

THE M-THEORY GEOMETRY

OF GENERALISED TORIC POLYGONS

STRINGS 2024 Gong Show - 4<sup>th</sup> June 2024

Guillermo Arias-Tamargo

Imperial College London

Based on 2403.09776 with S. Franco & D. Rodríguez-Gómez

## 5d SCFTs FROM STRING THEORY

- M-theory geometric engineering
- Brane webs in type IIB
- Duality: M-th on  $T^2 \leftrightarrow$  IIB on  $S^1$

# 5d SCFTs FROM STRING THEORY

- M-theory geometric engineering

- Brane webs in type IIB

- Duality: M-th on  $T^2 \leftrightarrow$  IIB on  $S^1$  (Leung, Vafa '97)

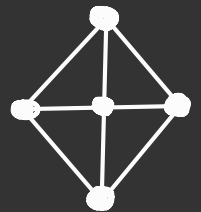


Toric case  $\rightarrow$

Combinatorial data  
"toric diagram"

$\nearrow$  Brane web

$\searrow$  Toric geometry



- Brane manipulations in webs with 7-branes

# 5d SCFTs FROM STRING THEORY

- M-theory geometric engineering

- Brane webs in type IIB

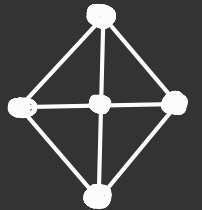
- Duality: M-th on  $T^2 \leftrightarrow$  IIB on  $S^1$  (Leung, Vafa '97)



Toric case  $\rightarrow$  Combinatorial data  
"toric diagram"

Brane web

Toric geometry



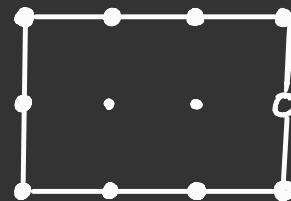
- Brane manipulations in webs with 7-branes

