From matrix model to string theory and M-theory

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July 12, 2023 @ CERN

Ultimate goal (not today)

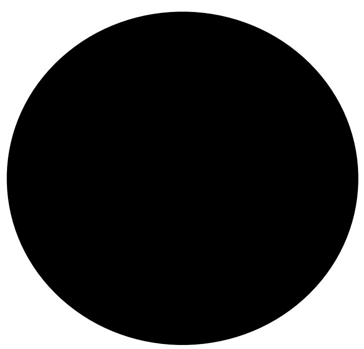
- We will play with our own black hole at home (or at lab).

Holography + Quantum Technology

(Gauge/Gravity Duality)

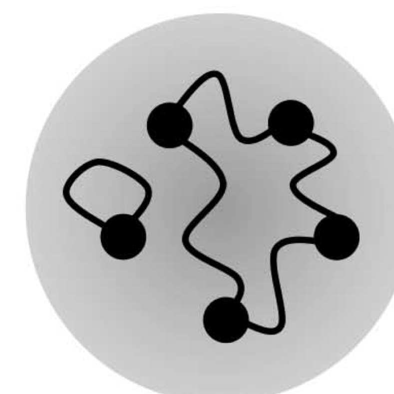
AdS/CFT Duality

(Maldacena1997)



IIB string
on $AdS_5 \times S^5$

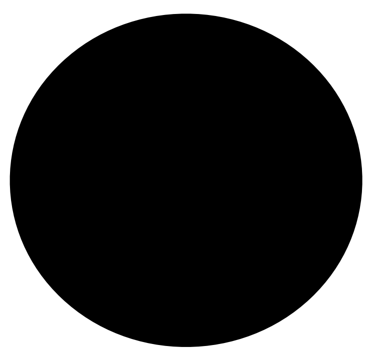
equivalent



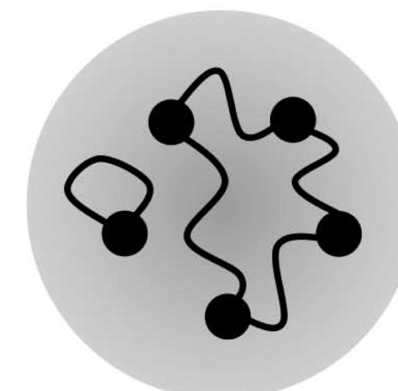
(3+1)-d U(N)
maximal SYM

≒
QCD

Gauge/Gravity Duality



dual



Black p-brane
in IIA/IIB string

(p+1)-d U(N)SYM
(Dp-branes+strings)

$p=3 \rightarrow AdS_5 \times S^5$

$p=0,1,2,3,\ldots$

Maldacena, 1997;
Itzhaki-Maldacena-Sonnenschein-Yankielowicz, 1998

High Energy Physics – Theory

[Submitted on 27 Nov 1997 (v1), last revised 22 Jan 1998 (this version, v3)]

The Large N Limit of Superconformal Field Theories and Supergravity

Juan M. Maldacena

We show that the large N limit of certain conformal field theories in various dimensions include in their Hilbert space a sector desc
spacetimes, spheres and other compact manifolds. This is shown by taking some branes in the full M/string theory and then takin
decouples from the bulk. We observe that, in this limit, we can still trust the near horizon geometry for large N. The enhanced sup
to the extra supersymmetry generators present in the superconformal group (as opposed to just the super-Poincare group). The 't

6. Discussion, relation to matrix theory

By deriving various field theories from string theory and considering their large N limit we have shown that they contain in their Hilbert space excitations describing supergravity on various spacetimes. We further conjectured that the field theories are dual to the full quantum M/string theory on various spacetimes. In principle, we can use this duality to give a definition of M/string theory on flat spacetime as (a region of) the large N limit of the field theories. Notice that this is a non-perturbative proposal for defining such theories, since the corresponding field theories can, *in principle*, be defined non-perturbatively. We are only scratching the surface and there are many things to be worked out. In [61] it has been proposed that the large N limit of D0 brane quantum mechanics would describe eleven dimensional M-theory. The large N limits discussed above, also provide a definition of M-theory. An obvious difference with the matrix model of [61] is that here N is not interpreted as the momentum along a compact direction. In our case, N is related to the curvature and the size of the space where the theory is defined. In both cases, in the large N limit we expect to get flat, non-compact spaces. The matrix model [61] gives us a prescription to build asymptotic states, we have not shown here how to construct graviton states, and this is a very interesting problem. On the other hand, with the present proposal it is more clear that we recover supergravity in the large N limit.

Nonperturbative formulation

Black Hole

Quantum gravity

duality

Non-gravitational systems

Matrix Model
Super Yang-Mills
SYK model
...

BH

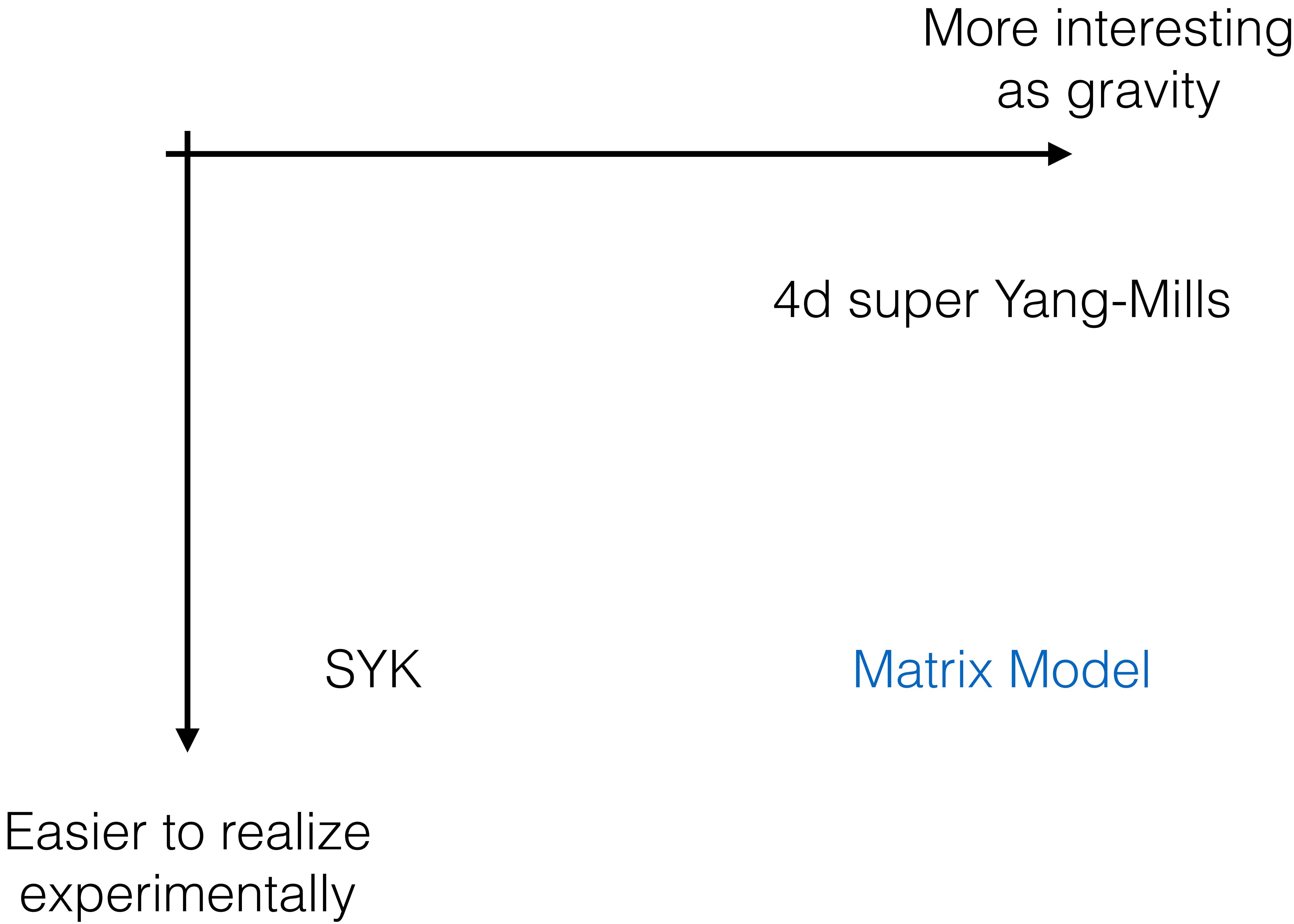
Building 

is equivalent to creating a black hole.

In principle, we can create 

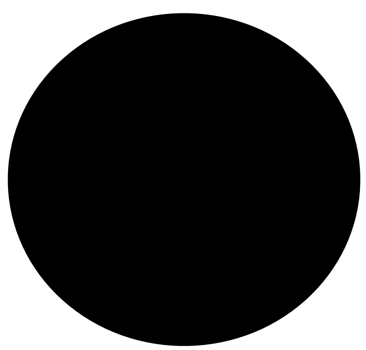
on analog or digital quantum simulators.

What is the best setup?


More interesting
as gravity
4d super Yang-Mills
SYK
Matrix Model
Easier to realize
experimentally

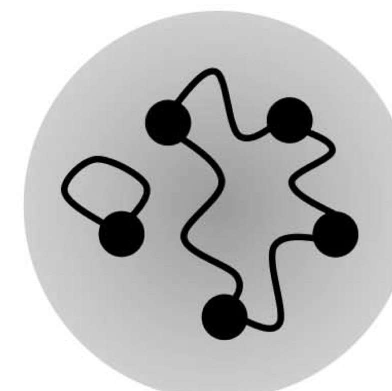
Gauge/Gravity Duality

(Maldacena1997, Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)



IIA/IIB string around black p-brane

equivalent



(p+1)-d U(N)SYM (Dp-branes+strings)

black 3-brane = $AdS_5 \times S^5$

p=0,1,2,3

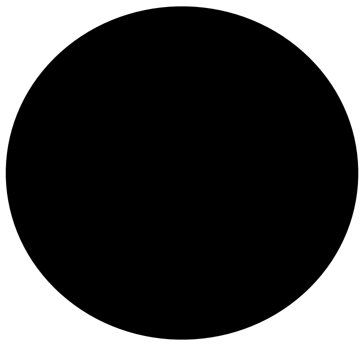
.II˙

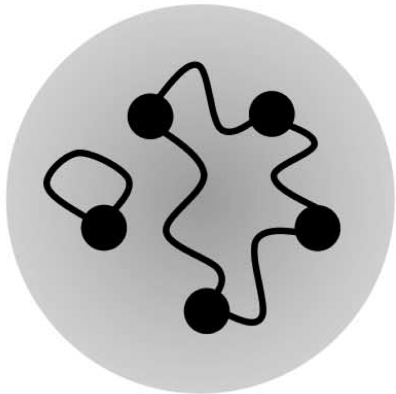
QCD

p=0 → matrix model

Black p-brane = bunch of Dp-branes

(+ strings between them)



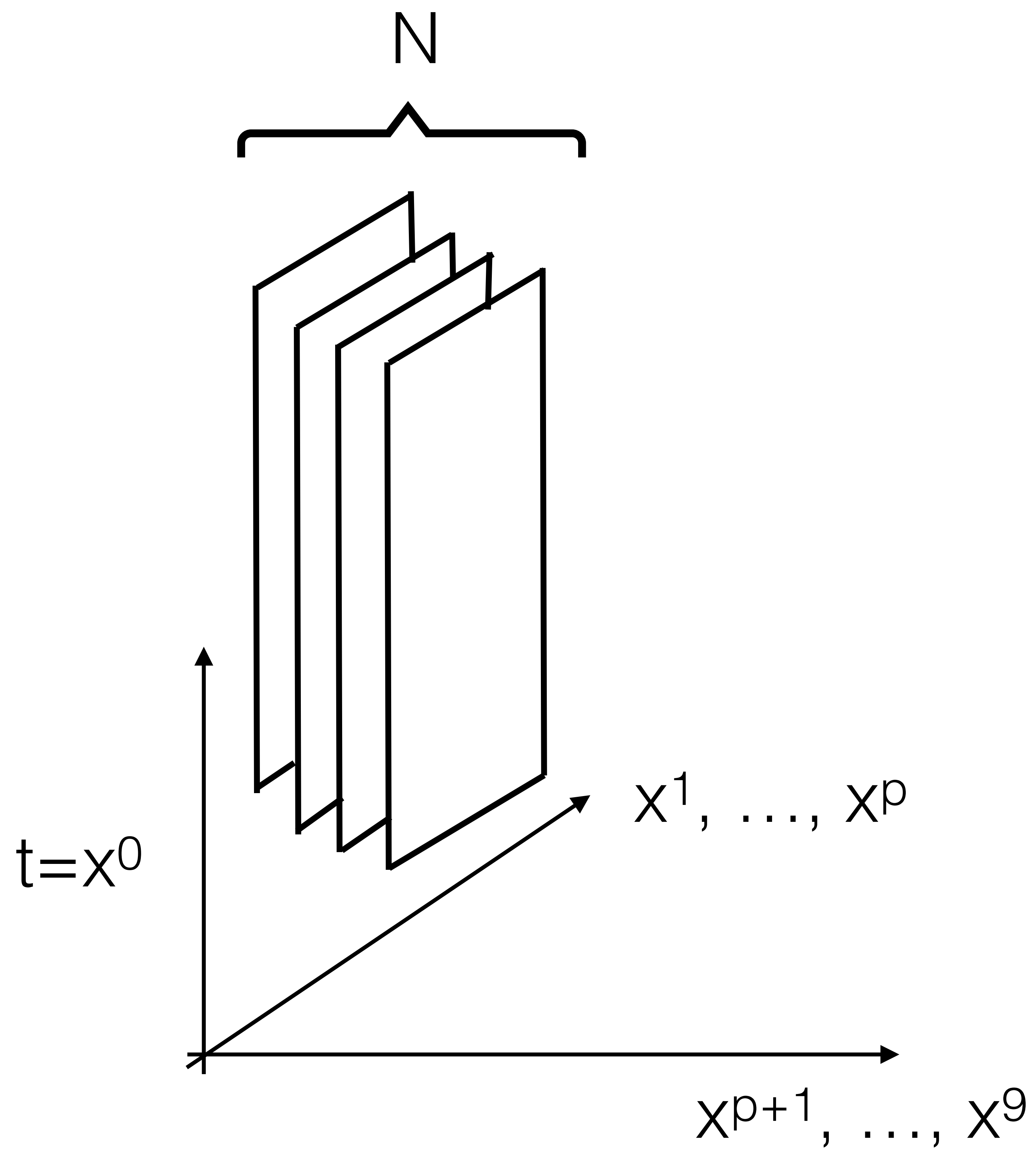


- Dp-brane : (p+1)-d object
- Open string connects Dp-branes

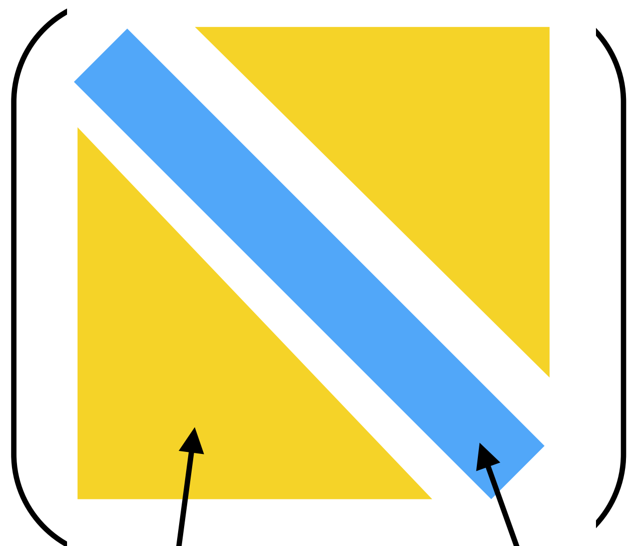
low-energy effective theory of Dp-branes
= (p+1)-d SYM

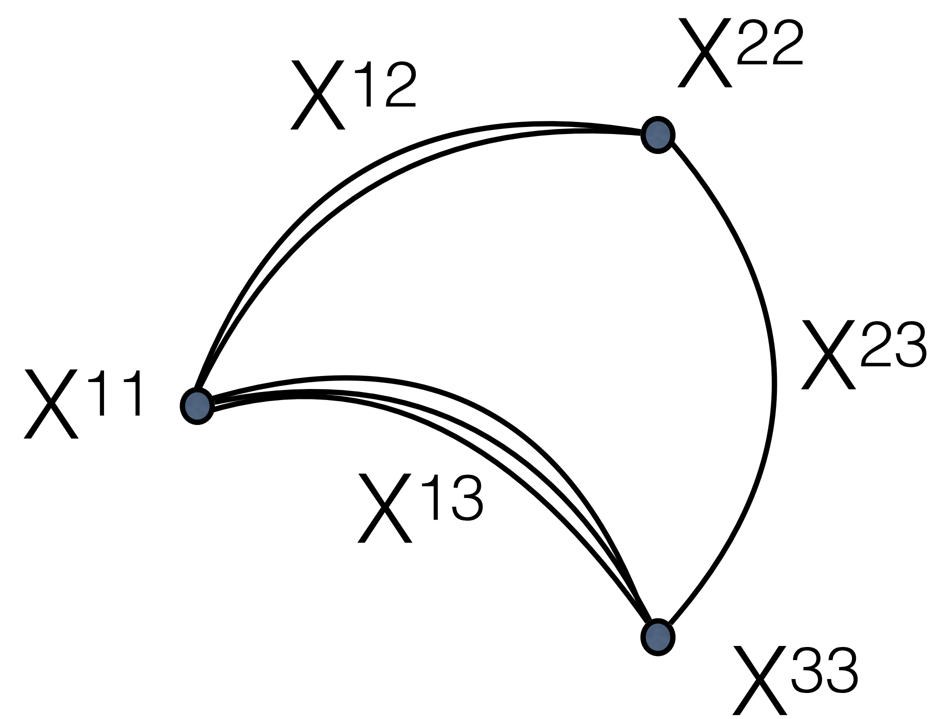
p=0 → matrix model

SU(N)   N = number of D-branes


N
t=x0
x1, ..., xp
xp+1, ..., x9

Dp-brane bound state and Gauge Theory

$X_M =$ 



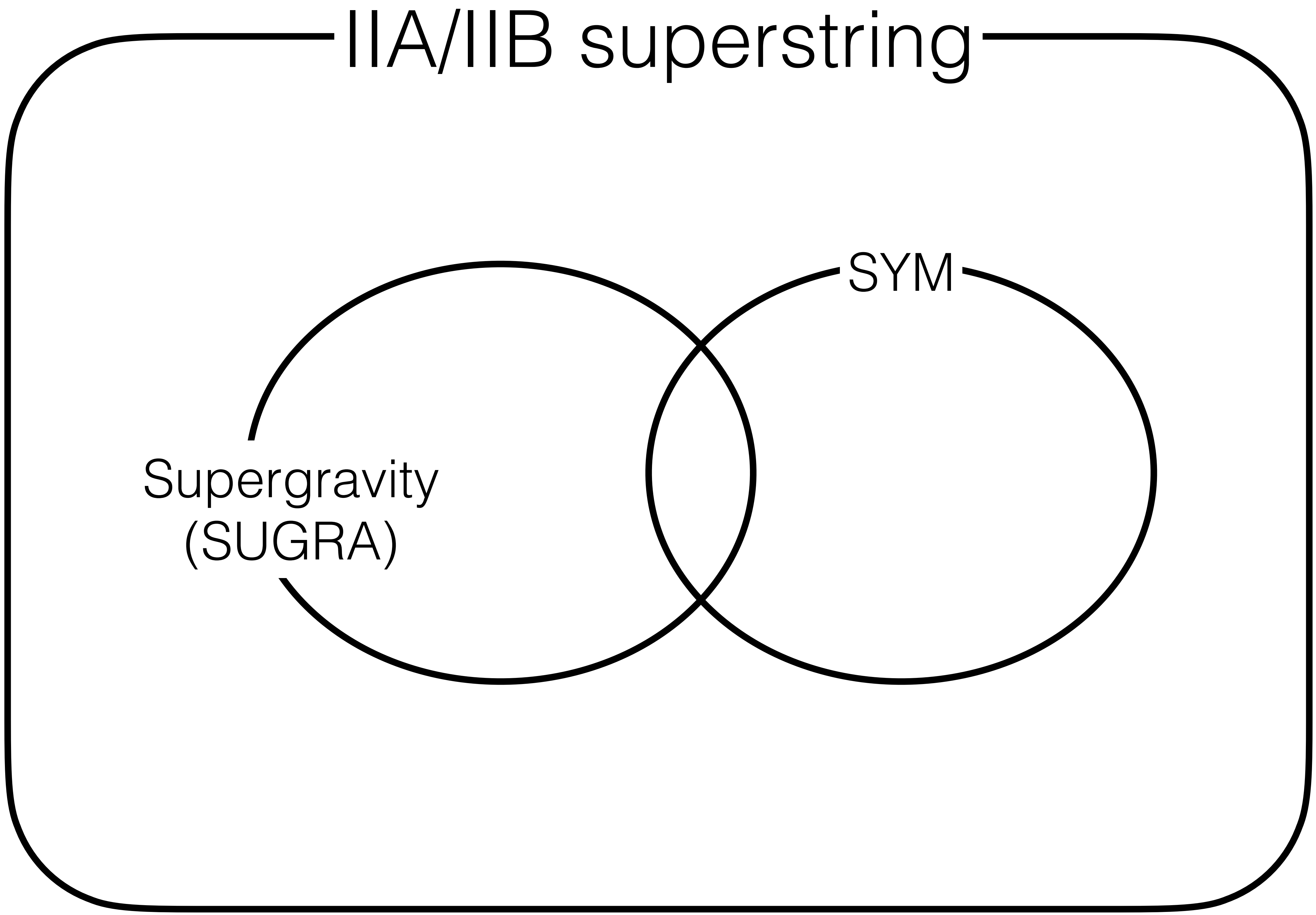
$(X_1^{ii}, X_2^{ii}, \ldots, X_{9-p}^{ii})$

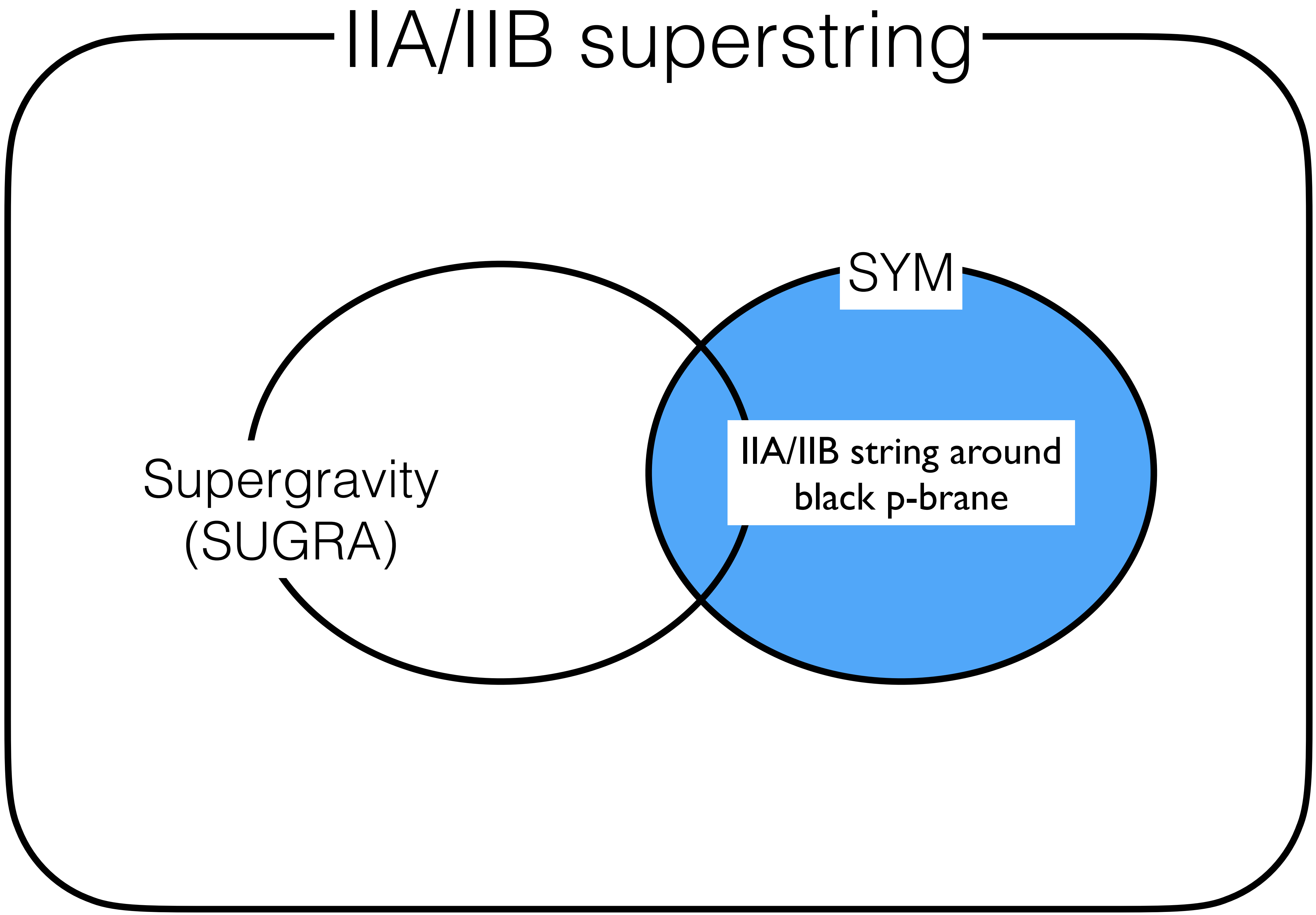
position of i-th Dp-brane in R^{9-p}

X_M^{ij} : open strings connecting i-th and j-th D0-branes.

