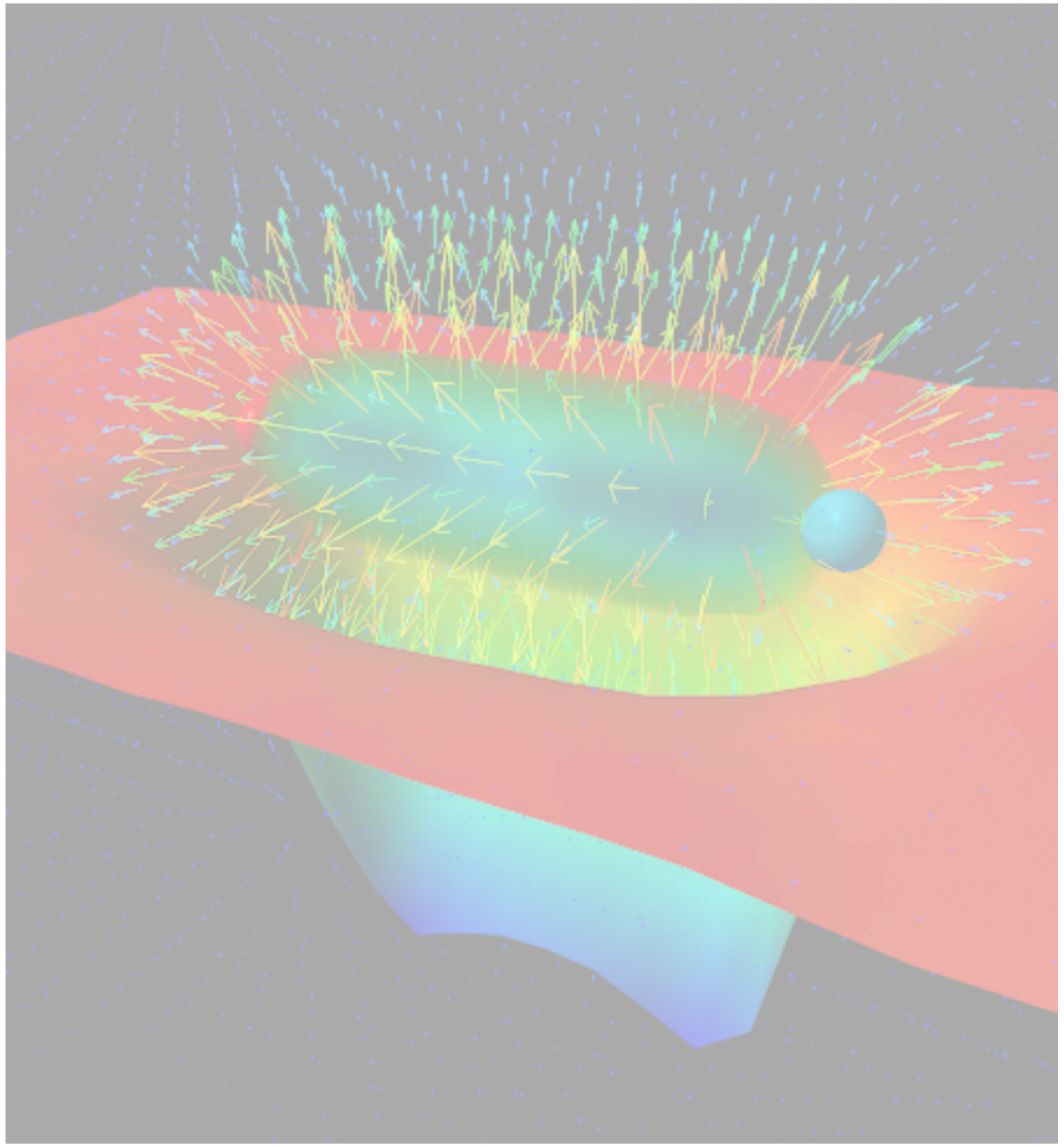
Bootstrapping Strings

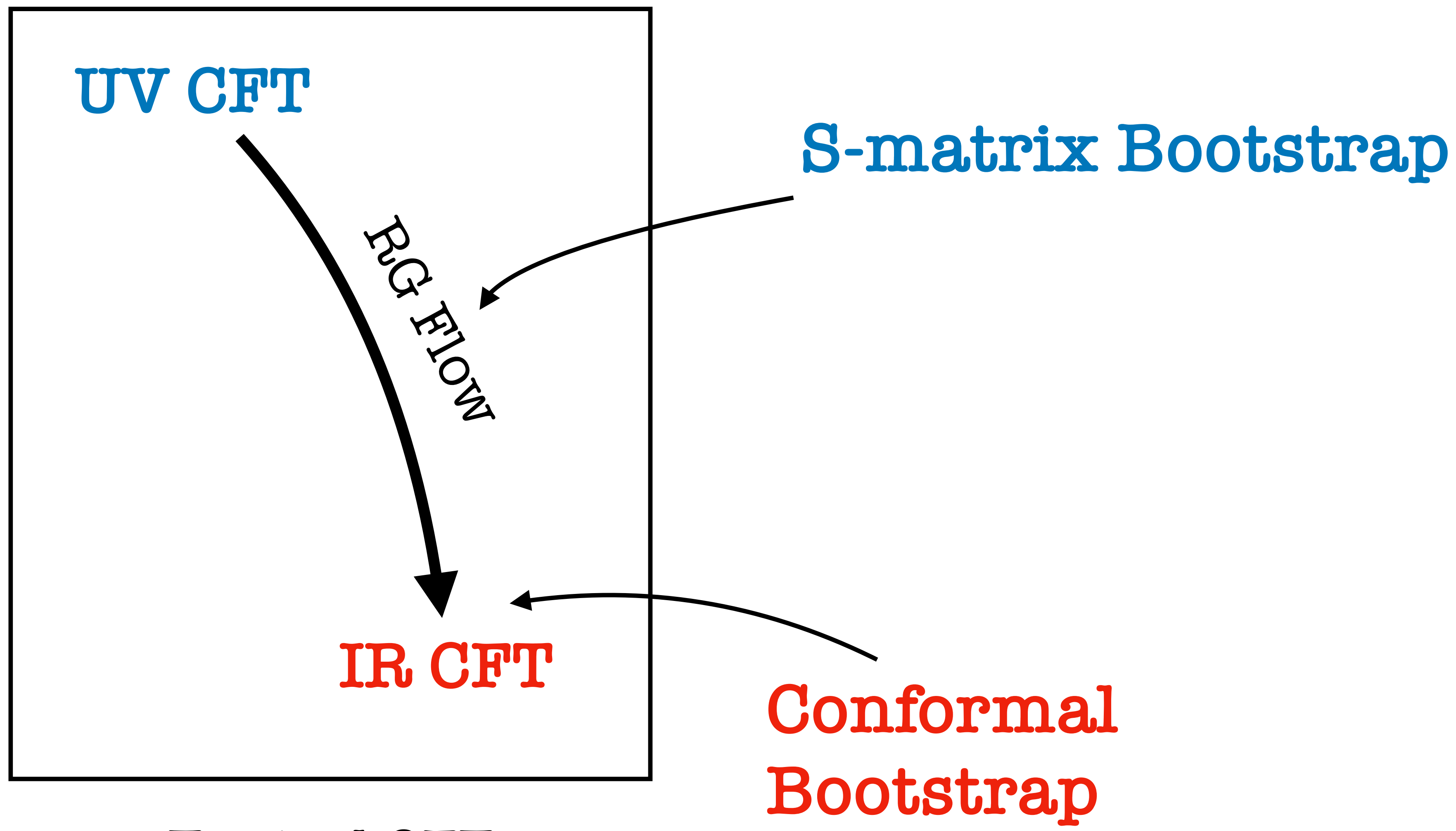


Andrea Guerrieri

June 6, 2024

Bootstrap:
What is (im)possible in the Space of QFTs and Strings

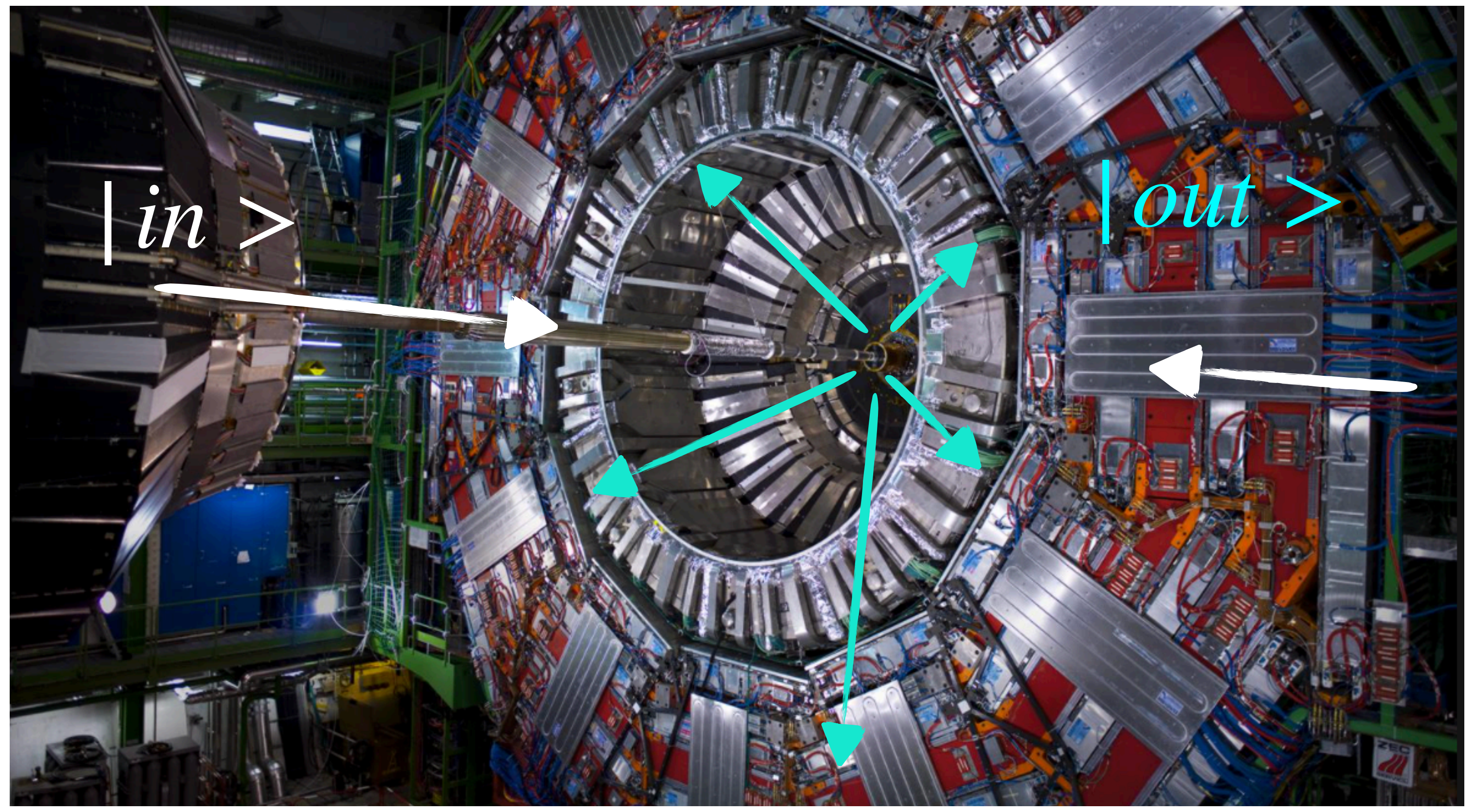
Use causality, crossing, and unitarity to constrain the space of physical observables



Typical QFT

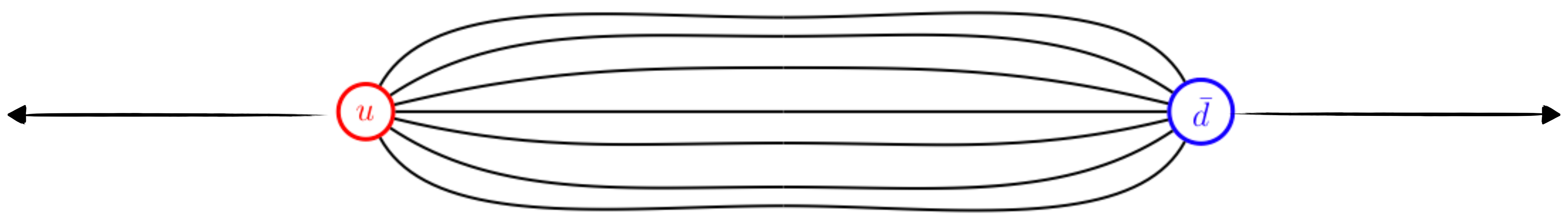
The S-matrix measures the probability of a scattering process

$$\mathcal{S}_{in \to out} \equiv < in \, | \, out >$$



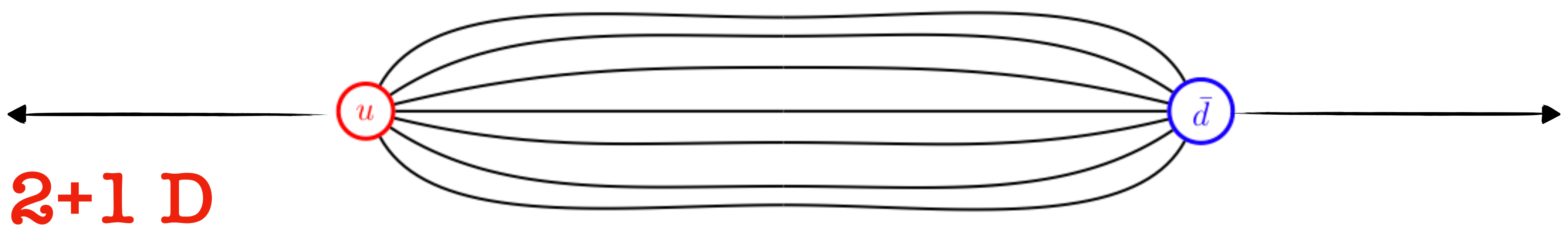
Bounds on "real world" confining strings

Distance between quarks $R/\ell_s \to \infty$



Bounds on “real world” confining strings

Distance between quarks $R/\ell_s \to \infty$



Flux Tube ground state in 2+1 D

$$E_0(R) = \frac{R}{\ell_s^2} - \frac{\pi}{6R} - \frac{\pi^2 \ell_s^2}{72R^3} - \frac{\pi^3 \ell_s^4}{432R^5} + \frac{\Delta_3 \ell_s^6}{R^7} + \mathcal{O}\left(\frac{\ell_s^8}{R^9}\right)$$

$$\boxed{\Delta_3 \leq \frac{\pi^6}{5400} - \frac{5\pi^4}{10368}}$$

String tension $\sigma = \ell_s^{-2}$

Lüscher term ’80, Polchinski, Strominger ‘91

Lüscher, Weisz, Drummond ‘04

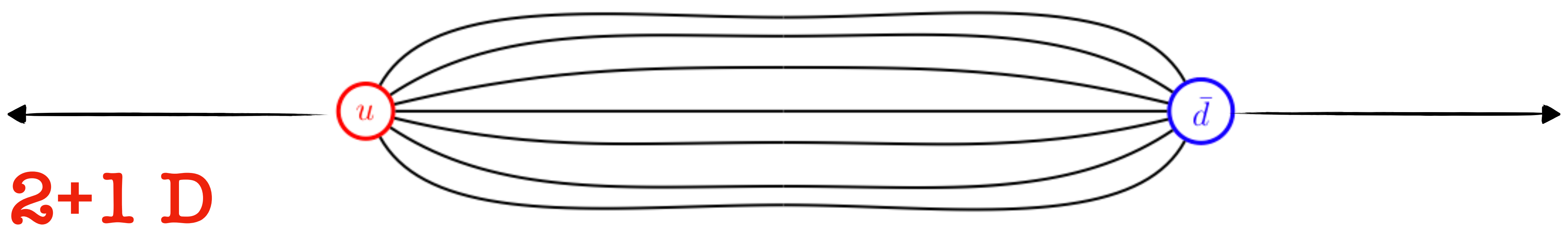
Aharony, Komargodsky, Dubovsky, Flauger, Gorbenko,... ‘15

Theory-dependent, but bounded

Elias-Miró, ALG, Hebbar, Penedones, and Vieira, ’19

Bounds on “real world” confining strings

Distance between quarks $R/\ell_s \to \infty$



Flux Tube ground state in 2+1 D

$$E_0(R) = \frac{R}{\ell_s^2} - \frac{\pi}{6R} - \frac{\pi^2 \ell_s^2}{72R^3} - \frac{\pi^3 \ell_s^4}{432R^5} + \frac{\Delta_3 \ell_s^6}{R^7} + \mathcal{O}\left(\frac{\ell_s^8}{R^9}\right)$$

$$\boxed{\Delta_3 \leq \frac{\pi^6}{5400} - \frac{5\pi^4}{10368}}$$

String tension $\sigma = \ell_s^{-2}$

Lüscher term ’80, Polchinski, Strominger ‘91

Lüscher, Weisz, Drummond ‘04

Aharony, Komargodsky, Dubovsky, Flauger, Gorbenko,... ‘15

Theory-dependent, but bounded

Elias-Miró, ALG, Hebbar, Penedones, and Vieira, ’19

$$\Delta_3 = -\frac{32\gamma_3\pi^6}{225} - \frac{5\pi^4}{10368}$$

Lattice simulations agree with the bound!

[4] Baffigo, Caselle ‘23

[5] Caristo, Caselle, Magnoli, Nada, Panero ‘21

gauge group	$\mathbb{Z}_2$	$SU(2)$	$SU(6)$	$SU(\infty)$
$\gamma_3 \times 768$	-0.4 [4]	-0.3 [5]	0.2 [1, 6]	0.3

Spectrum from Bootstrap

Let's now consider a toy model for $\pi\pi$ scattering

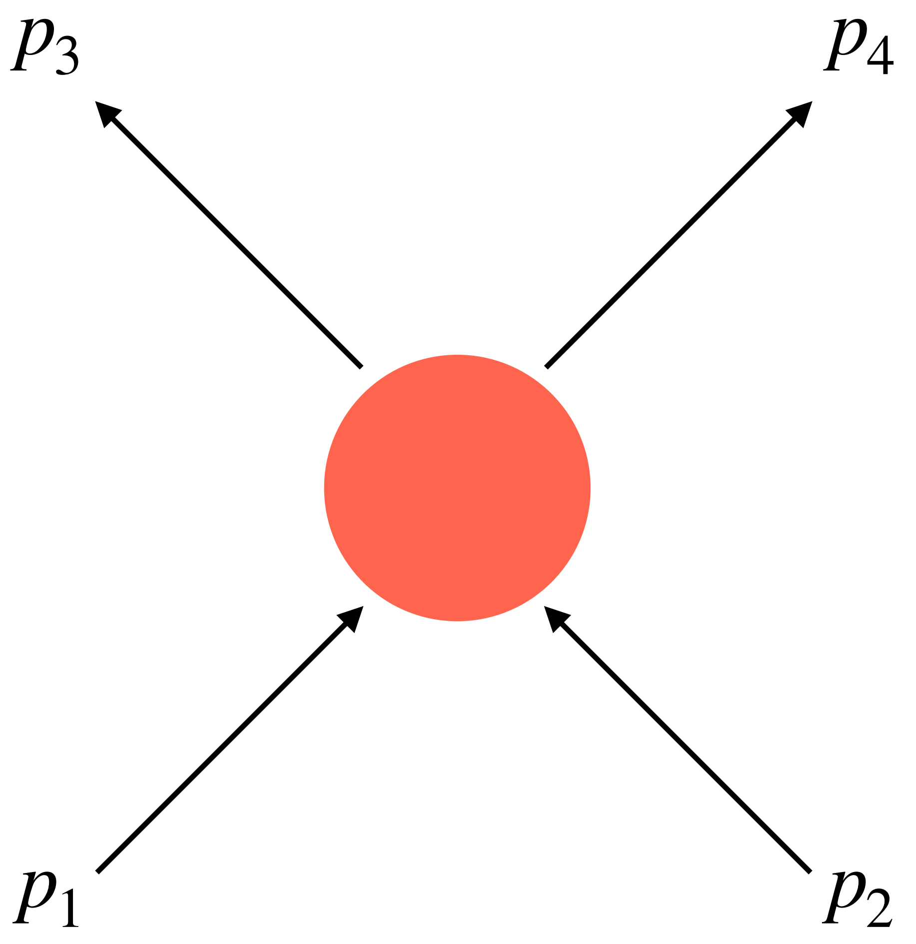
Spectrum from Bootstrap

Let's now consider a toy model for $\pi\pi$ scattering

Scalar field ϕ in 3+1 D with $m = 0$, Goldstone of spontaneously broken U(1)

$$\mathscr{L} \supset (\partial\phi)^4 + (\partial\phi)^6 + \partial^6\phi^4 + g_4\partial^8\phi^4 + \ldots$$

$T(s, t, u) =$



$$s = (p_1 + p_2)^2, t = (p_1 - p_4)^2, s + t + u = 0$$

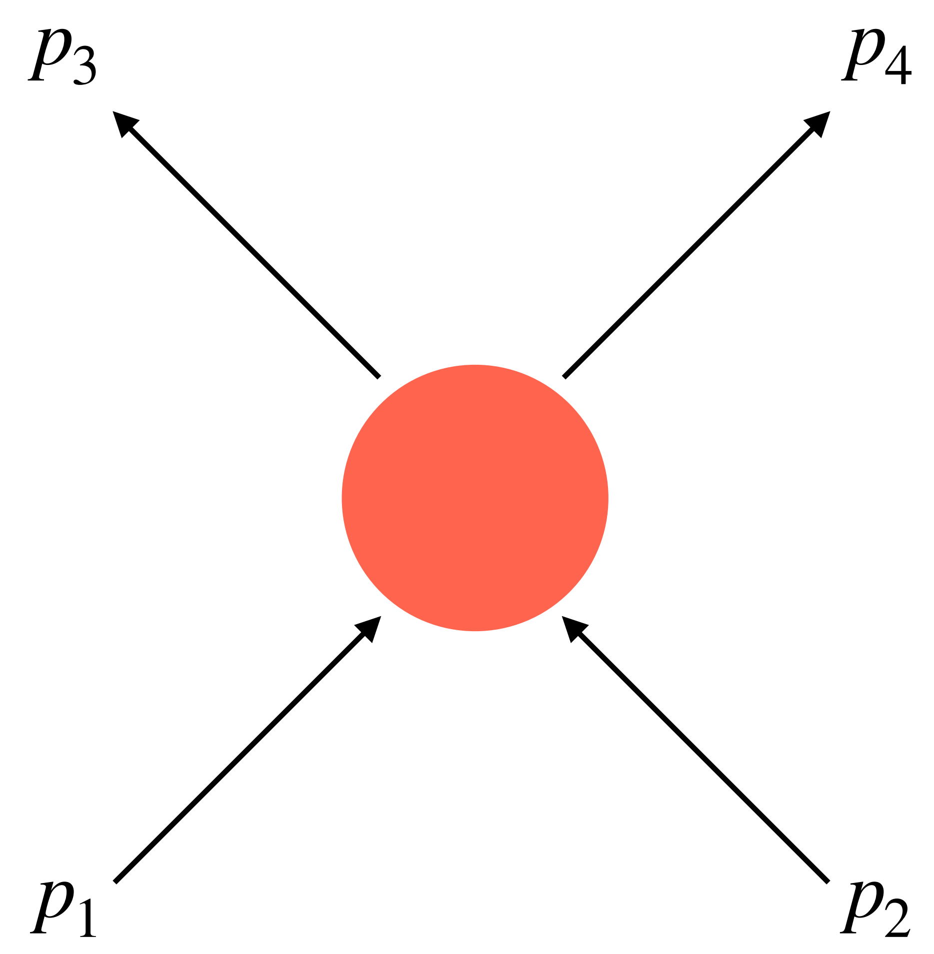
Spectrum from Bootstrap

Let's now consider a toy model for $\pi\pi$ scattering

Scalar field ϕ in 3+1 D with $m = 0$, Goldstone of spontaneously broken U(1)

$$\mathscr{L} \supset (\partial\phi)^4 + (\partial\phi)^6 + \partial^6\phi^4 + g_4\partial^8\phi^4 + \ldots$$

