How the Hilbert space of two-sides black holes factorise?

Guanda Lin

UC Berkeley

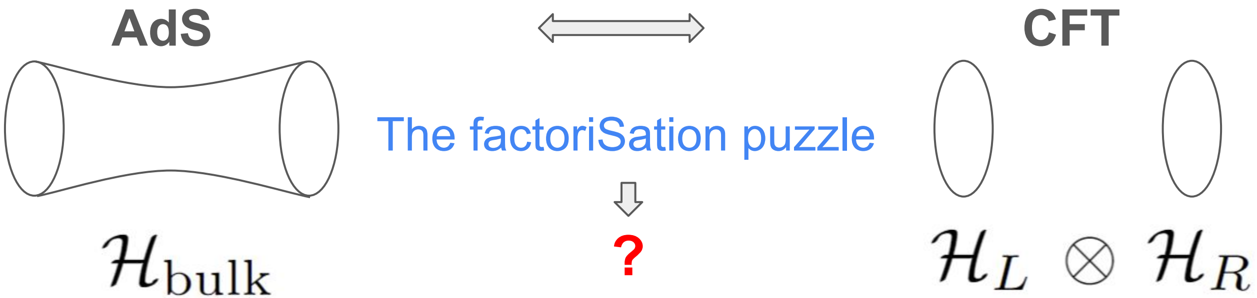
Gong Show Talk, Strings2024

Based on an upcoming work with Jan Boruch, Luca Iliesiu and Cynthia Yan

Motivation

What is the Hilbert space of a two-sided black hole?

- In AdS/CFT, how does gravity know factorisation?



- More broadly, what is the algebra type for one-sided observables?

QFT	**Pert. Grav.**	**Full Quant. Grav.**
Type-III	Type-II	**Type I?**

Main result

- “Puzzle” mostly at the perturbative level

$$\mathcal{H}_{\text{pert.}} \sim \mathcal{H}_{\text{grav.}} \times \mathcal{H}_{\text{mat.}}$$

- We prove that **non-perturbative corrections** will provide resolutions

[non-pert.] $$\mathrm{Tr}_{\mathcal{H}_{\text{bulk}}}(k_L k_R) = \mathrm{Tr}_{\mathcal{H}_L}(k_L) \times \mathrm{Tr}_{\mathcal{H}_R}(k_R)$$

$$\Downarrow$$

[non-pert.] $$\mathcal{H}_{\text{bulk}} = \mathcal{H}_L \otimes \mathcal{H}_R$$

- In particular, **Wormhole contributions** to gravitational path integral are crucial

Set-up

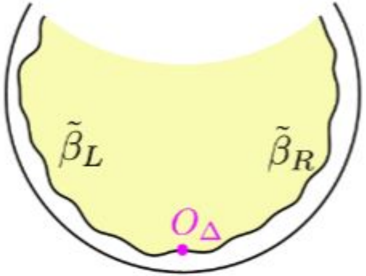
- JT+matter

break Hamiltonian constraint

$$I = -S_0\chi(\mathcal{M}) - \frac{1}{2}\left(\int_{\mathcal{M}} \phi(R+2) + 2\int_{\partial\mathcal{M}} \phi_b(K-1)\right) + I_{\text{matter}}$$

