Constraints on the Species Scale and the Spectrum of States in Quantum Gravity

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Based on:

2310.07213, 2305.07701, **2303.13580,** 2212.06841

with
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— and —

2405.00083

with
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Introduction: Quantum Gravity Cutoff/Species Scale

- **General expectation:** Energy scale at which quantum gravitational effects become relevant

$$M_{\rm pl} \sim 10^{19}\ \text{GeV}$$

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- In the presence of large number of *light species* of states
  → Quantum Gravity Cutoff parametrically below Planck scale

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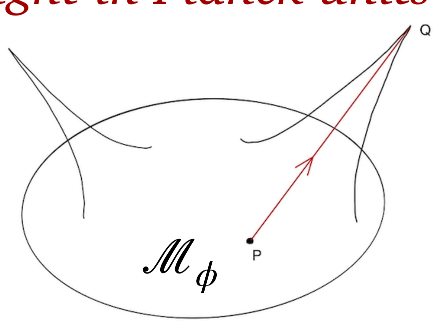
Distance Conjecture [Ooguri, Vafa '06]

Along paths in scalar field space traversing distances $d \gg l_p$ an infinite tower of states becomes light in Planck units as

$$\frac{M(Q)}{M_{pl}} \sim e^{-Ad(P,Q)}$$



- In *asymptotic* limits $N_{\text{light}} \to \infty$
  $\Leftrightarrow\ \Lambda_s/M_{\text{pl}} \to 0$

- Tower *known* in explicit cases
  → can compute Λ_s in these limits!

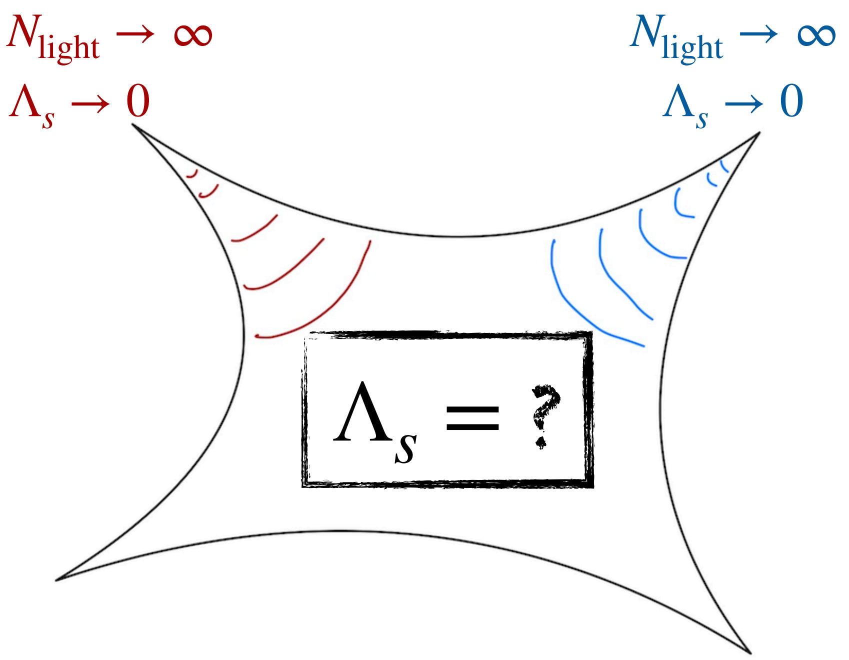
For reviews see [Palti '19;
van Beest, Calderon-Infante, Mirfendereski, Valenzuela '21
Agmon, Bedroya, Kang, Vafa '22]

Recent works on species scale: [(Calderon-Infante), Castellano, Herraez, Ibanez '22,'23; Melotti, Marchesano '22; v.d. Heisteeg, Vafa, MW, Wu '22,'23; Cribiori, Lust, (Staudt) '22,'23; Cribiori, Lust, Montella '23; Castellano, Ruiz, Valenzuela '23; Calderon-Infante, Delgado, Uranga '23; Basile, Lust, Montella '23; Cota, Mininno, Weigand, MW '22,'23; Basile, Cribiori, Lust, Montella '24; Bedroya, Vafa, Wu '24; Bedroya, Mishra, MW '24; Aoufia, Basile, Leone '24]

Λ_s without counting light states

- **Question:** How do we compute the QG cutoff/species scale away from asymptotic limit?

  - Would need to compute light spectrum at any point in field space…
  - Computation of exact spectrum at strong coupling difficult
    $\rightarrow$ need different way to compute Λ_s



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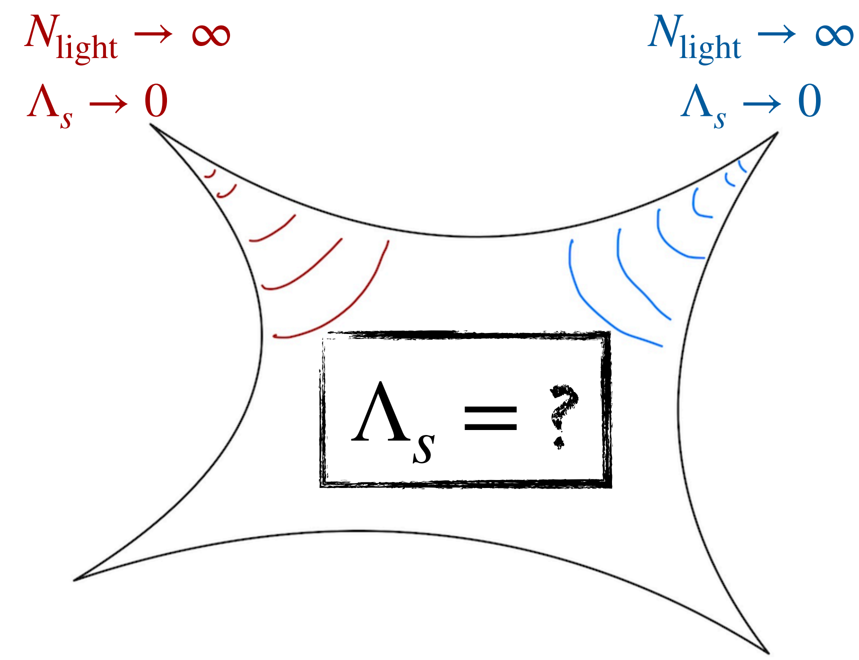

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- **Proposal:** QG cutoff captured by gravitational higher-derivative corrections to Einstein-Hilbert action. [v.d. Heisteeg, Vafa, MW, (Wu) '23]

$$S_{\text{corr.}} = \frac{M_{\text{pl}}^{d-2}}{2}\int \mathrm{d}^d x\sqrt{-g}\left(\mathcal{R} + \frac{1}{2}(\partial\phi)^2 + \ldots + \sum_{n=1}^{\infty} a_n(\phi)\frac{\mathcal{O}_{2n+2}(\mathcal{R},\partial)}{M_{\text{pl}}^{2n}}\right)$$

Scalar fields

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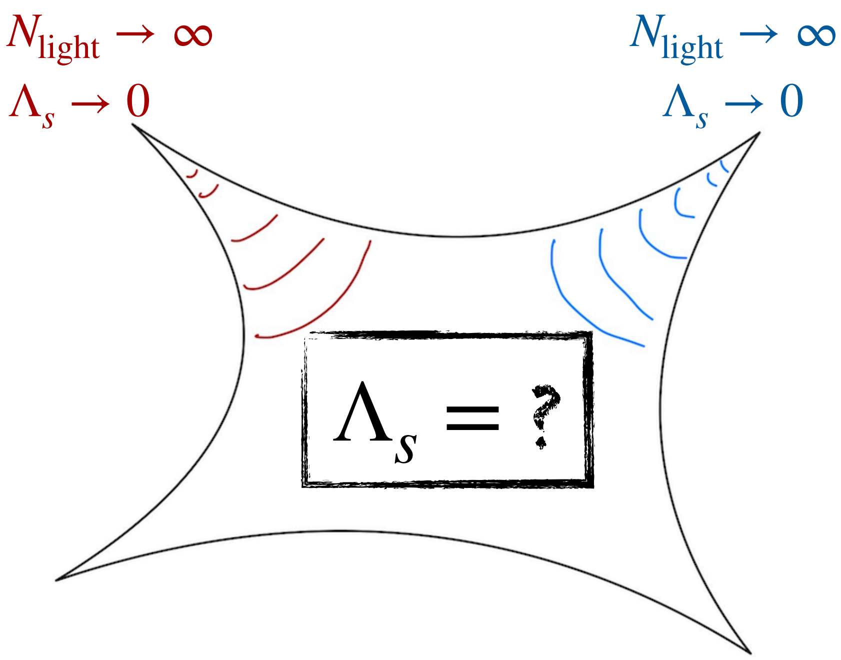

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Scalar fields

  - Wilson coefficients of higher-derivative terms give species scale:

$$\Lambda_s(\phi) \sim \frac{M_{\text{pl}}}{a_n(\phi)^{\frac{1}{2n}}}$$

“Minimal” Black hole and Λ_s

- **What is the physical meaning of the scale suppressing the higher-derivative corrections?**

- Compare to Field Theory (e.g. electrodynamics in d dimensions):
  → higher-derivative corrections sensitive to particles of mass m that have been integrated out.

$$\mathscr{L}_{\text{corr}} \supset \frac{1}{m^{4n-d}}(F^2)^n$$

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- Still: higher-derivative corrections encode imprint of “minimal black hole”: → cf. original motiviation for species scale in [Dvali ’07]

see also [Cribiori, Lüst, Staudt ’22; Calderon-Infante, Delgado, Uranga ’23]

  - Minimal black hole = smallest black hole describable by some effective field theory
    → mass M_{min} and radius $r_H^{\text{min}} = \Lambda_{\text{min}}^{-1}$ (EFT cutoff) (in general $\Lambda_{\text{min}} \neq M_{\text{min}}$)

  - Consider $2 \to 2$ scattering amplitudes and match higher-derivative expansion of action with contribution from minimal black hole!