$T(s, t, u) =$

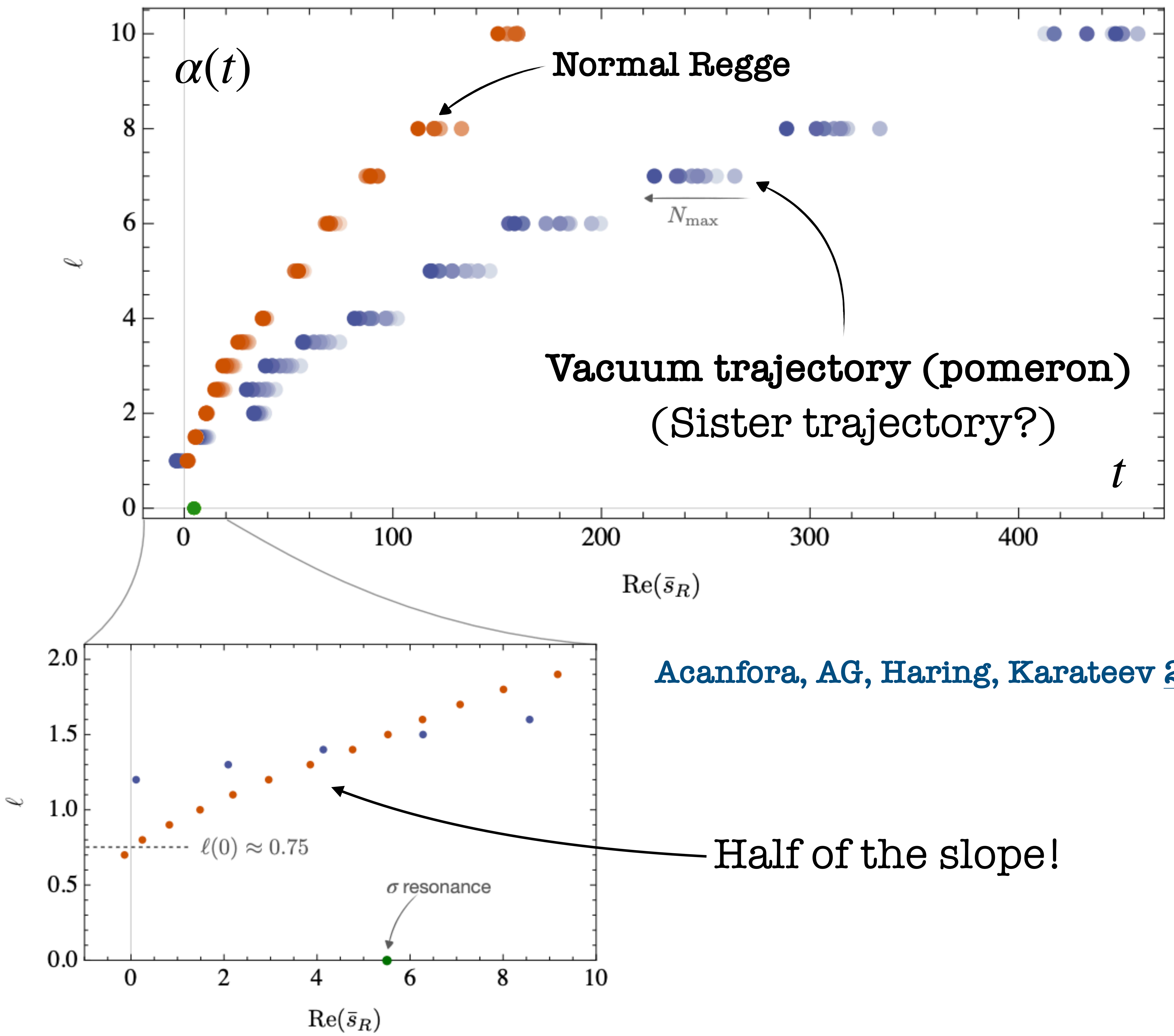


$$s = (p_1 + p_2)^2, t = (p_1 - p_4)^2, s + t + u = 0$$

Consider the amplitude that minimizes the $\partial^8\phi^4$ coupling g_4

Spectrum from Bootstrap

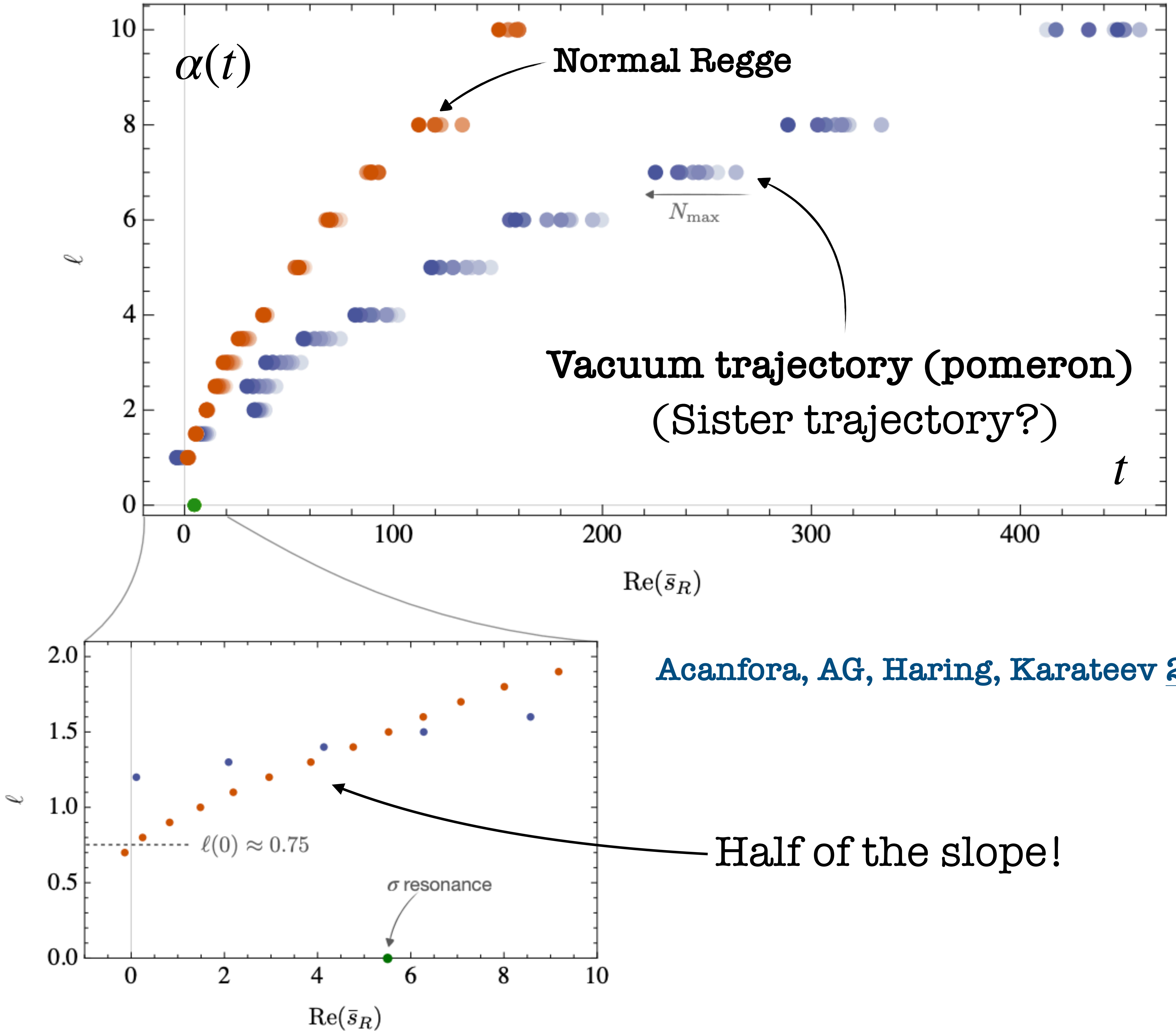
Let's have a look at the amplitude that minimizes the g_4 coupling



Acanfora, AG, Haring, Karateev 2310.06027

Spectrum from Bootstrap

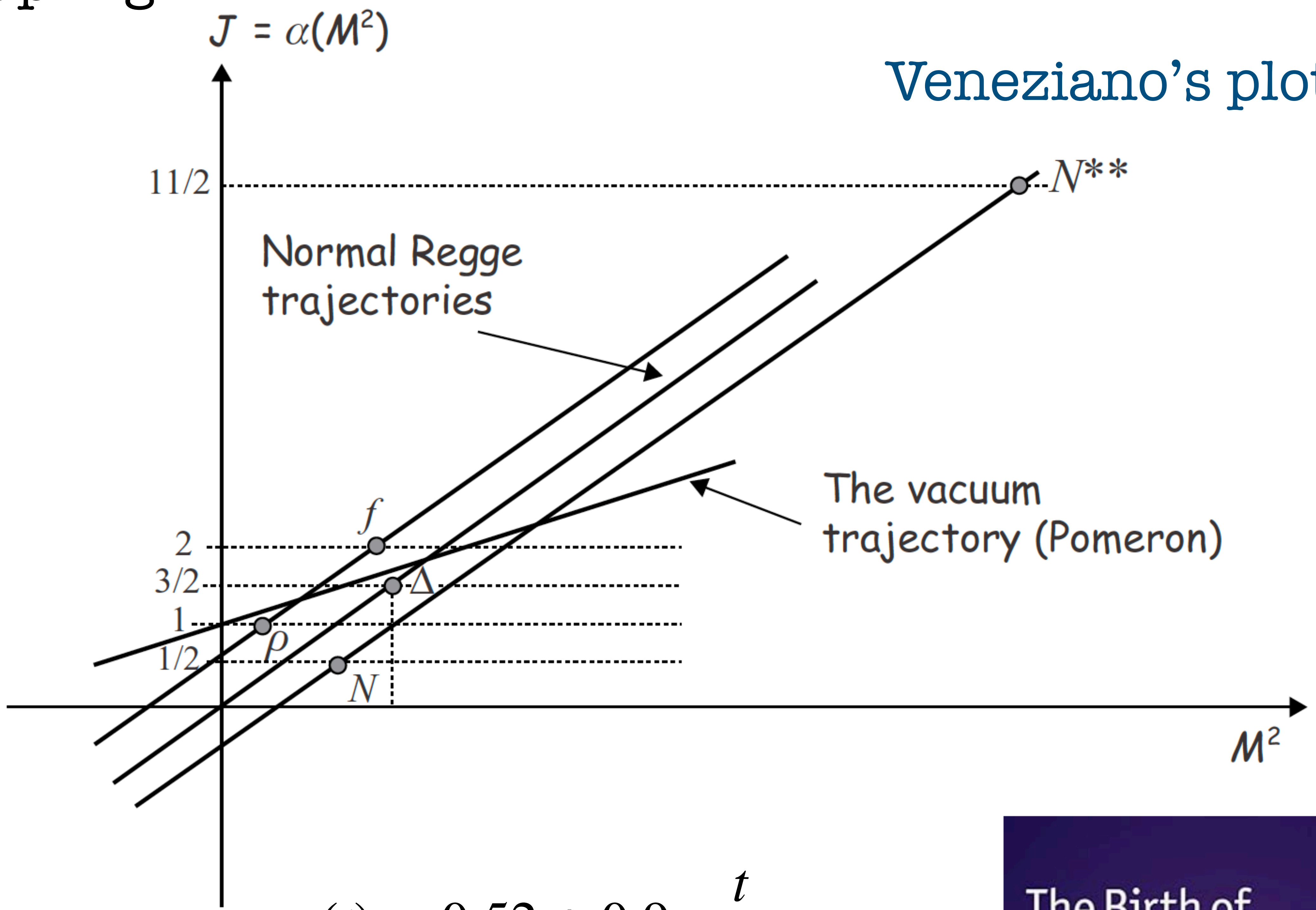
Let's have a look at the amplitude that minimizes the g_4 coupling



Half of the slope!

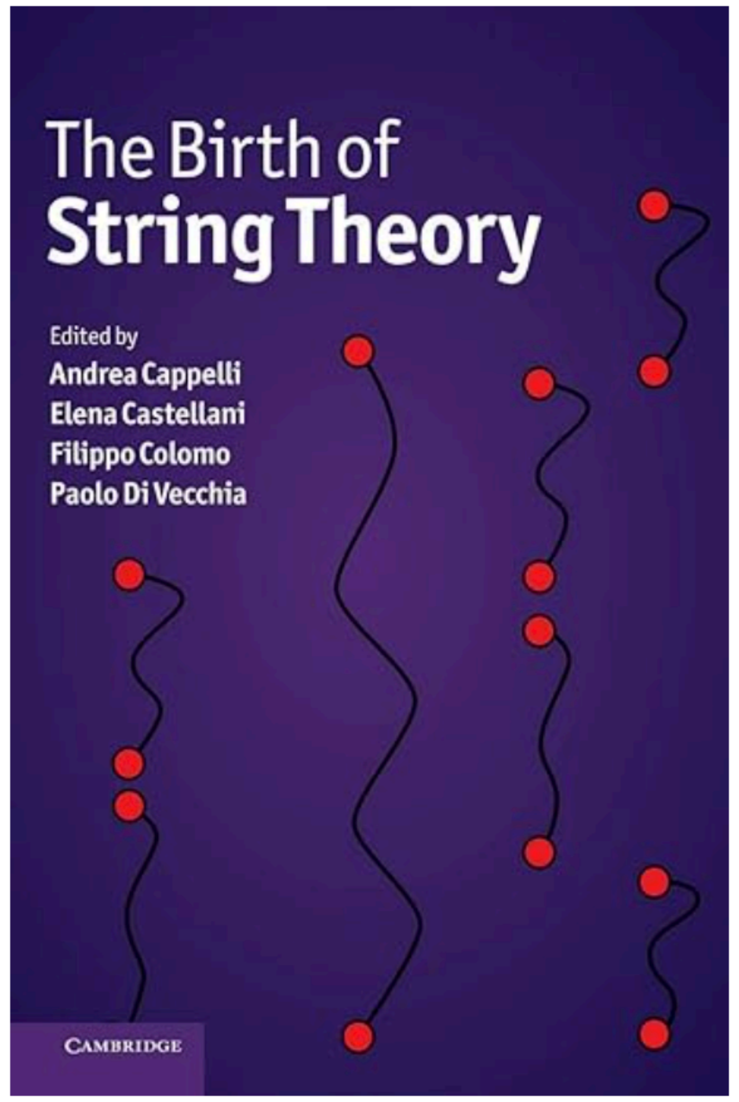
Acanfora, AG, Haring, Karateev 2310.06027

Veneziano's plot



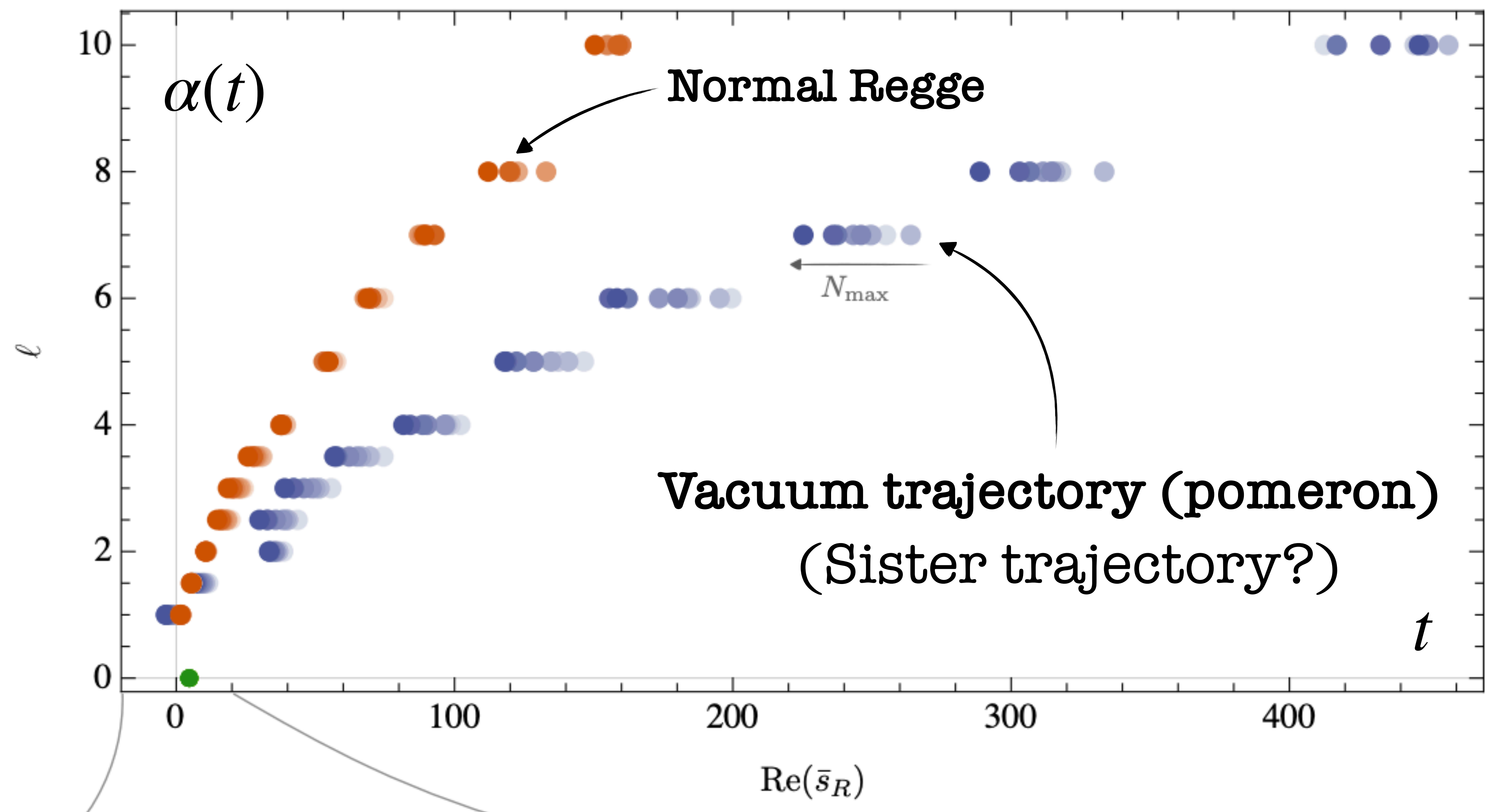
$$\alpha_\rho(t) = 0.52 + 0.9\frac{t}{GeV^2}$$

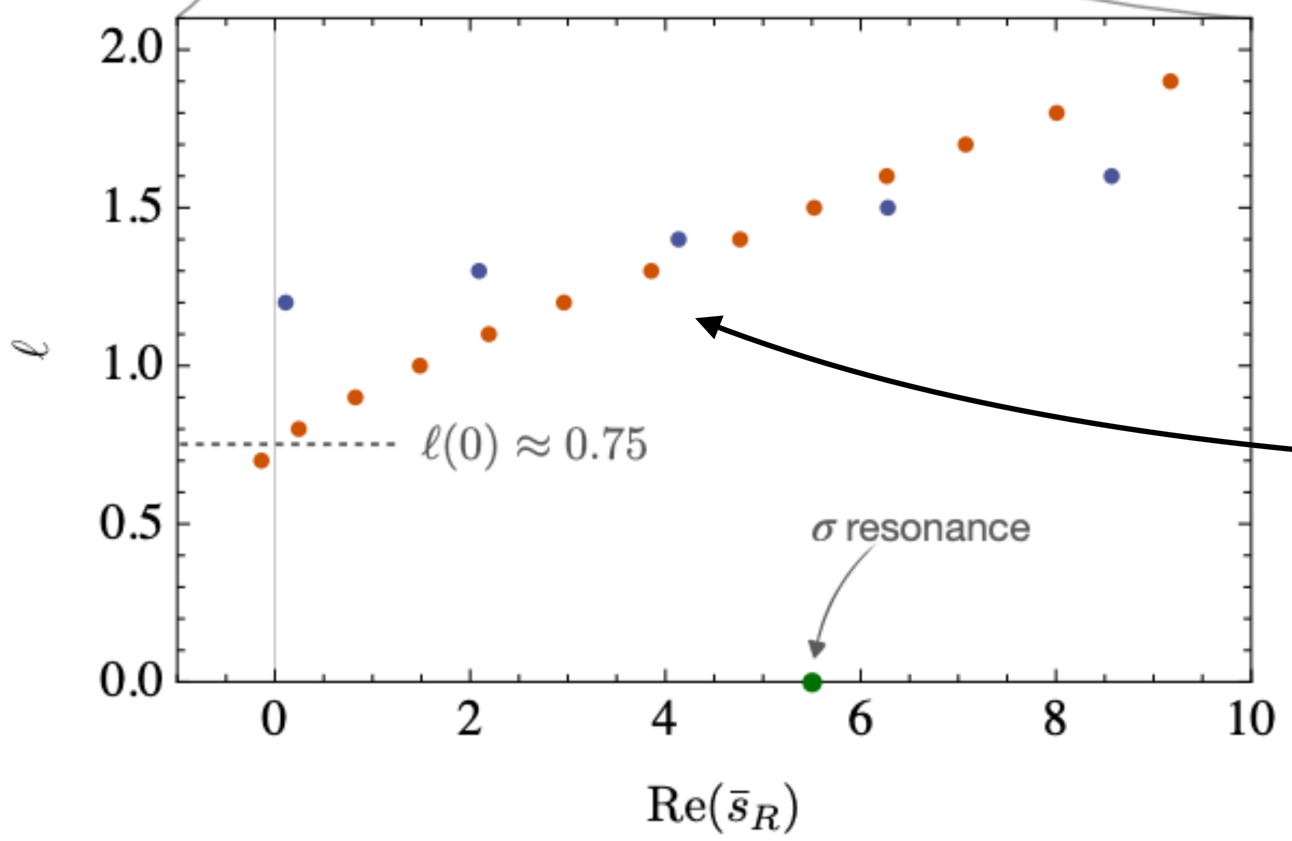
$$\alpha_{Pomeron}(t) = 1.08 + 0.25\frac{t}{GeV^2}$$



Spectrum from Bootstrap

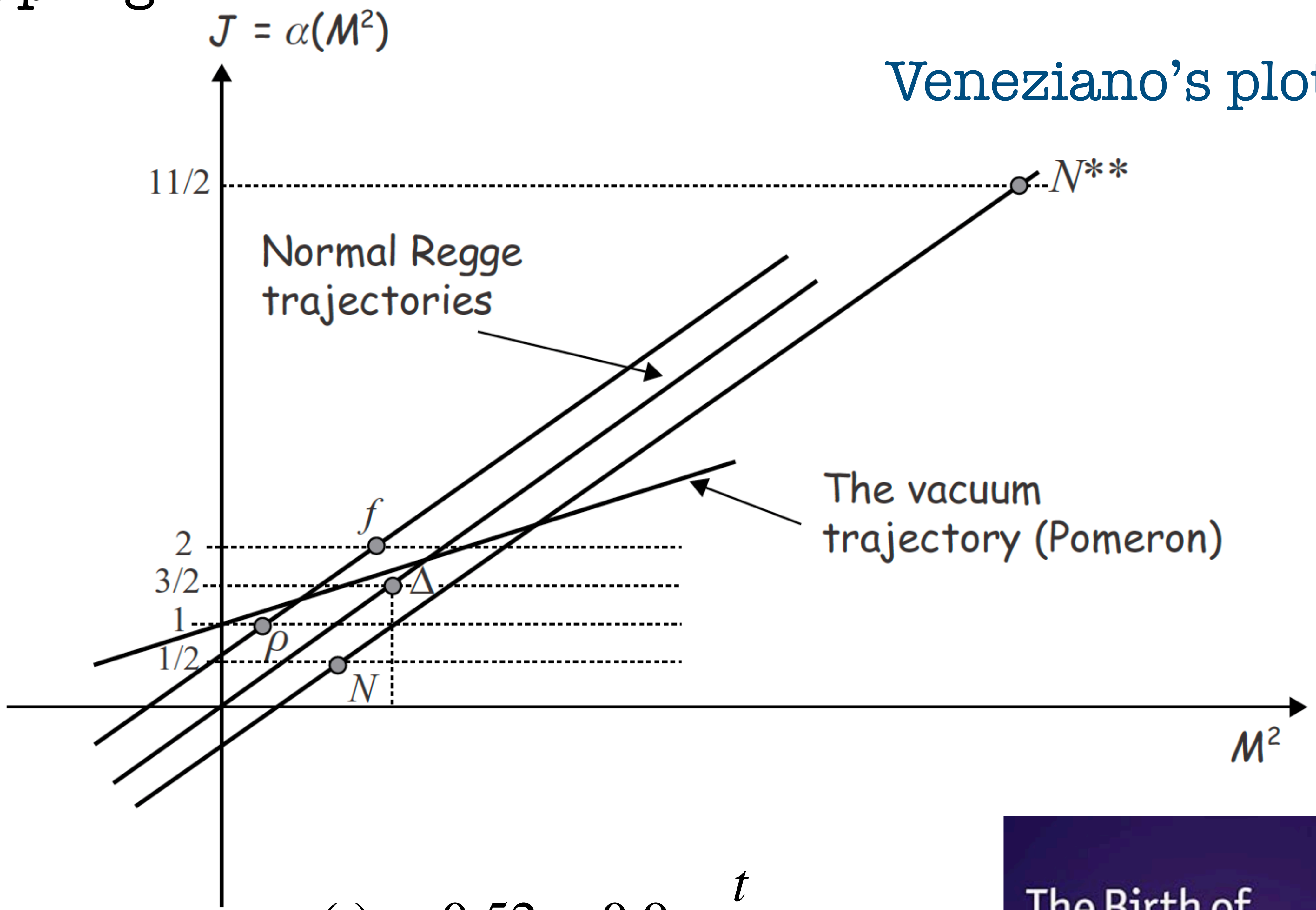
Let's have a look at the amplitude that minimizes the g_4 coupling





Half of the slope!

