Higgsing SCFTs

-- Decay and Fission of Magnetic Quivers

Zhenghao Zhong

In collaboration with Antoine Bourget and Marcus Sperling

Based on Phys. Rev. Lett. **132,** 221603 (2024)

University of Oxford

UNIVERSITY OF OXFORD

Strings 2024

NEW COLLEGE OXFORD

Superconformal Field Theories (SCFTs) with eight supercharges in space-time dimensions $d = 3, 4, 5, 6$

Higgsing SCFTs along the Higgs branch

Give VEV to scalars that parameterize the Higgs branch

4d $\mathcal{N} = 2$ example:

$SU(3)$ gauge theory $N_f = 6$ Flavors $\xrightarrow{\text{Higgsing}}$ $SU(2)$ gauge theory $N_f = 4$ Flavors $\xrightarrow{\text{Higgsing}}$ Trivial

Most SCFTs don't have known Lagrangian description $\rightarrow$ Higgsing is HARD!

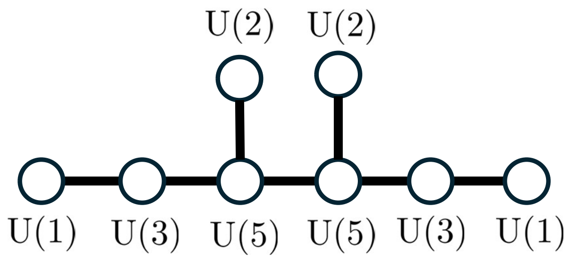
Example: 5d $\mathcal{N}=1$ Low-energy: $SU(5)_1$, $N_f = 4$, $N_{\mathrm{AS}} = 2$

Superconformal fixed point: No known Lagrangian,
Higgs branch enhanced by massless instanton operators

To study Higgs branch: Magnetic Quiver

Definition: An auxillary combinatorial object that encodes the Higgs branch of $d = 3, 4, 5, 6$ SCFT with eight supercharges

[Akhond, Arias-Tamargo, Bhardwaj, Bourget, Bullimore, Carta, Closset, Dwivedi, Eckhard, Fazzi, Ferlito, Ferarri, Giacomelli, Giri, Gledhill, Grimminger, Guhesh, Hanany, Hayashi, Kalveks, Kim, Lawrie, Li, Liu, Mansi, Mekareeya, Miketa, Nawata, Pasquarella, Pini, Sacchi, Schäfer-Nameki, Sperling, Van Beest, Wang, Wang, Yagi, Zafrir, Zajac, Zhong '18- '24]

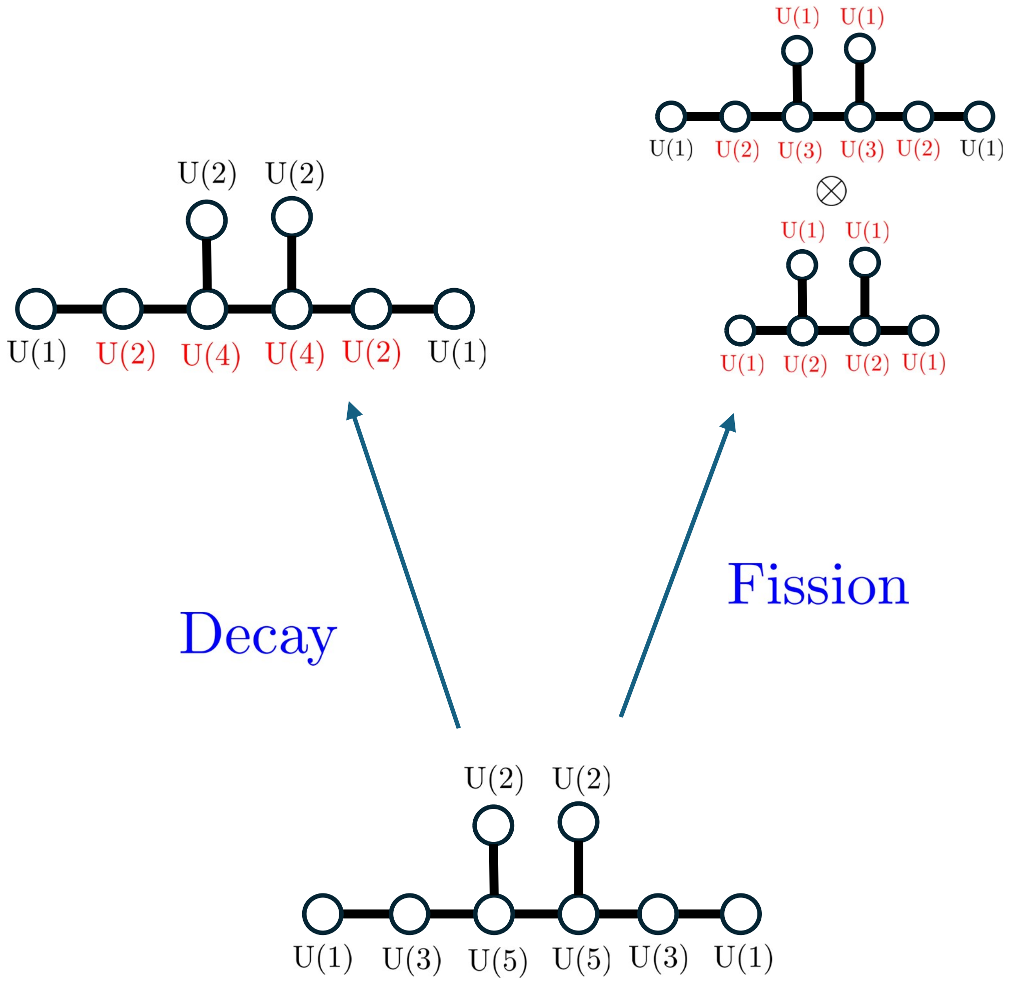


Magnetic Quiver from:
[Van Beest, Bourget, Eckhard, Schäfer-Nameki '20]

To perform Higgsing: Decay and Fission algorithm

A Higgsing algorithm: Decay and Fission of magnetic quivers

Magnetic Quivers



SCFTs

$SU(3)_1^{g_\infty}$, $N_f = 6$
$N_{\mathrm{AS}} = 0$

$\otimes$

$SU(2)_1^{g_\infty}$, $N_f = 4$
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$SU(4)_1^{g_\infty}$, $N_f = 4$
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Higgsing

Higgsing

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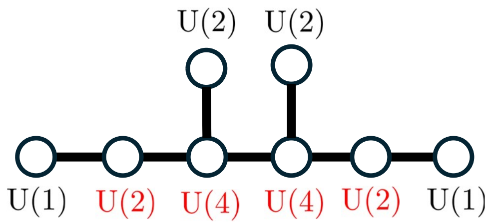
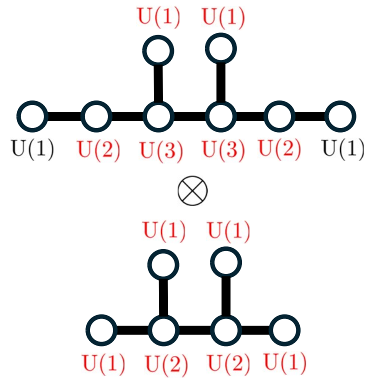
Magnetic Quivers

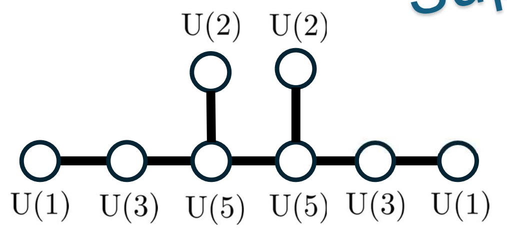




Decay

Fission

Super efficient code!!!



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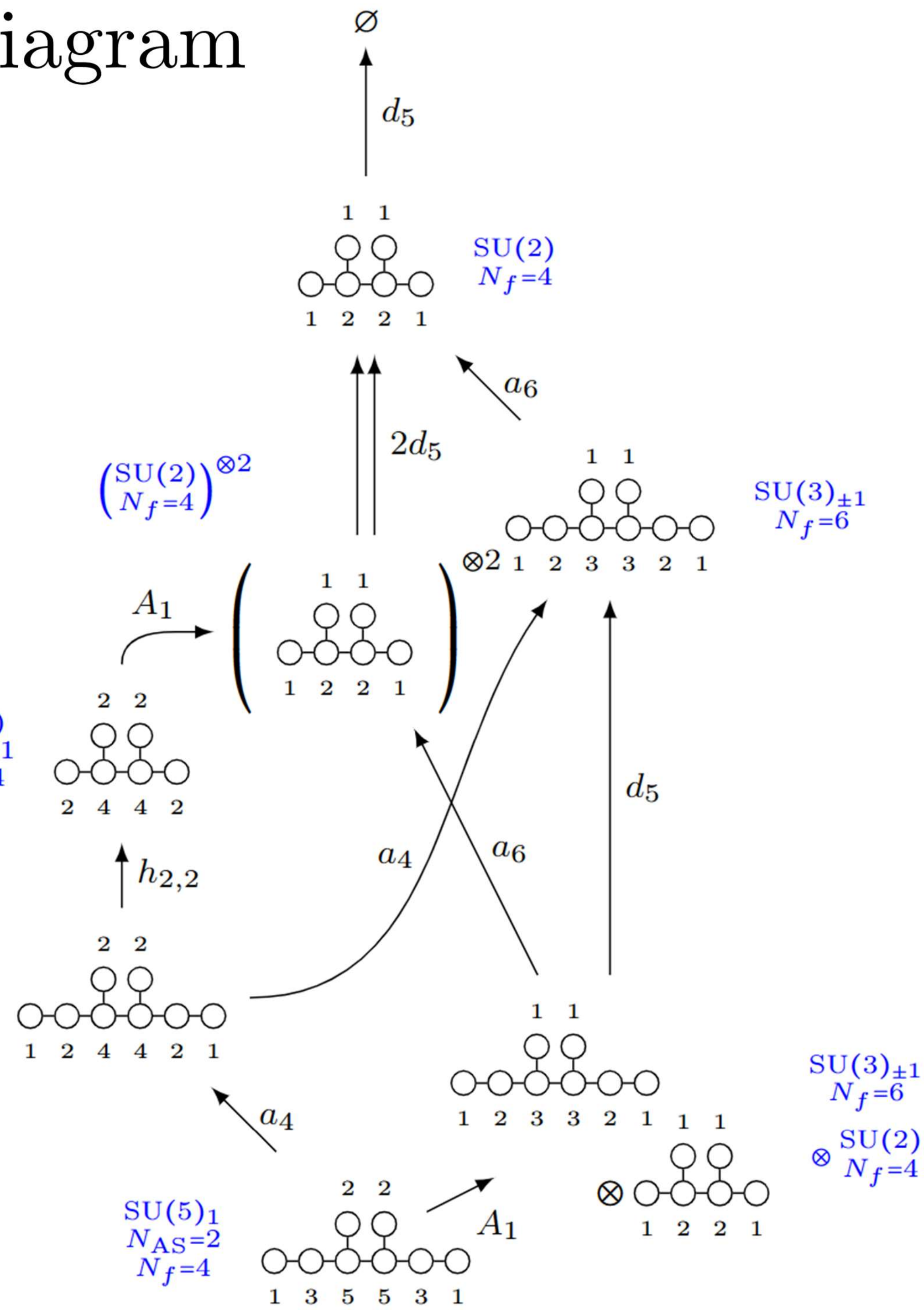
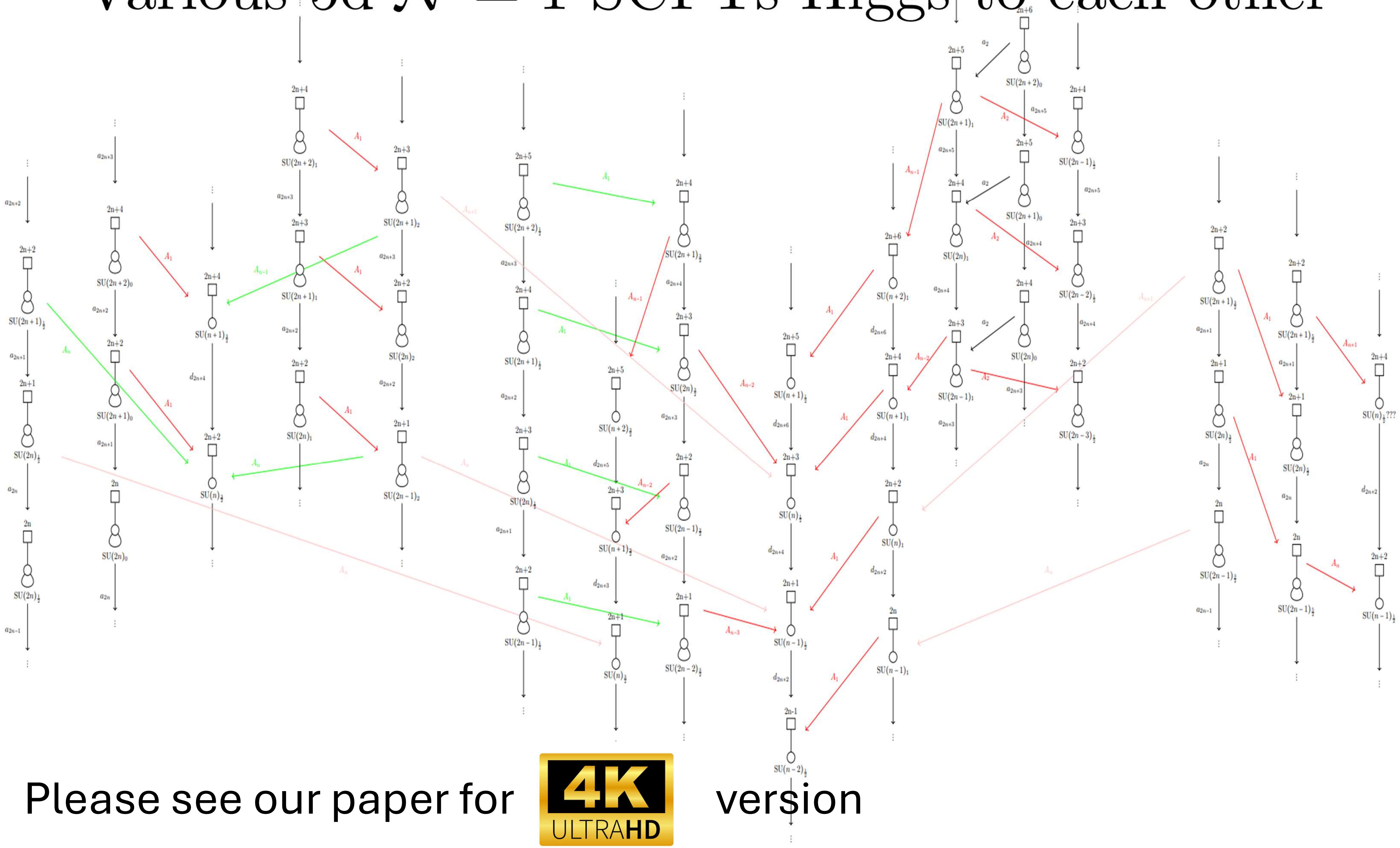
Higgsing phase diagram



Various 5d $\mathcal{N}=1$ SCFTs Higgs to each other



Please see our paper for 4K ULTRAHD version

Long version:
[2401.08757]

What we Higgsed so far

	SCFTs + more
3d $\mathcal{N}=4$	T_ρ^σ theories Mixed U(n) and SU(n) quiver theories
4d $\mathcal{N}=2$	Class $\mathcal{S}$ theories Argyres-Douglas theories More exotic SCFTs
5d $\mathcal{N}=1$	SQCD theories
6d $\mathcal{N}=(1,0)$	Orbi-instanton Higher-rank E-string Little string theories

THE M-THEORY GEOMETRY OF GENERALISED TORIC POLYGONS

STRINGS 2024 Gong Show - 4th June 2024

Guillermo Arias-Tamargo
Imperial College London

Based on 2403.09776 with S. Franco & D. Rodríguez-Gómez

5d SCFTs FROM STRING THEORY

- M-theory geometric engineering

- Brane webs in type IIB

- Duality: M-th on $T^2 \leftrightarrow$ IIB on S^1

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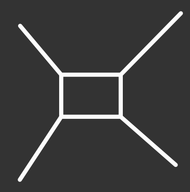
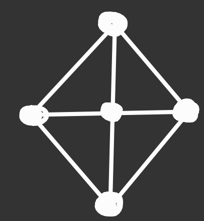


Toric case → Combinatorial data "toric diagram" → Brane web / Toric geometry



- Brane manipulations in webs with 7-branes

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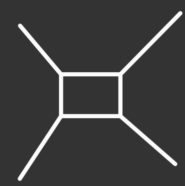

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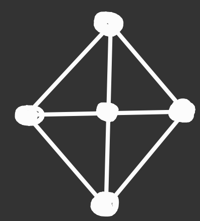
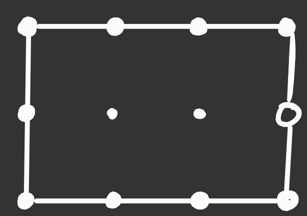


- Brane manipulations in webs with 7-branes

  - Hanany-Witten moves
  - Higgs branch flows

$\rightarrow$ Generalised Toric Polygon



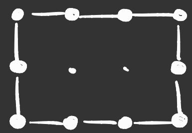
(Benini, Benvenuti, Tachikawa '09)

GENERAL IDEA

Brane web in IIB

(Aharony, Hanany, Kol '97)

Toric diagram



(Hori, Vafa '00)

M-theory on toric CY $\mathcal{M}$

"original"

M-theory on mirror CY $\mathcal{W}$

"mirror"

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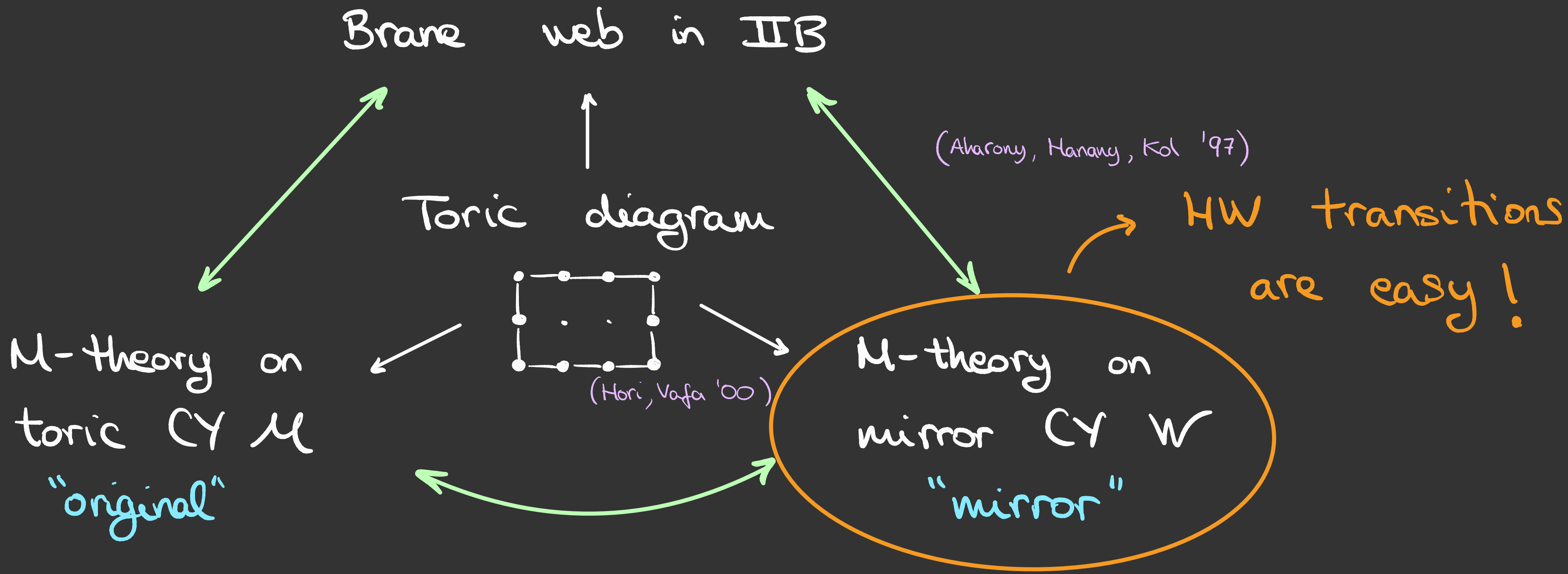

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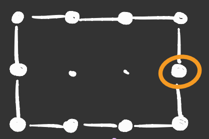

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HW transitions
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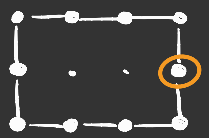

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Constraint for Kähler moduli

(Bourget, Collinucci, Schafer-Nameki '23)

White dot = Constraint for complex structure moduli

- Connection to mutations in math $\longrightarrow$ invariants

(Franco, Seong '23; Cremonesi, Sá '23)

Punchline and Outlook

PUNCHLINE AND OUTLOOK

- GTP geometries as non-isolated "frozen" singularities

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  - Period $\pi(t) = \int \frac{dx\,dy}{xy} \frac{1}{1 - tP(x,y)}$ (Akhtar, Coates, Galkin, Kasprzyk, ... '12–'22)

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  → Useful tool for classification of brane webs

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- BPS quivers for 5d theory on S^1

THANK YOU FOR

YOUR ATTENTION !