Scattering Amplitudes and Λ_s

[Bedroya, Mishra, MW '24]

- **Result**: minimal black hole contribution to $2 \to 2$ scattering amplitudes can only be reproduced by higher-derivative corrections of the form

$$\mathscr{L}_{\text{corr}} \supset \hat{a}_n \frac{M_{\text{pl,d}}^{d-2}}{\Lambda_{\min}^{2n}} \mathscr{R} \square^{n-1} \mathscr{R} \quad \Rightarrow \quad \Lambda_s = \Lambda_{\min} = (r_H^{\min})^{-1}$$

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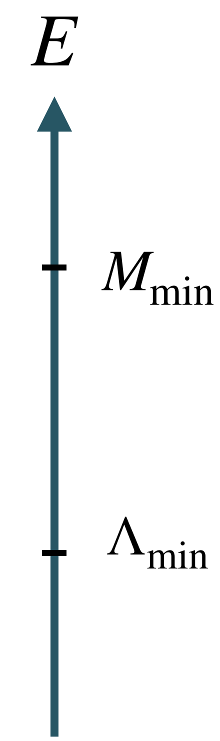
    → amplitude is exponentially suppressed by black hole entropy



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[Giddings, Srednicki '07]

Horizon radius of Schwarzschild black hole with mass E

$\rightarrow$ becomes exponentially large upon continuation to unphysical regime ($t \gg 0$)

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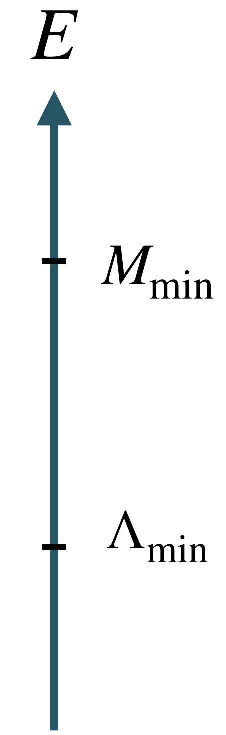

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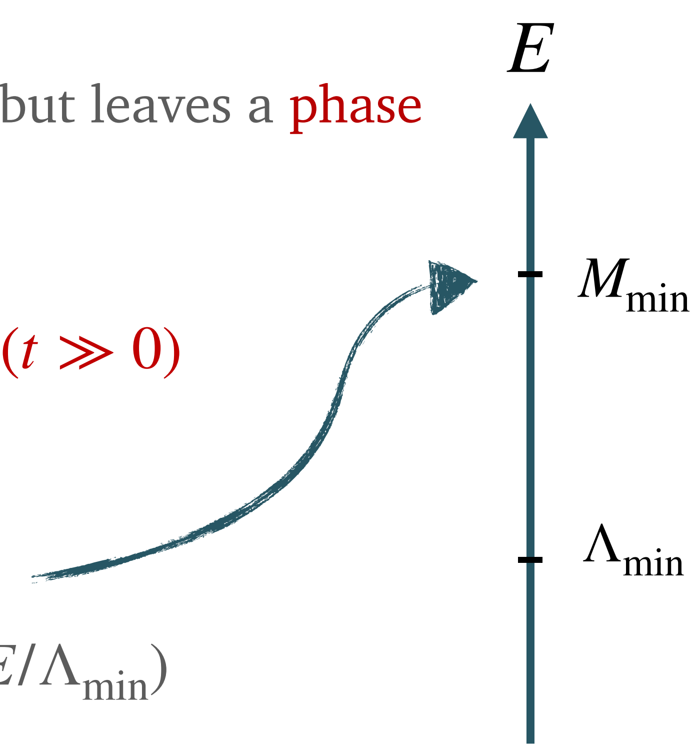

- Can argue: at energies $\Lambda_{\rm min} < E < M_{\rm min}$ replace r_H by $b_c(E) = \Lambda_{\rm min}^{-1} \mathcal{O}(E/\Lambda_{\rm min})$

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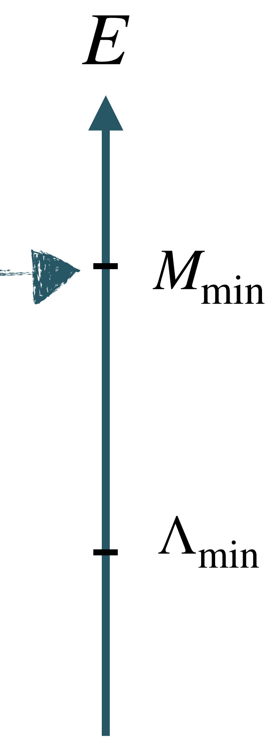

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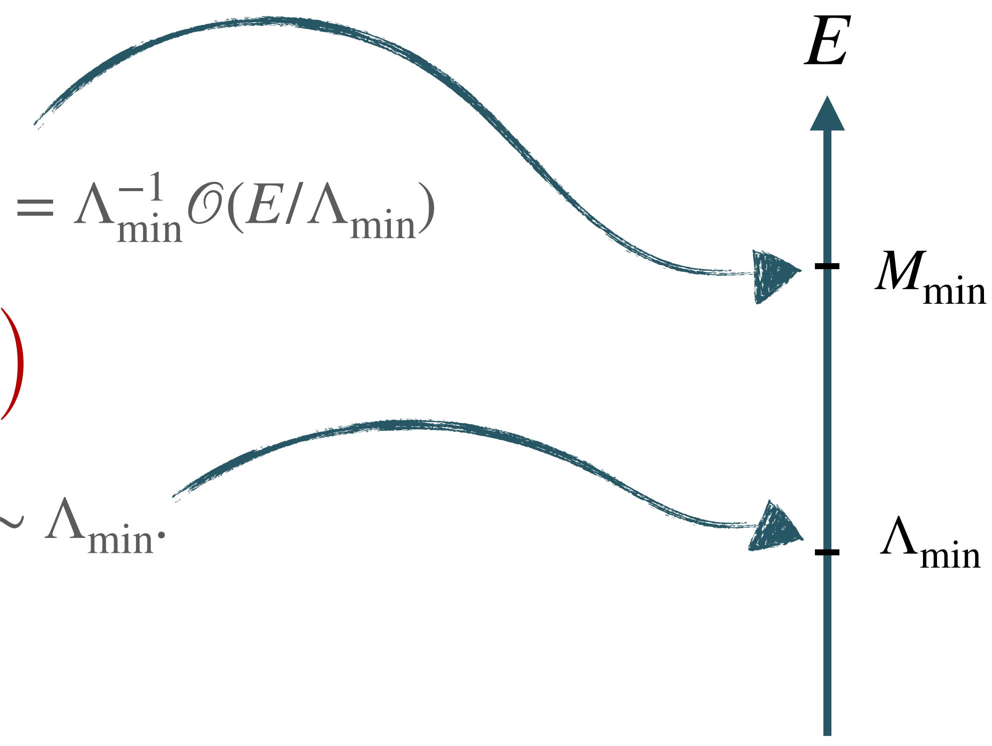
  - To reproduce exponential behavior of $\mathcal{A}$ for $t \gg 0$ need:

$$\mathcal{L}_{\rm eff} \supset M_{\rm pl}^{d-2}\frac{\hat{a}_n}{\Lambda_{\rm min}^{2n}}\mathcal{R}\,\Box^{n-1}\,\mathcal{R} \qquad \text{with} \qquad \hat{a}_n \sim \frac{1}{(2n)!(n-1)!}$$



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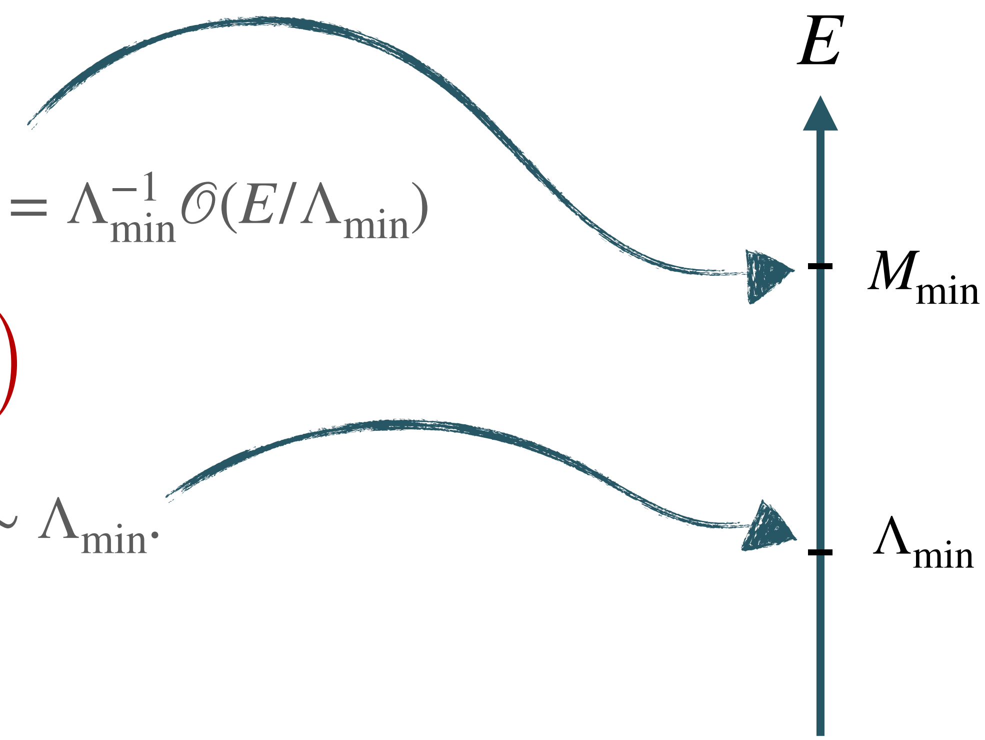

- Compare to species scale definition: $\quad \mathscr{L}_{\text{corr}}^{\text{grav}} \supset \hat{a}_n \frac{M_{\text{pl,d}}^{d-2}}{\Lambda_s^{2n}}\mathscr{R}\,\Box^{n-1}\,\mathscr{R}$

Upshot: $\Lambda_s = \Lambda_{\text{min}} \rightarrow$ the QG cutoff corresponds to the radius of the smallest black hole in the theory!

Species Scale from Higher-derivative Corrections

[v.d. Heisteeg, Vafa, MW, Wu '23]

- Consider higher-derivative corrections to effective action, e.g.

$$S = \int d^d x \sqrt{-g} \left[\frac{M_{\rm pl}^{d-2}}{2} \left(R + \frac{1}{2}(\partial\phi)^2 + a_2(\phi)R^2 + a_3(\phi)R^3 + a_4(\phi)R^4 + \dots \right) \right].$$

- Wilson coefficients encode field dependence of scale

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- Obtain an **upper bound** for the species scale *everywhere* in moduli space!

Species Scale from Higher-derivative Corrections

[v.d. Heisteeg, Vafa, MW, Wu '23]

- As an example consider 10d Type IIA string theory → single modulus ≙ string coupling $\phi = \log(g_s)$:

- First **non-vanishing** term of the higher-derivative corrections is $t_8 t_8 R^4$-coupling.

$$S_{10,R^4} = \frac{M_{\rm pl}^2}{2} \int \mathrm{d}^{10}x \sqrt{-g}\, a_4(\phi)\, t_8 t_8 R^4$$ [Green, Vanhove '97]

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- For the species scale this means:

