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Black Holes from Supercharge Cohomology

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- Talk based on collaboration with
  - Xi Yin (Harvard University)
  - Ying-Hsuan Lin (Harvard University)
  - Jingxiang Wu (Oxford University)
  - Li Feng (Northeastern University)
  - Yi-Xiao Tao (Tsinghua University)

“1/16 BPS states in N = 4 super-Yang-Mills theory” arXiv:1305.6314

“Words to describe a black hole” arXiv:2209.06728

“Decoding stringy near-supersymmetric black holes” arXiv:2306.04673

“Holographic covering and the fortuity of black holes” arXiv:2402.10129

“On 1/8-BPS black holes and the chiral algebra of N=4 SYM” arXiv:2310.20086

See also:

- Choi, Choi, Kim, Lee, Lee, Lee, Park [CCKLLLP]

  “The shape of non-graviton operators for SU(2)” arXiv:2209.12696

  “Towards quantum black hole microstates” arXiv:2304.10155

  “Finite N black hole cohomologies” arXiv:2312.16443

- Budzik, Gaiotto, Kulp, Williams, Wu, Yu [BGKWWY] and Budzik, Murali, Vieira [BMV]

  “Semi-Chiral Operators in 4d N=1 Gauge Theories” arXiv:2306.01039

  “Following Black Hole States” arXiv:2306.04693

- Talks in previous Strings:

  2013: Xi Yin “Comments on BPS states in N=4 SYM”

  2023: Seok Kim “Black hole cohomologies in N=4 Yang-Mills”

Motivation

- There are large BPS black holes in AdS_5 [Gutowski-Reall ’04, Chong-Cvetic-Lu-Pope ’05, …]. We would like to understand their microstates using the dual CFT.
- The N^2 growth of their entropy can be reproduced by the superconformal index. [(Cabo-Bizet)-Cassani-Martelli-Murthy, Choi-Kim-Kim-Nahmgoong, Benini-Milan ‘18]
- Given a non-perturbatively complete framework of AdS/CFT, we should be able to answer more refined questions:
  - What are the wave functions and dynamics of the microstates?
  - How do we distinguish a weakly bound state of N^2 gravitons from typical black hole microstates?

Motivation

- Obstacle: Holography is a strong/weak duality. Black holes exist at large N and strong 't Hooft coupling, and are hard to study from the CFT side.
- Two ways to proceed:
  - Study toy models, such as lower-dimensional holography
  - Focus on physical quantities protected by supersymmetry
- Supercharge cohomology is semi-protected by SUSY and reveals much richer information beyond state counting.

Motivation

- Semi-protection by SUSY is due to a non-renormalization conjecture
  - Supercharge cohomology is invariant along the conformal manifold of the boundary CFT away from free points.
  - Evidence: 1. Matching BPS spectrum at infinite N (see later)

    2. Consistency with S-duality of N=4 SYM and N=2 theories

    [**CC**-Choi-Dong-Yan WIP]
- New information beyond the superconformal index:
  - Complete BPS spectrum (It counts BPS states without $(-1)^F$.)
  - Information on the wave functions of BPS states (modulo exact terms in cohomology) and how they are related in theories at different ranks N.

Outline

- Introduction to supercharge cohomology
- A classification: monotone (graviton) vs. fortuitous (black holes)
- Bulk duals of supercharge cohomologies
- Fortuitous supercharge cohomologies and black holes

Supercharge Cohomology

Supercharge Cohomology

- Supercharge cohomology can be defined very generally. It only needs a supercharge Q that is nilpotent $Q^2 = 0$.

  The main focus in this talk:

  4d maximal SYM with $\mathrm{SU}(N)$ gauge group

- Pick a pair Q & $S = Q^\dagger$ out of 16 Q and 16 S.

