Emergence of kinetic terms in String Theory

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Central Swampland constraints relate kinetic terms to towers of states

Weak Gravity Conjecture (magnetic)

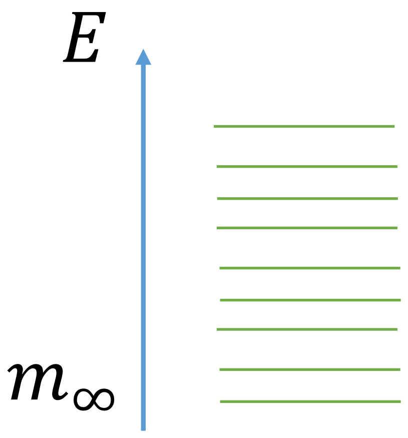
$$\mathcal{L} = -\frac{1}{4g^2}F^2 \qquad m_\infty \sim g\, M_p$$

[Arkani-Hamed, Motl, Nicolis, Vafa '06] + …

Distance Conjecture

$$\mathcal{L} = -g_{\phi\phi}\partial_\mu\phi\partial^\mu\phi \qquad m_\infty \sim \mathrm{Exp}[-\alpha \int \sqrt{g_{\phi\phi}\frac{d\phi}{d\tau}\frac{d\phi}{d\tau}}\, d\tau]\, M_p$$

[Ooguri, Vafa '06] + …



Emergence Proposal: Swampland conjectures are manifestations of emergent kinetic terms for fields

[Harlow '16; Heidenreich, Reece, Rudelius '17+'18; Grimm, EP, Valenzuela '18; EP '19]

[Basile, Blumenhagen, Calderón-Infante, Castellano, Cribiori, Corvilian, Cota, Delgado, EP, Grimm, Gligovic, Herraez, Heidenreich, Ibanez, Lee, Lerche, Li, Marchesano, Melotti, Mininno, Paraskevopoulou, Paoloni, Reece, Rudelius, Ruiz, Uranga, Valenzuela, Weigand, Wiesner, + …]

A good toy model of such a connection to keep in mind is the $\mathbb{CP}^N$ model

[D'Adda, Luscher, Di Vecchia '78]…[Harlow '16]

In the ultraviolet we have N scalars z^i with Lagrangian :

$$\mathcal{L} = -\frac{N}{c^2} \left(D_\mu z_i\right)^* \left(D^\mu z^i\right) \; . \qquad A_\mu \equiv \frac{1}{2iN} \left(z_i^* \partial_\mu z^i - z^i \partial_\mu z_i^*\right) \; .$$

Integrating out the massive degrees of freedom leads, in the infrared, to emergent dynamics for the field

