Determinants and Branes in Twisted Holography

Kasia Budzik

Strings 2024, CERN

arXiv:2106.14859 [KB, D. Gaiotto]
arXiv:2211.01419 [KB, D. Gaiotto]
arXiv:2306.01039 [KB, D. Gaiotto, J. Kulp, B. Williams, J. Wu, M. Yu]
$+$ work in progress [KB]

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Eg. protected subsector of $\text{AdS}_5/\text{CFT}_4$: [Costello, Gaiotto '18]

$$\begin{tikzcd}
\text{4d } \mathcal{N}=4 \text{ SYM} \arrow[d, "\substack{\text{``}Q+S\text{'' twist} \\ \text{[Beem et al. '13]}}"'] & \approx & \text{type IIB on AdS}_5 \times S^5 \arrow[d, "\substack{\text{twisted strings} \\ \text{[Costello, Li '16]}}"] \\
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- More mathematically rigorous: **homological algebra**

In this talk

Examples of dualities in B-model:

- 2d chiral algebra $\iff$ B-model on $SL(2, \mathbb{C})$
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- Determinant modifications and open strings on D1 and D3-branes

Twisting Supersymmetric QFTs

Twisting SQFT is the procedure of passing to a cohomology of a supercharge:

$$[\boldsymbol{Q}, \phi] = 0 \qquad\qquad (\boldsymbol{Q}\text{-closed})$$

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- Extra math structure — Gwilliam, Saberi, Williams, ...
  - ∞-dim symmetry algebras

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- The **worldvolume theory of N D1-branes** in B-model $\mathbb{C} \subset \mathbb{C}^3$

Backreaction

- The stack sources a **Beltrami differential** which deforms complex structure:

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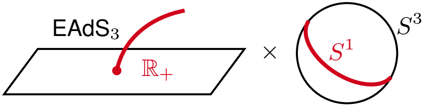
- Non-conformal vacua $\longleftrightarrow$ "Multicenter" asymptotically $SL(2,\mathbb{C})$ geometries

Giant Gravitons

- Determinant operator in the chiral algebra

$$\det(m + X(z) + uY(z)), \qquad m \in \mathbb{C}$$

is dual to a D1-brane wrapping $\mathbb{C}^* \cong \mathbb{R}_+ \times S^1$ in $SL(2,\mathbb{C}) \cong \mathrm{EAdS}_3 \times S^3$



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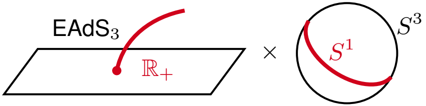
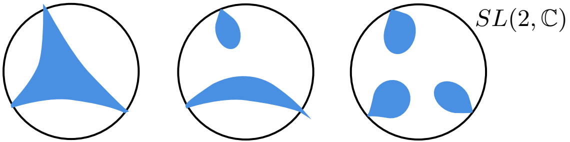


- Many possible brane configurations with the same boundary behaviour



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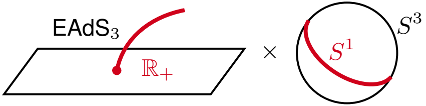

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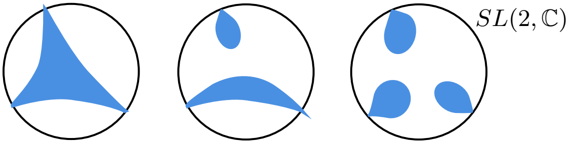

- Match saddles of determinant correlation functions with brane configurations

Determinant correlation functions

[Jiang, Komatsu, Vescovi '19]

- Rewrite correlators using **auxiliary bosonic variables** ρ^i_j for $i \neq j$, $\rho^i_i \equiv m_i$

$$\Big\langle \prod_i^k \det(m_i + X(z_i) + u_i Y(z_i)) \Big\rangle \sim \int \mathrm{d}\rho \, e^{N S[\rho]}$$

with action

$$S[\rho] = \frac{1}{2} \sum_{i \neq j} \frac{z_i - z_j}{u_i - u_j} \rho^i_j \rho^j_i + \log\det\rho$$

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- Saddles can be matched to **holomorphic curves** in $SL(2, \mathbb{C})$ using a **spectral curve** construction [KB, Gaiotto '21]

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- Determinant modifications correspond to brane excitations [Berenstein '03]

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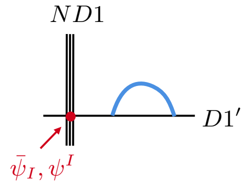

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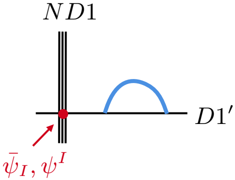

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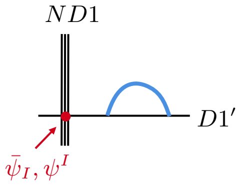

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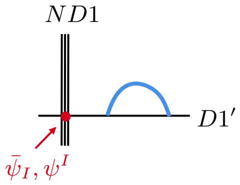

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- Can be extended to powers of determinants $(\det X)^k$ and multiple branes k D1'

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- The wordlvolume theory on the probe D1' brane is a (second) **chiral algebra**

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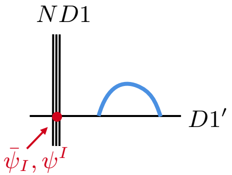

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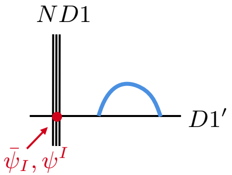

- Differential $\widetilde{Q}_{\text{BRST}}$ of the (second) chiral algebra can be mapped to a dual differential on **multiple** mesons [WIP]

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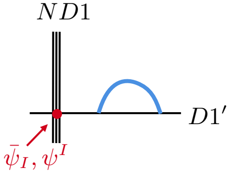

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- Twisted theory is **4d holomorphic**

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- Extra math structure: [Gwilliam, Williams '21] [Saberi, Williams '20]
  - ∞-dim symmetry algebra

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- The holomorphic twist of **pure 4d** $\mathcal{N} = 1$ **SYM** is the holomorphic BF theory:

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- $\boldsymbol{Q}$-cohomology of $\mathcal{N}=1$ SYM is equivalent to the BRST cohomology of holomorphic BF theory

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[KB, Gaiotto, Kulp, Williams, Wu, Yu '23]

$$\begin{array}{ccc} \text{B-model on } \mathbb{C}^3 + N \text{ D3-branes} & \longrightarrow & \text{B-model on } \mathbb{C}^3 \setminus \mathbb{C}^2 + \eta \\ \uparrow & & \\ \text{holomorphic BF theory} & & \end{array}$$

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- **Classification of determinant modifications** in $\mathcal{N} = 4$ SYM (also BMN subsector and holomorphic BF theory) using correspondence with Giant Graviton branes
[WIP]

Future directions

- Determinant modifications give families(N) of BPS operators in $\mathcal{N} = 4$ SYM (and BMN subsector). Do they account for some fortuitous/non-multigraviton operators? [Chang, Lin '22 '24] [Choi, Kim, Lee, Lee, Park '23] [Choi, Choi, Kim, Lee, Lee '23]

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Thank you!