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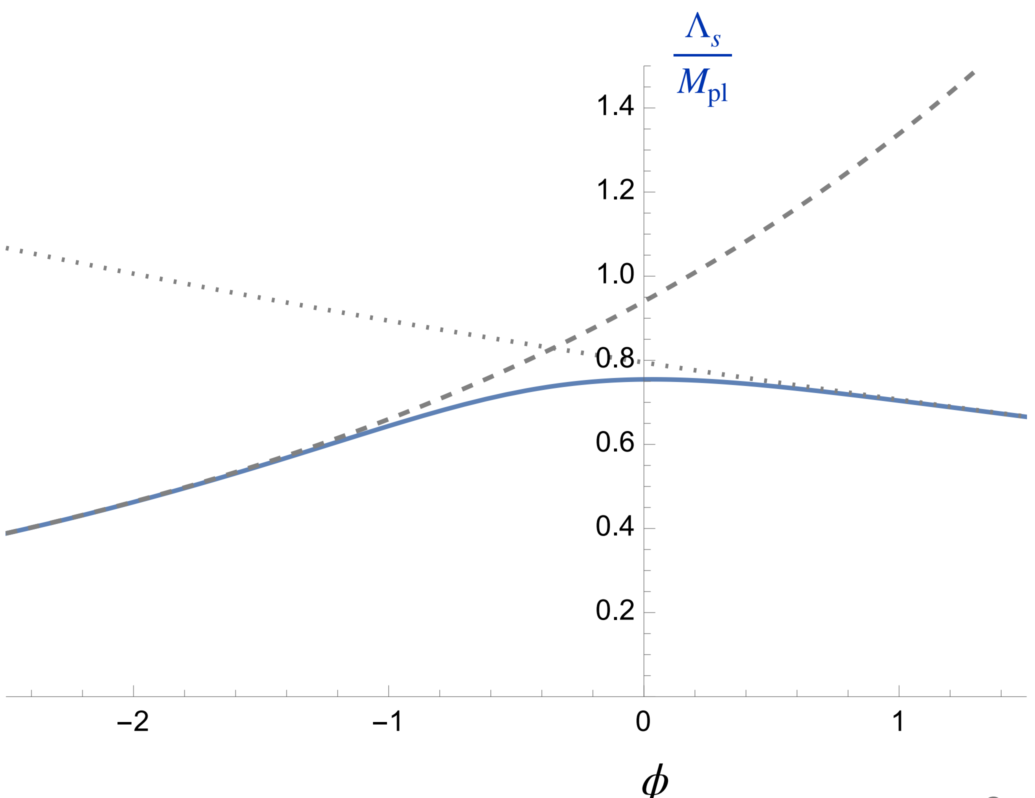

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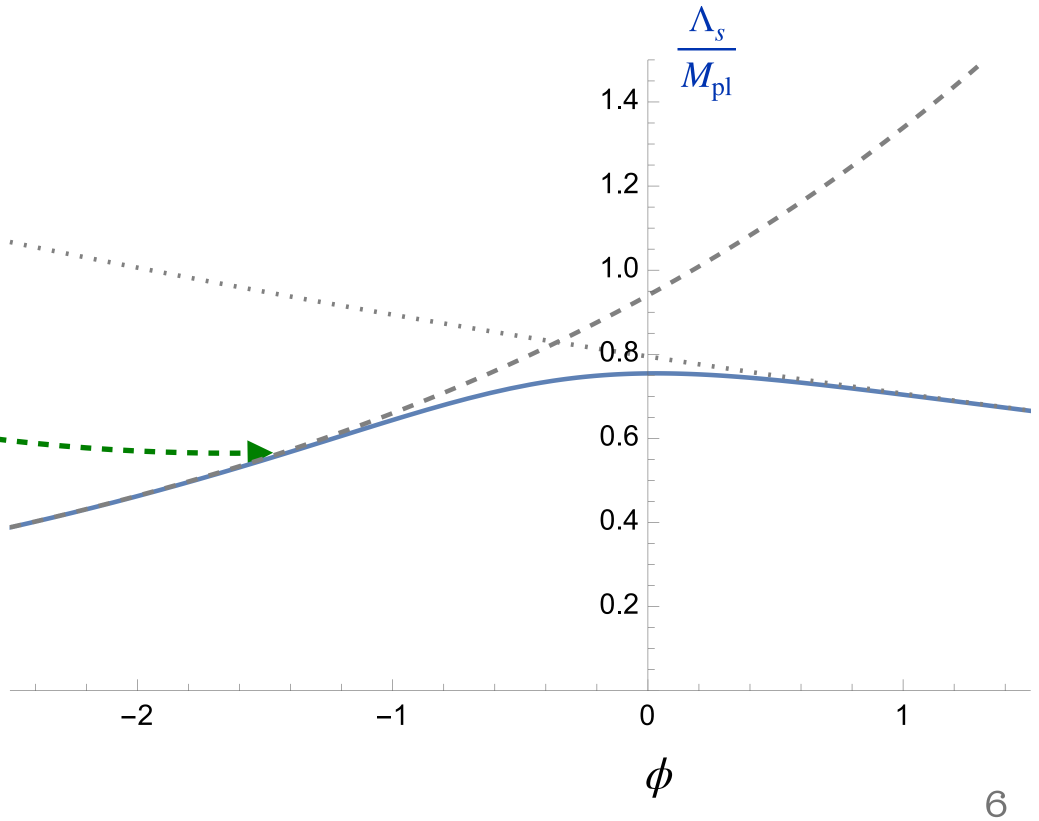

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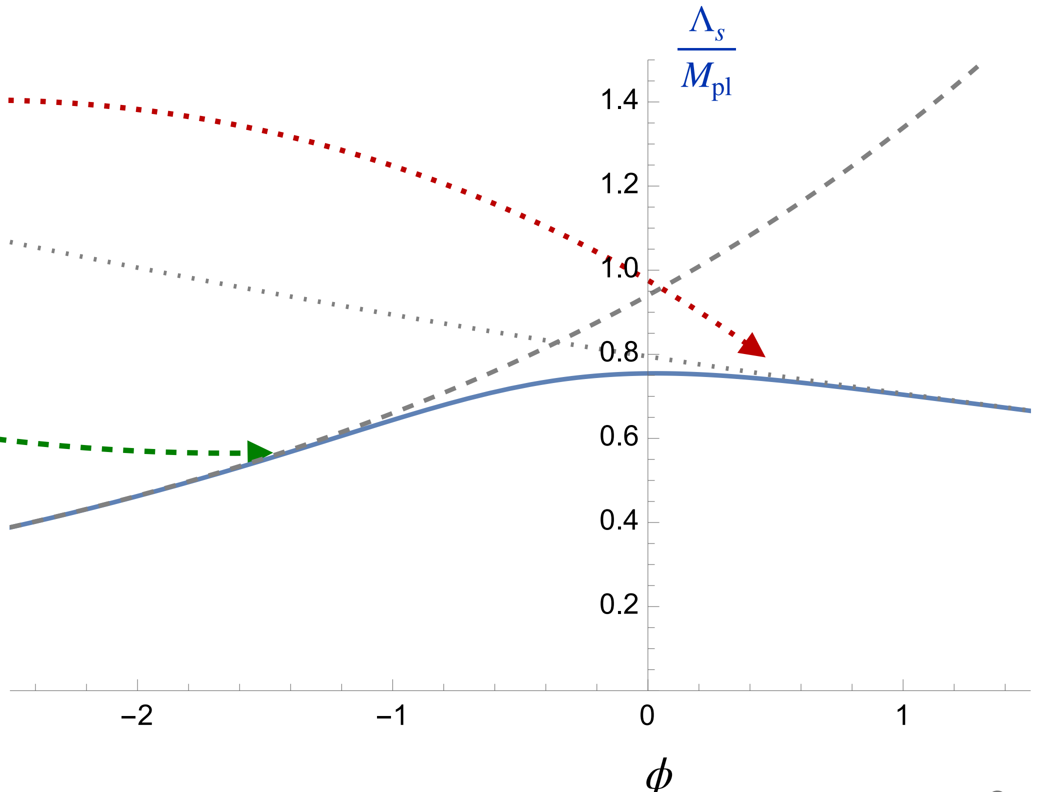

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- For $\phi \to +\infty$: one-loop dominates → species scale agrees with expectation from 11d M-theory

The Desert of the Moduli Space

[v.d. Heisteeg, Vafa, MW, Wu '23]

- Can repeat this in a large class of examples:

  → 32 supercharges; *e.g. M-theory on* T^n

  see also [Cribiori, Lüst '23; Castellano, Herraez, Ibanez '23]

  → 16 supercharges; *e.g. Heterotic/Type I on* T^n

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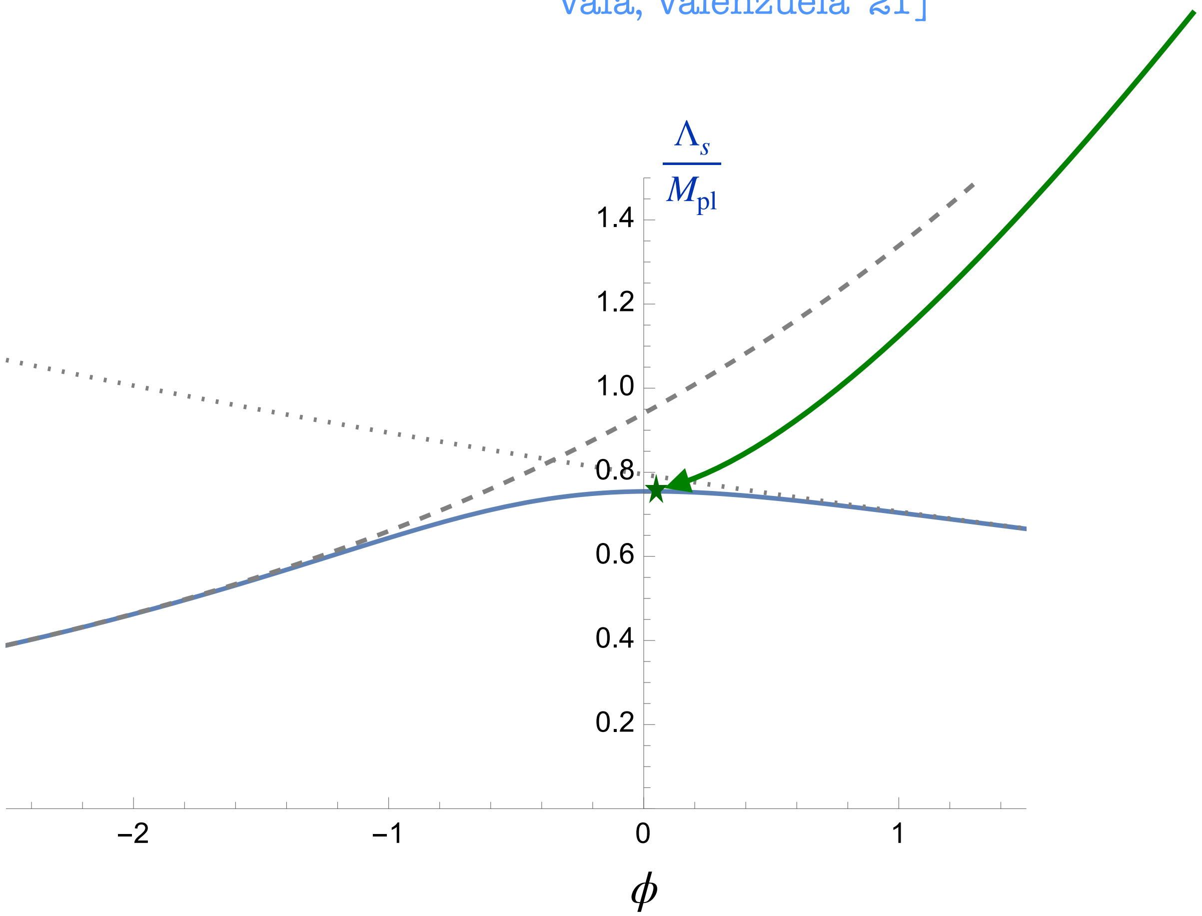