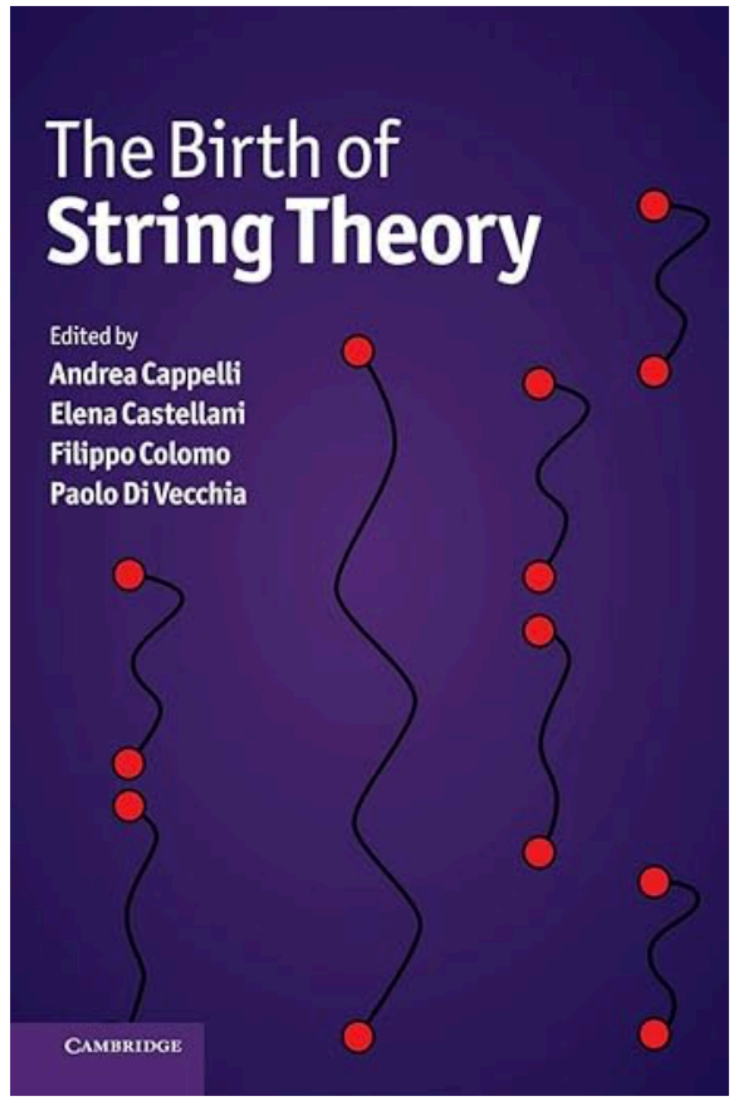
Acanfora, AG, Haring, Karateev 2310.06027



$$\alpha_\rho(t) = 0.52 + 0.9\frac{t}{GeV^2}$$

$$\alpha_{Pomeron}(t) = 1.08 + 0.25\frac{t}{GeV^2}$$

See Jan's Talk



Plan of the Talk

1) $D \geq 9$, Quantum Gravity S-matrix Bootstrap

AG, J. Penedones, P. Vieira 2102.02847

AG, H. Murali, J. Penedones, P. Vieira 2212.00151

2) Multi-particle scattering on confining Strings

AG, A. Homrich, P. Vieira 2404.10812

Supergravity amplitudes in $D \geq 9$

Supergravity amplitudes in D≥9

Maximal Susy, turn off all couplings except G_D

$$A_{QG} = \int \sqrt{-g}(R + 0 \times R^2 + 0 \times R^3 + \alpha_D R^4 + \ldots)$$

Supergravity amplitudes in D≥9

Maximal Susy, turn off all couplings except G_D

$$A_{QG} = \int \sqrt{-g}(R + 0 \times R^2 + 0 \times R^3 + \alpha_D R^4 + \ldots)$$

Scattering amplitude of particles in the gravity multiplet

$$\mathbb{A}_{2\to 2} = \mathbf{R}^4 A(s,t,u)$$

s^4
Axi-dilaton

$s^4 + t^4 + u^4$
Dilaton

2->2 Graviton

2 dilaton->2 Graviton

…

Supergravity amplitudes in D≥9

Maximal Susy, turn off all couplings except G_D

$$A_{QG} = \int \sqrt{-g}(R + 0 \times R^2 + 0 \times R^3 + \alpha_D R^4 + \ldots)$$

Scattering amplitude of particles in the gravity multiplet

$$\mathbb{A}_{2\to 2} = \mathbf{R}^4 A(s,t,u)$$

s^4
Axi-dilaton

$s^4 + t^4 + u^4$
Dilaton

2->2 Graviton

...
2 dilaton->2 Graviton

At low energy and for $D \geq 9$

$$A(s,t,u) = 8\pi G_D \left(\underbrace{\frac{1}{stu}}_{\text{Sugra}} + \alpha_D \ell_P^6 + \mathcal{O}(s \log s) \right)$$

First quantum correction $\alpha_D R^4$

Can α_D take any value?

Supergravity amplitudes in D≥9

Maximal Susy, turn off all couplings except G_D

$$A_{QG} = \int \sqrt{-g}(R + 0 \times R^2 + 0 \times R^3 + \alpha_D R^4 + \ldots)$$

Scattering amplitude of particles in the gravity multiplet

$$\mathbb{A}_{2\to 2} = \mathbf{R}^4 A(s,t,u)$$

s^4 Axi-dilaton

$s^4 + t^4 + u^4$ Dilaton

2->2 Graviton

2 dilaton->2 Graviton

...

At low energy and for $D \geq 9$

$$A(s,t,u) = 8\pi G_D \left(\underbrace{\frac{1}{stu}}_{\text{Sugra}} + \alpha_D \ell_P^6 + \mathcal{O}(s \log s) \right)$$

First quantum correction $\alpha_D R^4$

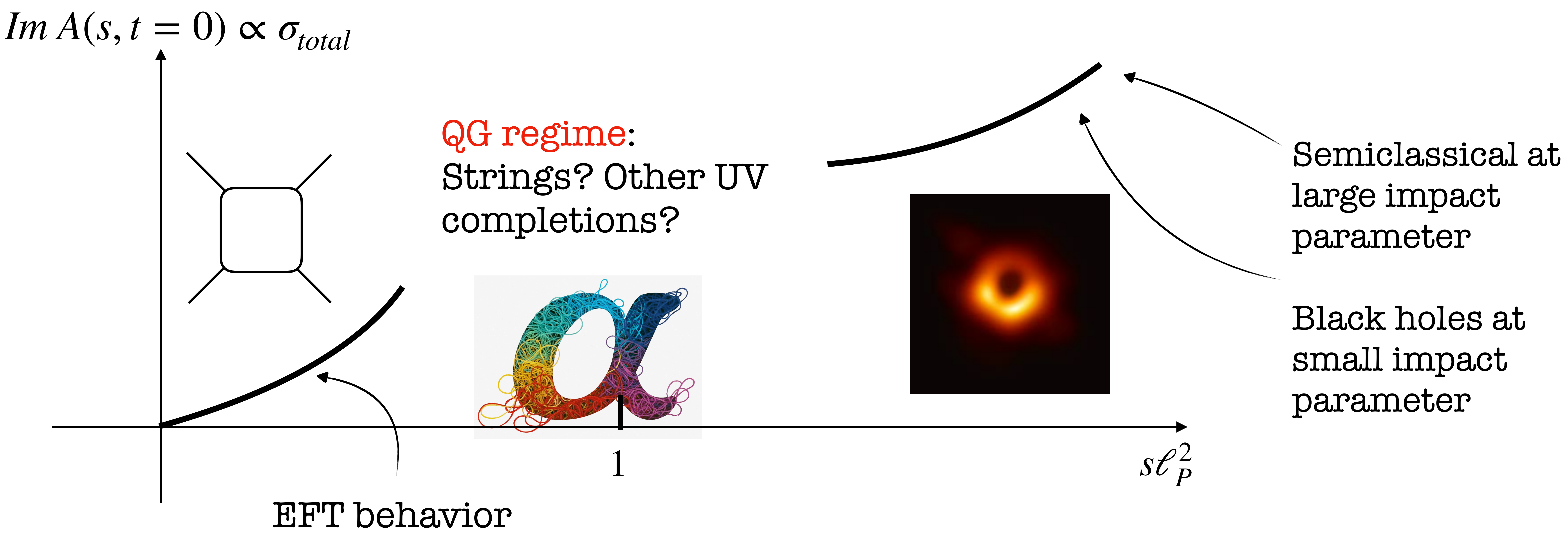
Can α_D take any value?

$$\alpha_D \propto \int_0^\infty \frac{Im A(s,t=0)}{s} ds \geq 0$$

Supergravity amplitudes in D≥9

α_D knows about the theory at all scales

$$\alpha_D \propto \int_0^\infty \frac{ImA(s,t=0)}{s} ds \geq 0$$



We will only assume Maximal Susy, causality, crossing, and unitarity!

Non-perturbative S-matrix Bootstrap

- Consider an ansatz for the non-perturbative amplitude which is **crossing symmetric** and **analytic**

Non-perturbative S-matrix Bootstrap

- Consider an ansatz for the non-perturbative amplitude which is **crossing symmetric** and **analytic**

$$A(s,t,u) = \underbrace{\frac{8\pi G_D}{stu}}_{\text{Sugra}} + \underbrace{\prod_{X=s,t,u} (\rho_X + 1)^2 \sum_{a+b+c\leq N} \nu_{(a,b,c)} \rho_s^a \rho_t^b \rho_u^c}_{\text{UV completion}}$$

Crossing

Analyticity

Non-perturbative S-matrix Bootstrap

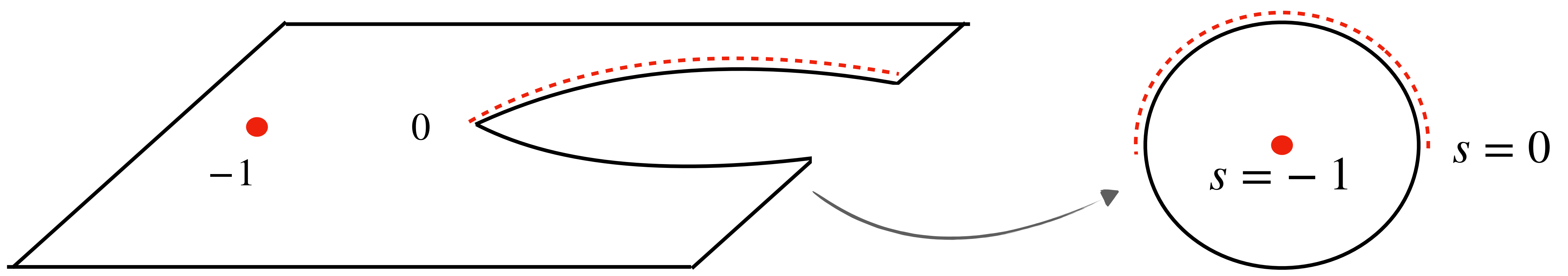
- Consider an ansatz for the non-perturbative amplitude which is **crossing symmetric** and **analytic**

Crossing

$$A(s,t,u) = \underbrace{\frac{8\pi G_D}{stu}}_{\text{Sugra}} + \underbrace{\prod_{X=s,t,u} (\rho_X+1)^2 \sum_{a+b+c\leq N} \nu_{(a,b,c)} \rho_s^a \rho_t^b \rho_u^c}_{\text{UV completion}}$$

Analyticity

$$\rho(s) = \frac{1-\sqrt{-s}}{1+\sqrt{-s}}$$



- When $N \to \infty$, this expansion covers all non-perturbative amplitudes

Non-perturbative S-matrix Bootstrap

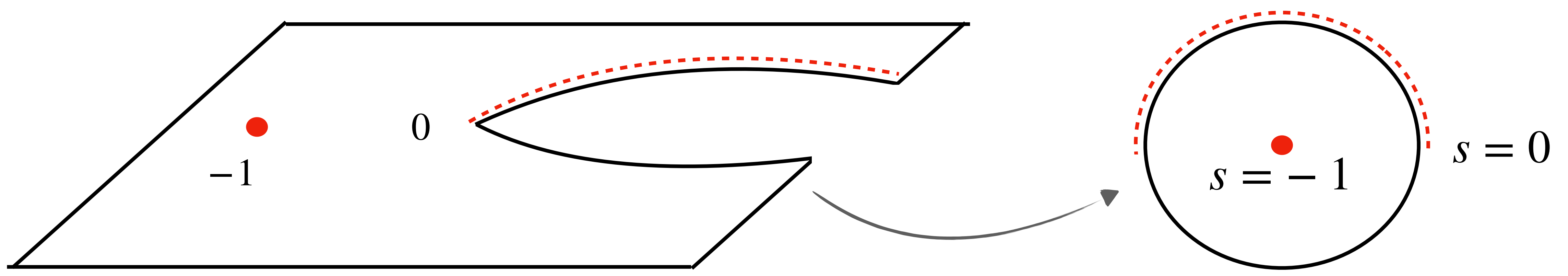
- Consider an ansatz for the non-perturbative amplitude which is **crossing symmetric** and **analytic**

Crossing

$$A(s,t,u) = \underbrace{\frac{8\pi G_D}{stu}}_{\text{Sugra}} + \underbrace{\prod_{X=s,t,u} (\rho_X + 1)^2 \sum_{a+b+c\leq N} \nu_{(a,b,c)} \rho_s^a \rho_t^b \rho_u^c}_{\texttt{UV completion}}$$

Analyticity

$$\rho(s) = \frac{1-\sqrt{-s}}{1+\sqrt{-s}}$$



- When $N \to \infty$, this expansion covers all non-perturbative amplitudes

What is the minimum of $\alpha_D = 2^6 \sum_{a+b+c\leq N} \nu_{(a,b,c)}$**?**

Partial Wave Unitarity

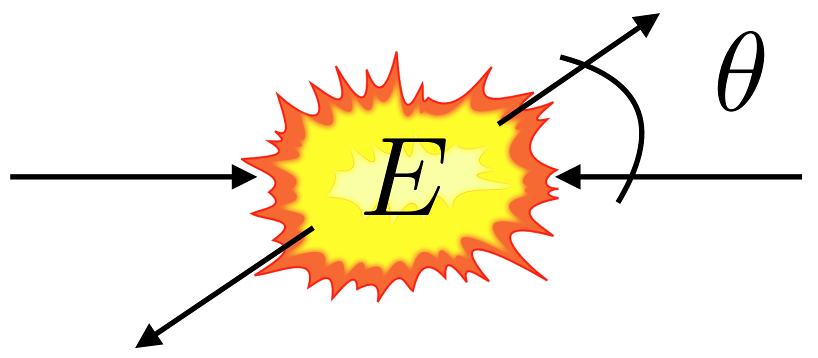
- No unitarity for generic $\nu_{(a,b,c)}$

$$A(s,t,u) = \frac{8\pi G_D}{stu} + \prod_{X=s,t,u} (\rho_X + 1)^2 \sum_{a+b+c\leq N} \nu_{(a,b,c)} \rho_s^a \rho_t^b \rho_u^c$$

- We must impose **unitarity** numerically on $T(s,t,u) = s^4 A(s,t,u)$

$$x = \cos\theta$$

$$T\left(s, t = -\frac{s}{2}(1-\cos\theta), u = -\frac{s}{2}(1+\cos\theta)\right)$$



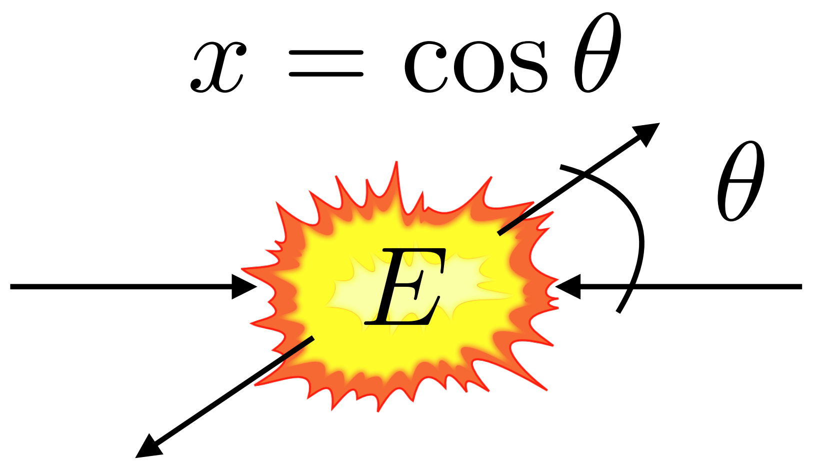
Partial Wave Unitarity

- No unitarity for generic $\nu_{(a,b,c)}$

$$A(s,t,u) = \frac{8\pi G_D}{stu} + \prod_{X=s,t,u} (\rho_X + 1)^2 \sum_{a+b+c\le N} \nu_{(a,b,c)} \rho_s^a \rho_t^b \rho_u^c$$

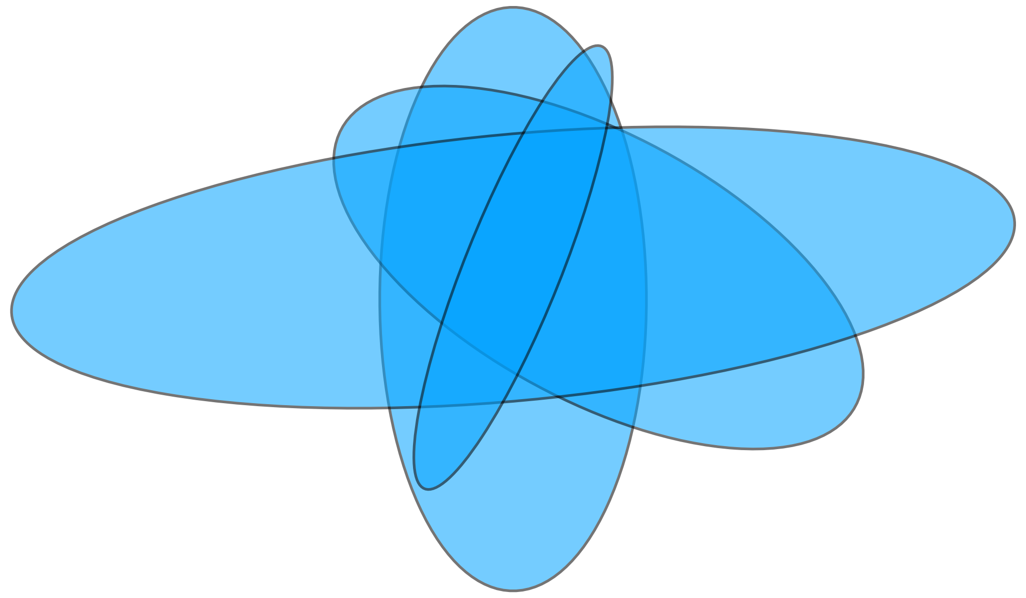
- We must impose **unitarity** numerically on $T(s,t,u) = s^4 A(s,t,u)$

$$T\left(s, t = -\frac{s}{2}(1-\cos\theta), u = -\frac{s}{2}(1+\cos\theta)\right)$$



Example: 10D

$$\boxed{S_\ell(s) = 1 + i\frac{s^3}{2^{18}3\pi^4}\int_{-1}^{1}(1-x^2)^3\frac{C_\ell^{7/2}(x)}{C_\ell^{7/2}(1)}T(s,x)}$$



$\nu_{(a,b,c)}$ live inside the intersection of ellipses

- Unitarity as an inequality is valid at all energies and spins

$$|S_\ell(s)|^2 \le 1, \quad s > 0, \quad \ell = 0,2,\ldots,\infty$$

Partial Wave Unitarity

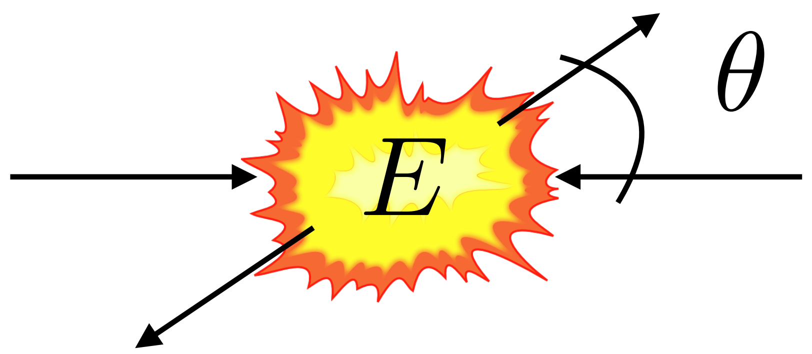
- No unitarity for generic $\nu_{(a,b,c)}$

$$A(s,t,u) = \frac{8\pi G_D}{stu} + \prod_{X=s,t,u} (\rho_X + 1)^2 \sum_{a+b+c\leq N} \nu_{(a,b,c)} \rho_s^a \rho_t^b \rho_u^c$$