Comparing the decoherence effects due to black holes versus ordinary matter

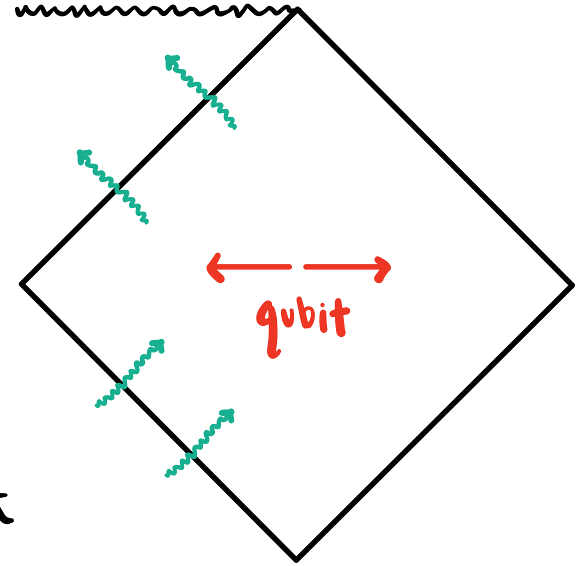
based on 2405.02227 with J. Maldacena

Anna Biggs
Princeton University
Strings 2024

A thought experiment was recently discussed which involves the decoherence of a quantum system due to a black hole.

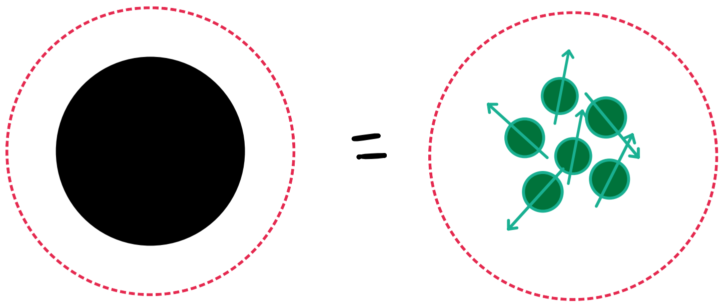
Wald, Satishandran, Danielson 2205.06279, 2301.00026

Gralla, Wei 2311.11461



It has been suggested that this effect may be unique to black holes and/or be of fundamental importance for their quantum description.

On the other hand, we expect that a black hole, when viewed from the outside, is described by an ordinary quantum system evolving unitarily.

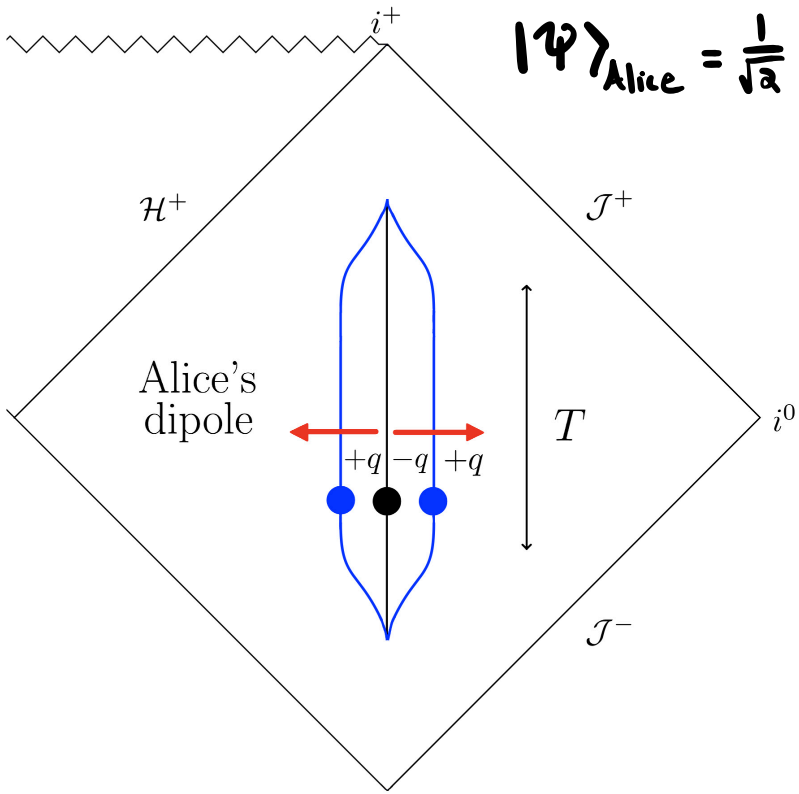


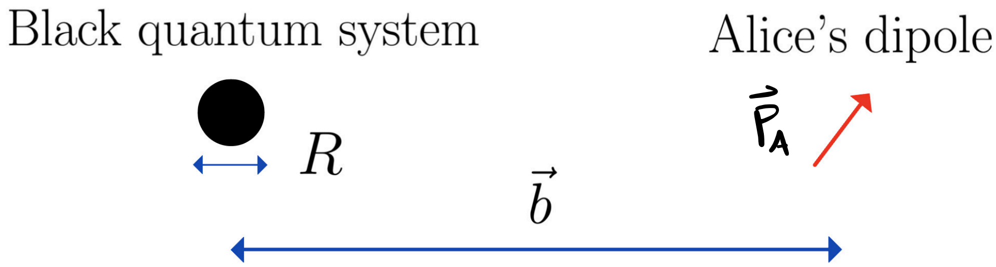
Today: we replace the black hole by an ordinary quantum system at finite temperature and obtain the same qualitative effect.

The idea is to analyze the problem in terms of an effective theory that applies equally well for the black hole case as for an ordinary matter system.

Review of setup for decoherence thought experiment

$$|\psi\rangle_{\text{Alice}} = \frac{1}{\sqrt{2}}\left(|-\tfrac{d}{2}\rangle + |+\tfrac{d}{2}\rangle\right)$$



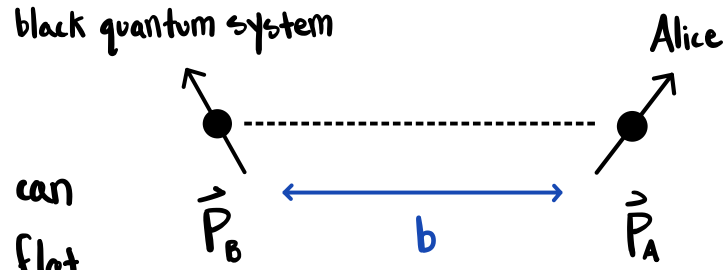


$$T \gg b \gg R$$

The black hole will destroy the coherence of the superposition at a constant rate, i.e. the off-diagonal elements of Alice's density matrix decay as $e^{-\Gamma t}$ for some constant Γ. Wald, Satishandran, Danielson 2205.06279, 2301.00026

There is an analogous gravitational version of the effect (which involves the superposition of a massive particle)

Effective theory picture



The fields sourced by $\vec{P}_A$ have wavelength $\lambda \sim T \gg R$, so we can approximate the black quantum system by a point particle in flat space. Interactions are captured by multipole operators living on the worldline.

$$S_{\text{int}} = -\frac{e^2}{4\pi b^3}\int dt\left(\vec{P}_A\cdot\vec{P}_B - 3\left(\vec{P}_A\cdot\hat{b}\right)\left(\vec{P}_B\cdot\hat{b}\right)\right)\sigma_3$$

$R\omega \ll 1$

$T \gg b$

Classical vector, indicating the size of Alice's dipole

Operator acting on the Hilbert space of the black quantum system

Pauli matrix acting on Alice's system (parameterizing the two configurations)

When we compute the time evolution of Alice's density matrix under this interaction, we find a constant decoherence rate of the form

$$\Gamma \propto \left(\frac{e^2}{b^3}\right)^2 P_A^2 \int dt\, \langle P_B(t) P_B(0)\rangle$$

In this framework, the decoherence is arising from thermal or quantum fluctuations of the electric dipole operator describing the black quantum system.

The Fluctuation-Dissipation Theorem

To determine $G_W(\omega) \equiv \int dt\, e^{i\omega t} \langle P_B(t) P_B(0) \rangle$ for various systems of interest, we compute the response function $\chi(t) \equiv i\theta(t) \langle [P_B(t), P_B(0)] \rangle$ which is related to $G_W(\omega)$ by the fluctuation-dissipation theorem:

$$G_W(\omega) = 2(n_B(\omega)+1)\, \mathrm{Im}\, \chi(\omega), \quad n_b(\omega) \equiv \frac{1}{e^{\beta\omega}-1} \overset{\beta\omega \ll 1}{\approx} \frac{1}{\beta\omega}$$

For an ordinary matter system, $\chi(\omega)$ depends on transport coefficients such as the conductivity and viscosity.

For black holes, the low-frequency expansion of $\chi(\omega)$ is given by the (static and dynamical) "Love numbers",

$$\chi(\omega) = A + i\omega B + \mathcal{O}(\omega^2)$$

$A = 0$ for black holes in 4d, but $B \neq 0$.

From a computation of the Love numbers one can read off $G_W(\omega)$ and reproduce the black hole decoherence rates of Wald, Satishandran, Danielson

Comparison to ordinary matter

The response functions set the decoherence rate, so comparing χ_{BH} and χ_{matter} is the same as comparing the decoherence effects.

We compare objects of the same size and at the same temperature.

Response function of spherical conductor with resistivity ρ:

$$\operatorname{Im} \chi_{\text{cond}}(\omega) \sim \frac{1}{e^4}\omega\rho R^3$$

BH response function:

$$\operatorname{Im} \chi^{e}_{\text{BH}}(\omega) \sim \frac{1}{e^2}\omega r_s^4$$

To be comparable to a black hole, $\rho/R \sim e^2$.

A small ball of some impure metal will do (ex. Al alloy)

Comparison to ordinary matter - gravitational case

We can make a similar comparison for the gravitational effect.

(Ex. Compare to a self-gravitating fluid or an elastic solid with some viscosity)

An ordinary object of the same mass as the BH typically has a larger decoherence effect, while the opposite is true for an object of the same size (when all comparisons are made at the same temperature).

Summary

- The decoherence effect is consistent with the hypothesis that, from the outside, black holes are described by ordinary quantum systems.

- It is qualitatively present for ordinary matter at finite temperature.

- The decoherence arises from thermal fluctuations of the multipole moments of the black hole/matter system.

- For the electromagnetic effect, the decoherence can be of equal magnitude for black holes and ordinary objects.

- For the gravitational effect, ordinary matter produces a weaker effect, if we compare objects of the same size and temperature.

The connection to absorption

$G_w(\omega)$ also governs the absorption of low-frequency fields.

Let us discuss scalar fields for simplicity, where

operator describing black quantum system

$$S_{int} = \int dt\, \mathcal{O}(t)\, \phi(t)$$

bulk scalar field

Amplitude for transition from $|i\rangle$ to $|f\rangle$ of black quantum system:

$$\mathcal{M}_{i\to f} = \frac{1}{\sqrt{2\omega}} \int_0^T dt e^{-i\omega t} \langle f|\mathcal{O}(t)|i\rangle$$

$\Gamma_{i\to f} = \sum_f |\mathcal{M}_{i\to f}|^2$, so

$$\Gamma_{i\to f} = \frac{T}{2\omega} \int_0^T dt e^{i\omega t} \langle \mathcal{O}(t)\mathcal{O}(0)\rangle$$

For σ_{abs}, divide $\Gamma_{i\to f}$ by T and incoming particle flux.

$$\sigma_{\text{abs}}(\omega) = \frac{1}{2\omega} \int dt e^{i\omega t} \langle \mathcal{O}(t)\mathcal{O}(0)\rangle = \frac{1}{2\omega} G_w(\omega) \qquad \omega R \ll 1$$

σ_{abs} is related to the classical absorption cross section σ_{abs}^{clas} by

$$\sigma_{\text{abs}}(\omega) = (n_b + 1)\sigma_{\text{abs}}^{\text{clas}}(\omega) \implies \sigma_{abs}^{clas}(\omega) \sim \frac{\beta}{2} G_w(\omega) \quad \text{for } \beta\omega \ll 1$$

Comparison to ordinary matter - more details...

Gravitational case:

BH response function: $\mathrm{Im}\,\chi^g_{\mathrm{BH}}(\omega) \propto \omega \dfrac{r_s^6}{G}$

a) Same mass and same temperature comparison:

$\nu \equiv$ kinematic viscosity
$R_s \equiv$ Schwarzschild radius of fluid

Response function of self-gravitating fluid: $\mathrm{Im}\,\chi(\omega) \sim \omega \dfrac{\nu R^6}{G R_s}$

Example: self-gravitating ball of water at room temp. $r_s \sim 0.6\,\mu\mathrm{m}$, $R \sim 5\times 10^5$ m

$$\frac{\mathrm{Im}\chi_{\mathrm{fluid}}}{\mathrm{Im}\chi^g_{\mathrm{BH}}} \sim \frac{\nu}{\nu_{\mathrm{BH}}}\left(\frac{R}{r_s}\right)^6 \sim 10^{64} \qquad \frac{R}{r_s} \sim 10^{12} \qquad \frac{\nu}{\nu_{\mathrm{BH}}} \sim 10^{-8}$$

A ball of water absorbs gravitons more easily than a black hole of the same mass.

b) Same size and same temperature comparison:

Response function of elastic solid with some viscosity ν: $\mathrm{Im}\chi(\omega)_{\mathrm{metal}} \sim \omega\nu \dfrac{R^5}{G}\left(\dfrac{R_s}{R}\right)\left(\dfrac{c}{c_s}\right)^4$

Example: lead ball at ~ 1.5 K. $r_s \sim 100\,\mu\mathrm{m}$, $R_s \sim \ell_p$

$R_s \equiv$ Schwarzschild radius of the solid
$c_s \equiv$ speed of sound in solid

$$\frac{\mathrm{Im}\chi_{\mathrm{metal}}}{\mathrm{Im}\chi^g_{\mathrm{BH}}} \sim 10^{-19} \qquad \frac{\nu}{\nu_{\mathrm{BH}}} \sim 10^{-10} \qquad \frac{R_s}{R} \sim 10^{-30} \qquad \left(\frac{c}{c_s}\right)^4 \sim 10^{21}$$

The metal absorbs fewer gravitons than the black hole.

Zero temperature black holes

The near-horizon geometry of an extremal black hole develops an approximate $SL(2)$ symmetry that fixes the form of the correlators.

Scalar effect: the decoherence is not linear in T but $\propto \ln T$

A massless field corresponds to an operator of dimension $\Delta = 1$ so that

$$\int_0^T dt \int_0^T dt' \langle \mathcal{O}(t)\mathcal{O}(t')\rangle \propto \int_0^T dt \int_0^T dt' \frac{1}{(t-t'-i\epsilon)^2}$$

$$\propto \ln T + \text{endpoints}$$

This answer was derived using different methods for a Kerr black hole in Gralla, Wei 2311.11461

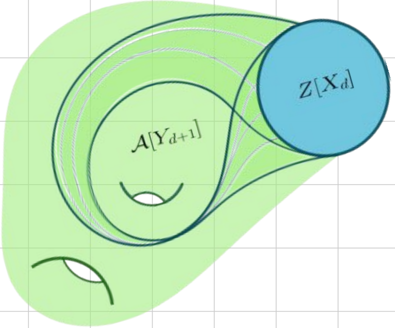
For operators with dimension $\Delta > 1$,

$$\int_0^T dt \int_0^T dt' \langle \mathcal{O}(t)\mathcal{O}(t')\rangle \propto \int_0^T dt \int_0^T dt' \frac{1}{(t-t'-i\epsilon)^{2\Delta}}$$

$$\propto \left(\frac{1}{T}\right)^{2(\Delta-1)} + \text{end points}$$

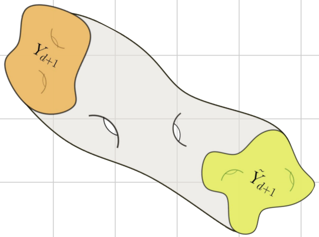
The T-dependent term $\to 0$ in the $T \to \infty$ limit.

No T-dependent contribution to the decoherence at long times.



ANOMALIES & BORDISMS
OF NON-SUPERSYMMETRIC STRINGS

Matilda Delgado



Based on:
[2310.06895] I. Basile, A. Debray, M.D., M. Montero

ift

BIG PICTURE

Our world is non-supersymmetric

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(**at least** at low energies)

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It is crucial for phenomenology to understand String Theory (QG) in setups without supersymmetry!

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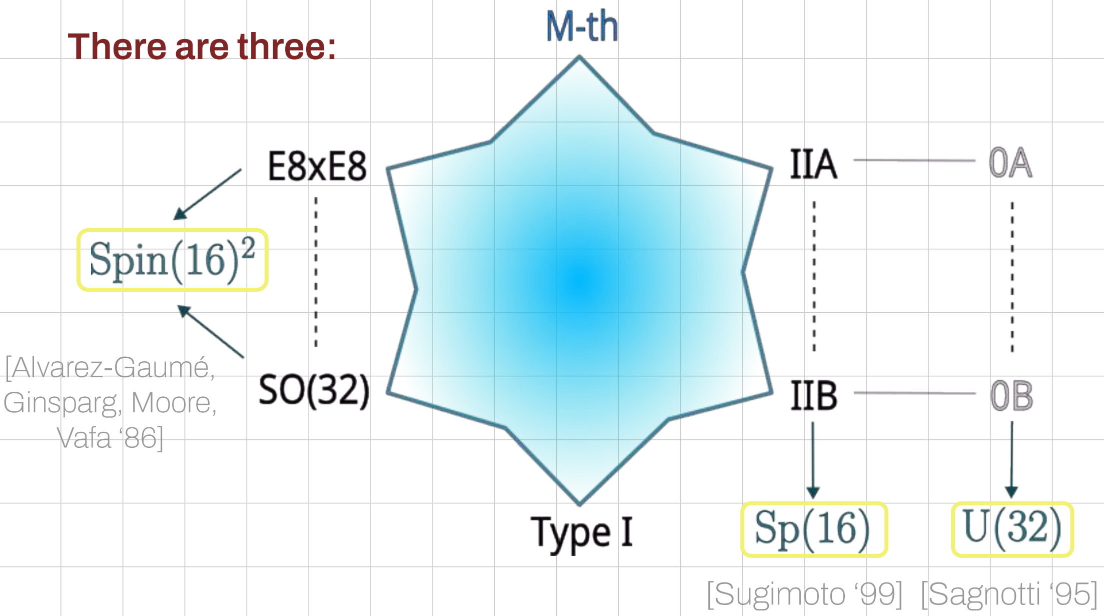
How?

10D NON-SUPERSYMMETRIC STRING THEORIES

There are three:



Arguably the most natural way to study QG away from SUSY !!

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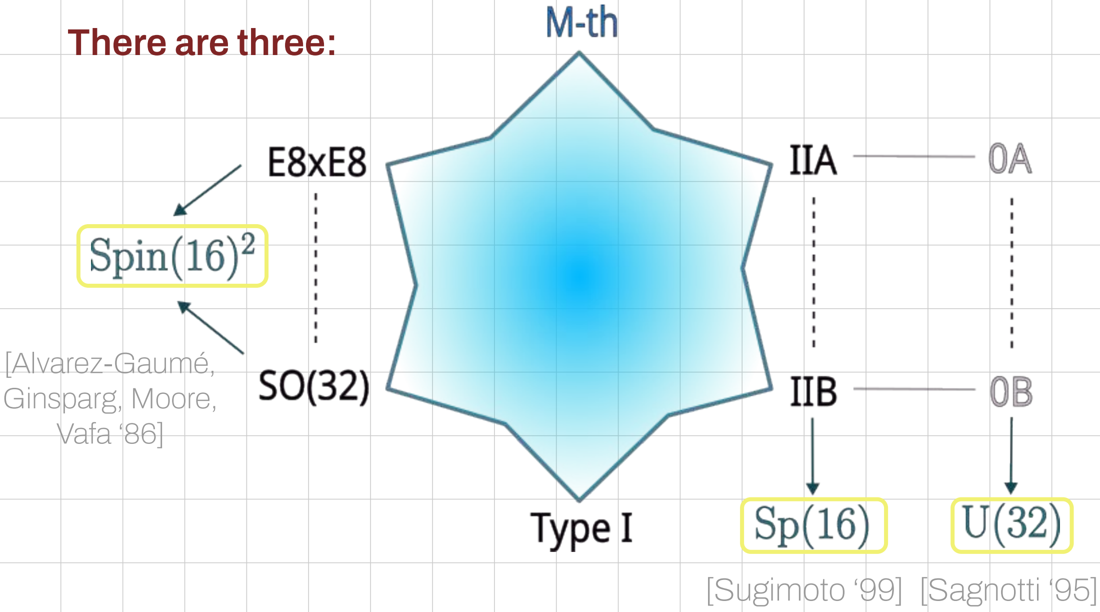

Not a lot is known about these theories..