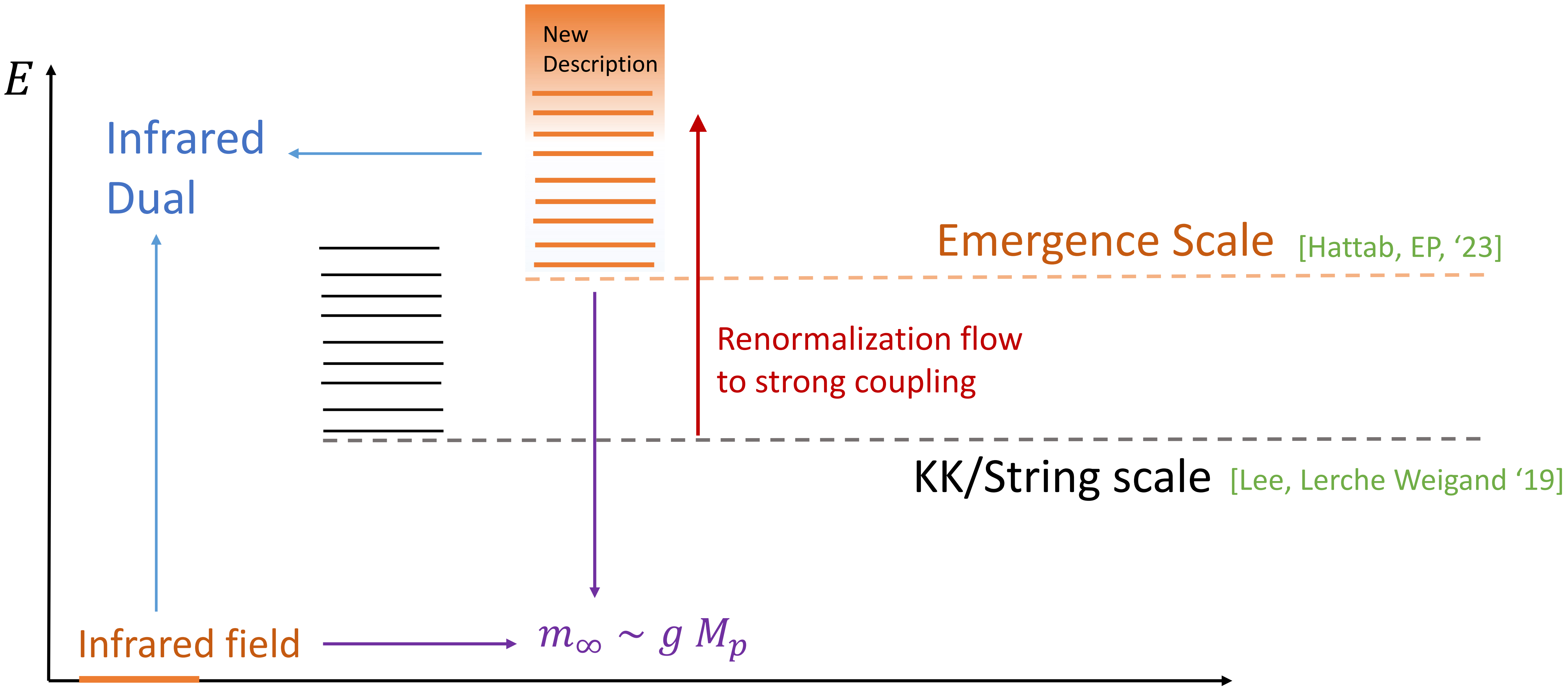
$$\mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} \; , \qquad \frac{1}{g^2} = \frac{N}{12\pi^2} \log\left(\frac{\Lambda_{CP}}{m_z}\right) \; .$$

Infrared emergent gauge coupling appears purely 1-loop

Emergence proposal in string theory

[Grimm, EP, Valenzuela ’18][EP ‘19][Hattab, EP, ‘23]

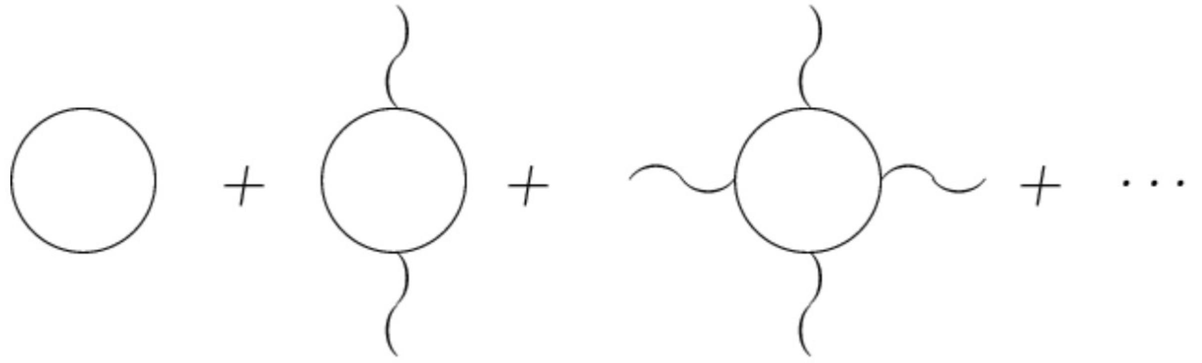
Proposal: In string theory, all fields are infrared dual to towers of states