- Hanany-Witten moves

- Higgs branch flows



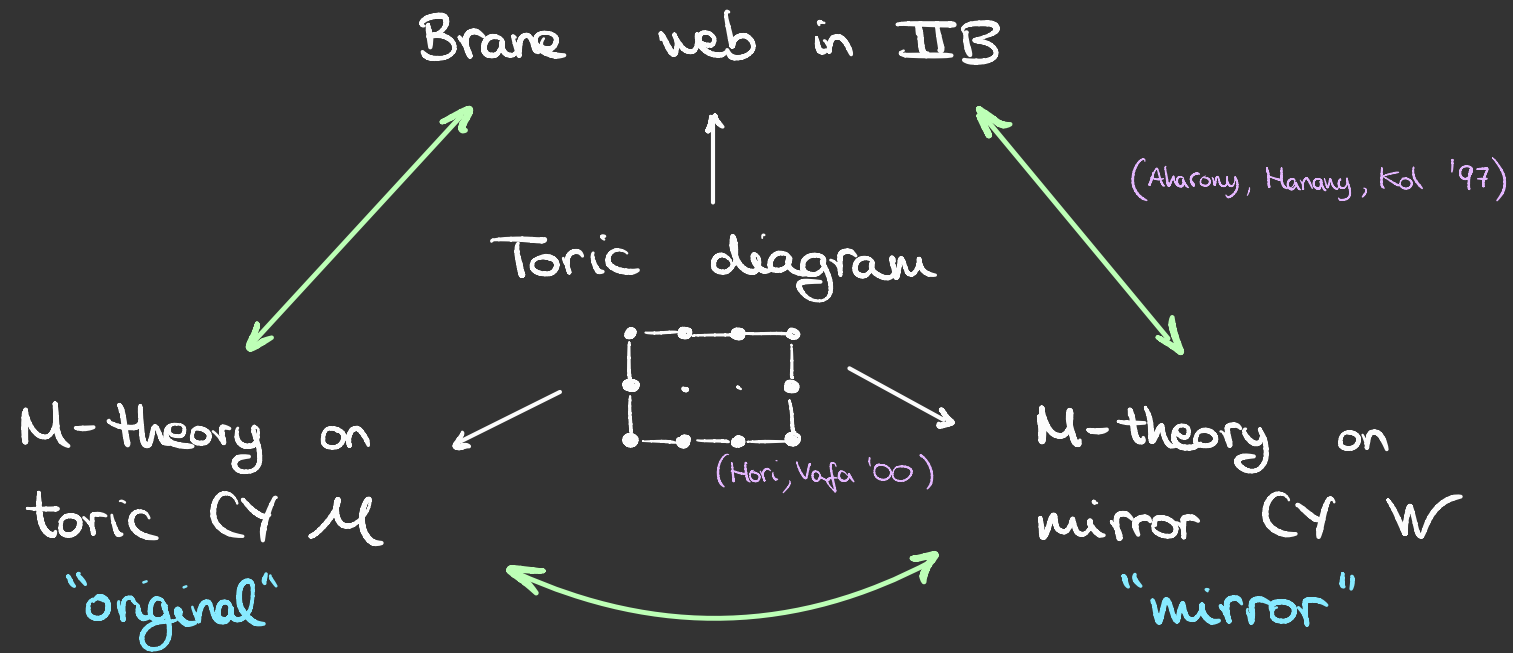
Generalised Toric Polygon



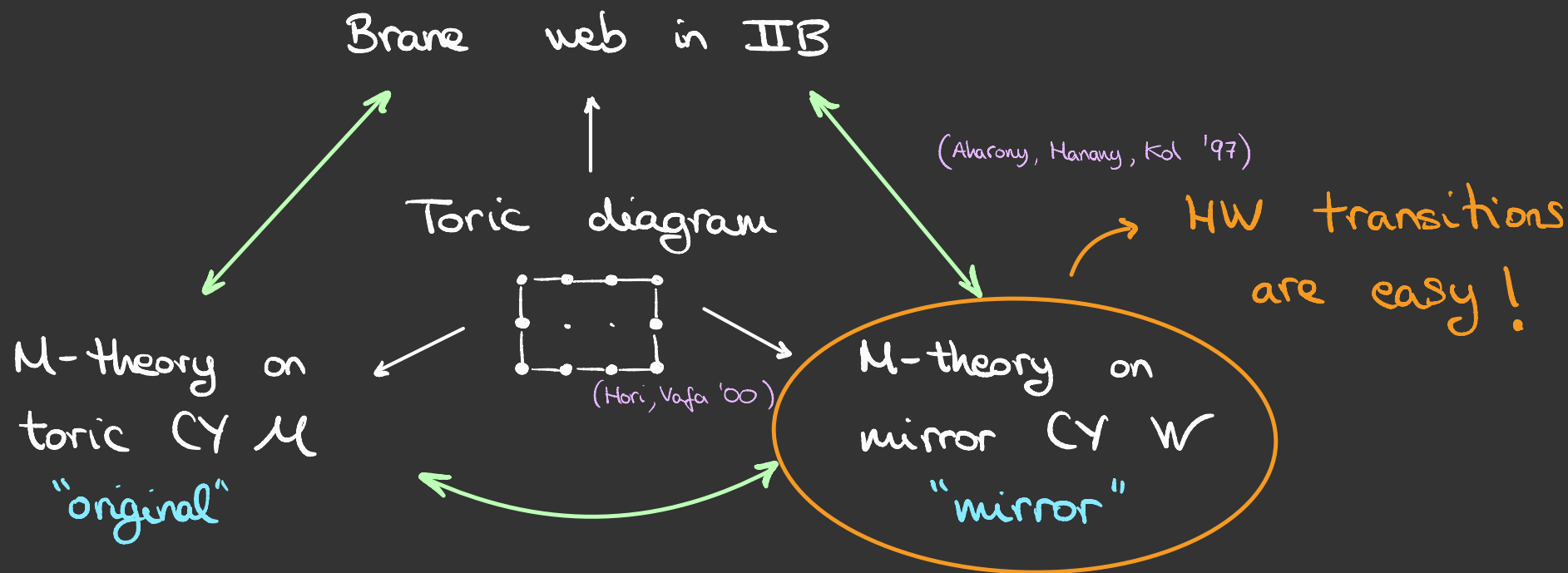
(Benini, Bonanuti, Tachikawa '09)