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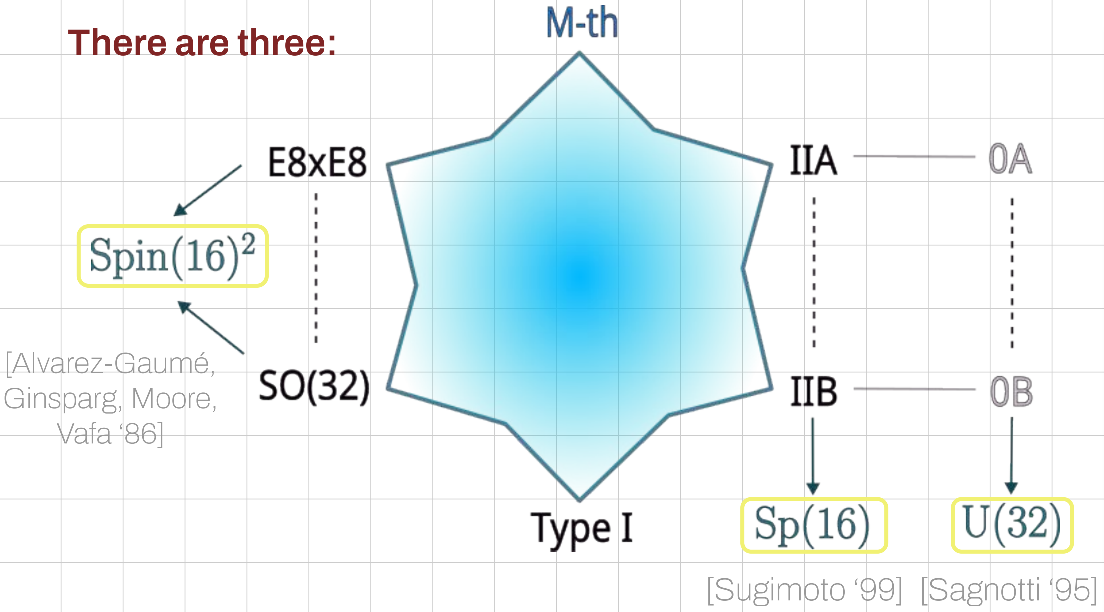

Not a lot is known about these theories..

e.g. do gauge/gravitational anomalies cancel?

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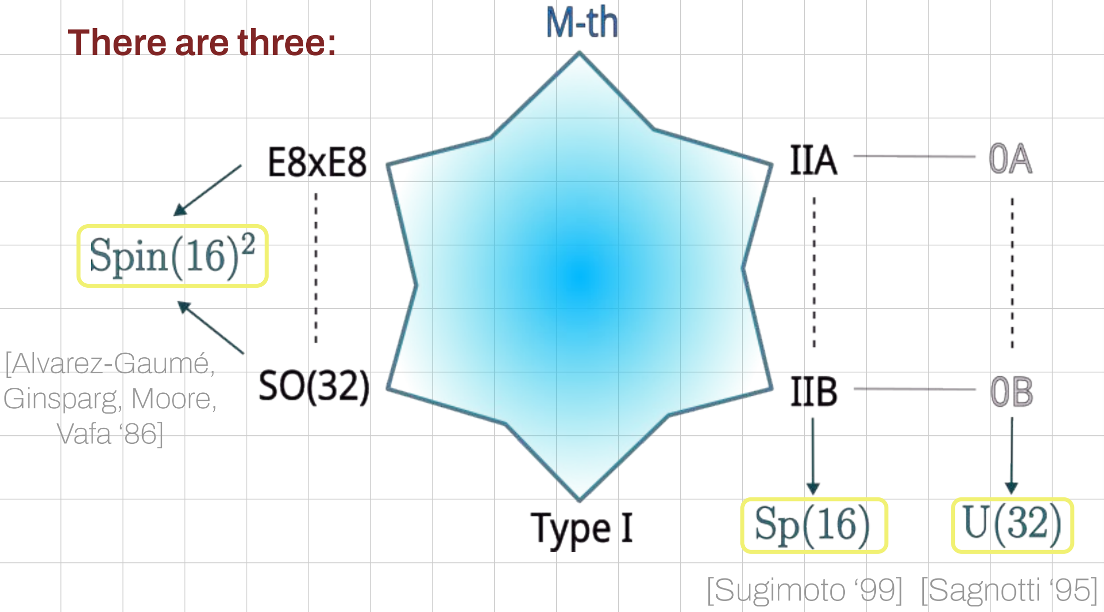

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10D NON-SUPERSYMMETRIC STRING THEORIES

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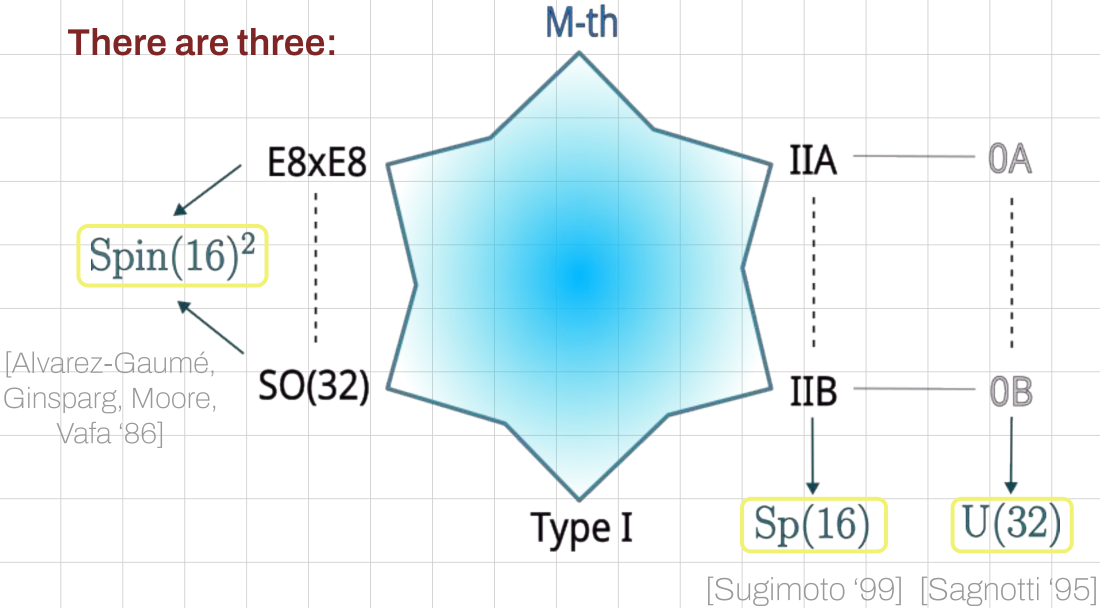

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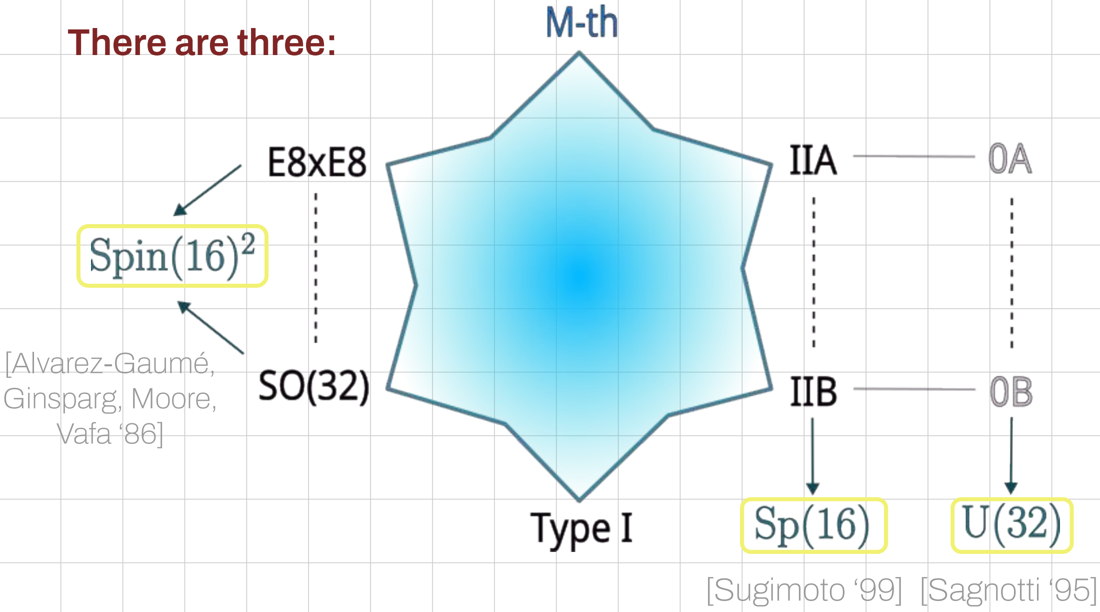

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e.g. do gauge/gravitational anomalies cancel?

- local anomaly cancellation ✅

- **global anomaly cancellation** ❓

Our work answers this question

HOW?

by computing the relevant bordism groups for these theories

[e.g. Álvarez-Gaumé, Vázquez-Mozo '22]

A CRASH COURSE ON GLOBAL ANOMALIES

An anomaly in a **gauge** transformation or diffeomorphism is:

$$Z[X_d] \Longrightarrow \tilde{Z}[X_d] \neq Z[X_d]$$

Global anomalies = associated to a transformation that cannot be *deformed to the identity*

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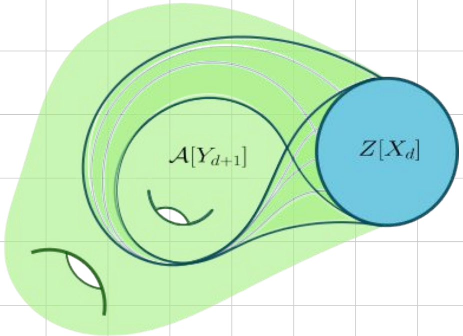
[eg. García-Etxebarria, Montero '18]



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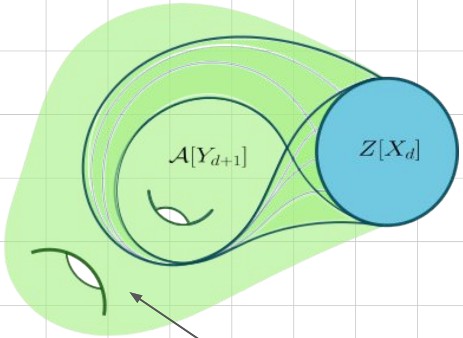

In QG, allow for topology-change ⇒ Dai-Freed anomalies

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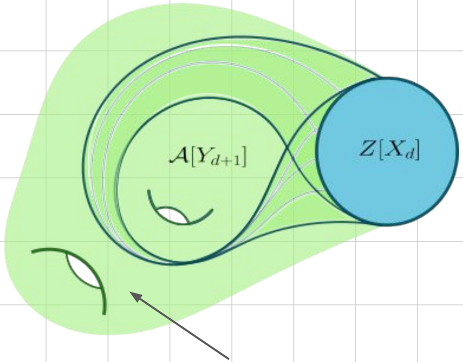

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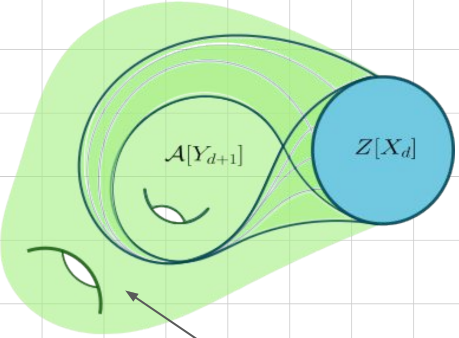

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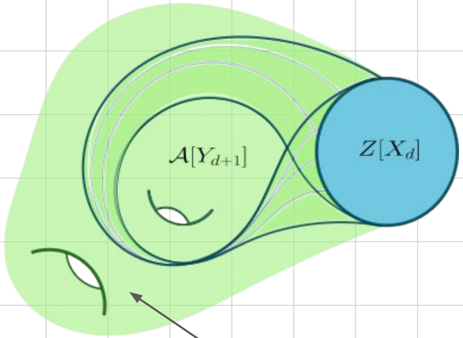

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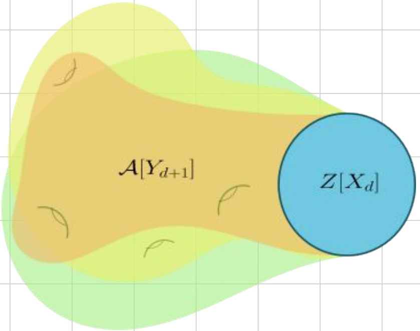
—> the anomaly is much easier to compute this way because:

The anomaly itself is a bordism invariant of these (d+1)-manifolds
→ we "just" have to compute 11D bordism groups for our three 10D theories

WHAT ARE THE BORDISM GROUPS?

What are the relevant bordism groups for these theories?
The background must satisfy:

$$dH \sim \mathrm{tr}F^2 - \mathrm{tr}R^2 = 0$$



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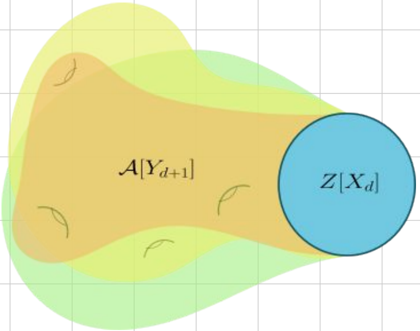

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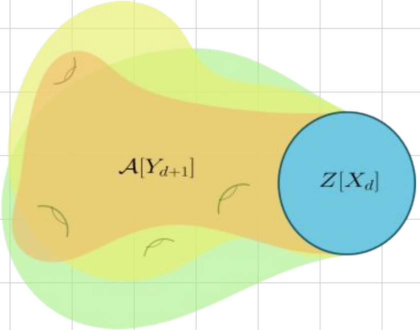

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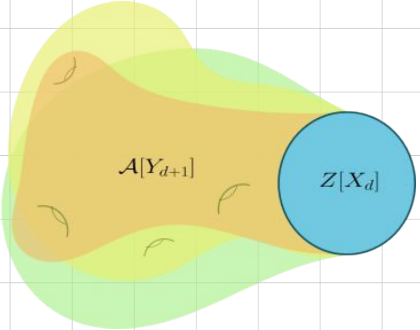
(up to a technical subtlety for the Sagnotti string)



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CONCLUSIONS

We showed there are no global anomalies for the three 10D non-supersymmetric string theories

Huge consistency check!

But that's not all…

➥ We used **anomaly inflow** to shed light on the chiral content of worldvolumes of branes in these theories

➥ We also computed **lower-dimensional cobordism** groups for these theories:

$$\begin{array}{ll}
\Omega_0^{\text{String-}Sp(16)} \cong \mathbb{Z} & \Omega_6^{\text{String-}Sp(16)} \cong \mathbb{Z}_2 \\
\Omega_1^{\text{String-}Sp(16)} \cong \mathbb{Z}_2 & \Omega_7^{\text{String-}Sp(16)} \cong \mathbb{Z}_4 \\
\Omega_2^{\text{String-}Sp(16)} \cong \mathbb{Z}_2 & \Omega_8^{\text{String-}Sp(16)} \cong \mathbb{Z}^{\oplus 3} \oplus \mathbb{Z}_2 \\
\Omega_3^{\text{String-}Sp(16)} \cong 0 & \Omega_9^{\text{String-}Sp(16)} \cong (\mathbb{Z}_2)^{\oplus 3} \\
\Omega_4^{\text{String-}Sp(16)} \cong \mathbb{Z} & \Omega_{10}^{\text{String-}Sp(16)} \cong (\mathbb{Z}_2)^{\oplus 3} \\
\Omega_5^{\text{String-}Sp(16)} \cong \mathbb{Z}_2 & \Omega_{11}^{\text{String-}Sp(16)} \cong 0.
\end{array}$$

Example: Sugimoto

No Global Symmetries in QG implies that all of these bordism classes have to **trivialize in QG**.
[Cobordism Conjecture by McNamara, Vafa '19]

We can predict the existence of **new extended objects** that trivialize these classes!

… more info on arXiv:2310.06895

THANKS!

Stringy Horizons

Elliott Gesteau

Caltech

Strings 2024, Gong Show

To appear with Hong Liu (MIT)

A question

- Consider a quantum dynamical system. What is the **minimal amount** of time $\mathcal{T}$ for which we need to look at the system in order to be able to predict all the rest of its evolution?

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- For systems **with information loss**, it can be that

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- Algebraically, $\mathcal{T} \neq 0$ means that algebras of operators supported on a **time band** may be inequivalent to the full algebra of the theory.

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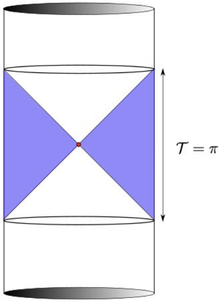

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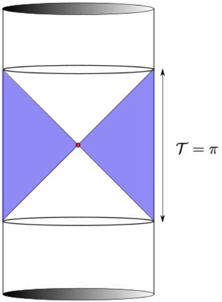

- $\mathcal{T} = \pi \neq 0$: emergence of a **radial direction**.

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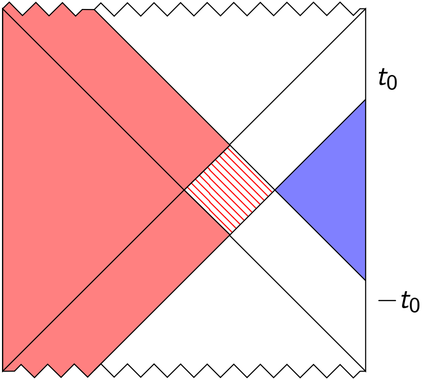

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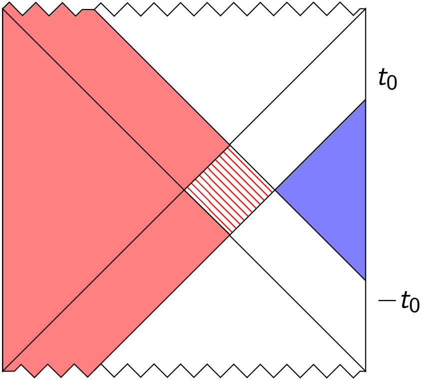

$\mathcal{T} = \infty$: emergence of a **bifurcate horizon**.

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- There is an emergent **stringy horizon** at high temperature even at weak coupling!

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Stringy holography

- $\mathcal{T}$ can be computed for a **wide range** of theories.
- It can also be used to detect violations of the **equivalence principle** in the stringy regime.
- When applied to **modular time** instead of boundary time, $\mathcal{T}$ can also be used to diagnose the presence of a **stringy QES** rather than a stringy horizon.

The jump to $\mathcal{T} \neq 0$ at large N is the basic mechanism allowing for the emergence of a radial direction and horizons in AdS/CFT, even in the stringy regime.

Thank you!