Probe emergence by integrating out in a background field

Simplest model: Euler-Heisenberg Lagrangian

$$\mathcal{L}_{\mathrm{EH}} = -\frac{1}{4}F^2 + \frac{e^2}{32\pi^2}\int_\epsilon^\infty \frac{ds}{s} e^{-sm^2}\left[\frac{\mathrm{Re}\ \cosh(esF)}{\mathrm{Im}\ \cosh(esF)} F\tilde{F} + \frac{4}{e^2 s^2} + \frac{3}{2}F^2\right]$$



Emergence would have the full action coming from the 1-loop term

Vector multiplets sector of type IIA string theory on Calabi-Yau manifolds

For simplicity consider one vector multiplet with scalar component

$$T = t + ib$$

Two-derivative Vector multiplets sector controlled by a prepotential F_0 :

$$F_0 = \mathrm{a}\,\mathrm{T}^3 + \mathrm{b}\,\mathrm{T}^2 + \mathrm{c}\,\mathrm{T} + \mathrm{d} + \sum_{\gamma \in \mathbb{N}} n_\gamma e^{-2\pi\gamma T}$$

Polynomial / Tree-level   Exponential / Non-perturbative

Higher derivative terms correspond to higher genus prepotentials

$$\sum_{g \geq 1} F_g(T) W^{2g-2} R_+^2$$

A graviphoton background field calculation can reproduce the non-perturbative parts of the prepotential from integrating out D2-D0 branes

$$F^{NP} = \frac{1}{g_s^2}\sum_{g=0}^{\infty} F_g^{NP}(T)(g_s W)^{2g} = \sum_{\beta,r,n}\int_{\epsilon}^{\infty}\frac{ds}{s}\frac{W^2}{\left(2\sinh\frac{s g_s W}{2}\right)^{2-2r}} e^{-sZ(T)_{\beta,n}}$$

[Gopakumar, Vafa ’98]

$$F^{NP}(T,W) = \mathcal{L}_{\text{eff}}^{IR}(T,W)$$

- β is the wrapping number of the D2
- n is the D0 charge
- r is the genus of the wrapped curve

Emergence proposal predicts that the full prepotential should arise from integrating out the non-perturbative (dual) tower of states

Indeed, the idea of emergence of moduli space was considered already in the initial formulation of the distance conjecture. However, it was dismissed is because the polynomial piece was not reproduced from integrating out the D-branes

[Ooguri, Vafa ’06]

We propose evidence that the polynomial piece does arise from integrating out non-perturbative ultraviolet degrees of freedom

We propose:

$$F^{NP}(T,W) = \mathcal{L}^{IR}_{\text{eff}}(T,W)$$

$$\Big\downarrow \; \epsilon = 0$$

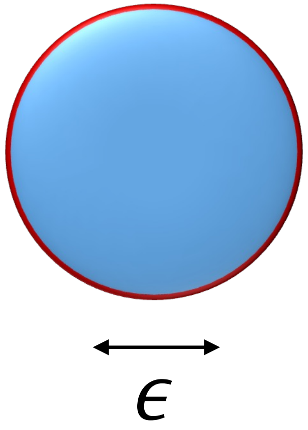
$$F(T,W) = \mathcal{L}_{\text{eff}}(T,W)$$

See also [Blumenhagen, Cribiori, Gligovic, Paraskevopoulou ’23]

$$F^{NP}(T,W) = \sum_{\beta,r,n} \int_{\epsilon}^{\infty} \frac{ds}{s} [\ldots] e^{-sZ(T)_{\beta,n}}$$

But how to UV complete the integral?

