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# LHC Study of Third-Generation Scalar Leptoquarks with Machine-Learned Likelihoods

*In collaboration with E. Arganda, D. A. Díaz, R. M., A. D. Perez, A. Szykman  
2309.05407*



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XV CPAN DAYS

*October 2, 2023*

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# Outline

## Model and Final state

- Signal: Third generation scalar-leptoquarks
- Backgrounds and event selection criteria

## Set up: From Machine Learning to significances

- ML classifiers and binned vs. unbinned statistical tests

## Results @ 13 TeV LHC (full Run 2 dataset)

- 95% C.L. expected exclusion limits for  $m$  ( $LQ^{u/d}$ ) vs.  $BR(LQ^{u/d} \rightarrow q\ell)$

## Estimation of the impact of systematics (NEW!)

## Prospects @ 14TeV LHC (NEW!)

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# Motivation:

- Third-generation scalar leptoquarks could provide an explanation to B-anomalies!
- The search of LQs at the LHC represents a very extensive program at the LHC, both in the ATLAS and CMS Collaborations.
- None of these experimental analysis, even with (binned) multivariate analysis, has been able to find significant deviations from the SM prediction.

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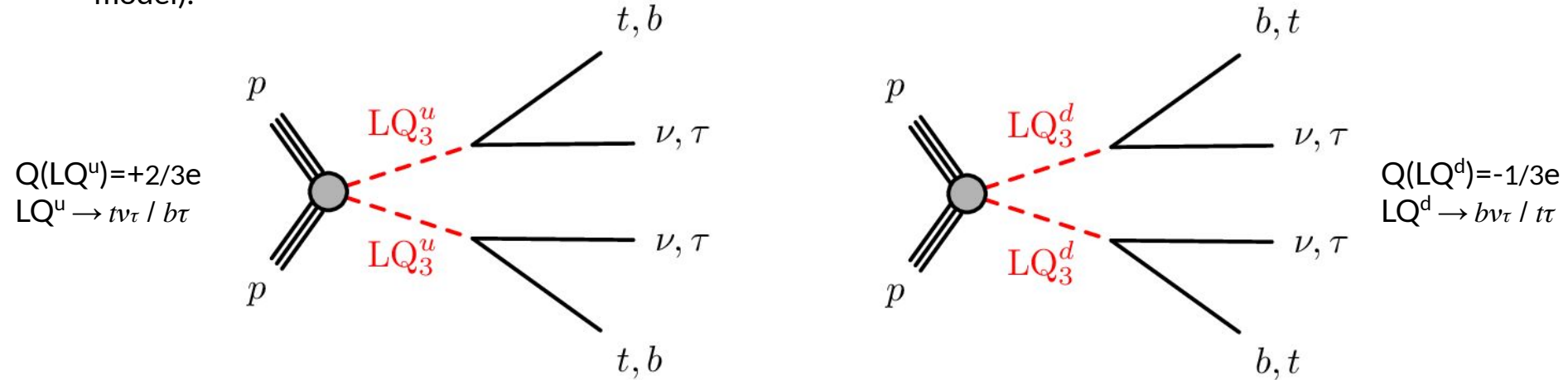
**A ML unbinned approach would potentially improve results?**

# Model and Final state

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# Signal: pair production of scalar-leptoquarks

Leptoquarks are motivated by many extensions of the SM (used to explain  $(g-2)_\mu$ , B-anomalies, ...). Focus only on decays into third-generation leptons and quarks (minimal Buchmüller–Rückl–Wyler model).

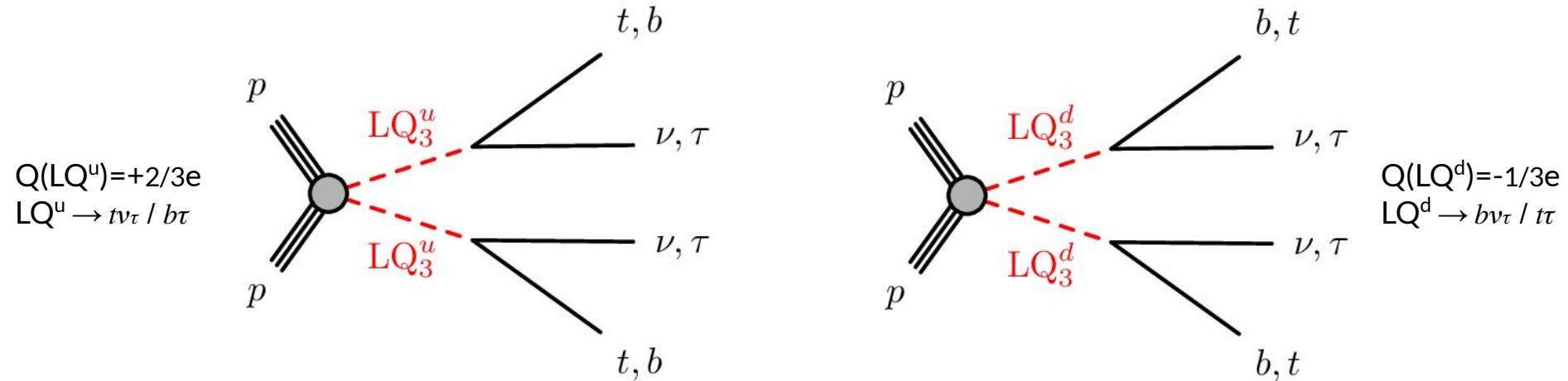


2 parameters:  $m(LQ^{u/d})$  leptoquark mass

$BR(LQ^{u/d} \rightarrow q\ell)$  the branching fraction into a quark and a charged lepton

$$BR(LQ^{u/d} \rightarrow q\nu) = 1 - BR(LQ^{u/d} \rightarrow q\ell)$$

# Signal: pair production of scalar-leptoquarks



For a  $BR(LQ^{u/d} \rightarrow q\ell) \sim 0.5$  most of the decays of the pair of third-generation leptoquarks yield a final state with one tau lepton, two  $b$ -jets and large MET from the tau neutrino.

# Event selection criteria

ATLAS analysis, Phys. Rev. D 104, (2021) 112005.

Single tau SR definition:

$$\begin{aligned} & 1 \tau_{\text{had}} (p_{\text{T}} > 10 \text{ GeV}, |\eta| < 2.5) \\ & N_{b\text{-jets}} \geq 2 (p_{\text{T}} > 20 \text{ GeV}, |\eta| < 2.5) \\ & E_{\text{T}}^{\text{miss}} > 250 \text{ GeV} \\ & \text{Light leptons veto } (e/\mu) \end{aligned}$$

ATLAS extra selection criteria:

$$\Sigma m_{\text{T}}(b_{1,2}) = m_{\text{T}}(b_1) + m_{\text{T}}(b_2) > 700 \text{ GeV}$$

$$E_{\text{T}}^{\text{miss}} > 280 \text{ GeV}$$

$$p_{\text{T}}(\tau_{\text{had}}) > 50 \text{ GeV} \quad (\text{multi-bin SR})$$

$$m_{\text{T}}(\tau_{\text{had}}) > 150 \text{ GeV} \quad (\text{multi-bin SR})$$

$$s_{\text{T}} = p_{\text{T}}(\tau_{\text{had}}) + p_{\text{T}}(j_1) + p_{\text{T}}(j_2) > 600 \text{ GeV} \quad (\text{multi-bin SR})$$

where  $m_{\text{T}}^2(A) = m_{\text{T}}^2(p_{\text{T}}^A, E_{\text{T}}^{\text{miss}}) = 2 p_{\text{T}}^A E_{\text{T}}^{\text{miss}} (1 - \cos \Delta\Phi(p_{\text{T}}^A, E_{\text{T}}^{\text{miss}}))$  is the stransverse mass



# Background: SM

ATLAS analysis, Phys. Rev. D 104, (2021) 112005.