Tensionless strings on AdS_3 orbifolds

Bin Guo

IPhT, Saclay

Strings Gong Show

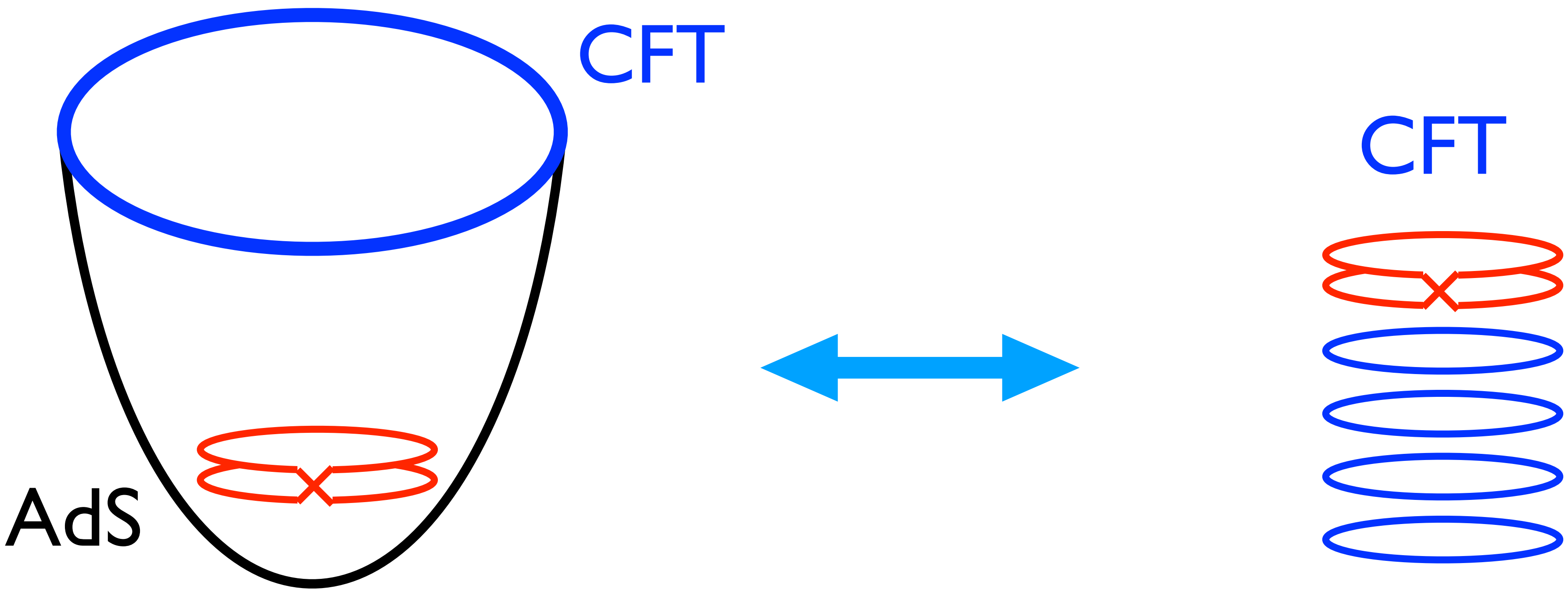
June 4, 2024

Based on arXiv: 2312.01348 and wip with M. R. Gaberdiel and S. D. Mathur

Tensionless strings on AdS_3 $\quad \ell_s \sim \ell_{AdS}$

Tensionless string theory on $AdS_3 \times S^3 \times T^4$ $\longleftrightarrow$ Free symmetric orbifold $(T^4)^N/S_N$ (D1D5 CFT)

(Eberhardt, Gaberdiel and Gopakumar, 18…)



$AdS_3 \times S^3 \times T^4$ background $\leftrightarrow$ 1-cycles

A string with winding w $\leftrightarrow$ a w-cycle

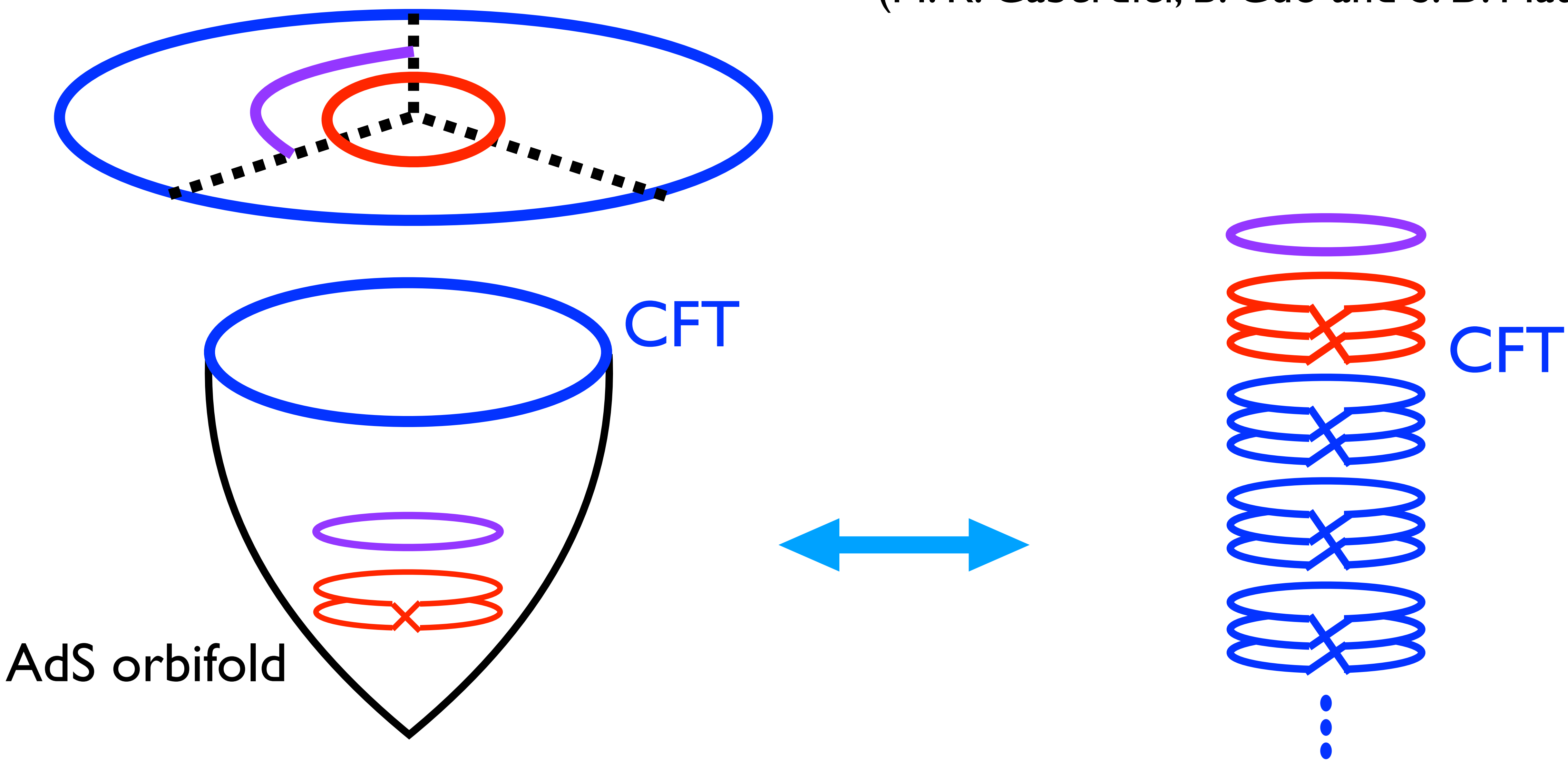
Questions

Other backgrounds?

Black hole?

Tensionless strings on $(\mathrm{AdS}_3 \times \mathrm{S}^3)/\mathbb{Z}_k \times \mathrm{T}^4$: spectrum

(M. R. Gaberdiel, B. Guo and S. D. Mathur, 23)



$(\mathrm{AdS}_3 \times \mathrm{S}^3)/\mathbb{Z}_k \times \mathrm{T}^4$ background $\leftrightarrow$ k-cycles

A string with winding w (in 1/k) $\leftrightarrow$ a kw-cycle

Condensation of winding k strings in $AdS_3 \times S^3 \times T^4$ produces $(\mathrm{AdS}_3 \times \mathrm{S}^3)/\mathbb{Z}_k \times \mathrm{T}^4$.

(Eberhardt, 21)

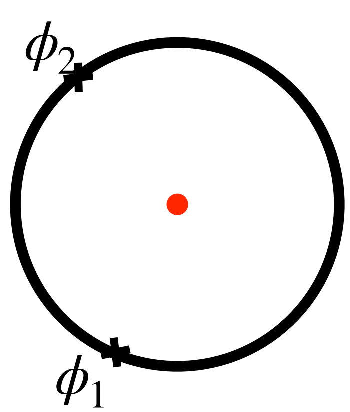
Correlator (work in progress)

String worldsheet correlators ⟷ BCFT correlators.

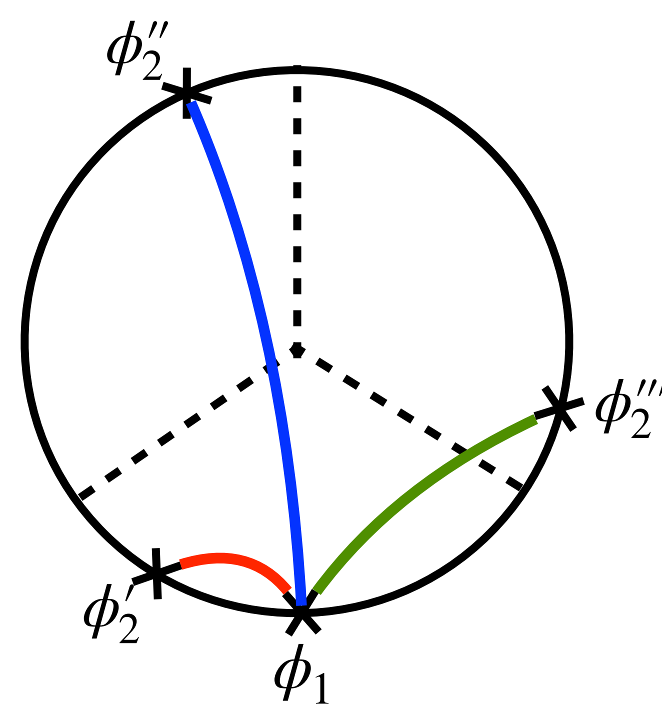
Correlator of untwisted strings at genus-0

Intermediate strings are untwisted

→ Semiclassical orbifold geometry: summing over images



(see also Bufalini, Iguri, Kovensky and Turton, 22)



⟷ BCFT correlator at leading order in 1/N

Correlator of untwisted strings at higher genus

Twisted intermediate strings contribute

→ Beyond semiclassical orbifold geometry

From the BCFT, the genus expansion parameter is $\frac{k}{\sqrt{N}}$.

$$k \sim \sqrt{N}$$

$$M_{AdS3/\mathbb{Z}_k} = -\frac{N}{2k^2} \sim -O(1)$$

→ Twisted intermediate strings are important during BH formation.

Future direction

(Martinec, 23)

AdS_3 orbifolds by Kleinian group (multiple conical defects)

$\downarrow$

Bag of gold geometry (beyond semiclassical geometry due to twisted strings)

Thank you!

Taming Mass Gap with Anti-de Sitter Space

Based on 2312.09277 with ***Christian Copetti, Lorenzo Di Pietro, Shota Komatsu***

Ziming Ji
(SISSA)

Ziming Ji

zji@sissa.it

CERN, 4 June 2024

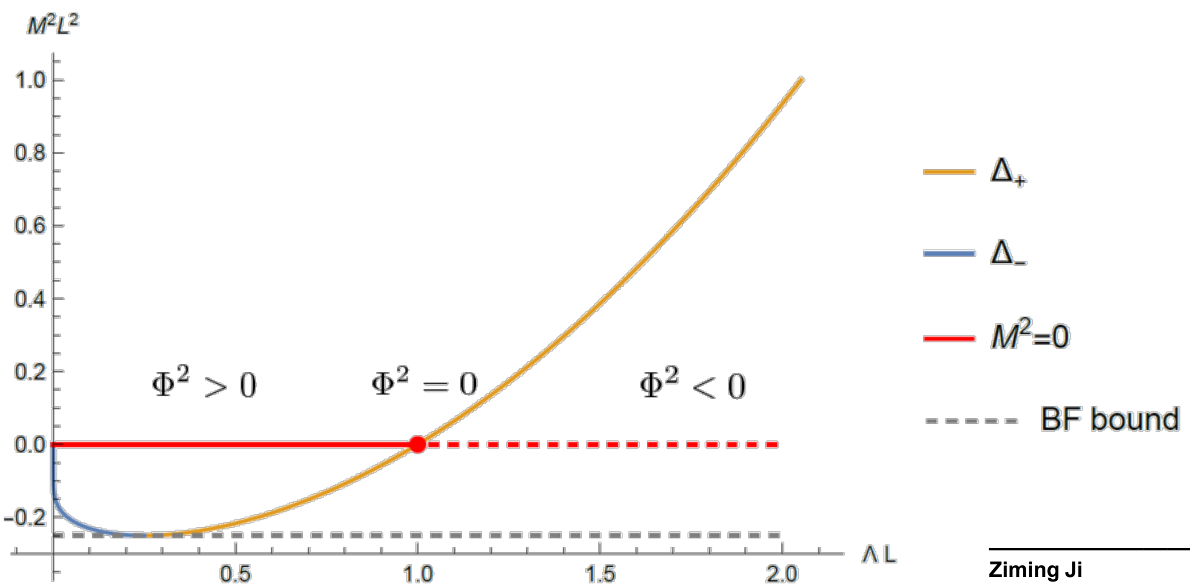
The Focus:

Boundary Diagnostic of the Mass Gap

- Asymptotic free theories with dynamically generated mass gaps in the IR in flat space.
- In AdS, by choosing different boundary conditions, we realize gapped and gapless phases.
- As we vary the AdS radius L, the theory interpolates between weak coupling and strong coupling.
- Consistency with flat space limit demands gapless phases to disappear at large enough L.
- Can we see this gapless-gapped transition from the boundary? How?

$O(N)$ NLSM at large N

- In flat space, $O(N)$ symmetric gap $M \sim \Lambda$. No SSB in 2d.
- In AdS, analog of SSB can happen due to IR regularity.
- At large N, bulk phases at all values of ΛL can be found by solving the gap equations.



Ziming Ji

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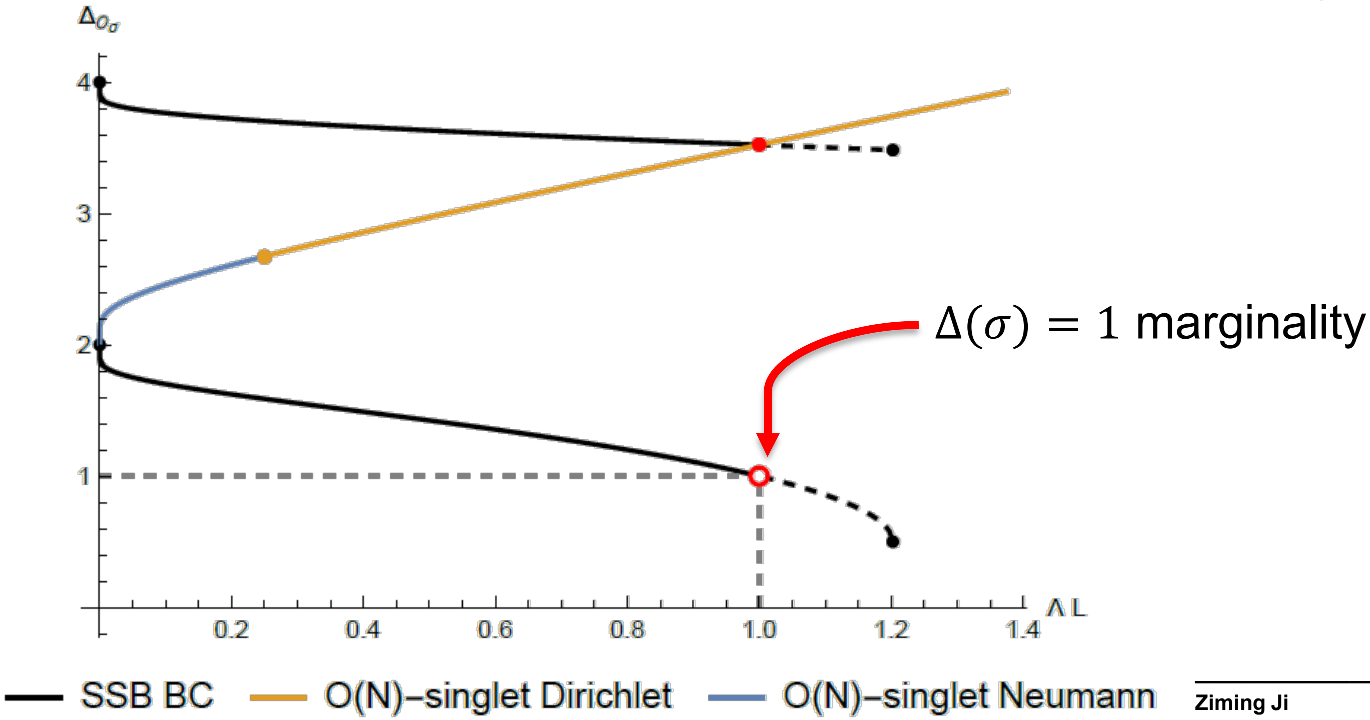
$O(N)$ NLSM at large N

- What is the boundary signal?
- Look at the lightest operator in the boundary spectrum of the Hubbard–Stratonovich field σ which is a singlet.



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Gross-Neveu at large N

- In flat space, vector symmetric gap $M \sim \Lambda$. The discrete axial symmetry is spontaneously broken.
- In AdS, we define massive vector boundary conditions and massless axial preserving boundary conditions.
- At large N, bulk phases at all values of ΛL can be found by solving the gap equations.
- No signal of gapless-gapped transition from the bulk gap equations.
- However, the gapless axial preserving boundary condition should disappear at large enough ΛL.

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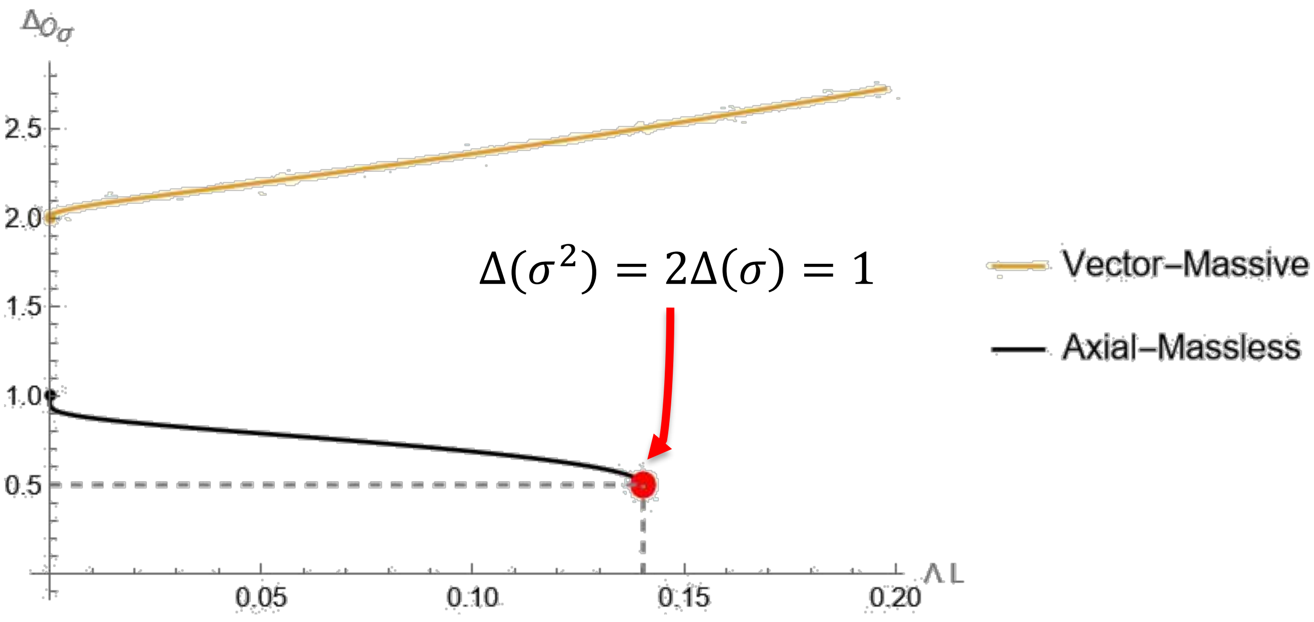
Gross-Neveu at large N

- Try to look at the boundary data!
- Look at the lightest operator in the boundary spectrum of the Hubbard–Stratonovich field σ. σ^2 is axial singlet.



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The conjecture:

Boundary Singlet Marginal Gaps!

- Yang-Mills in four-dimensional Anti-de Sitter Space.
- Dirichlet boundary condition with boundary global symmetry and boundary conserved currents dual to gluons should disappear before the flat space limit.
- We conjecture that some singlet operator on the boundary will become marginal at some ΛL and destabilize this Dirichlet boundary condition, mediating a quantum phase transition to confinement.

Thank you!

Towards a String theory for 2D YM theory

Suman Kundu

Weizmann Institute of Science, Israel

2312.12266

With Ofer Aharony, Tal Sheaffer (WIS)

Review: 2D Yang-Mills & large N

- 2D Yang-Mills is exactly solvable. (Migdal '75, Kazakov-Kostov '80, Rusakov '90, Fine '90, Witten '92, Blau '91 …)
- Large N expansion of Partition function $\mathscr{Z}$, and $\langle WL \rangle$ organizes into sum over world sheet maps. (Gross-Taylor '93)
- No known world sheet action.
- Only particular kind of maps contribute.

Previous Attempts:

Topological String theory at $g_{YM} = 0$

Cordes, Moore, Ramgoolam ’94

- Localization to ‘holomorphic maps’.
- Contributes to only ‘chiral’ part (chiral YM).

Horava ’96

- Localization to ‘Extremal area maps’.
- Solution to Nambu-Goto equation of motion.
- Includes ‘non-chiral’ maps.

Both gives vague proposal for finite ’t Hooft coupling $\lambda \ (= g_{YM}^2 N)$.

Our work:

Topological String theory at $g_{YM} = 0$

Horava '96

- A term in Horava action vanishes identically.
- Moduli space integral is ill-defined.

We found a non-vanishing replacement: gives correct measure on the moduli space for $\lambda = 0$ (topological YM theory).

- We reformulated this action as a Polyakov-type path integral,

$$S = -i\frac{t}{2}\int d^2\sigma d^2\theta \sqrt{H} H^{ab}\partial_a X \,.\, \partial_b X + \frac{1}{2}\int d^2\sigma d^2\theta \partial_\theta X^\mu K_{\mu\nu}[X]\partial_{\bar{\theta}} X^\nu$$

2312.12266

- This regulates some of the ill-behaved non-chiral maps.

Future Directions:

- $\langle WL \rangle$: Adding boundaries to these string world-sheet maps we can match the Wilson loop expectation values.

- Finite λ : Add corrections ($\propto \lambda$) to the action to match the finite coupling contributions. Note, the theory is no longer topological.