large value → a lot of strings are excited

(Witten, 1994)


IIA/IIB superstring
SYM
Supergravity
(SUGRA)


IIA/IIB superstring
SYM
Supergravity
(SUGRA)
IIA/IIB string around
black p-brane

Outline

- Precision test via Monte Carlo simulation
- New phase: "confinement" at low energy
- Confinement ~ M-theory ? (somewhat speculative)
- Toward quantum simulation
  (backup slides)

p=0

D0-brane matrix model
(BFSS model)

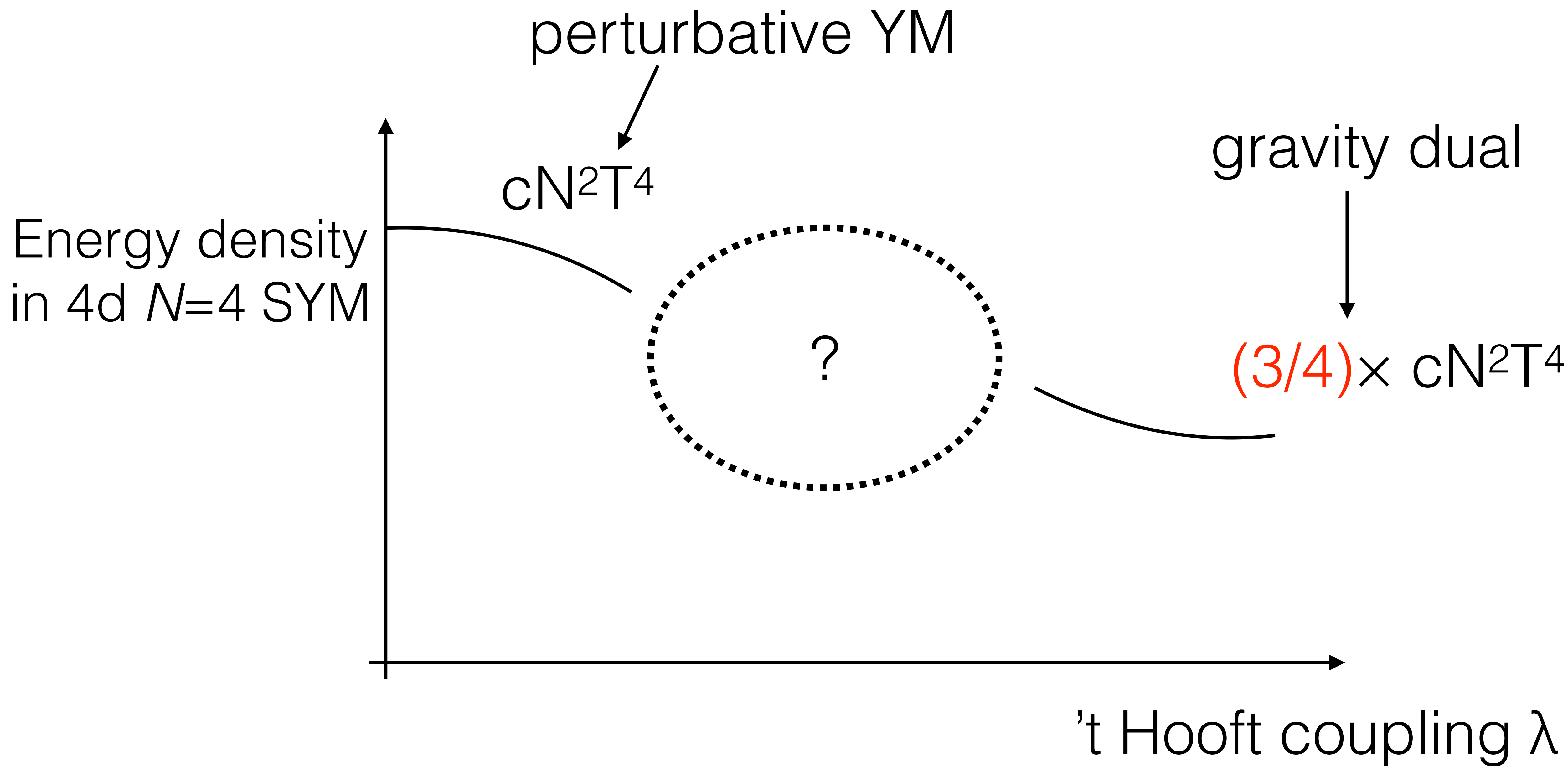
$$L = \frac{1}{2g_{YM}^2}\mathrm{Tr}\left\{(D_t X_M)^2 + [X_M, X_{M'}]^2 + i\bar{\psi}^\alpha D_t \psi^\beta + \bar{\psi}^\alpha \gamma^M_{\alpha\beta}[X_M, \psi^\beta]\right\}$$

M=1,2,..,9; α=1,2,...,16

- Dimensional reduction of 4d maximal SYM
- Low-energy description of D0 and strings
  Witten, 1995
- Matrix regularization of supermembrane
  de Wit-Hoppe-Nicolai, 1988
- Matrix Model of M-theory Banks-Fischler-Shenker-Susskind, 1996
- Dual to type IIA black zero-brane near 't Hooft limit
  Itzhaki-Maldacena-Sonnenschein-Yankielowicz, 1998

3/4-problem in 4d $\mathcal{N}$=4 SYM

p=3



Let's solve the D0-brane version of this problem.

arXiv.org > hep-th > arXiv:hep-th/9802042

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High Energy Physics – Theory

[Submitted on 7 Feb 1998 (v1), last revised 26 Sep 2000 (this version, v3)]

Supergravity and The Large N Limit of Theories With Sixteen Supercharges

Nissan Itzhaki, Juan M. Maldacena, Jacob Sonnenschein, Shimon Yankielowicz

$$\lambda = g_{\rm YM}^2 N = ({\rm mass})^3$$

$$\lambda E^{-3} \sim \text{dimensionless coupling}$$

i.e., low energy ⇔ strong coupling

Black 0-brane in type IIA SUGRA

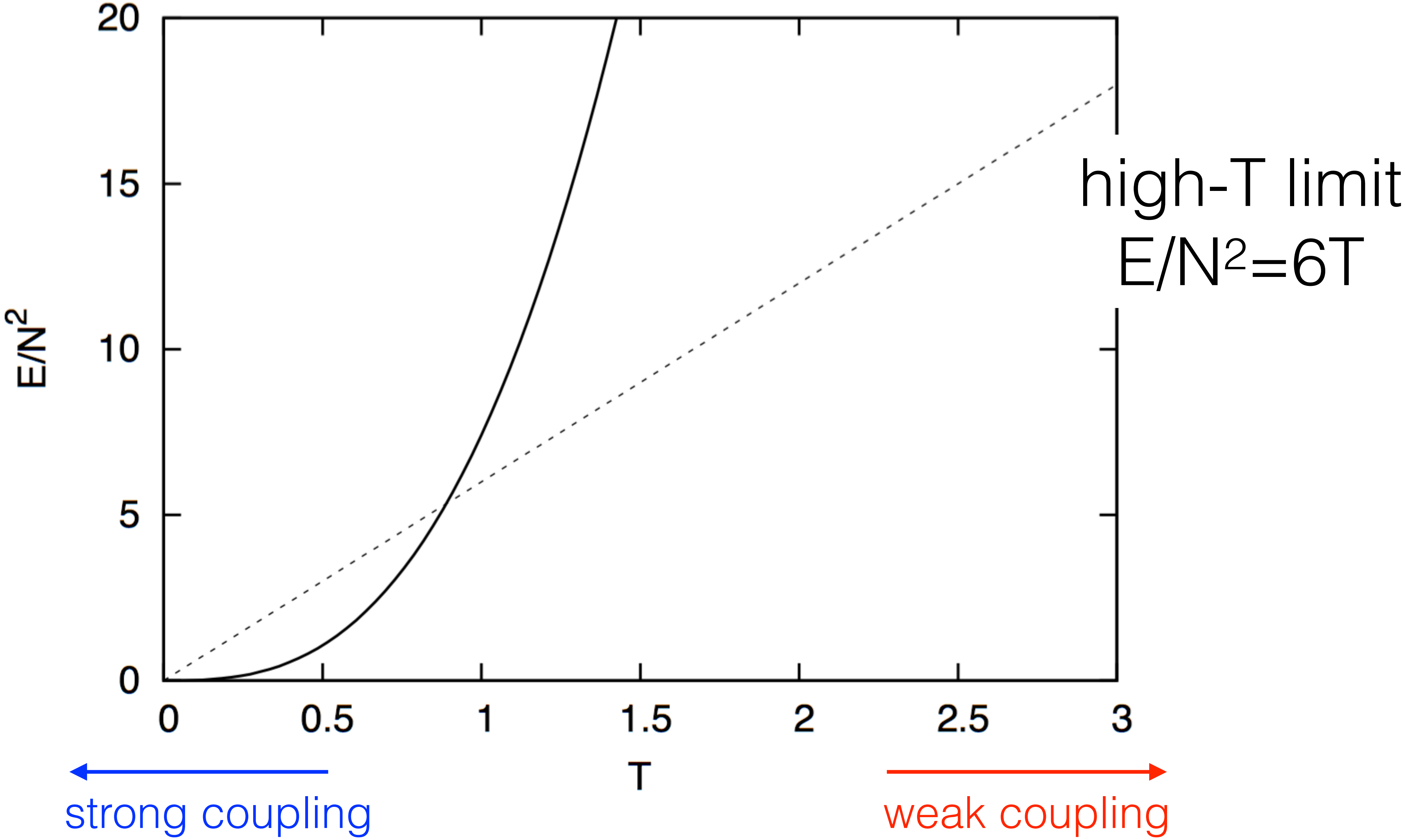
$$E = 7.4 N^2 \lambda^{-3/5} T^{14/5}$$

deconfined at any T>0 ($E \sim N^2$)

λ=1 from now on

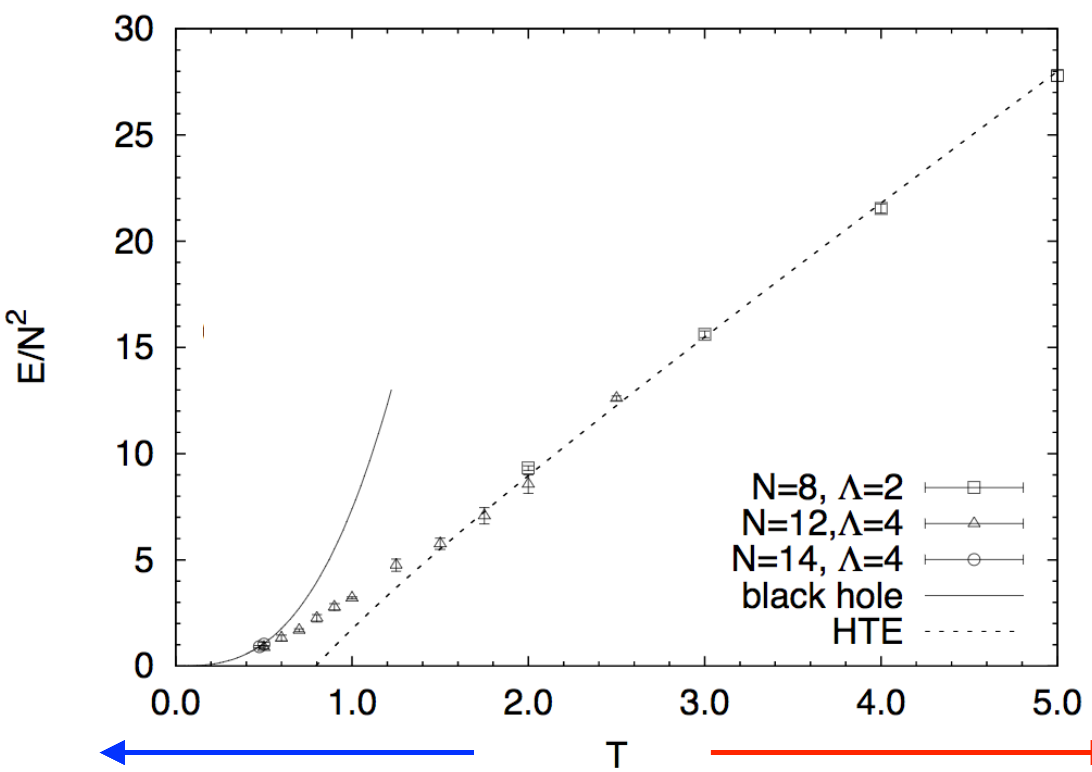
SUGRA (low-T limit)

$E/N^2 = 7.4T^{2.8}$



Let’s see how they are interpolated.

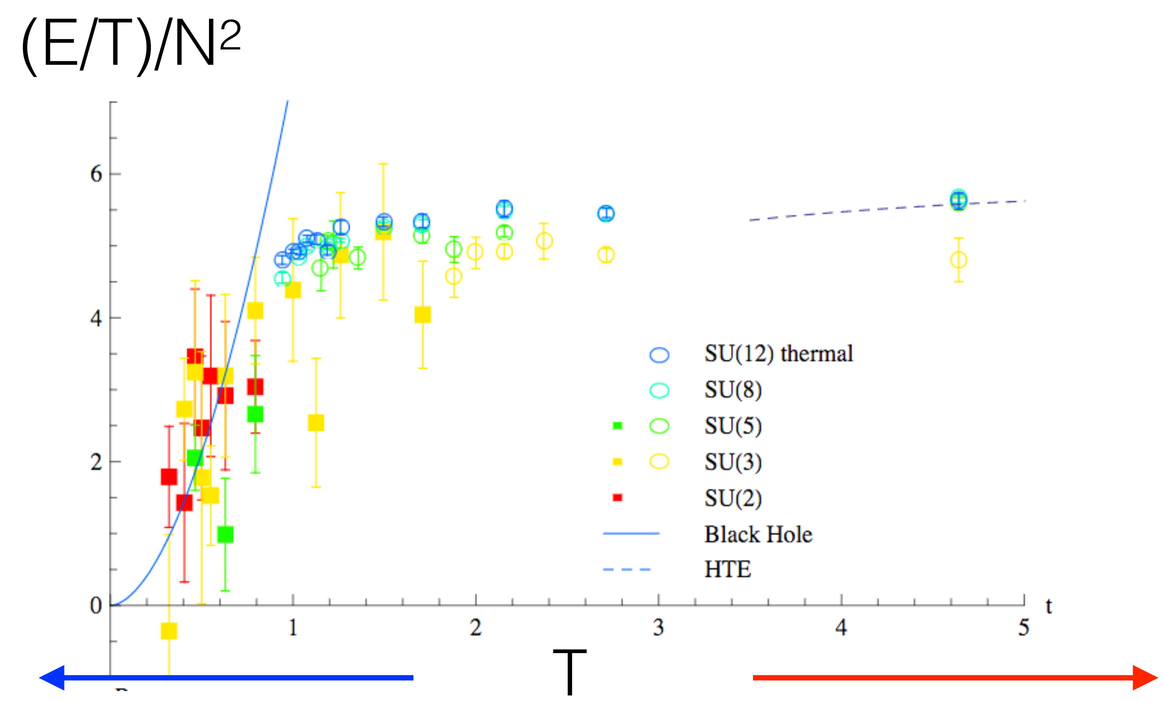
An earlier attempt
with a mean-field method:
Kabat-Lifschytz, 2001



strong coupling

weak coupling

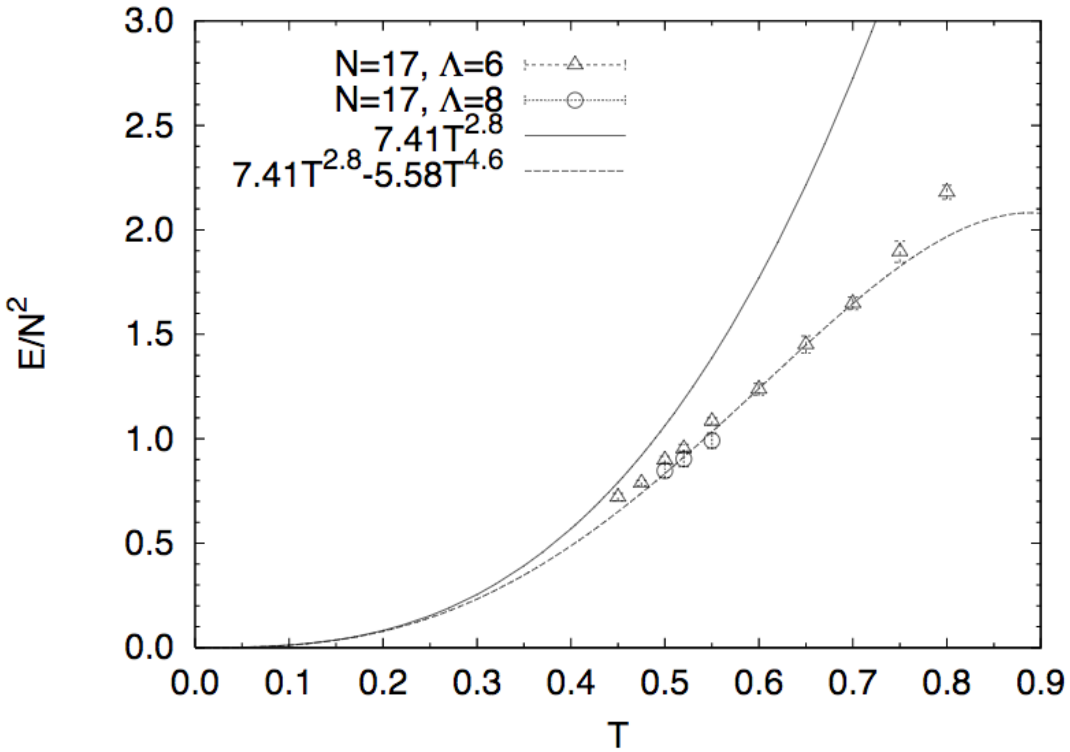
Anagnostopoulos-M.H.-Nishimura-Takeuchi, 0707.4454 [hep-th]



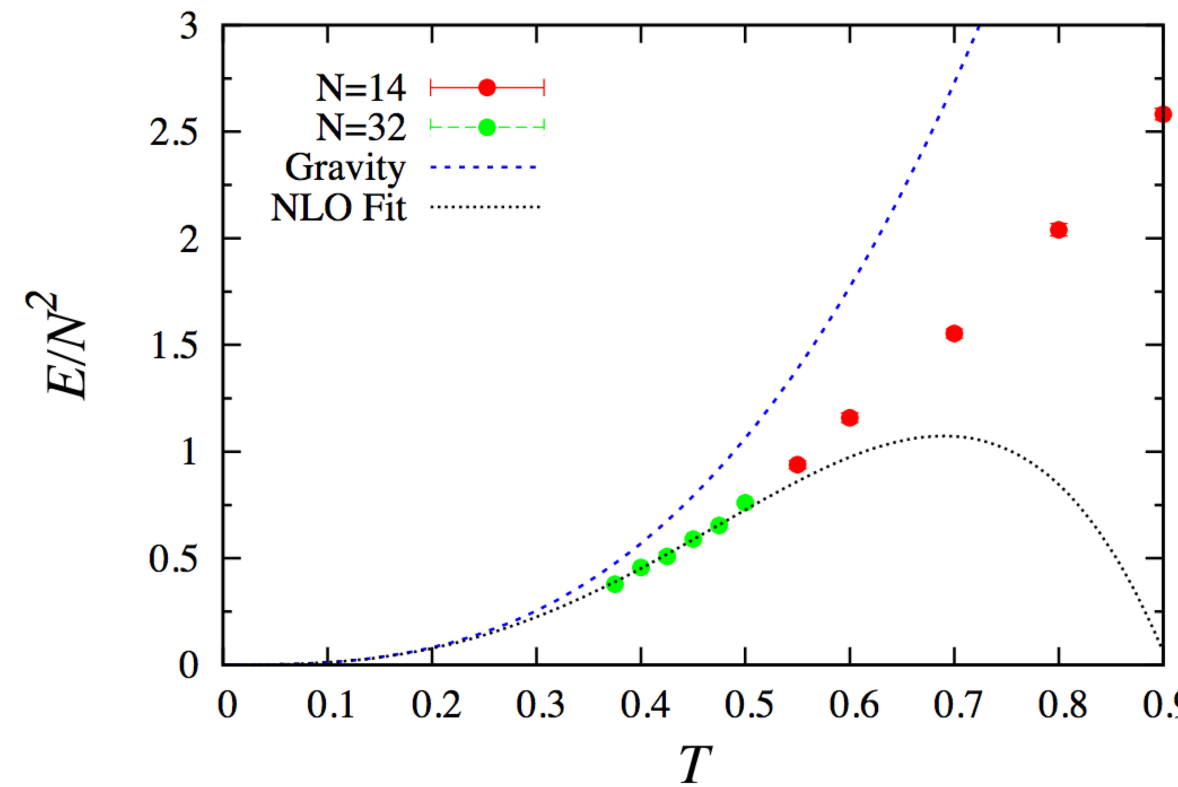
strong coupling

weak coupling

Catterall-Wiseman, 0803.4273 [hep-th]



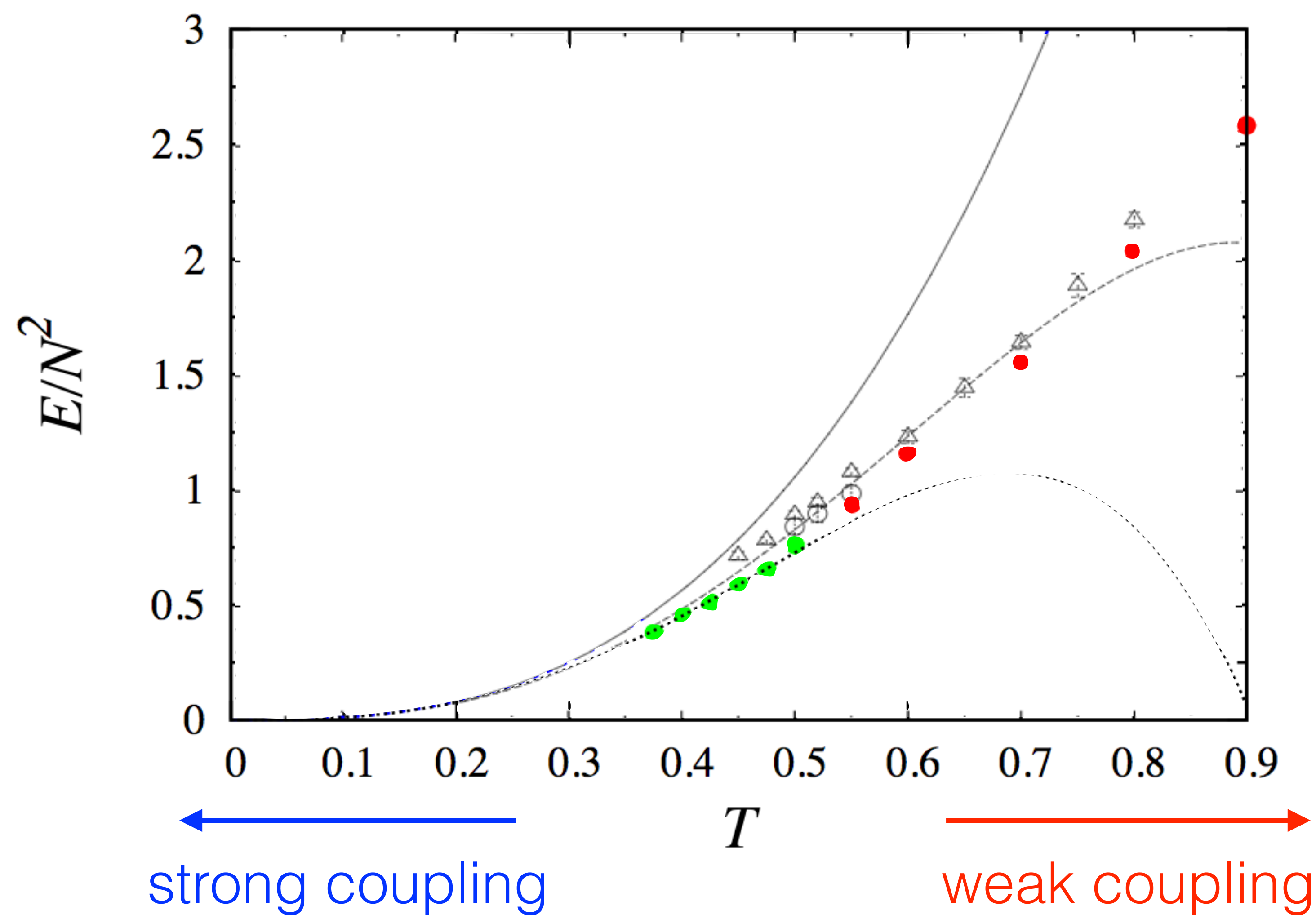
M.H.-Hyakutake-Nishimura-Takeuchi, 0811.3102 [hep-th]