	Single-tau SR (binned in $p_T(\tau)$ )					
	[50, 100] GeV		[100, 200] GeV		> 200 GeV	
Observed	8		6		2	
Total bkg.	10.1	$\pm 1.8$	5.1	$\pm 1.1$	2.05	$\pm 0.64$
$t\bar{t}$ (2 real $\tau$ )	—		—		—	
$t\bar{t}$ (1 real $\tau$ )	4.8	$\pm 1.2$	2.69	$\pm 0.88$	0.64	$\pm 0.29$
$t\bar{t}$ -fake	2.83	$\pm 0.87$	0.66	$\pm 0.17$	0.185	$\pm 0.072$
Single top	0.85	$^{+0.86}_{-0.85}$	0.54	$\pm 0.54$	0.57	$\pm 0.56$
$W$ + jets	0.34	$\pm 0.12$	0.64	$\pm 0.24$	0.37	$\pm 0.12$
$Z$ + jets	$0.275 \pm 0.081$		$0.043 \pm 0.022$		$0.123 \pm 0.048$	
Multiboson	$0.163 \pm 0.037$		$0.111 \pm 0.030$		$0.030^{+0.032}_{-0.030}$	
$t\bar{t}$ + $V$	0.65	$\pm 0.16$	0.31	$\pm 0.12$	0.092	$\pm 0.035$
$t\bar{t}$ + $H$	0.10	$\pm 0.10$	$0.060^{+0.061}_{-0.060}$		0.028	$^{+0.029}_{-0.028}$
Other top	$0.096 \pm 0.074$		$0.091 \pm 0.049$		$0.0120 \pm 0.0084$	

# “Loose” event selection criteria

ATLAS analysis, Phys. Rev. D 104, (2021) 112005.

Single tau SR definition:

$$\begin{aligned} & 1 \tau_{\text{had}} (p_{\text{T}} > 20 \text{ GeV}, |\eta| < 2.5) \\ & N_{b\text{-jets}} \geq 2 (p_{\text{T}} > 20 \text{ GeV}, |\eta| < 2.5) \\ & E_{\text{T}}^{\text{miss}} > 280 \text{ GeV} \\ & \text{Light leptons veto } (e/\mu) \end{aligned}$$

~~ATLAS extra selection criteria:~~

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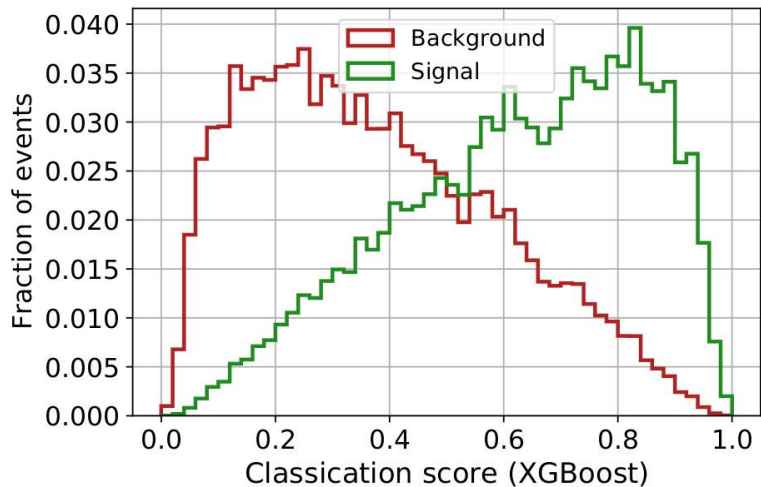
where  $m_{\text{T}}^2(A) = m_{\text{T}}^2(p_{\text{T}}^A, E_{\text{T}}^{\text{miss}}) = 2 p_{\text{T}}^A E_{\text{T}}^{\text{miss}} (1 - \cos \Delta\Phi(p_{\text{T}}^A, E_{\text{T}}^{\text{miss}}))$  is the transverse mass

# From ML to significances

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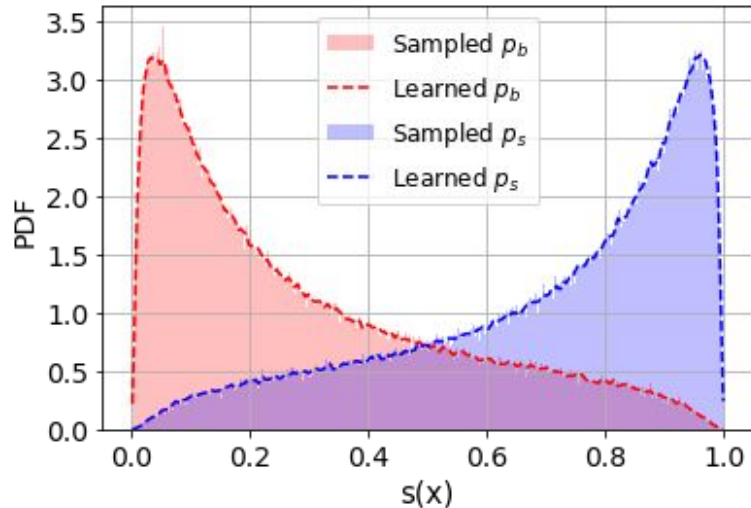
# Expected significance

We used the **full 1D ML classifier output**  $o(x)$  with the standard statistical tests (without defining a working point) to compute the significance



## $o(\bar{x})$ Binned Likelihood method

$$\text{med} [Z_0^{\text{binned}}|1] = \left[ 2 \sum_{d=1}^D \left( (S_d + B_d) \text{Ln} \left( 1 + \frac{S_d}{B_d} \right) - S_d \right) \right]^{1/2}$$



## $o(\bar{x})$ Unbinned method

Kernel Density Estimation (KDE) to estimate the B and S PDFs.

↳ Calculate Z building pseudo-experiments

# Set up:

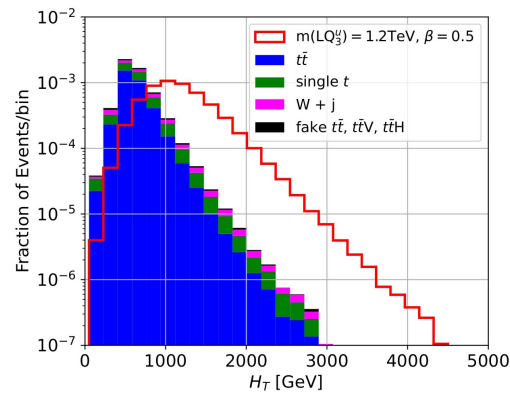
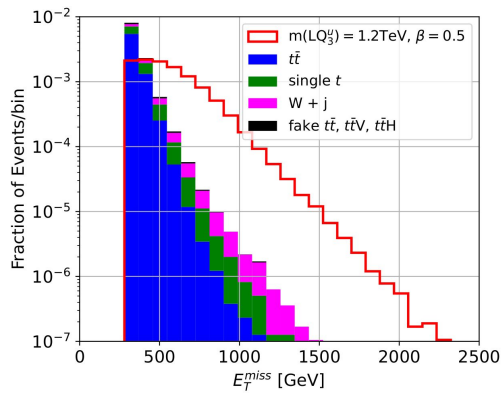
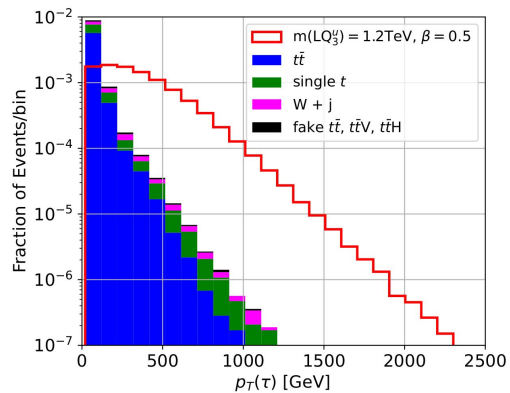
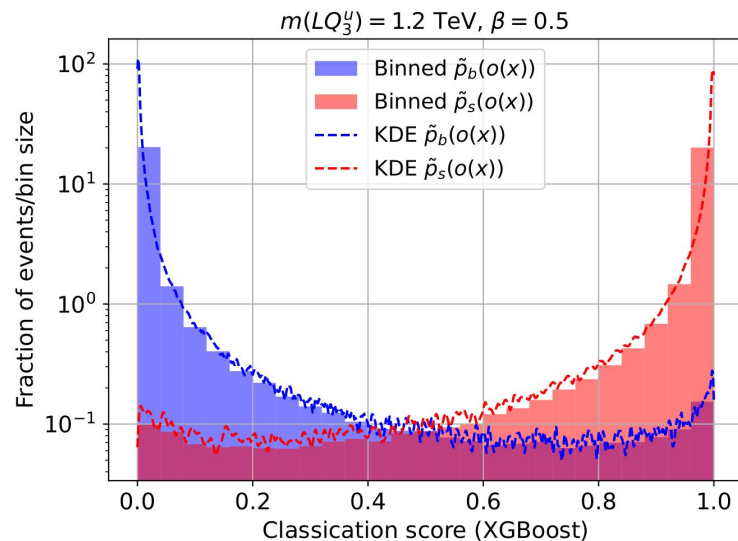
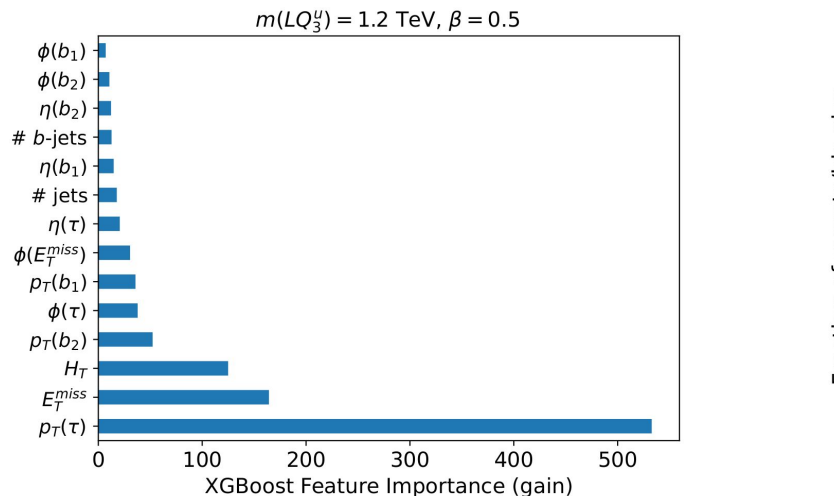
- We generate events with *MadGraph5\_aMC@NLO+Pythia8+Delphes*.
- We simulated signal samples with  $m(\text{LQ}^{u/d}) \in [800, 1800]$  GeV and a fixed value  $BR(\text{LQ}^{u/d} \rightarrow q\ell) = 0.5$ , selecting BPs with a step of 200 GeV.
- We apply the “loose” event selection criteria.
- We train one supervised binary XGBoost classifier with 500k events per class (background and signal) for each BP, using simple discriminating variables (object multiplicities and low-level kinematic variables). We consider the relative weight of each background in the total background sample.
- We estimate the significance with the binned likelihood method, and for the unbinned approach, using KDEs.
- We extend results to different branching fractions.