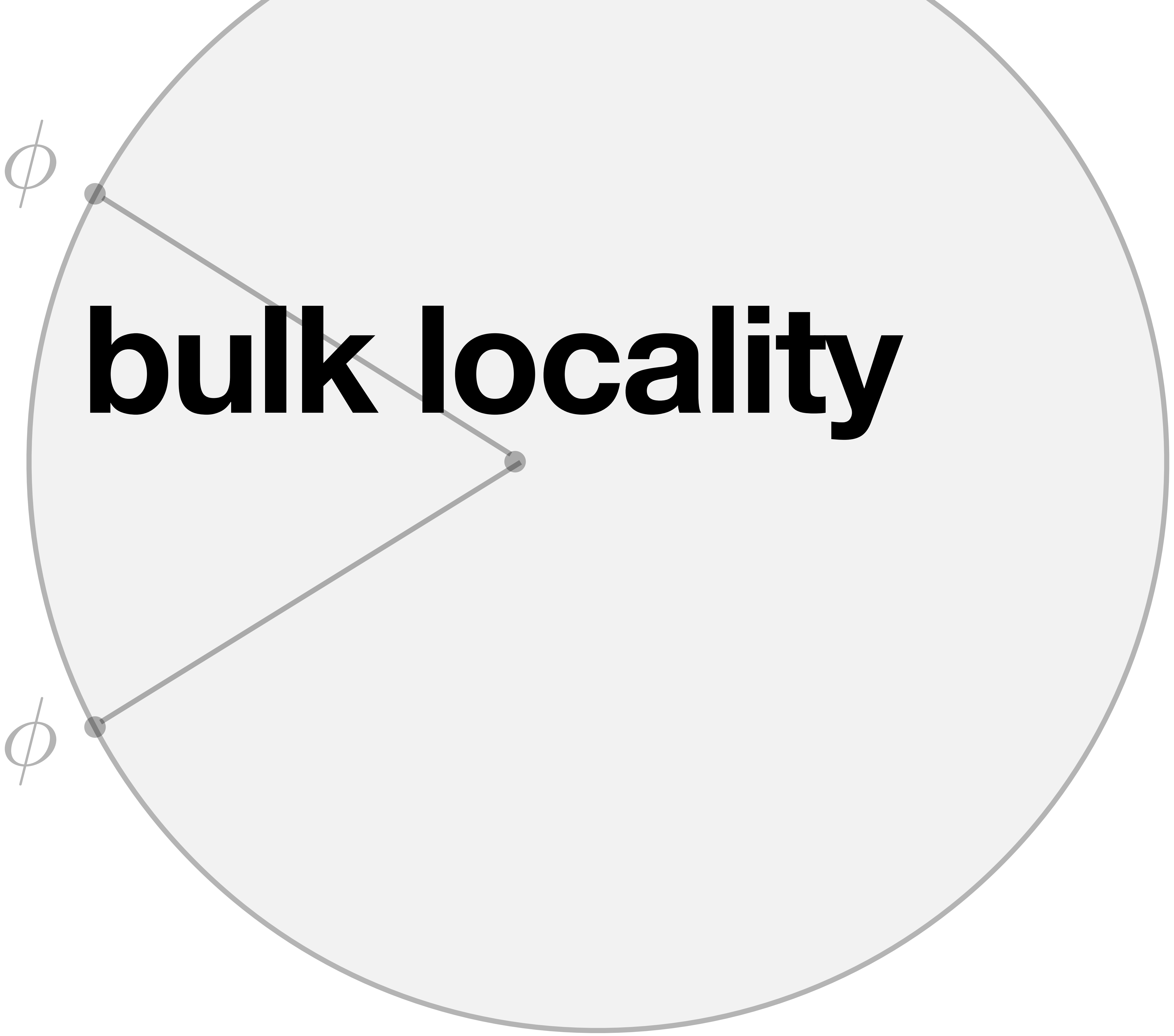
- 't Hooft meson spectrum;

- Adjoint particles: as an extra bid on the strings.

- Higher D.

Thank you!

Bootstrapping bulk locality



Nat Levine

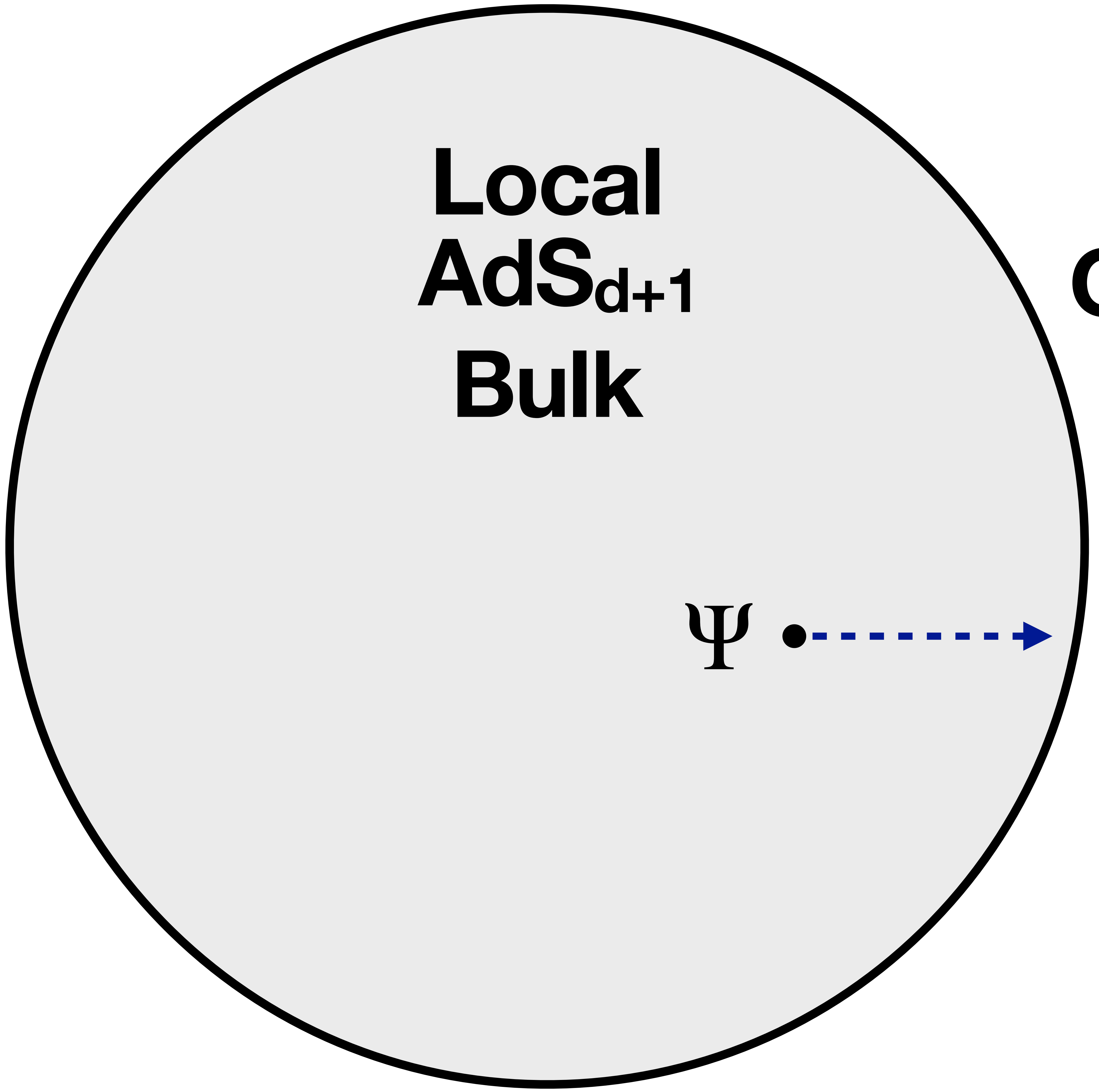
École Normale Supérieure

Strings 2024 Gong Show
CERN

[Part I: 2305.07078]
[Part 2: 24xx.xxxxx]

with Miguel Paulos

Setup

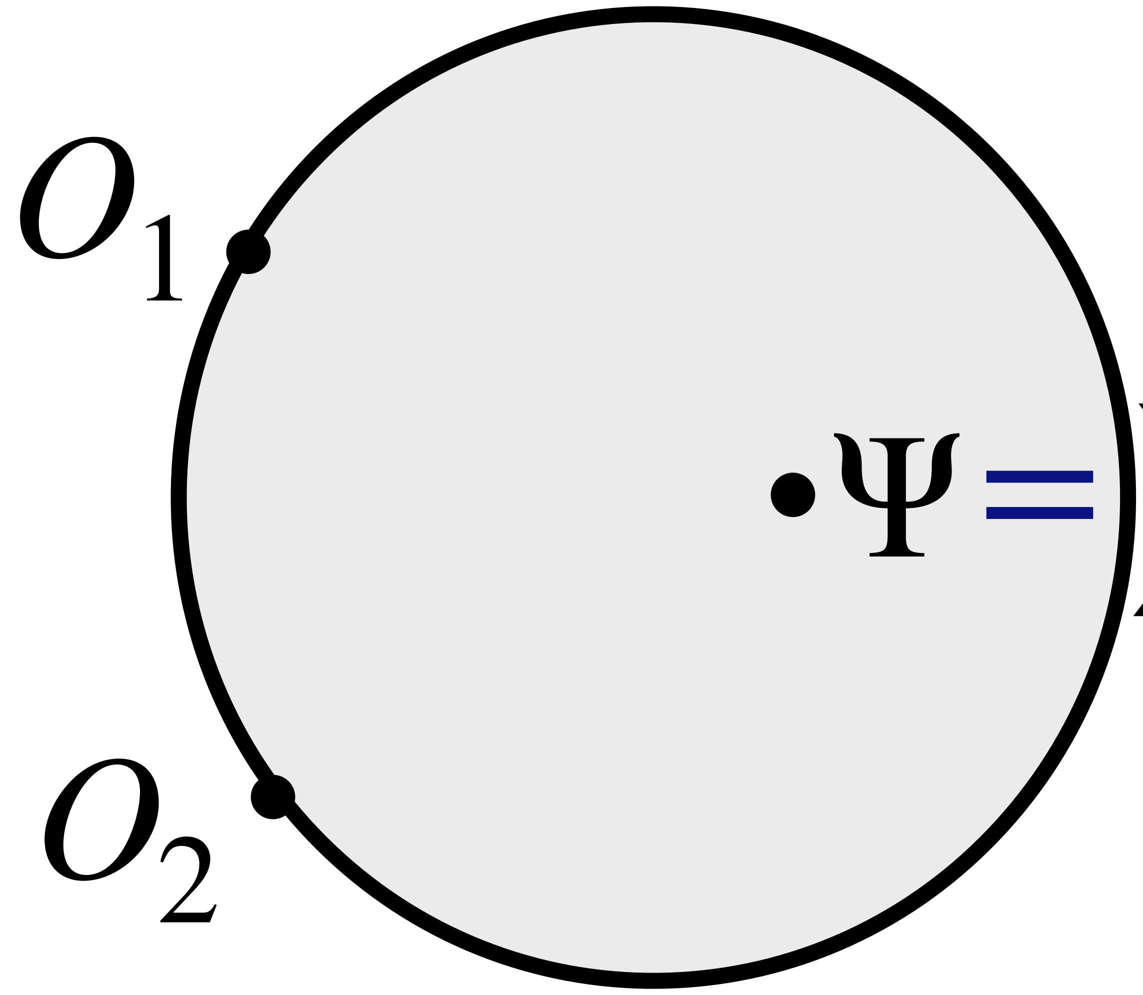


C(F)T$_d$

$$= \text{“}\sum_{\Delta} \mu_{\Delta} O_{\Delta}(x)\text{”}$$

cf. [Bena]
[Hamilton Kabat Lyfchitz Lowe]

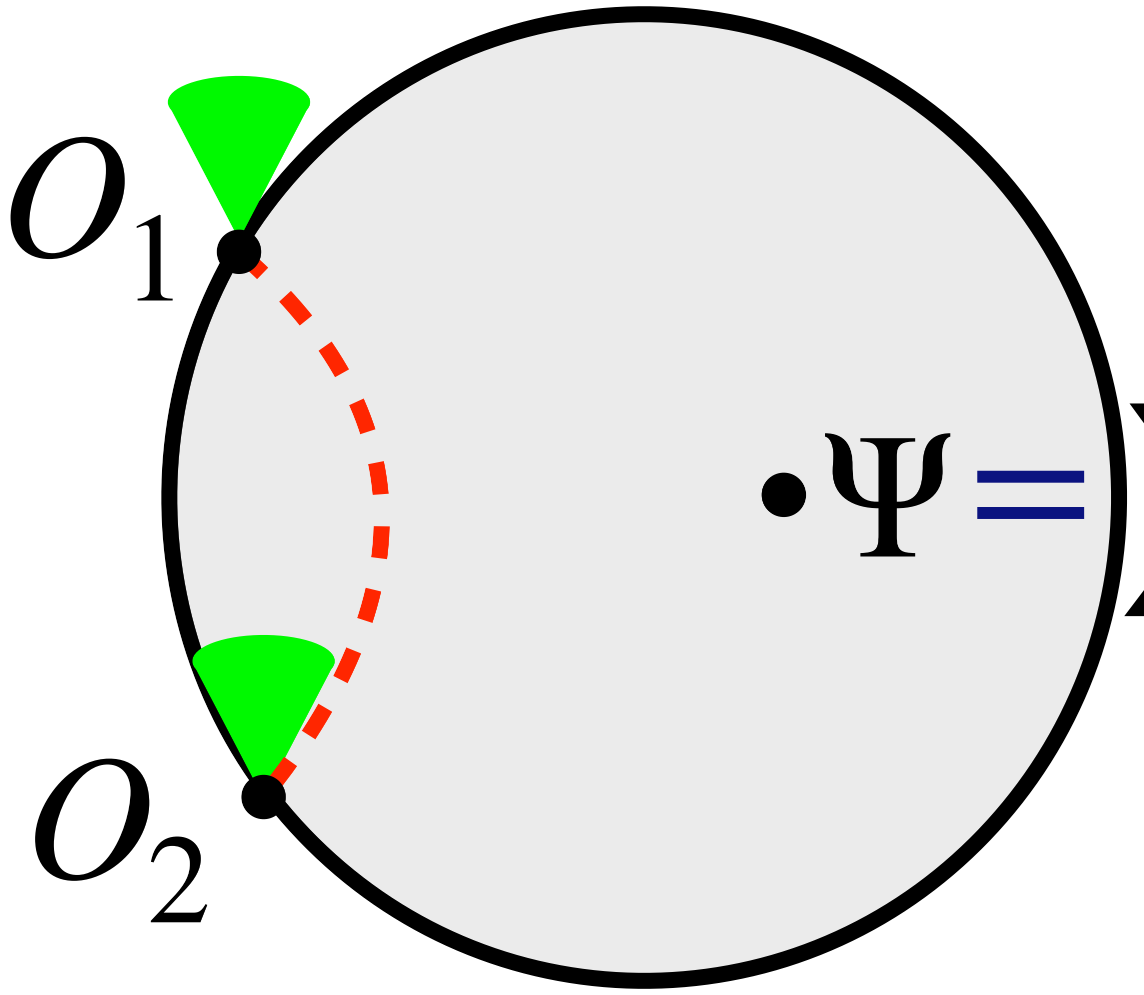
Setup



$$\Psi = \sum_{\Delta} \mu_{\Delta} O_{\Delta}(x)$$

$$\langle \Psi \,|\, O_1 O_2 \rangle = F(z) = \sum_{\Delta} c_{\Delta}\, g_{\Delta}(z) \qquad (c_{\Delta} = \mu_{\Delta}\, \lambda_{\Delta}^{12})$$

Setup



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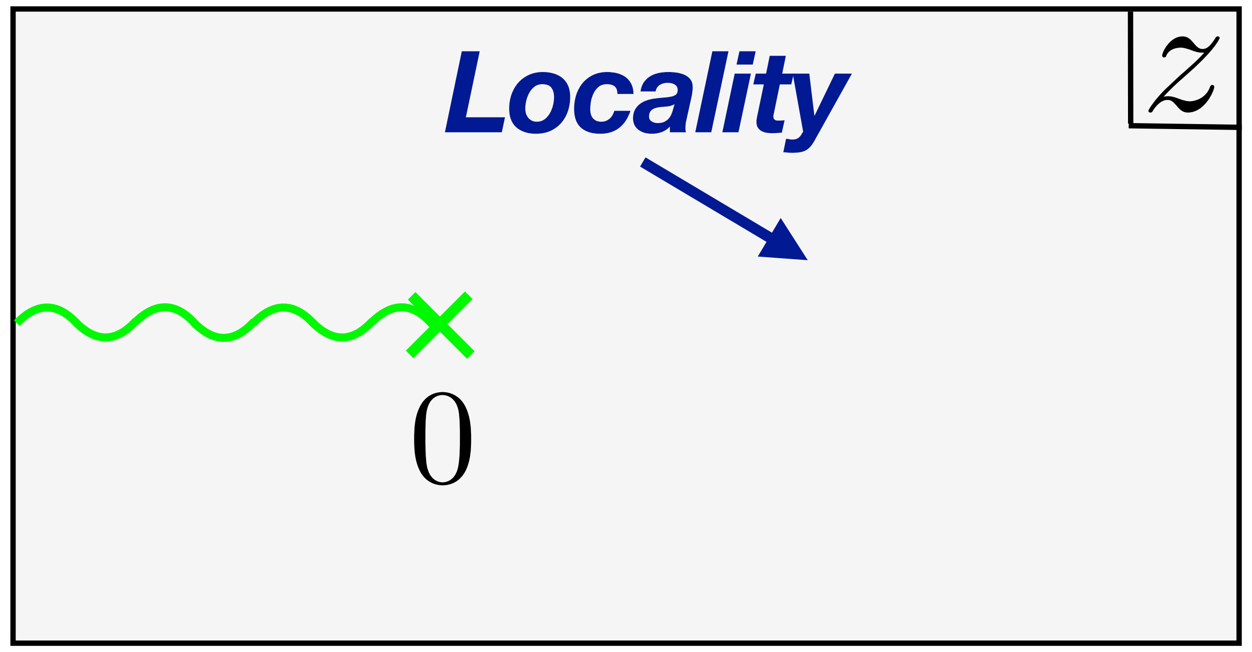
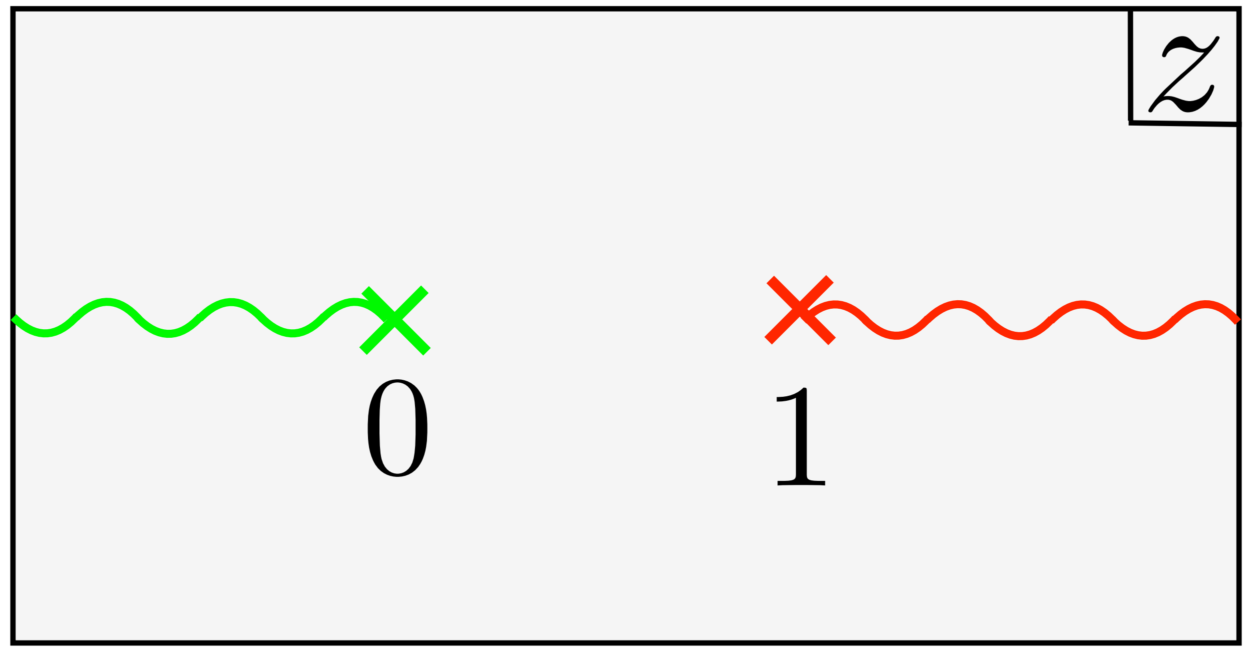
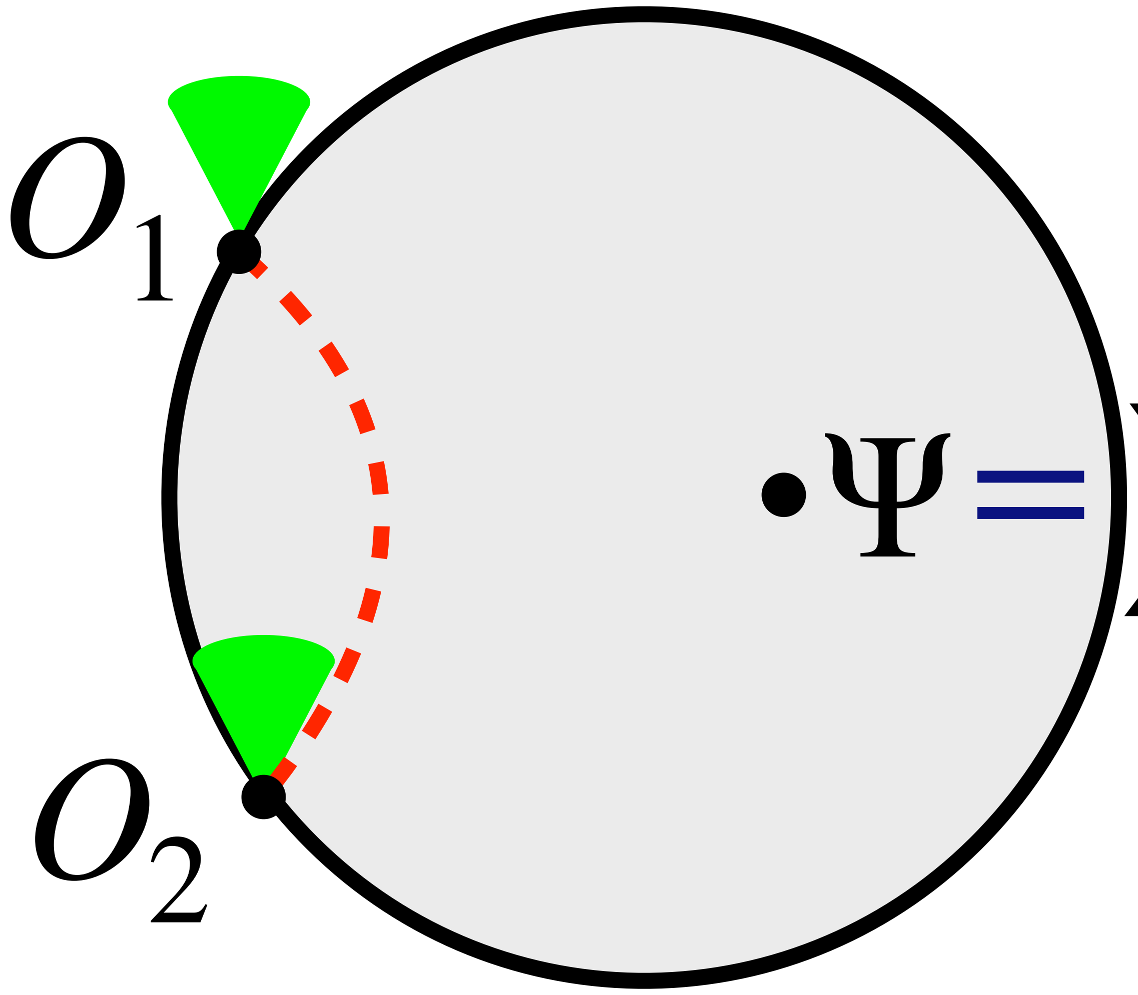