Kadoh-Kamata, 1503.08499 [hep-lat]

M.H.-Hyakutake-Nishimura-Takeuchi, 0811.3102 [hep-th]

Kadoh-Kamata, 1503.08499 [hep-lat]



Some disagreement;

We need large-N and continuum result!

arXiv > hep-lat > arXiv:1606.04951

High Energy Physics – Lattice

[Submitted on 15 Jun 2016]

Precision lattice test of the gauge/gravity duality at large-N

Evan Berkowitz, Enrico Rinaldi, Masanori Hanada, Goro Ishiki, Shinji Shimasaki, Pavlos Vranas

arXiv > hep-th > arXiv:2210.04881

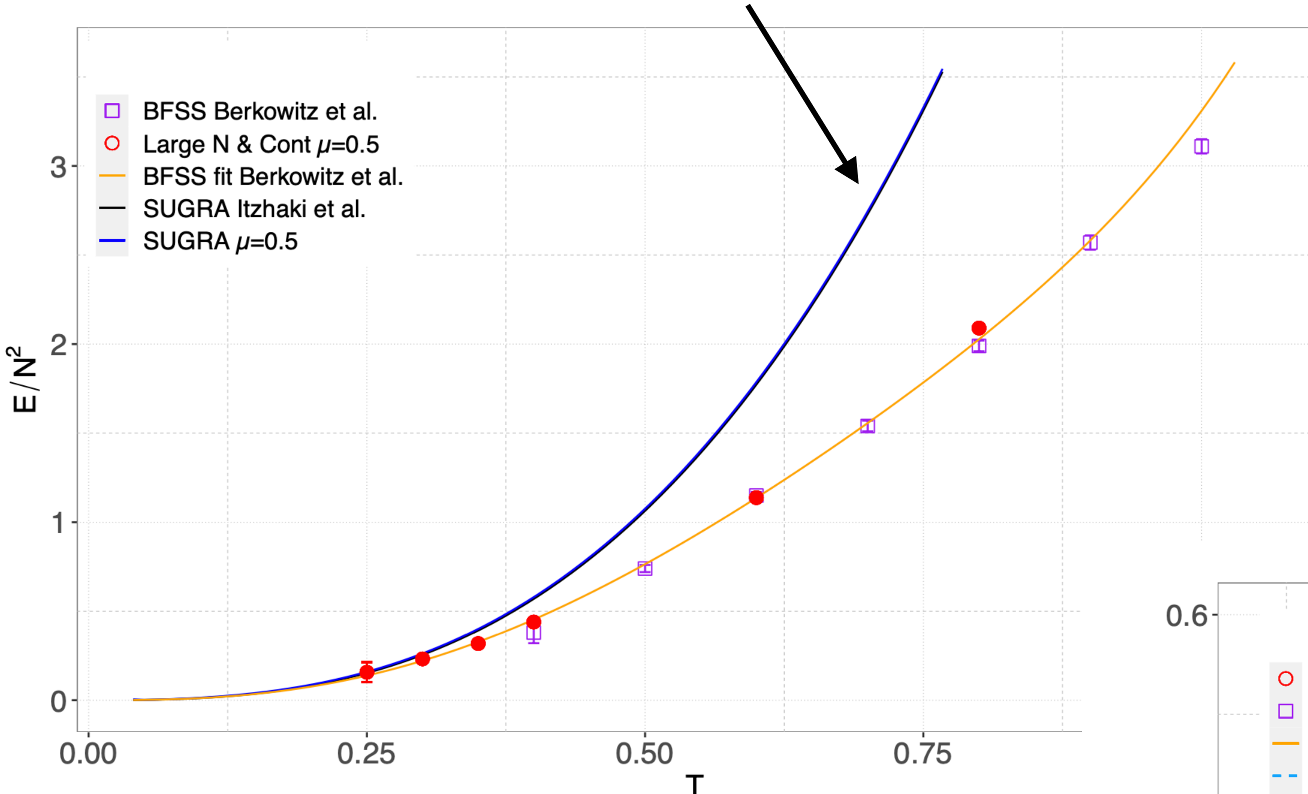
High Energy Physics – Theory

[Submitted on 10 Oct 2022]

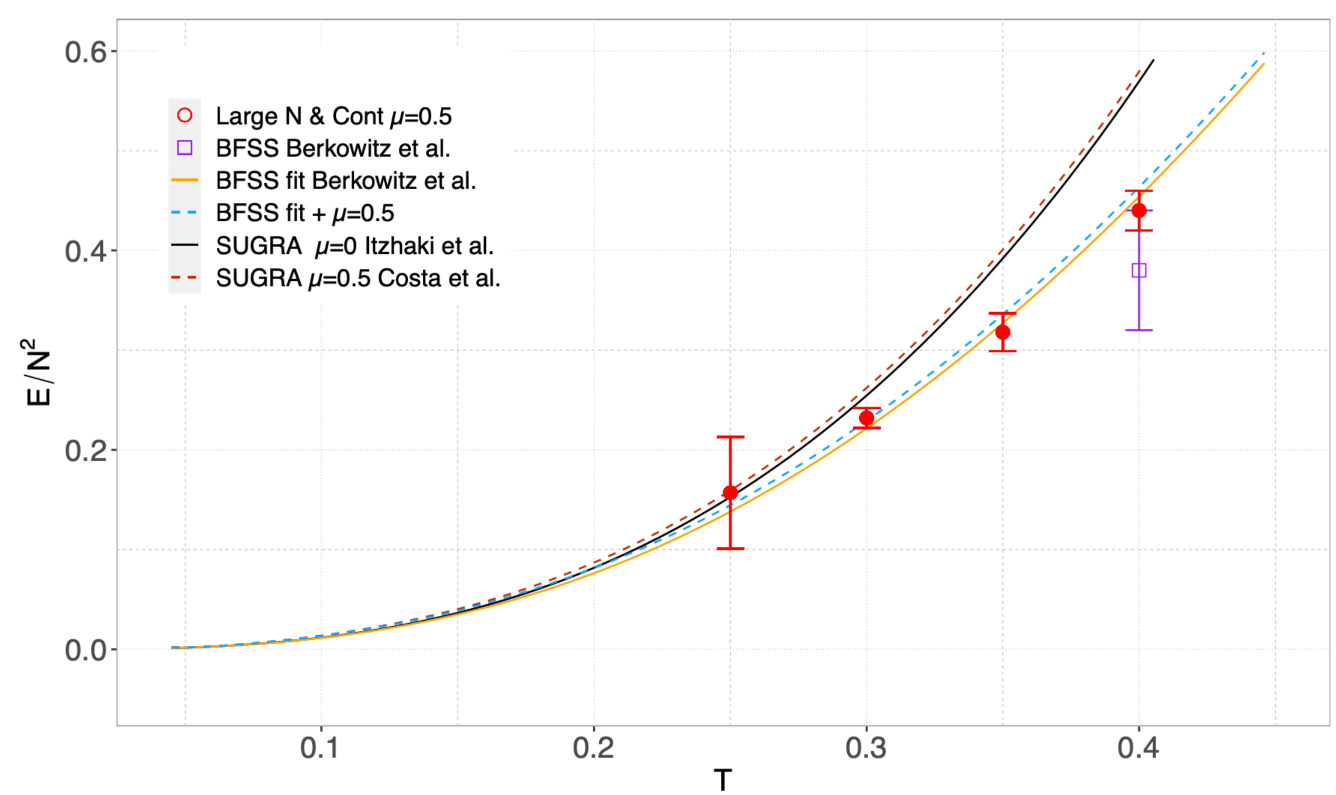
Precision test of gauge/gravity duality in D0-brane matrix model at low temperature

Stratos Pateloudis, Georg Bergner, Masanori Hanada, Enrico Rinaldi, Andreas Schäfer, Pavlos Vranas, Hiromasa Watanabe, Norbert Bodendorfer

Gravity dual (Itzhaki et al 1998, Costa et al 2014)



BMN matrix model was used in the 2022 paper



Flat direction

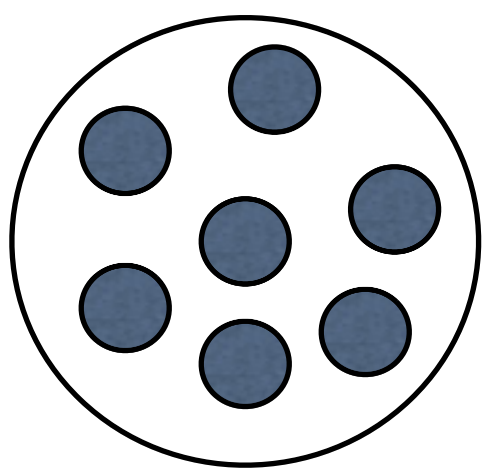
$$S = \frac{N}{\lambda} \int dt \; Tr\Big\{ \frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2$$
$$+ \frac{1}{2}\bar{\psi} D_t \psi - \frac{1}{2}\bar{\psi}\gamma^i [X_i, \psi] \Big\}$$

There is a flat direction even at quantum level.

$$[X_i, X_j] = 0$$

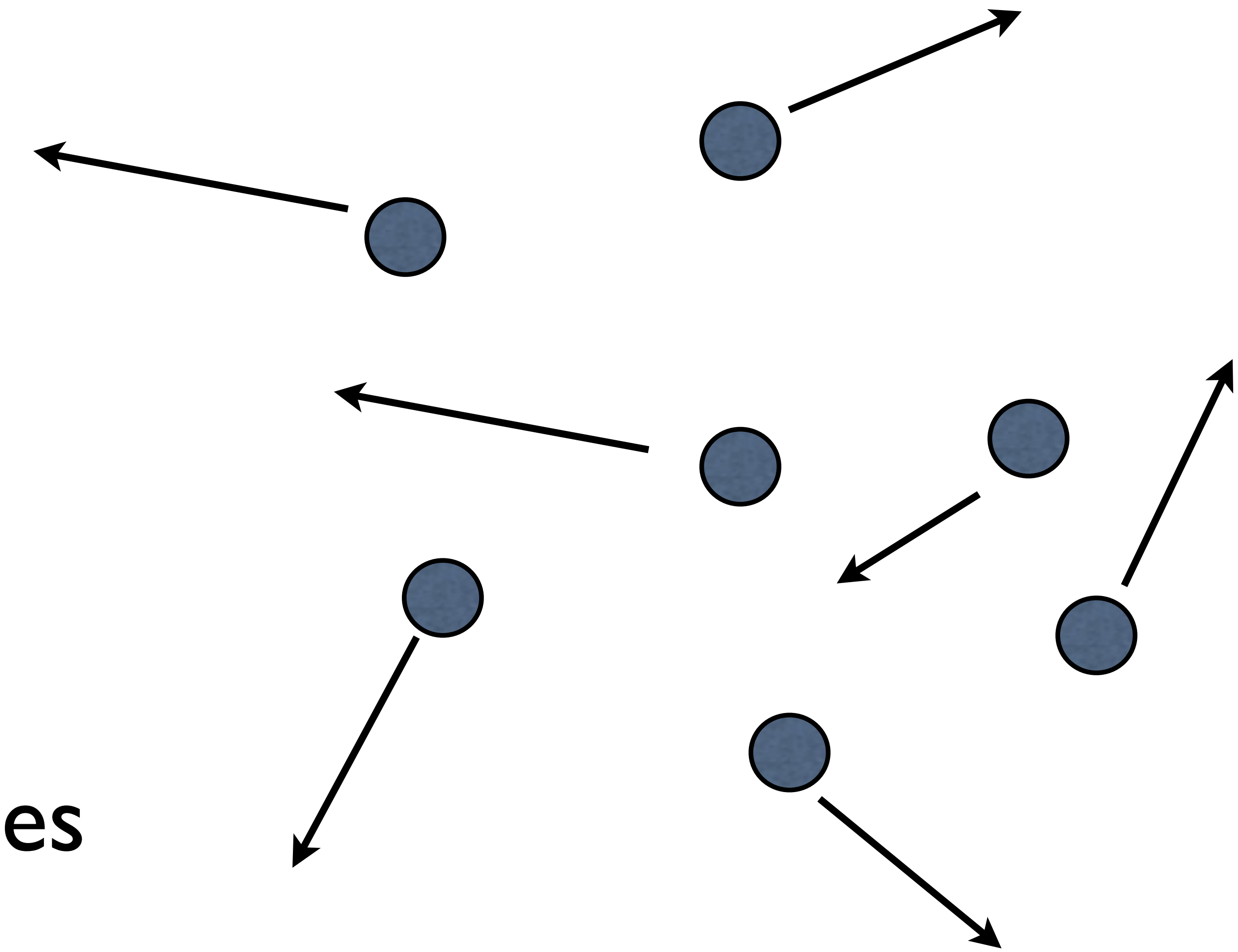
‘eigenvalues’ = position of D0-branes

(Witten, 1995)



bound state of eigenvalues
= black hole

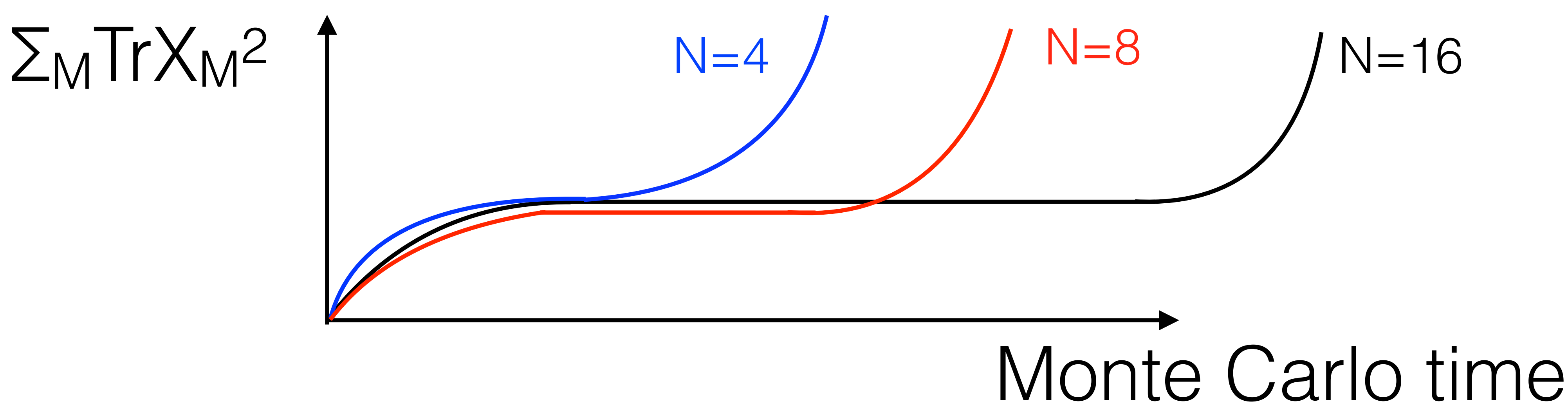
More stable at larger N



flat direction
$\sim$ gas of D0-branes

In string theory, this BH is stable at g_s=0.

In the gauge theory, bound state should become more stable as N becomes larger



(Flat direction is less serious in BMN matrix model)

Monte Carlo String/M-theory Collaboration (MCSMC)

arXiv > hep-lat > arXiv:1606.04951

High Energy Physics – Lattice

[Submitted on 15 Jun 2016]

Precision lattice test of the gauge/gravity duality at large-N

Evan Berkowitz, Enrico Rinaldi, Masanori Hanada, Goro Ishiki, Shinji Shimasaki, Pavlos Vranas

Enrico Rinaldi

Evan Berkowitz

arXiv > hep-th > arXiv:2210.04881

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High Energy Physics – Theory

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Precision test of gauge/gravity duality in D0-brane matrix model at low temperature

Stratos Pateloudis, Georg Bergner, Masanori Hanada, Enrico Rinaldi, Andreas Schäfer, Pavlos Vranas, Hiromasa Watanabe, Norbert Bodendorfer

Stratos Pateloudis

Norbert Bodendorfer

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Typically 256 –4096 core parallel

O(100) parameters (N, T, lattice size)

(rather modest compared to lattice QCD)

Enrico Rinaldi

Evan Berkowitz

Vulcan
(LLNL, Livermore, USA)

in 2019

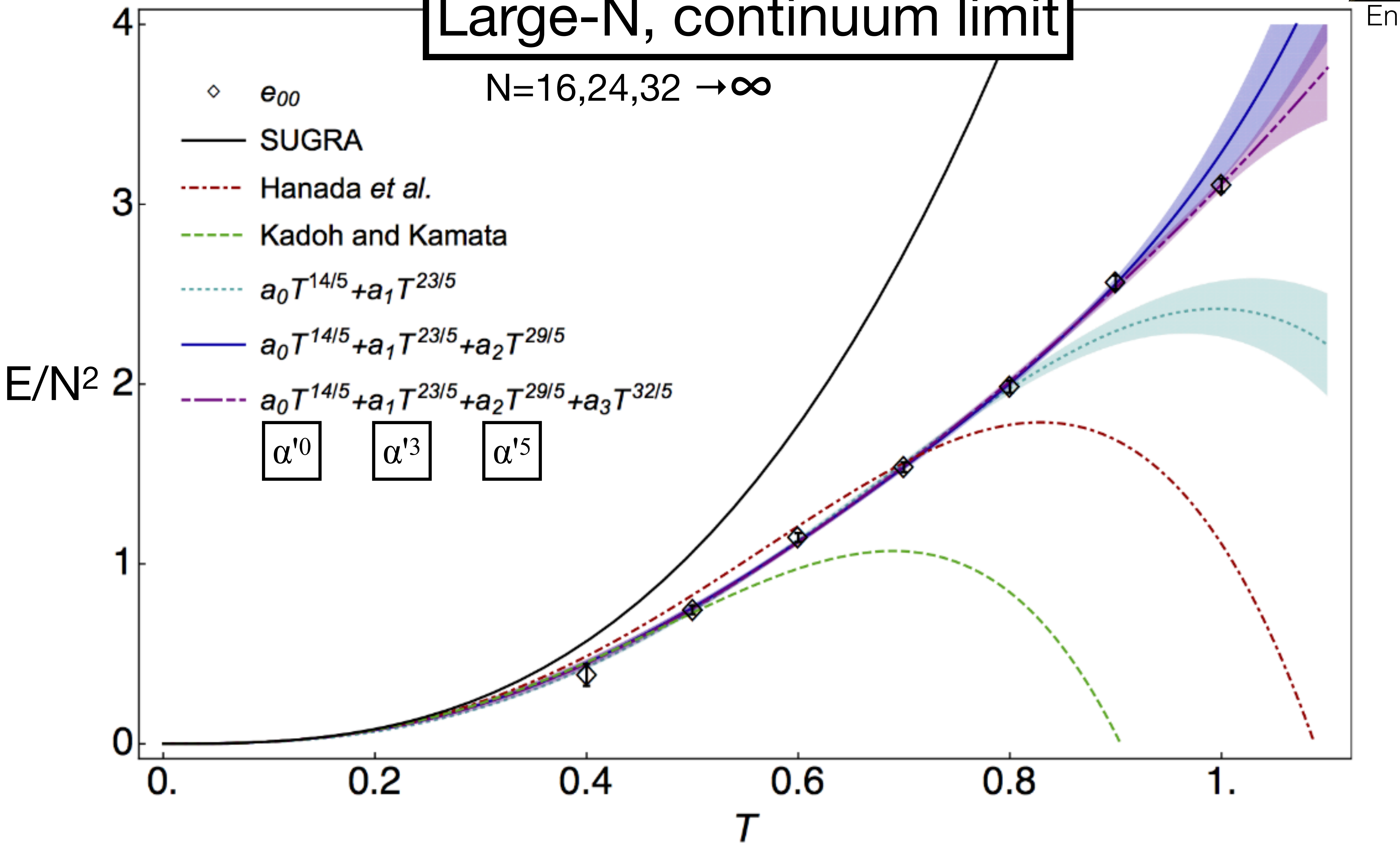
1606.04951[hep-lat]


Evan Berkowitz


Enrico Rinaldi

Large-N, continuum limit

N=16,24,32 →∞



← **strong coupling**

weak coupling → (α' corrections large)

SUGRA vs Matrix Model

$E/N^2 = aT^{14/5} + b\,T^{23/5} + c\,T^{29/5}$

3-parameter fit
(4-parameter is too much)

$a = 7.33 +/- 0.35$

1606.04951 [hep-lat] + a bit more data

$b = -10.0 +/- 0.4$
$c = 5.8 +/- 0.5$

$E/N^2 = 7.41T^{14/5} + b\,T^{23/5} + c\,T^{29/5} + \ldots + O(1/N^2)$

α'^0 α'^3 α'^5

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Von Neumann's elephant

1 language

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From Wikipedia, the free encyclopedia

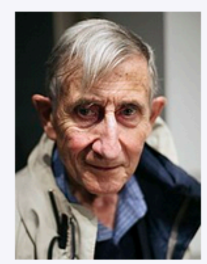
Von Neumann's elephant is a problem in recreational mathematics, consisting of constructing a planar curve in the shape of an elephant from only four fixed parameters. It originated from a discussion between physicists John von Neumann and Enrico Fermi.

History [edit]

In a 2004 article in the journal *Nature*, Freeman Dyson recounts his meeting with Fermi in 1953. Fermi evokes his friend von Neumann who, when asking him how many arbitrary parameters he used for his calculations, replied, "With four parameters I can fit an elephant, and with five I can make him wiggle his trunk." By this he meant that the Fermi simulations relied on too many input parameters, presupposing an overfitting phenomenon.[1]

John von Neumann

Enrico Fermi



Freeman Dyson in 2005

Solving the problem (defining four complex numbers to draw an elephantine shape) subsequently became an active research subject of recreational mathematics. A 1975 attempt through least-squares function approximation required dozens of terms.[2] The best approximation was found by three physicists in 2010.[3]

"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk."

(So we shouldn't introduce too many fit parameters.)