# Results @ 13 TeV LHC

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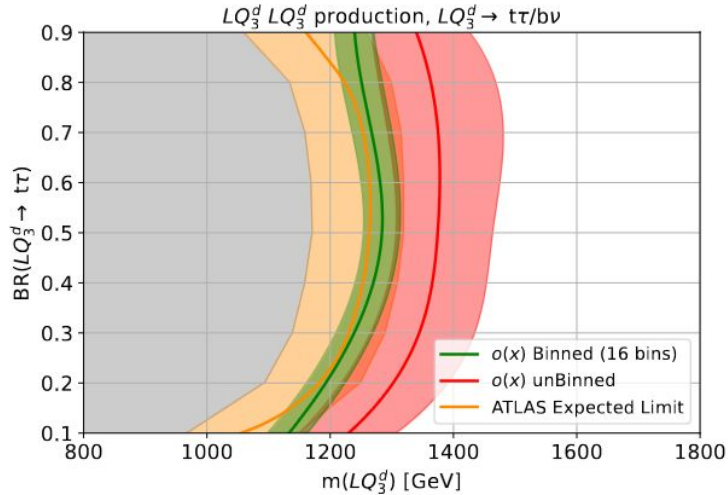
# Results

BP with  $m(\text{LQ}^u) = 1200 \text{ GeV}$  and  $\text{BR}(\text{LQ}^u \rightarrow q\ell) = 0.5$

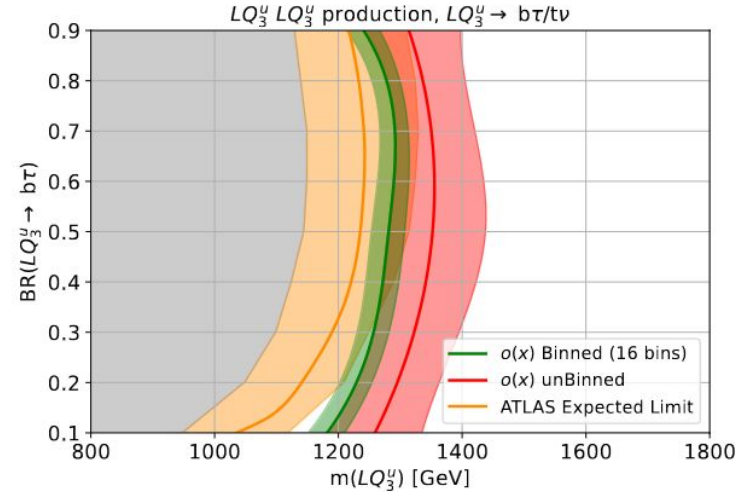


# Results

## Down-type Leptoquarks



## Up-type Leptoquarks



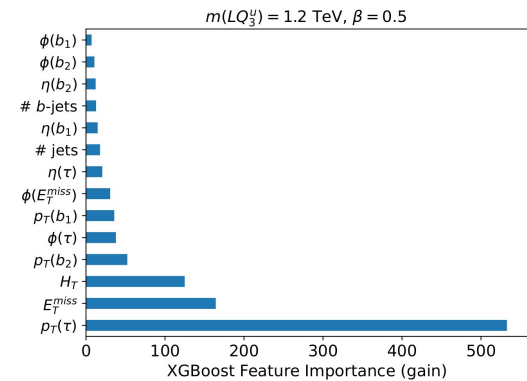
- For  $139 \text{ fb}^{-1}$  and both types of scalar LQs, the expected exclusion limits extend to  $\sim 1.3 \text{ TeV}$  (binned ML) and  $\sim 1.35 \text{ TeV}$  (unbinned ML) at 95% CL for intermediate values of  $BR(LQ^{u/d} \rightarrow q\ell)$ .
- The unbinned method provides more stringent bounds, but is computationally more expensive and has larger statistical uncertainty region.



# Estimation of the impact of systematics

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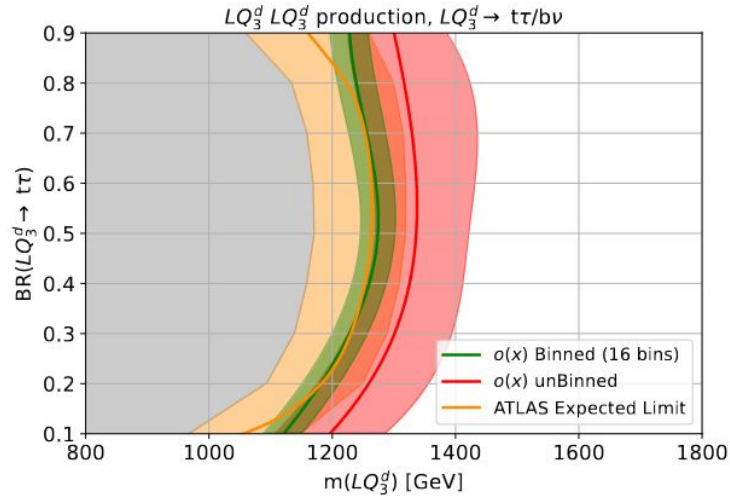
# Idea:



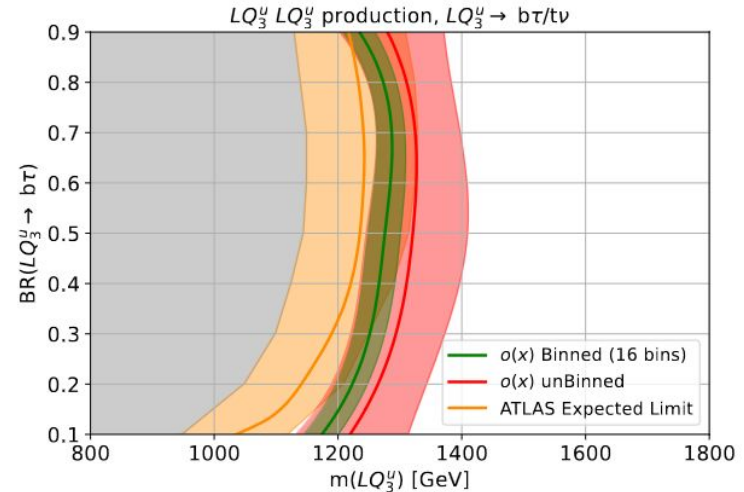
- We consider uncertainties only in the most relevant features for training ( $p_T(\tau)$ ,  $E_T^{\text{miss}}$  and  $H_T$ ) and take correlations among them as not significant.
- We consider shifts of 5-10% of these variables.
- Taking as example  $p_T(\tau)$ , we take the ML algorithm trained with no uncertainties, and evaluate it with two new samples with all variables unchanged but  $p_T(\tau) + \Delta p_T(\tau)$ , and obtained two  $o^{\pm}(x)$ , respectively.
- For the binned method, the uncertainty in each bin  $d$  is  $\Delta o(x_d) = |o(x_d)^+ - o(x_d)^-|$  for analytical formula.
- For the unbinned method, we repeat the entire procedure with  $o(x_d)^{\pm}$  and take as “modified result” the outcome with less restrictive limits.

