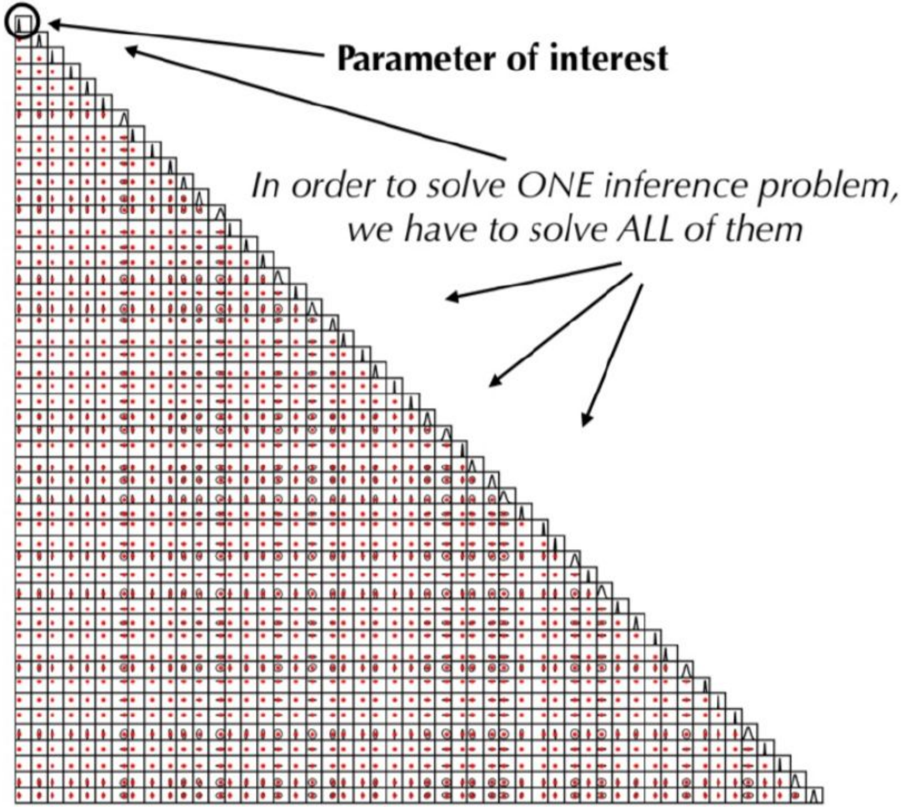
Strategy: *We estimate posteriors-to-prior ratio by training a binary classifier to discriminate between matching and scrambled (data, parameter) pairs.*

MARGINAL

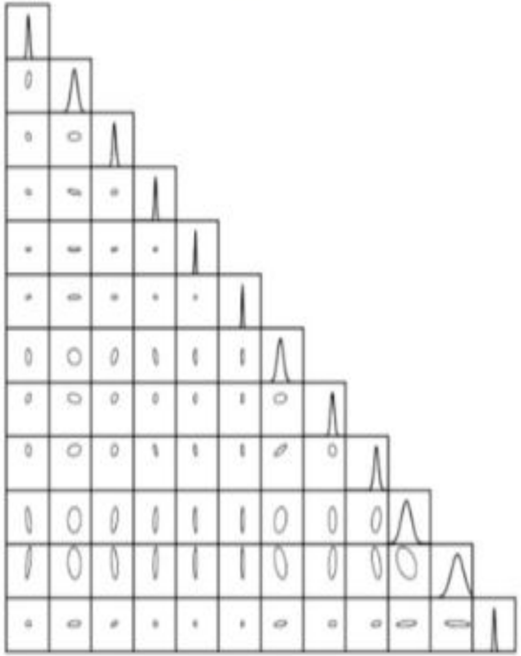
- MCMC or Nested sampling methods produce samples from the **posterior distribution**.

- Classical methods require sampling the **full joint posterior**, so that they are slow to converge.

- Novel approaches in the field of *simulation-based inference* (SBI) are starting to overcome these obstacles.