AAPT PHYSICS EDUCATION

American Journal of Physics

Volume 78, Issue 6
June 2010

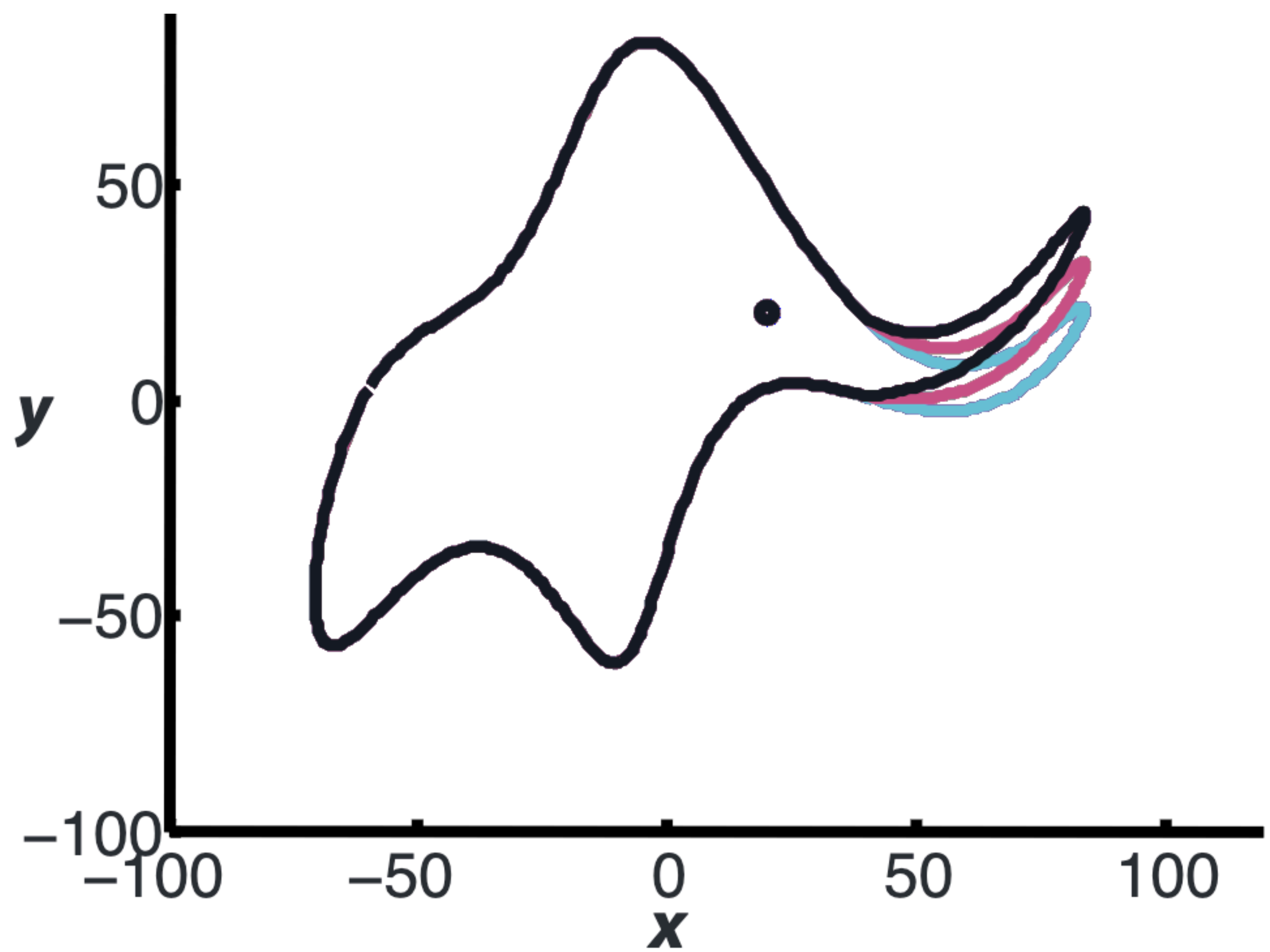
JUNE 01 2010

Drawing an elephant with four complex parameters

Jürgen Mayer; Khaled Khairy; Jonathon Howard

American Journal of Physics 78, 648–649 (2010)
https://doi.org/10.1119/1.3254017 **Article history**

We define four complex numbers representing the parameters needed to specify an elephantine shape. The real and imaginary parts of these complex numbers are the coefficients of a Fourier coordinate expansion, a powerful tool for reducing the data required to define shapes.



STRING vs Matrix Model

$E/N^2 = 7.41 T^{14/5} + b\, T^{p} + c\, T^{p+6/5}$ 3-parameter fit

(4-parameter is too much)

$p = 4.6 +/- 0.3$

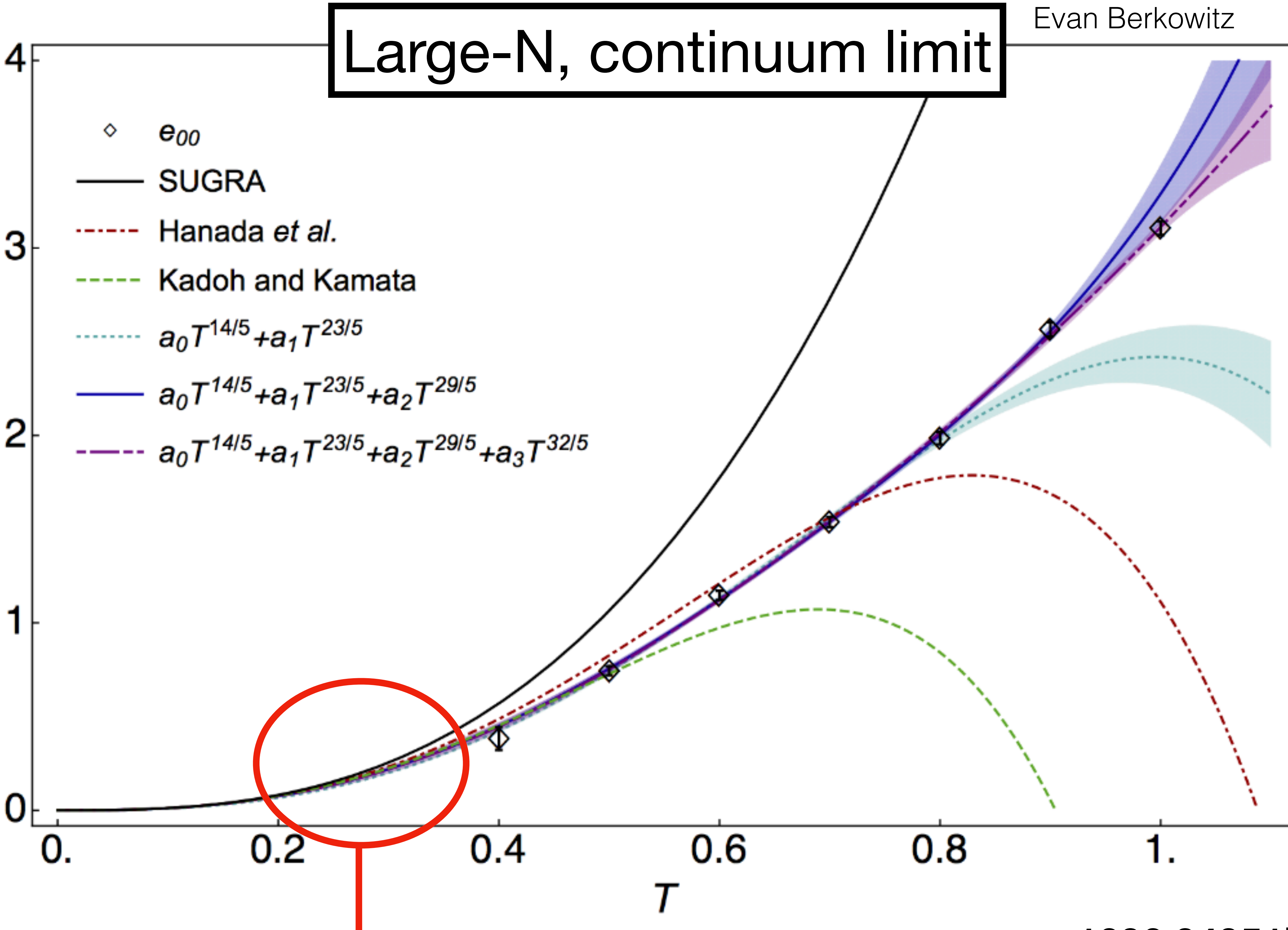
1606.04951 [hep-lat] + a bit more data

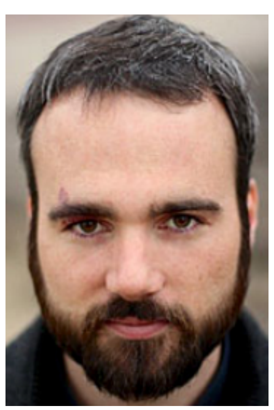
$E/N^2 = 7.41 T^{14/5} + b\, T^{23/5} + c\, T^{29/5} + \ldots + O(1/N^2)$

α'^{0} α'^{3} α'^{5}

Evan Berkowitz

Enrico Rinaldi





Norbert Bodendorfer

Stratos Pateloudis

Added points here.

1606.04951[hep-th]

arXiv > hep-lat > arXiv:1606.04951

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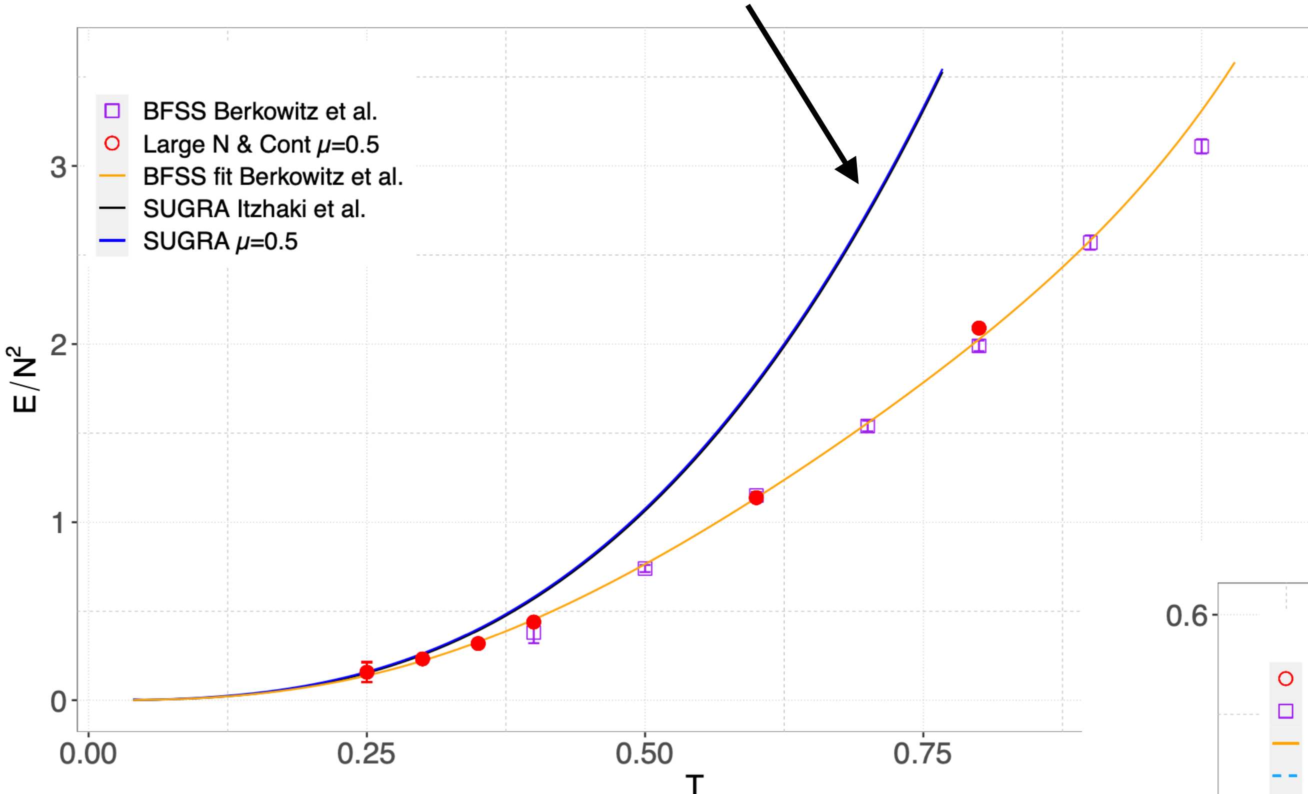
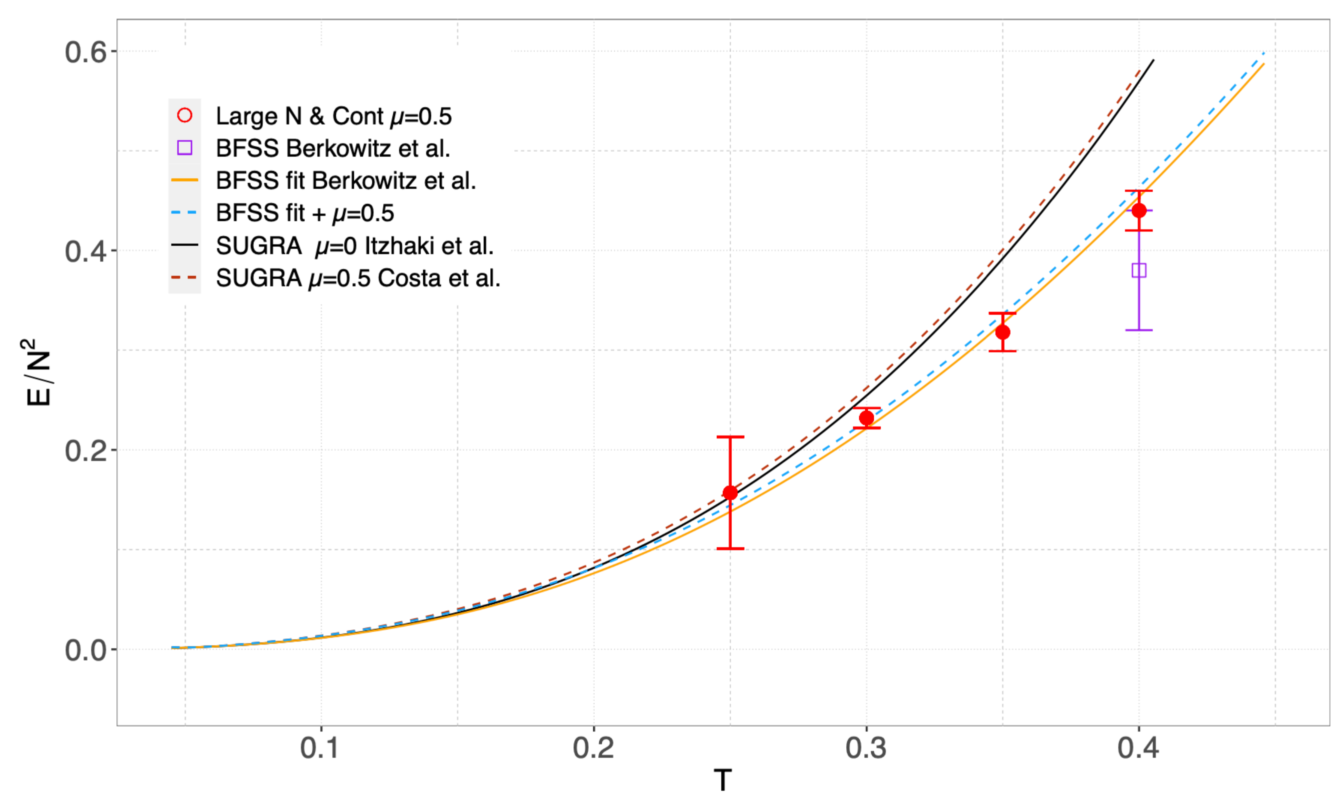


BMN matrix model was used in the 2022 paper



BMN matrix model

Berenstein-Maldacena-Nastase, 2002

$$S = \underbrace{S_b + S_f}_{\text{BFSS}} + \Delta S_b + \Delta S_f$$

$$S_b = \frac{N}{\lambda}\int_0^\beta dt\ \mathrm{Tr}\left\{\frac{1}{2}\sum_{I=1}^{9}(D_t X_I)^2 - \frac{1}{4}\sum_{I,J=1}^{9}[X_I, X_J]^2\right\} ,$$

$$S_f = \frac{N}{\lambda}\int_0^\beta dt\ \mathrm{Tr}\left\{i\bar{\psi}\gamma^{10}D_t\psi - \sum_{I=1}^{9}\bar{\psi}\gamma^I[X_I,\psi]\right\} ,$$

$$\Delta S_b = \frac{N}{\lambda}\int_0^\beta dt\ \mathrm{Tr}\left\{\frac{\mu^2}{2}\sum_{i=1}^{3}X_i^2 + \frac{\mu^2}{8}\sum_{a=4}^{9}X_a^2 + i\sum_{i,j,k=1}^{3}\mu\epsilon^{ijk}X_iX_jX_k\right\} ,$$

$$\Delta S_f = \frac{3i\mu}{4\lambda}\cdot N\int_0^\beta dt\ \mathrm{Tr}\left(\bar{\psi}\gamma^{123}\psi\right) .$$

BMN matrix model

Berenstein-Maldacena-Nastase, 2002

- Supersymmetric deformation of BFSS.
- Flat direction is lifted.
- Various fuzzy sphere vacua exist.

$$X_i = \frac{\mu}{3} J_i \ (i = 1,2,3) \ , \qquad X_a = 0 \ (a = 4, \cdots, 9) \ , \qquad \psi = 0$$

SU(2) generator

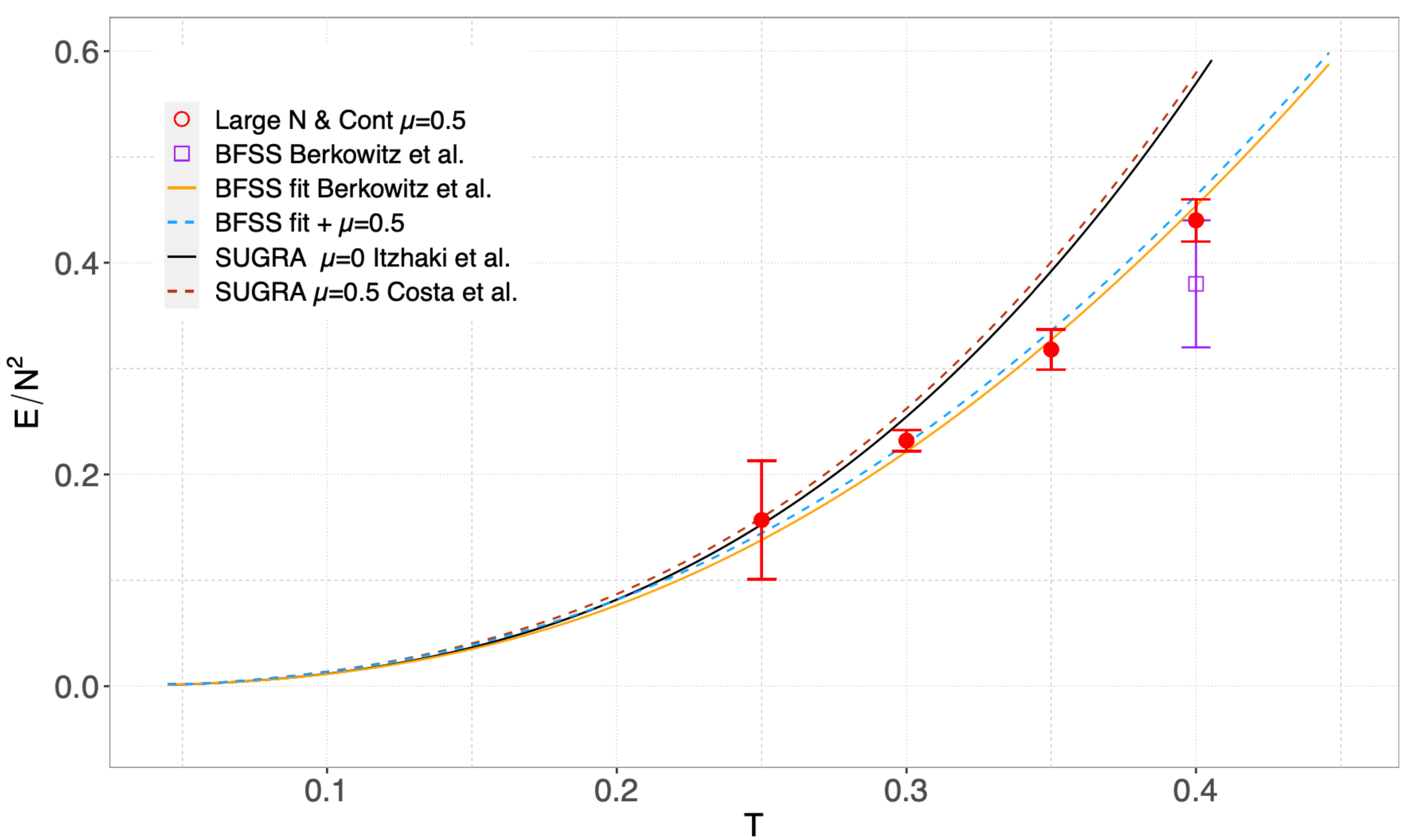
(+ quantum fluctuation)

- We study 'trivial' vacuum.

$$X_1 = X_2 = \cdots = X_9 = 0 \ , \qquad \psi = 0$$

(+ quantum fluctuation)

- Flat direction is tamed rather well.
- Smaller N can be used; simulation cost is smaller.
- Finite-μ effect is very small at μ < 1 (SUGRA: Costa, Greenspan, Penedones, Santos 2014)



Outline

- Precision test via Monte Carlo simulation
- New phase: "confinement" at low energy
- Confinement ~ M-theory ? (somewhat speculative)
- Toward quantum simulation

  (backup slides)

Polyakov loop $P = 0$

Energy $E/N^2 = 0$

arXiv > hep-th > arXiv:2110.01312

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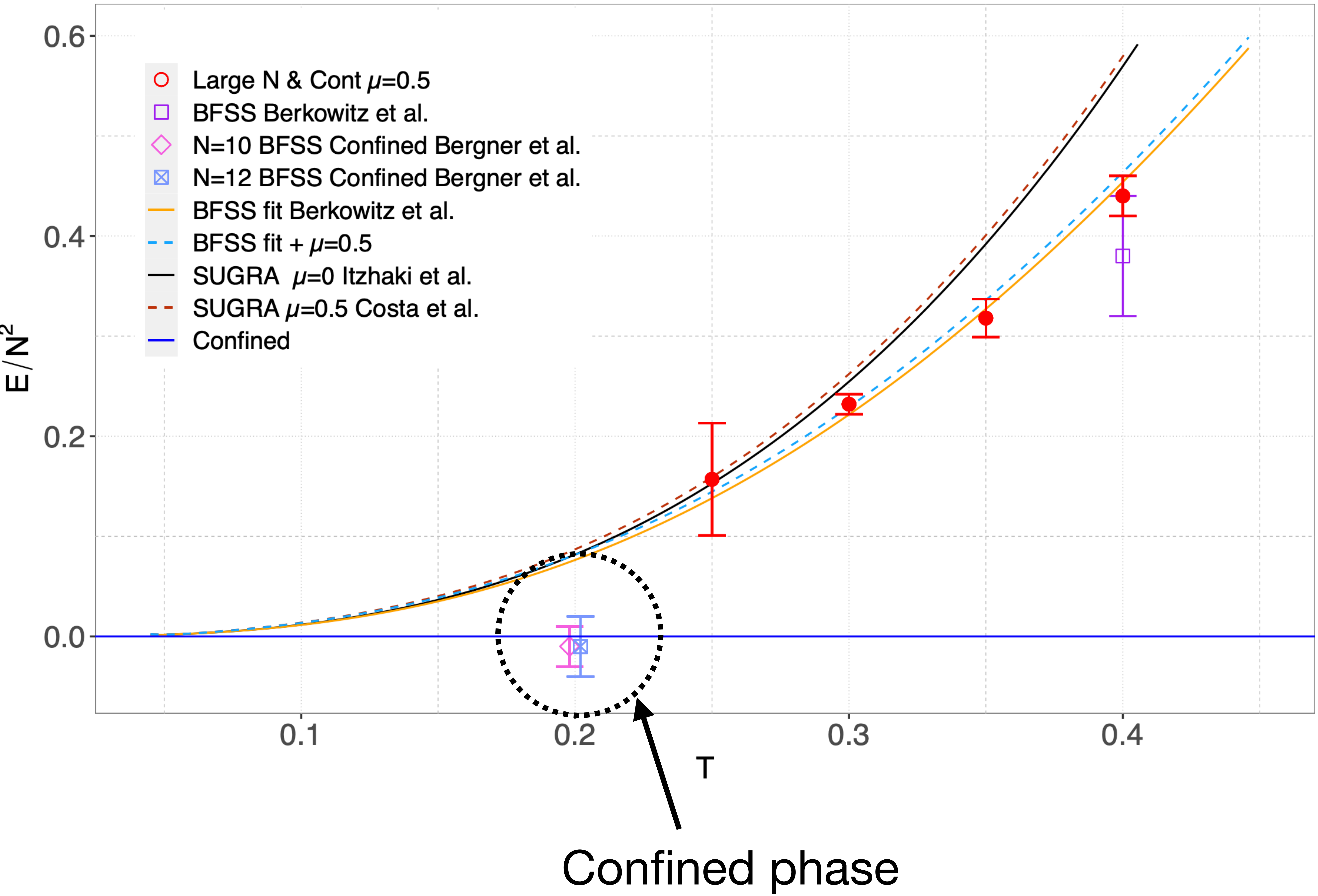
High Energy Physics – Theory

[Submitted on 4 Oct 2021 (v1), last revised 18 May 2022 (this version, v2)]

Confinement/deconfinement transition in the D0-brane matrix model -- A signature of M-theory?