$$= \sum_{\Delta} c_\Delta$$



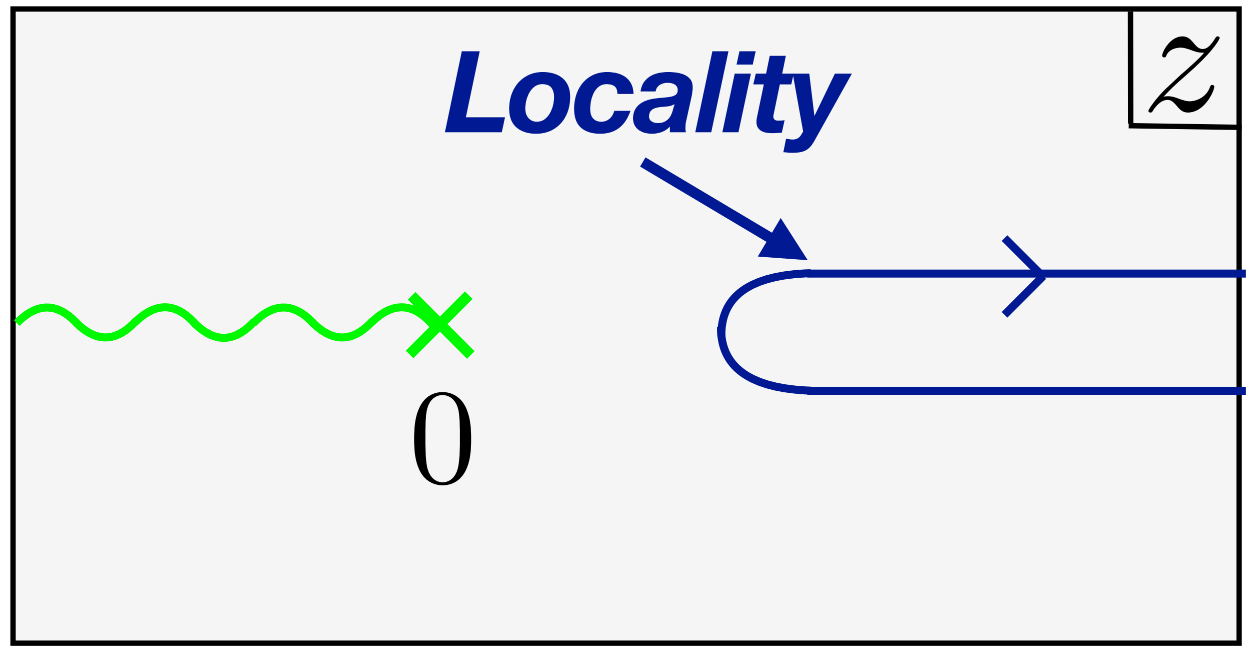
[Kabat Lifschytz]

Setup

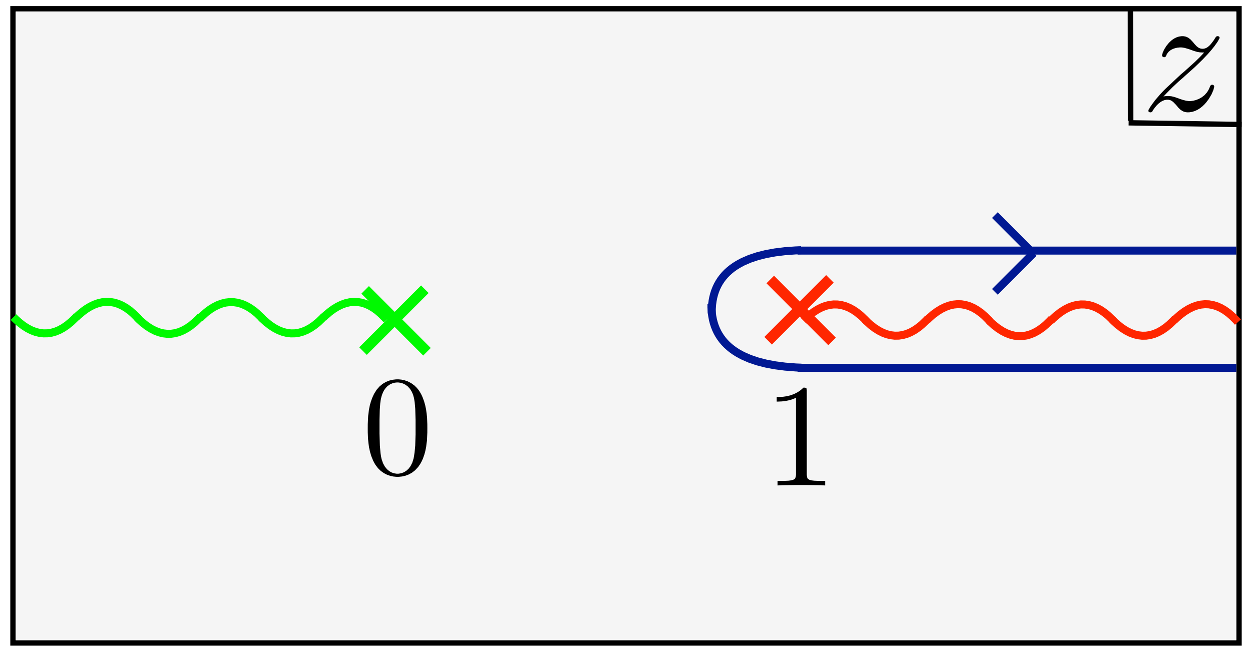


$$\Psi = \sum_\Delta \mu_\Delta O_\Delta(x)$$

$$\langle \Psi | O_1 O_2 \rangle = F(z) = \sum_\Delta c_\Delta \, g_\Delta(z) \qquad (c_\Delta = \mu_\Delta \, \lambda_\Delta^{12})$$



$$= \sum_\Delta c_\Delta$$



[Kabat Lifschytz]

functionals → **sum rules**

$$\theta_f[\,-\,] = \oint dz\, f(z)\,(\,-\,)$$

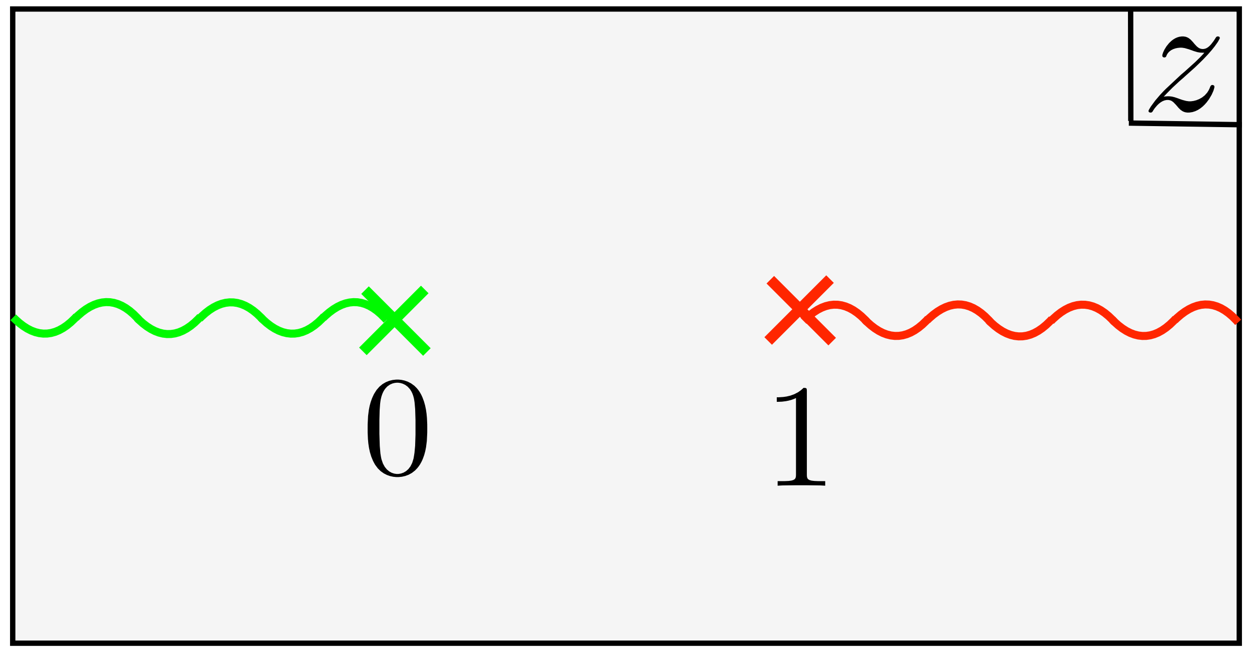
$$\theta_f[F] = \sum_\Delta c_\Delta \, \theta_f[g_\Delta] = 0$$

Why?

- Additional constraints on top of crossing

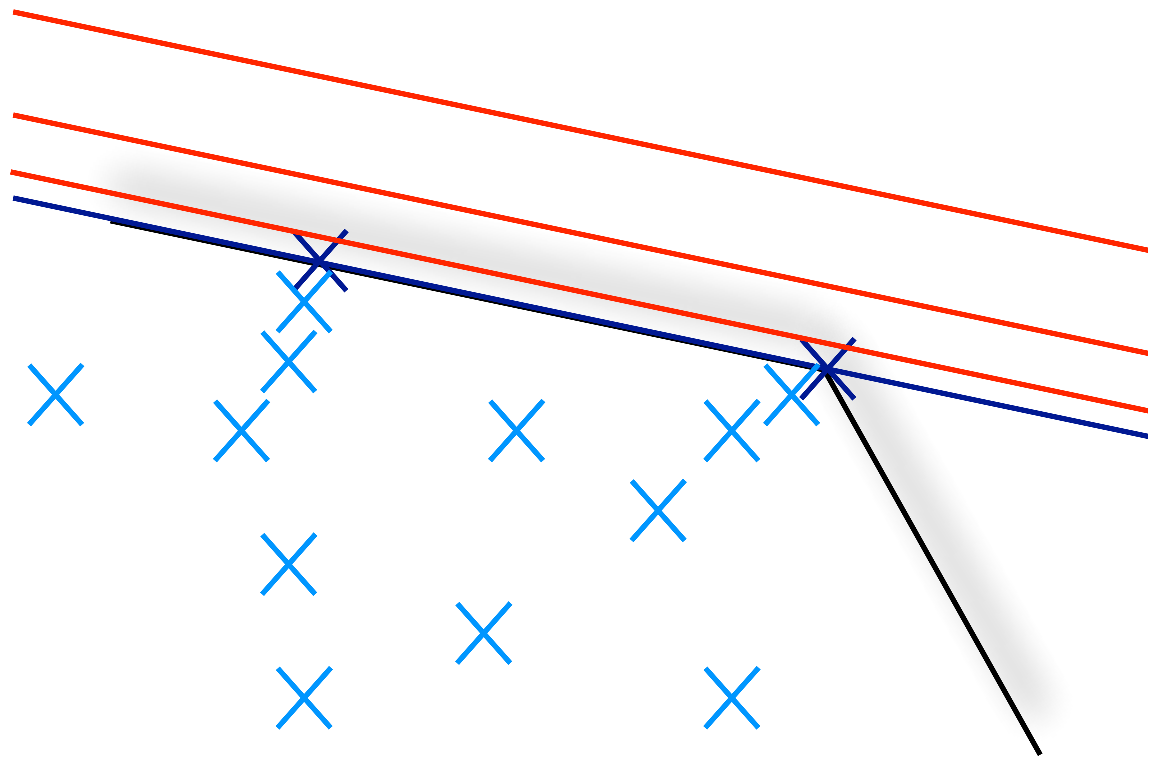
$$\theta_f[F] = \sum_{\Delta} c_\Delta\, \theta_f[g_\Delta] = 0$$

- Toy model for 1d crossing



Want to understand **"extremal"** solutions

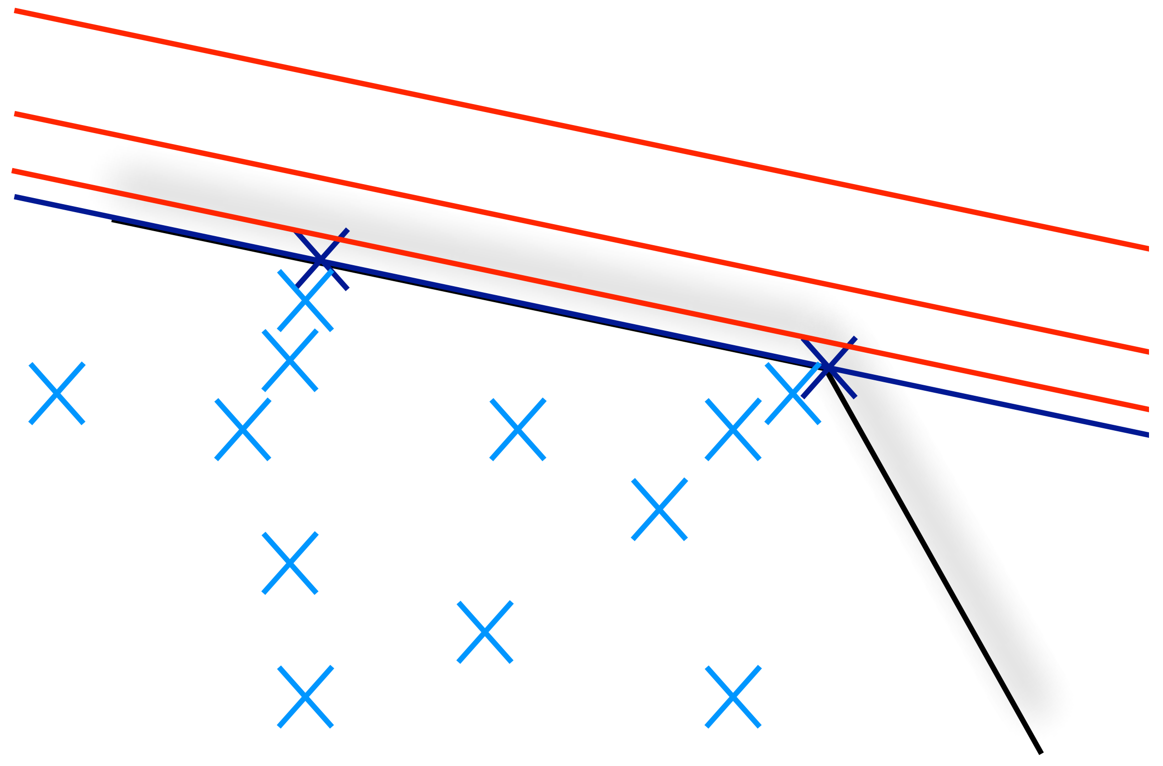
[El-Showk Paulos] [Mazac] [Mazac Paulos] [Paulos Zan]



Why?