# Results

## Down-type Leptoquarks



## Up-type Leptoquarks



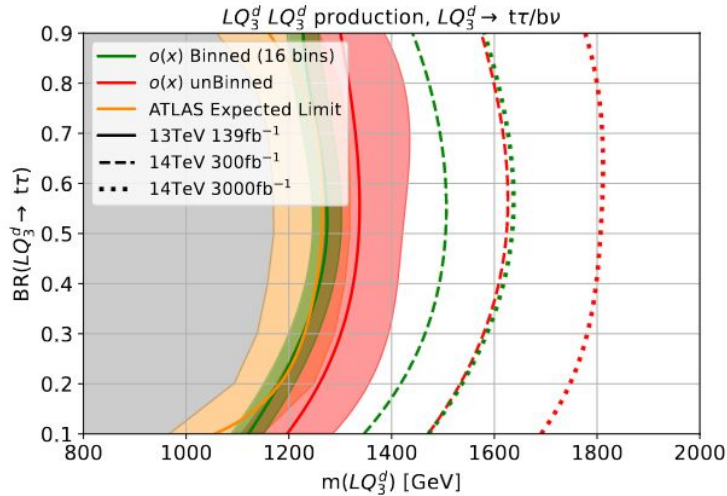
- The impact on the exclusion contours is only of a few percent, and the effect in both methods is similar  $\rightarrow$  the treatment for the unbinned case provides a good numerical approximation.
- Including variations in other features does not impact significantly the results.
- Still necessary full treatment including all sources and correlations!

# Prospects @ 14 TeV LHC

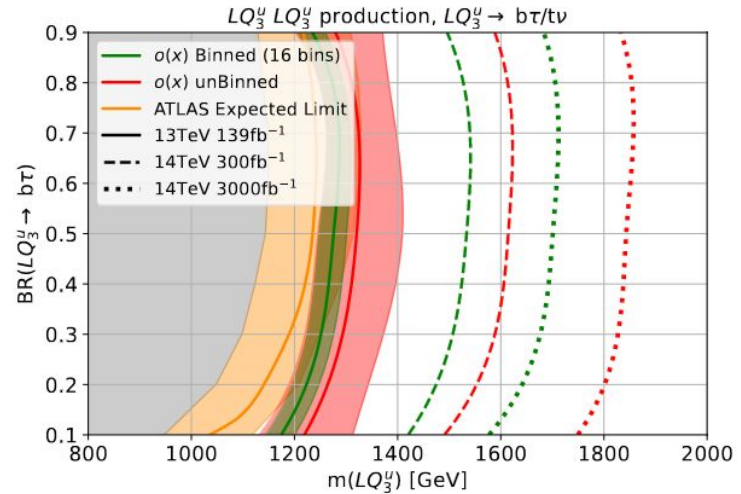
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# Results

## Down-type Leptoquarks



## Up-type Leptoquarks



- For 300 fb<sup>-1</sup> and both types of scalar LQs, the expected exclusion limits extend to ~1.5 TeV (binned ML) and ~1.6 TeV (unbinned ML) at 95% CL for intermediate values of  $BR(LQ^{u/d} \rightarrow q\ell)$ , while for 3000 fb<sup>-1</sup> extend to ~1.65 TeV and ~1.8 TeV, respectively.
- Results include the “naive” approach for the inclusion of systematic uncertainties!

# Conclusions

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# Conclusions

Search for third-generation scalar leptoquarks in final states with one hadronically decaying tau lepton,  $b$ -jets and large missing transverse momentum. As a *proof of concept* we used ML algorithms with a binned and an unbinned likelihood approach and simple selection cuts.

- Tendency towards a potential improvement of the exclusion limits @13 TeV and  $139 \text{ fb}^{-1}$  reported in the ATLAS search used as reference, specially for unbinned analysis!
  - The unbinned method however is more computationally expensive and has larger statistical uncertainties.

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# Conclusions

- Results stable when including a naive estimation of systematic uncertainties
  - The impact is slight and similar for both the binned and unbinned approaches.
  - Still necessary a full analysis with all sources and correlations!
- Promising prospects for @14 TeV and both 300 and 3000 fb<sup>-1</sup>
  - For both types of scalar leptoquarks, possible to exclude masses up to ~1.6 and ~1.8 TeV respectively with the unbinned approach at 95% CL for intermediate values of  $BR(LQ^{u/d} \rightarrow q\ell)$ .



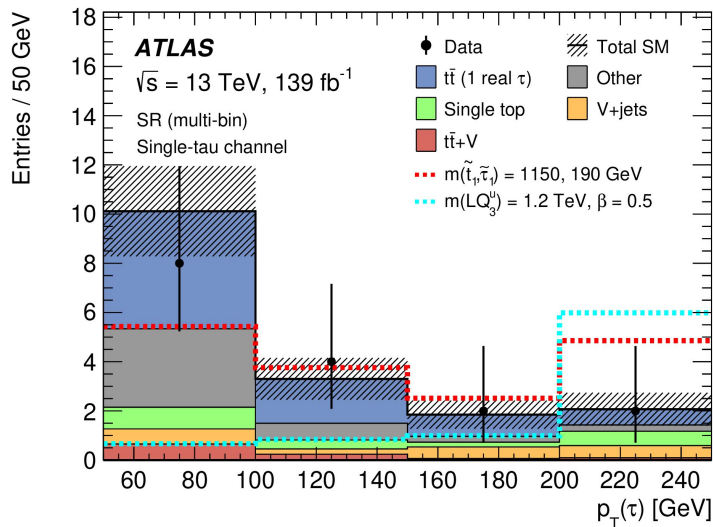
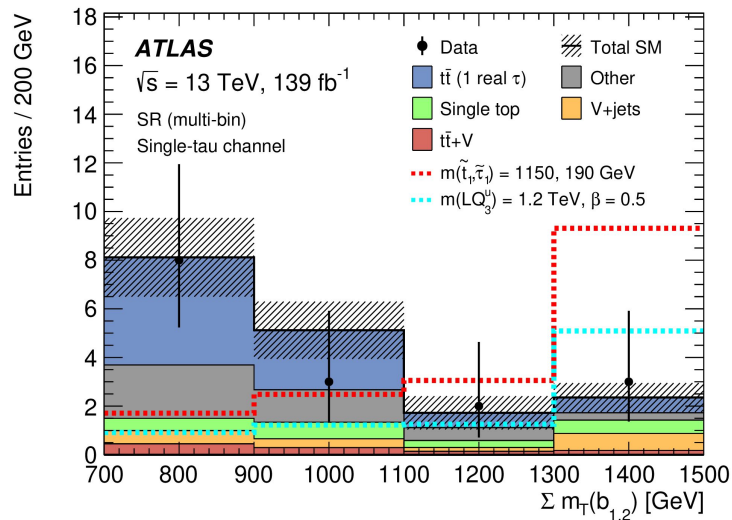
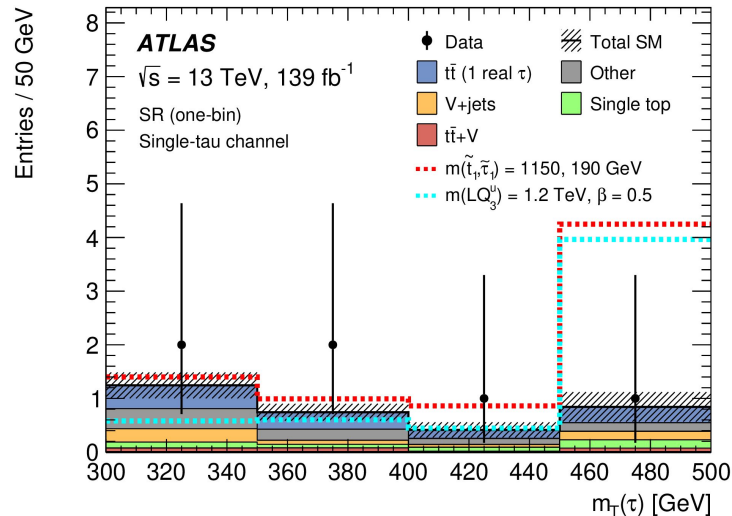
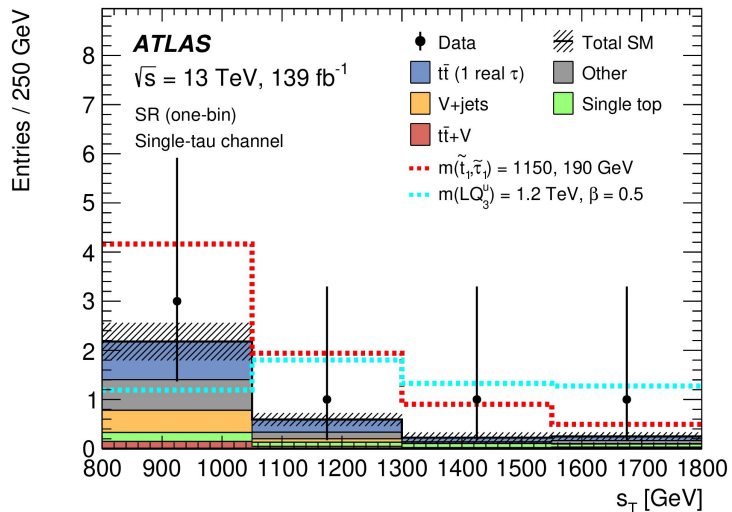
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# Thank you!