Georg Bergner, Norbert Bodendorfer, Masanori Hanada, Stratos Pateloudis, Enrico Rinaldi, Andreas Schäfer, Pavlos Vranas, Hiromasa Watanabe


0.6
0.4
0.2
0.0
0.1
0.2
0.3
0.4
T
E/N^2
Large N & Cont μ=0.5
BFSS Berkowitz et al.
N=10 BFSS Confined Bergner et al.
N=12 BFSS Confined Bergner et al.
BFSS fit Berkowitz et al.
BFSS fit + μ=0.5
SUGRA μ=0 Itzhaki et al.
SUGRA μ=0.5 Costa et al.
Confined
Confined phase

Confined phase

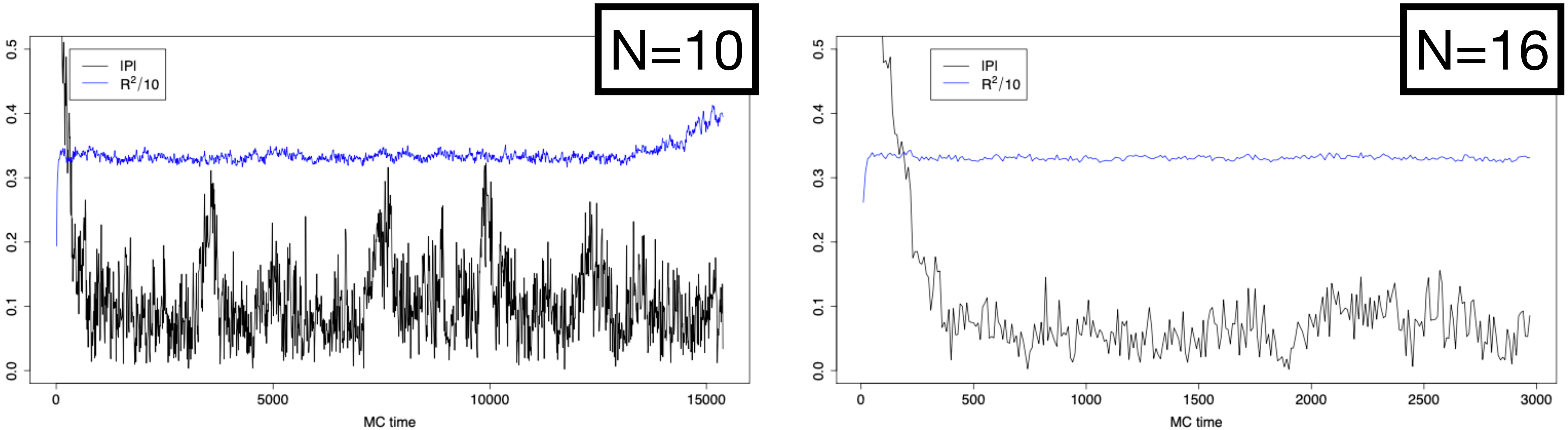
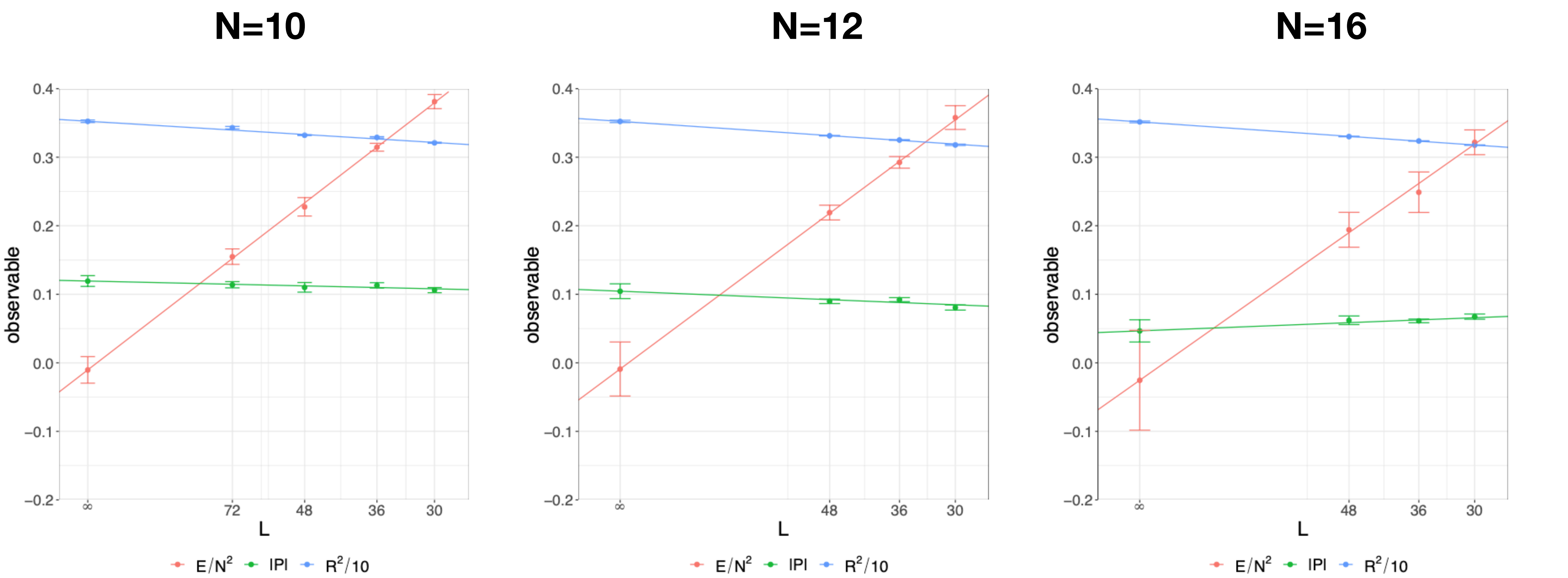


Figure 27: Monte Carlo histories from cold starts ($X_1 = X_2 = \cdots = X_9 = 0$) for $\mu = 0$, $T = 0.2$, $L = 48$, $N = 10$ (left) and $N = 16$ (right). For $N = 10$, the onset of the run-away behavior (i.e., the increase of R^2) can be seen at late time.

Confined phase is more stable

$$R^2 = \frac{1}{N\beta}\int_0^\beta dt \sum_{I=1}^{9} \mathrm{Tr} X_I^2 \qquad P = \frac{1}{N}\mathrm{Tr}\left(\mathcal{P}\exp\left(i\int_0^\beta dt A_t\right)\right)$$

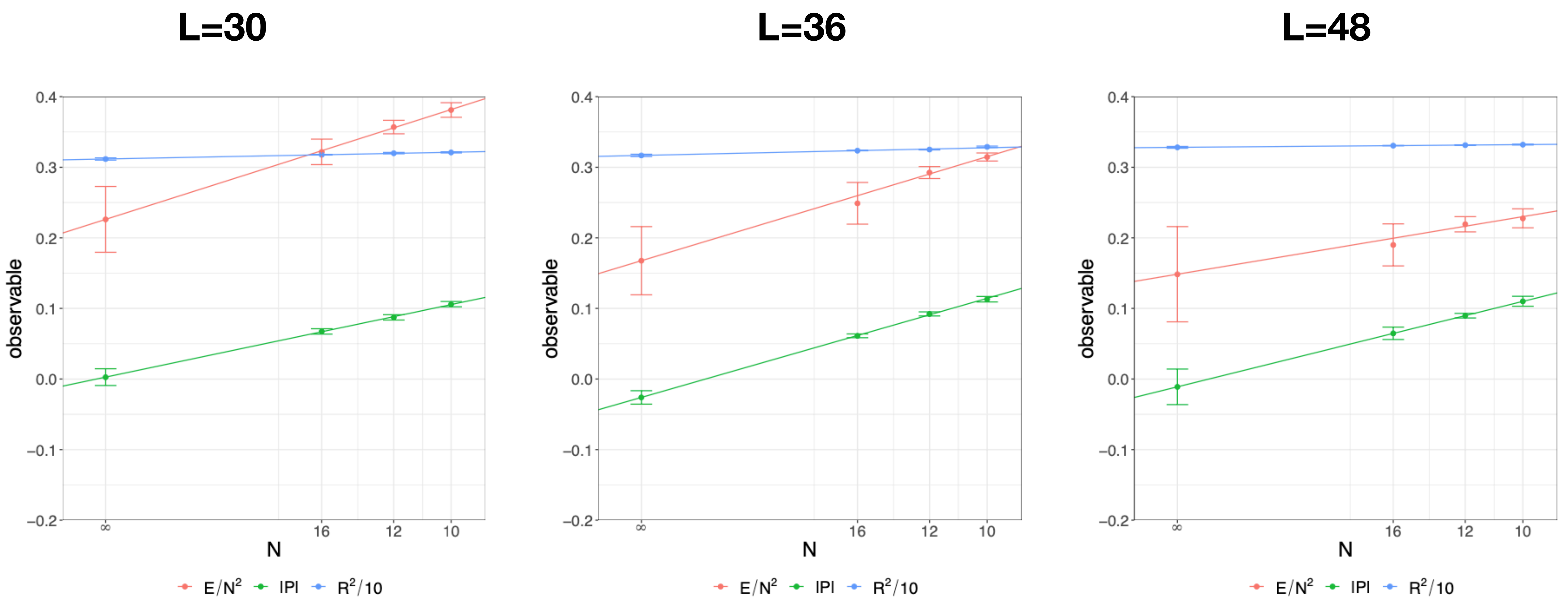
Confined phase



E/N² → 0 as L = # of lattice points → ∞

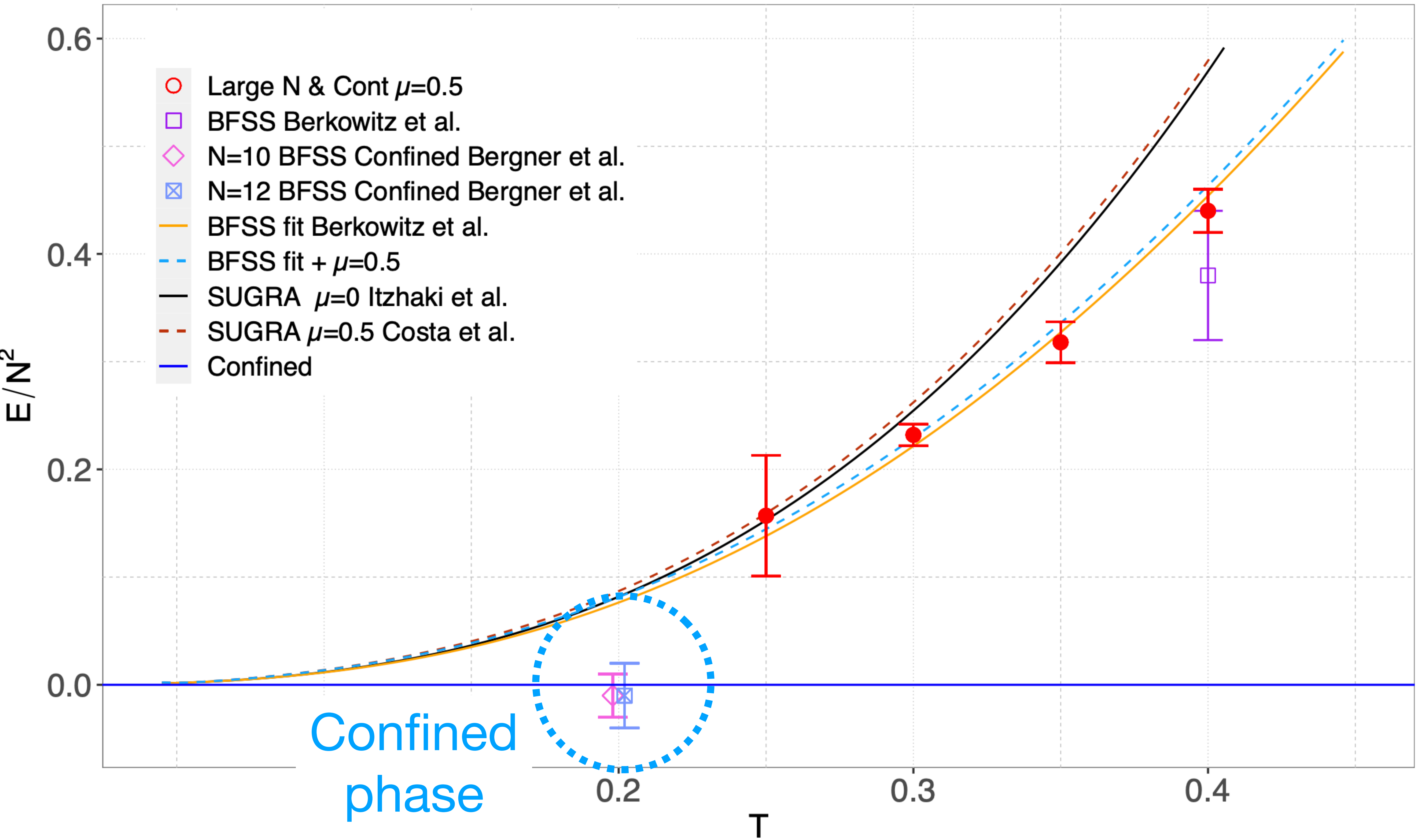
Deconfined phase: $\frac{E}{N^2} \simeq 7.41 T^{14/5} \simeq 0.0818$ at $T = 0.2$

Confined phase



$P \to 0$ as $N \to \infty$

Deconfined phase: $P \simeq 0.51$ at $T = 0.2$ (extrapolation from higher T)



Large N & Cont μ=0.5
BFSS Berkowitz et al.
N=10 BFSS Confined Bergner et al.
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BFSS fit Berkowitz et al.
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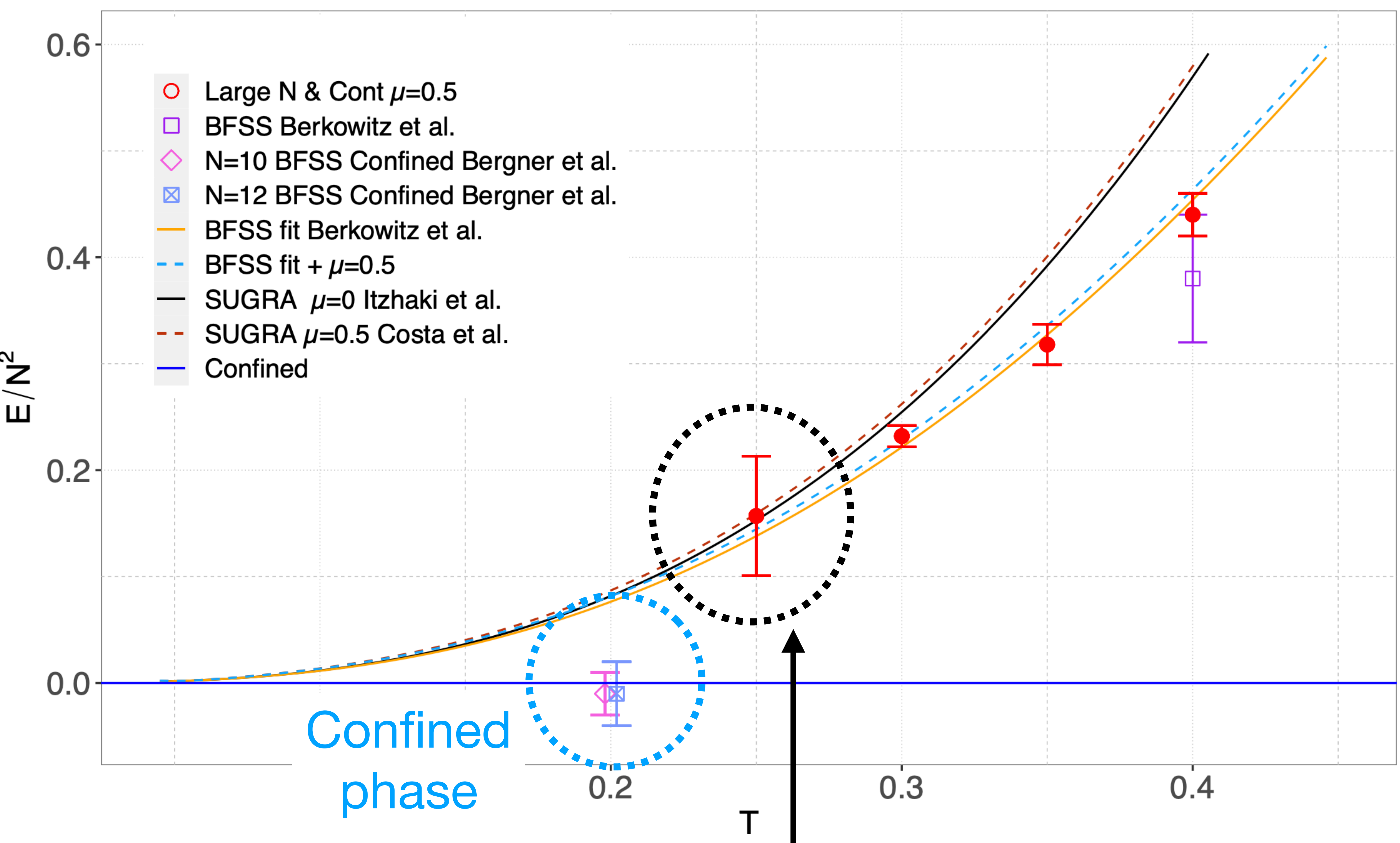
Confined
E/N^2
T
0.6
0.4
0.2
0.0
0.2
0.3
0.4
Confined phase



Tunneling between confined and deconfined phases was observed.
Only deconfined configurations were used.

Outline

- Precision test via Monte Carlo simulation
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  (backup slides)

arXiv > hep-th > arXiv:2110.01312

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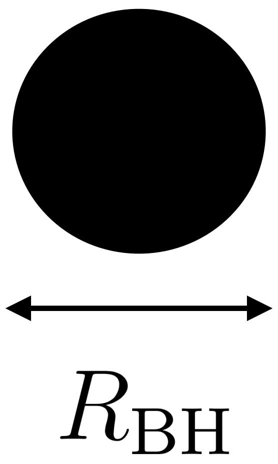
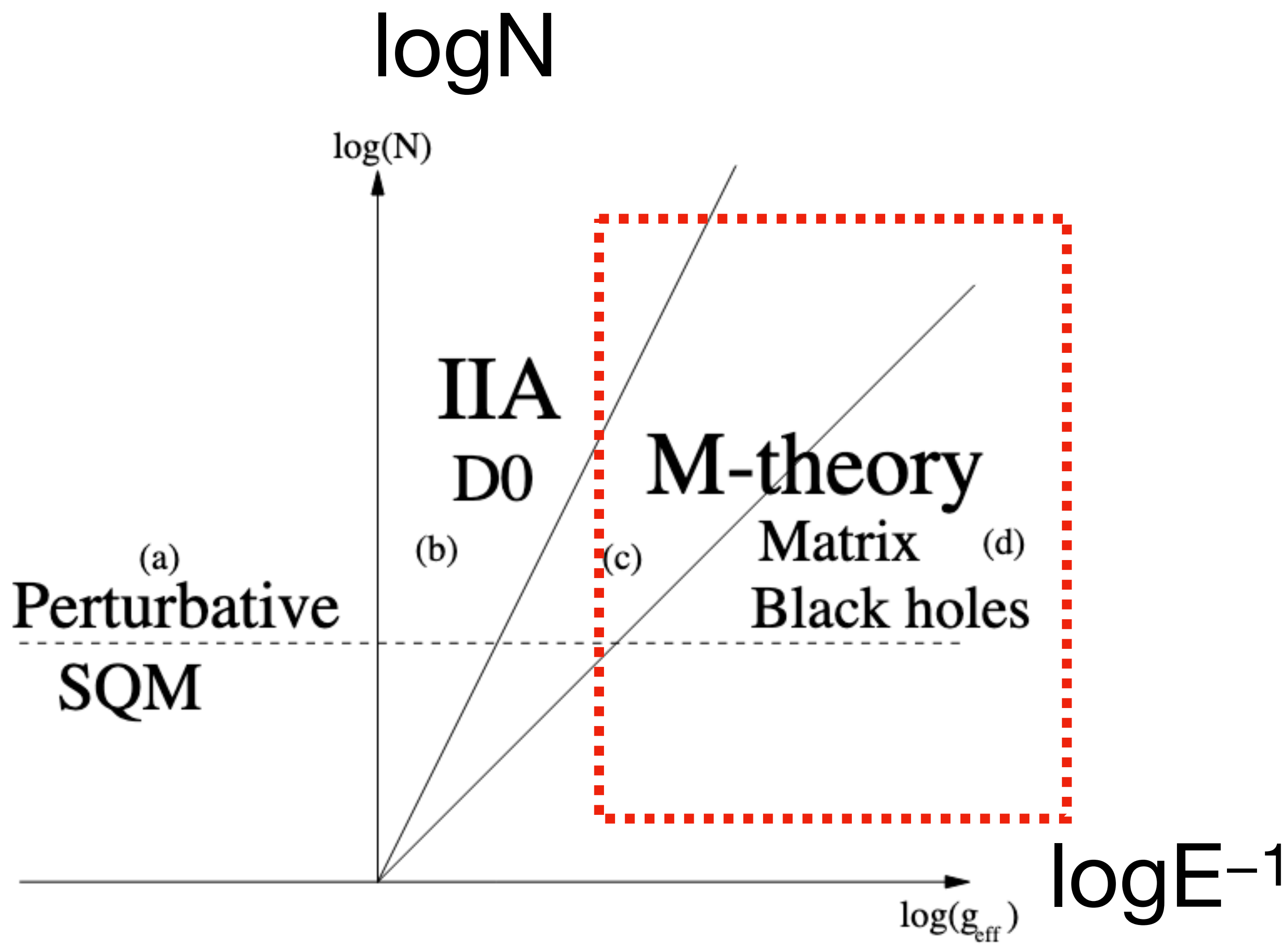
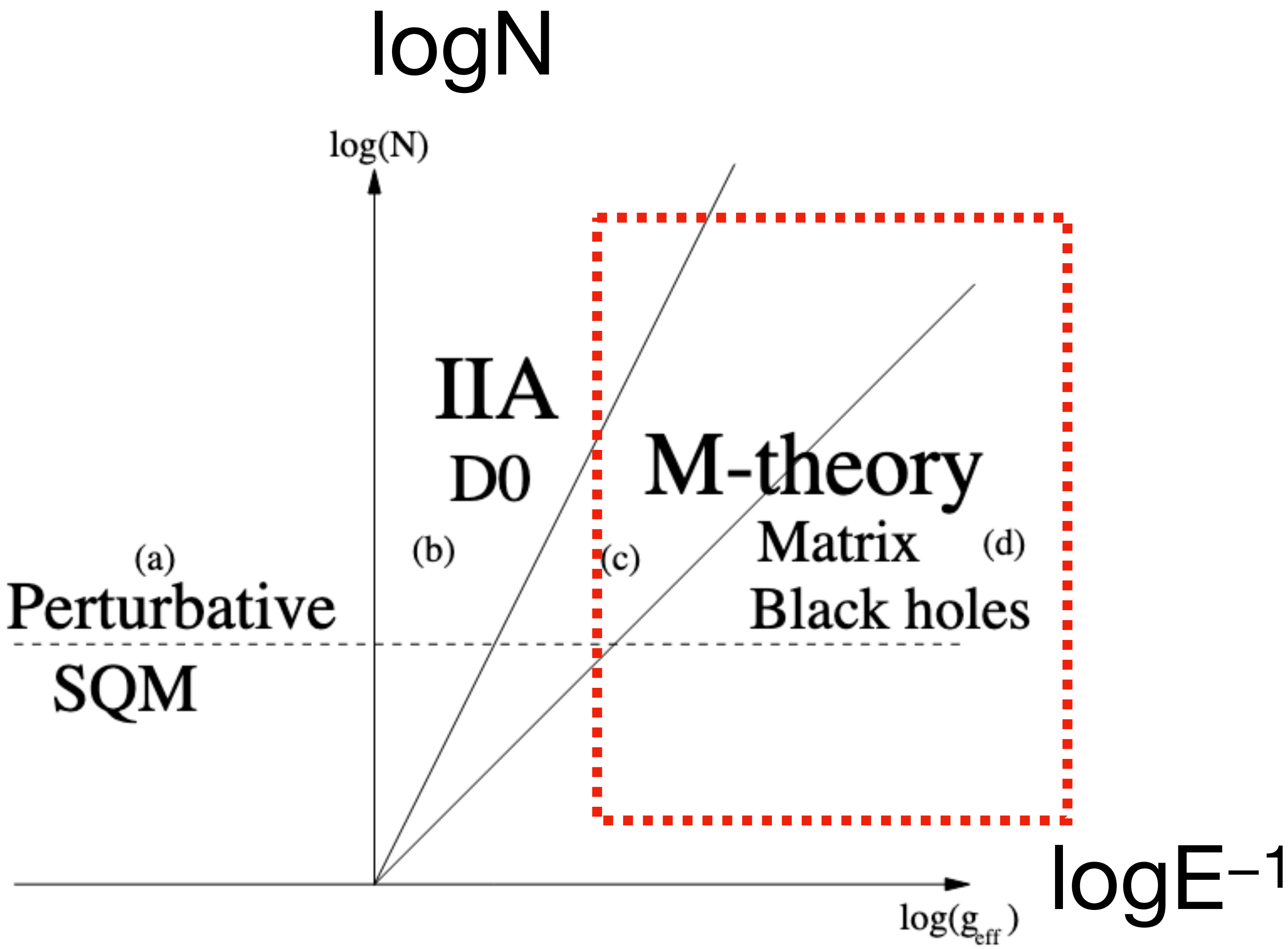