- Integral is UV divergent (doubly)
- We cannot treat the wrapped branes as particles



Simple example: The resolved conifold (a non-compact setting with a single 2-cycle)

The genus-0 prepotential is:

$$\mathcal{F}_0 = -\frac{(2\pi i)^3}{12}\left[iT^3 + \frac{3}{2}(1+4m)T^2 - \frac{i}{2}T\right] - \zeta(3) + \mathrm{Li}_3\left(e^{-2\pi T}\right) \ ,$$

[Gopakumar, Vafa '98]

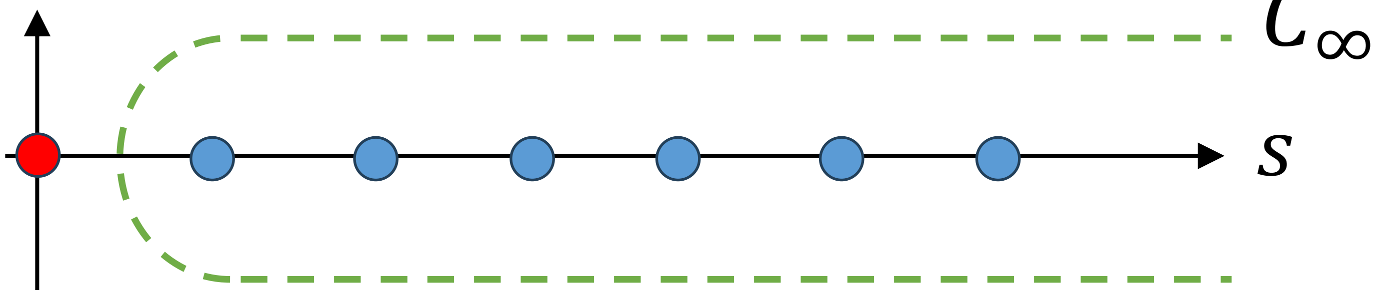
The instanton contributions are recovered as usual:

$$\sum_{n\in\mathbb{Z}} \int_\epsilon^\infty \frac{ds}{s^3} e^{-s2\pi(T+in)} = \sum_{k\geq 1} \frac{1}{k^3} e^{-2\pi kT} = \mathrm{Li}_3\left(e^{-2\pi T}\right) \ .$$

It is possible to repackage the instanton integral into a contour integral

$$F_0^{Inst} = \sum_{n\in\mathbb{Z}} \int_\epsilon^\infty \frac{ds}{s^3} e^{-2\pi s(T+in)} = \sum_{k\in\mathbb{Z}} \int_\epsilon^\infty \frac{ds}{s^3} \delta(s-k)\, e^{-2\pi sT} = \oint_{C_\infty} \frac{dz}{z^3} \frac{e^{-2\pi zT}}{1-e^{-2\pi iz}}$$