- We must impose **unitarity** numerically on $T(s,t,u) = s^4 A(s,t,u)$

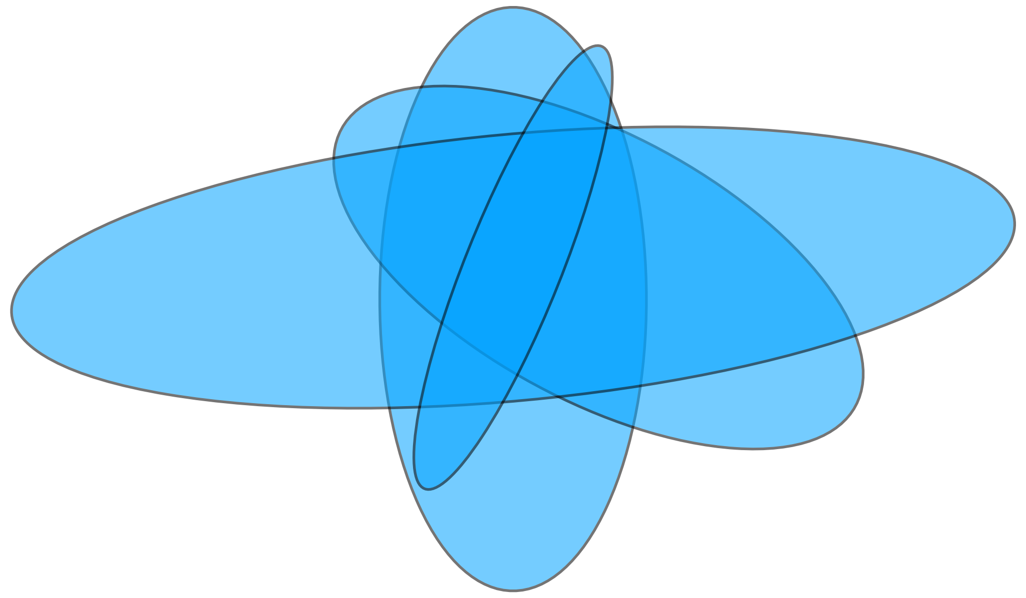
$$T\left(s, t = -\frac{s}{2}(1-\cos\theta), u = -\frac{s}{2}(1+\cos\theta)\right)$$

$x = \cos\theta$



Example: 10D

$$\boxed{S_\ell(s) = 1 + i\frac{s^3}{2^{18}3\pi^4}\int_{-1}^{1}(1-x^2)^3 \frac{C_\ell^{7/2}(x)}{C_\ell^{7/2}(1)} T(s,x)}$$



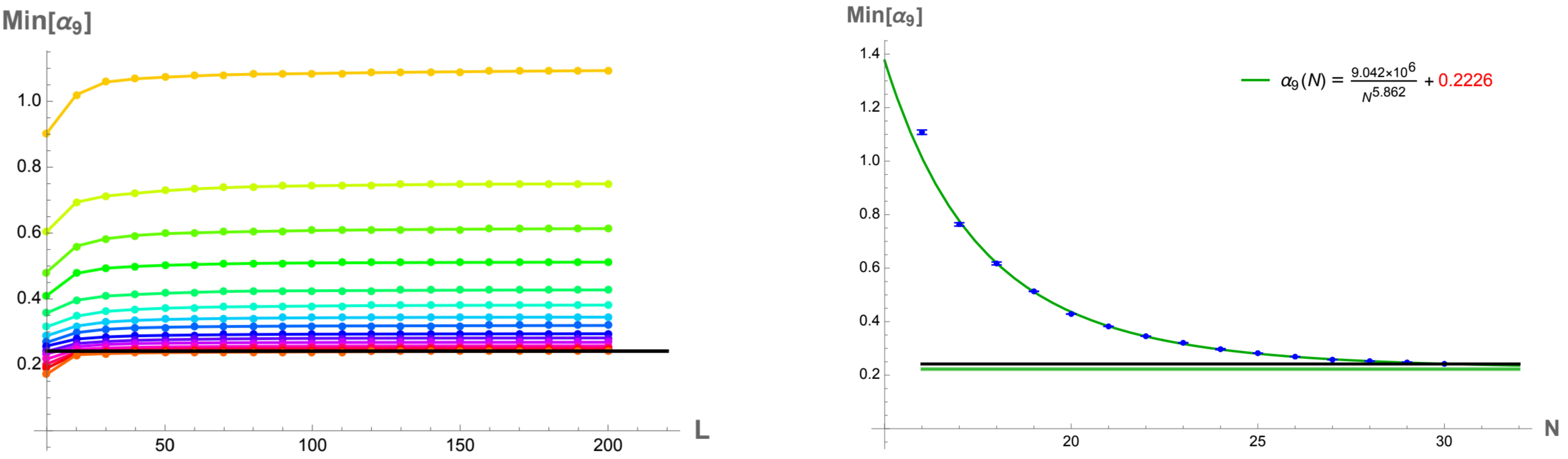
$\nu_{(a,b,c)}$ live inside the intersection of ellipses

- Unitarity as an inequality is valid at all energies and spins

$$|S_\ell(s)|^2 \leq 1, \quad s > 0, \quad \ell = 0,2,\ldots,\infty$$

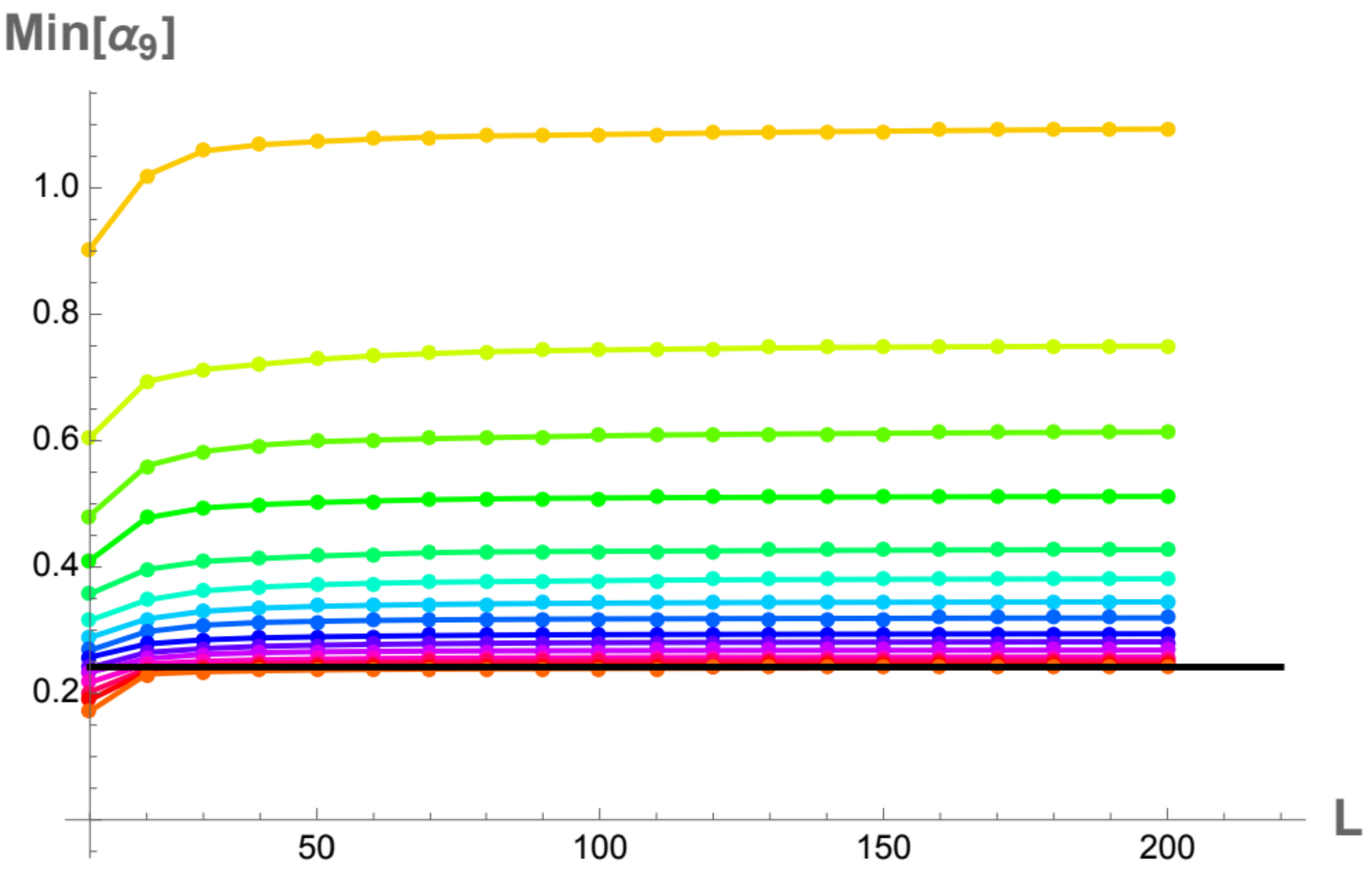
$$\ell < L_{max}$$

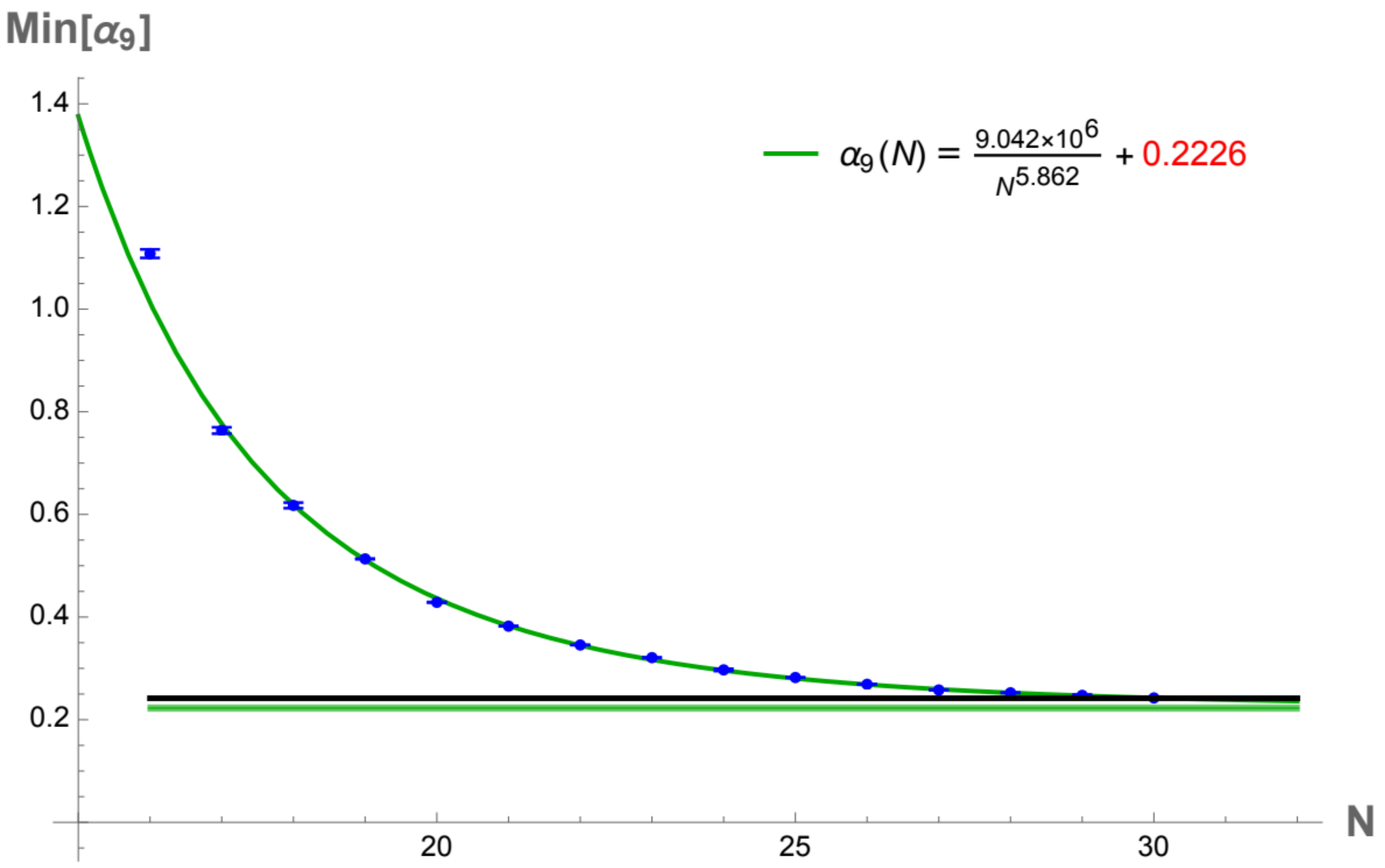
QG Bootstrap: the bounds



Existence of Universal lower bound depending on low energy SUGRA, analyticity, crossing, and unitarity

QG Bootstrap: the bounds





Existence of Universal lower bound depending on low energy SUGRA, analyticity, crossing, and unitarity

1. The Bound on α_D

$$\alpha_D^{\text{min}} < \alpha_D < \infty$$

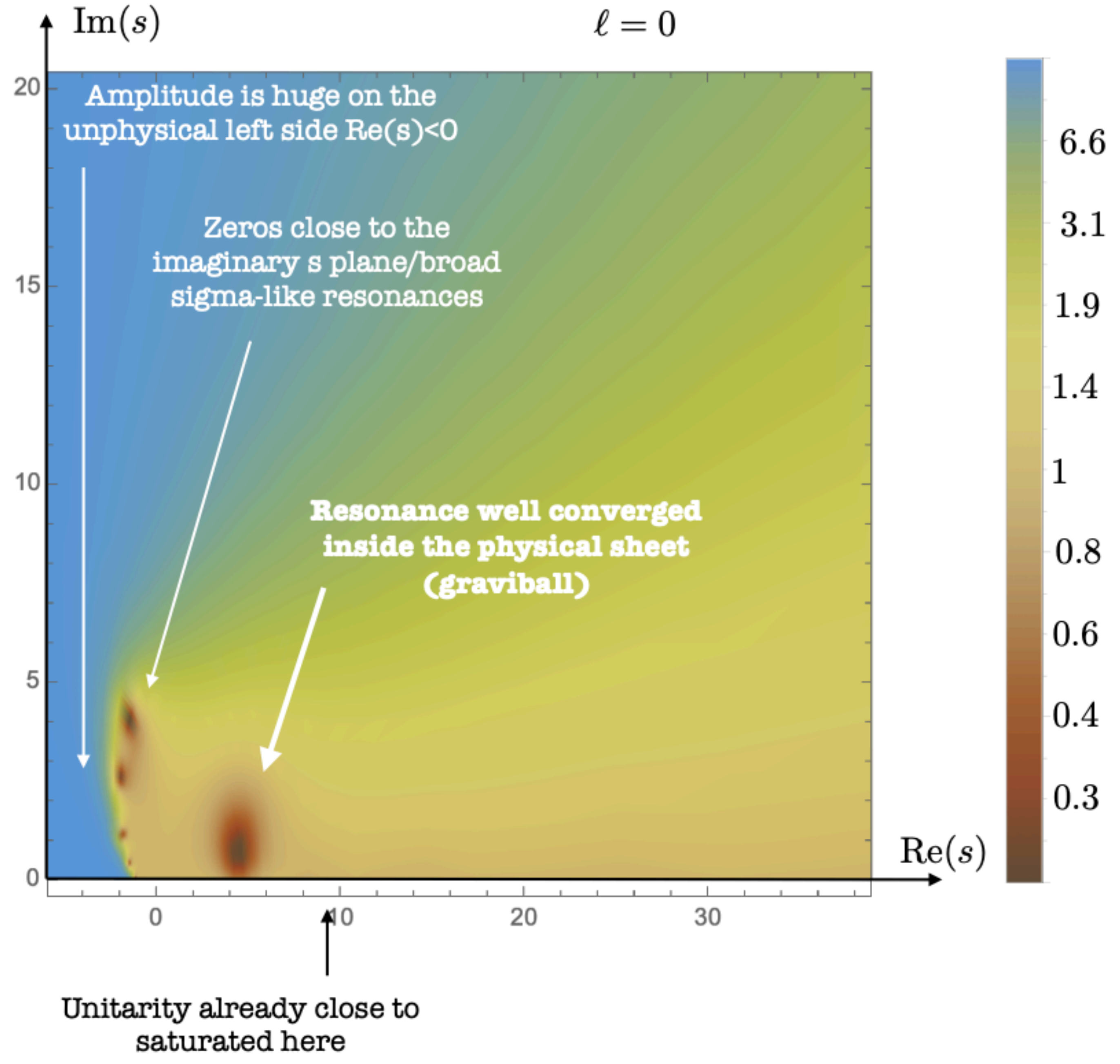
Dimension	String/M theory	Bootstrap α_D^{min}
9	≥ 0.2411	0.223 ± 0.002
10	≥ 0.1389	0.124 ± 0.003
11	0.1304	0.101 ± 0.005

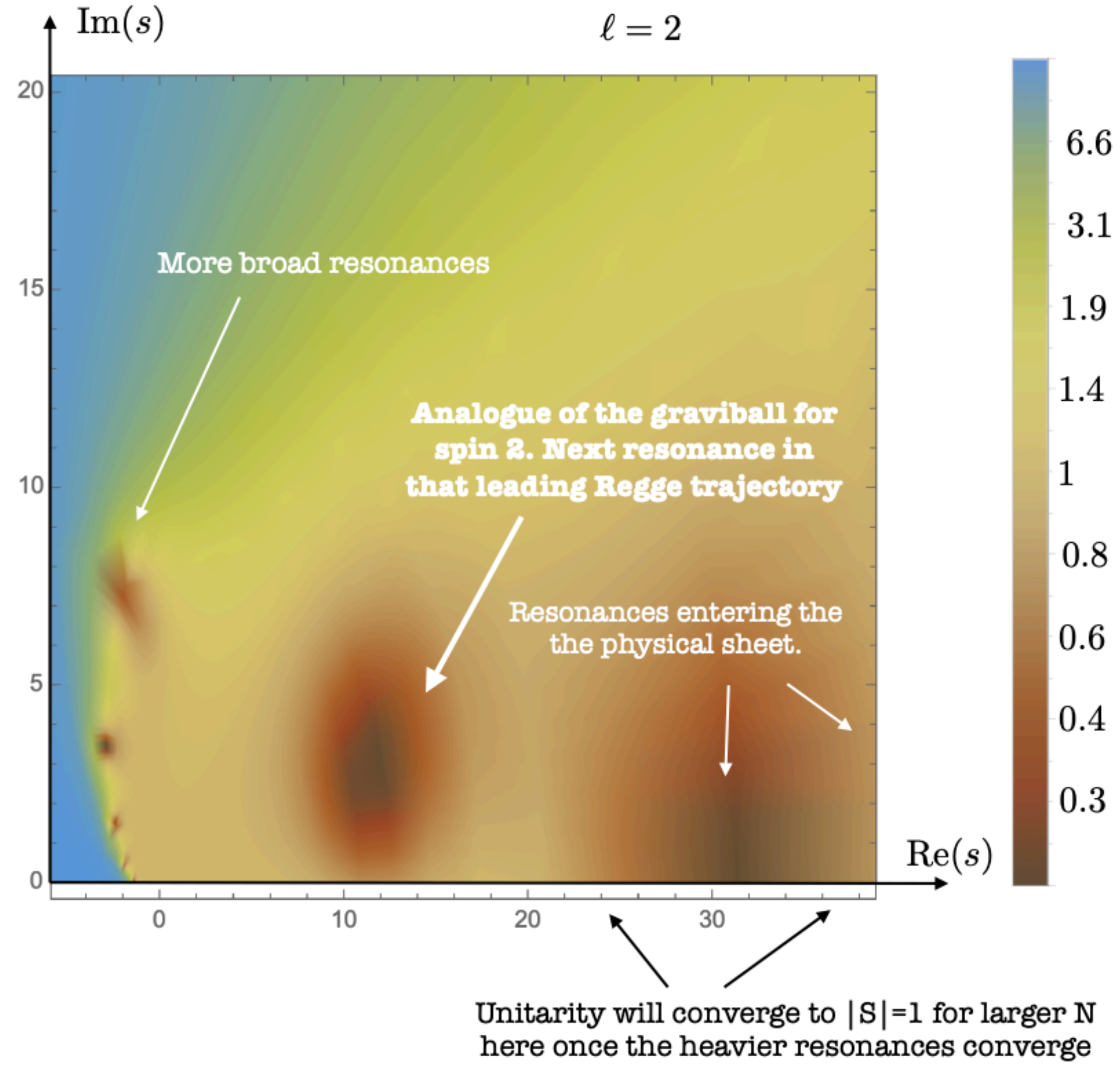
a) D=9, 10 String Theory almost saturates the allowed region for α

b) α for M-theory is close to the boundary of the allowed region

QG Bootstrap: the non-perturbative amplitude

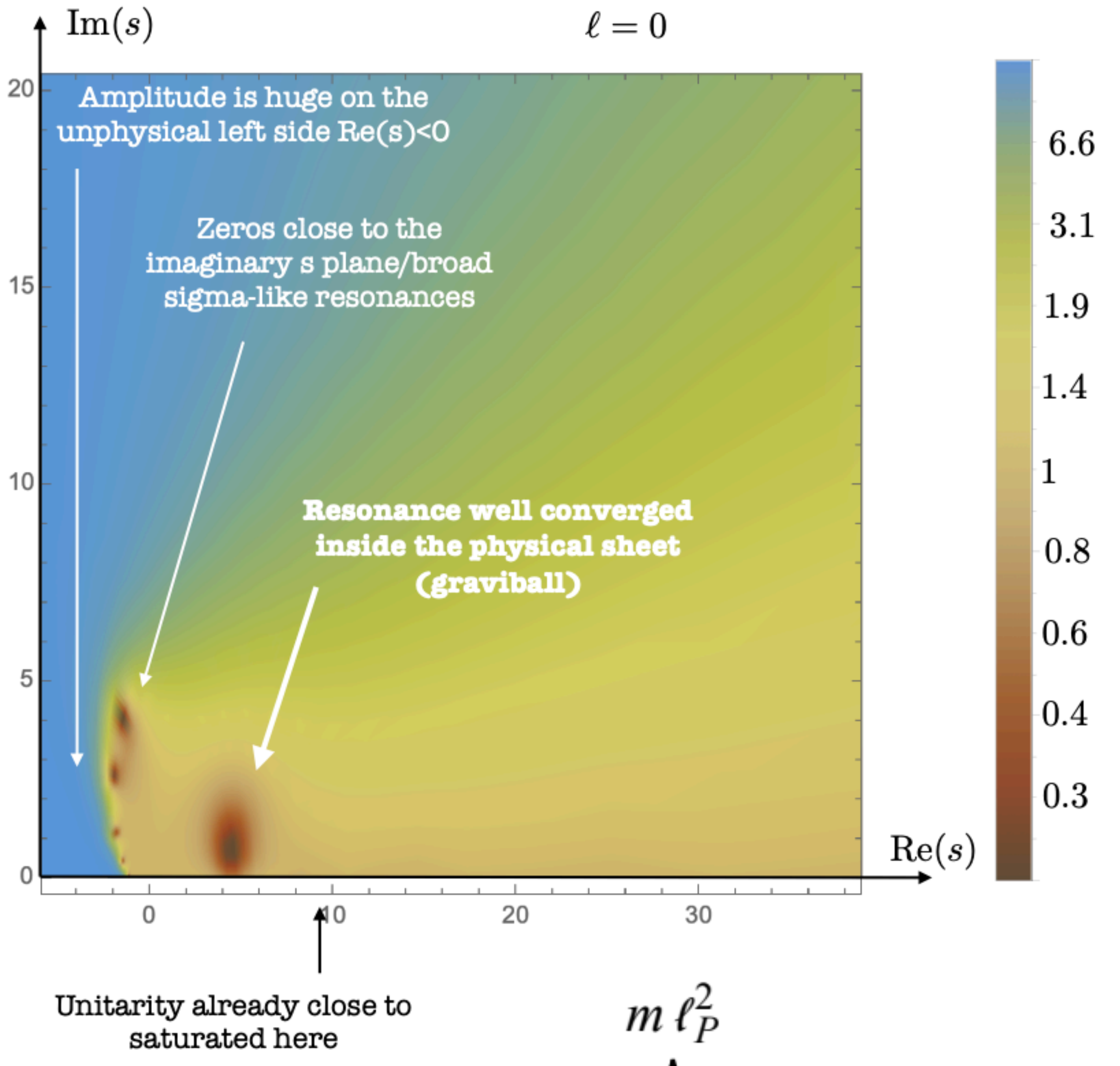
The function that minimizes α_D has an intricate structures of zeros, some of those are resonances, others we don't know!