Want to understand **"extremal"** solutions

[El-Showk Paulos] [Mazac] [Mazac Paulos] [Paulos Zan]



Recipe

basis of functionals $\theta_n[\,-\,] = \oint dz\, f_n(z)\,(\,-\,)$

1. **Complete**
2. **Dual** to a "sparse" spectrum: $\theta_n[g_{\Delta_m}] = \delta_{mn}$

$\longrightarrow$

"extremal" solution

$$F_\Delta^{\Delta_n} = g_\Delta - \sum_n \theta_n[g_\Delta]\, g_{\Delta_n}$$

Result

Explicit bases of functionals θ_n

dual to any $\Delta_n = 2\Delta_\phi + 2n + \gamma_n$

$\sim n^{-\epsilon}$ for large n

analytic for large n

Result

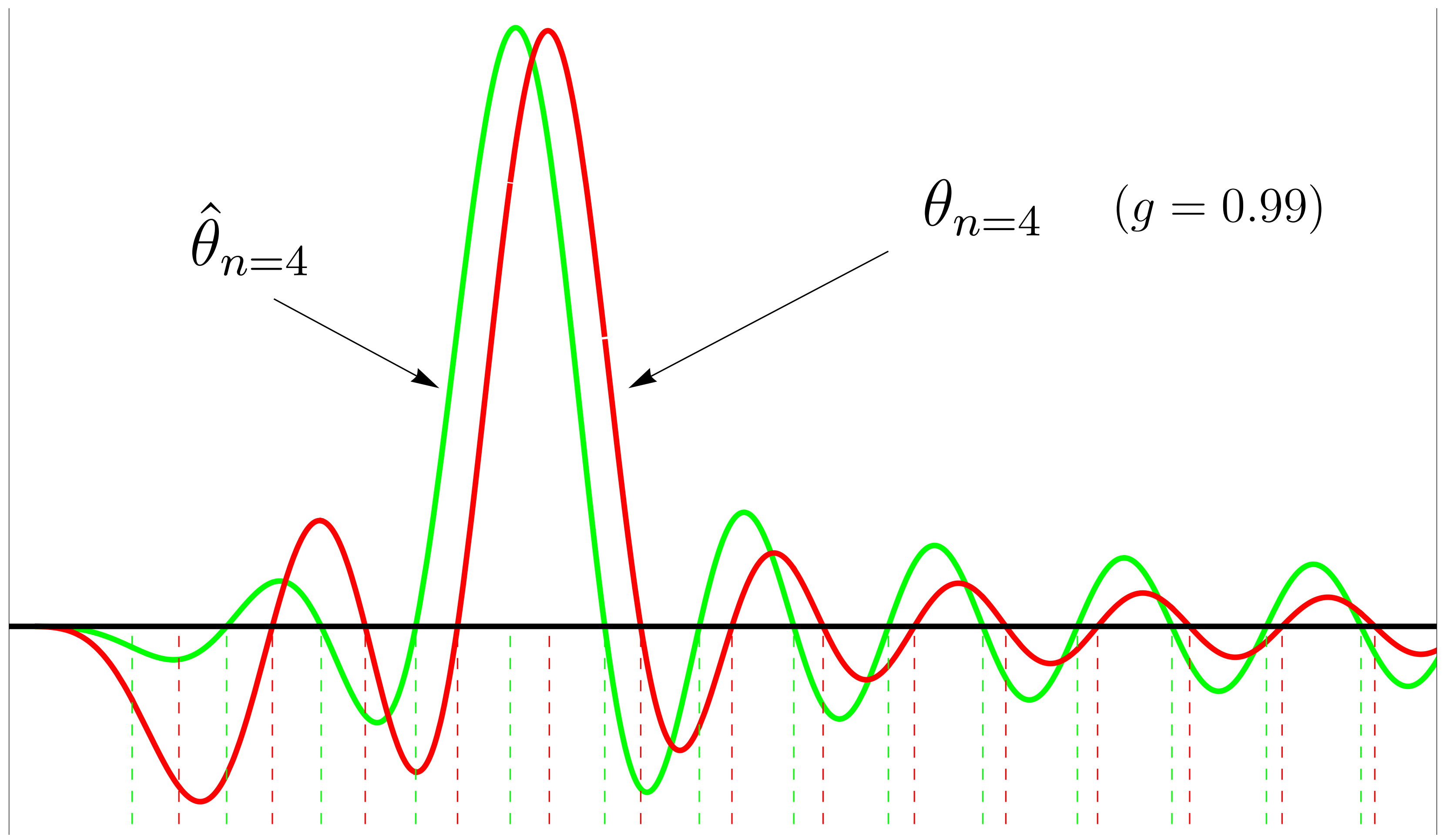
Explicit bases of functionals θ_n

dual to any $\Delta_n = 2\Delta_\phi + 2n + \gamma_n$

$\sim n^{-\epsilon}$ for large n

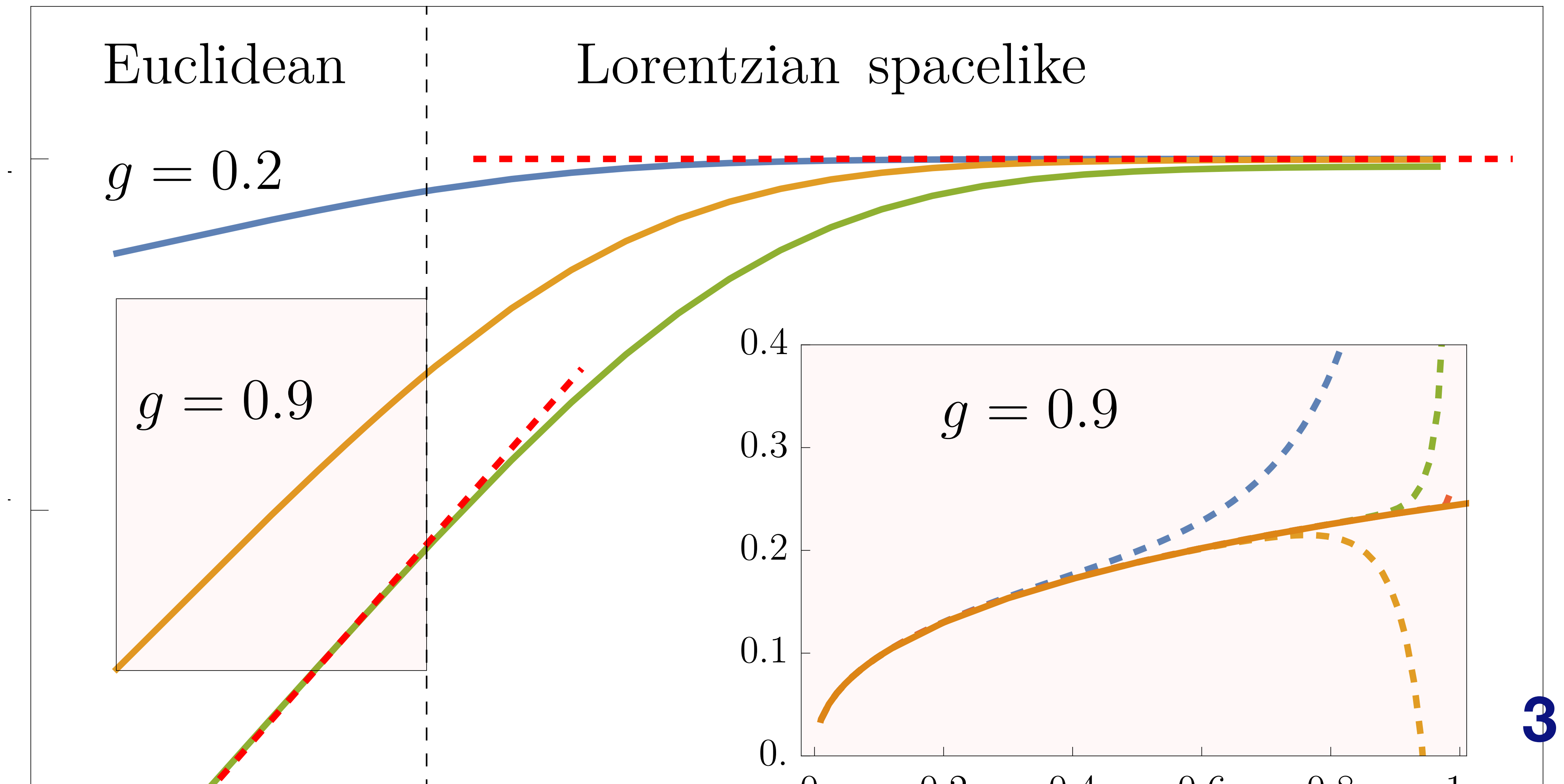
analytic for large n

- Explicit functional actions: $$\theta_n[g_\Delta] = \prod_{m\neq n}\left(\frac{\Delta - \Delta_m}{\Delta_n - \Delta_m}\right)$$



- Interacting "extremal" solutions:

$$F^{\Delta_n}_{\Delta} = g_\Delta - \sum_n \left[\prod_{m\neq n}\left(\frac{\Delta - \Delta_m}{\Delta_n - \Delta_m}\right)\right] g_{\Delta_n}$$



How?

$$\theta_n[F] = \oint dz\, f_n(z)\, F(z) = (f_n, \text{Disc}(F))$$

Dual

$$f_n \qquad\qquad\qquad\qquad \text{Disc}(F)$$

$$\Cap \qquad\qquad\qquad\qquad \Cap$$

$$\{\text{real-analytic on } (1,\infty)\} \qquad \{\text{hyperfunctions on } (1,\infty)\}$$

test functions ***distributions***

How?

$$\theta_n[F] = \oint dz\, f_n(z)\, F(z) = (f_n, \text{Disc}(F))$$

Dual

$$\begin{array}{ccccc} f_n & & & & \text{Disc}(F) \\ \Cap & & & & \Cap \\ \{\text{real-analytic on } (1,\infty)\} & \subset & L^2 & \subset & \{\text{hyperfunctions on } (1,\infty)\} \end{array}$$

test functions ***distributions***

Constructed Schauder bases $\{f_n\}$ $\xleftrightarrow{\textbf{Dual}}$ $\{\text{Disc}(g_{\Delta_n})\}$

Paley-Wiener theorem: $\theta_n(\Delta) := \theta_n[g_\Delta]$ entire function

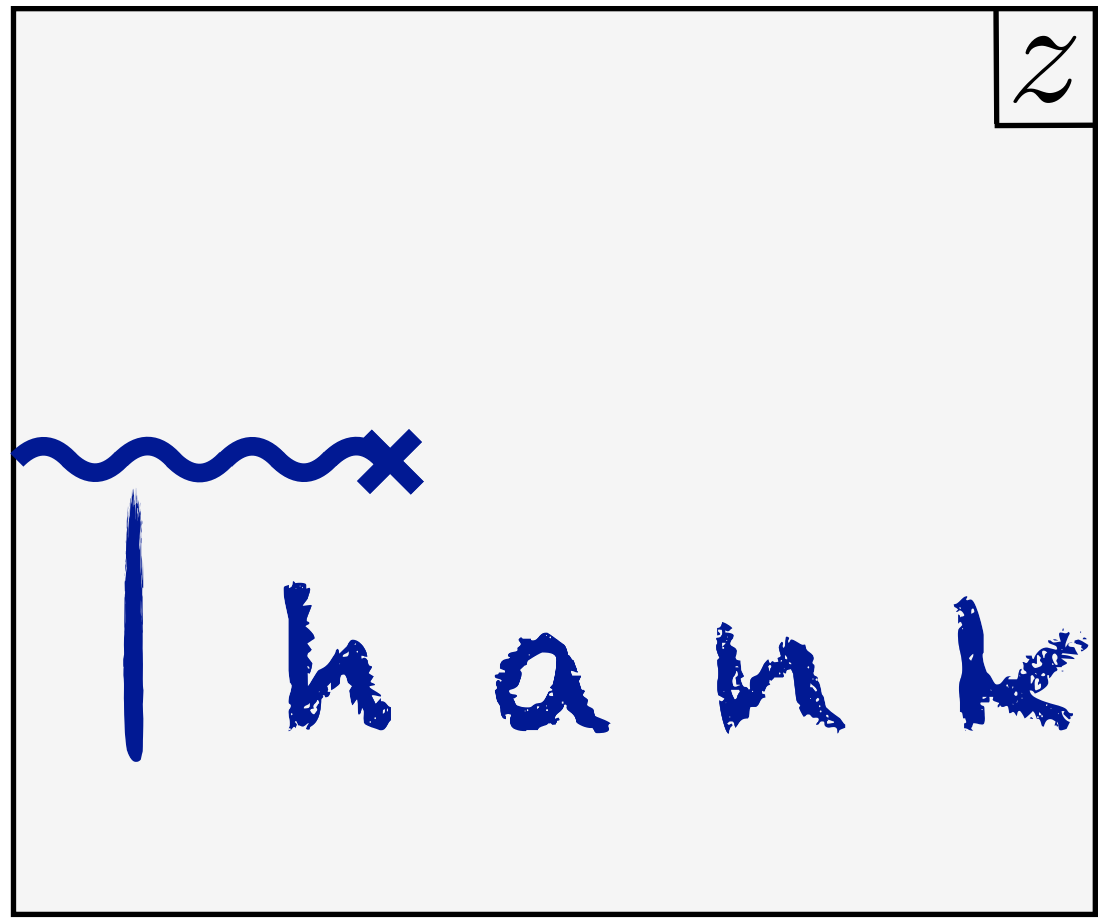
$$= \prod_{m \neq n} \left(\frac{\Delta - \Delta_m}{\Delta_n - \Delta_m} \right)$$

Future directions…

1. *Numerics: Crossing+Locality*

2. *Analytic extremal solutions of 1d crossing equation?*

3. *Modified locality with gauge or gravitational dressing*



Thank you!

How the Hilbert space of two-sides black holes factorise?

Guanda Lin

UC Berkeley

Gong Show Talk, Strings2024

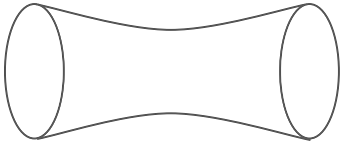
Based on an upcoming work with Jan Boruch, Luca Iliesiu and Cynthia Yan

Motivation

What is the Hilbert space of a two-sided black hole?

- In AdS/CFT, how does gravity know factorisation?

AdS $\Longleftrightarrow$ **CFT**



The factoriSation puzzle

$\mathcal{H}_{\mathrm{bulk}}$ $\Downarrow$ **?** $\mathcal{H}_L \otimes \mathcal{H}_R$

- More broadly, what is the algebra type for one-sided observables?

QFT	**Pert. Grav.**	**Full Quant. Grav.**
Type-III	Type-II	**Type I?**

Main result

- "Puzzle" mostly at the perturbative level

$$\mathcal{H}_{\text{pert.}} \sim \mathcal{H}_{\text{grav.}} \times \mathcal{H}_{\text{mat.}}$$

- We prove that **non-perturbative corrections** will provide resolutions

[non-pert.] $$\mathrm{Tr}_{\mathcal{H}_{\text{bulk}}}(k_L k_R) = \mathrm{Tr}_{\mathcal{H}_L}(k_L) \times \mathrm{Tr}_{\mathcal{H}_R}(k_R)$$

$$\Downarrow$$

[non-pert.] $$\mathcal{H}_{\text{bulk}} = \mathcal{H}_L \otimes \mathcal{H}_R$$

- In particular, **Wormhole contributions** to gravitational path integral are crucial

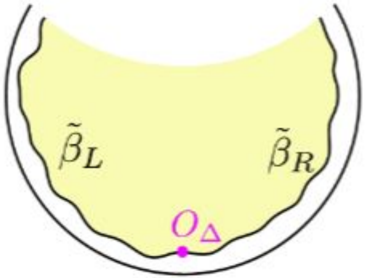
Set-up

- JT+matter

break Hamiltonian constraint

$$I = -S_0\chi(\mathcal{M}) - \frac{1}{2}\left(\int_{\mathcal{M}} \phi(R+2) + 2\int_{\partial\mathcal{M}} \phi_b(K-1)\right) + I_{\text{matter}}$$

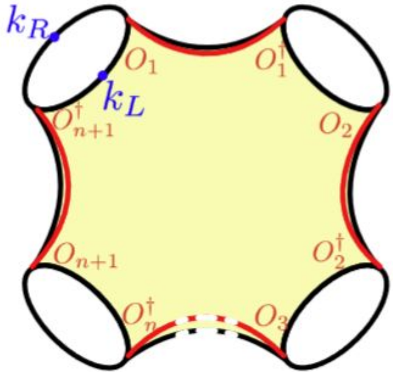
- Basis of Hilbert space

$|q_i\rangle = O_i|HH\rangle =$ 

$\mathcal{H}_{\text{bulk}} = \text{Span}\{|q_i\rangle, i = 1,\ldots,K\}$

- The bulk trace and replica wormholes

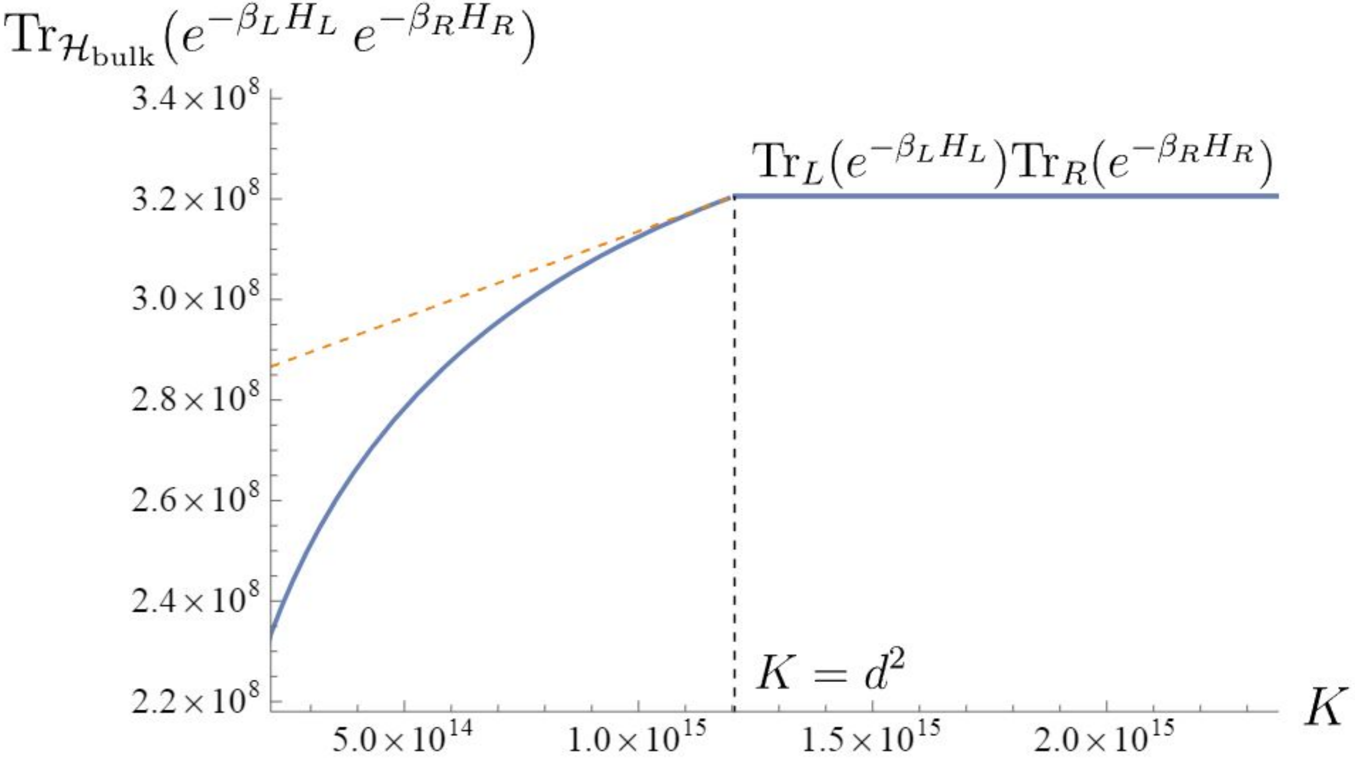
$$\text{Tr}_{\mathcal{H}_{\text{bulk}}}(k_L\, k_R) = \lim_{n\to -1} \langle q_i|q_j\rangle^n \langle q_i|k_L k_R|q_j\rangle$$



Probing factorisation

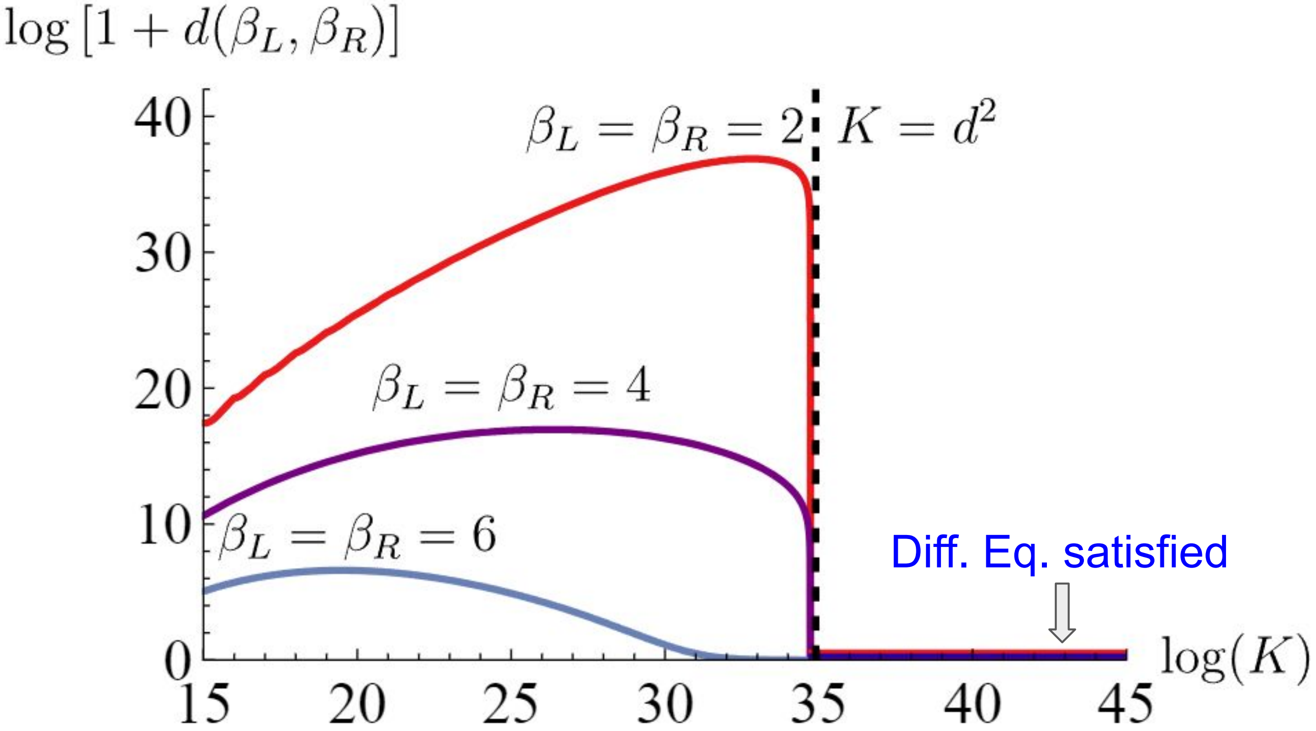
- **Bulk trace** $\mathrm{Tr}_{\mathcal{H}_{\rm bulk}}(k_L k_R) = \mathrm{Tr}_{\mathcal{H}_L}(k_L) \times \mathrm{Tr}_{\mathcal{H}_R}(k_R)$

  more precisely $Z_{\rm bulk}(\beta_L, \beta_R) = \mathrm{Tr}_{\mathcal{H}_{\rm bulk}}(e^{-\beta_L H_L}\, e^{-\beta_R H_R})$



Probing factorisation

- **Differential equation** $\overline{d(\beta_L, \beta_R)} = \overline{d(\beta_L, \beta_R)^2} = 0$



$$d(\beta_L, \beta_R) = Z_{\text{bulk}}(\beta_L, \beta_R)\partial_{\beta_L}\partial_{\beta_R} Z_{\text{bulk}}(\beta_L, \beta_R) - \partial_{\beta_L} Z_{\text{bulk}}(\beta_L, \beta_R)\partial_{\beta_R} Z_{\text{bulk}}(\beta_L, \beta_R)$$

Thank you!