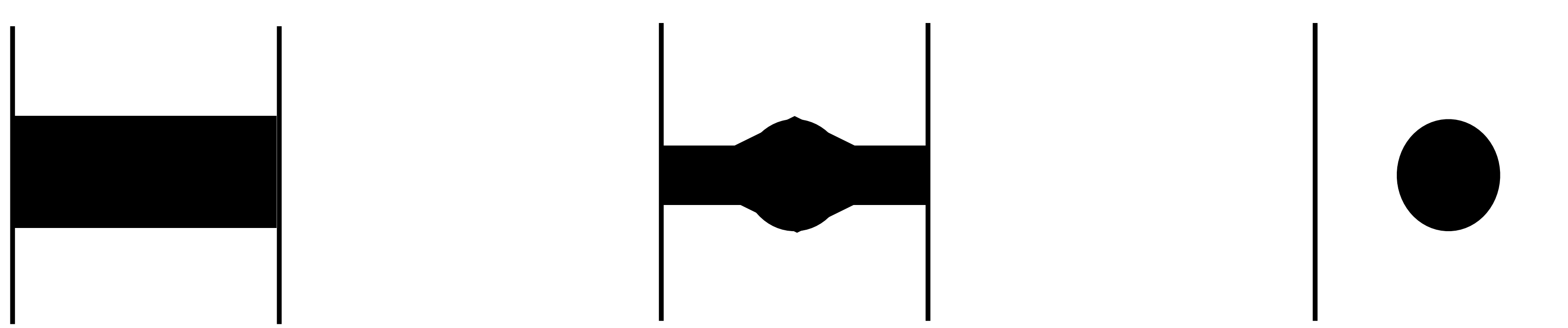
$$R_{\rm BH} \text{ small} \leftrightarrow g_{\rm eff} \text{ large}$$

$$e^{\phi}\Big|_{\rm horizon} \sim \frac{g_{\rm eff}^{7/2}}{N} \sim \frac{1}{N}\left(\frac{\lambda^{-1/3}E}{N^2}\right)^{-3/4}$$



(picture from Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)





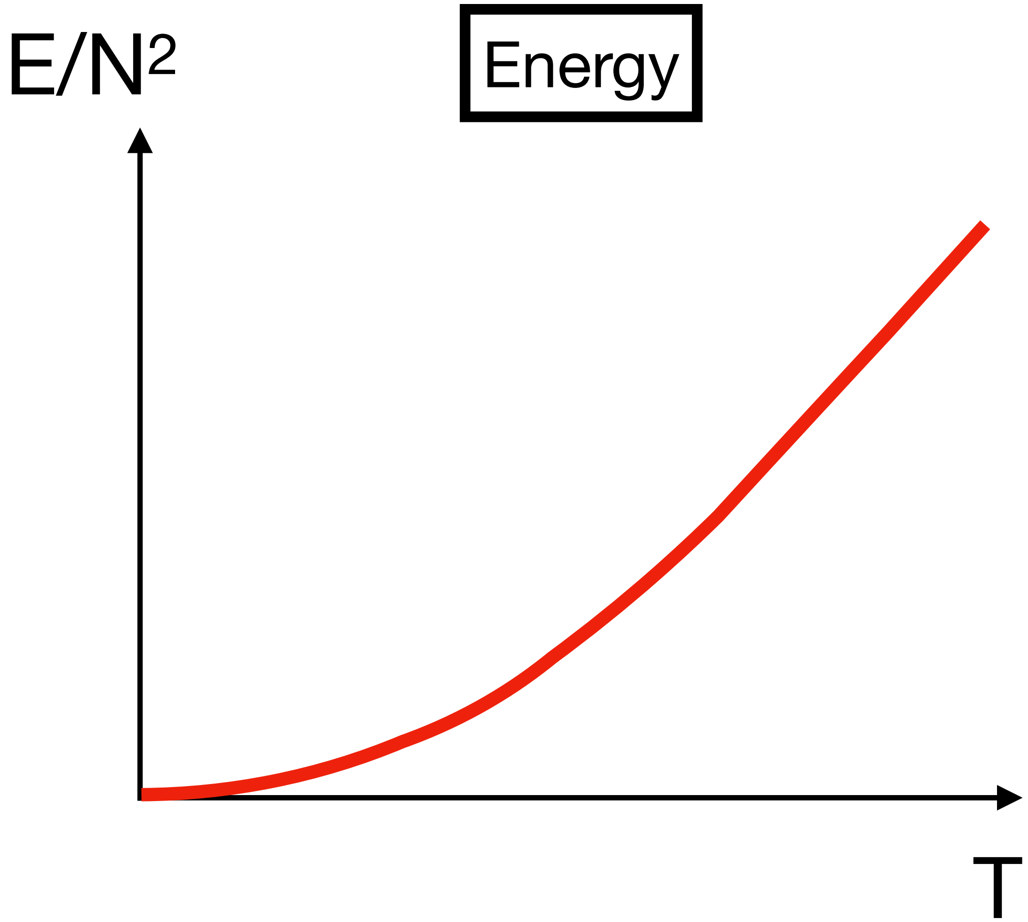
uniform string

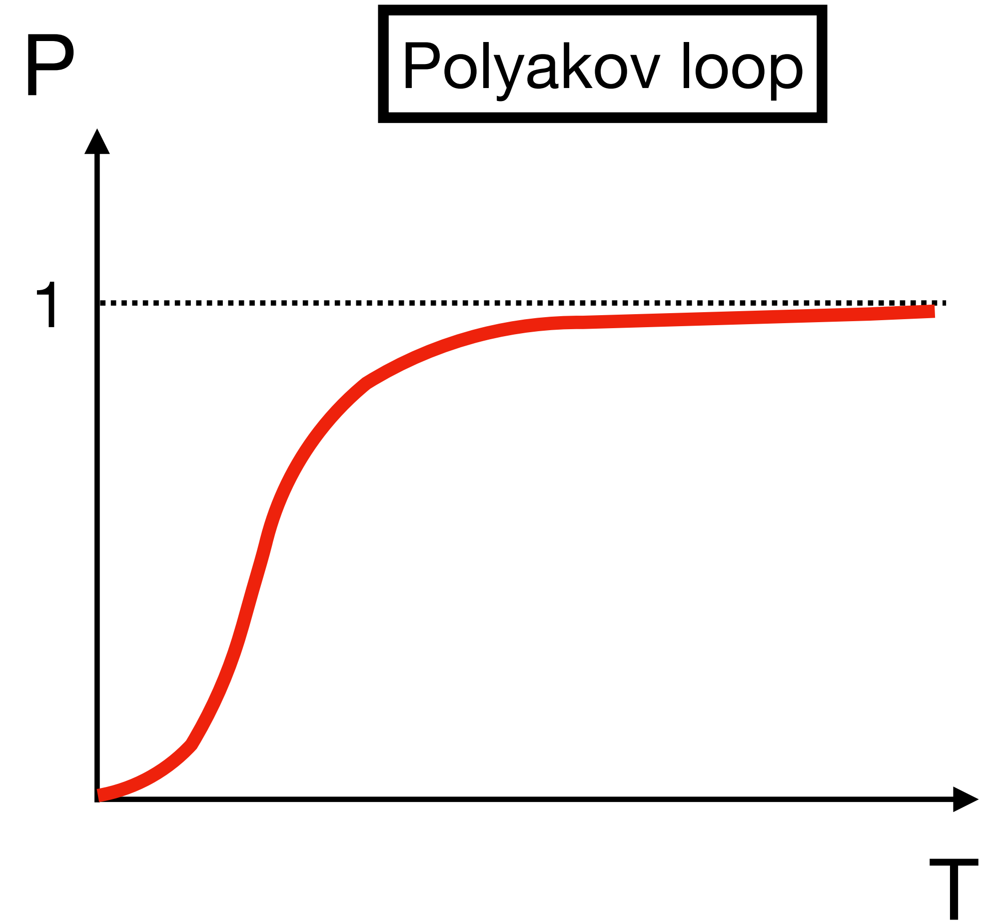
nonuniform string

11d black hole

't Hooft large-N limit (T fix)

$$P = \frac{1}{N}\mathrm{Tr}\left(\mathcal{P}\exp\left(i\int_0^\beta dt A_t\right)\right)$$

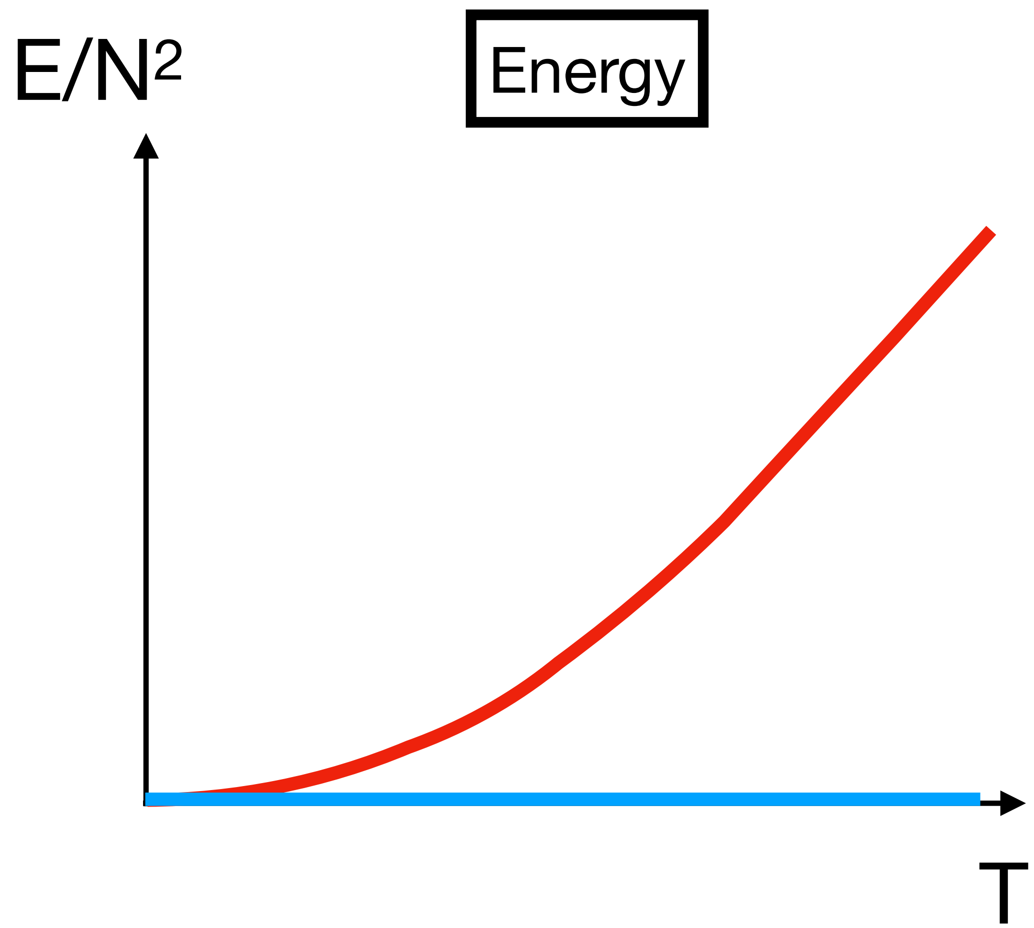


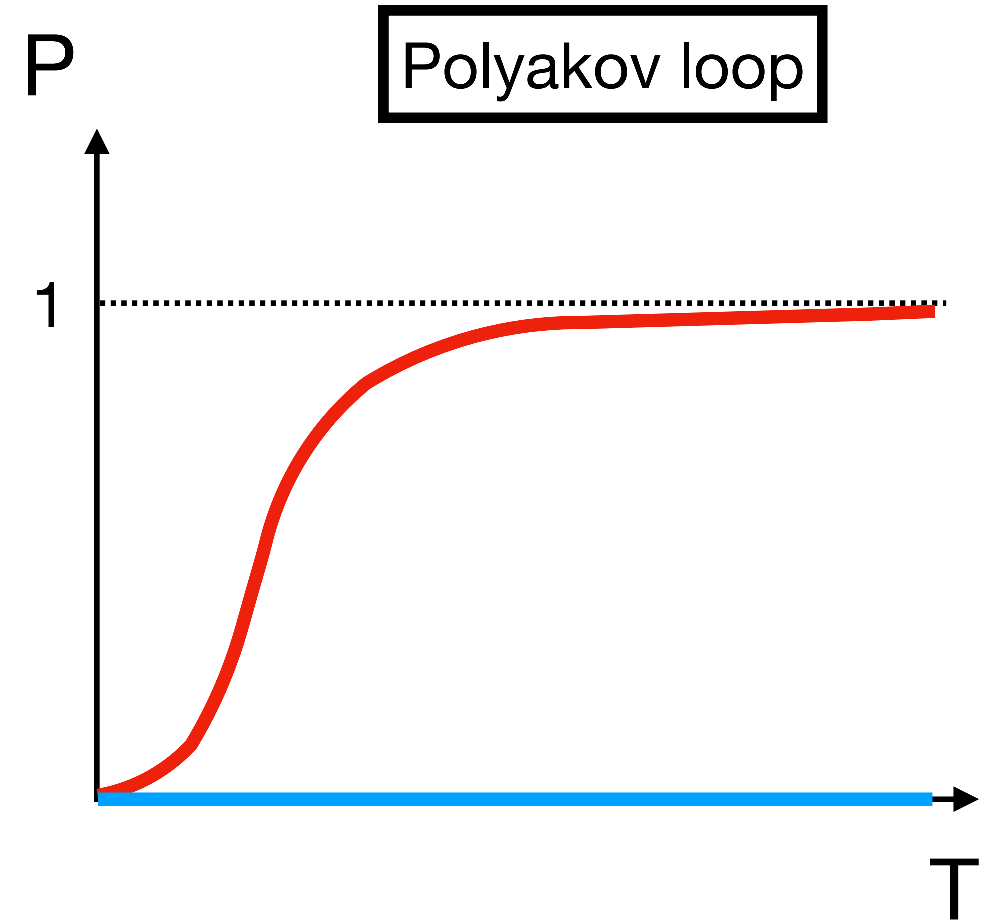


Deconfined at any temperature **(type IIA)**

't Hooft large-N limit (T fix)

$$P = \frac{1}{N}\mathrm{Tr}\left(\mathcal{P}\exp\left(i\int_0^\beta dt A_t\right)\right)$$





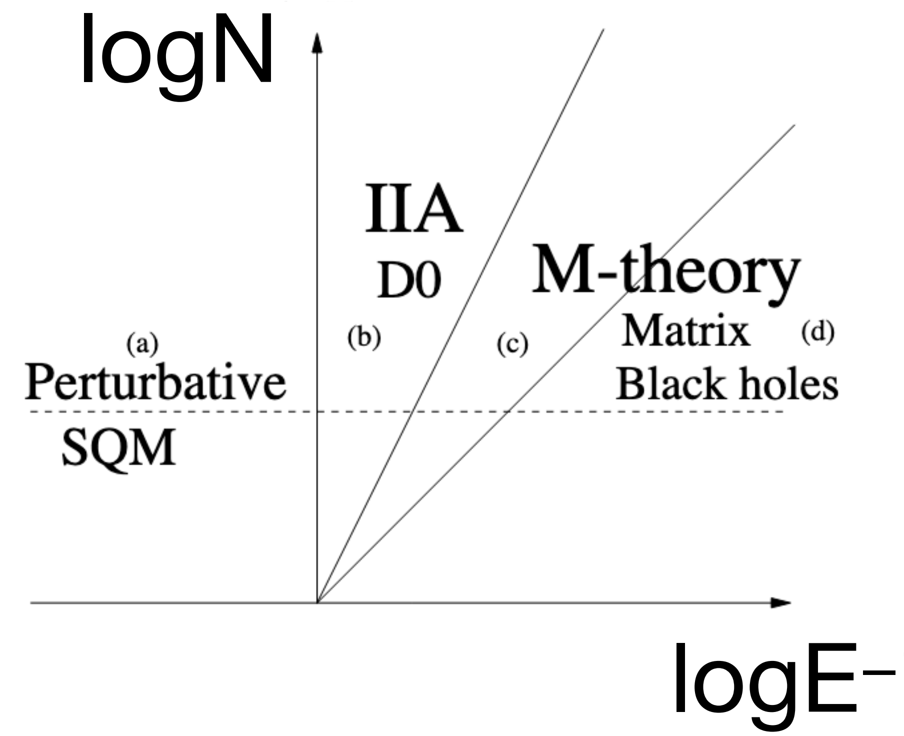
Deconfined at any temperature **(type IIA)**

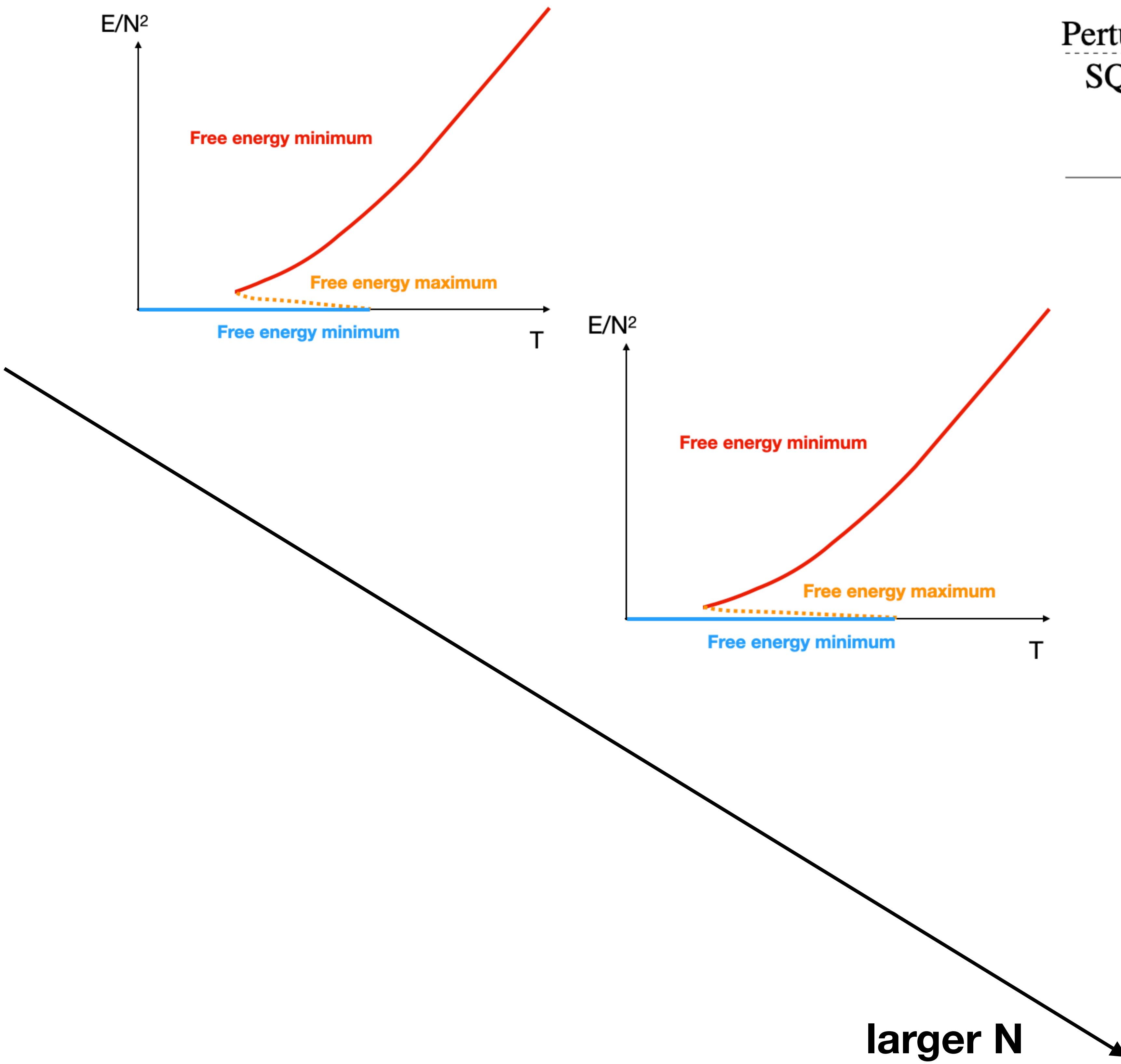
Our proposal

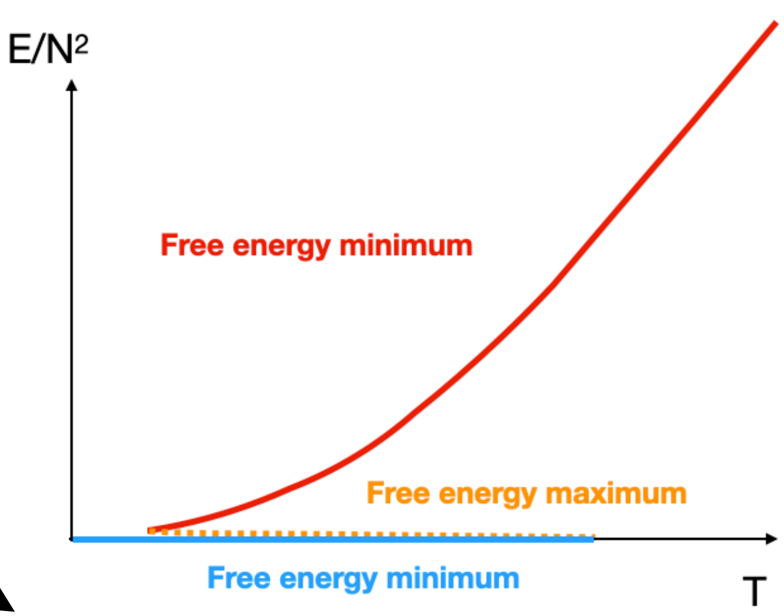
Confined phase should exist as well **(M theory)**

($E/N^2 = 0$)