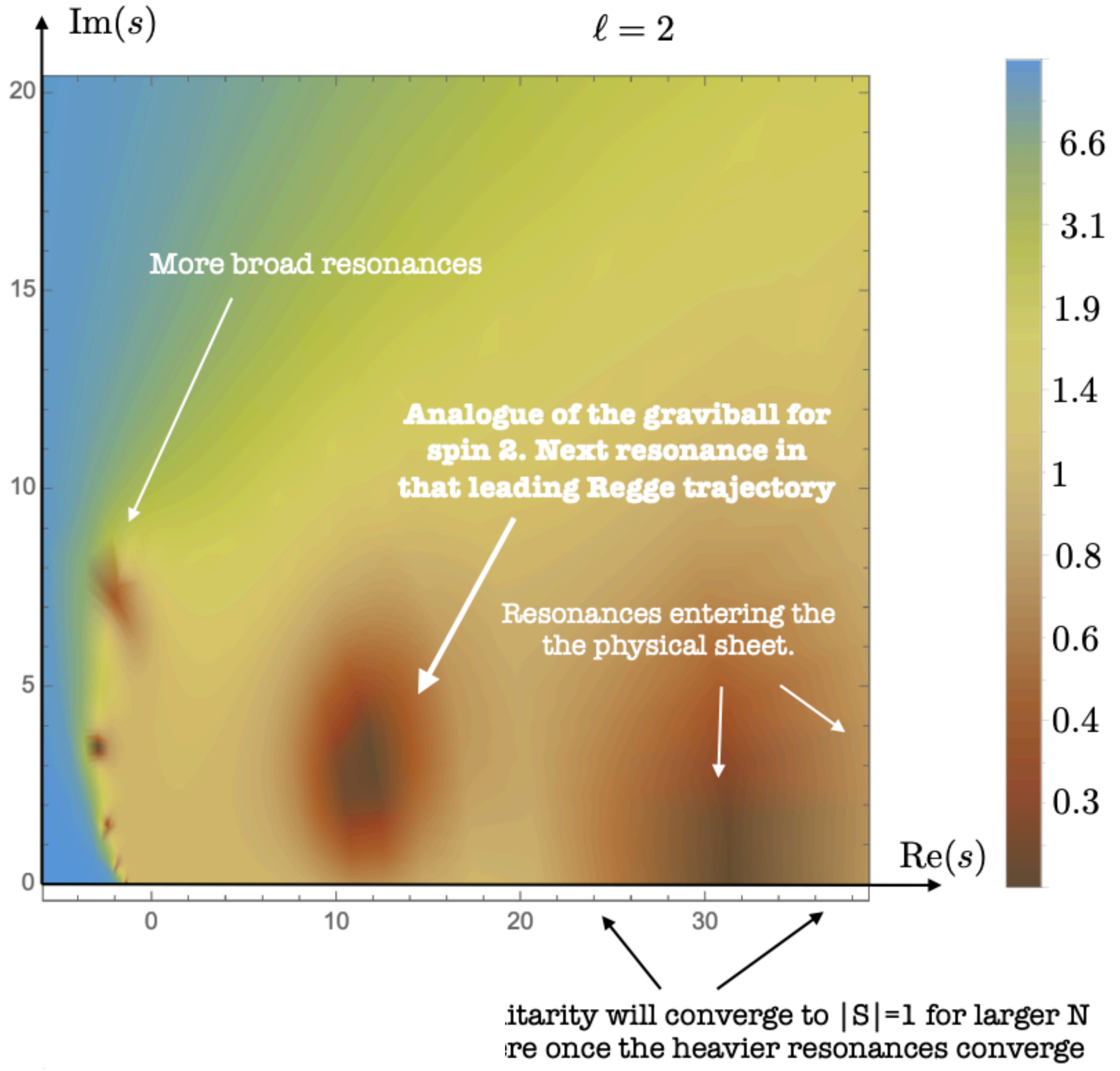


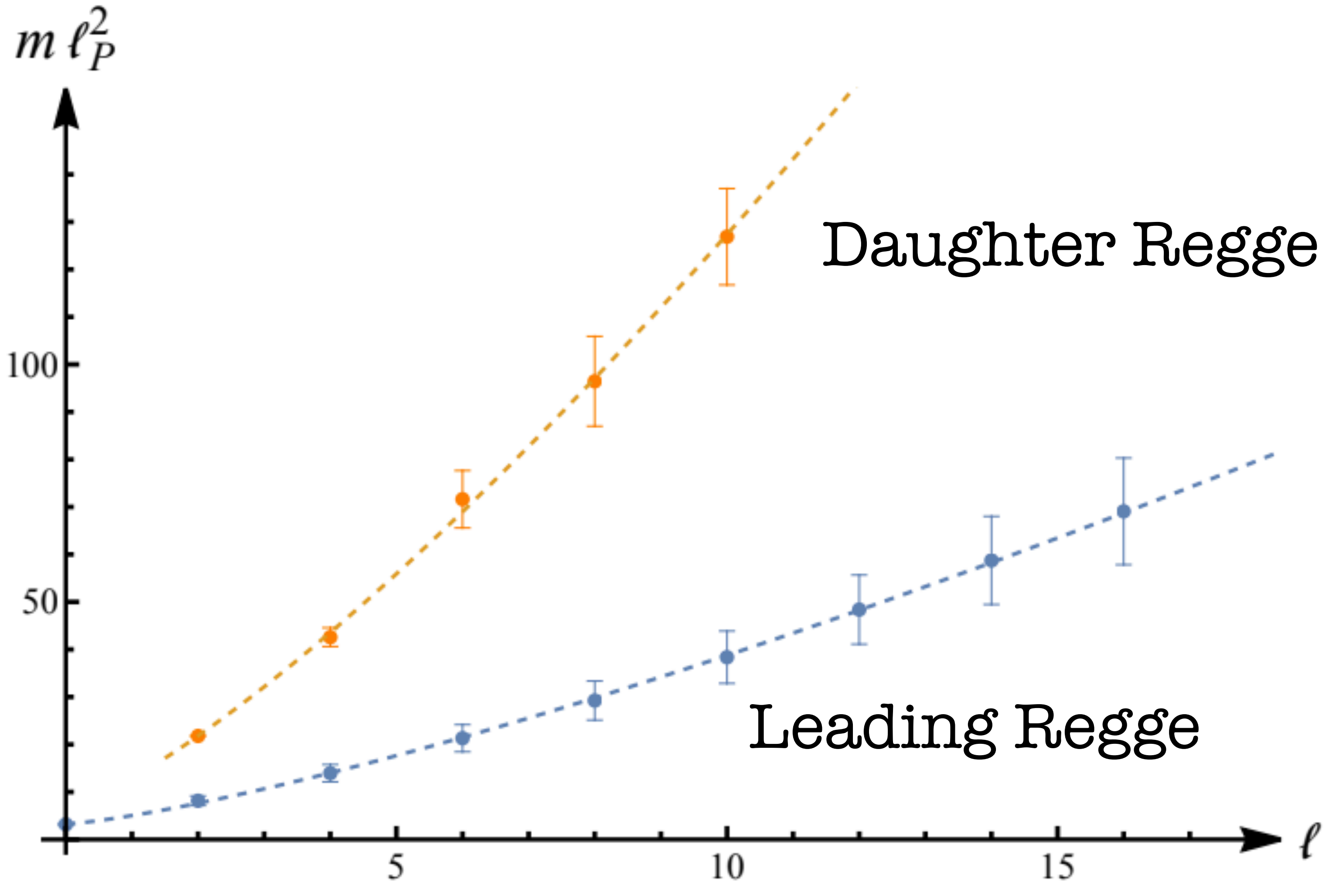


QG Bootstrap: the non-perturbative amplitude

The function that minimizes α_D has an intricate structures of zeros, some of those are resonances, others we don't know!







Resonance spectrum organizes in (curved) Regge trajectories

Stringy Spectrum although there is no assumption about the UV completion

Is String Theory the only consistent theory of QG?

1. The Bound on α_D

$$\alpha_D^{\text{min}} < \alpha_D < \infty$$

Dimension	String/M theory	Bootstrap α_D^{min}
9	≥ 0.2411	0.223 ± 0.002
10	≥ 0.1389	0.124 ± 0.003
11	0.1304	0.101 ± 0.005

α

String Theory

More exotic UV completions?

The Desert

Is String Theory the only consistent theory of QG?

1. The Bound on α_D

$$\alpha_D^{\min} < \alpha_D < \infty$$

Dimension	String/M theory	Bootstrap $\alpha_D^{\min}$
9	≥ 0.2411	0.223 ± 0.002
10	≥ 0.1389	0.124 ± 0.003
11	0.1304	0.101 ± 0.005

α

String Theory

More exotic UV completions?

The Desert

Can we close the gap with Black-Holes Production?

$$\boxed{\alpha \geq \frac{16}{3\pi^4 \ell_P^{14}} \sum_l (l+1)_6 (2l+7) \int_0^\infty ds \frac{\eta_l(s)}{s^8}}$$

α **increases if we add inelasticity:** $\eta = 1 - Prob(2 \to 2)$

Is String Theory the only consistent theory of QG?

1. The Bound on α_D

$$\alpha_D^{\text{min}} < \alpha_D < \infty$$

Dimension	String/M theory	Bootstrap α_D^{min}
9	≥ 0.2411	0.223 ± 0.002
10	≥ 0.1389	0.124 ± 0.003
11	0.1304	0.101 ± 0.005

α

String Theory

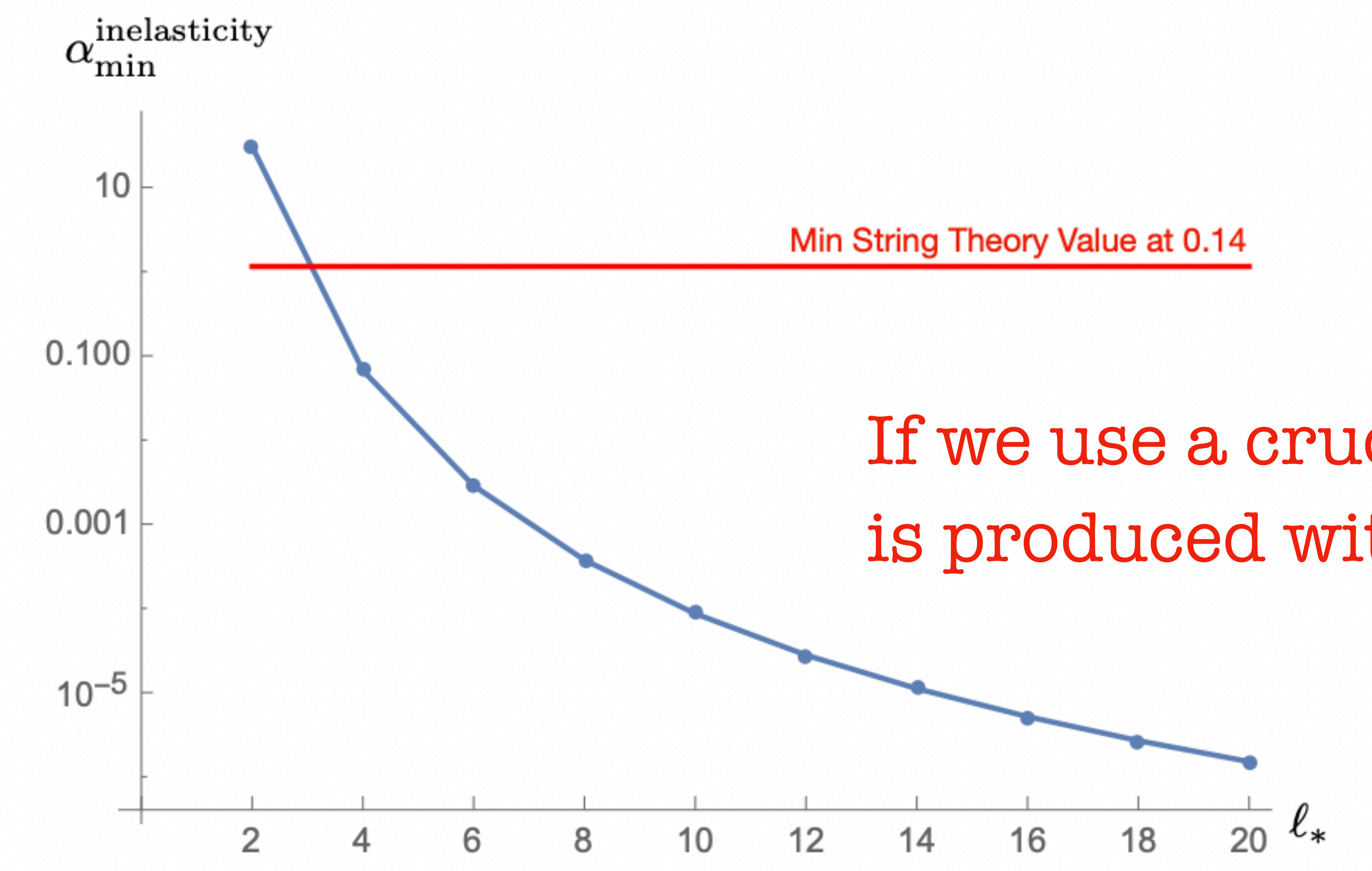
More exotic UV completions?

The Desert

Can we close the gap with Black-Holes Production?

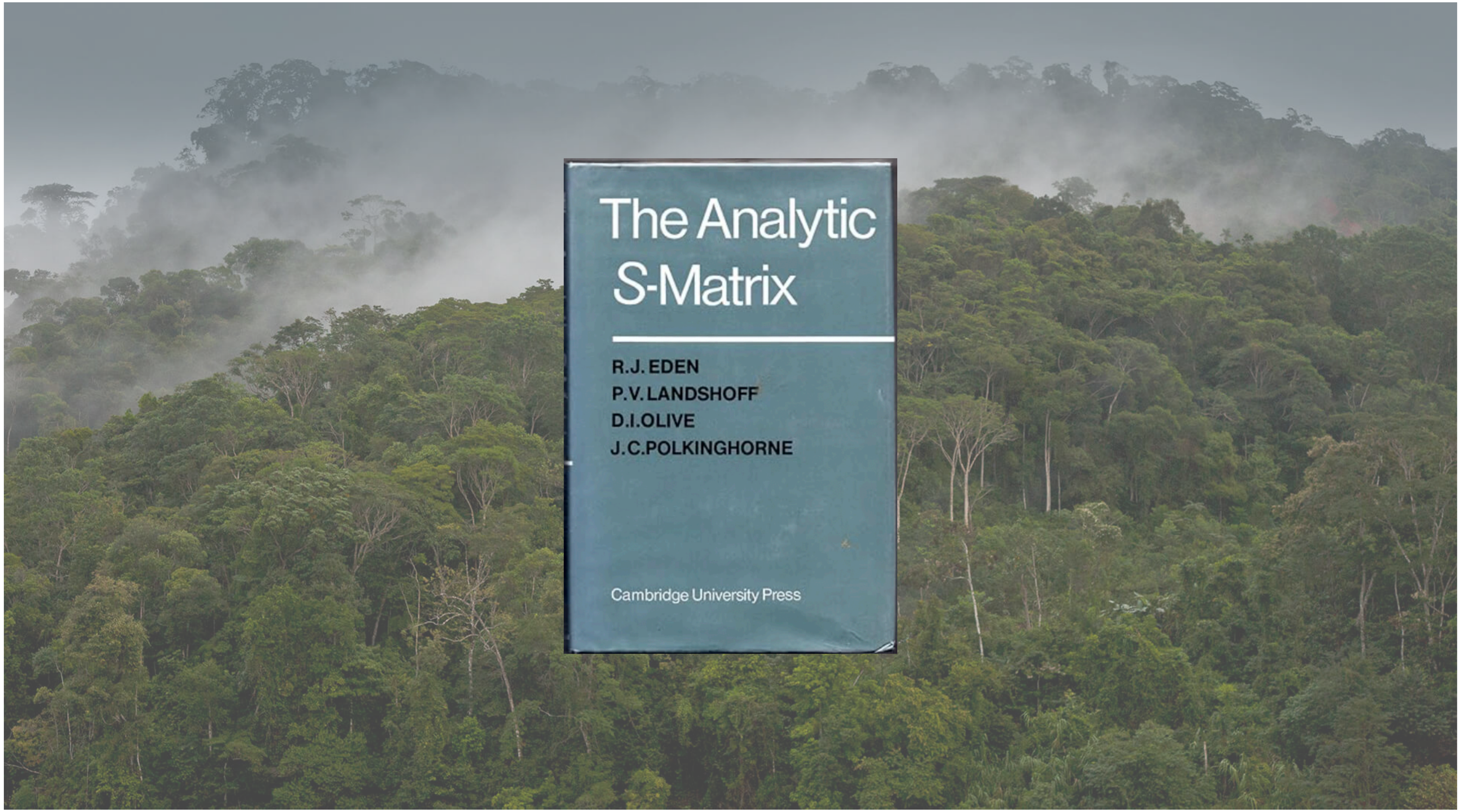
$$\boxed{\alpha \geq \frac{16}{3\pi^4 \ell_P^{14}} \sum_l (l+1)_6(2l+7) \int_0^\infty ds \frac{\eta_l(s)}{s^8}}$$

α **increases if we add inelasticity:** $\eta = 1 - Prob(2 \to 2)$



If we use a crude model: $\eta \sim 1$ when a BH is produced with spin $\ell > \ell_*$

To understand inelasticity we have to step in the multi-particle world



The simplest non-integrable S-matrix

The simplest theory of quantum gravity

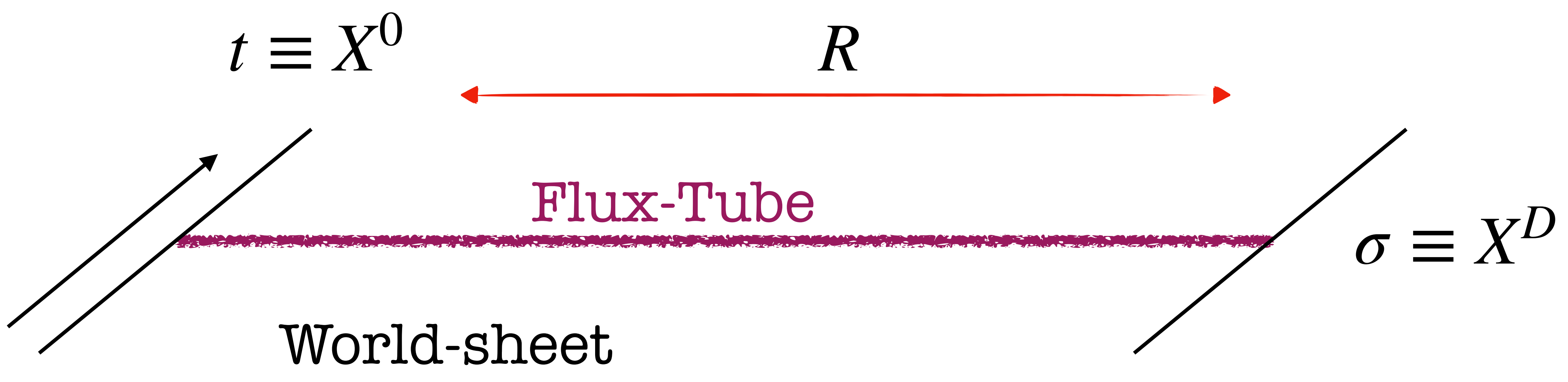
Dubovsky, Gorbenko, Flauger 1205.6805

The simplest non-integrable S-matrix

The simplest theory of quantum gravity

Dubovsky, Gorbenko, Flauger 1205.6805

The theory of the confining strings in 2+1 dimensions



Physical Degrees of freedom:

X^i with i=2,…,D massless Goldstones

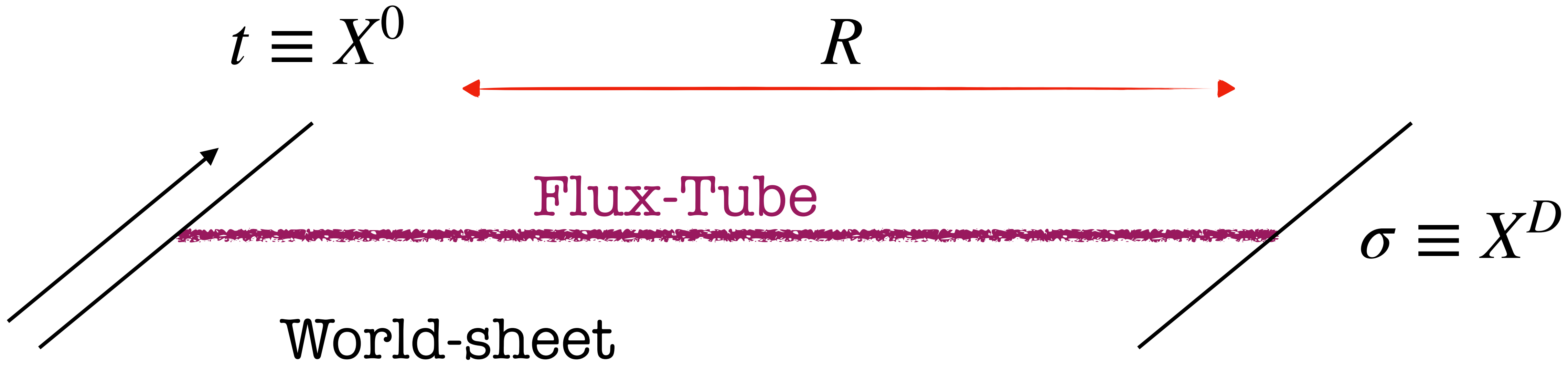
$SO(1,D-1) \to SO(1,1) \times O(D-2)$

The simplest non-integrable S-matrix

The simplest theory of quantum gravity

Dubovsky, Gorbenko, Flauger 1205.6805

The theory of the confining strings in 2+1 dimensions



Physical Degrees of freedom:

X^i with i=2,...,D massless Goldstones

$SO(1,D-1) \to SO(1,1) \times O(D-2)$

In D=3 we have a single Goldstone X

$$\mathcal{A}_{EFF} = \int d^2\sigma \sqrt{-h} \left(\frac{1}{\ell_s^2} + \gamma_2 \ell_s^2 R + \gamma_3 \ell_s^4 R^2 + \dots \right)$$

In $D \neq 26$, infinite corrections

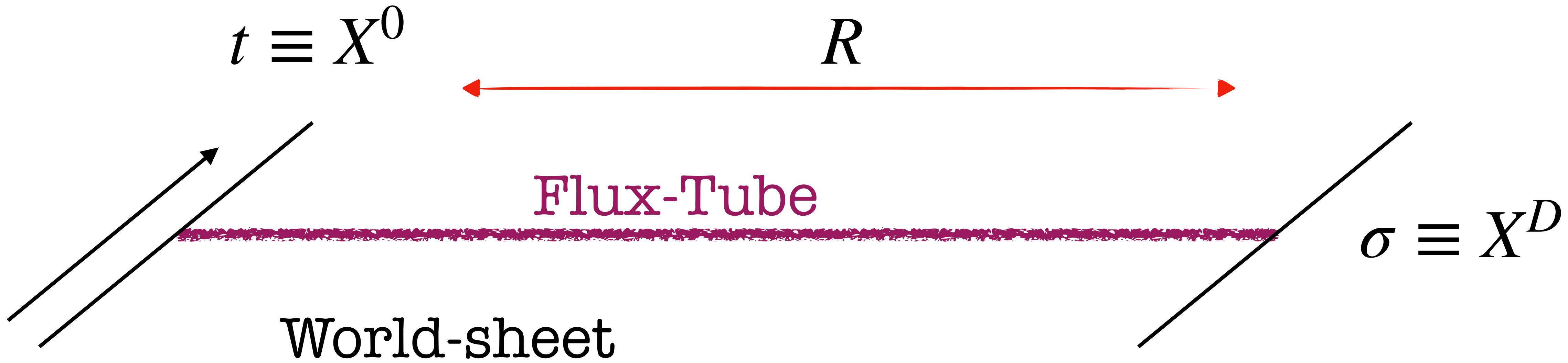
γ_3 parametrizes the leading violation from String universality