Extra slides

Barred v.s. unbarred

Leading order in K

$$\overline{\mathrm{Tr}_{\mathcal{H}_{bulk}(K)}(k_L k_R)} = \overline{\mathrm{Tr}_{\mathcal{H}_L}(k_L)\,\mathrm{Tr}_{\mathcal{H}_R}(k_R)} \Leftrightarrow \mathrm{Tr}_{\mathcal{H}_{\mathrm{bulk}}(K)}(k_L k_R) = \mathrm{Tr}_{\mathcal{H}_L}(k_L)\,\mathrm{Tr}_{\mathcal{H}_R}(k_R)$$

In general, the barred one is correct because of the non-trivial statistics of energy levels

Dimension of the Hilbert Space

$$\dim_{\mathcal{H}_{\mathrm{bulk}}} = \overline{d^2} = \int_{\mathcal{E}} \rho(E_L)\rho(E_R) \quad \text{with } \mathcal{E} \text{ energy cut-off}$$

Note that even with a cut-off, $\mathcal{H}_{\mathrm{pert.}}$ is infinite dimensional

Irrelevance of UV divergence and higher dimensional generalization

(1) very general symmetry property argument for diff. eq.

(2) the matter supported wormholes are saddles and not the ones causing UV div. In JT+matter

K independence

K is a parameter in the technique, irrelevant to the property of the actual $\mathcal{H}_{\mathrm{bulk}}$

checked $1/K$ expansions, no effect; expect exp. small # of outliers in which $\mathcal{H}_{\mathrm{bulk}}$ is not spanned

Basis independence

checked using operators with different conformal dimensions, length basis, etc

About gauge symmetry

Want no energy degeneracy so that the bulk trace factorisation tells about factorised basis

No boundary global symmetry and no bulk gauge symmetry

The end of talk brane

Massless Lifshitz Field Theory for Arbitrary z

Himanshu Parihar

National Center for Theoretical Sciences,
National Tsing-Hua University
Taiwan

In collaboration with Jaydeep Kumar Basak, Adrita Chakraborty,
Chong-Sun Chu and Dimitrios Giataganas

JHEP 05 (2024) 284 [2312.16284]

Strings 2024
CERN, Switzerland

Lifshitz field theory

- LFTs are a class of non-relativistic field theories which are spatially isotropic, homogeneous and admits the scaling symmetry

$$t \to \lambda^z t, \quad x^i \to \lambda x^i, \quad \lambda > 0.$$

- For $z = 2$, Lifshitz scalar field theory in (2+1) dimensions known as Quantum Lifshitz model (QLM) describe the critical point of the well-known Rokhsar-Kivelson Quantum dimer model. [Moessner, Sondhi and Fradkin '01] [Ardonne, Fendley and Fradkin '04]
- Various entanglement measures such as entanglement entropy [Fradkin, Moore, Hsu, Thorlacius....], entanglement negativity [Angel-Ramelli et al. '20], reflected entropy and Markov gap [Berthiere, Chen and Chen '23] have been studied mostly for integer z.
- We employ the notion of fractional derivatives to study the massless Lifshitz theory for arbitrary values of z in any dimensions.

Massless Lifshitz scalar theory and Lifshitz ground state

- Consider the action for the massless Lifshitz scalar field theory in (1+1)-dimensions for arbitrary $z > 1$

$$S = \frac{1}{2} \int dt dx \left[(\partial_t \phi)^2 - \kappa^2 (\nabla_x^z \phi)^2 \right].$$

- In our work, we use the following definition of fractional derivative ∇_x^z

$$\nabla_x^z e^{ikx} \equiv (ik)^z e^{ikx}.$$

- Then, the fractional derivative of any arbitrary function can be obtained using the Fourier analysis with appropriate choice of integral contour

$$\nabla_x^z F(x) = \int_C \mathcal{F}(k) (ik)^z e^{ikx} dk.$$

- The ground state of the Lifshitz theory is given by

$$|\Psi_0\rangle = \frac{1}{\sqrt{\mathcal{Z}}} \int \mathcal{D}\phi \, e^{-S_{\rm cl}[\phi]/2} |\phi\rangle, \quad S_{\rm cl}[\phi] = \kappa \int \left(\nabla_x^{\frac{z}{2}} \phi \right)^2 dx.$$

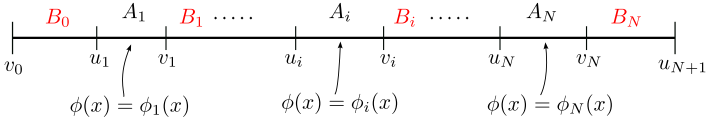
- This ground state takes the form of RK vacuum, it is given by a superposition of quantum states with a quantum mechanical amplitude $c[\phi] \propto e^{-S_{\rm cl}[\phi]/2}$.
- The propagator of the theory is given by

$$K(\phi_i, \phi_f; x_i, x_f) = \int_{\phi(x_i)=\phi_i}^{\phi(x_f)=\phi_f} \mathcal{D}\phi \exp\left(-\kappa \int_{x_i}^{x_f} \left(\nabla_x^{\frac{z}{2}} \phi\right)^2 dx\right).$$

- Usually the integral can be evaluated by integrating out the fluctuations around the classical solution ϕ_c and expressed as

$$K(\phi_i, \phi_f; l) = \sqrt{\frac{\gamma}{\pi l^{z-1}}} e^{-\gamma(\phi_f - \phi_i)^2/l^{z-1}}.$$

- Consider a subsystem $A \equiv \bigcup_{i=1}^{N} A_i$



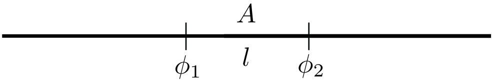
- The trace $\mathcal{Z}_n \equiv \int \mathcal{D}\phi_A (\rho_A^n)_{\phi_A, \phi_A}$ is given by

$$\mathcal{Z}_n = \frac{1}{\mathcal{Z}^n} \int_{-\infty}^{\infty} d\alpha_1 d\beta_1 \cdots d\alpha_N d\beta_N \prod_{i=1}^{N} K^n(u_i, v_i) \prod_{i=1}^{N} K^n(v_i, u_{i+1}).$$

Entanglement entropy

- For a finite subsystem A of length l in an infinite system, the trace $\mathcal{Z}_n$ is given by

$$\mathcal{Z}_n = \mathcal{Z}^{-n} \int d\phi_1 \int d\phi_2 K(\phi_1, \phi_2; l)^n .$$



- Using the form of the propagator, the Rényi entropy may be expressed as

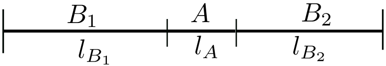
$$S_n(A) = \frac{z-1}{2} \log \frac{l}{\epsilon} + \frac{c_n}{2} .$$

- This is different from the usual case of a conformal vacuum where the UV parts are proportional with a nontrivial n-dependent coefficient

$$[S_n(A)]_{\mathrm{UV}} = \frac{1}{2}(1 + 1/n)[S(A)]_{\mathrm{UV}} .$$

- We observe that the Lifshitz vacuum is different from the vacuum of the CFT.

Mutual information



- The mutual information between B_1 and B_2 is given by

$$I(B_1 : B_2) = \frac{1}{2} \log \frac{\left(l_{B_1}^{z-1} + l_A^{z-1}\right)\left(l_{B_2}^{z-1} + l_A^{z-1}\right)}{l_A^{z-1}\left(l_{B_1}^{z-1} + l_A^{z-1} + l_{B_2}^{z-1}\right)} = \frac{1}{2} \log \frac{1}{1-\tilde{\eta}}.$$

- Here the cross-ratio $\tilde{\eta}(z)$ is given by

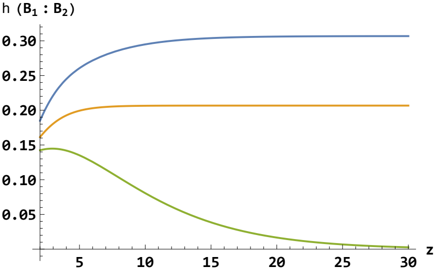
$$\tilde{\eta}(z) := \frac{\left(l_{B_1} l_{B_2}\right)^{z-1}}{\left(l_{B_1}^{z-1} + l_A^{z-1}\right)\left(l_{B_2}^{z-1} + l_A^{z-1}\right)}.$$

- When $l_A \ll l_{B_i}$, then $\tilde{\eta}(z) \to 1$. It happens same for $l_A < l_{B_i}$ and $z \gg 1$.
- The mutual information maximizes in these cases which is expected since the interactions of the theory have increasing range while the length l_A is small compared to the rest subsystems sizes.

Reflected entropy and Markov gap

- The Markov gap for the configuration of disjoint intervals can be obtained as

$$h(B_1 : B_2) = \frac{1}{\sqrt{1-\tilde{\eta}}} \log\left(\frac{1+\sqrt{1-\tilde{\eta}}}{\sqrt{\tilde{\eta}}}\right) - \log\left(\frac{2(1-\tilde{\eta})}{\sqrt{\tilde{\eta}}}\right).$$



- For $l_A \leq \min\{l_{B_1}, l_{B_2}\}$, $h(B_1 : B_2)$ increases up to a constant value whereas for $l_A > \min\{l_{B_1}, l_{B_2}\}$, $h(B_1 : B_2)$ decays to zero.
- We observe that with increasing degrees of anisotropy of the Lifshitz field theory, the tripartite entanglement can be enhanced or completely destroyed depending on the sizes of the partitions.

THANK YOU!

Interacting fields at spatial infinity

Priyadarshi Paul

International Centre for Theoretical Sciences (ICTS-TIFR), Bengaluru

Strings 2024, CERN

4th June, 2024

References

Anupam A.H

Athira P.V.

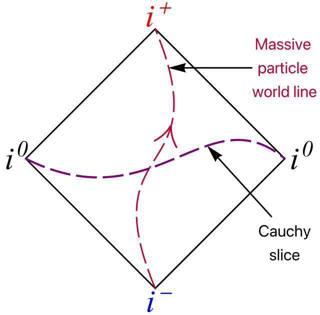
Suvrat Raju

arXiv:2405.20326 with Anupam A.H , Athira P.V and Suvrat Raju

Massive fields at spatial infinity $\longrightarrow$ Holography

Holography of information : "In any theory of quantum gravity in flat space (massless fields) & AdS, information that is available in the bulk of a Cauchy slice is also available near its boundary."

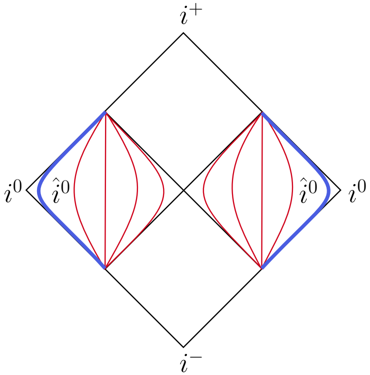
[*Laddha, Prabhu, Raju, Shrivastava*; 2002.02448]
[*Raju*; 2012.05770]



Massive particles go from i^- to i^+ (not natural for holography).

i^0 is the boundary of the Cauchy slice.

Blow up of spatial infinity ($\hat{i}^0$)



Take de Sitter slicing of flat space.

$$t = \rho \sinh \tau \quad , \quad r = \rho \cosh \tau$$

$$ds^2 = d\rho^2 + \rho^2 \underbrace{\left(-d\tau^2 + \cosh^2 \tau \, d\Omega^2\right)}_{dS_3 \text{ metric}}$$

The slice at $\rho \to \infty$ is $\hat{i}^0$ (blue slice).

Free field theory at blow up of spatial infinity ($\hat{i}^0$)

Massive scalar field decays as

$$\phi \to \rho^{-\frac{3}{2}} e^{-m\rho}.$$

Define extrapolated boundary operators

$$\mathcal{Z}(\tau, \Omega) = \lim_{\rho \to \infty} \sqrt{\frac{2}{\pi}}\, \rho \sqrt{m\rho}\, e^{m\rho}\, \phi(\rho, \tau, \Omega).$$

We smear the fields with smearing function analytic in $\mathrm{Im}[\tau] \in (-\frac{\pi}{2}, \frac{\pi}{2})$.

$$\mathcal{Z}(g) = \int [d\mu]_{\tau,\Omega}\, g(\tau, \Omega)\, \mathcal{Z}(\tau, \Omega)$$

In free theory, smeared two point function $\langle \mathcal{Z}(g)\mathcal{Z}(f) \rangle$ is well defined.

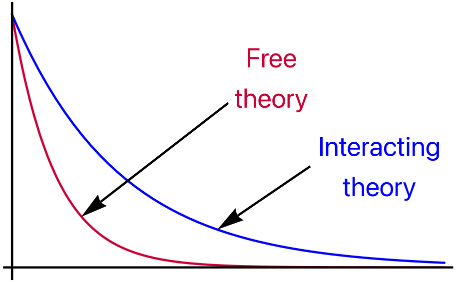
[*Laddha, Prabhu, Raju, Shrivastava*; 2207.06406]

Interacting field theory : Wightman functions

For perturbation theory we choose Wightman functions (as we are smearing over time).

Interacting Wightman correlators can have slowly decaying parts than $e^{-m\rho}$.

$$\int [d\mu]_{\tau,\Omega}\, g(\tau,\Omega)\, W^{\Psi_1,\Psi_2}(\{\rho,\tau,\Omega\},\ldots) \to \int da\, G(a)\, \rho^{-\frac{3}{2}} e^{-a\rho}$$

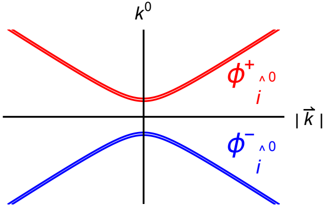


On-shell Wightman functions

Proposal : Extract the on-shell part of the bulk Wightman functions, which has correct extrapolate limit.

$$W^{\Psi_1,\Psi_2}(k_1,\dots,k_n) = G^{\Psi_1,\Psi_2}(k_1,\dots,k_n)(2\pi)\delta(k_1^2+m^2)\dots(2\pi)\delta(k_n^2+m^2)+\dots$$

In the momentum space Feynman rules, replace all the external propagators with their on-shell parts.



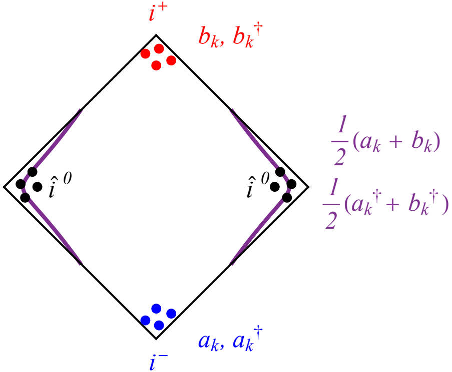
Extract on-shell part $\phi^+_{\hat{i}0}(\vec{k})$ & $\phi^-_{\hat{i}0}(\vec{k})$ from the single Heisenberg operator $\Phi(k)$. Smeared field can be written as

$$\mathcal{Z}(g) = \int \frac{d^3\vec{k}}{(2\pi)^3 2\omega_k}\left(\phi^+_{\hat{i}0}(\vec{k})\tilde{g}^+(\vec{k}) + \phi^-_{\hat{i}0}(\vec{k})\tilde{g}^-(\vec{k})\right).$$

Algebra at $\hat{i}^0$

The operators at $\hat{i}^0$ are average of “in” and “out” operators.

$$\phi^+_{\hat{i}^0}(k) = \frac{1}{2}(a_k + b_k)$$
$$\phi^-_{\hat{i}^0}(k) = \frac{1}{2}(a^\dagger_k + b^\dagger_k)$$



[*Caron-Huot, Giroux, Hannesdottir, Mizera*; 2308.02125]

Sample computations

4 point vacuum correlator :

$$\langle\Omega|\phi^{+}_{\hat{j}0}(k_1)\phi^{+}_{\hat{j}0}(k_2)\phi^{-}_{\hat{j}0}(k_3)\phi^{-}_{\hat{j}0}(k_4)|\Omega\rangle_{\text{connected}} = -\frac{1}{2}\,\mathrm{Im}\left(T_{\vec{k}_1\vec{k}_2\leftarrow\vec{k}_3,\vec{k}_4}\right)$$

5 point vacuum correlator :

$$\langle\Omega|\phi^{+}_{\hat{j}0}(k_1)\phi^{+}_{\hat{j}0}(k_2)\phi^{+}_{\hat{j}0}(k_3)\phi^{-}_{\hat{j}0}(k_4)\phi^{-}_{\hat{j}0}(k_5)|\Omega\rangle_{\text{connected}}$$
$$= -\frac{1}{2}\,\mathrm{Im}\left(T_{\vec{k}_1\vec{k}_2\vec{k}_3\leftarrow\vec{k}_4,\vec{k}_5}\right) + \frac{1}{4}\,\mathrm{Re}\left(\sum_X (T_{X\leftarrow\vec{k}_1,\vec{k}_2})^*\,T_{\vec{k}_3,X\leftarrow\vec{k}_4,\vec{k}_5}\right)$$

$$\langle\Omega|\phi^{+}_{\hat{j}0}(k_1)\phi^{+}_{\hat{j}0}(k_2)\phi^{-}_{\hat{j}0}(k_3)\phi^{-}_{\hat{j}0}(k_4)\phi^{-}_{\hat{j}0}(k_5)|\Omega\rangle_{\text{connected}}$$
$$= -\frac{1}{2}\,\mathrm{Im}\left(T_{\vec{k}_1\vec{k}_2\leftarrow\vec{k}_3,\vec{k}_4,\vec{k}_5}\right) + \frac{1}{4}\,\mathrm{Re}\left(\sum_X (T_{\vec{k}_3,X\leftarrow\vec{k}_1,\vec{k}_2})^*\,T_{X\leftarrow\vec{k}_4,\vec{k}_5}\right).$$

where,

$$S = 1 + i\,T$$

Outlook

- Holography for asymptotically flat space when massive fields couple with dynamical gravity ?

- Relation to celestial CFT amplitudes ?

- Bootstrapping correlators at $\hat{i}^0$ and derive consequences about bulk theory ?

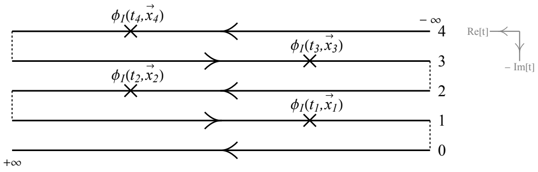
- Relation to the recent understanding of S matrix from the perspective of path integral by *Jain, Kundu, Minwalla, Parrikar, Prabhu, Shrivastava* [2311.03443] ?

Thank you

For details, please look at

arXiv:2405.20326

Backup slide 1: Standard Wightman function Feynman rules



- Consider propagator $D_{ij}(k)$ between two points which are at ith contour & jth contour. Momentum k is flowing from j to i.

- Propagator rules :

$$T_{ij}(k) = \frac{-i}{k^2 + m^2 - i\epsilon} + \ldots, \qquad i = j\,;\ (n-i) = \text{even}$$

$$\overline{T}_{ij}(k) = \frac{i}{k^2 + m^2 + i\epsilon} + \ldots \qquad i = j\,;\ (n-i) = \text{odd}$$

$$W_{ij}(k) = 2\pi\theta(k^0)\delta(k^2 + m^2) + \ldots \qquad i < j$$

$$\overline{W}_{ij}(k) = 2\pi\theta(-k^0)\delta(k^2 + m^2) + \ldots \qquad i > j$$

- Vertex Factor is $(-iH_I)$ for $(n-i)$ is even, $(+iH_I)$ for $(n-i)$ odd.

Backup slide 2 : Modified Feynman Rules at $\hat{i}^0$

- If we want to calculate

$$W^{\Psi_1,\Psi_2}_{\hat{i}^0}(k_1 \ldots k_n) = \langle \Psi_1 | \phi_{\hat{i}^0}(k_1) \ldots \phi_{\hat{i}^0}(k_n) | \Psi_2 \rangle$$

directly, we put external time-ordered and anti-time-ordered propagators on-shell.

$$\boxed{\frac{\pm i}{k^2 + m^2 \pm i\epsilon} = \pm i \left\{ \text{P.V} \left(\frac{1}{k^2 + m^2} \right) \mp i\pi\delta(k^2 + m^2) \right\}}$$

- New Feynman rules for the external propagators

$$\begin{array}{ll}
\mathbb{T}_{ij}(k) = \pi\delta(k^2 + m^2) & i = j\,;\ (n - i) = \text{even} \\
\overline{\mathbb{T}}_{ij}(k) = \pi\delta(k^2 + m^2) & i = j\,;\ (n - i) = \text{odd} \\
\mathbb{W}_{ij}(k) = 2\pi\theta(k^0)\delta(k^2 + m^2) & i < j \\
\overline{\mathbb{W}}_{ij}(k) = 2\pi\theta(-k^0)\delta(k^2 + m^2) & i > j
\end{array}$$