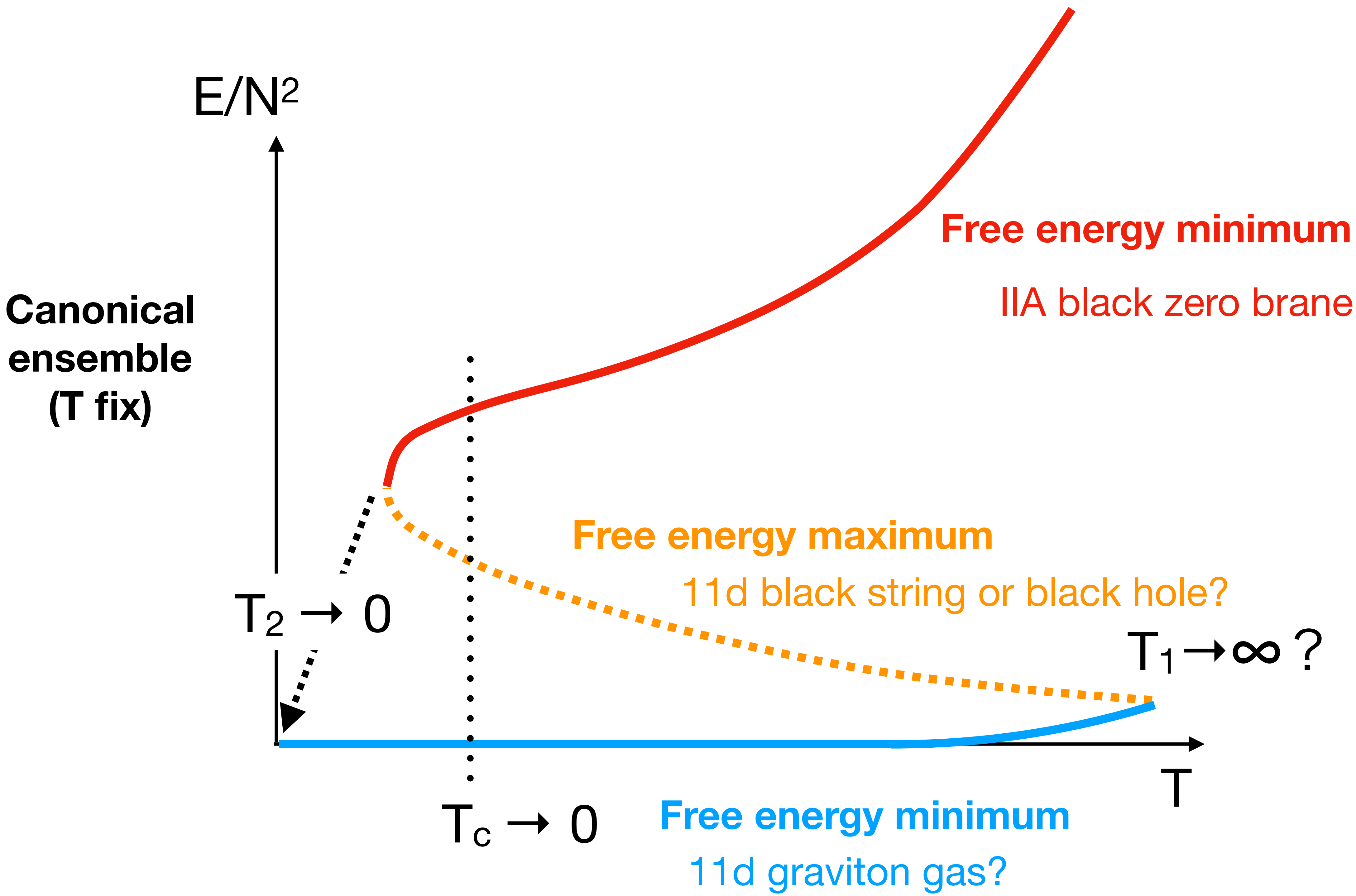
Our proposal

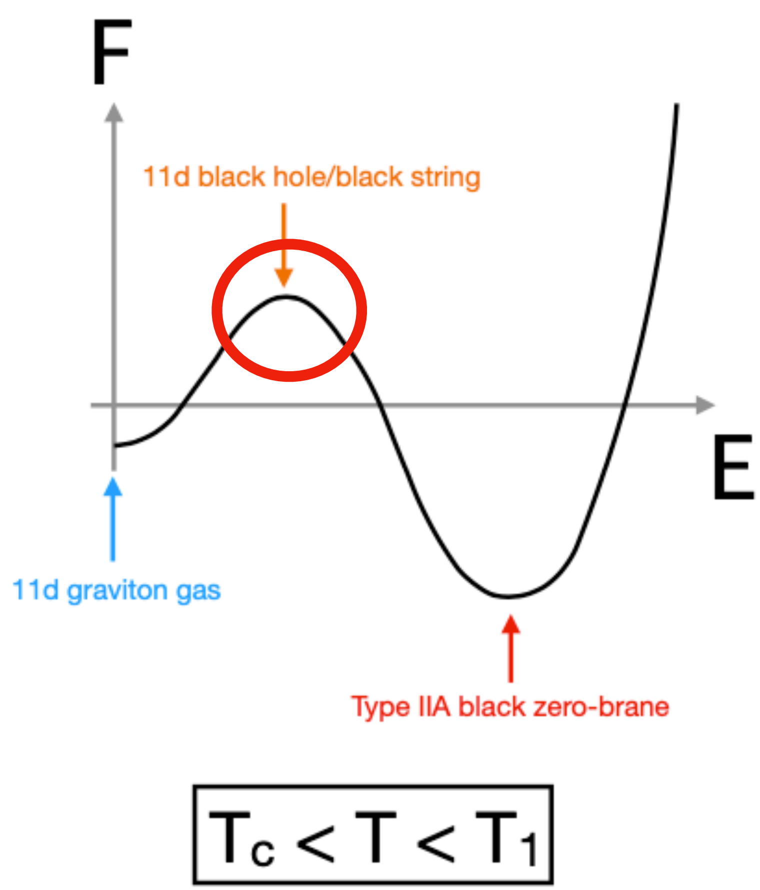


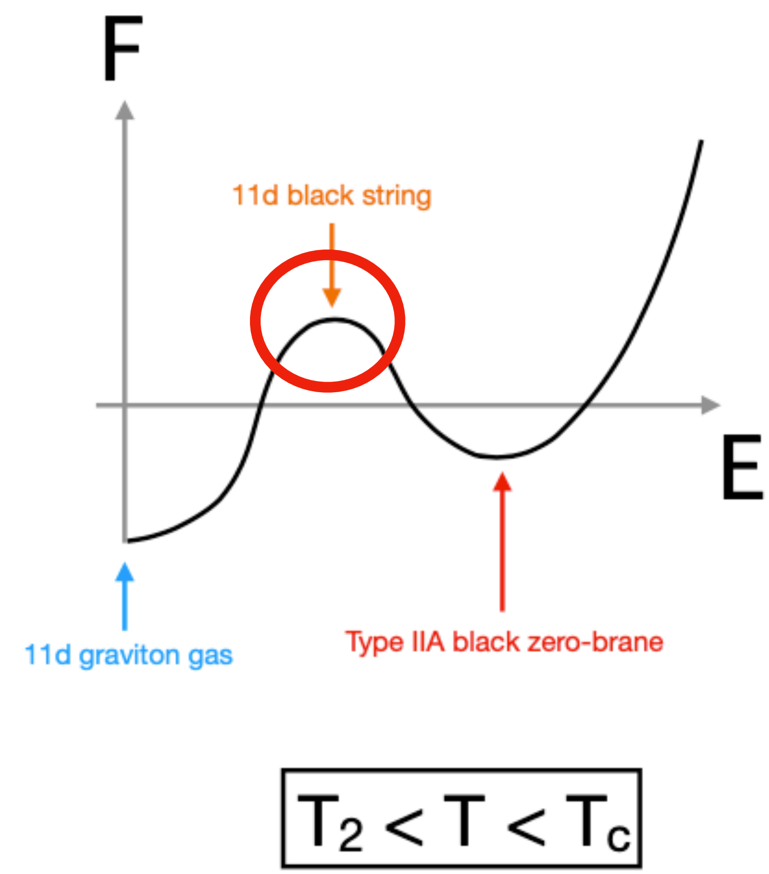




BFSS phase diagram?




F
E
11d black hole/black string
11d graviton gas
Type IIA black zero-brane
$T_c < T < T_1$

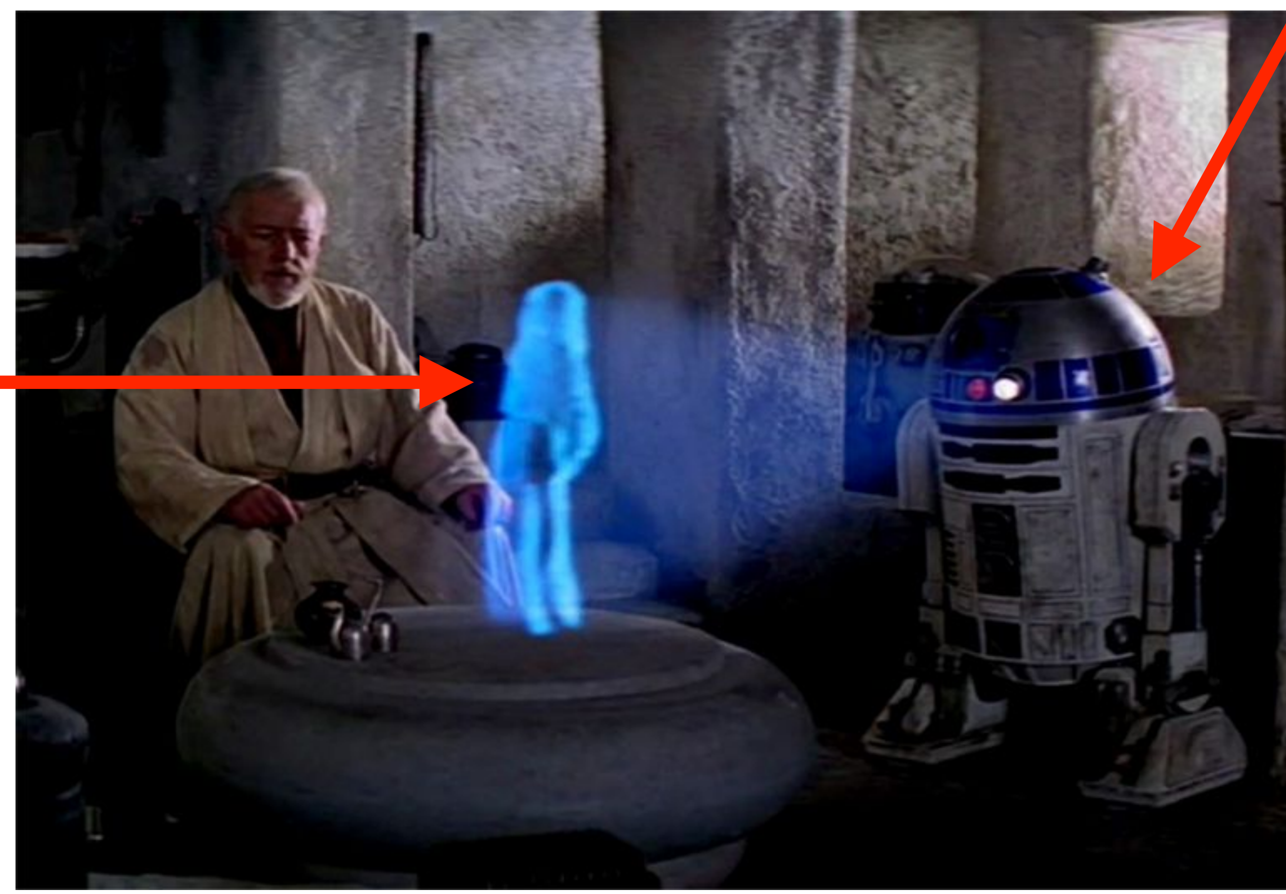

F
E
11d black string
11d graviton gas
Type IIA black zero-brane
$T_2 < T < T_c$

Summary

- Matrix model and type IIA string agrees perfectly.
- Duality is supported including stringy corrections.
- Phase transition between IIA string and M-theory?
- We might be able to create type IIA black zero-brane (charged black hole) and 11d Schwarzschild black hole.
- 11d Schwarzschild → black hole evaporation. Complete resolution of Hawking's paradox?

Matrix Model

11d Schwarzschild type IIA zero-brane



Backup slides

Let's get intuition from

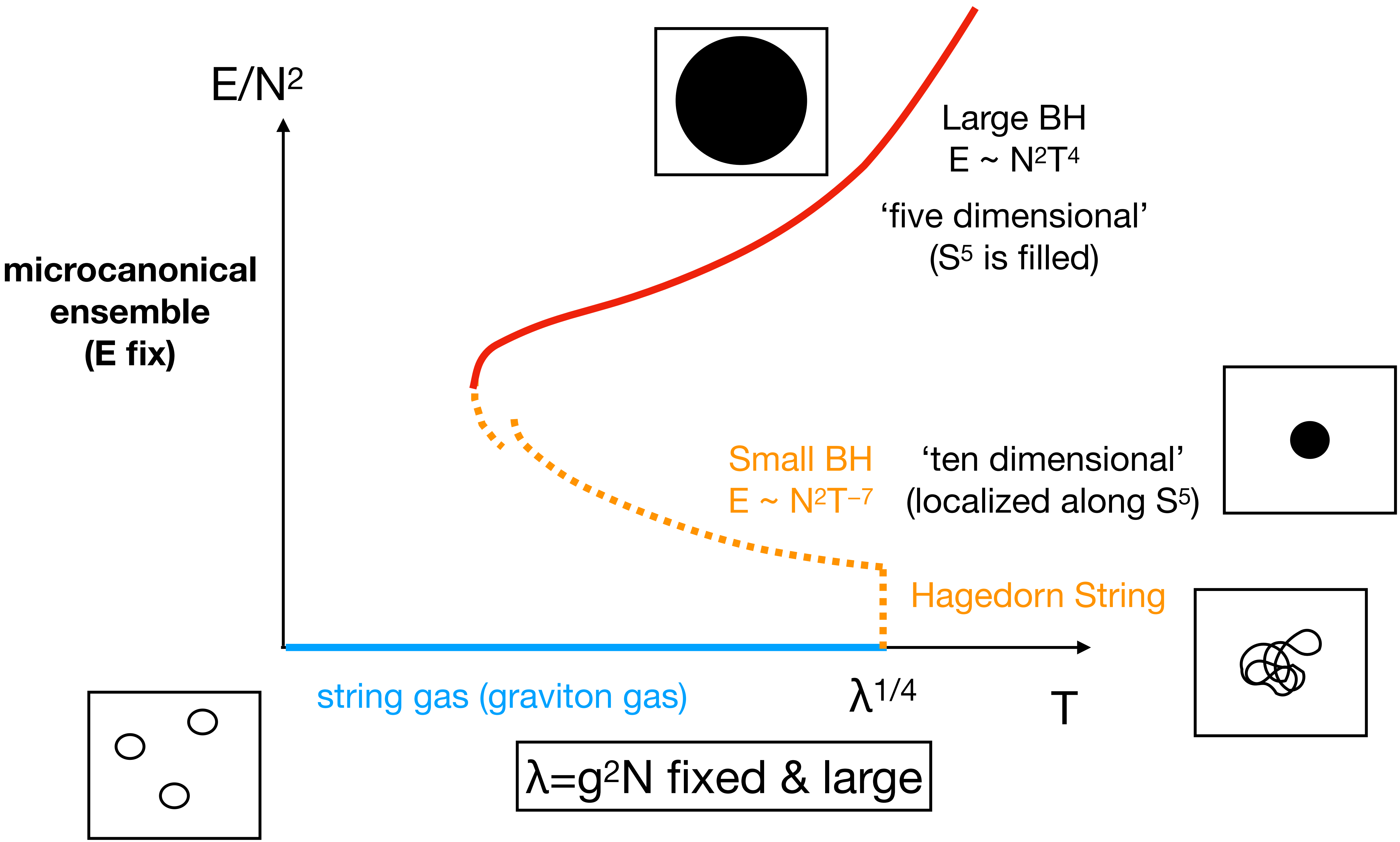
4d SYM on S^3

vs

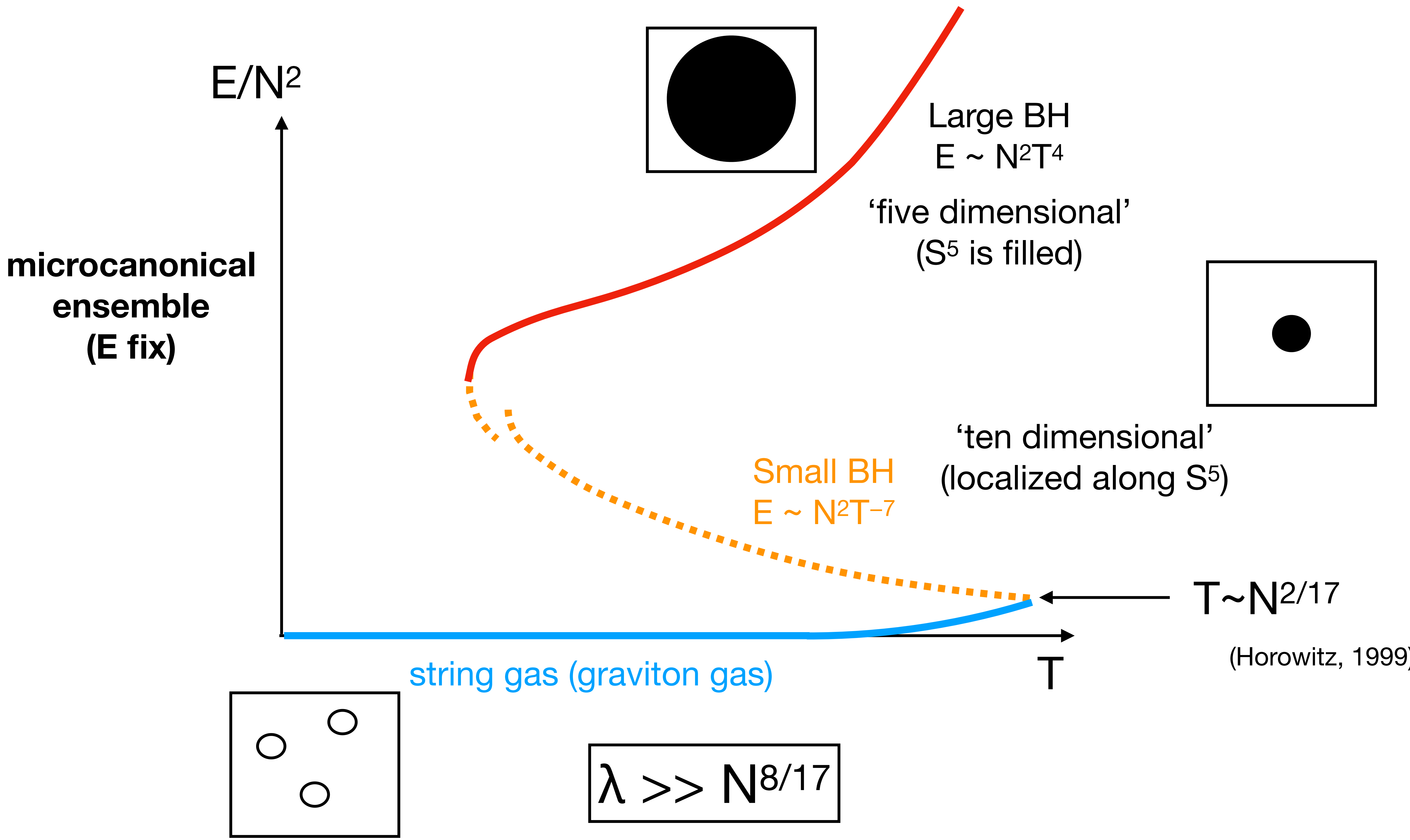
type IIB string on $AdS_5 \times S^5$

(Witten 1998, Horowitz 1999, ...)

BH thermodynamics in $AdS_5 \times S^5$

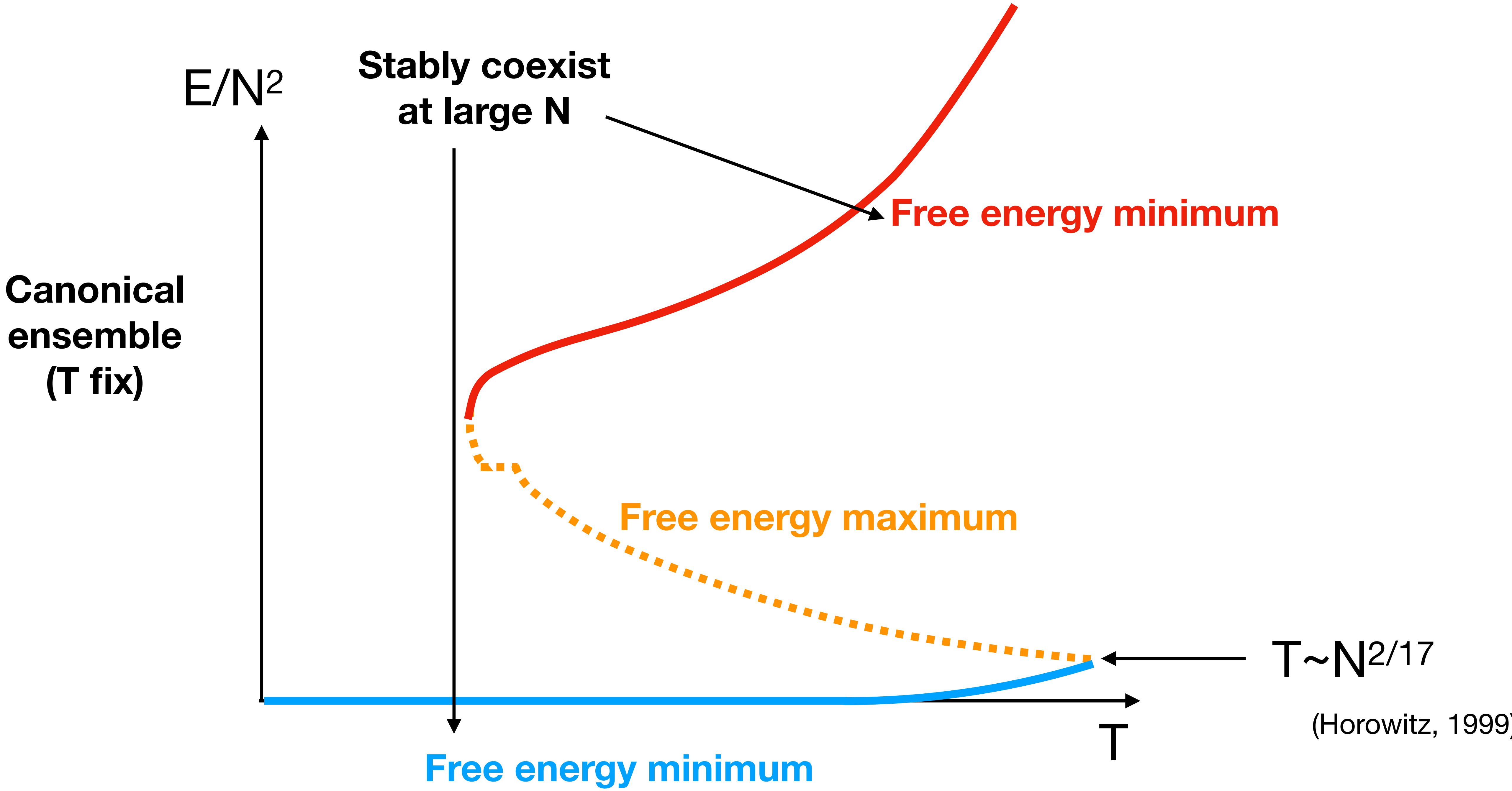


BH thermodynamics in $AdS_5 \times S^5$



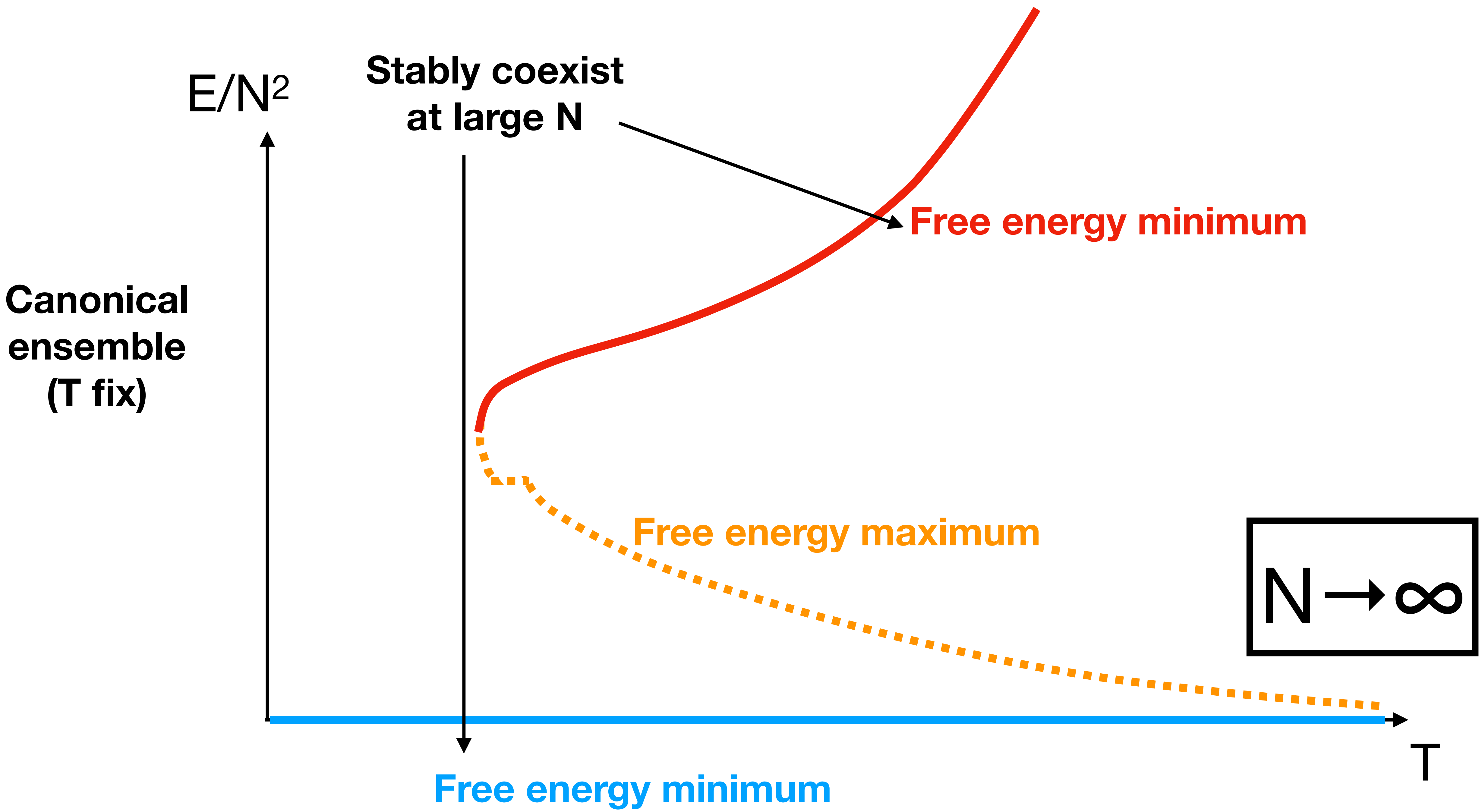
$\lambda >> N^{8/17}$

BH thermodynamics in $AdS_5 \times S^5$



$\lambda >> N^{8/17}$

BH thermodynamics in $AdS_5 \times S^5$



$\lambda >> N^{8/17}$

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Explicit construction of ground-state wave function