The simplest non-integrable S-matrix

The simplest theory of quantum gravity

Dubovsky, Gorbenko, Flauger 1205.6805

The theory of the confining strings in 2+1 dimensions



Physical Degrees of freedom:

X^i with i=2,...,D massless Goldstones

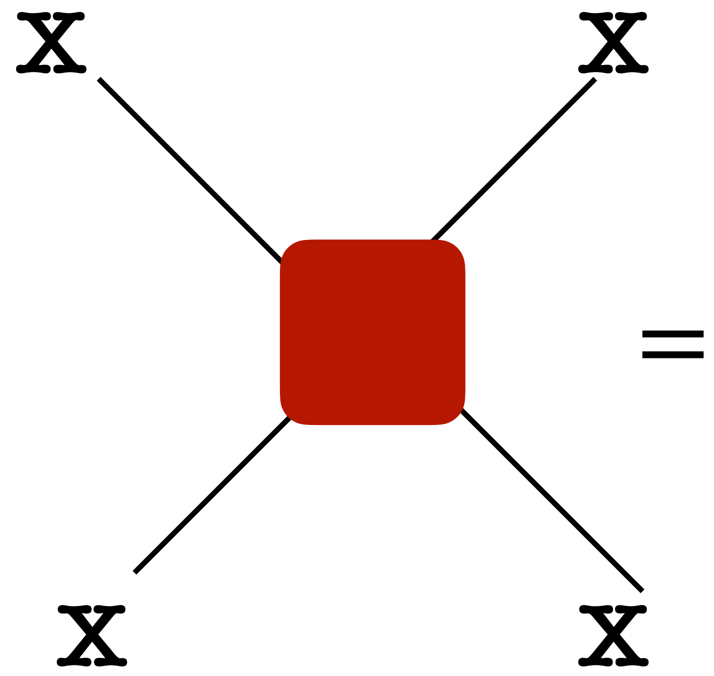
$SO(1,D-1) \to SO(1,1) \times O(D-2)$

In D=3 we have a single Goldstone X

$$\mathcal{A}_{EFF} = \int d^2\sigma \sqrt{-h} \left(\frac{1}{\ell_s^2} + \gamma_2 \ell_s^2 R + \gamma_3 \ell_s^4 R^2 + \dots \right)$$

In $D \neq 26$, infinite corrections

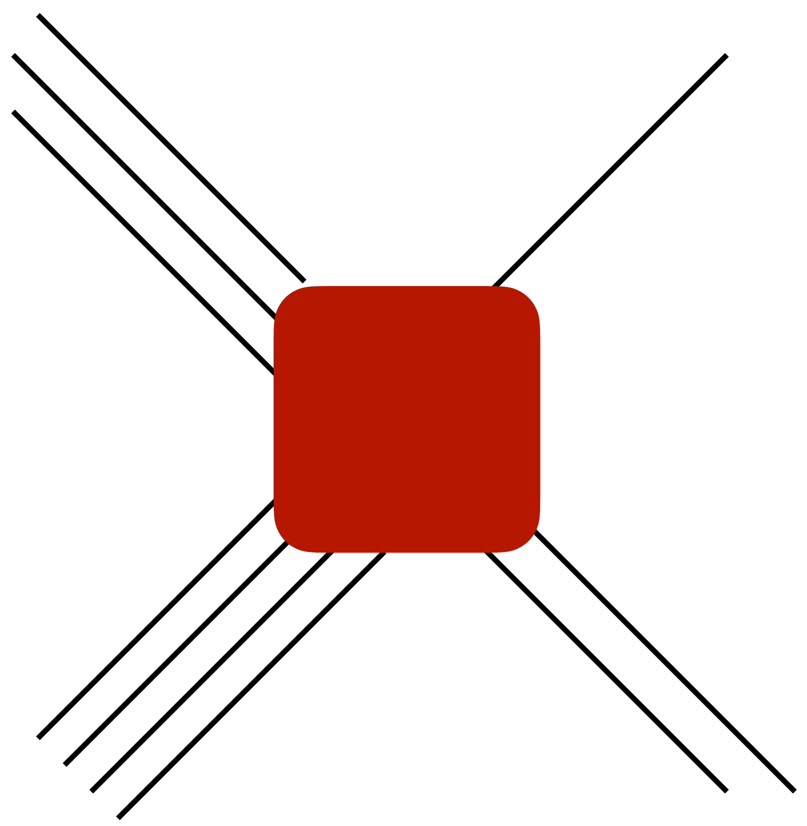
γ_3 parametrizes the leading violation from String universality



$$= \frac{s^2}{4} + i\frac{s^3}{16} + \left(2\gamma_3 - \frac{1}{192}\right) + \dots$$

Analytic S-matrix Bootstrap: $\gamma_3 \geq -\frac{1}{768}$

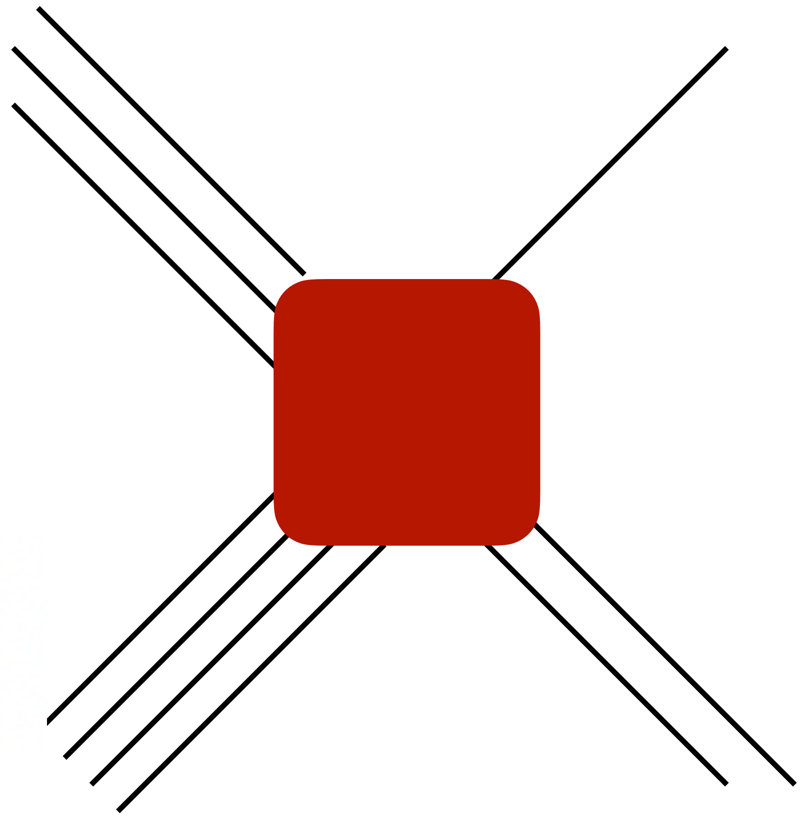
Jet-Basis for multi-particle amplitudes



Consider an $n \to m$ process

In 1+1 dimensions particles can be Left or Right movers

Jet-Basis for multi-particle amplitudes



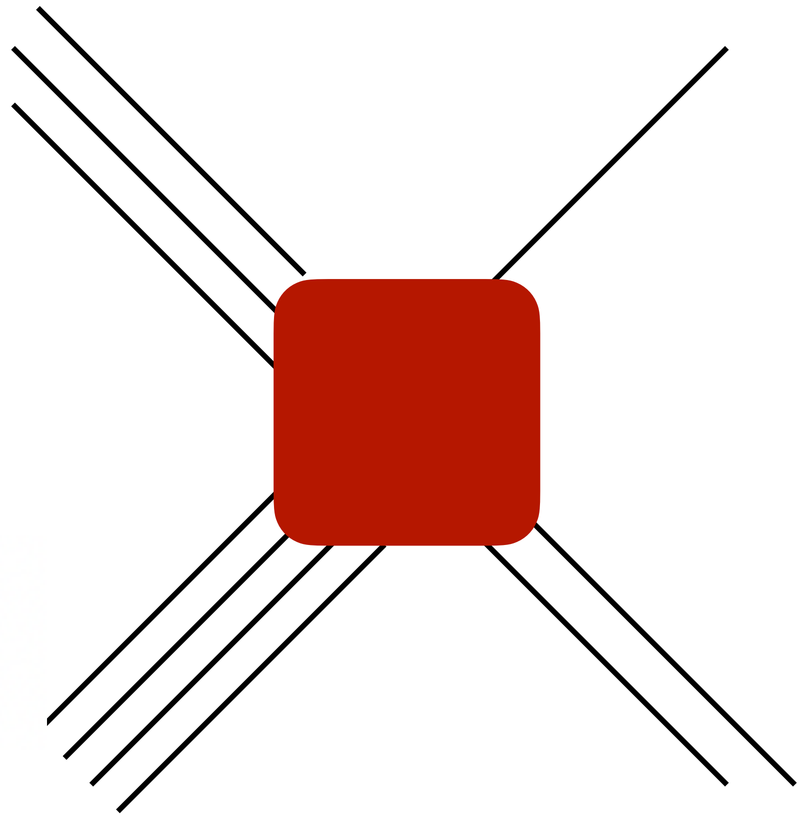
Consider an $n \to m$ process

In 1+1 dimensions particles can be Left or Right movers

Consider a jet of n collinear particles $|\alpha_1, \ldots, \alpha_n, P\rangle_n \equiv |\alpha_1 \vec{P}, \alpha_2 \vec{P}, \ldots, \alpha_n \vec{P}\rangle$

$$\alpha_1 + \ldots + \alpha_n = 1$$

Jet-Basis for multi-particle amplitudes



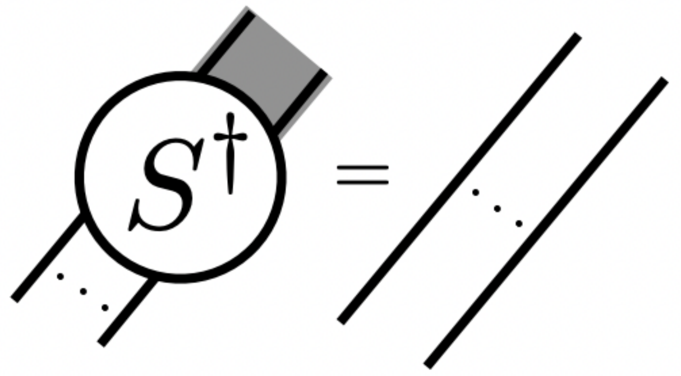
Consider an $n \to m$ process

In 1+1 dimensions particles can be Left or Right movers

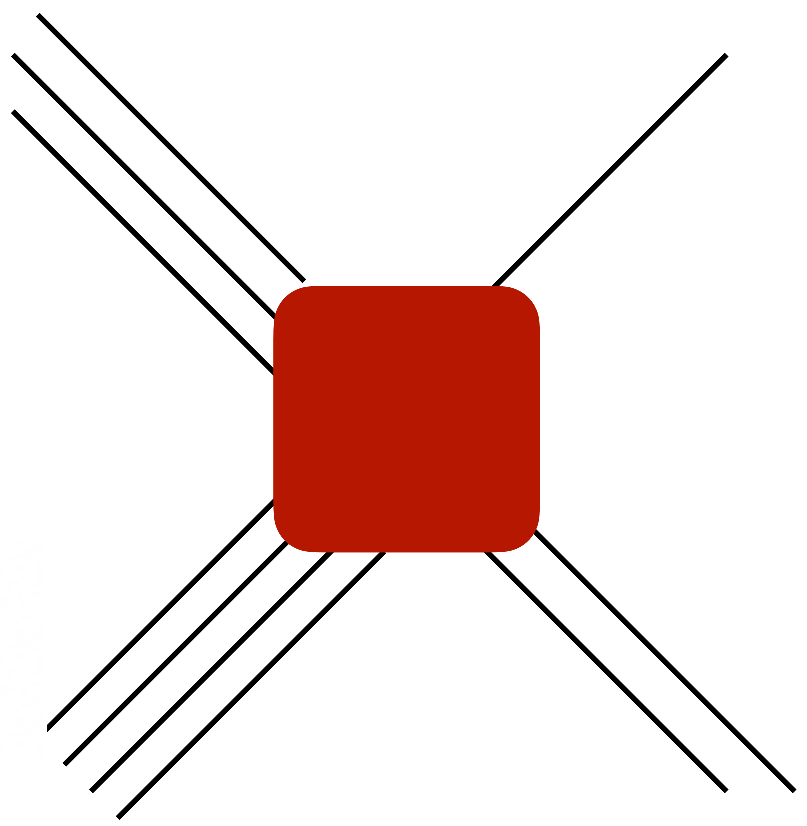
Consider a jet of n collinear particles $|\alpha_1, \ldots, \alpha_n, P\rangle_n \equiv |\alpha_1 \vec{P}, \alpha_2 \vec{P}, \ldots, \alpha_n \vec{P}\rangle$

$$\alpha_1 + \ldots + \alpha_n = 1$$

Such a state is stable because collinear scattering is trivial at all energies $S_{LL'} = S_{RR'} = 1$



Jet-Basis for multi-particle amplitudes



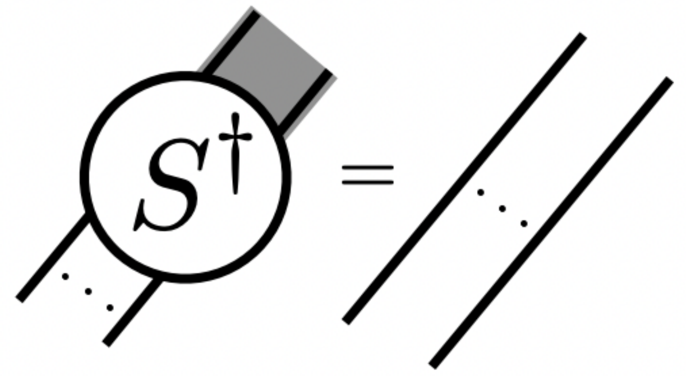
Consider an $n \to m$ process

In 1+1 dimensions particles can be Left or Right movers

Consider a jet of n collinear particles $|\alpha_1, \ldots, \alpha_n, P\rangle_n \equiv |\alpha_1 \vec{P}, \alpha_2 \vec{P}, \ldots, \alpha_n \vec{P}\rangle$

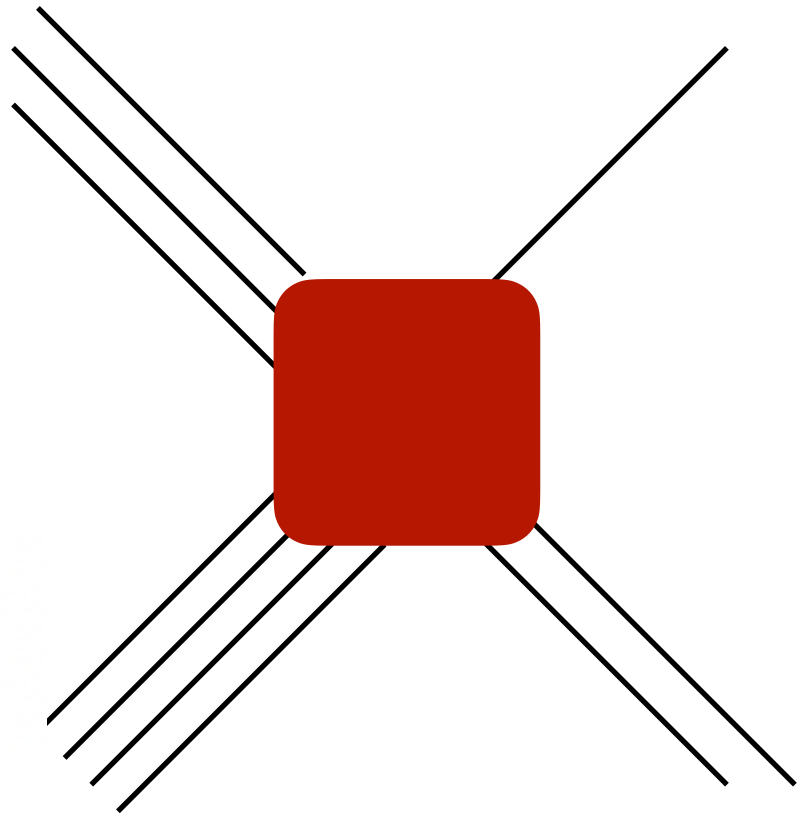
$$\alpha_1 + \ldots + \alpha_n = 1$$

Such a state is stable because collinear scattering is trivial at all energies $S_{LL'} = S_{RR'} = 1$



Goal: find an orthonormal basis of functions in $\alpha_1, \ldots, \alpha_{n-1}$ on the unit simplex, such that jet-states are normalized as single particle states

Jet-Basis for multi-particle amplitudes



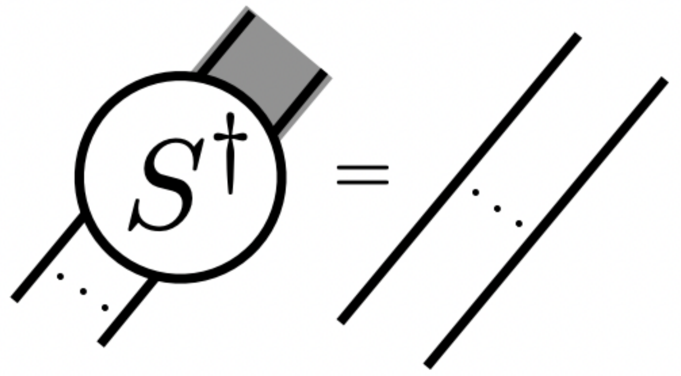
Consider an $n \to m$ process

In 1+1 dimensions particles can be Left or Right movers

Consider a jet of n collinear particles $|\alpha_1, \ldots, \alpha_n, P\rangle_n \equiv |\alpha_1 \vec{P}, \alpha_2 \vec{P}, \ldots, \alpha_n \vec{P}\rangle$

$$\alpha_1 + \ldots + \alpha_n = 1$$

Such a state is stable because collinear scattering is trivial at all energies $S_{LL'} = S_{RR'} = 1$



Goal: find an orthonormal basis of functions in $\alpha_1, \ldots, \alpha_{n-1}$ on the unit simplex, such that jet-states are normalized as single particle states

E.g: 2-jet state of species n

$$|n, P\rangle \equiv \sqrt{2n+1} \int_0^1 d\alpha \frac{P_n(2\alpha - 1)}{\sqrt{8\pi\alpha(1-\alpha)}} |\alpha, (1-\alpha), P\rangle_2$$

$$\langle m, Q | n, P\rangle = 4\pi P_0 \delta_{n,m} \delta(\vec{Q} - \vec{P})\,.$$

Unitarity and Analyticity in the Jet Basis

Call **1** the branon state, and **2** the 2-jet state with $n = 0$

$$\begin{pmatrix} 1 & S_{11\to 11} & 0 & S_{11\to 22} \\ S^*_{11\to 11} & 1 & S^*_{22\to 11} & 0 \\ 0 & S_{22\to 11} & 1 & S_{22\to 22} \\ S^*_{11\to 22} & 0 & S^*_{22\to 22} & 1 \end{pmatrix} \succeq 0$$

Unitarity and Analyticity in the Jet Basis

Call **1** the branon state, and **2** the 2-jet state with $n = 0$

$$1 - |S_{11\to 11}|^2 \geq 0$$

$$\begin{pmatrix} \boxed{\begin{matrix} 1 & S_{11\to 11} \\ S^*_{11\to 11} & 1 \end{matrix}} & \begin{matrix} 0 & S_{11\to 22} \\ S^*_{22\to 11} & 0 \end{matrix} \\ \begin{matrix} 0 & S_{22\to 11} \\ S^*_{11\to 22} & 0 \end{matrix} & \begin{matrix} 1 & S_{22\to 22} \\ S^*_{22\to 22} & 1 \end{matrix} \end{pmatrix} \succeq 0$$

Unitarity and Analyticity in the Jet Basis

Call **1** the branon state, and **2** the 2-jet state with $n = 0$

$1 - |S_{11\to 11}|^2 - |S_{11\to 22}|^2 \geq 0$

$1 - |S_{11\to 11}|^2 \geq 0$

$$\begin{pmatrix} 1 & S_{11\to 11} & 0 & S_{11\to 22} \\ S^*_{11\to 11} & 1 & S^*_{22\to 11} & 0 \\ 0 & S_{22\to 11} & 1 & S_{22\to 22} \\ S^*_{11\to 22} & 0 & S^*_{22\to 22} & 1 \end{pmatrix} \succeq 0$$

Unitarity and Analyticity in the Jet Basis

Call **1** the branon state, and **2** the 2-jet state with $n = 0$

$$1 - |S_{11\to 11}|^2 \geq 0$$

$$1 - |S_{11\to 11}|^2 - |S_{11\to 22}|^2 \geq 0$$

$$\begin{pmatrix} 1 & S_{11\to11} & 0 & S_{11\to22} \\ S^*_{11\to11} & 1 & S^*_{22\to11} & 0 \\ 0 & S_{22\to11} & 1 & S_{22\to22} \\ S^*_{11\to22} & 0 & S^*_{22\to22} & 1 \end{pmatrix} \succeq 0$$

We argue that $S_{nm\to jk}(s)$ is an analytic function of s in the upper half-plane

AG, A. Homrich, and P. Vieira 2404.10812

Physically, this follows from the fact that a jet is indistinguishable from a single particle state

Unitarity and Analyticity in the Jet Basis

Call **1** the branon state, and **2** the 2-jet state with $n = 0$

$$1 - |S_{11\to 11}|^2 - |S_{11\to 22}|^2 \geq 0$$

$$1 - |S_{11\to 11}|^2 \geq 0$$

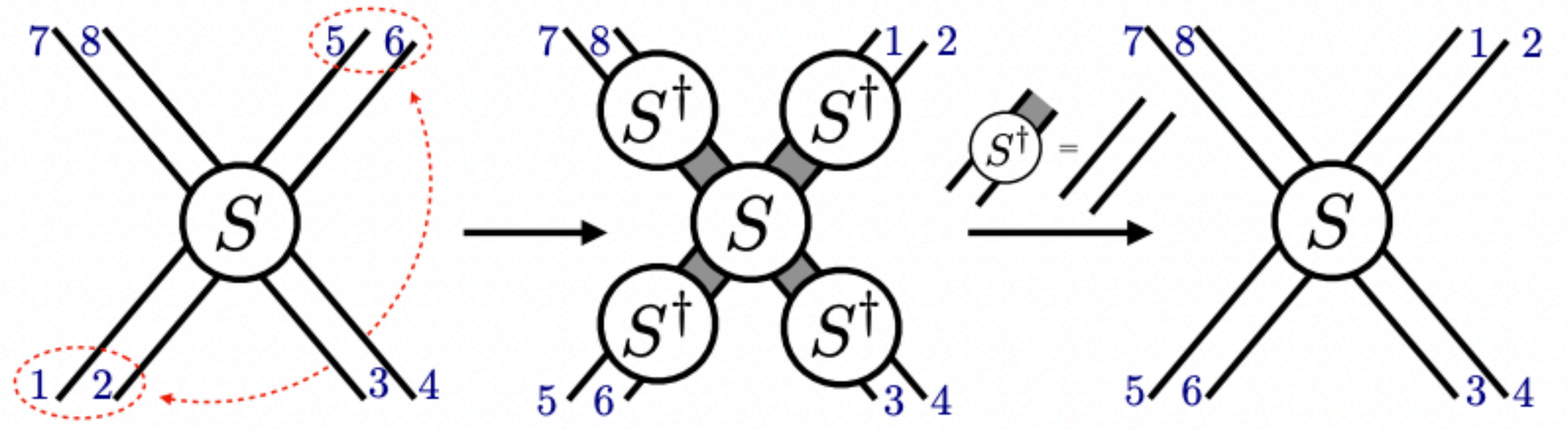
$$\begin{pmatrix} 1 & S_{11\to 11} & 0 & S_{11\to 22} \\ S^*_{11\to 11} & 1 & S^*_{22\to 11} & 0 \\ 0 & S_{22\to 11} & 1 & S_{22\to 22} \\ S^*_{11\to 22} & 0 & S^*_{22\to 22} & 1 \end{pmatrix} \succeq 0$$