  BPS bound: $\Delta = 2\{Q, Q^\dagger\} = E - (J_1 + J_2 + q_1 + q_2 + q_3) \geq 0$

  BPS states are states with $\Delta = 0$. (spins J_i , R-charges q_i)

Supercharge Cohomology

- Nilpotency $Q^2 = 0$.

$$Q\text{-cohomology} = \frac{\{O \,|\, QO = 0\}}{\{O \,|\, O = QO'\}}$$

- Hodge theory argument:

$$Q\text{-cohomology classes} \overset{\text{1 to 1}}{\longleftrightarrow} \text{BPS states } (\Delta = 2\{Q, Q^\dagger\} = 0)$$

- The non-renormalization conjecture (Q-cohomology is independent of $g_{\rm YM} \neq 0$) allows us to compute Q-cohomology at weak coupling $g_{\rm YM} \ll 1$

Weak-Coupling Setup

- At weak couplings, local operators could be written in terms of multitraces of fundamental fields with covariant derivatives (both $N \times N$ matrices) and modulo trace relations.

- Trace relations play an important role in the study of supercharge cohomology. A simple example of trace relations is

  e.g. for any 2×2 matrix X, $2\mathrm{Tr}\,X^3 = 3\mathrm{Tr}\,X\,\mathrm{Tr}\,X^2 - (\mathrm{Tr}\,X)^3$.

- A property of trace relations we will use later:
  - I_N = (space of trace relations at rank N) . We have $I_{N+1} \subsetneq I_N$

Weak-Coupling Setup

- At weak coupling, it suffices to work with fundamental fields that are BPS in free theory. They can be assembled into a superfield $\Psi(z^\alpha, \theta_i)$ on superspace $\mathbb{C}^{2|3}$ with two bosonic coordinates z^α $(\alpha = \pm)$ and three fermionic coordinates θ_i $(i = 1,2,3)$. [**CC**-Yin '13]

$$\Psi(z^\alpha, \theta_i) \sim \lambda_\alpha z^\alpha + \phi^i \theta_i + \epsilon^{ijk} \psi_i \theta_j \theta_k + F_{++} \theta^3 + \cdots,$$

λ_α gauginos
$\phi^i = (X, Y, Z)$ complex scalars
ψ_i complex fermions
F_{++} self-dual field strength
$(\cdots)$ covariant derivatives