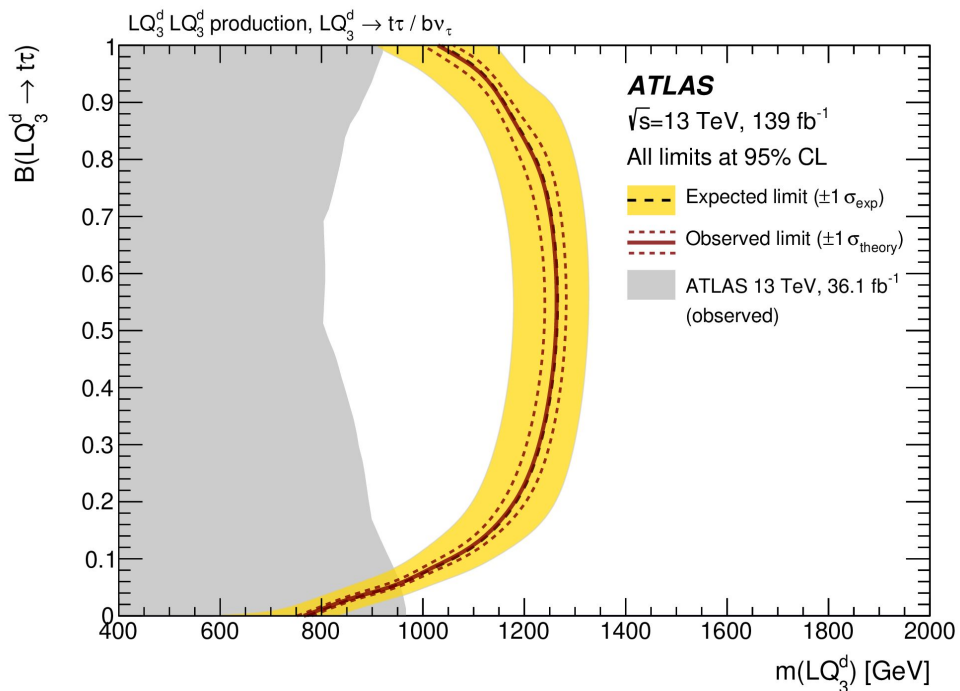
*(Special thanks to A. D. Perez!)*

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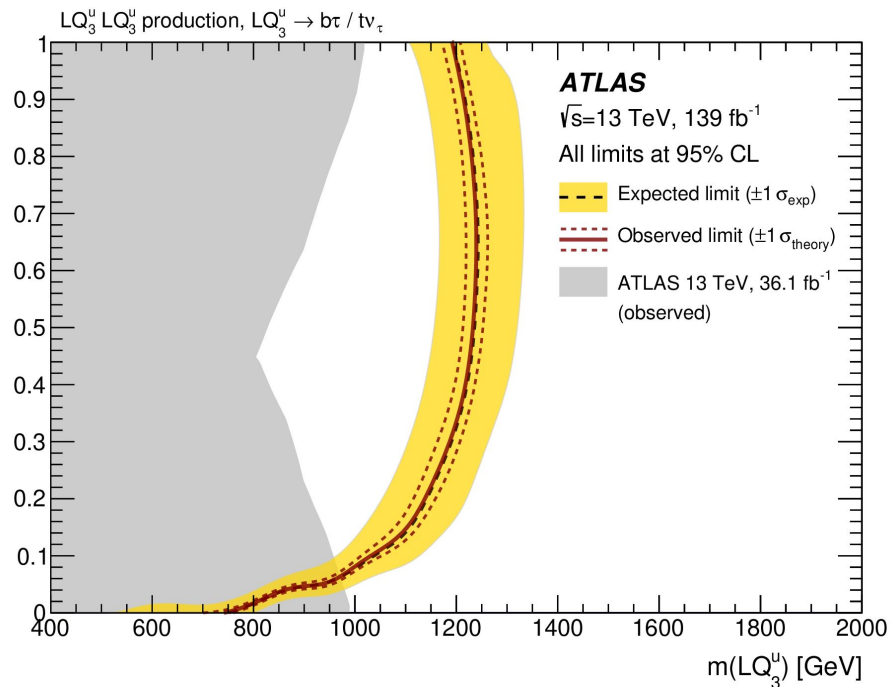
# Back up



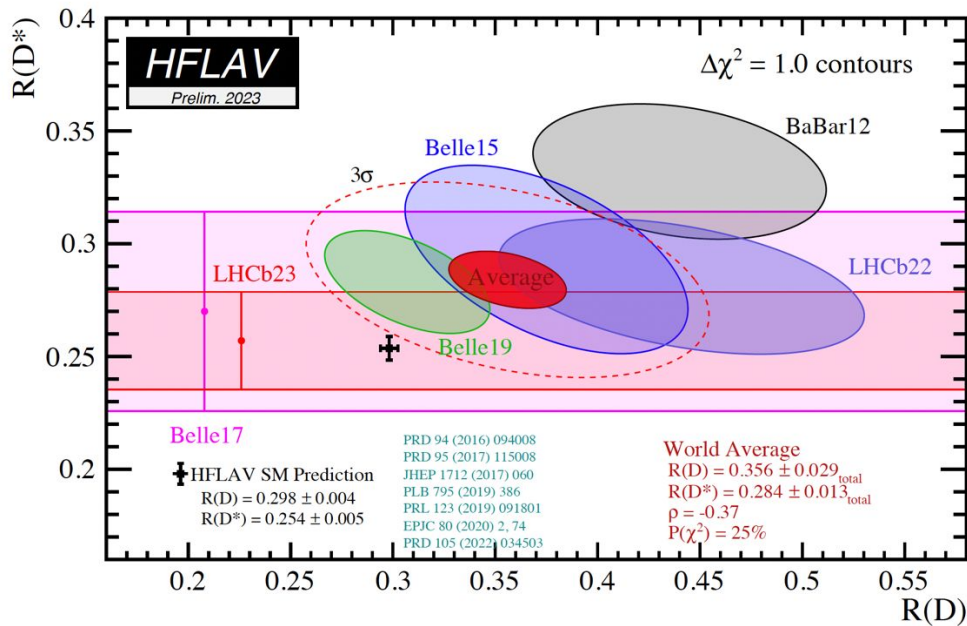
## Down-type Leptoquarks



## Up-type Leptoquarks

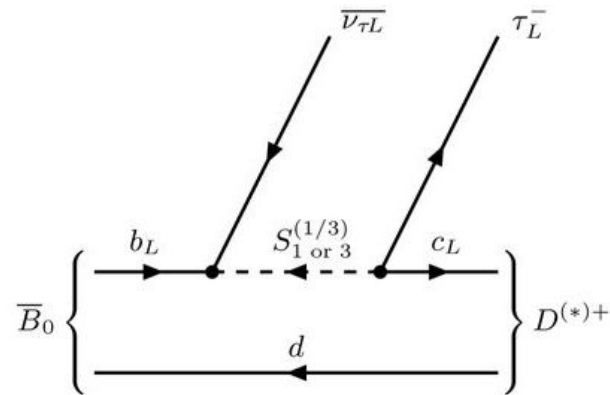
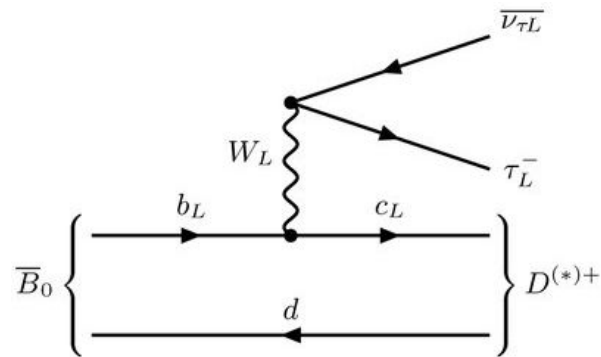