$$\sum_{k\in\mathbb{Z}} \delta(s-k) = \sum_{n\in\mathbb{Z}} e^{2\pi ins},$$

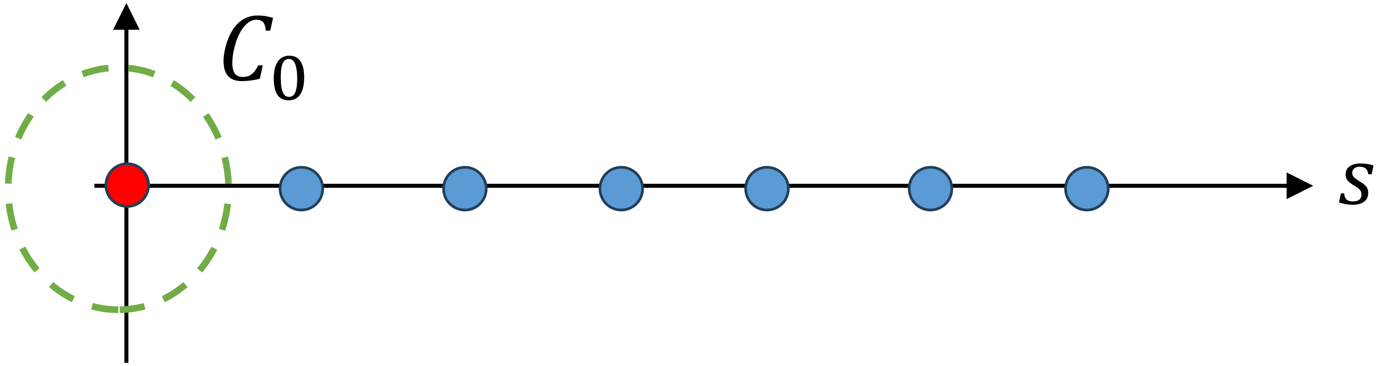


This can be thought of as an analytic continuation of the Schwinger proper time

$$s \to z$$

It is now possible to evaluate the ultraviolet pole at the origin

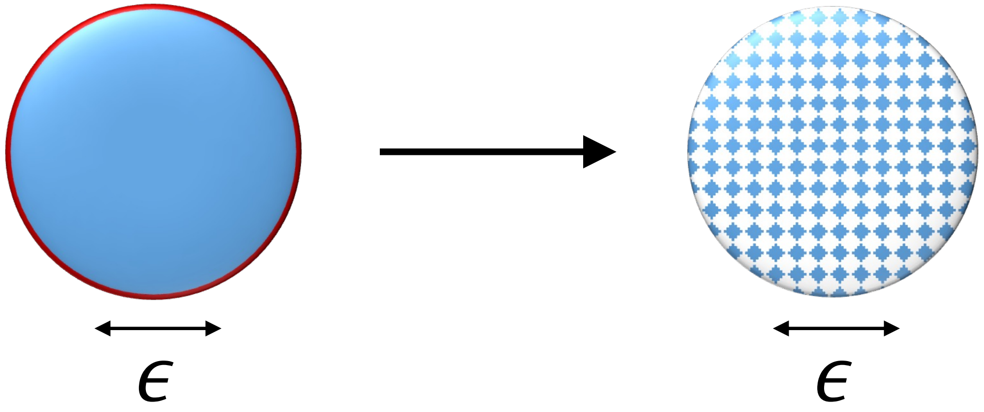
$$F_0^{Poly} = \frac{1}{2}\oint_{C_0} \frac{dz}{z^3}\frac{e^{-2\pi z T}}{1-e^{-2\pi i z}} = -\frac{(2\pi i)^3}{12}\left[iT^3+\frac{3}{2}T^2-\frac{i}{2}T\right]$$



Our interpretation is that indeed the full prepotential arises from integrating out the states

Can we see integrating out the ultraviolet more directly?

Would need to resolve the wrapped branes into constituents



Can be done by switching an expansion in g_s for one in

$$\hbar = \frac{(2\pi)^2 i}{g_s}$$

[Marino, Putrov '11]

Leads to a Fermi-Gas picture of certain type IIA backgrounds

The prototypical example is the Blown-up Conifold : A non-compact cone over a base $B = \mathbb{CP}^1 \times \mathbb{CP}^1$

(Geometric dual to ABJM on S^3)

Known that the exact all-genus, fully non-perturbative, prepotential can be matched onto the Grand Canonical potential (associated to a single-particle Hamiltonian) of a free Fermi-Gas of a one-dimensional Fermion in an external potential:

$$F(T, g_s) = J(\mu, \hbar) \qquad\qquad T = \frac{4\pi\mu}{\hbar} - \pi i$$

[Marino, Putrov '11]

This includes the tree-level polynomial parts of the prepotential

Our proposal is that this should be understood as an integrating out calculation, just like Gopakumar-Vafa