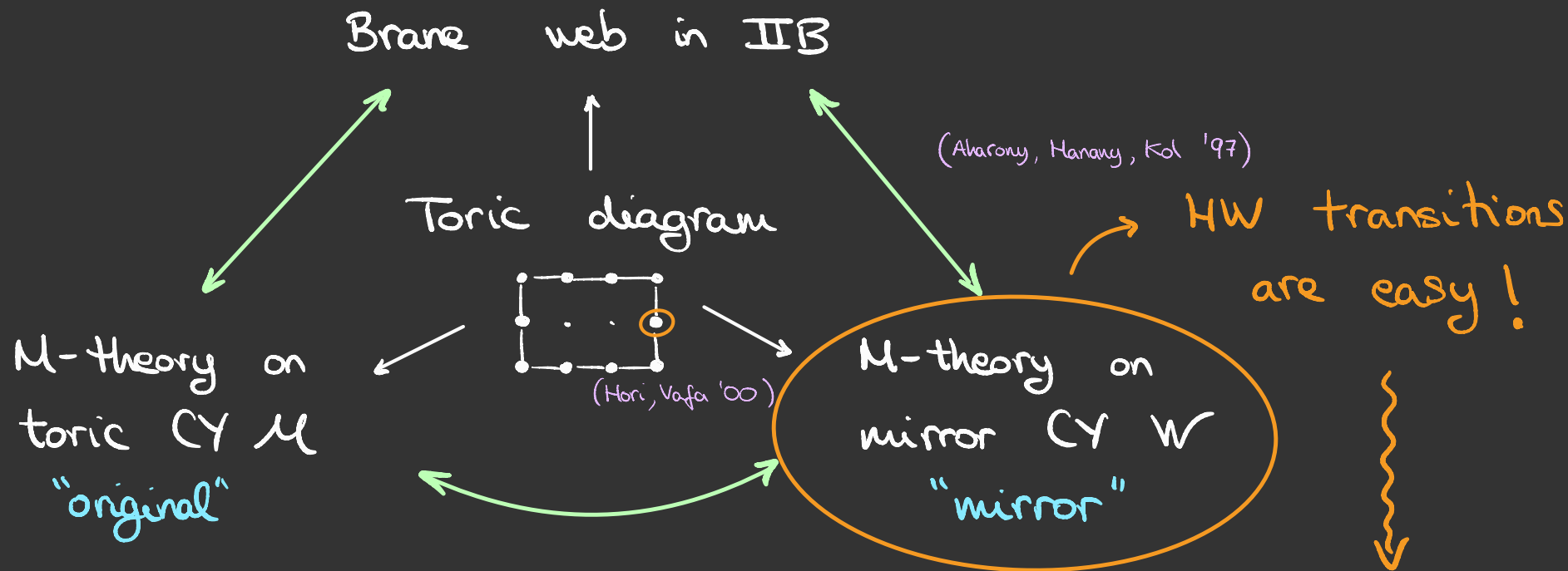
# GENERAL IDEA



# GENERAL IDEA

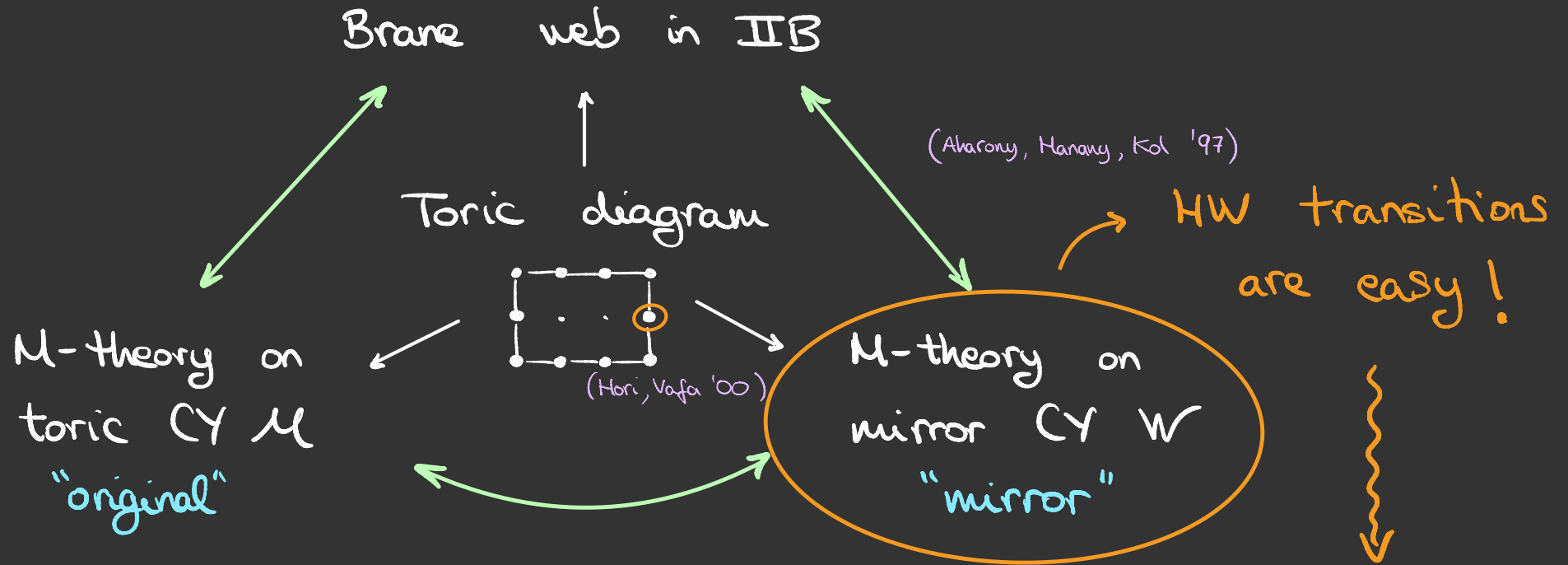


# GENERAL IDEA



White dot = Constraint for complex structure moduli

# GENERAL IDEA



Constraint for Kähler moduli

(Borger, Collinucci, Schafer-Nameki '23)

White dot = Constraint for complex structure moduli

- Connection to mutations in math  $\rightarrow$  invariants

(Franco, Seiberg '23; Cremonesi, Sca '23)

# PUNCHLINE AND OUTLOOK

## PUNCHLINE AND OUTLOOK

- GTP geometries as non-isolated "frozen" singularities

## PUNCHLINE AND OUTLOOK

- GTP geometries as non-isolated "frozen" singularities
- Conserved quantities under mutations

- Period  $\pi(t) = \int \frac{dx dy}{xy} \frac{1}{1 - tP(x,y)}$  (Akhtar, Coates, Galkin, Kasprzyk, ... '12-'22)

$$\pi_{\Delta_1} = \pi_{\Delta_2} \Leftrightarrow \Delta_1, \Delta_2 \text{ connected by mutation}$$

↪ Useful tool for classification of brane webs

## PUNCHLINE AND OUTLOOK

- GTP geometries as non-isolated "frozen" singularities