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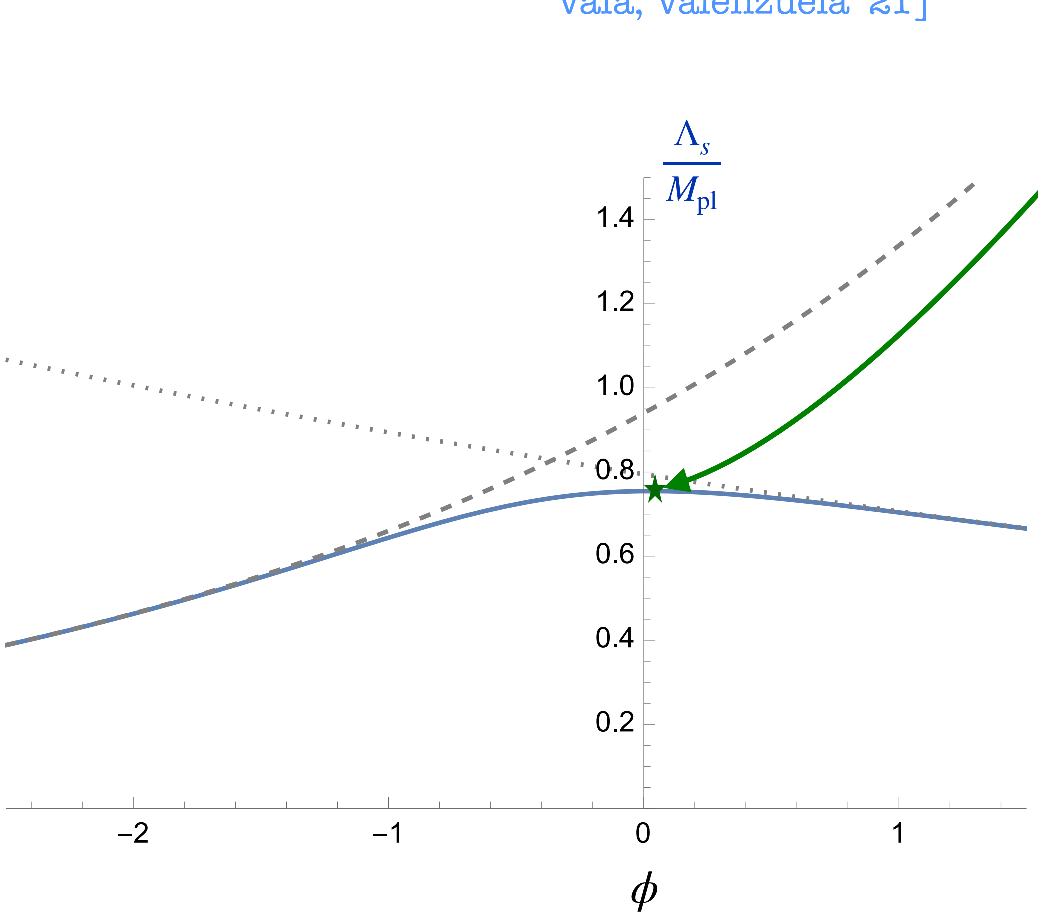

$\Lambda_s^{max} < M_{pl}$

Example	Λ_s^{max}/M_{pl}
10d IIA	0.755
10d IIB	0.756
M-theory on T^2	0.513
M-theory on T^3	0.504
10d Heterotic $E_8 \times E_8$	0.823
10d Heterotic SO(32)	0.822
F-theory on $\mathbb{F}_{n\leq 2}$	$2^{-3/4}$
M-theory on $X_{2,86}$	0.490

Bound on the Slope of Λ_s

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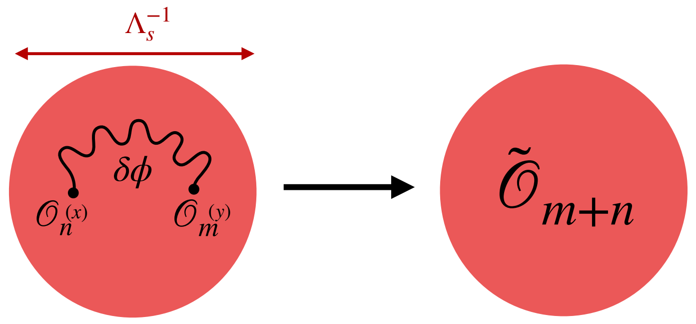
$$S_{\text{grav}} = \int d^d x \sqrt{-g} \left[\frac{M_{\text{pl}}^{d-2}}{2} \left(R + \sum_n \frac{\mathcal{O}_n(R)}{\Lambda_s^{n-2}(\phi)} + \dots \right) \right]$$

- Integrate out high-energy modes of $\phi = \phi_0 + \delta\phi$



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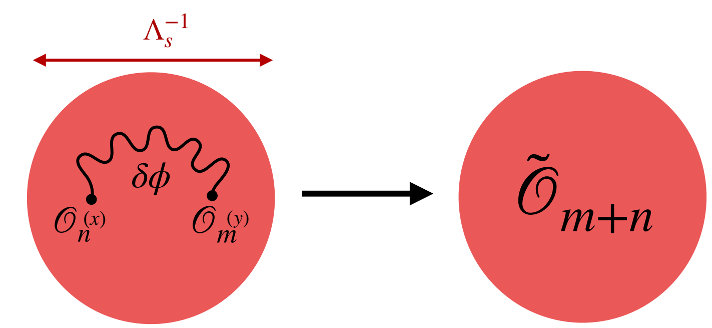

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→ Consistency of effective higher-derivative expansion leads:

$$\left| \frac{\nabla \Lambda_s(\phi_0)}{\Lambda_s(\phi_0)} \right|^2 \leq \frac{c}{M_{\text{pl}}^{(d-2)}}\,, \quad c \sim \mathcal{O}(1)$$

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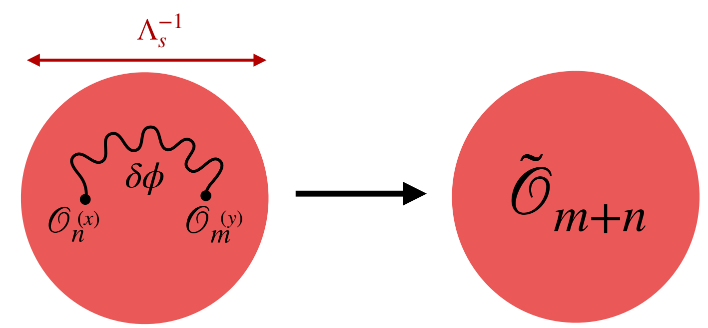

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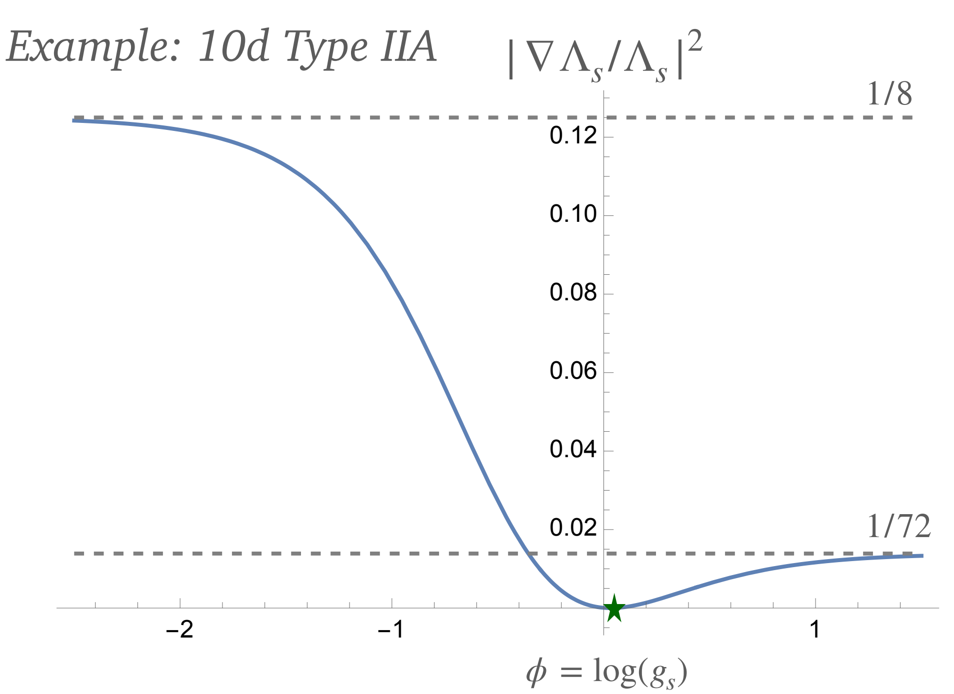

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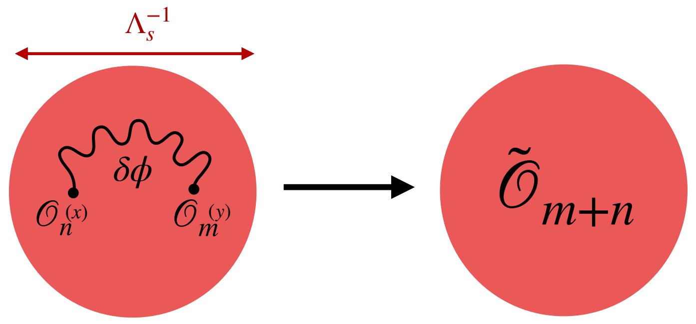

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- From explicit examples:

  → slope maximized in asymptotic limits and bound satisfied with $$c = \frac{1}{d-2}$$

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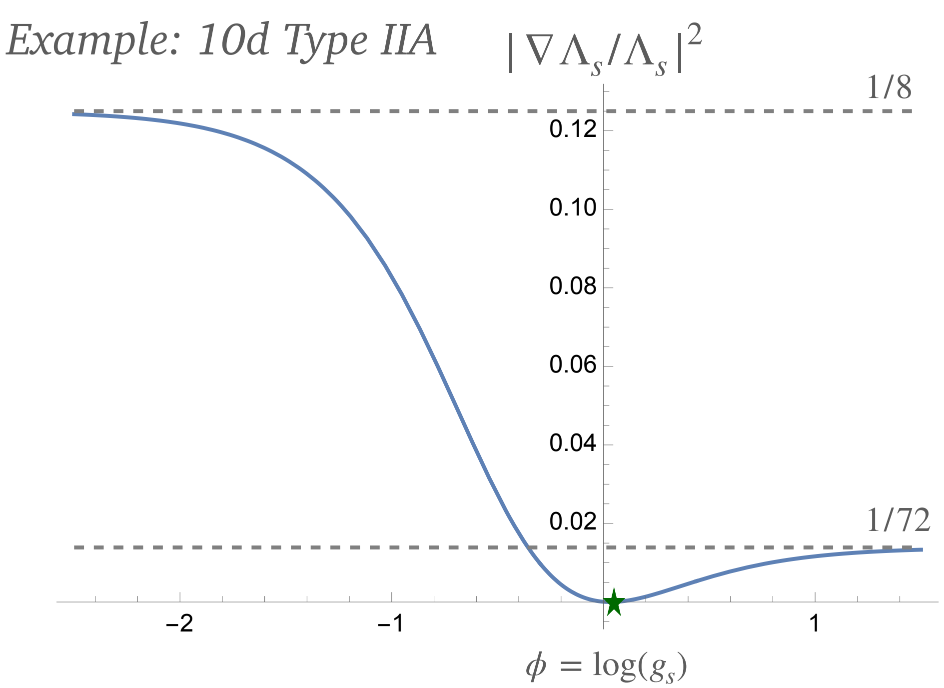

Species Scale and Emergent String Conjecture

[v.d. Heisteeg, Vafa, MW, Wu '23]

- Species Scale varies most rapidly in asymptotic limits

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- Emergent string limit gives most extreme variation of Λ_s

  ⟶ Emergent String Conjecture gives sharp bound on slope of species scale! $\left|\frac{\nabla\Lambda_s}{\Lambda_s}\right|^2 \leq \frac{1}{d-2}$

Species Scale — Overview

[v.d. Heisteeg, Vafa, MW, Wu '23]

- Species Scale can be computed from higher-derivative corrections to Einstein-Hilbert action ($\rightarrow$ corresponding to scale of minimal black hole)

$$S_{\text{corr.}} = \frac{M_{\text{pl}}^{d-2}}{2} \int \mathrm{d}^d x \sqrt{-g} \left(\mathcal{R} + \frac{1}{2}(\partial\phi)^2 + \ldots + \sum_{n=1}^{\infty} a_n(\phi) \frac{\mathcal{O}_{2n+2}(\mathcal{R}, \partial)}{M_{\text{pl}}^{2n}} \right) \longrightarrow \Lambda_s(\phi) \sim \frac{M_{\text{pl}}}{a_n(\phi)^{\frac{1}{2n}}}$$

- In explicit examples can give an *upper bound* on Λ_s from terms protected e.g. by supersymmetry $\rightarrow$ bound on the maximally possible value for QG cutoff (Desert point)

$$\Lambda_s^{\max} < M_{\text{pl}}$$

- Slope of species scale bounded from above everywhere in moduli space.