We argue that $S_{nm\to jk}(s)$ is an analytic function of s in the upper half-plane

AG, A. Homrich, and P. Vieira 2404.10812

Physically, this follows from the fact that a jet is indistinguishable from a single particle state

Crossing is also satisfied thanks to $S_{LL'} = S_{RR'} = 1$



The Matrioska

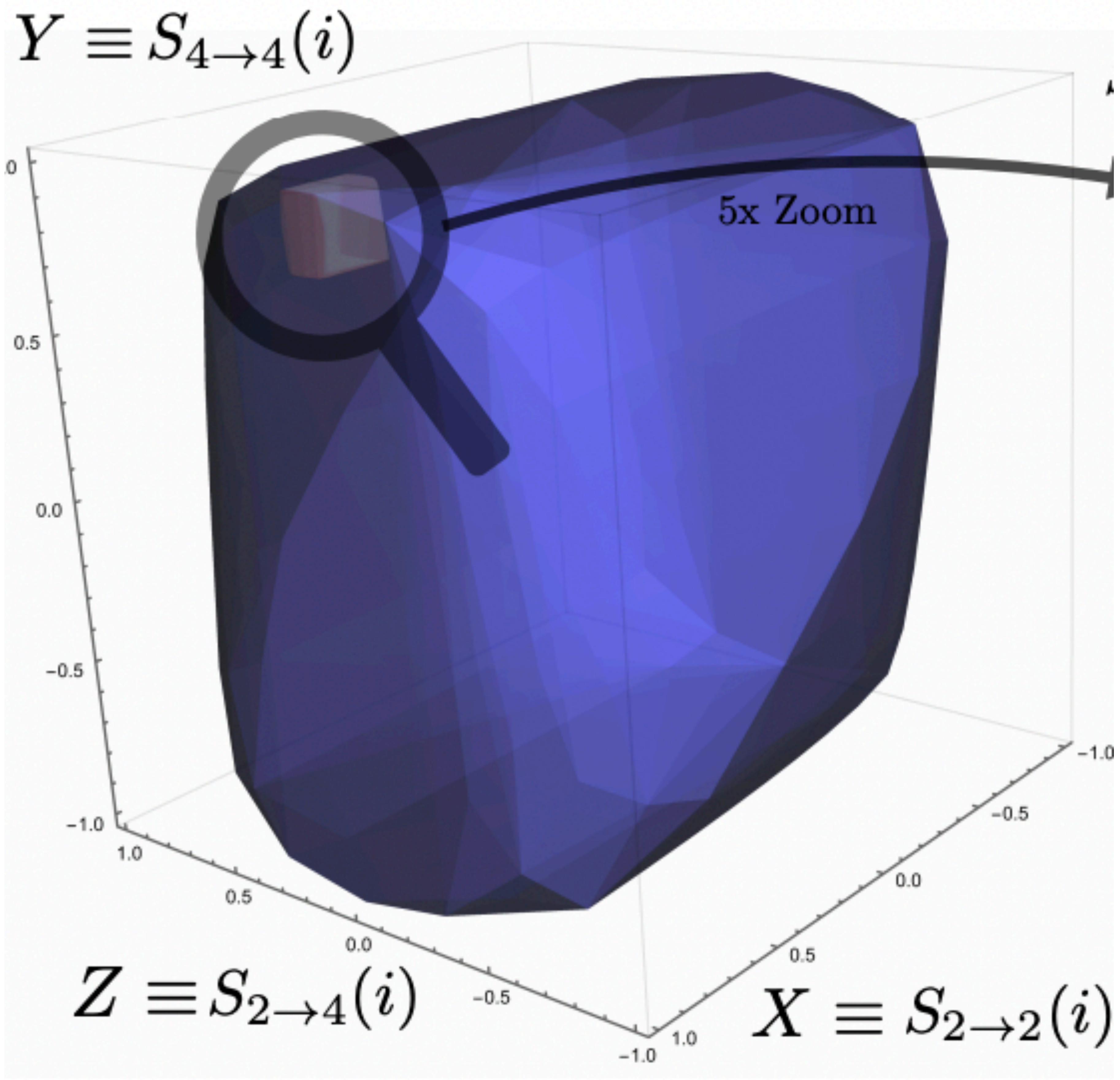
The observables we Bootstrap are effective 4, 6, and 8 particles couplings

$$(X, Y, Z) \equiv (S_{11\to 11}(i), S_{22\to 22}(i), \mathrm{Re}(S_{11\to 22}(i)))$$

The Matrioska

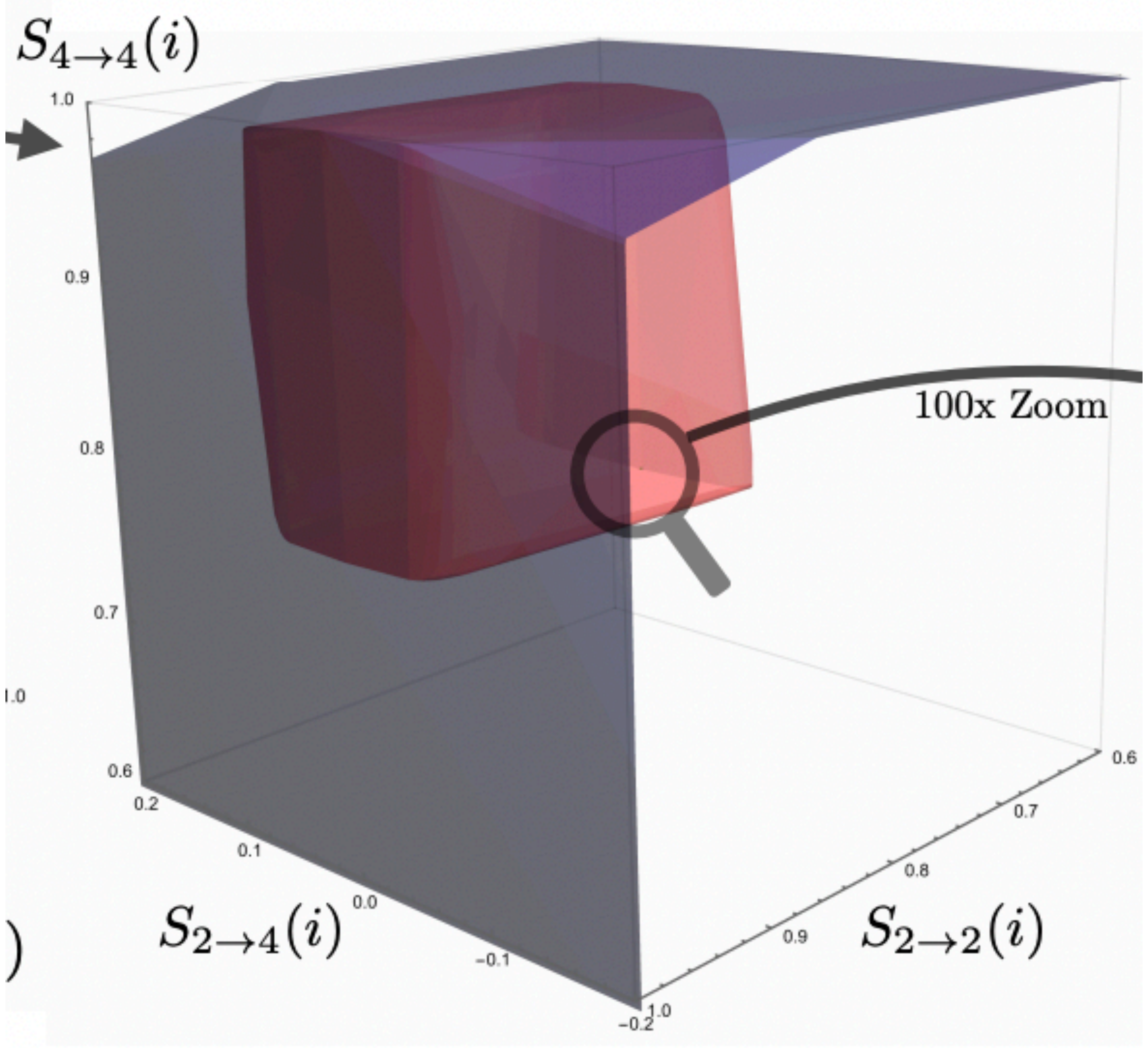
The observables we Bootstrap are effective 4, 6, and 8 particles couplings

$$(X, Y, Z) \equiv (S_{11\to11}(i), S_{22\to22}(i), \mathrm{Re}(S_{11\to22}(i)))$$



The Matrioska

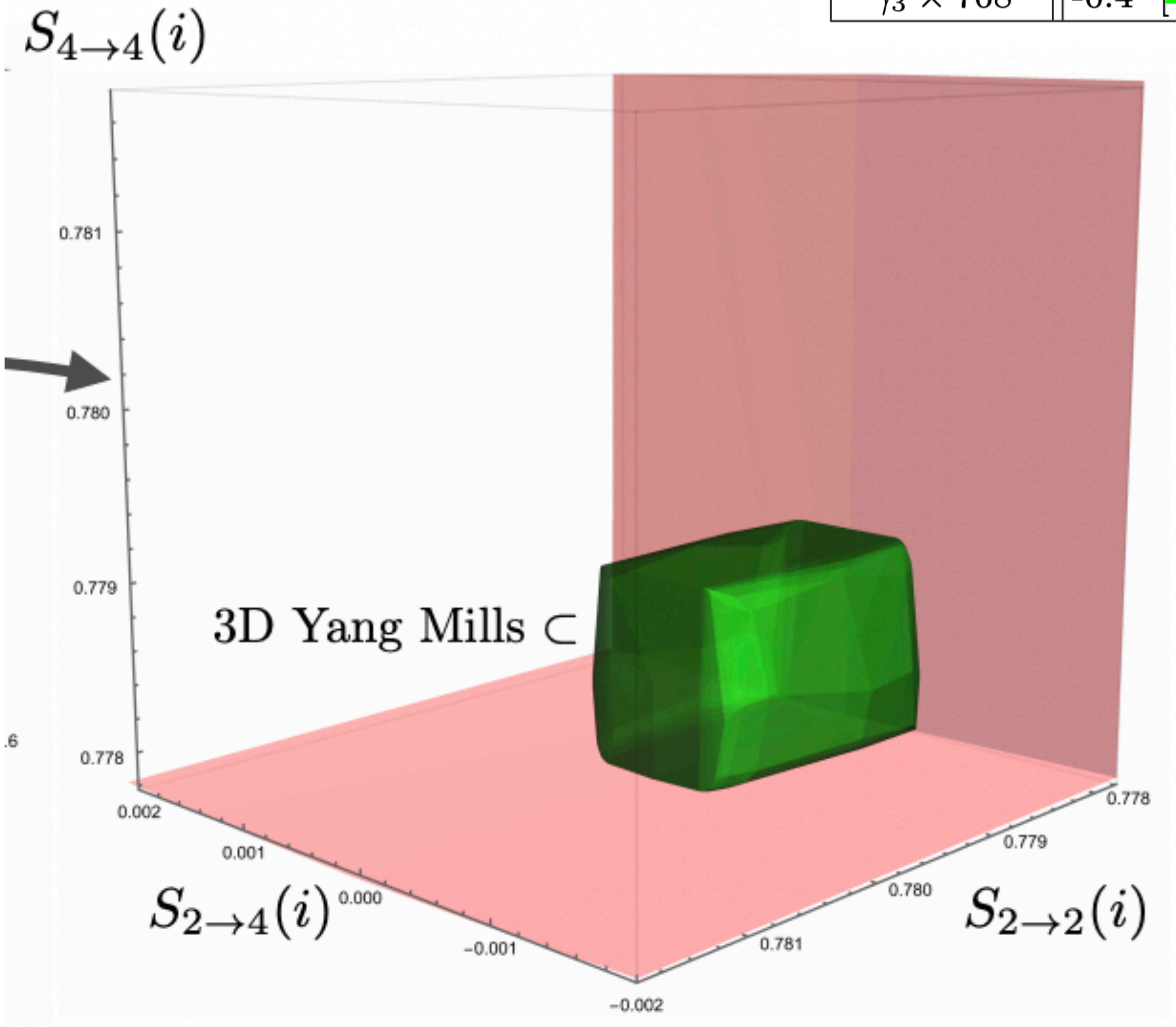
(X,Y,Z) space imposing leading Nambu-Goto behaviour $S_{11\to11} = S_{22\to22} = S_{12\to12} = 1 + is/4 + \ldots,$ $S_{11\to22} = S_{12\to21} = \mathcal{O}(s^2)$



The Matrioska

(X,Y,Z) space imposing NLNG behaviour and $\gamma_3 \times 768 < 0.7$

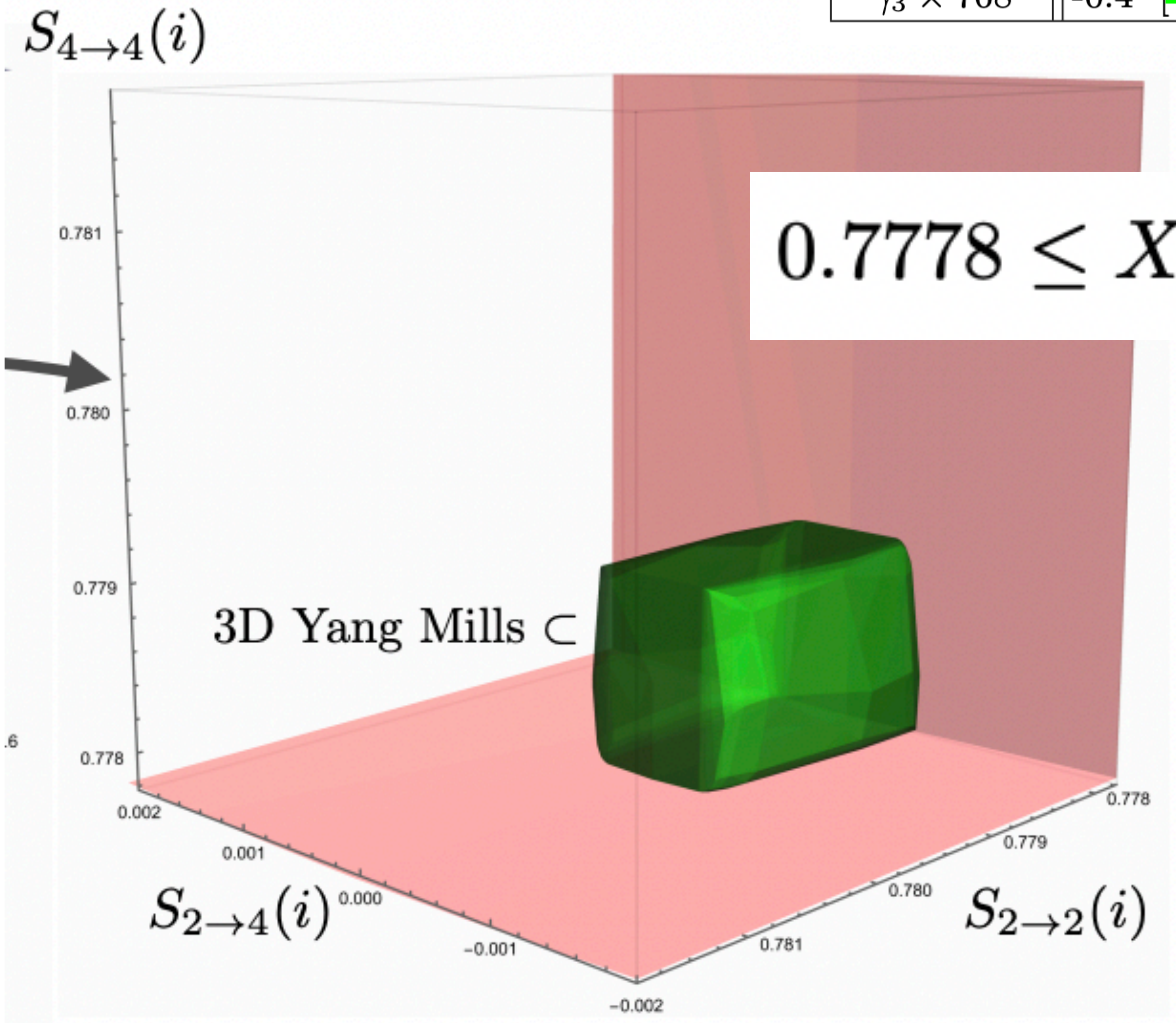
gauge group	$\mathbb{Z}_2$	$SU(2)$	$SU(6)$	$SU(\infty)$
$\gamma_3 \times 768$	-0.4 [4]	-0.3 [5]	0.2 [1, 6]	0.3



The Matrioska

(X,Y,Z) space imposing NLNG behaviour and $\gamma_3 \times 768 < 0.7$

gauge group	$\mathbb{Z}_2$	$SU(2)$	$SU(6)$	$SU(\infty)$
$\gamma_3 \times 768$	-0.4 [4]	-0.3 [5]	0.2 [1, 6]	0.3



$$0.7778 \leq X \leq 0.7796$$

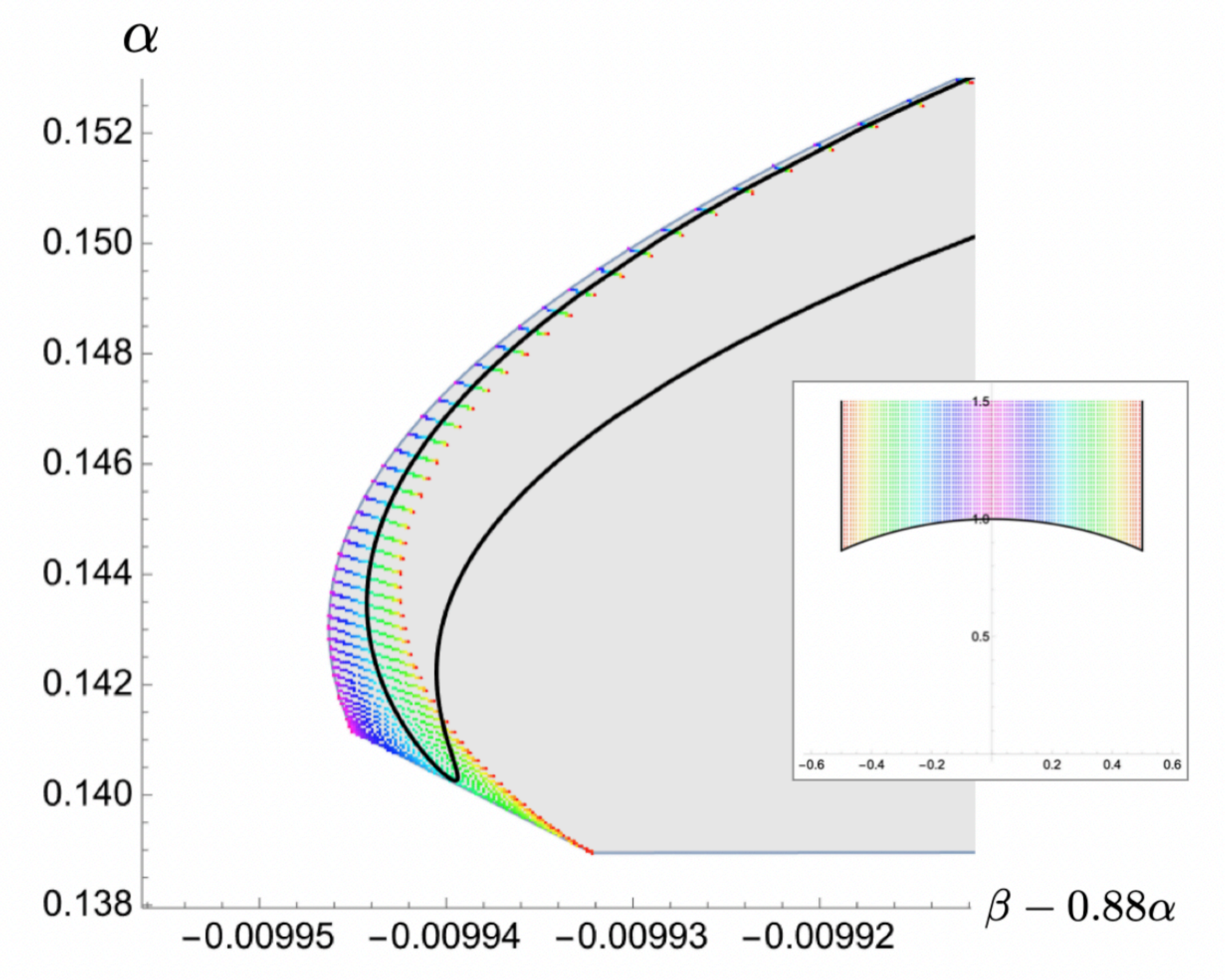
All physical confining flux-tube theories must satisfy this bound!

Possible future directions

Extend the bounds on α_D for $D \leq 8$. In $D \leq 7$ we expect loop effects to be so strong that $\alpha_D < 0$!

Bossard, Loty 2308.02847

Bound also β_D!



Study multi-particles on confining strings in 4 dimensions where particle production is universal and there is an axion on the world-sheet!

Extend the multi-particle Bootstrap to more complicated cases.