Simple toy model of a two-dimensional Fermion (second quantized) field (in $S^1 \times \mathbb{R}$) over all points in four-dimensional spacetime

$$S_\psi^{(6)} = \int d^4x d^2y \mathcal{L}_\psi^{(2)} \, . \qquad \mathcal{L}_\psi^{(2)} = \bar{\psi}_x(y) i \not{D}_E \psi_x(y) + m \, \bar{\psi}_x(y) \psi_x(y) \, . \qquad A_\tau = \mu(x)$$

Integrating out the Fermion gives an effective action for the chemical potential

$$\mathcal{L}_{eff}(\mu) = J(\mu)$$

* Up to some irrelevant contributions due to toy model

We can understand this as a toy model for the integrating out calculation performed in a finite graviphoton background

Gopakumar-Vafa: $g_s \to \infty$ $W \to 0$ $g_s W \to 0$

Here: $g_s \to \infty$ $W \to 1$ $g_s W \to \infty$

We can replace $g_s W \to g_s$ and recover the sought-after relation:

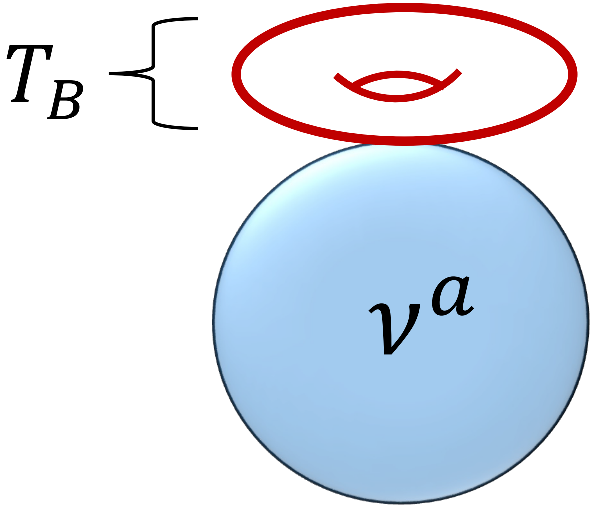
$$F(T, g_s) = J(T, g_s) = \mathcal{L}_{\text{eff}}(T, g_s)$$

Therefore, the full prepotential is understood as arising from integrating out the Fermi gas and the D2-D0 states are solitonic contributions in this picture

Can this work for compact models with dynamical gravity?

(Based on the non-compact cone setting [Grassi, Hatsuda, Marino, ‘11])

Consider an elliptic fibration over a reflexive Toric base



Fermi gas Hamiltonian:

$$e^H = \sum_{a=1}^{k+2} \mathrm{Exp}\left[\nu_1^a q + \nu_2^a p + f^a\right]$$

Reproduces the cubic coefficient for the fibre modulus T_B via the map

$$T_B = \frac{2\pi\mu}{\hbar} + \ \ldots \qquad\qquad F(T_B)_{\text{cubic}} = J(\mu)_{\text{cubic}}$$

More generally, we propose that for any compact one-parameter CY we identify the period with the classical Grand Canonical potential of the UV

$$\frac{x_1^3}{3}+\frac{x_2^3}{3}+\frac{x_3^3}{3}+\psi x_1 x_2 x_3 = 0\ ,$$
$$\frac{x_4^3}{3}+\frac{x_5^3}{3}+\frac{x_6^3}{3}+\psi x_4 x_5 x_6 = 0\ ,$$

$$\mu = 2\log\left(3\psi\right)$$

$$J_0(\mu) = -\int_C \frac{dz}{2\pi i}\frac{9e^{3\mu z}}{3^{6z}}\left[\frac{\Gamma\left(\frac{1}{3}-z\right)^2\Gamma(z)^4}{\Gamma\left(\frac{1}{3}+z\right)^2}+\frac{\Gamma\left(\frac{2}{3}-z\right)^2\Gamma(z)^4}{\Gamma\left(\frac{2}{3}+z\right)^2}\right.$$
$$-\frac{(-1)^z(3i+\sqrt{3})\pi\Gamma\left(\frac{2}{3}-z\right)\Gamma(z)^4}{\Gamma\left(\frac{1}{3}+z\right)\Gamma(\frac{2}{3}+z)^2}$$
$$\left.+\frac{(-1)^z(3i-\sqrt{3})\pi\Gamma\left(\frac{1}{3}-z\right)\Gamma(z)^4}{\Gamma\left(\frac{1}{3}+z\right)^2\Gamma\left(\frac{2}{3}+z\right)}\right].$$