- Basis of Hilbert space

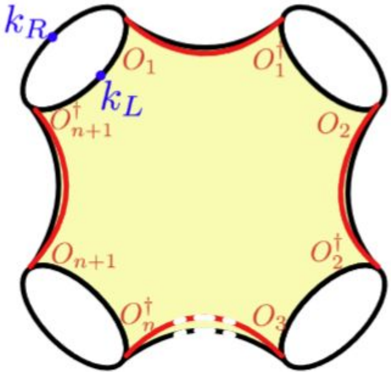
$$|q_i\rangle = O_i|HH\rangle =$$



$$\mathcal{H}_{\text{bulk}} = \text{Span}\{|q_i\rangle, i = 1, \ldots, K\}$$

- The bulk trace and replica wormholes

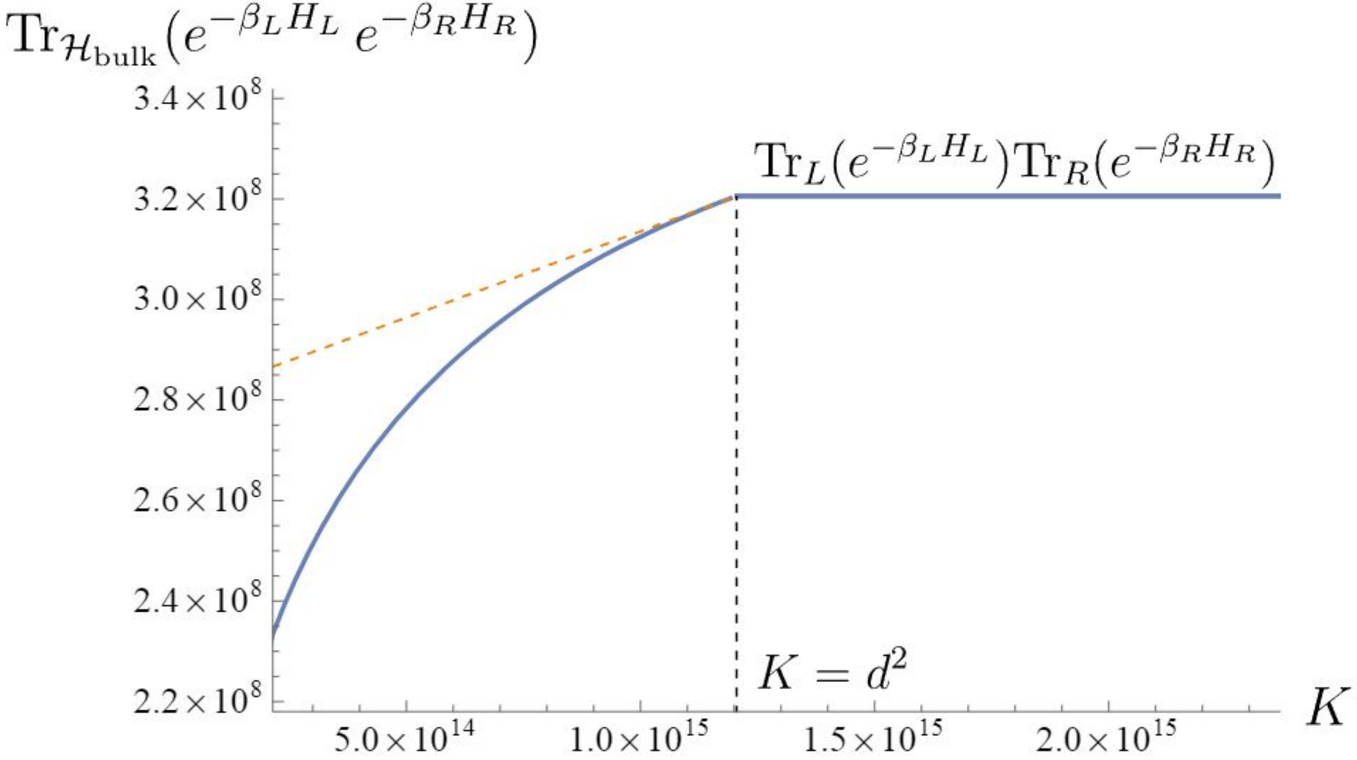
$$\text{Tr}_{\mathcal{H}_{\text{bulk}}}(k_L\, k_R) = \lim_{n\to -1} \langle q_i|q_j\rangle^n \langle q_i|k_L k_R|q_j\rangle$$



