Interacting fields at spatial infinity

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References

Anupam A.H

Athira P.V.

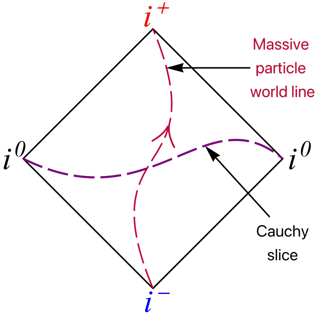
Suvrat Raju

arXiv:2405.20326 with Anupam A.H , Athira P.V and Suvrat Raju

Massive fields at spatial infinity ⟶ Holography

Holography of information : “In any theory of quantum gravity in flat space (massless fields) & AdS, information that is available in the bulk of a Cauchy slice is also available near its boundary.”

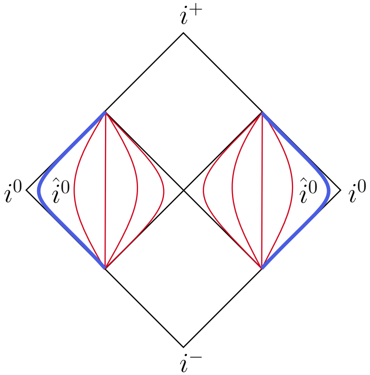
[*Laddha, Prabhu, Raju, Shrivastava*; 2002.02448]
[*Raju*; 2012.05770]



Massive particles go from i^- to i^+ (not natural for holography).

i^0 is the boundary of the Cauchy slice.

Blow up of spatial infinity ($\hat{i}^0$)



Take de Sitter slicing of flat space.

$$t = \rho \sinh \tau \quad , \quad r = \rho \cosh \tau$$

$$ds^2 = d\rho^2 + \rho^2 \underbrace{\left(-d\tau^2 + \cosh^2 \tau \; d\Omega^2\right)}_{dS_3 \text{ metric}}$$

The slice at $\rho \to \infty$ is $\hat{i}^0$ (blue slice).

Free field theory at blow up of spatial infinity ($\hat{i}^0$)

Massive scalar field decays as

$$\phi \to \rho^{-\frac{3}{2}} e^{-m\rho}.$$

Define extrapolated boundary operators

$$\mathcal{Z}(\tau, \Omega) = \lim_{\rho \to \infty} \sqrt{\frac{2}{\pi}}\, \rho \sqrt{m\rho}\, e^{m\rho}\, \phi(\rho, \tau, \Omega).$$

We smear the fields with smearing function analytic in $\mathrm{Im}[\tau] \in (-\frac{\pi}{2}, \frac{\pi}{2})$.

$$\mathcal{Z}(g) = \int [d\mu]_{\tau,\Omega}\, g(\tau, \Omega)\, \mathcal{Z}(\tau, \Omega)$$

In free theory, smeared two point function $\langle \mathcal{Z}(g) \mathcal{Z}(f) \rangle$ is well defined.

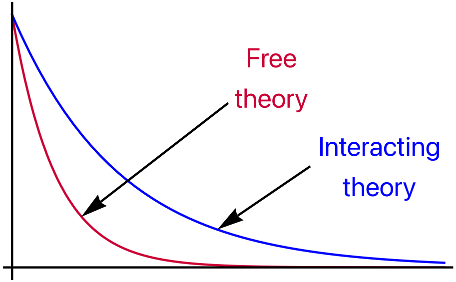
[*Laddha, Prabhu, Raju, Shrivastava*; 2207.06406]

Interacting field theory : Wightman functions

For perturbation theory we choose Wightman functions (as we are smearing over time).