We can then understand this as an effective action for μ coming from integrating out the UV (a la Schwinger). This is then related to the effective action for the Kahler modulus through a leading quantum map:

$$T_B = \frac{6\pi\mu}{\hbar}+\dots\ . \qquad\qquad J\left(\mu\right) = F\left(T_B\right)\ ,$$

Summary

- Argued that tree-level kinetic terms in type IIA string theory, determined by the cubic prepotential, should be thought of as emergent from integrating out non-perturbative states

- Proposed to UV complete the Schwinger integral by analytic continuation

- More microscopic understanding through strong-coupling Fermi-gas picture: the prepotential is determined by the effective action after integrating out a (second-quantized) two-dimensional free Fermion

- Began applying these ideas to compact cases with dynamical gravity

- Many open questions remain about realizing in all string theory settings

Thank You

In fact, it turns out that one can recover the full non-perturbative all-genus prepotential when we recall that

$$z^2 \to \left(2i\sinh\left(\frac{g_s z}{2}\right)\right)^2$$

So that we have

$$F = \oint \frac{dz}{z} \frac{e^{-z(2\pi T - i\pi)}}{(1 - e^{-2\pi i z})\left(2i\sinh\left(\frac{g_s z}{2}\right)\right)^2}$$

The imaginary poles at $z = \frac{2\pi i n}{g_s}$ now give the full completion as calculated through the refined topological string

[Alim ’21]

Proposal formulated following a series of investigations [EP ’19]

- Factorisation in AdS/CFT suggests gauge fields are emergent from charged states at high energies

  [Harlow ’16]

- Toy model calculation suggests Swampland conjectures are imposing unification at the Species scale (weak emergence)

  [Heidenreich, Reece, Rudelius ’17; Grimm, EP, Valenzuela ’18; Heidenreich, Reece, Rudelius ’18]

- Proposal that type IIB complex-structure moduli space arises fully from integrating out wrapped D3 branes: So, emergence is an exact Infrared Duality (strong emergence)

  [Grimm, EP, Valenzuela ’18]

[Basile, Blumenhagen, Calderón-Infante, Castellano, Cribiori, Corvilian, Cota, Delgado, EP, Grimm, Gligovic, Herraez, Heidenreich, Ibanez, Lee, Lerche, Li, Marchesano, Melotti, Mininno, Paraskevopoulou, Paoloni, Reece, Rudelius, Ruiz, Uranga, Valenzuela, Weigand, Wiesner, + …]

Can this work for compact models with dynamical gravity?

For non-compact cones of reflexive Toric surfaces, a Fermi gas Hamiltonian matches the classical part of the Grand canonical potential with the period

$$J_0(\mu) = \Pi_0(T_B) = 2\mathcal{F}_0\left(T_B\right) - T_B \frac{\partial \mathcal{F}_0\left(T_B\right)}{\partial T_B} . \qquad J_0(\mu)|_{\text{cubic}} = -\left.\mathcal{F}_0\left(T_B\right)\right|_{\text{cubic}}$$

[Grassi, Hatsuda, Marino, '11]