- The Q action takes a very concise form

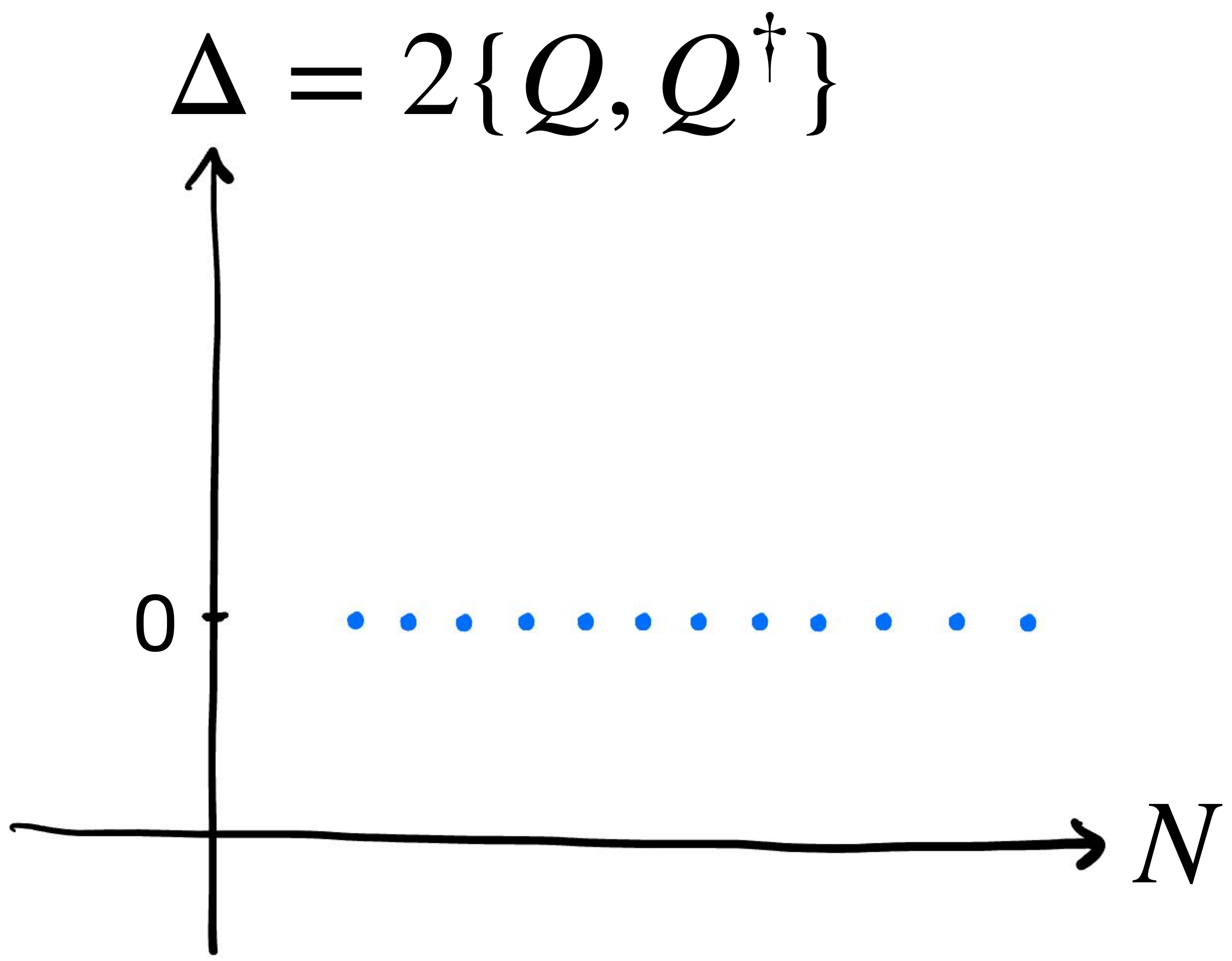
$$Q(\Psi) = \Psi^2$$

A Classification of Cohomologies: Monotone vs. Fortuitous

A Classification of Cohomologies

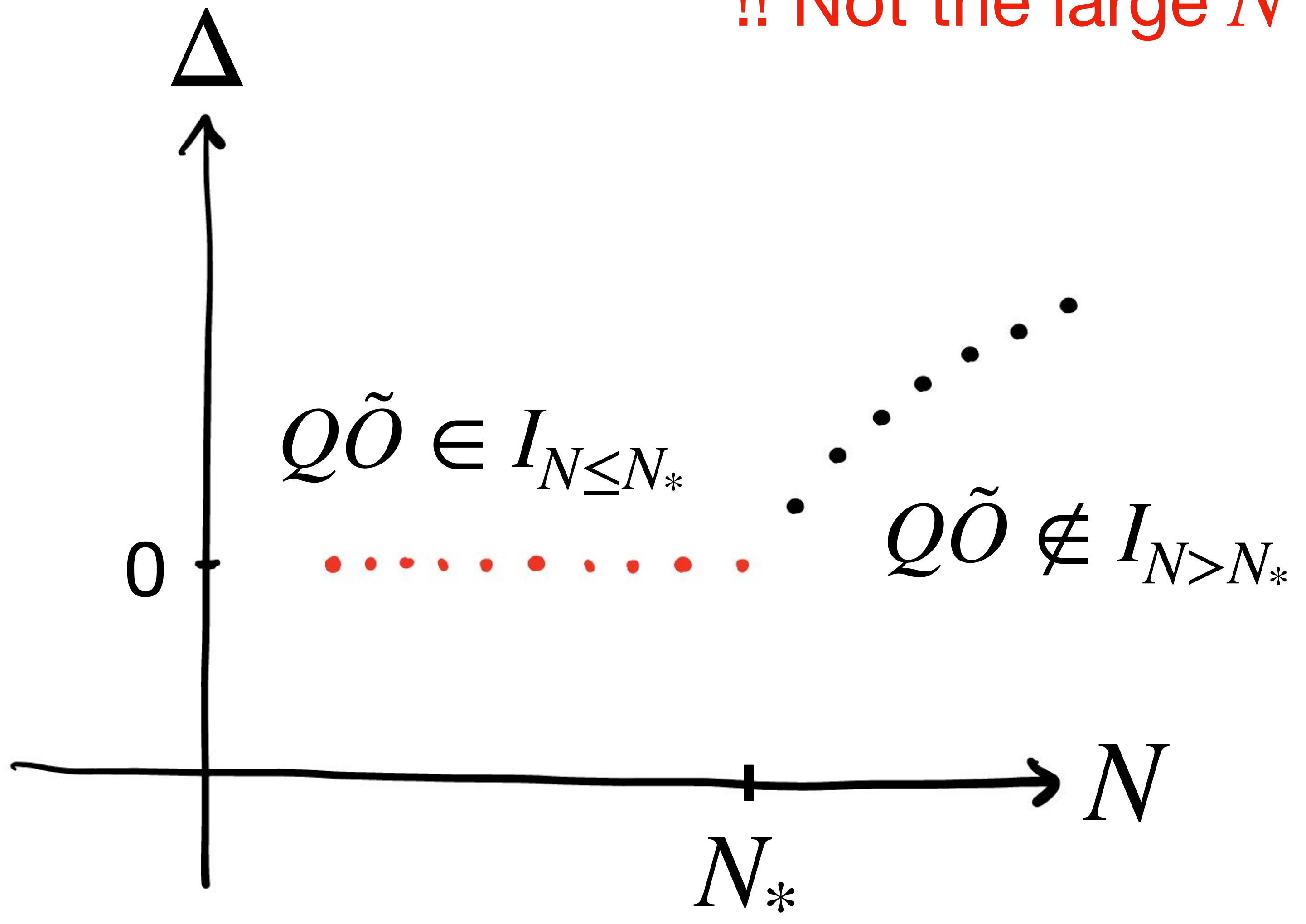
- Let O represent a Q-cohomology class, and write O non-uniquely as a multitrace $\tilde{O}$.