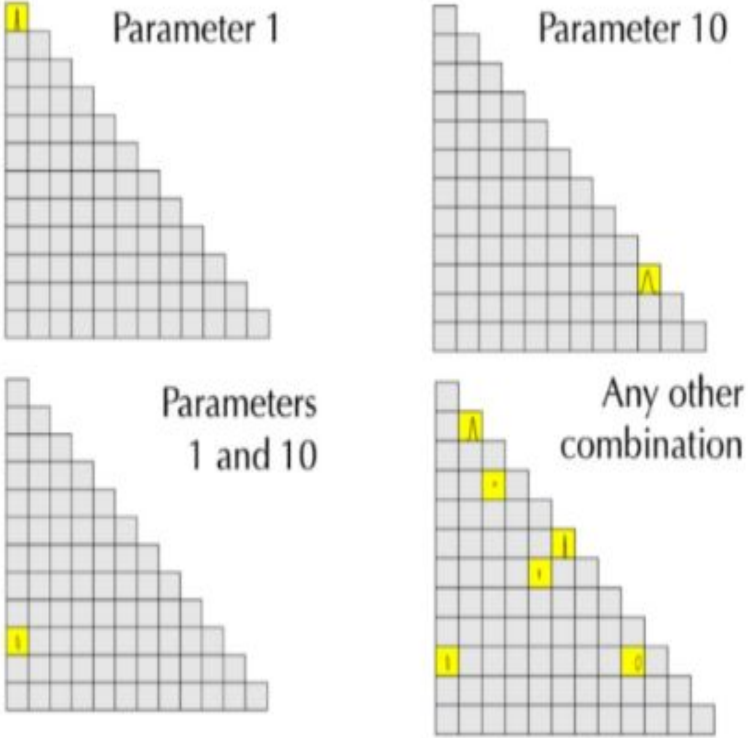


Instead of estimating all parameters…



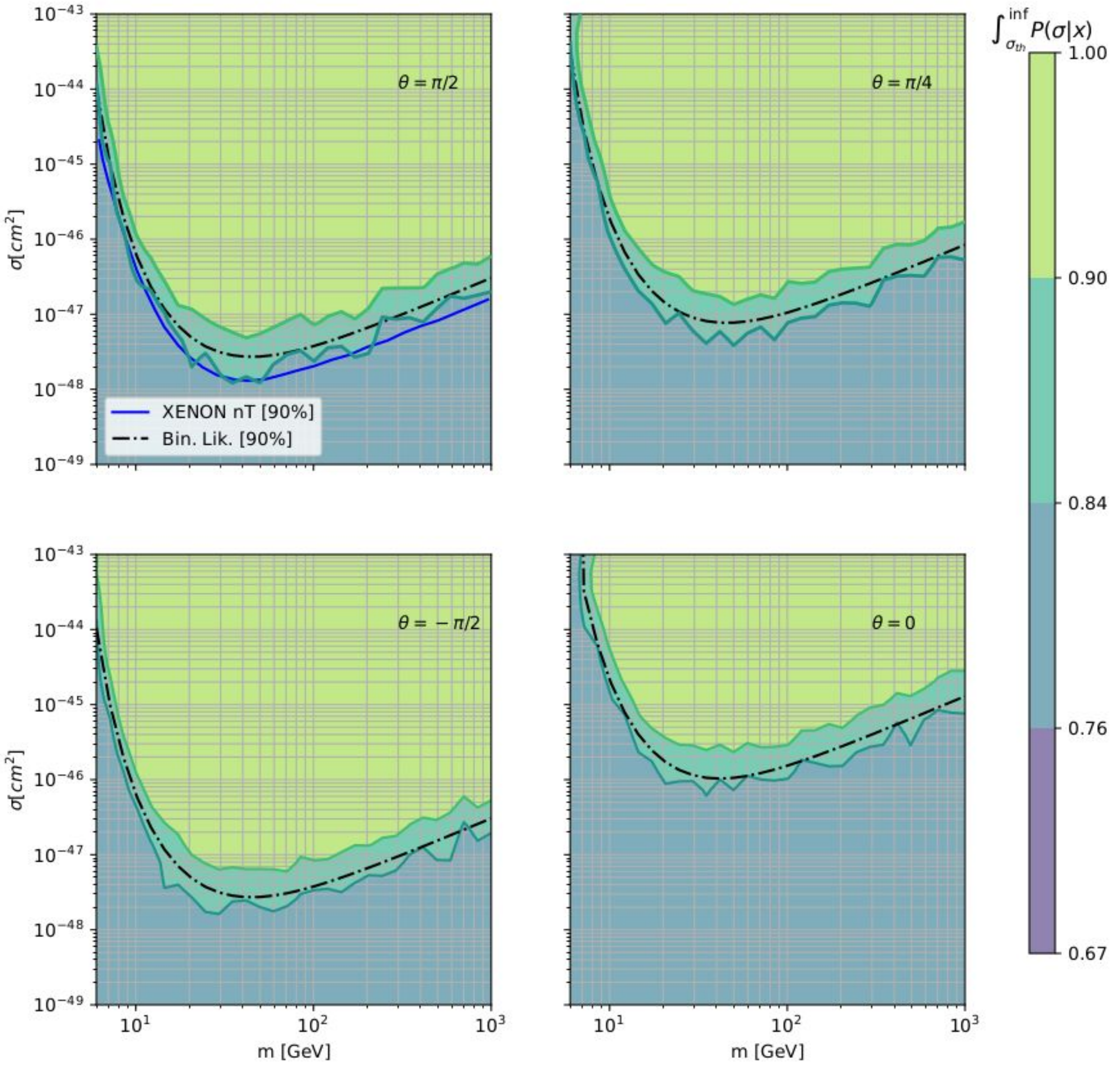
50 parameters ~ 100 Million simulations

…we can choose what we care about



Depending on which parameter is scrambled

Data:
the entire cS1 vs cS2 plane



Data: the total number of events