- Bound saturated in *asymptotic* limits where Emergent String Conjecture predicts *universal* bound:

$$\left| \frac{\nabla \Lambda_s}{\Lambda_s} \right|^2 \leq \frac{M_{\text{pl}}^{2-d}}{d-2}$$

Emergent Strings — Bottom-Up?

- **Emergent string conjecture** states that asymptotic regimes where $\Lambda_s \ll M_{\rm pl}$ have universal properties!

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see for example [**Lee, Lerche, Weigand** '18,**'19**,'21; Baume, Marchesano, MW '19; Xu '20; Klaewer, Lee, Weigand, MW '20, Alvarez-Garcia, Klaewer, Weigand '21; MW '22; Etheredge, Heidenreich, McNamara, Rudelius, Ruiz, Valenzuela '23, '24; Alvarez-Garcia, Lee, Weigand '23]

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Hagedorn growth as for perturbative string!

see [Basile, Montella, Lüst '23] for complementary bottom-up approach to Emergent String Conjecture!

One-Particle Density of States

[Bedroya, Mishra, MW '24]

- **Central object for our analysis:** density of one particle states ρ

- In limit $\Lambda_s \ll M_{\rm pl}$ a good estimate for $\rho(E)$ for $E \gg \Lambda_s$ is given in terms of high-energy $2 \to 2$ scattering amplitudes at fixed, small impact parameter:

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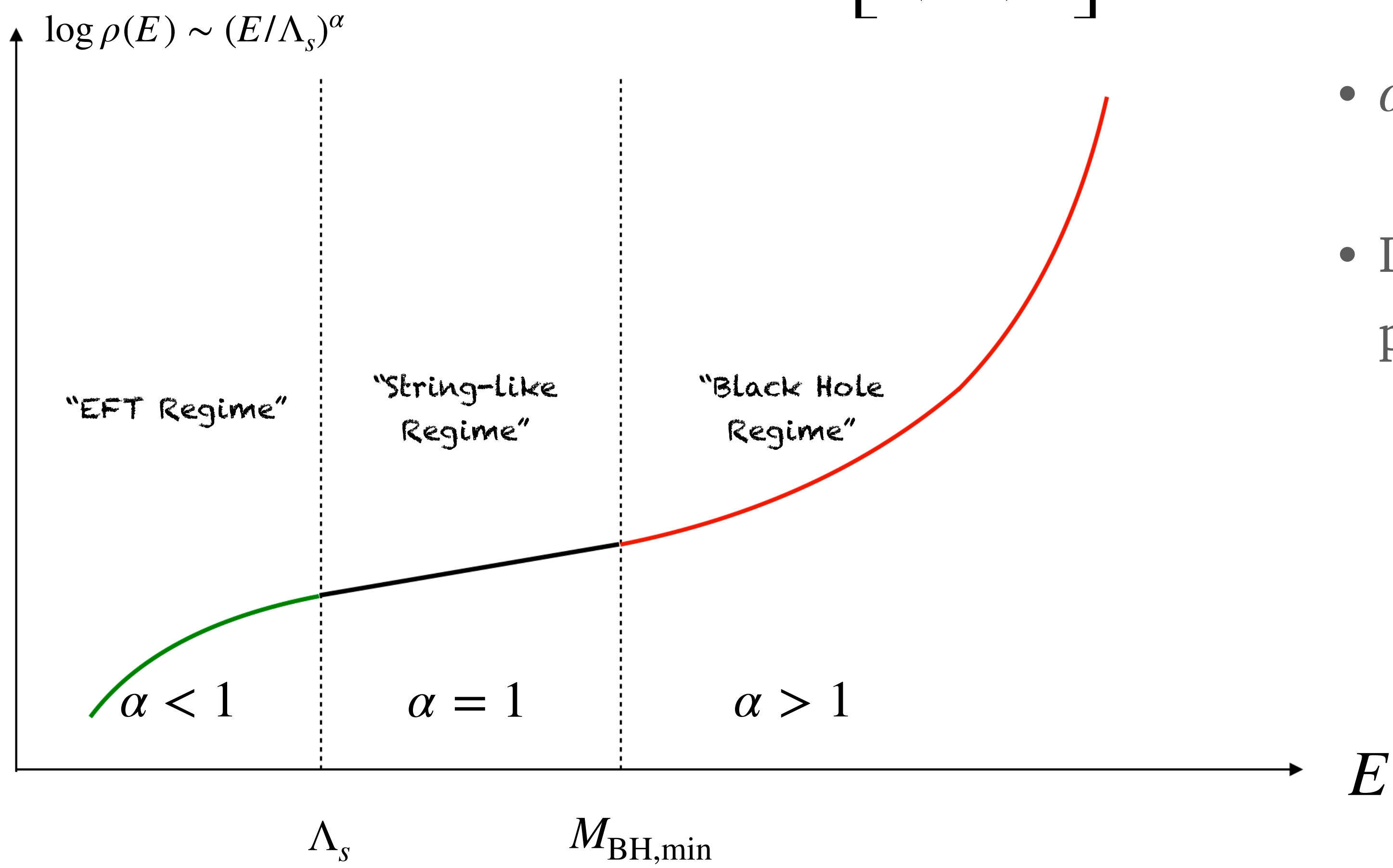

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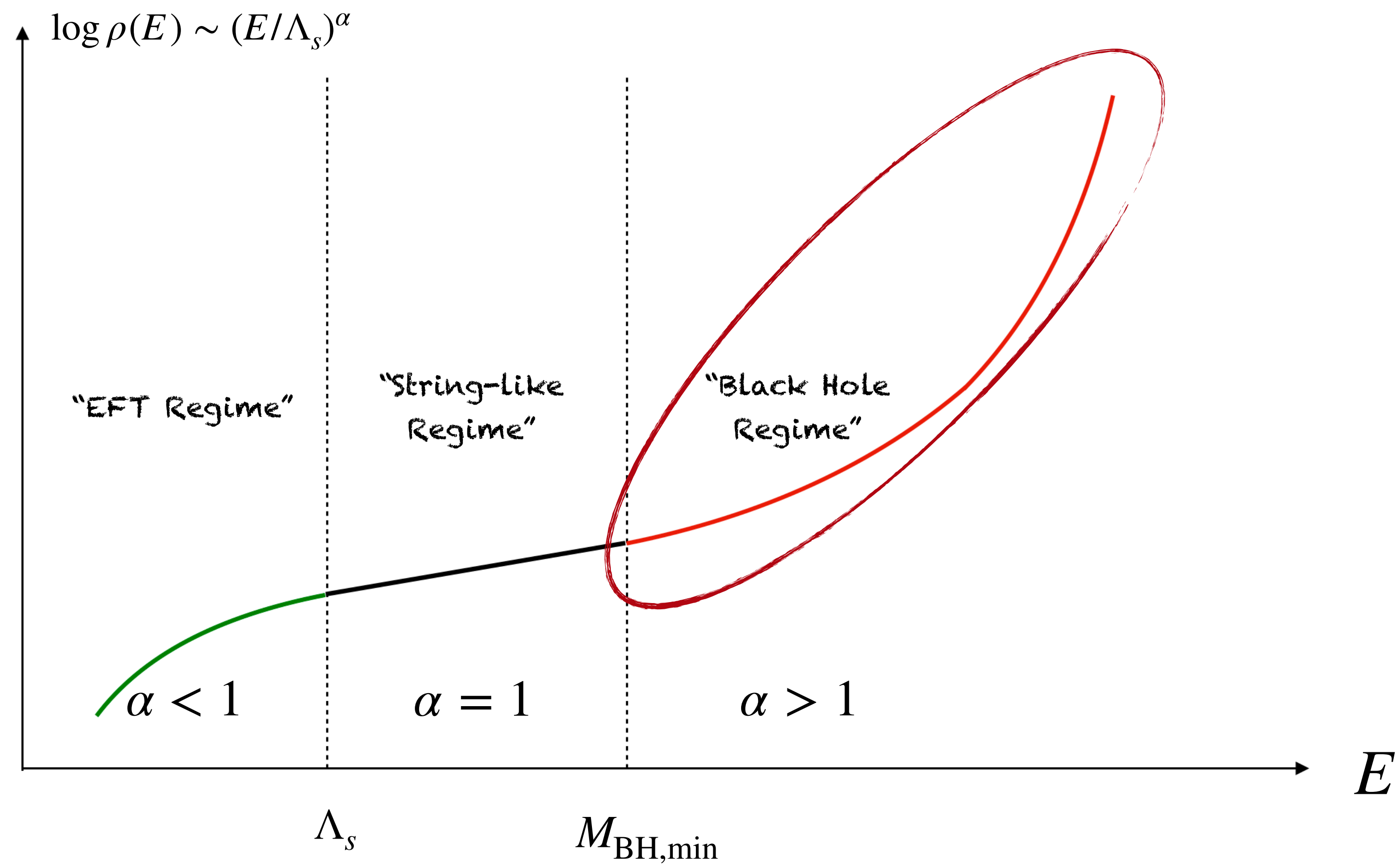
Black hole (UV) regime

- At very high energies the density of one-particle states is dominated by black hole microstates.
- Entropy of Schwarzschild black holes $S = \log \rho \sim \left(\frac{E}{M_{\rm pl}}\right)^{\frac{d-2}{d-3}} \quad \Rightarrow \quad \alpha > 1$
- More generally: one-particle states are black hole microstates $\Leftrightarrow \alpha > 1$ [Bedroya, Mishra, MW '24]

  $\rightarrow$ energy scale at which $\alpha \leq 1 \rightarrow \alpha > 1$ corresponds to mass of minimal black hole $M_{\rm BH,min}$.