Probing factorisation

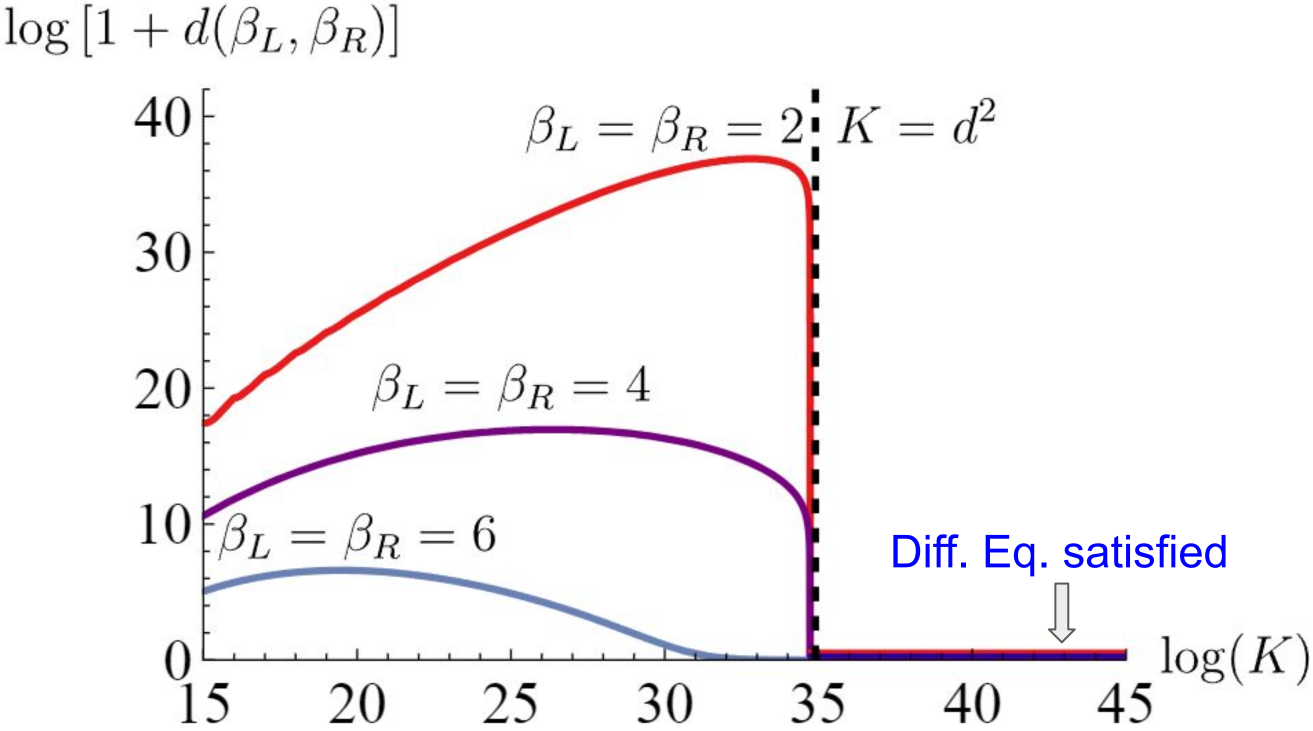
- **Bulk trace** $\mathrm{Tr}_{\mathcal{H}_{\rm bulk}}(k_L k_R) = \mathrm{Tr}_{\mathcal{H}_L}(k_L) \times \mathrm{Tr}_{\mathcal{H}_R}(k_R)$

  more precisely $Z_{\rm bulk}(\beta_L, \beta_R) = \mathrm{Tr}_{\mathcal{H}_{\rm bulk}}(e^{-\beta_L H_L}\, e^{-\beta_R H_R})$



Probing factorisation

- **Differential equation** $\overline{d(\beta_L, \beta_R)} = \overline{d(\beta_L, \beta_R)^2} = 0$



$$d(\beta_L, \beta_R) = Z_{\text{bulk}}(\beta_L, \beta_R)\partial_{\beta_L}\partial_{\beta_R}Z_{\text{bulk}}(\beta_L, \beta_R) - \partial_{\beta_L}Z_{\text{bulk}}(\beta_L, \beta_R)\partial_{\beta_R}Z_{\text{bulk}}(\beta_L, \beta_R)$$

Thank you!

Extra slides

Barred v.s. unbarred

Leading order in K

$$\overline{\mathrm{Tr}_{\mathcal{H}_{bulk}(K)}(k_L k_R)} = \overline{\mathrm{Tr}_{\mathcal{H}_L}(k_L)\,\mathrm{Tr}_{\mathcal{H}_R}(k_R)} \Leftrightarrow \mathrm{Tr}_{\mathcal{H}_{\mathrm{bulk}}(K)}(k_L k_R) = \mathrm{Tr}_{\mathcal{H}_L}(k_L)\,\mathrm{Tr}_{\mathcal{H}_R}(k_R)$$

In general, the barred one is correct because of the non-trivial statistics of energy levels

Dimension of the Hilbert Space

$$\dim_{\mathcal{H}_{\mathrm{bulk}}} = \overline{d^2} = \int_{\mathcal{E}} \rho(E_L)\rho(E_R) \quad \text{with } \mathcal{E} \text{ energy cut-off}$$

Note that even with a cut-off, $\mathcal{H}_{\mathrm{pert.}}$ is infinite dimensional

Irrelevance of UV divergence and higher dimensional generalization

(1) very general symmetry property argument for diff. eq.

(2) the matter supported wormholes are saddles and not the ones causing UV div. In JT+matter

K independence

K is a parameter in the technique, irrelevant to the property of the actual $\mathcal{H}_{\mathrm{bulk}}$

checked $1/K$ expansions, no effect; expect exp. small # of outliers in which $\mathcal{H}_{\mathrm{bulk}}$ is not spanned

Basis independence

checked using operators with different conformal dimensions, length basis, etc

About gauge symmetry

Want no energy degeneracy so that the bulk trace factorisation tells about factorised basis

No boundary global symmetry and no bulk gauge symmetry

The end of talk brane