# B-anomalies



Status of the charged-current LFU ratios  $R(D)$  and  $R(D^*)$ .

$$R_{D^{(*)}} = \left. \frac{\text{Br}(B \rightarrow D^{(*)} \tau \bar{\nu})}{\text{Br}(B \rightarrow D^{(*)} \ell \bar{\nu})} \right|_{\ell \in \{e, \mu\}}$$



# Supervised Learning

## Input

Labeled data  $D = \{(\bar{x}_1, t_1), \dots, (\bar{x}_n, t_n)\}$

$\{\bar{x}_i\}$ : features, e.g.  $p_T, \Delta\phi_{12}, E_t^{\text{miss}}$

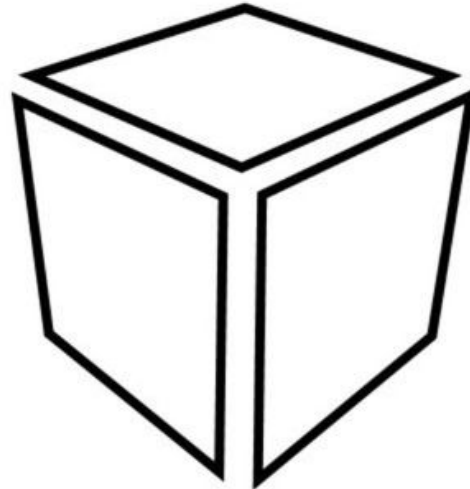
$\{t_i\}$ : target, e.g. for classification:

1 for *signal*

0 for *background*

Train  
dataset  $\bar{X}$

dataset  $\bar{X}$   
Train



ML classifier

## Output

The algorithm finds a mapping:

ideally  $o(\bar{x}_i) = t_i$

for classification:  $o(\bar{x}_i) \in [0, 1]$

ML output  
 $o(\bar{x})$

$o(\bar{x})$   
ML output

Always 1D

# Supervised Learning

## New data

Data sample that we do not know if it is Signal or Background

S or B  
label ??

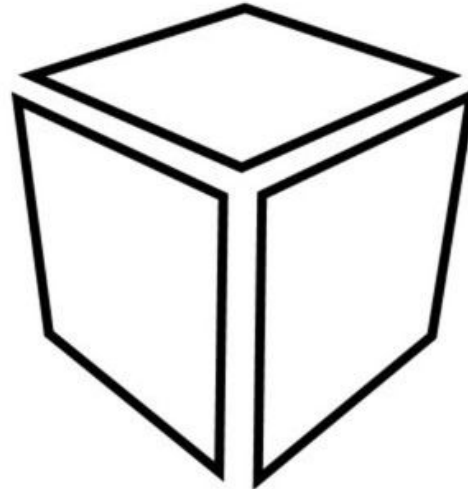
## Prediction

To assign a label a threshold or working point (WP) is needed

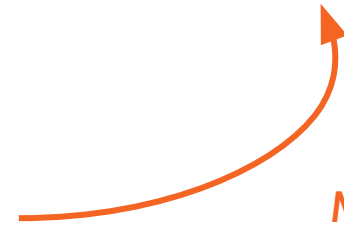
if  $o(x) < WP$  label  $\rightarrow$  '0'  $\rightarrow$  **B**  
if  $o(x) > WP$  label  $\rightarrow$  '1'  $\rightarrow$  **S**

Test  
dataset  $\bar{x}$

dataset  $\bar{x}$   
test



ML classifier

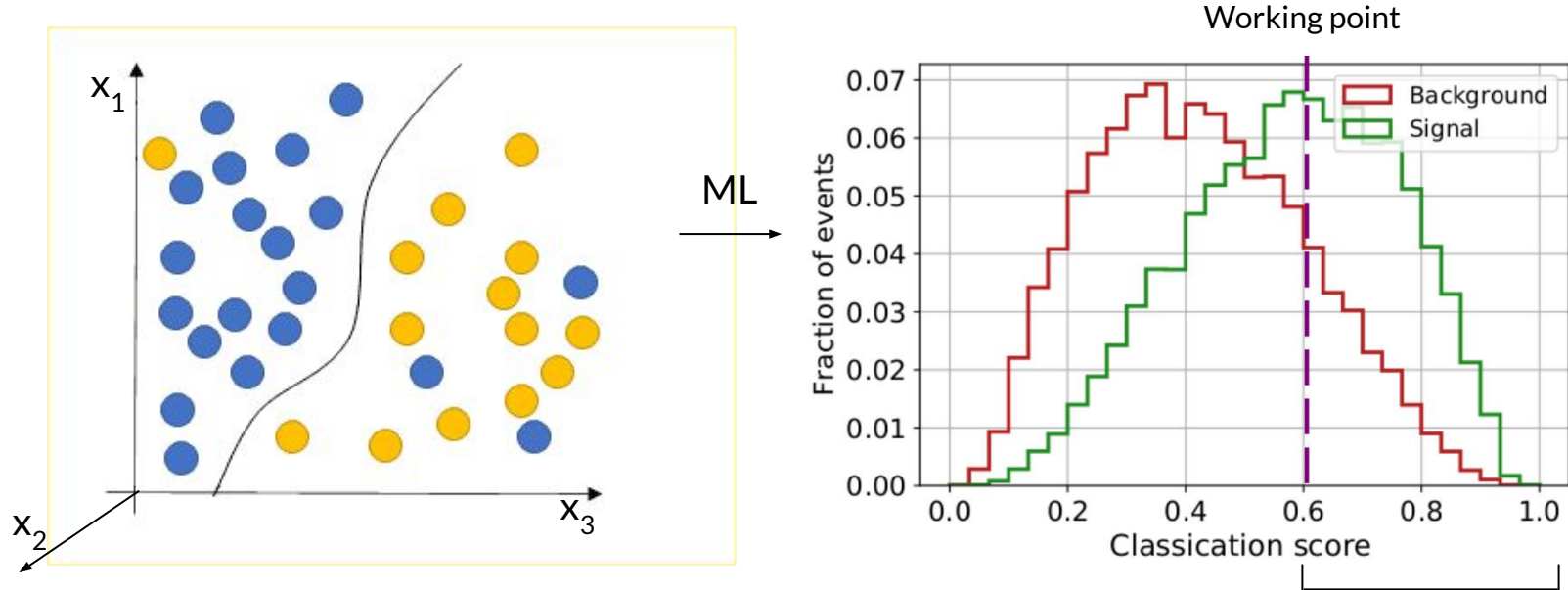


ML output  
 $o(\bar{x})$

$o(\bar{x})$   
ML output

Always 1D

# “Naive” expected significance



Defining a working point ~ Defining signal enriched region

In the defined area  
(we discard events outside of it)

→

$$\text{Significance} \sim \frac{S}{\sqrt{B}}$$



# Machine Learned-Likelihood (MLL)

Likelihood to define the statistical model for  $N$  independent measurements, with a set of observables  $x_i$

$$\mathcal{L}(\mu, s, b) = p(N, \{x_i, i = 1, \dots, N\} | \mu, s, b) = \underbrace{\text{Pois}(N | \mu S + B)}_{\sim \text{global info ensemble factor}} \underbrace{\prod_{i=1}^N p(x_i | \mu, s, b)}_{\sim \text{local info event-by-event}}$$

with:

- $S$  the expected total signal yield
- $B$  the expected total background yield

$$\bullet \quad p(x | \mu, s, b) = \frac{B}{\mu S + B} p_b(x) + \frac{\mu S}{\mu S + B} p_s(x)$$

$$p_s(x) = p(x | s)$$

$$p_b(x) = p(x | b)$$

- $\mu$  the signal strength defines the hypothesis we are testing for:

background-only hypothesis  $\rightarrow \mu = 0$

background-plus-signal hypothesis  $\rightarrow \mu = 1$

# Machine Learned-Likelihood (MLL)

The relevant test statistic for **discovery** limits (very similar for exclusion):

