The gravitational dual of double-scaled SYK

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WIP

SYK model system of N Majorana fermions with J Gaussian random

$$H_{\mathsf{SYK}} = \sum_{i_1 < \dots < i_p} J_{i_1 \dots i_p} \psi_{i_1} \dots \psi_{i_p}$$

For **low energies** $N \to \infty$ SYK model is Schwarzian quantum mechanics and holographic dual to **JT gravity**[1]

$$S_{\mathsf{JT}} = -\frac{1}{2} \int \mathrm{d}x \sqrt{g}\, \Phi(R+2)$$

Taught us important lessons about gravity path integral $\sim$ wormholes

One could wonder **All energies holographic dual?**
Why would you care? Motivation follows soon
Interesting regime all energies called **double scaled SYK**

$$\boxed{|\log q| = \frac{p^2}{N}} \quad \text{finite} \quad N \to \infty$$

Correlators were exactly computed in this theory[2]

[1] Maldacena, Stanford, Yang, Engelsoy, Mertens, Verlinde, Jensen
[2] Berkooz, Isachenkov, Narovlansky, Torrents, Narayan, Simon.

Interested in correlators of following operators

$$\mathcal{O}_{\Delta} = \sum_{i_1 < \cdots < i_{\Delta p}} M_{i_1 \ldots i_{\Delta p}} \psi_{i_1 \ldots i_{\Delta p}}$$

Interesting evidence for relations with dS quantum gravity[3]
What is gravitational dual DSSYK?[4]

We claim **DSSYK holographically dual to sine dilaton gravity**

$$\boxed{S_{\text{sine}} = -\frac{1}{2} \int \mathrm{d}x \sqrt{g} \left(\Phi R + \frac{\sin(2|\log q|\Phi)}{|\log q|} \right)}$$

Before proving duality...
Discuss **surprising features** of DSSYK **semiclassics** from point of view of any potential simple gravitational dual

These **call for new ingredients gravity path integral from boundary**
Motivation for being interested in gravitational dual of DSSYK

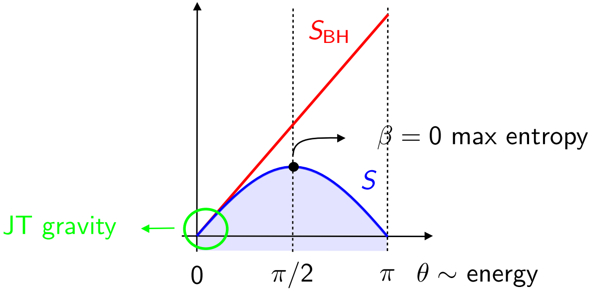
[3] Susskind, Lin, Rahman, Verlinde, Narovlansky.
[4] Verlinde, Narovlansky, Zhang, Almheiri, Popov, Goel, Hu

DSSYK semiclassical surprises

1. Consider DSSYK semiclassical **partition function**

$$\left\langle \mathrm{Tr}\left(e^{-\beta H_{\mathrm{SYK}}}\right)\right\rangle \overset{\text{class}}{=} \int_0^\pi \mathrm{d}\cos(\theta)\ \exp\left(\frac{\pi\theta-\theta^2}{|\log q|}+\beta\frac{\cos(\theta)}{2|\log q|}\right) \qquad \boxed{\beta=\frac{2\pi-4\theta}{\sin(\theta)}}$$

$$\boxed{S=\frac{\pi\theta-\theta^2}{|\log q|}}$$



Entropy **non-monotonic, surprising** versus **black hole area** growth

Will later argue

$$\frac{A_{\mathrm{BH}}}{4G_{\mathrm{N}}}=\boxed{S_{\mathrm{BH}}=\frac{\pi\theta}{|\log q|}<S}$$

Spectrum returning to zero sign of UV completeness finite dimensional $\mathcal{H}$
Gravity dual UV completeness? **What causes entropy decrease?**

2. Related **surprise** DSSYK semiclassical **correlator** versus black holes

$$G_\Delta(\tau) = \frac{1}{Z_{\text{DSSYK}}(\beta)} \left\langle \text{Tr}\left(e^{-\beta H_{\text{SYK}}} \mathcal{O}_\Delta(0) \mathcal{O}_\Delta(\tau)\right)\right\rangle$$

$$\boxed{G_\Delta(T) = \frac{\sin(\theta)^{2\Delta}}{\cosh(\sin(\theta) T/2)^{2\Delta}}} \sim \exp\left(-\Delta \frac{2\pi}{\beta_{\text{BH}}} T\right) \quad \boxed{\beta_{\text{BH}} = \frac{2\pi}{\sin(\theta)} > \beta}$$

Decay rate not controlled by temperature but by **fake temperature**[5]
For instance at $\theta = \pi/2$ temperature is infinite but **decay rate** is finite

$$\beta = 0 \quad \text{versus} \quad \beta_{\text{BH}} = 2\pi$$

What is gravity interpretation fake temperature?