Interacting Wightman correlators can have slowly decaying parts than $e^{-m\rho}$.

$$\int [d\mu]_{\tau,\Omega}\, g(\tau,\Omega)\; W^{\Psi_1,\Psi_2}(\{\rho,\tau,\Omega\},\ldots) \to \int da\, G(a)\, \rho^{-\frac{3}{2}} e^{-a\rho}$$

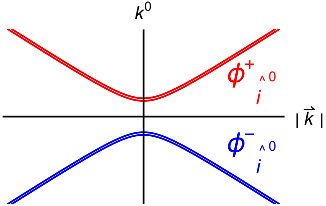


On-shell Wightman functions

Proposal : Extract the on-shell part of the bulk Wightman functions, which has correct extrapolate limit.

$$W^{\Psi_1,\Psi_2}(k_1,\ldots,k_n) = G^{\Psi_1,\Psi_2}(k_1,\ldots,k_n)(2\pi)\delta(k_1^2+m^2)\ldots(2\pi)\delta(k_n^2+m^2)+\ldots$$

In the momentum space Feynman rules, replace all the external propagators with their on-shell parts.



Extract on-shell part $\phi^+_{\hat{i}0}(\vec{k})$ & $\phi^-_{\hat{i}0}(\vec{k})$ from the single Heisenberg operator $\Phi(k)$. Smeared field can be written as

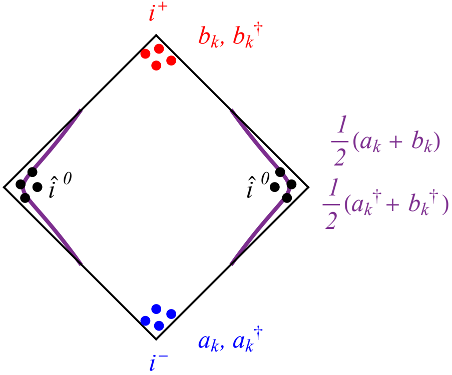
$$\mathcal{Z}(g) = \int \frac{d^3\vec{k}}{(2\pi)^3 2\omega_k}\left(\phi^+_{\hat{i}0}(\vec{k})\tilde{g}^+(\vec{k}) + \phi^-_{\hat{i}0}(\vec{k})\tilde{g}^-(\vec{k})\right).$$

Algebra at $\hat{i}^0$

The operators at $\hat{i}^0$ are average of “in” and “out” operators.

$$\phi^{+}_{\hat{i}^0}(k) = \frac{1}{2}(a_k + b_k)$$

$$\phi^{-}_{\hat{i}^0}(k) = \frac{1}{2}(a_k^\dagger + b_k^\dagger)$$



[*Caron-Huot, Giroux, Hannesdottir, Mizera*; 2308.02125]

Sample computations

4 point vacuum correlator :

$$\langle\Omega|\phi^{+}_{\hat{j}0}(k_1)\phi^{+}_{\hat{j}0}(k_2)\phi^{-}_{\hat{j}0}(k_3)\phi^{-}_{\hat{j}0}(k_4)|\Omega\rangle_{\text{connected}} = -\frac{1}{2}\,\mathrm{Im}\left(T_{\vec{k}_1\vec{k}_2\leftarrow\vec{k}_3,\vec{k}_4}\right)$$

5 point vacuum correlator :

$$\langle\Omega|\phi^{+}_{\hat{j}0}(k_1)\phi^{+}_{\hat{j}0}(k_2)\phi^{+}_{\hat{j}0}(k_3)\phi^{-}_{\hat{j}0}(k_4)\phi^{-}_{\hat{j}0}(k_5)|\Omega\rangle_{\text{connected}}$$
$$= -\frac{1}{2}\,\mathrm{Im}\left(T_{\vec{k}_1\vec{k}_2\vec{k}_3\leftarrow\vec{k}_4,\vec{k}_5}\right) + \frac{1}{4}\,\mathrm{Re}\left(\sum_X (T_{X\leftarrow\vec{k}_1,\vec{k}_2})^*\,T_{\vec{k}_3,X\leftarrow\vec{k}_4,\vec{k}_5}\right)$$