BMN, μ=0

ground state
= Gaussian wave function

↓ turn-on the interaction adiabatically

BMN, small μ
(μ=0 → BFSS)

Nontrivial ground state

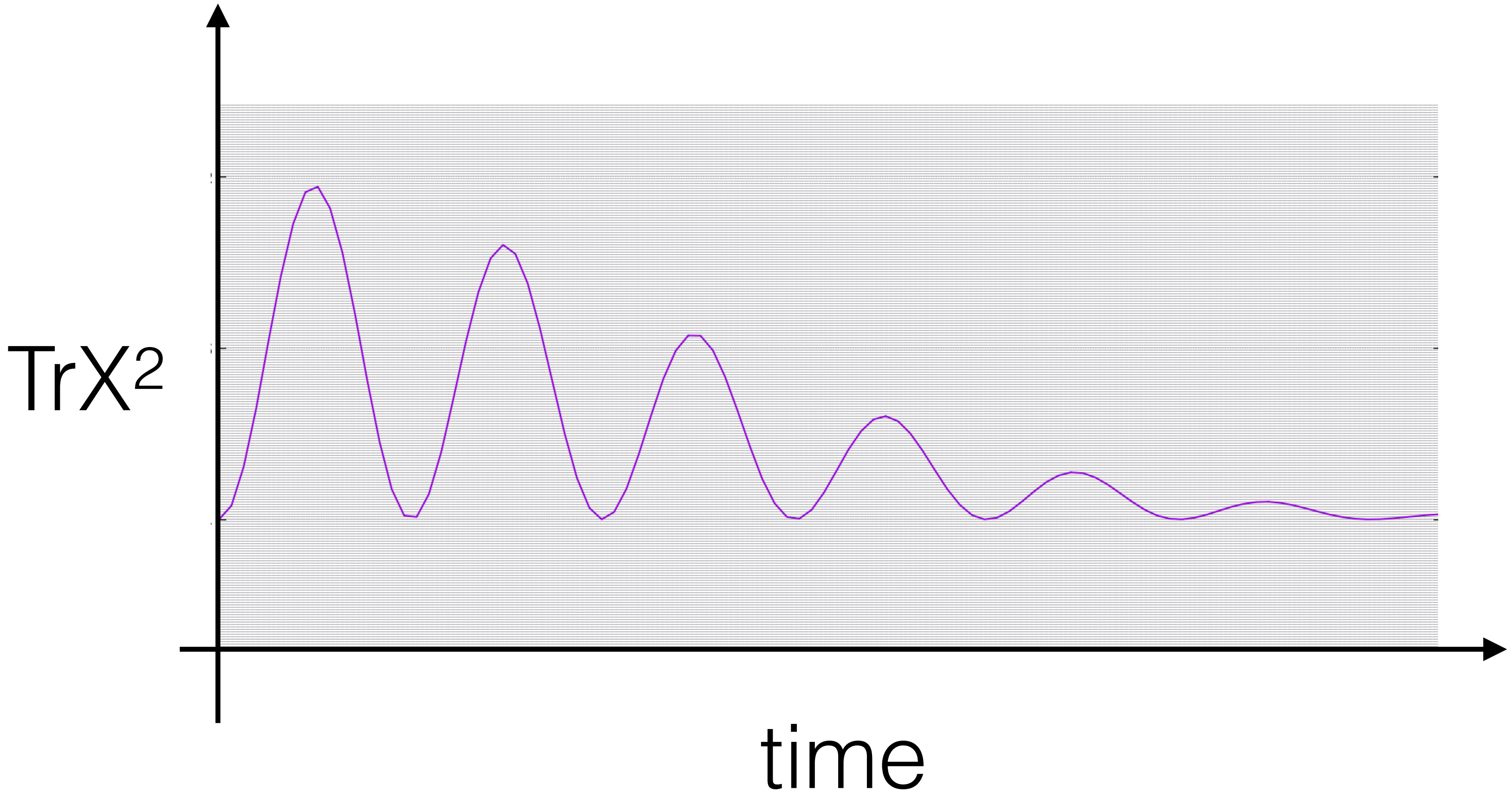
Black hole ring down

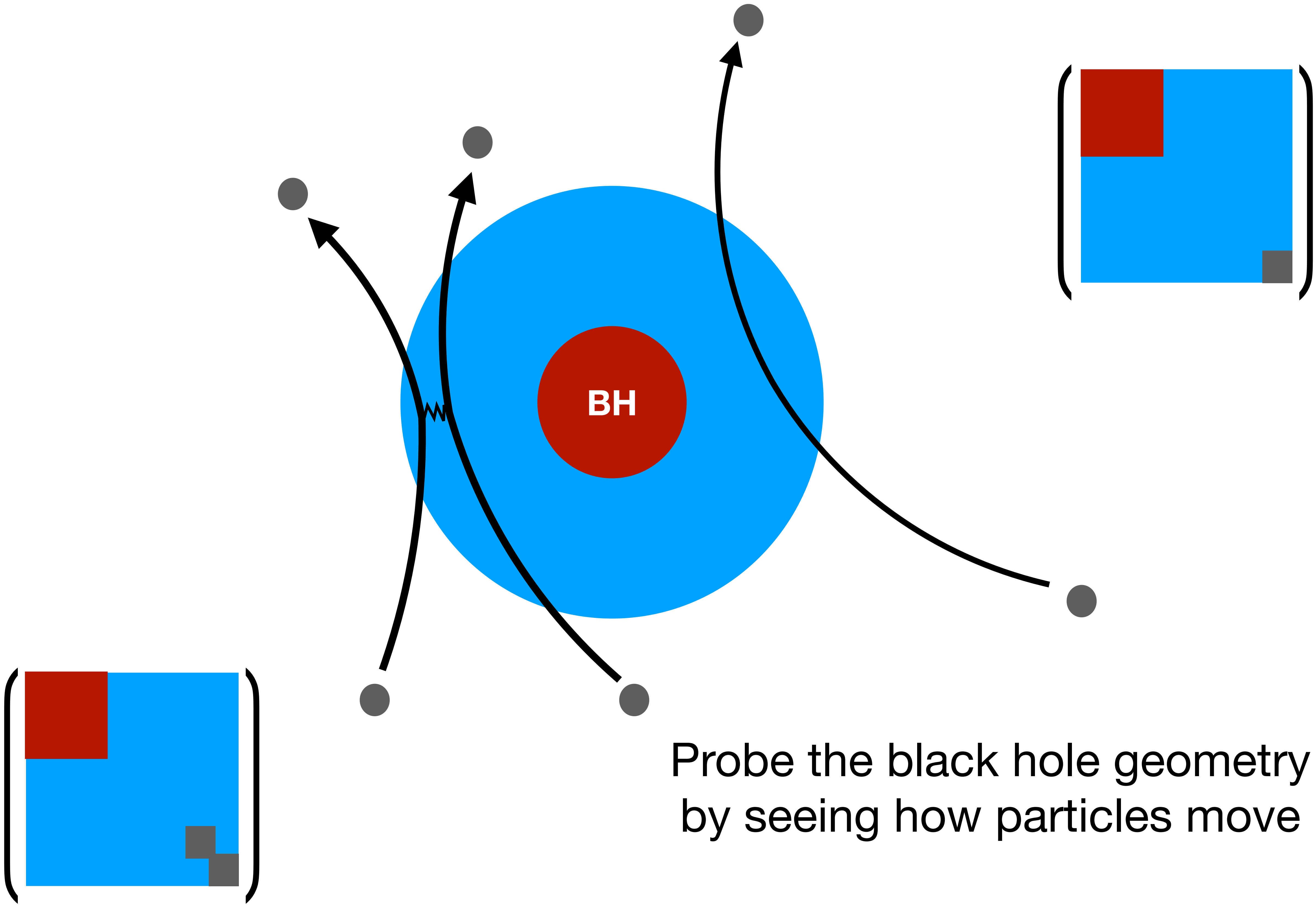
Random state

thermalize (Hamiltonian time evolution)

Black hole

Black hole
+
perturbation





Probe the black hole geometry
by seeing how particles move

Dirac-Born-Infeld action should
describe the motion of particles

Simulation on Quantum Computer

In the ideal world:

- Direct access to big Hilbert space (qubits).
- Any unitary time evolution can be programmed.

In the real world:

- How can we program the theory?
- How big resources?
- Fine tuning?

$$L = \mathrm{Tr}\left\{ \frac{1}{2}(D_t X_I)^2 + \frac{1}{2}\Psi^T D_t \Psi + \frac{g^2}{4}[X_I, X_J]^2 - \frac{ig}{2}\Psi^T \gamma_I [X_I, \Psi] \right.$$
$$\left. -\frac{\mu^2}{18} X_i^2 - \frac{\mu^2}{72} X_a^2 - \frac{\mu}{8}\Psi^T \gamma_{123} \Psi - \frac{i\mu g}{3}\epsilon^{ijk} X_i X_j X_k \right\},$$

$$\Big\downarrow$$

(modulo some field redefinitions)

$$\hat{H} = \mathrm{Tr}\left\{ \frac{1}{2}(\hat{P}_I)^2 - \frac{g^2}{4}[\hat{X}_I, \hat{X}_J]^2 + \frac{\mu^2}{18}\hat{X}_i^2 + \frac{\mu^2}{72}\hat{X}_a^2 + \frac{i\mu g}{3}\epsilon^{ijk}\hat{X}_i \hat{X}_j \hat{X}_k \right.$$
$$\left. + g\hat{\psi}^{\dagger Ip}\sigma^{i\,q}_{p}[\hat{X}_i, \hat{\psi}_{Iq}] - \frac{g}{2}\epsilon_{pq}\hat{\psi}^{\dagger Ip} g^a_{IJ}[\hat{X}_a, \hat{\psi}^{\dagger Jq}] + \frac{g}{2}\epsilon^{pq}\hat{\psi}_{Ip}(g^{a\dagger})^{IJ}[\hat{X}_a, \hat{\psi}_{Jq}] + \frac{\mu}{4}\hat{\psi}^{\dagger Ip}\hat{\psi}_{Ip} \right\}$$

Gauge-singlet constraint ($A_0=0$ gauge)

$$\hat{G}_\alpha |\mathrm{phys}\rangle = 0 \quad \text{with} \quad \hat{G}_\alpha \equiv \sum_{\beta,\gamma=1}^{N^2} f_{\alpha\beta\gamma}\left(\sum_{I=1}^{9} \hat{X}_I^\beta \hat{P}_I^\gamma + i\sum_{I,p} \hat{\psi}^{\dagger Ip\beta}\hat{\psi}^\gamma_{Ip} \right)$$

$$X_{I,ij} = \sum_{\alpha=1}^{N^2} X_I^\alpha \tau_{\alpha,ij}, P_{I,ij} = \sum_{\alpha=1}^{N^2} P_I^\alpha \tau_{\alpha,ij}$$

Coordinate basis $\hat{X}_I^\alpha \left|X\right\rangle = X_I^\alpha \left|X\right\rangle$

Momentum basis $\hat{P}_I^\alpha \left|P\right\rangle = P_I^\alpha \left|P\right\rangle$

$$\hat{g}\left|X\right\rangle = \left|g^{-1}Xg\right\rangle$$

Not SU(N)-invariant

$$\hat{g}\hat{X}_{I,ij}\hat{g}^{-1} = \sum_{k,l} g_{ik}\hat{X}_{I,kl}g^{-1}_{lj}$$

$$\mathcal{H}_{\mathrm{ext}} = \left\{\left|X\right\rangle; X \in \mathbb{R}^{9N^2}\right\} = \left\{\left|P\right\rangle; P \in \mathbb{R}^{9N^2}\right\}$$

Extended Hilbert Space

$$Z(T) = \int [dA_t][dX] e^{-S[A_t, X]}$$

↕ **Feynman's method**

$$Z(T) = \frac{1}{\mathrm{vol} G} \int_G dg \, \mathrm{Tr}_{\mathcal{H}_{\mathrm{ext}}} \left(\hat{g} e^{-\hat{H}/T} \right)$$

↕

$$G = \mathrm{SU}(N)$$

$$Z(T) = \mathrm{Tr}_{\mathcal{H}_{\mathrm{inv}}} (e^{-\hat{H}/T})$$

- We use the extended Hilbert Space.
- Singlet constraint is compatible with Hamiltonian time evolution.

$$[\hat{g}, \hat{H}] = [\hat{g}, e^{-i\hat{H}t}] = 0$$

$$L = \mathrm{Tr}\left\{ \frac{1}{2}(D_t X_I)^2 + \frac{1}{2}\Psi^T D_t \Psi + \frac{g^2}{4}[X_I, X_J]^2 - \frac{ig}{2}\Psi^T \gamma_I [X_I, \Psi] \right.$$
$$\left. -\frac{\mu^2}{18}X_i^2 - \frac{\mu^2}{72}X_a^2 - \frac{\mu}{8}\Psi^T \gamma_{123}\Psi - \frac{i\mu g}{3}\epsilon^{ijk}X_i X_j X_k \right\},$$

(modulo some field redefinitions)

$$\hat{H} = \mathrm{Tr}\left\{ \underline{\frac{1}{2}(\hat{P}_I)^2} - \frac{g^2}{4}[\hat{X}_I, \hat{X}_J]^2 + \underline{\frac{\mu^2}{18}\hat{X}_i^2 + \frac{\mu^2}{72}\hat{X}_a^2} + \frac{i\mu g}{3}\epsilon^{ijk}\hat{X}_i \hat{X}_j \hat{X}_k \right.$$
$$\left. + g\hat{\psi}^{\dagger Ip}\sigma^{i\,q}_{p}[\hat{X}_i, \hat{\psi}_{Iq}] - \frac{g}{2}\epsilon_{pq}\hat{\psi}^{\dagger Ip} g^a_{IJ}[\hat{X}_a, \hat{\psi}^{\dagger Jq}] + \frac{g}{2}\epsilon^{pq}\hat{\psi}_{Ip}(g^{a\dagger})^{IJ}[\hat{X}_a, \hat{\psi}_{Jq}] + \underline{\frac{\mu}{4}\hat{\psi}^{\dagger Ip}\hat{\psi}_{Ip}} \right\}$$

Free part (bosonic/fermionic harmonic oscillators)

Gauge-singlet constraint (A_0=0 gauge)

$$\hat{G}_\alpha |\mathrm{phys}\rangle = 0 \quad \text{with} \quad \hat{G}_\alpha \equiv \sum_{\beta,\gamma=1}^{N^2} f_{\alpha\beta\gamma}\left(\sum_{I=1}^{9} \hat{X}_I^\beta \hat{P}_I^\gamma + i\sum_{I,p} \hat{\psi}^{\dagger Ip\beta}\hat{\psi}^\gamma_{Ip} \right)$$

$$\hat{H} = \mathrm{Tr}\Big\{ \underline{\frac{1}{2}(\hat{P}_I)^2} - \frac{g^2}{4}[\hat{X}_I, \hat{X}_J]^2 + \underline{\frac{\mu^2}{18}\hat{X}_i^2 + \frac{\mu^2}{72}\hat{X}_a^2} + \frac{i\mu g}{3}\epsilon^{ijk}\hat{X}_i\hat{X}_j\hat{X}_k$$

$$+ g\hat{\psi}^{\dagger Ip}\sigma_p^{i\,q}[\hat{X}_i, \hat{\psi}_{Iq}] - \frac{g}{2}\epsilon_{pq}\hat{\psi}^{\dagger Ip} g^a_{IJ}[\hat{X}_a, \hat{\psi}^{\dagger Jq}] + \frac{g}{2}\epsilon^{pq}\hat{\psi}_{Ip}(g^{a\dagger})^{IJ}[\hat{X}_a, \hat{\psi}_{Jq}] + \underline{\frac{\mu}{4}\hat{\psi}^{\dagger Ip}\hat{\psi}_{Ip}} \Big\}$$

Free part (bosonic/fermionic harmonic oscillators)

Fock basis

$$X_I = \sum_{\alpha=1}^{N^2} X_I^\alpha \tau_\alpha, \quad \psi_{Ip} = \sum_{\alpha=1}^{N^2} \psi_{Ip}^\alpha \tau_\alpha \qquad [\tau_\alpha, \tau_\beta] = i f_{\alpha\beta\gamma}\tau_\gamma, \quad \mathrm{Tr}(\tau_\alpha \tau_\beta) = \delta_{\alpha\beta}$$

$$\hat{A}_{I\alpha} = \sqrt{\frac{\omega_I}{2}}\hat{X}_{I\alpha} + \frac{i\hat{P}_{I\alpha}}{\sqrt{2\omega_I}}, \qquad \hat{A}^\dagger_{I\alpha} = \sqrt{\frac{\omega_I}{2}}\hat{X}_{I\alpha} - \frac{i\hat{P}_{I\alpha}}{\sqrt{2\omega_I}}, \qquad \omega_I = \begin{cases} \frac{\mu}{3} & \text{for } I = 1,2,3 \\ \frac{\mu}{6} & \text{for } I = 4,5,\cdots,9 \end{cases}$$

$$|\{n_{I\alpha}\}\rangle \equiv \otimes_{I,\alpha}|n_{I\alpha}\rangle_{I\alpha} = \left(\prod_{I,\alpha}\frac{\hat{A}^{\dagger n_{I\alpha}}_{I\alpha}}{\sqrt{n_{I\alpha}!}}\right)|\mathrm{VAC}_{\mathrm{free}}\rangle, \qquad \hat{A}_{I\alpha}|\mathrm{VAC}_{\mathrm{free}}\rangle = 0.$$

Regularization: $0 \le n_{I\alpha} \le \Lambda - 1$

(No regularization needed for fermions)

$$\hat{a}^\dagger = \sum_{j=0}^{\Lambda-2} \sqrt{j+1}|j+1\rangle\langle j|$$

$$|j\rangle = |b_0\rangle\,|b_1\rangle \dots |b_{K-1}\rangle$$

b, b' = 0 or 1

$$|j+1\rangle = |b'_0\rangle\,|b'_1\rangle \dots \left|b'_{K-1}\right\rangle$$

$$|j+1\rangle\langle j| = \otimes_{l=0}^{K-1}\left(|b'_l\rangle\langle b_l|\right)$$

$K=\log_2\Lambda$

$$|0\rangle\langle 0| = \frac{\mathbf{1}_2 - \sigma_z}{2}, \qquad |1\rangle\langle 1| = \frac{\mathbf{1}_2 + \sigma_z}{2},$$
$$|0\rangle\langle 1| = \frac{\sigma_x + i\sigma_y}{2}, \qquad |1\rangle\langle 0| = \frac{\sigma_x - i\sigma_y}{2}.$$

H = Σ(Pauli strings)

$$\hat{a}^\dagger = \sum_{j=0}^{\Lambda-2} \sqrt{j+1}|j+1\rangle\langle j|$$

$\sim 2^K = \Lambda$ Pauli strings of length $K = \log_2\Lambda$ for each j

➡ $\sim\Lambda^2$ Pauli strings of length $K = \log_2\Lambda$

$$\sum_{I\neq J} \mathrm{Tr}[\hat{X}_I, \hat{X}_J]^2 = -\sum_{I\neq J} \sum_{\alpha,\beta,\gamma,\rho,\sigma=1}^{N^2} f_{\alpha\beta\sigma} f_{\gamma\rho\sigma} \hat{X}_I^\alpha \hat{X}_J^\beta \hat{X}_I^\gamma \hat{X}_J^\rho$$

$\sim N^4$ color combinations

$\sim\Lambda^8 N^4$ Pauli strings of length 4K

$$\dim\left(\mathcal{H}_{\mathrm{BMN}}\right)\big|_{\mathrm{regularized}} = \Lambda^{9N^2} \cdot 2^{8N^2}$$ (~N^4 nonzero components/row)

$$\hat{H} = \sum_{i=1}^{L} \alpha_i \hat{\Pi}_i, \qquad L \lesssim \Lambda^8 N^4$$

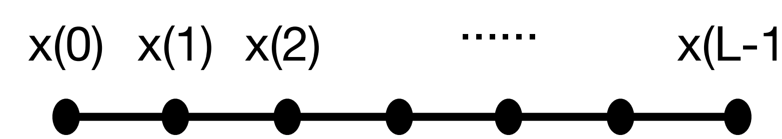
↖ Pauli strings

Coordinate basis

$$-R \leq x \leq R\,,$$

$$x(k) = -R + k a_{\rm dig}\,, \qquad a_{\rm dig} = \frac{2R}{L-1}\,, \qquad k = 0, 1, \cdots, L-1\,.$$

$$\hat{x} = \sum_{k=0}^{L-1} x(k)\,|k\rangle\,\langle k|\,.$$

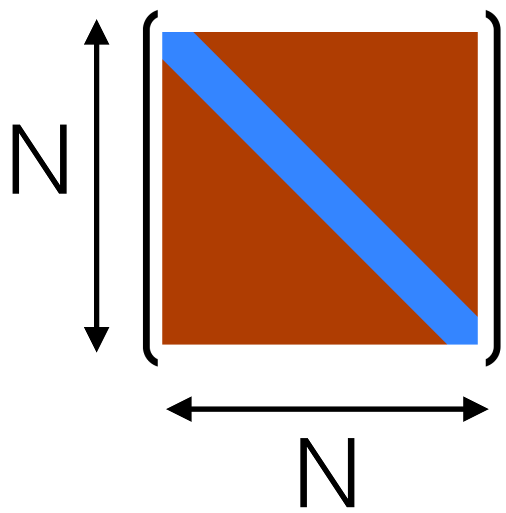


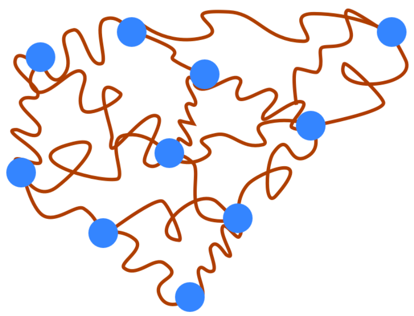
$$\hat{p}^2 = \frac{1}{a_{\rm dig}^2}\left\{\sum_{k=0}^{L-1} 2\,|k\rangle\,\langle k| - \sum_{k=0}^{L-2} |k+1\rangle\,\langle k| - \sum_{k=0}^{L-2} |k\rangle\,\langle k+1|\right\}\,.$$

H = Σ(Pauli strings)

How big Λ?

- Depend on the physics under consideration.
- Corrections to low-energy spectrum in the trivial vacuum ~ $\exp(-\Lambda)$
- # of (logical) qubits = $9N^2\log_2\Lambda + 8N^2$





<u>Free limit</u>

each matrix entry = harmonic oscillator

excitation level = # of strings

average excitation level < 1

For black zero-brane:

- N=8 or 12 are rather close to the large-N limit
- Λ=8 or 16 → 3 or 4 qubits/bosonic d.o.f.
- Similar estimate for the coordinate basis

$$9N^2\log_2\Lambda + 8N^2$$

$$9\times 8^\wedge 2\times \log_2 8 + 8\times 8^\wedge 2 = 2240$$

$$9\times 16^\wedge 2\times \log_2 16 + 8\times 16^\wedge 2 = 11264$$

Summary
(Quantum simulation part)

- In principle, matrix models can be studied on quantum computer in a straightforward manner.
- Many QFT can follow from matrix model.
- Interesting problems in holography
  – experimental quantum gravity!