- Monotone (graviton) cohomology:
  - $Q\tilde{O} = 0$ w/o using trace relations
  - Admit infinite N limits with **fixed** $\tilde{O}$



- Fortuitous (black hole) cohomology:
  - $Q\tilde{O} =$ (a nontrivial trace relation)
  - No infinite N limit (with fixed $\tilde{O}$)

!! Not the large N limit with $E \sim N^2$!!



- The precise definition is given by a long exact sequence. [**CC**-Lin '24]

Monotones in $\mathcal{N} = 4$ SYM

- Consider one-forms on the superspace $\mathbb{C}^{2|3}$: basis dz^α, $d\theta_i$

$$d\Psi \equiv dz^{\dot{\alpha}}\,\partial_{z^{\dot{\alpha}}}\Psi + d\theta_i\,\partial_{\theta_i}\Psi$$

Supercharge action: $Qd\Psi = [\Psi, d\Psi]$

- The multitrace $\mathrm{Tr}\,[(d\Psi)^{n_1}]\cdots\mathrm{Tr}\,[(d\Psi)^{n_L}]$ is Q-closed and not Q-exact without using trace relations.

- In fact, all monotone Q-cohomologies could be obtained by imposing trace relations at some finite N on $\mathrm{Tr}\,[(d\Psi)^{n_1}]\cdots\mathrm{Tr}\,[(d\Psi)^{n_L}]$. [**CC**-Yin '13] [BGKWWY '23]

Fortuitous in $\mathcal{N} = 4$ SYM

- By a brute force comprehensive search in the SU(2) theory up to high spin and R-charges, we found the first fortuitous Q-cohomology. [**CC**-Lin '22]
  - Very hard to find (1 in 10^5 cohomology classes) (Doesn't mean fortuitous are few!)
  - Explicit representative [Choi-Kim-Lee-Park '22] : $\partial^{i_1 \cdots i_n} \equiv \partial_{\theta_{i_1}} \cdots \partial_{\theta_{i_n}}$

$$O = \epsilon_{i_1 i_2 i_3} \epsilon_{j_1 j_2 j_3} \epsilon_{l_1 l_2 l_3} \epsilon_{m_1 m_2 m_3} \epsilon^{k_1 l_1 m_1} \mathrm{Tr}(\partial^{i_1} \Psi \partial^{k_2 k_3} \Psi) \mathrm{Tr}(\partial^{j_1} \Psi \partial^{l_2 l_3} \Psi) \mathrm{Tr}(\partial^{i_2 i_3} \Psi \partial^{j_2 j_3} \Psi \partial^{m_2 m_3} \Psi)$$

- Working in the BMN sector (only ∂_{θ_i} and no $\partial_{z^{\dot{\alpha}}}$), [CCKLLLP '22, '23] performed a more efficient search and achieved the following results:
  - SU(2) and SU(3): multiple infinite towers of fortuitous cohomologies
  - SU(4): leading fortuitous cohomology

Bulk Duals of Q-cohomologies

Intuitions for the bulk duals

- While supercharge cohomology is independent of the ’t Hooft coupling λ, we will focus on large λ and look for bulk duals in supergravity.