Here T_B is the Kahler modulus associated to the whole base B, and at large volume the (classical) map is

$$T_B = r\mu$$

Here r is a determined constant related to the choice of base

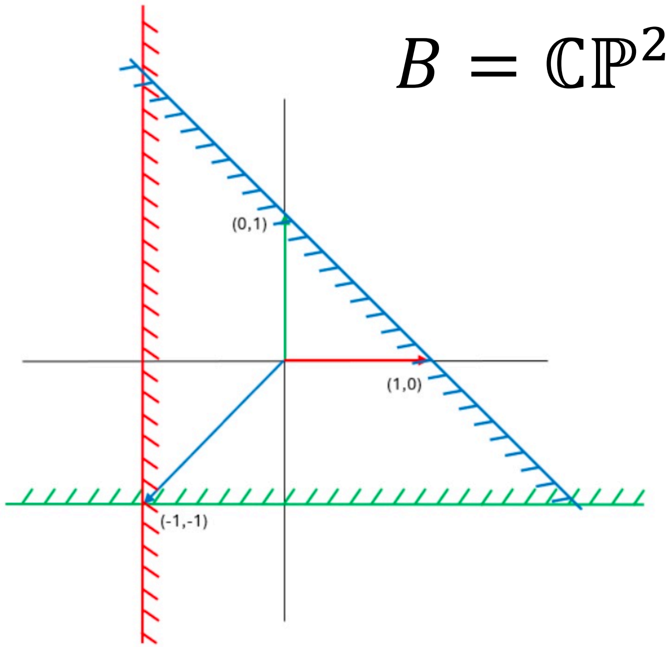
We propose that this is part of a full (quantum) map which relates the prepotential to the Grand Canonical potential

$$J(\mu) = F(T_B) \;, \qquad\qquad T_B = \left(\frac{2\pi}{\hbar}\right) r\mu + \ldots \,.$$

This leading behaviour reproduces the cubic term in the prepotential from integrating out the Fermi gas also in such cases

$B = \mathbb{CP}^2$

In the Fermi model, the cubic coefficient in the prepotential is calculated as the area of the toric diagram of the base



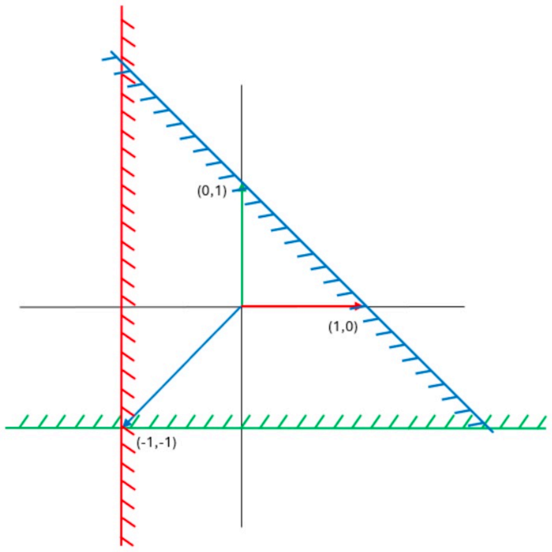
Instead of a non-compact cone over base B we can consider a compact elliptic fibration over it

The Fermi model, and the map, stay the same ($r = 1$)

Turns out that also in this case the cubic coefficient is given by the area of the base toric diagram, and therefore the Fermi model gives the correct answer

$$\kappa_B = (c_1(B))^2 \qquad c_1(B)^2 = 12 - \#\left(\partial\Delta \cap \mathrm{M}\right)$$

$$\#\left(\partial\Delta \cap \mathrm{M}\right) + \#\left(\partial\Delta^* \cap \mathrm{N}\right) = 12$$



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[Gopakumar, Vafa '98]

- β is the wrapping number of the D2
- n is the D0 charge
- g is the genus of the wrapped curve

$$F^{NP}(T, W) = \mathcal{L}_{\text{eff}}^{IR}(T, W)$$

Capturing a full calculation in superspace: $S_{\text{eff}}^{IR} = -\mathrm{i} \int d^4x \, d^4\theta \, F^{NP}(\boldsymbol{T}, \boldsymbol{W})$

[Dedushenko, Witten '14]