- Conserved quantities under mutations

- Period  $\pi(t) = \int \frac{dx dy}{xy} \frac{1}{1 - tP(x,y)}$  (Akhtar, Coates, Galkin, Kasprzyk, ... '12-'22)

$\pi_{\Delta_1} = \pi_{\Delta_2} \Leftrightarrow \Delta_1, \Delta_2$  connected by mutation  
↳ Useful tool for classification of brane webs

- General formula for (non-dynamical) deformation corresponding to HW move  $\rightarrow$  T-varieties (Iten '12; Coates, Heuberger, Kasprzyk '22)



## PUNCHLINE AND OUTLOOK

- GTP geometries as non-isolated "frozen" singularities
- Conserved quantities under mutations

- Period  $\pi(t) = \int \frac{dx dy}{xy} \frac{1}{1 - tP(x,y)}$  (Akhtar, Coates, Galkin, Kasprzyk, ... '12-'22)

$\pi_{\Delta_1} = \pi_{\Delta_2} \Leftrightarrow \Delta_1, \Delta_2$  connected by mutation  
↳ Useful tool for classification of brane webs

- General formula for (non-dynamical) deformation corresponding to HW move  $\rightarrow$  T-varieties (Iten '12; Coates, Heuberger, Kasprzyk '22)
- BPS quivers for 5d theory on  $S^1$

THANK YOU FOR

YOUR ATTENTION !