Notice replacing $S \to S_{\text{BH}}$ gives thermodynamics with $\beta \to \beta_{\text{BH}}$
These are identical surprises (require only one explanation)

[5]Susskind, Stanford, Lin

Naive sine dilaton semiclassical thermodynamics

Compare with sine dilaton gravity classical **black hole** solutions $\Phi = r$[6]

$$\mathrm{d}s^2 = F(r)\mathrm{d}\tau^2 + \frac{1}{F(r)}\mathrm{d}r^2$$
$$F(r) = -2\cos(r) + 2\cos(\theta) > 0$$

Resulting thermodynamics has **monotonic entropy** indeed

$$Z_{\text{sine naive}}(\beta_{\text{BH}}) \overset{\text{class}}{=} \int_0^\pi \mathrm{d}\cos(\theta)\,\exp\left(\boxed{\frac{\pi\theta}{|\log q|}} + \beta_{\text{BH}}\frac{\cos(\theta)}{2|\log q|}\right) \qquad \boxed{\beta_{\text{BH}} = \frac{2\pi}{\sin(\theta)}}$$

Different answers than DSSYK
Logical person might conclude sine dilaton simply not dual DSSYK

However, will now show sine dilaton and DSSYK **are identical systems**!
$\to$ **puzzle within gravity** **What causes entropy decrease?**
Which new ingredient gravity path integral overlooked in current analysis?

[6] Witten

Auxiliary q-Schwarzian quantum mechanics for DSSYK

To **prove sine dilaton equals DSSYK** requires **short detour**.

Recall DSSYK correlators computed via auxiliary quantum mechanics[7]

$$\left\langle \mathrm{Tr}\left(e^{-\beta H_{\text{SYK}}}\right)\right\rangle = \langle \mathbf{n}=0|\, e^{-\beta \mathbf{H}} \boxed{|\mathbf{n}=0\rangle}$$

$$\mathbf{H} = -\frac{\cos(\mathbf{p})}{2|\log q|} + \frac{1}{4|\log q|} e^{i\mathbf{p}} e^{-2|\log q|\mathbf{n}}\,, \quad [\mathbf{n},\mathbf{p}] = \mathrm{i}$$

Few things to know about this QM formulation

1. Notice **transition matrix element** starting from $\boxed{|\mathbf{n}=0\rangle}$.

2. Semiclassical correlator computes the **classical solution n**

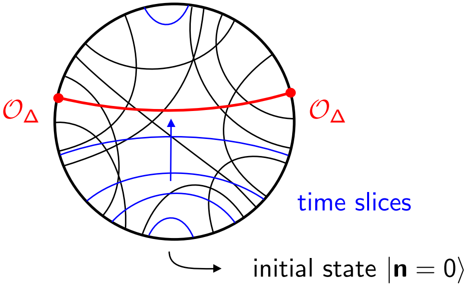
$$G_\Delta(T) = e^{-2\Delta|\log q|n} = \frac{\sin(\theta)^{2\Delta}}{\cosh(\sin(\theta)T/2)^{2\Delta}}$$

[7] Berkooz, Isachenkov, Narovlansky, Torrents, Narayan, Simon

3. **Variable n positive quantity** throughout derivation of QM

$$\boxed{\mathbf{n} \geq 0}$$

Indeed, so called **chord number n** crossing time slices **positive**



Canonical quantization sine dilaton equals DSSYK

Now show **canonical quantization** sine dilaton gives same QM

Inspired by

$$e^{-2|\log q|n} = \frac{\sin(\theta)^2}{\cosh(\sin(\theta)\,T/2)^2}$$

Consider **Weyl rescaled geodesic length** in sine dilaton

$$\boxed{L = \int \mathrm{d}s\, e^{-\mathrm{i}|\log q|\Phi}} \qquad \int \mathcal{D}x\, \exp\left(-m\int_{x(\tau)} \mathrm{d}s\, e^{-\mathrm{i}|\log q|\Phi}\right)$$

Action for matter particles coupled non-minimally to dilaton gravity

The classical saddles action are geodesics in effective geometry

$$\mathrm{d}s^2 e^{-2\mathrm{i}|\log q|\Phi} = \sin(\theta)^2 \sinh(\rho)^2 \mathrm{d}\tau^2 + \mathrm{d}\rho^2$$

AdS_2 black hole with $\beta_{\mathrm{AdS}} = 2\pi/\sin(\theta)$ with **length of two-sided ERB**

$$\boxed{e^{-L} = \frac{\sin(\theta)^2}{\cosh(\sin(\theta)\,T/2)^2}}$$

Notice similarity with DSSYK semiclassical n **suggests dictionary**

$$\boxed{\mathbf{L} \overset{?}{=} 2|\log q|\mathbf{n}}$$

$\rightarrow$ natural to do canonical quantization with L as a phase space variable?