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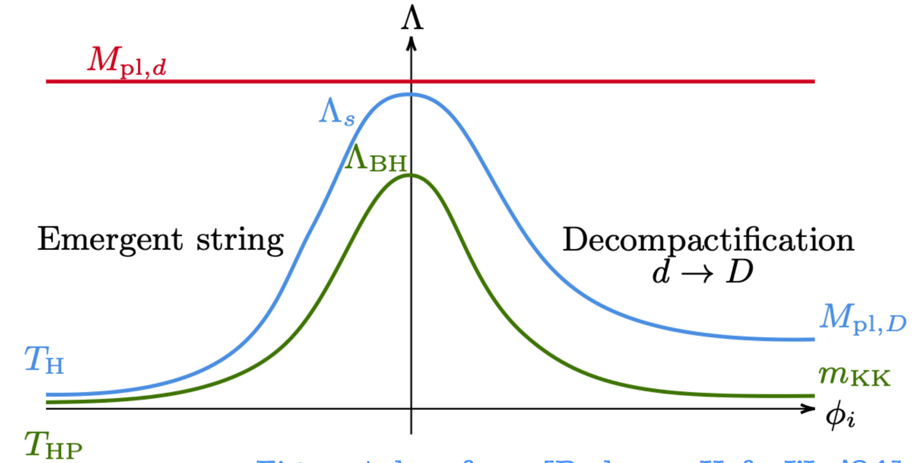

Figure taken from [Bedroya, Vafa, Wu '24]

  - Scale $\Lambda_{\text{BH}} \equiv r_*^{-1}$ related to scale of lightest tower in asymptotic regimes: $\Lambda_{\text{BH}} \lesssim \Lambda_s \lesssim M_{\text{pl}}$

Towers of lights States from Black Holes

[Bedroya, Mishra, MW '24]

- **Question:** What do I need to get a black hole that is entropically dominates over a d-dimensional Schwarzschild black hole?

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- In asymptotically flat space can use **Weak Energy Condition:**

  - Consider spherically symmetric black hole: $ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2 d\Omega_{d-2}^2$ such that $\lim_{r\to\infty}(1-e^{2\nu(r)})r^{d-3} = \lim_{r\to\infty}(1-e^{-2\lambda(r)})r^{d-3} = \frac{\kappa M}{4\pi}$

  - **Weak Energy Condition** ($T^0_0 \leq 0$) requires: $\frac{(d-3)}{2}\left(-1+e^{2\lambda}\right) + r\lambda' \geq 0$

  - The boundary conditions imply $e^{2\lambda} \leq e^{2\lambda_{\text{Schwarzschild}}}$ for every r
  $\to r_H \leq r_H^{\text{Schwarzschild}} \to$ no such black hole will have **bigger** entropy!

Towers of lights States from Black Holes

[Bedroya, Mishra, MW '24]

- **Question:** What do I need to get a black hole that is entropically dominates over a d-dimensional Schwarzschild black hole?

- Consider a d-dimensional EFT of gravity in *asymptotically flat spacetime* with finitely many weakly coupled states.

- In asymptotically flat space can use **Weak Energy Condition:**

  - Consider spherically symmetric black hole: $ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2 d\Omega_{d-2}^2$ such that $\lim_{r\to\infty}(1 - e^{2\nu(r)})r^{d-3} = \lim_{r\to\infty}(1 - e^{-2\lambda(r)})r^{d-3} = \frac{\kappa M}{4\pi}$

  - **Weak Energy Condition** ($T_0^0 \leq 0$) requires: $$\frac{(d-3)}{2}\left(-1 + e^{2\lambda}\right) + r\lambda' \geq 0$$

  - The boundary conditions imply $e^{2\lambda} \leq e^{2\lambda_{\rm Schwarzschild}}$ for every r
    $\rightarrow r_H \leq r_H^{\rm Schwarzschild} \rightarrow$ no such black hole will have **bigger** entropy!

- To get entropically favored state need ***infinitely*** many additional states!

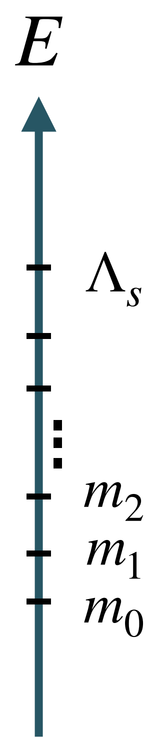
  $\rightarrow$ Transition scale $\Lambda_{\rm BH}$ is *indeed* associated to mass scale of tower of states!

consistent with proposal in [Bedroya, Vafa, Wu '24]

Towers of Weakly Coupled States Below Λ_s

- **Tower of States:** Infinite family of states with mass $m_n = m_0 + f(n)\,\Delta m$

- What kind of **towers of weakly coupled states** can we get with mass scale $m_0 \ll \Lambda_s$?



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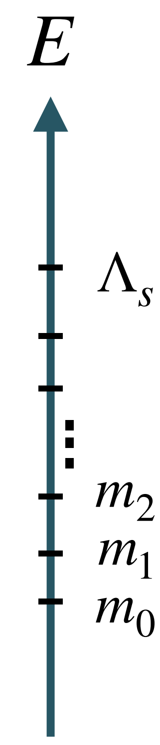
- Basic properties of species scale $\rightarrow \exists$ EFT with cutoff Λ_s
  - EFT described in terms of finitely many fundamental fields and defects $\rightarrow$ collectively account for all states below Λ_s
  - Weakly coupled defect can give rise to tower of weakly coupled states if in closed configuration (e.g. upon compactification)



"**Weakly coupled**" brane: self-energy is negligible compared to its tension
$\rightarrow$ e.g. extrinsic curvature smaller than tension.

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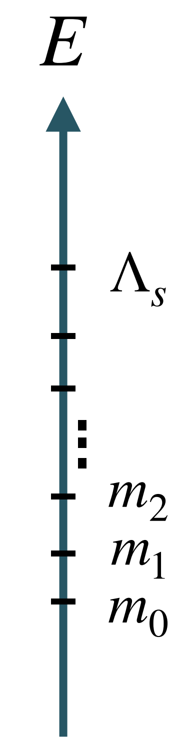

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- **Result:** Tension $\mathcal{T}$ of weakly coupled p-branes ($p \geq 1$) bounded as $\mathcal{T} \gtrsim \Lambda_s^{p+1}$
  [Bedroya, Mishra, MW '24]
  - Obtained by considering contribution of these branes to scattering amplitude.
  - Weakly-coupled defects cannot give tower of weakly coupled states with $m_0 \ll \Lambda_s$.

Towers of Weakly Coupled States Below Λ_s

[Bedroya, Mishra, MW '24]

- **What possibilities for weakly coupled towers with $m_0 \ll \Lambda_s$ are left?**

- Consider EFT that is valid at Λ_s and take weak-coupling limit $\Lambda_s/M_{\rm pl,d} \to 0$.

- Since EFT is valid at energies Λ, the partition function has to be finite if we put it on a thermal circle of circumference $\beta \gtrsim \Lambda^{-1}$ *(fixed in units of* Λ^{-1}*)*.

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- Suppose we number of weakly coupled states with mass below Λ is N, then we have

$$\mathcal{Z}(\beta) \geq \int_0^{\Lambda} dm \;\, e^{-\beta m} \rho(m) \geq e^{-\beta\Lambda} \int_0^{\Lambda} dm \;\, \rho(m) = e^{-\beta\Lambda} N$$

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  - ***N* needs to be finite!** $\longrightarrow$ EFT valid at Λ_s has finitely many weakly coupled states.

  - Only way to get states below Λ_s is by compactification of theory

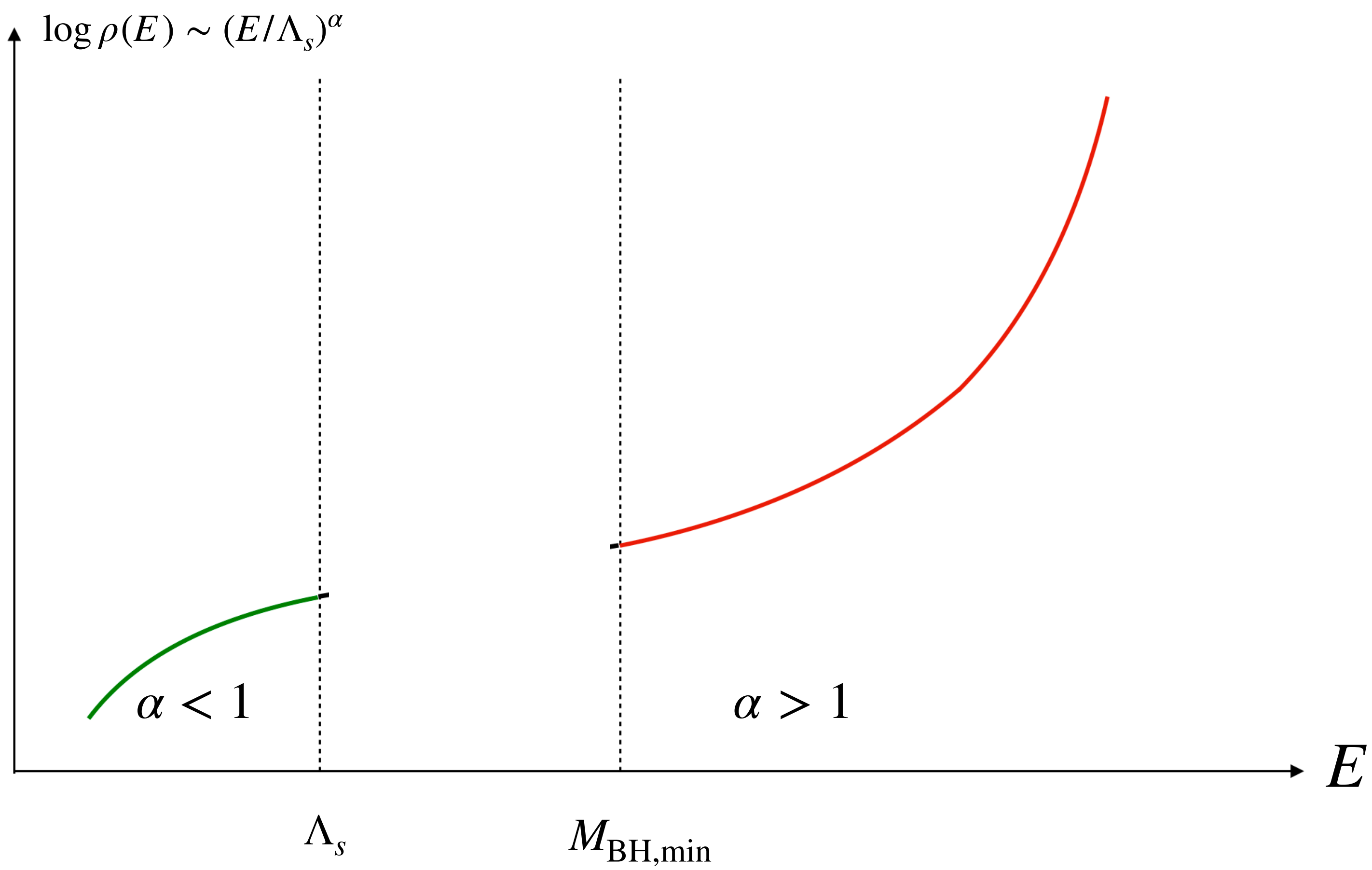
$\rightarrow$ Only possible tower of states with mass scale $m_0 \ll \Lambda_s$ are KK states.