$$\langle\Omega|\phi^{+}_{\hat{j}0}(k_1)\phi^{+}_{\hat{j}0}(k_2)\phi^{-}_{\hat{j}0}(k_3)\phi^{-}_{\hat{j}0}(k_4)\phi^{-}_{\hat{j}0}(k_5)|\Omega\rangle_{\text{connected}}$$
$$= -\frac{1}{2}\,\mathrm{Im}\left(T_{\vec{k}_1\vec{k}_2\leftarrow\vec{k}_3,\vec{k}_4,\vec{k}_5}\right) + \frac{1}{4}\,\mathrm{Re}\left(\sum_X (T_{\vec{k}_3,X\leftarrow\vec{k}_1,\vec{k}_2})^*\,T_{X\leftarrow\vec{k}_4,\vec{k}_5}\right).$$

where,

$$S = 1 + i\,T$$

Outlook

- Holography for asymptotically flat space when massive fields couple with dynamical gravity ?

- Relation to celestial CFT amplitudes ?

- Bootstrapping correlators at $\hat{i}^0$ and derive consequences about bulk theory ?

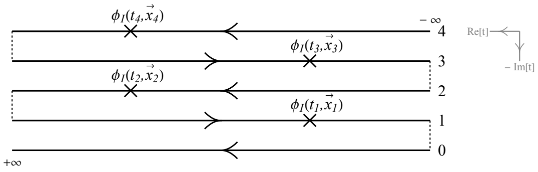
- Relation to the recent understanding of S matrix from the perspective of path integral by *Jain, Kundu, Minwalla, Parrikar, Prabhu, Shrivastava* [2311.03443] ?

Thank you

For details, please look at

arXiv:2405.20326

Backup slide 1: Standard Wightman function Feynman rules



- Consider propagator $D_{ij}(k)$ between two points which are at ith contour & jth contour. Momentum k is flowing from j to i.

- Propagator rules :

$$T_{ij}(k) = \frac{-i}{k^2+m^2-i\epsilon} + \ldots, \qquad i=j\,;\ (n-i) = \text{even}$$

$$\overline{T}_{ij}(k) = \frac{i}{k^2+m^2+i\epsilon} + \ldots \qquad i=j\,;\ (n-i) = \text{odd}$$

$$W_{ij}(k) = 2\pi\theta(k^0)\delta(k^2+m^2) + \ldots \qquad i<j$$

$$\overline{W}_{ij}(k) = 2\pi\theta(-k^0)\delta(k^2+m^2) + \ldots \qquad i>j$$

- Vertex Factor is $(-iH_I)$ for $(n-i)$ is even, $(+iH_I)$ for $(n-i)$ odd.

Backup slide 2 : Modified Feynman Rules at $\hat{i}^0$

- If we want to calculate

$$W_{\hat{i}^0}^{\Psi_1,\Psi_2}(k_1 \ldots k_n) = \langle \Psi_1 | \phi_{\hat{i}^0}(k_1) \ldots \phi_{\hat{i}^0}(k_n) | \Psi_2 \rangle$$

directly, we put external time-ordered and anti-time-ordered propagators on-shell.

$$\frac{\pm i}{k^2 + m^2 \pm i\epsilon} = \pm i \left\{ \text{P.V} \left(\frac{1}{k^2 + m^2} \right) \mp i\pi\delta(k^2 + m^2) \right\}$$

- New Feynman rules for the external propagators

$$\mathbb{T}_{ij}(k) = \pi\delta(k^2 + m^2) \qquad i = j\,;\ (n-i) = \text{even}$$

$$\overline{\mathbb{T}}_{ij}(k) = \pi\delta(k^2 + m^2) \qquad i = j\,;\ (n-i) = \text{odd}$$

$$\mathbb{W}_{ij}(k) = 2\pi\theta(k^0)\delta(k^2 + m^2) \qquad i < j$$

$$\overline{\mathbb{W}}_{ij}(k) = 2\pi\theta(-k^0)\delta(k^2 + m^2) \qquad i > j$$