Now Harlow-Jafferis-type canonical quantization for sine dilaton

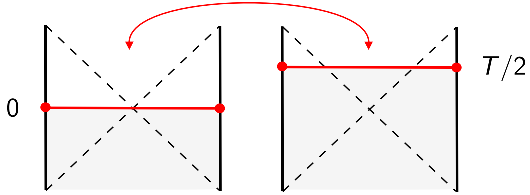
2d phase space sine dilaton $\sim$ horizon radius θ and two-sided time T[8]

$$F(r) = -2\cos(r) + 2\cos(\theta)$$

Kruskal extension spacetime with identical θ,

different **global time** slice

$=$ **different initial conditions**



[8] Harlow, Jafferis

Combined with expression for sine dilaton ADM energy this results in simple Hamiltonian system for sine dilaton gravity

$$\dot{T} = 1$$

$$\dot{\theta} = 0$$

$$H_{\text{grav}} = \boxed{-\frac{\cos(\theta)}{2|\log q|} = E_{\text{ADM}}}$$

$$\omega = \frac{\sin(\theta)}{2|\log q|}\,\mathrm{d}T \wedge \mathrm{d}\theta = \mathrm{d}T \wedge \mathrm{d}H_{\text{grav}} = \frac{1}{2|\log q|}\boxed{\mathrm{d}L \wedge \mathrm{d}P}$$

Following Harlow-Jafferis, task is **finding conjugate P** to length function

$$e^{-L} = \frac{\sin(\theta)^2}{\cosh(\sin(\theta)\,T/2)^2}$$

Then **inverting** (solving) expressions L and P for Hamiltonian $H_{\text{grav}} \to$

$$\boxed{\mathbf{H}_{\text{grav}} = -\frac{\cos(\mathbf{P})}{2|\log q|} + \frac{1}{4|\log q|}\,e^{i\mathbf{P}}\,e^{-\mathbf{L}}}$$

This is exactly the same quantum mechanics as DSSYK □

We must impose length positivity $L \geq 0$ in gravity

For instance following gravity correlator manifestly matches DSSYK

$$G_\Delta(\tau) = \langle \mathbf{L} = 0 |\, e^{-\tau \mathbf{H}_{\text{grav}}}\, e^{-\Delta \mathbf{L}}\, e^{-(\beta-\tau)\mathbf{H}_{\text{grav}}} \boxed{|\mathbf{L} = 0\rangle}$$

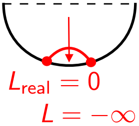
Then what about earlier sine gravity partition function calculation?
Missing ingredient naive sine gravity path integral?
In sine dilaton classical calculation we did not impose **length positivity**!

$$\boxed{\mathbf{L} \geq 0}$$

Indeed, L is a **holo renormalized geodesic distance** in AdS_2 with range

$$-\infty < L < +\infty = L_{\text{real}} - 2\log 1/\varepsilon$$

Indeed for Euclidean operator separation geometry approaches $L = -\infty$

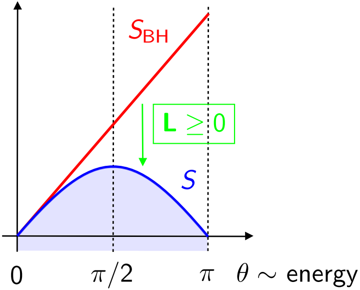


So length positivity $L \geq 0$ indeed **nontrivial geometric constraint**

Imposing length positivity $\mathbf{L} \geq 0$ does cause entropy decrease

Indeed, one finds **two quantization options** for the same Hamiltonian

$$\mathbf{H}_{\text{grav}} = -\frac{\cos(\mathbf{P})}{2|\log q|} + \frac{1}{4|\log q|} e^{i\mathbf{P}} e^{-\mathbf{L}}$$



Imposing **length positivity $\mathbf{L} \geq 0$** find q-Hermite wavefunctions DSSYK, **without constraint** find other wavefunctions with **black hole entropy**

$$\mathrm{d}E\rho(E) = \mathrm{d}\theta\, (e^{\pm 2\mathrm{i}\theta}; q^2)_\infty \quad \text{versus} \quad \mathrm{d}E\rho_{\text{BH}}(E) = \mathrm{d}\theta\, \sin(\theta)\, \sinh\left(\frac{\pi\theta}{|\log q|}\right)$$

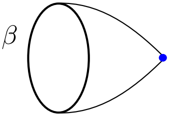
1. Can impose $L \geq 0$ classical q-Schwarzian solutions same conclusions
2. Density of states $\rho_{\text{BH}}(E)$ also found via Liouville-dS CFT perspective
3. Also **length discretization $\mathbf{L} = 2|\log q|\mathbf{n}$** however in classical regime $|\log q| \to 0$ invisible therefore does not address entropy differences

Lorentzian geometry is black hole with fake temperature

Having understood this we return to second DSSYK semiclassical surprise

What is gravity interpretation fake temperature?