**discovery** corresponds to studying background-only hypothesis  $\mu = 0$

using the Likelihood

$$q_0 = \begin{cases} -2 \text{Ln} \frac{\mathcal{L}(0, s, b)}{\mathcal{L}(\hat{\mu}, s, b)} & \text{if } \hat{\mu} \geq 0, \\ 0 & \text{if } \hat{\mu} < 0, \end{cases}$$

$$q_0 = \begin{cases} -2\hat{\mu}S + 2 \sum_{i=1}^N \text{Ln} \left( 1 + \frac{\hat{\mu}S}{B} \frac{p_s(x_i)}{p_b(x_i)} \right) & \text{if } \hat{\mu} \geq 0, \\ 0 & \text{if } \hat{\mu} < 0. \end{cases}$$

where  $\hat{\mu}$  is the parameter that maximizes the likelihood

$$\sum_{i=1}^N \frac{p_s(x_i)}{\hat{\mu}S p_s(x_i) + B p_b(x_i)} = 1.$$

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$$\sum_{i=1}^N \frac{p_s(x_i)}{\hat{\mu}S p_s(x_i) + B p_b(x_i)} = 1.$$

**We need**

$$p_s(x) = p(x|s)$$

$$p_b(x) = p(x|b)$$

# Machine Learned-Likelihood (MLL)

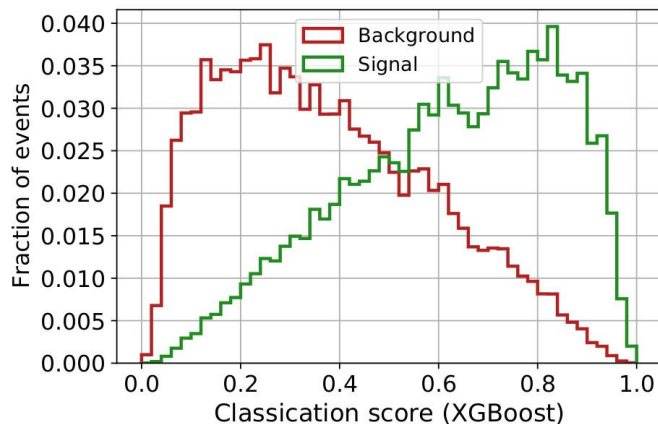
Replace the densities for the one-dimensional manifolds obtained with a machine-learning classifier.

The classification score that maximizes the binary cross-entropy approaches:

$$o(x) = \frac{p_s(x)}{p_s(x) + p_b(x)}$$

Dimensional reduction by dealing with  $o(x)$  instead of  $x$

$$p_b(x) \rightarrow \tilde{p}_b(o(x)), \quad \text{and} \quad p_s(x) \rightarrow \tilde{p}_s(o(x))$$



where  $\tilde{p}_{s,b}(o(x))$  are the distributions of  $o(x)$  for signal and background, obtained by evaluating the classifier on a set of pure signal or background events, respectively.

# Machine Learned-Likelihood (MLL)

Then, the relevant test statistic for **discovery** limits

$$q_0 = \begin{cases} -2\hat{\mu}S + 2 \sum_{i=1}^N \text{Ln} \left( 1 + \frac{\hat{\mu}S}{B} \frac{\tilde{p}_s(o(x_i))}{\tilde{p}_b(o(x_i))} \right) & \text{if } \hat{\mu} \geq 0, \\ 0 & \text{if } \hat{\mu} < 0, \end{cases}$$

with  $\hat{\mu}$  the parameter that maximizes the likelihood

$$\sum_{i=1}^N \frac{\tilde{p}_s(o(x_i))}{\hat{\mu}S \tilde{p}_s(o(x_i)) + B \tilde{p}_b(o(x_i))} = 1$$

We can estimate numerically the  $q_0$  distribution.

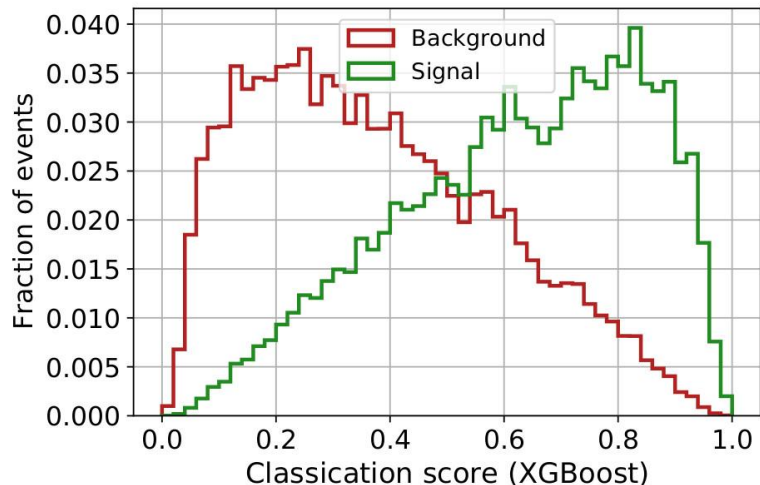
The median expected significance assuming signal-plus-background hypothesis ( $\mu'=1$ ) is

$$Z_0 \rightarrow \text{med} [Z_0|1] = \sqrt{\text{med} [q_0|1]}$$

# Density estimation

We want to retrieve the density function from which the samples were generated

$$p_b(x) \rightarrow \tilde{p}_b(o(x)), \quad \text{and} \quad p_s(x) \rightarrow \tilde{p}_s(o(x))$$



The original space,  $x$ , can be high-dimensional but the classifier output  $\mathbf{o}(x)$  is **always one-dimensional**

- To avoid binning, we use a non-parametric method:

**Kernel Density Estimation (KDE)**

# Kernel Density Estimation (KDE)

$$p_{s,b}(o(\mathbf{x})) = \frac{1}{N} \sum_i^N \kappa_\epsilon [o(\mathbf{x}) - o(\mathbf{x}_i)]$$

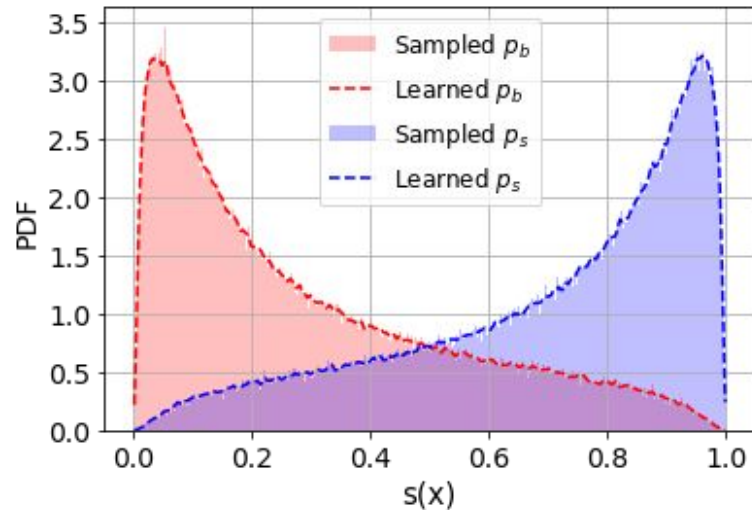
where  $\kappa_\epsilon$  is a kernel function that depends on the "smoothing" scale, or bandwidth parameter  $\epsilon$ .

We use the Epanechnikov kernel

$$\kappa_\epsilon(u) = \begin{cases} \frac{1}{\epsilon} \frac{3}{4} (1 - (u/\epsilon)^2), & \text{if } |u| \leq \epsilon \\ 0, & \text{otherwise} \end{cases}$$

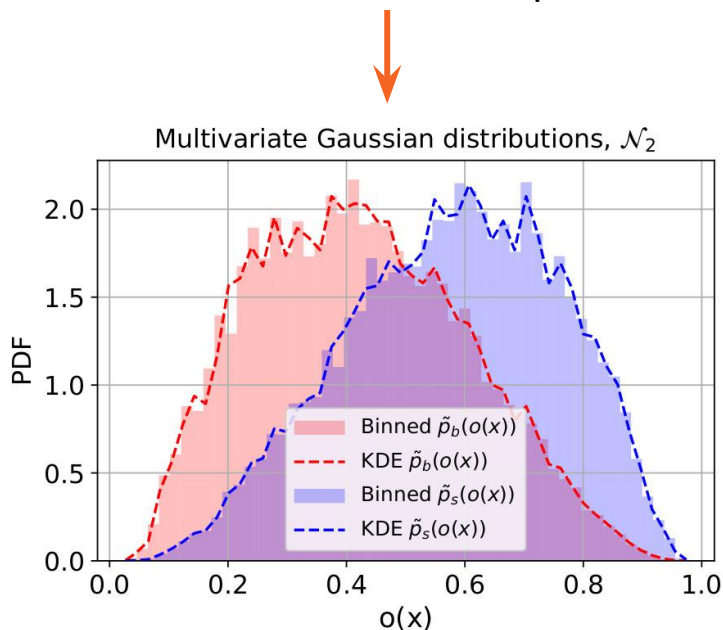
The bandwidth parameter  $\epsilon$  is key