- Given some classical BPS solutions in supergravity, consider $G_N \to 0$ $(N \to \infty)$ limit with fixed spins and charges (and energy by BPS condition).
  - Smooth horizonless solutions: Remain smooth horizonless and become perturbative particles in AdS — similar to monotone cohomologies.
  - Black hole solutions: $S = A/4G_N$ fixed as $G_N \to 0$. The solutions become naked singularities — similar to fortuitous cohomologies.

Bulk Duals of Monotones

- Monotone Q-cohomologies at infinite N are dual to BPS multi-particles in $\mathrm{AdS}_5 \times \mathrm{S}^5$.
  - Evidence: The counting of the multitraces $\mathrm{Tr}\,[(d\Psi)^{n_1}]\cdots\mathrm{Tr}\,[(d\Psi)^{n_L}]$ matches with the BPS multi-graviton partition function. [**CC**-Yin '13]

- Conjecture [**CC**-Lin '24] : Monotone Q-cohomologies at finite N are dual to quantizations of smooth horizonless solutions in supergravity.
  - Evidence: The partition function of half-BPS operators (all monotone) can be reproduced by a quantization of the Lin-Lunin-Maldacena (LLM) geometries (smooth horizonless half-BPS geometries).

Bulk Duals of Fortuitous?

- There are exponentially more fortuitous Q-cohomologies than monotone ones.

A typical black hole microstate is fortuitous.

- A bound on the number of monotone Q-cohomologies:

$$\boxed{\begin{array}{l}\text{\# monotones}\\ \text{at finite } N\end{array}} < \boxed{\begin{array}{l}\text{\# monotones}\\ \text{at infinite } N\end{array}} = \boxed{\begin{array}{l}\text{\# BPS multi}\\ \text{particle states}\end{array}} < \boxed{\begin{array}{l}\text{\# all multi}\\ \text{particle states}\end{array}} \sim e^{E^{\frac{9}{10}}} \sim e^{N^{\frac{9}{5}}}$$

trace relations

Entropy of gas of free particles $\sim E^{\frac{9}{10}}$

BH energy $E \sim N^2$

- The growth of the total number of all BPS states can be estimated by the superconformal index [CCMM, CKKN, BM ’18].