For this propose **interpretation** of $\mathbf{L} \geq 0$ via **conical defect** Why soon!

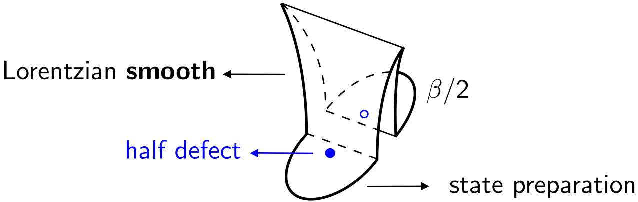


$$\int \mathrm{d}x \sqrt{g}\, e^{-(2\pi-\gamma)\Phi}\,, \quad \gamma = 2\pi - 4\theta = \frac{\beta}{\beta_{\mathrm{BH}}}$$

Such a defect indeed **corrects** semiclassical **entropy** $\boxed{S_{\mathrm{BH}} \to S}$

Key: Lorentzian continuation defect spacetime is completely **smooth black hole** with **Hawking temperature** equal to **fake temperature**[9]

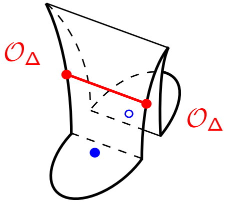
$$\boxed{\beta_{\mathrm{BH}} = \frac{2\pi}{\sin(\theta)}}$$



[9] Dong, Marolf, Rath, Tajdini, Wang

So **matter correlators** in Lorentzian region of defect spacetime **probe black holes with fake temperature**

Defect picture then indeed correctly **reproduces DSSYK correlator**



$$= \boxed{\frac{\sin^{2\Delta}(\theta)}{\cosh^{2\Delta}(\sin(\theta)\,T/2)}}$$

Crossing particles Lorentzian shockwaves $\sim$ expects **maximal Lyapunov** at **fake temperature**, as indeed found in DSSYK[10]

$$\boxed{\lambda_{\mathsf{L}} = \frac{2\pi}{\beta_{\mathsf{BH}}}} = \sin(\theta)$$

Notice that this is sub-maximal chaos from boundary perspective

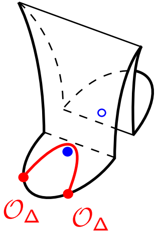
$$\beta_{\mathsf{BH}} > \beta \to \lambda_{\mathsf{L}} < \lambda_{\mathsf{L\,max}} = \frac{2\pi}{\beta}$$

[10] Maldacena, Stanford, Streicher, Choi, Mezei, Sarosi

Wait sorry but where did the defect come from?

The defect semiclassically imposes length positivity $\mathbf{L} \geq 0$ in gravity

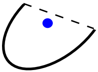
Indeed, renormalized length Euclidean geodesic around half-defect $\boxed{L \geq 0}$



$$= \frac{\sin^{2\Delta}(\theta)}{\sin^{2\Delta}(\sin(\theta)\tau/2 + \theta)} = e^{-\Delta L} \leq 1$$

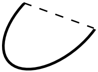
So defect semiclassically implements **initial** and final **state** $|\mathbf{L} = 0\rangle$

$$e^{-\beta \mathbf{H}_{\text{grav}}/2} |\mathbf{L} = 0\rangle \overset{\text{class}}{=}$$



This should be **contrasted with** the usual **no-boundary state**

$$e^{-\beta \mathbf{H}_{\text{grav}}/2} |\mathbf{L} = -\infty\rangle \overset{\text{class}}{=}$$



Conclusion

1. **Sine dilaton gravity is the gravitational hologram of DSSYK**

2. Canonical quantization gravity must impose **length positivity** $\mathbf{L} \geq 0$

3. Hawking temperature bulk black hole is fake temperature DSSYK

Open questions

1. **Exact gravity path integral formulation of length positivity?**
Non-Hamiltonian formulation

2. Various **relations with dS space** should be understood better
Not discussed today see paper

3. q-AdS/**q-conformal symmetry**[11]

Thanks.

[11] Almheiri, Popov, Goel, Hu