- if  $\epsilon$  is too low the model may overfit
- if  $\epsilon$  is too high the model may underfit



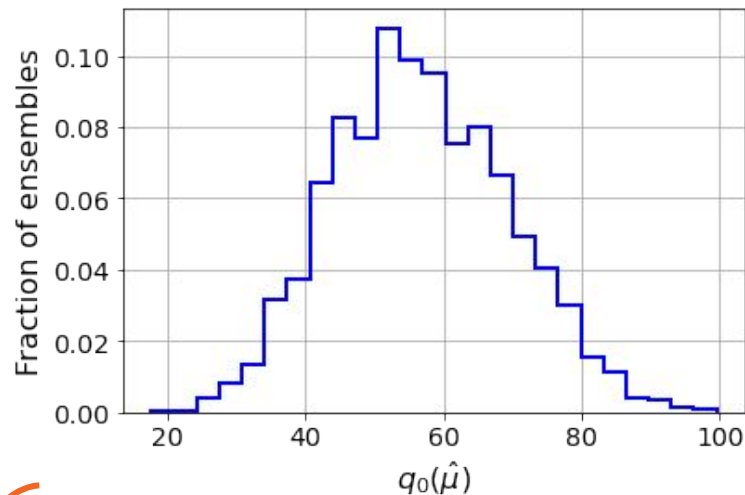
# Machine Learned-Likelihood (MLL)

Train supervised per-even classifier:  
XGBoost with 1M events per class



Evaluate  $o(x)$  with the test data-set  
Find the distributions with KDE

Build toy ensembles of fixed B and S (each one represent a possible experimental result)  
and evaluate the test statistic  $q_0$



Calculate the significance

$$Z = \sqrt{\text{med}[q_0]}$$



# Machine Learned-Likelihood (MLL)

First find  $\hat{\mu}$  (for each ensemble)

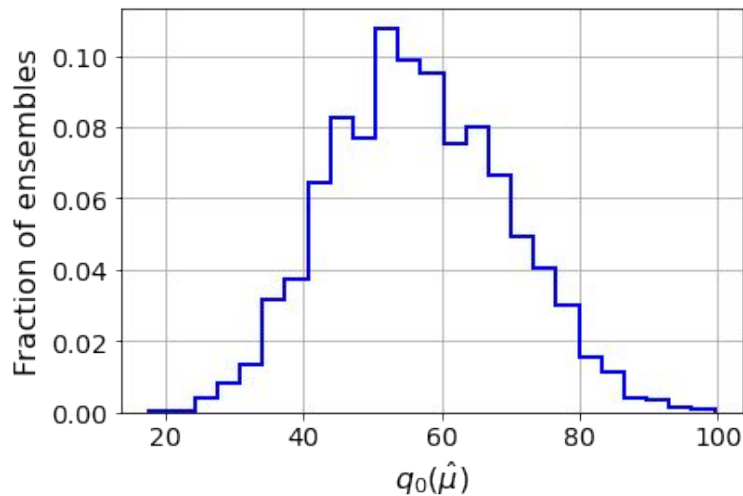
$$\sum_{i=1}^N \frac{\tilde{p}_s(o(x_i))}{\hat{\mu}S \tilde{p}_s(o(x_i)) + B \tilde{p}_b(o(x_i))} = 1$$

↓  
summation over the events of  
each ensemble (build a lot)

Estimate numerically the test  
statistic (for each ensemble)

$$q_0 = \begin{cases} -2\hat{\mu}S + 2 \sum_{i=1}^N \text{Ln} \left( 1 + \frac{\hat{\mu}S \tilde{p}_s(o(x_i))}{B \tilde{p}_b(o(x_i))} \right) & \text{if } \hat{\mu} \geq 0, \\ 0 & \text{if } \hat{\mu} < 0, \end{cases}$$

Build toy ensembles of fixed B and S (each one  
represent a possible experimental result)  
and evaluate the test statistic  $q_0$



Calculate the significance

$$Z = \sqrt{\text{med}[q_0]}$$

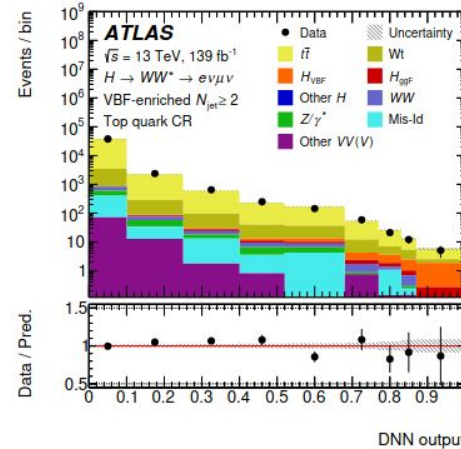
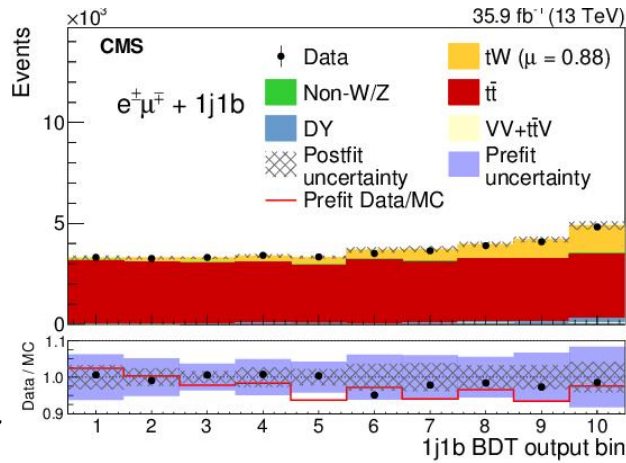
# Traditional Binned-Likelihood (BL) method

The Likelihood for  $D$  bins, where in each bin  $d$ ,  $B_d$ : the expected number of background events,  $S_d$ : the expected number of signal events, and  $N_d$ : the measured number of events,

$$\mathcal{L}(\mu, s, b) = \prod_{d=1}^D \text{Pois}(N_d | \mu S_d + B_d)$$

The median discovery significance

$$\text{med} [Z_0^{\text{binned}} | 1] = \left[ 2 \sum_{d=1}^D \left( (S_d + B_d) \text{Ln} \left( 1 + \frac{S_d}{B_d} \right) - S_d \right) \right]^{1/2} \xrightarrow[\sqrt{B} \gg 1]{S \ll B} \frac{S}{\sqrt{B}}$$



# Machine Learned-Likelihood (MLL)

The relevant test statistic for **exclusion limits**:

using the Likelihood

$$\tilde{q}_\mu = \begin{cases} 0 & \text{if } \hat{\mu} > \mu, \\ -2 \text{Ln} \frac{\mathcal{L}(\mu, s, b)}{\mathcal{L}(\hat{\mu}, s, b)} & \text{if } 0 \leq \hat{\mu} \leq \mu, \\ -2 \text{Ln} \frac{\mathcal{L}(\mu, s, b)}{\mathcal{L}(0, s, b)} & \text{if } \hat{\mu} < 0, \end{cases}$$

$$\tilde{q}_\mu = \begin{cases} 0 & \text{if } \hat{\mu} > \mu \\ 2(\mu - \hat{\mu})S - 2 \sum_{i=1}^N \text{Ln} \left( \frac{B p_b(x_i) + \mu S p_s(x_i)}{B p_b(x_i) + \hat{\mu} S p_s(x_i)} \right) & \text{if } 0 \leq \hat{\mu} \leq \mu \\ 2\mu S - 2 \sum_{i=1}^N \text{Ln} \left( 1 + \frac{\mu S p_s(x_i)}{B p_b(x_i)} \right) & \text{if } \hat{\mu} < 0; \end{cases}$$

where  $\hat{\mu}$  is the parameter that maximizes the likelihood

$$\sum_{i=1}^N \frac{p_s(x_i)}{\hat{\mu} S p_s(x_i) + B p_b(x_i)} = 1.$$

$$\begin{aligned} & \text{if } \hat{\mu} > \mu, \\ & \text{if } 0 \leq \hat{\mu} \leq \mu, \\ & \text{if } \hat{\mu} < 0, \end{aligned}$$

$$\begin{aligned} & \text{if } \hat{\mu} > \mu \\ & \text{if } 0 \leq \hat{\mu} \leq \mu \\ & \text{if } \hat{\mu} < 0; \end{aligned}$$

**exclusion** corresponds to studying signal+background hypothesis  $\mu = 1$

The median expected significance assuming background-only hypothesis ( $\mu'=0$ ) is

$$Z_\mu \rightarrow \text{med} [Z_\mu | 0] = \sqrt{\text{med} [\tilde{q}_\mu | 0]}$$