Universality of Hagedorn behavior

[Bedroya, Mishra, MW '24]

- So far: considered the regimes
  $E \lesssim \Lambda_s$ and $E \gtrsim \Lambda_s$
  (IR) *(UV)*

- Have EFT description valid up to Λ_s and black hole description above $M_{\rm BH,min}$.
  $\rightarrow$ consider temperature diagram



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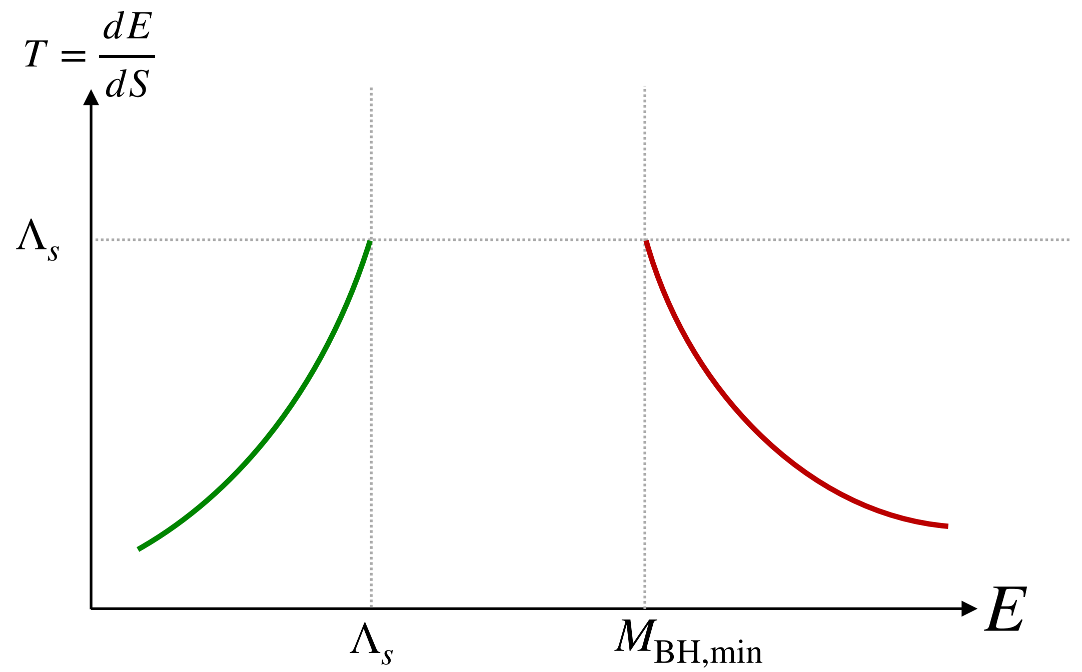

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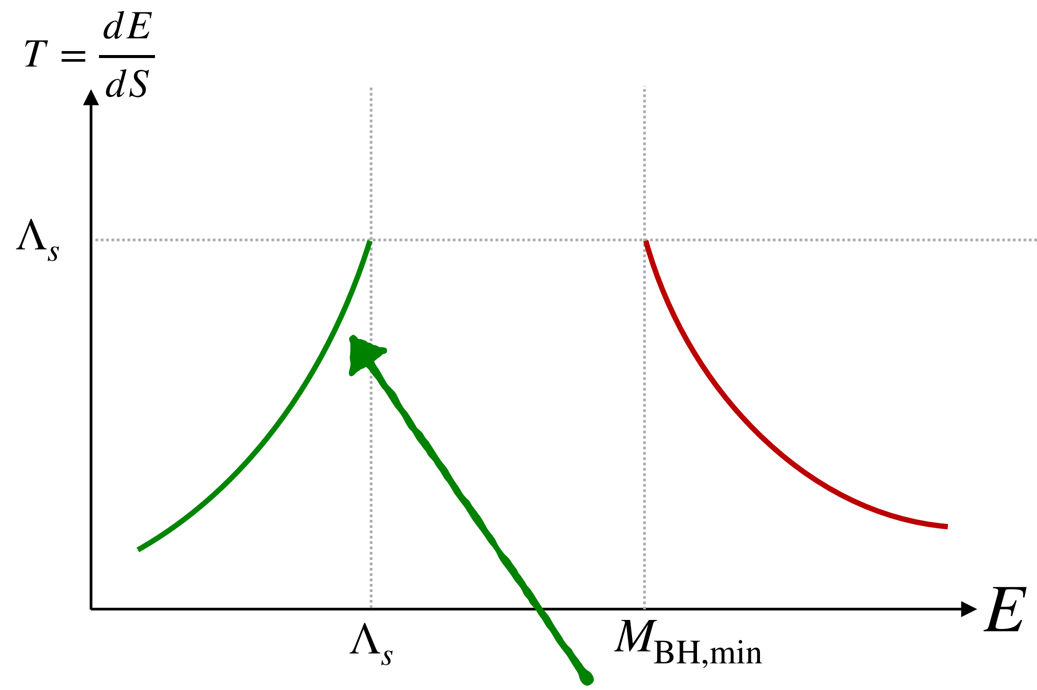

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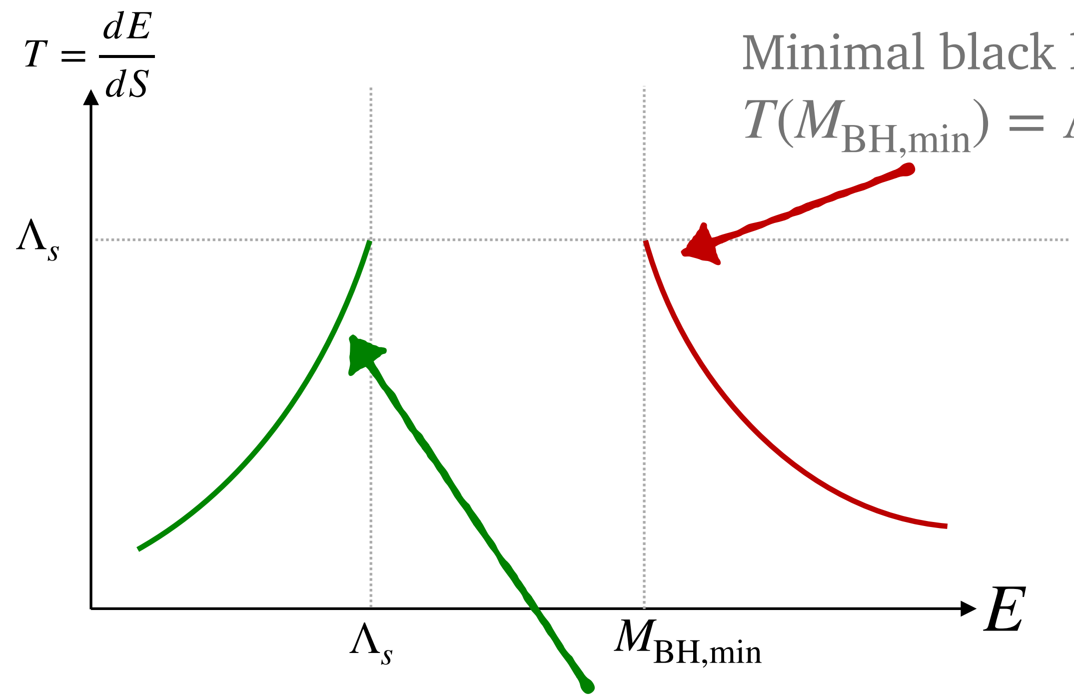

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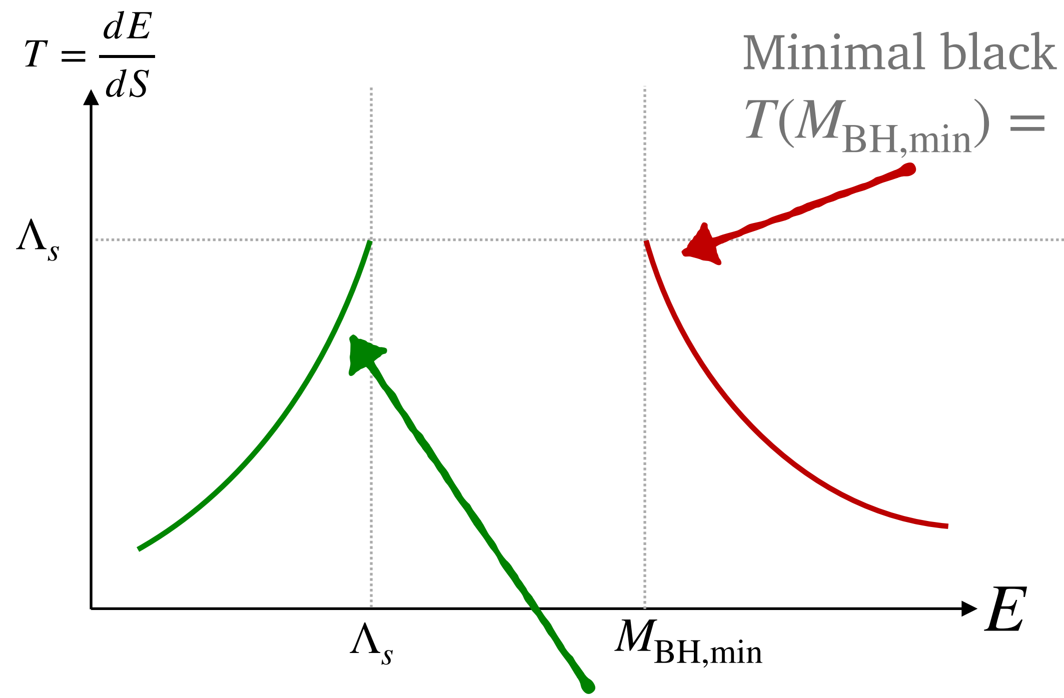

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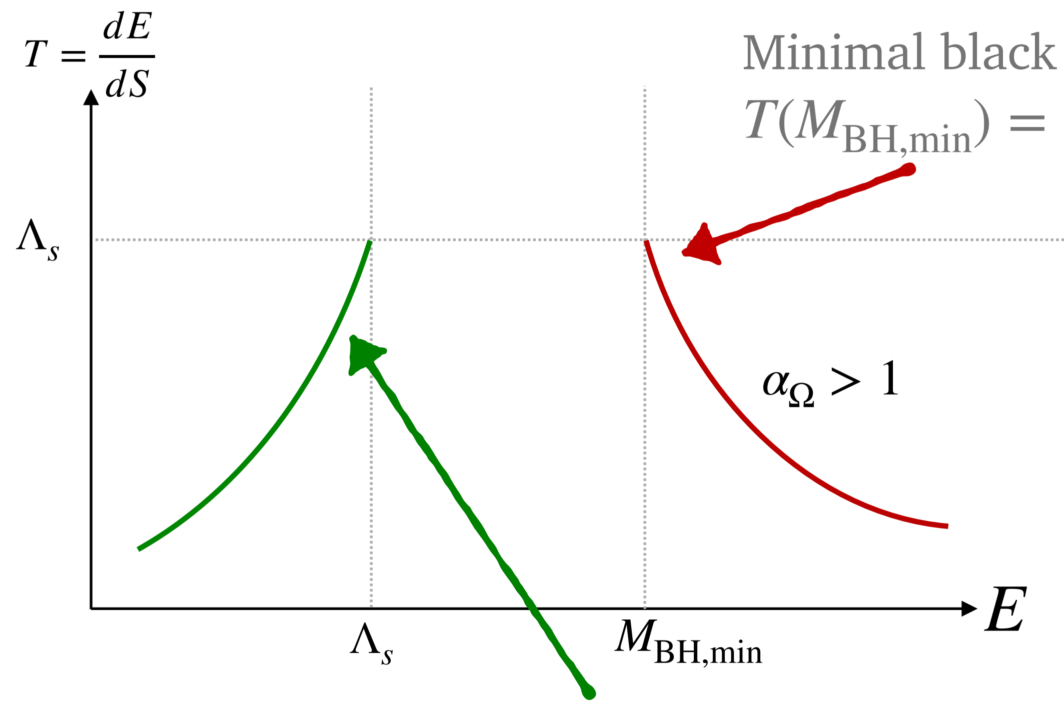