(# monotones) + (# fortuitous) = (# all BPS) $\sim e^{N^2}$

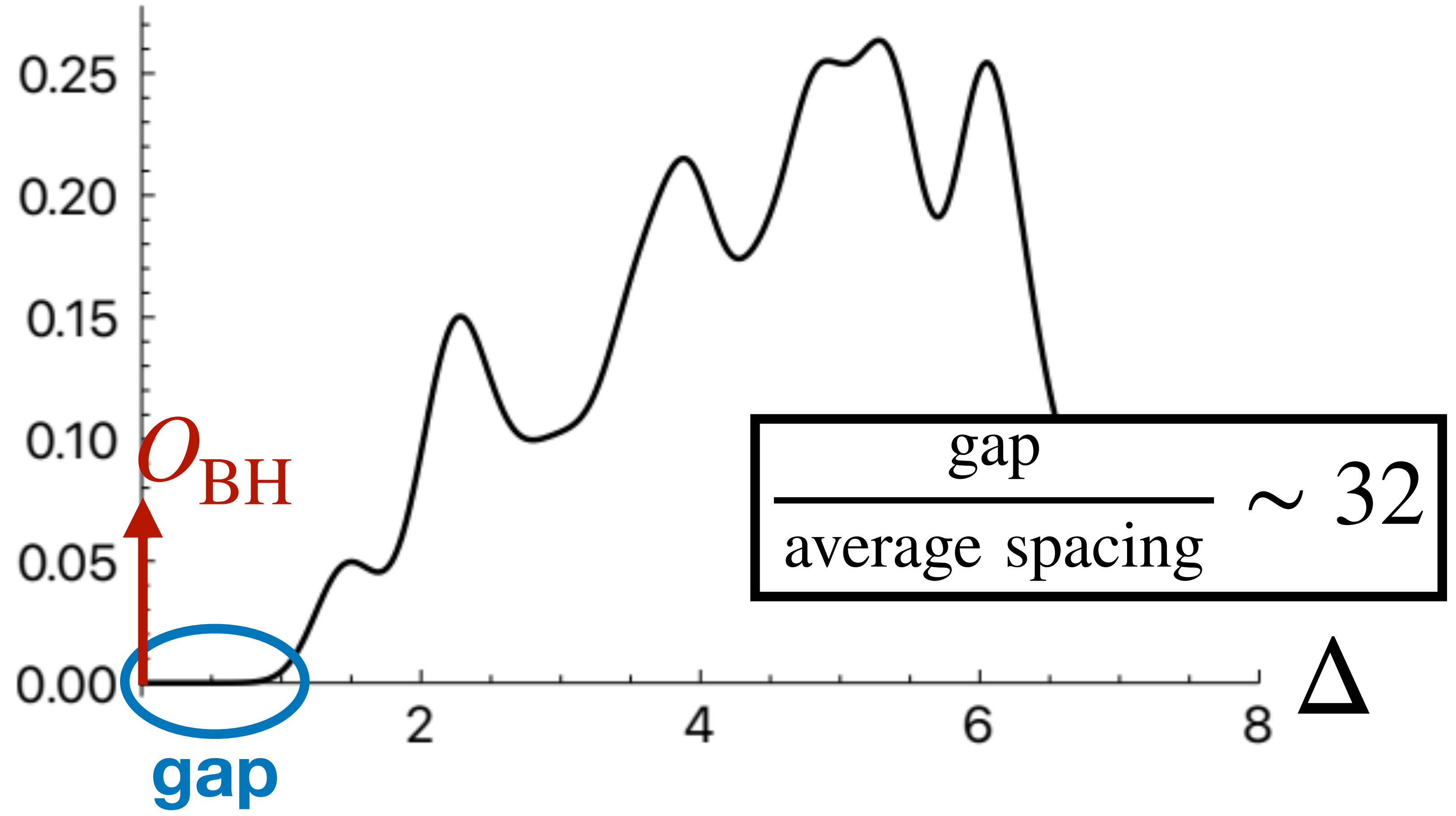
More on Fortuitous Cohomologies and Black Holes

Near-BPS States Above a Fortuitous State

- Small ’t Hooft coupling λ and N
  - O: representative of the previous fortuitous cohomology in SU(2) SYM.
  - $\mathscr{H}_{\mathrm{BH}} = \{O + QO'\}$ for QO' with the same J_i, q_i, and classical E as O.
  - Diagonalize the one-loop $\Delta_{1-\mathrm{loop}} = 2\{Q, Q^\dagger\}$. (ground state: O_{BH})

⇘

density of states (total 186 states)



[**CC**-Feng-Lin-Tao ’23]

- Large ’t Hooft coupling λ and N
  - JT sugra on the near-horizon AdS_2
  - a gap of order N^{-2} above the BPS states.

$$\frac{\text{gap}}{\text{average spacing}} \sim \frac{N^{-2}}{e^{-N^2}}\Big|_{N=2} \sim 14$$

[Boruch-Heydeman-Iliesiu-Turiaci ’22] [Stanford-Witten’17]

Future Direction

- What is the bulk dual of an individual fortuitous BPS state? What is a typical black hole microstate?
  - Construct fortuitous Q-cohomology at larger N (so far only $N = 2, 3, 4$)
  - $N \gtrsim 6$ maybe enough as S/N^2 from SCI already shows convergence at $N \sim 6$
  - A proposal: The bulk dual of a fortuitous BPS state is a coupled system of D-branes in a supergravity background at strong 't Hooft coupling.

- Generalizations: supercharge cohomology in D1-D5 CFTs [**CC**-Lin-Zhang WIP], 4d N=2 SCFTs [**CC**-Choi-Dong-Yan WIP], BMN matrix quantum mechanics, …

- There is much more to be learned about black holes from supercharge cohomology!

Thank you