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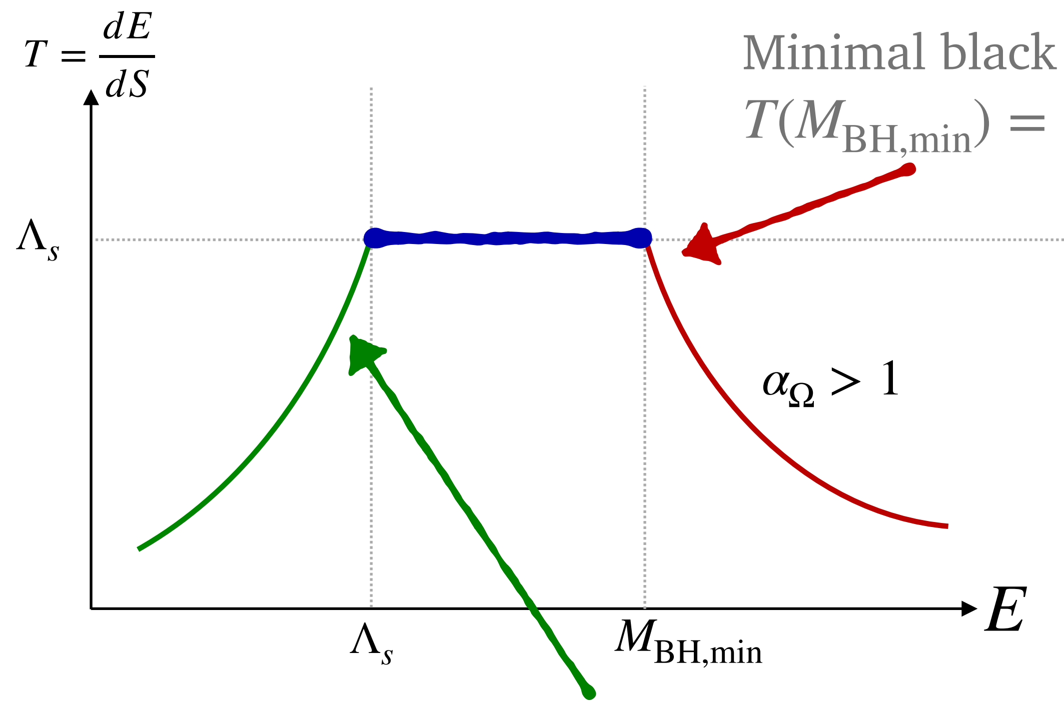

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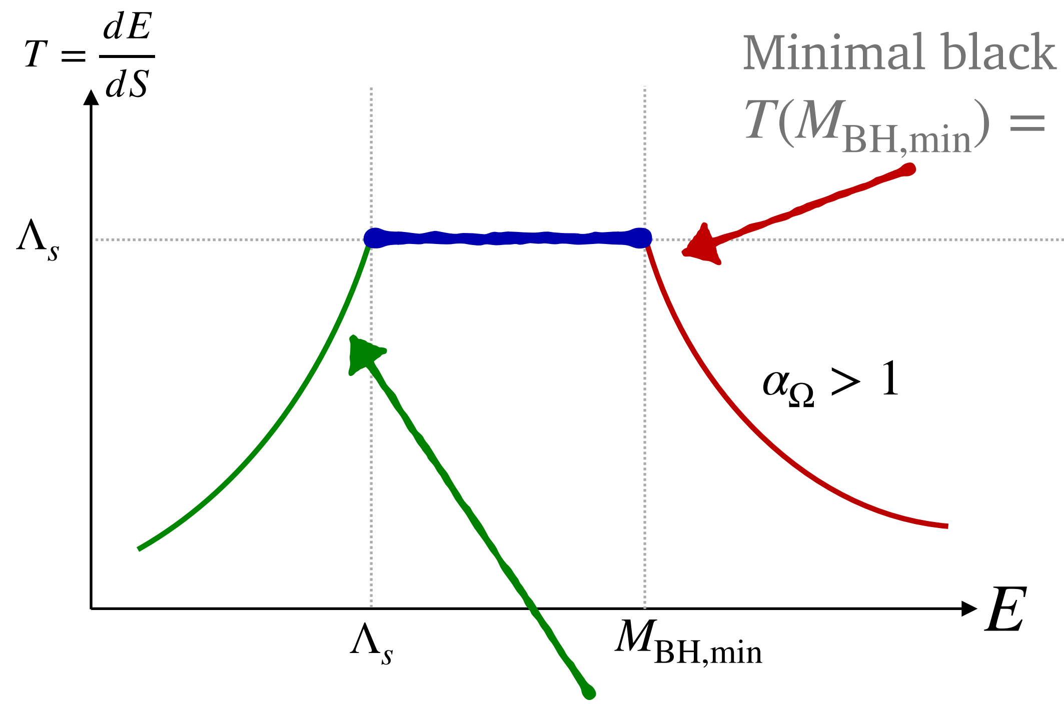

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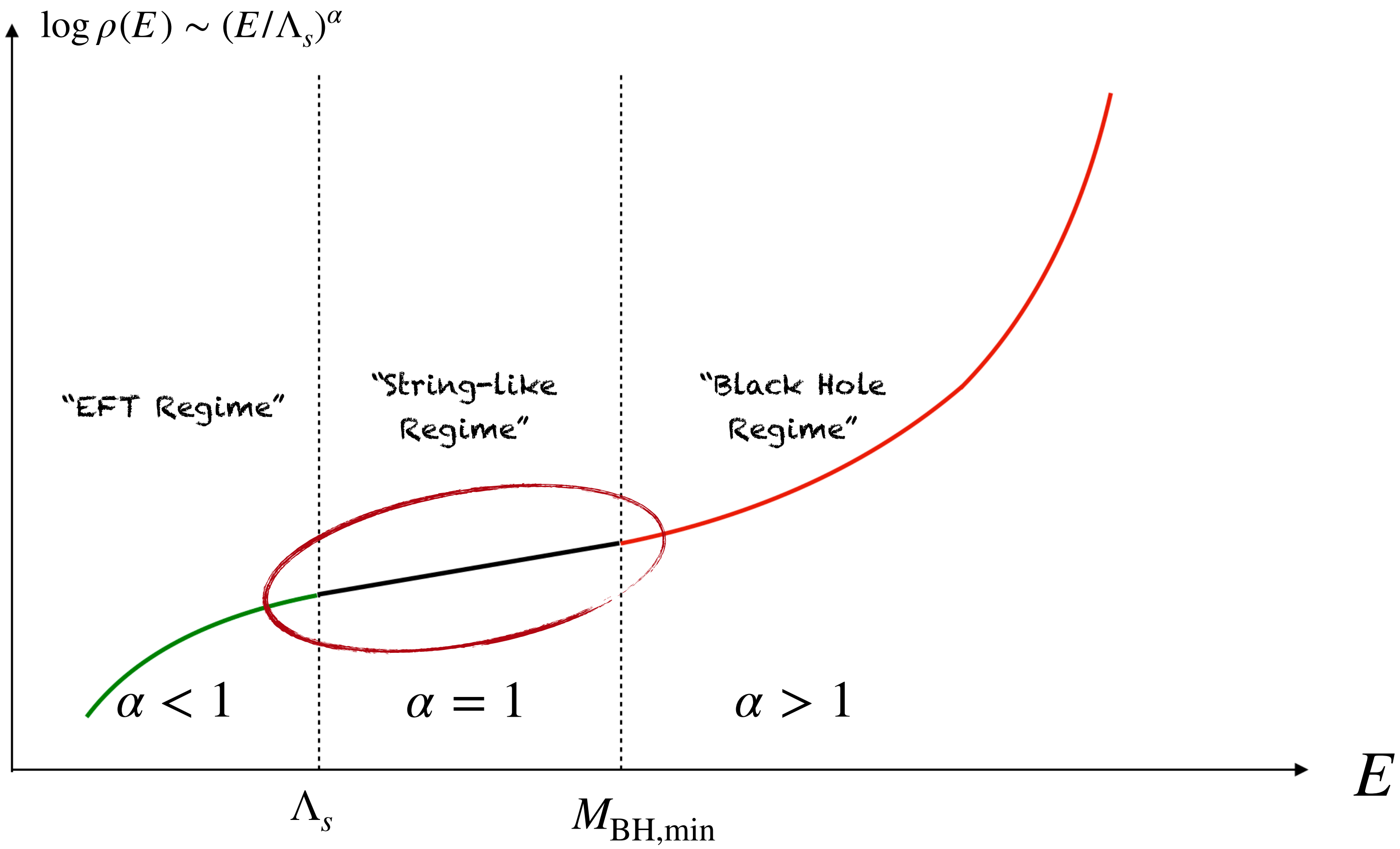

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Relation to Emergent String Conjecture

What do our results imply for the *light* towers of states?

- There are two possibilities in gravitational weak-coupling limit ($\Lambda_s \ll M_{\rm pl}$):

  1. Lightest tower of states is KK tower with mass $m \ll \Lambda_s$.

  2. In absence of KK tower, lightest tower of states has exponential degeneracy $\rho(E) \sim e^{E/\Lambda_s}$.

- Exponential degeneracy reminiscent of excitations of critical string! $\log \rho(E) \sim E/M_s$

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- Compare to **Emergent String Conjecture**: [Lee, Lerche, Weigand '19]

  Lightest tower of states in infinite distance limits is either a) a KK-tower, or b) the excitation tower of a critical string.

- Our results provide bottom-up evidence for such a ***binary*** choice!
  (Though from bottom-up we do not see that states always have to come from a fundamental string).

Summary

- Species Scale encodes crucial information about quantum gravity and is calculable via higher-derivative terms.

- In explicit examples can give an *upper bound* on Λ_s from terms protected e.g. by supersymmetry
  → can give a bound on the maximally possible value for QG cutoff (Desert point).

$$\Lambda_s^{\text{max}} < M_{\text{pl}}$$

- Slope of species scale bounded **everywhere** in moduli space: $\left|\frac{\nabla \Lambda_s}{\Lambda_s}\right|^2 \leq \frac{M_{\text{pl}}^{2-d}}{d-2}$

- In gravitational weak-coupling limit $\Lambda_s \ll M_{\text{pl}}$ → density of one-particle states $\rho(E)$ has
  has **universal behavior**!

- From basic properties of gravity (black hole thermodynamics, scattering amplitudes)
  → argue that lightest tower of states either KK-tower or has $\rho(E) \sim \exp(E/\Lambda_s)$

→ **Bottom-up evidence** for Emergent String Conjecture

Thank you!