Backup Slides

Effective multi-particle couplings

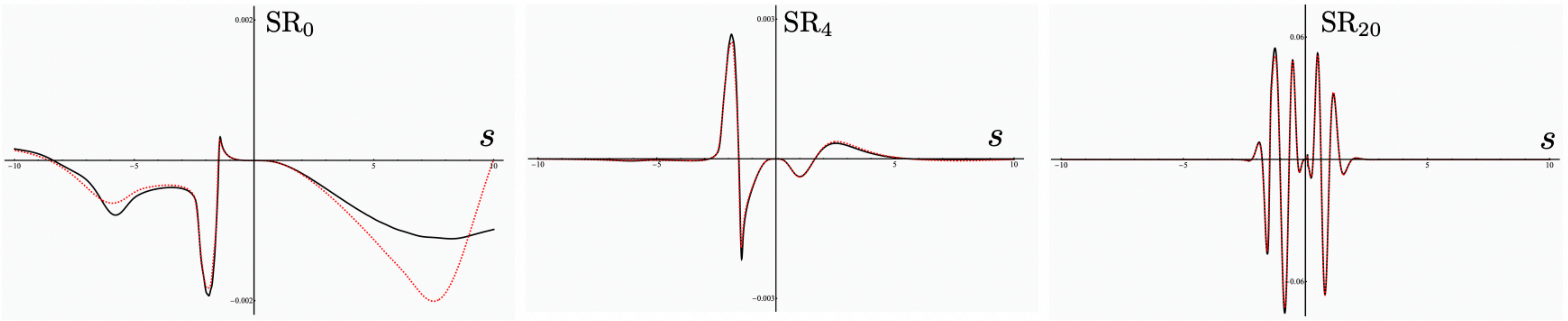
The observables we Bootstrap are effective 4, 6, and 8 particles couplings

$$(X, Y, Z) \equiv (S_{11\to 11}(i), S_{22\to 22}(i), \mathrm{Re}(S_{11\to 22}(i)))$$

This is clear in terms of dispersion relations $$X = \int_{\mathbb{R}} \frac{ds}{\pi(s^2+1)} S_{11\to 11}(s)$$

Phenomenologically not useful, so we introduce a set of dispersion relations for the same quantity

$$Z = SR_n \equiv \mathrm{Re} \int_{\mathbb{R}} \frac{ds}{\pi(s^2+1)} \left(\frac{3i}{s+2i}\right)^n S_{11\to 22}(s)$$

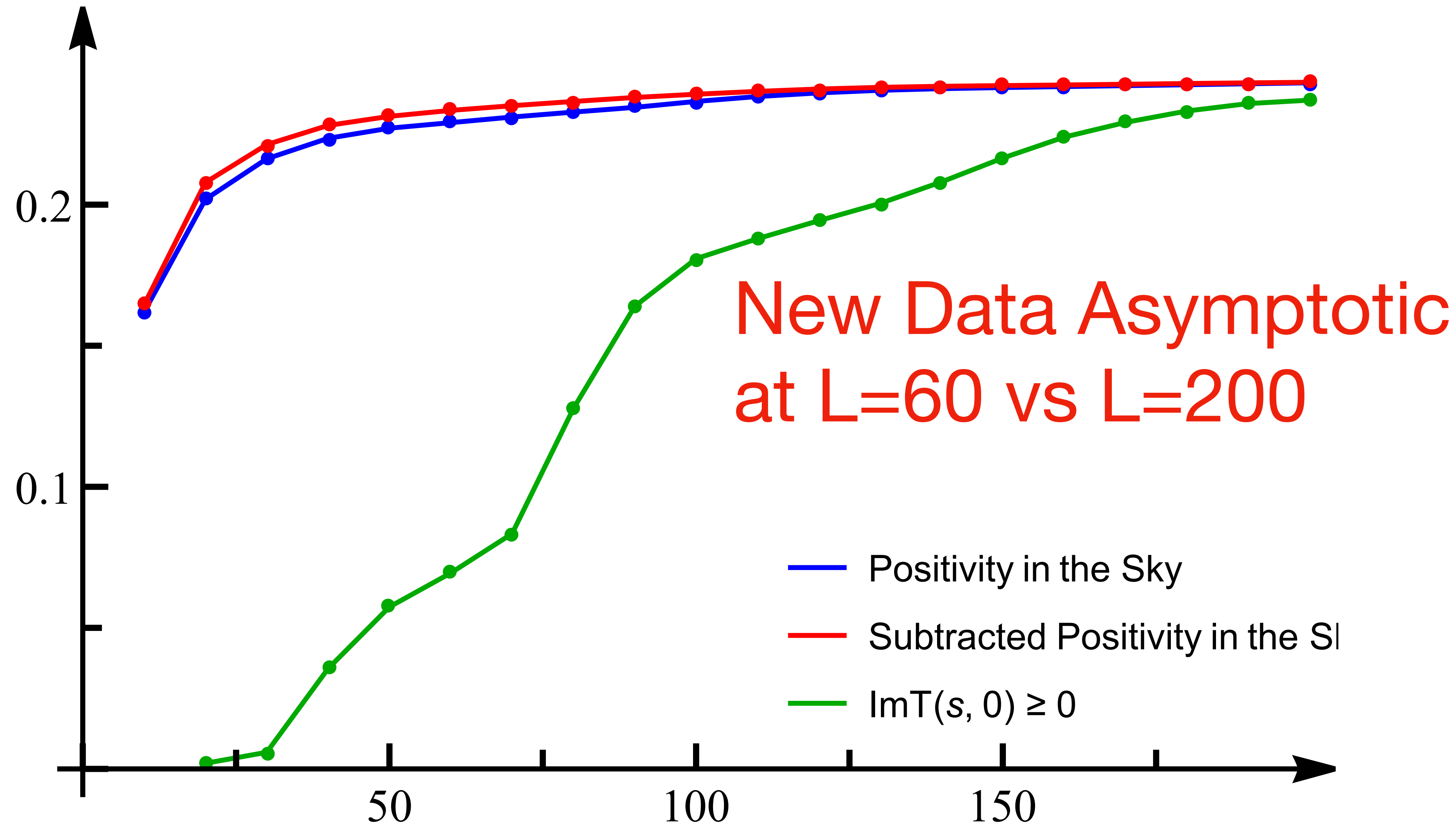


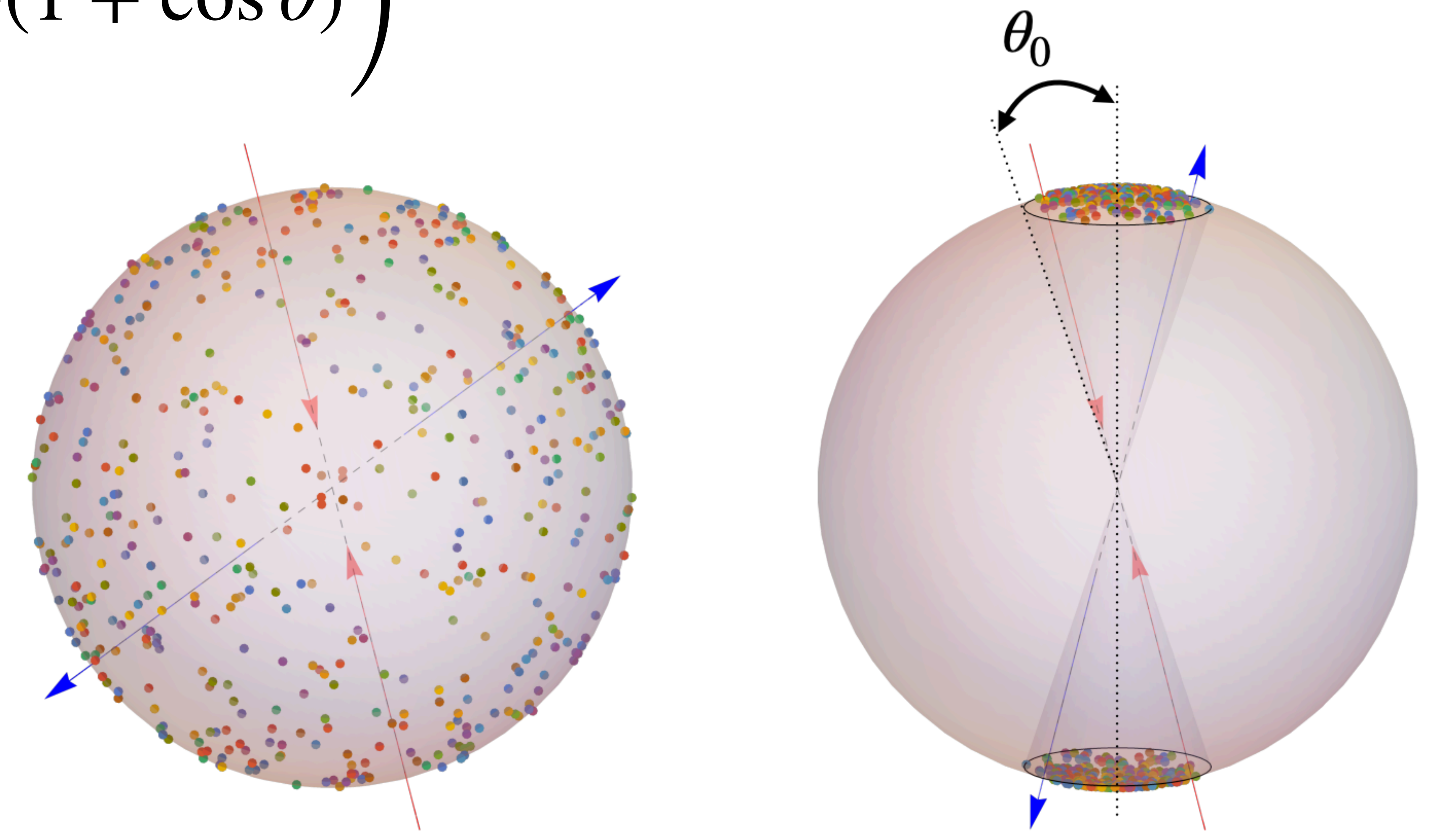
Positivity (Unitarity) in the Sky

Consider D random momenta with $p_i^2 = 0$ and $|\vec{p}_i| = \dfrac{\sqrt{s}}{2}$, and define the angle $\cos\theta_{ij} = \dfrac{p_i \cdot p_j}{|p_i|\,|\,|p_j|}$

In the centre of mass frame, to each θ_{ij} we associate a 2-particle state depends on

$$T\left(s, t = -\frac{s}{2}(1-\cos\theta), u = -\frac{s}{2}(1+\cos\theta)\right)$$





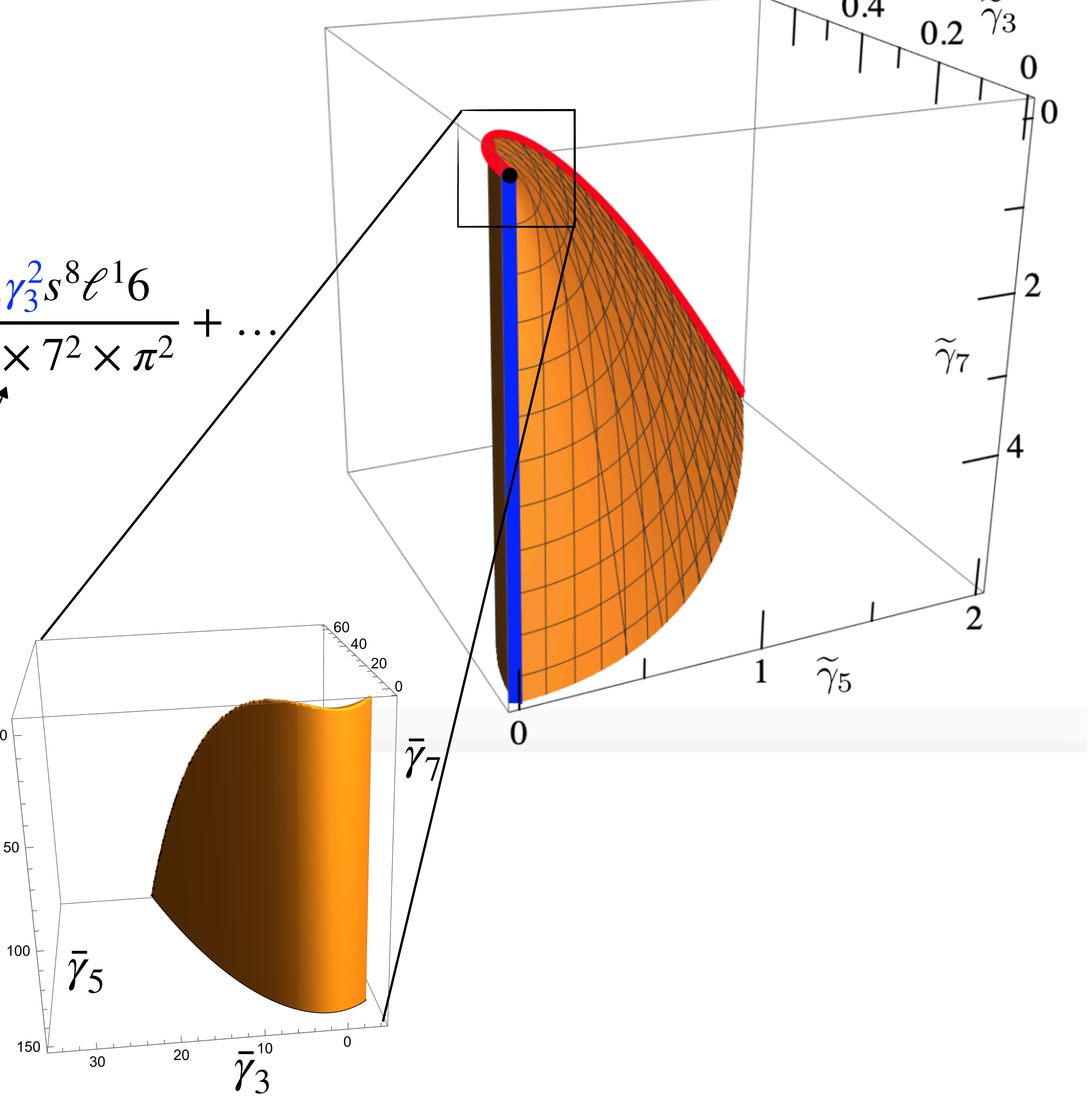
$$Im(T)_{ij} = \begin{pmatrix} ImT(s,1) & ImT(s,\cos\theta_{12}) & ImT(s,\cos\theta_{13}) & \dots \\ ImT(s,\cos\theta_{12}) & ImT(s,1) & ImT(s,\cos\theta_{23}) & \dots \\ ImT(s,\cos\theta_{13}) & ImT(s,\cos\theta_{23}) & ImT(s,1) & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix} \succeq 0$$

More Bounds in D=3

Elias-Miró, ALG, Hebbar, Penedones, and Vieira, ’21
Elias-Miró, ALG ‘21
Gaikwad, Gorbenko, ALG ’23 (axionic strings in 4D)

$$\frac{1}{2i}\log S(s) = \frac{s}{4}\ell_s^2 + \gamma_3 s^3 \ell_s^6 + \gamma_5 s^5 \ell_s^{10} + \gamma_7 s^7 \ell_s^{14} + i\frac{81\gamma_3^2 s^8 \ell^1 6}{2^{15}\times 7^2\times \pi^2} + \ldots$$

To go beyond we need to include particle production!

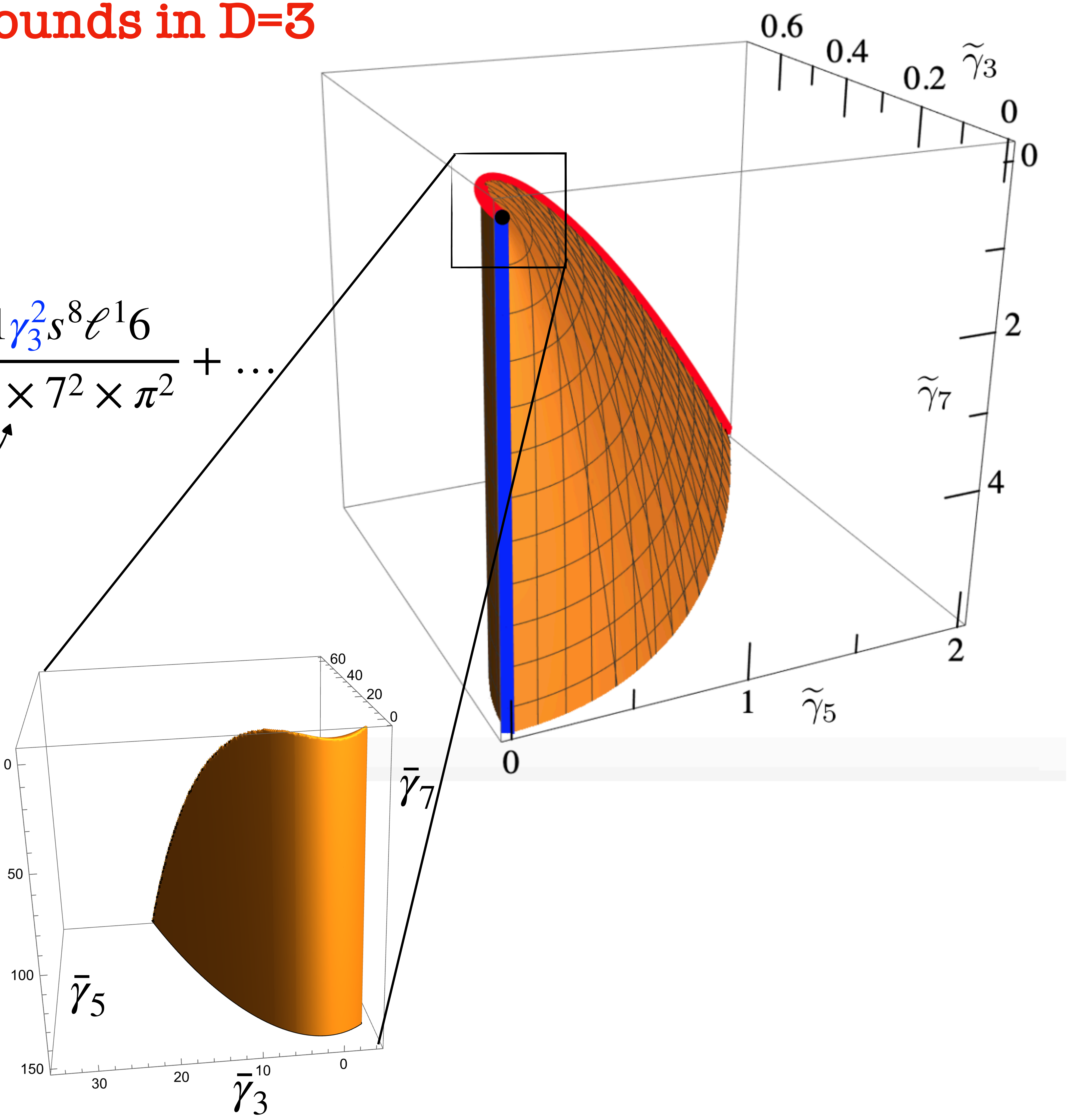


More Bounds in D=3

Elias-Miró, ALG, Hebbar, Penedones, and Vieira, '21
Elias-Miró, ALG '21
Gaikwad, Gorbenko, ALG '23 (axionic strings in 4D)

$$\frac{1}{2i}\log S(s) = \frac{s}{4}\ell_s^2 + \gamma_3 s^3 \ell_s^6 + \gamma_5 s^5 \ell_s^{10} + \gamma_7 s^7 \ell_s^{14} + i\frac{81\gamma_3^2 s^8 \ell^{1}6}{2^{15}\times 7^2 \times \pi^2} + \ldots$$

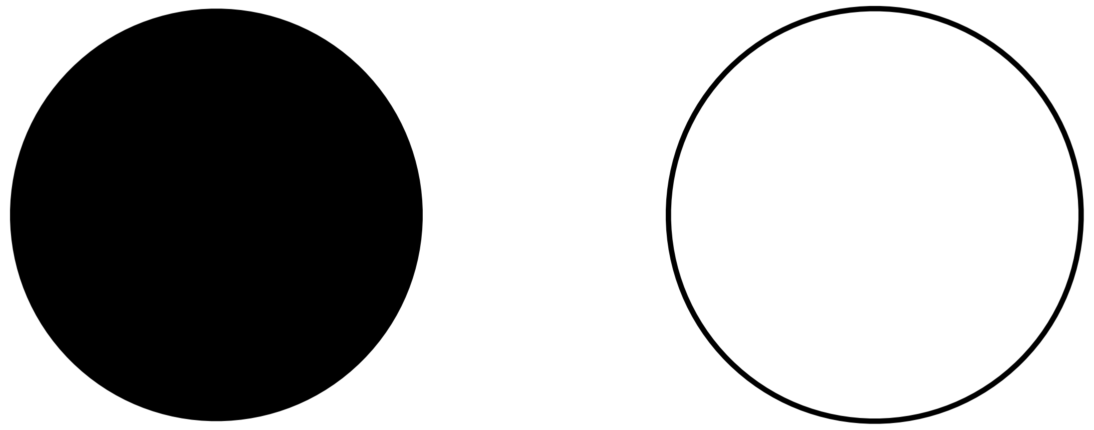
To go beyond we need to include particle production!



Non-convex!
Elastic unitarity is a non-convex constraint

$$\gamma[7] \geq \frac{\gamma t[5]^2}{\gamma t[3]} + \frac{1}{4096}\gamma t[3] + \frac{1}{64}\gamma t[5] - \frac{1}{16}\gamma t[3]^2 - \frac{1}{7\,340\,032}$$

Convex   Non-convex



The Hadronic String in 4D

D=4: X^1, X^2 Goldstones, deviations from Nambu-Goto α_3, β_3

$$\mathscr{A}_{EFF} = \int d^2\sigma\sqrt{-h}\left(\frac{1}{\ell_s^2} + \ldots + \alpha_3 \ell_s^6 K^4 + \beta_3 \ell_s^6 R^2 + \ldots\right)$$

$$(\gamma_3 = \alpha_3 - \beta_3)$$

New Effect in the amplitude: universal **Polchinski-Strominger** term at 1-loop $\propto \alpha_2 = \frac{D-26}{384\pi}$

The Hadronic String in 4D

D=4: X^1, X^2 Goldstones, deviations from Nambu-Goto α_3, β_3

$$\mathcal{A}_{EFF} = \int d^2\sigma\sqrt{-h}\left(\frac{1}{\ell_s^2} + \ldots + \alpha_3 \ell_s^6 K^4 + \beta_3 \ell_s^6 R^2 + \ldots \right)$$

$$(\gamma_3 = \alpha_3 - \beta_3)$$

New Effect in the amplitude: universal **Polchinski-Strominger** term at 1-loop $\propto \alpha_2 = \frac{D-26}{384\pi}$

PS term induces universal particle production at 1 Loop:
no hope for large N_c integrability, unless we add **massless** degrees of freedom to the world-sheet

The Hadronic String in 4D

D=4: X^1, X^2 Goldstones, deviations from Nambu-Goto α_3, β_3

$$\mathcal{A}_{EFF} = \int d^2\sigma\sqrt{-h}\left(\frac{1}{\ell_s^2} + \ldots + \alpha_3 \ell_s^6 K^4 + \beta_3 \ell_s^6 R^2 + \ldots \right. \\ \left. (\gamma_3 = \alpha_3 - \beta_3)\right)$$

New Effect in the amplitude: universal **Polchinski-Strominger** term at 1-loop $\propto \alpha_2 = \frac{D-26}{384\pi}$

PS term induces universal particle production at 1 Loop:
no hope for large N_c integrability, unless we add **massless** degrees of freedom to the world-sheet

E.g. we can add an axion

$$S_a = \int d^2\sigma \left[-\frac{1}{2}(\partial_\alpha a)^2 - \frac{1}{2}m_a a^2 - \ell_s^2 Q_a a \varepsilon^{ij}\varepsilon^{\alpha\beta}\partial_\alpha\partial_\gamma X^i \partial_\beta \partial^\gamma X^j + \ldots\right].$$

If we tune $Q_a = \frac{\sqrt{22}}{4\sqrt{3\pi}} \simeq 0.378$, and $m_a \to 0$, we can restore integrability

The Hadronic String in 4D

D=4: X^1, X^2 Goldstones, deviations from Nambu-Goto α_3, β_3

$$\mathcal{A}_{EFF} = \int d^2\sigma\sqrt{-h}\left(\frac{1}{\ell_s^2} + \ldots + \alpha_3\ell_s^6K^4 + \beta_3\ell_s^6R^2 + \ldots\right)$$

$$(\gamma_3 = \alpha_3 - \beta_3)$$

New Effect in the amplitude: universal **Polchinski-Strominger** term at 1-loop $\propto \alpha_2 = \frac{D-26}{384\pi}$

PS term induces universal particle production at 1 Loop:
no hope for large N_c integrability, unless we add **massless** degrees of freedom to the world-sheet

E.g. we can add an axion

$$S_a = \int d^2\sigma \left[-\frac{1}{2}\left(\partial_\alpha a\right)^2 - \frac{1}{2}m_a a^2 - \ell_s^2 Q_a a \varepsilon^{ij}\varepsilon^{\alpha\beta}\partial_\alpha\partial_\gamma X^i \partial_\beta \partial^\gamma X^j + \ldots\right].$$

If we tune $Q_a = \frac{\sqrt{22}}{4\sqrt{3\pi}} \simeq 0.378$, and $m_a \to 0$, we can restore integrability

Lattice results show the presence of an axion resonance with the correct coupling, but massive at large N_c

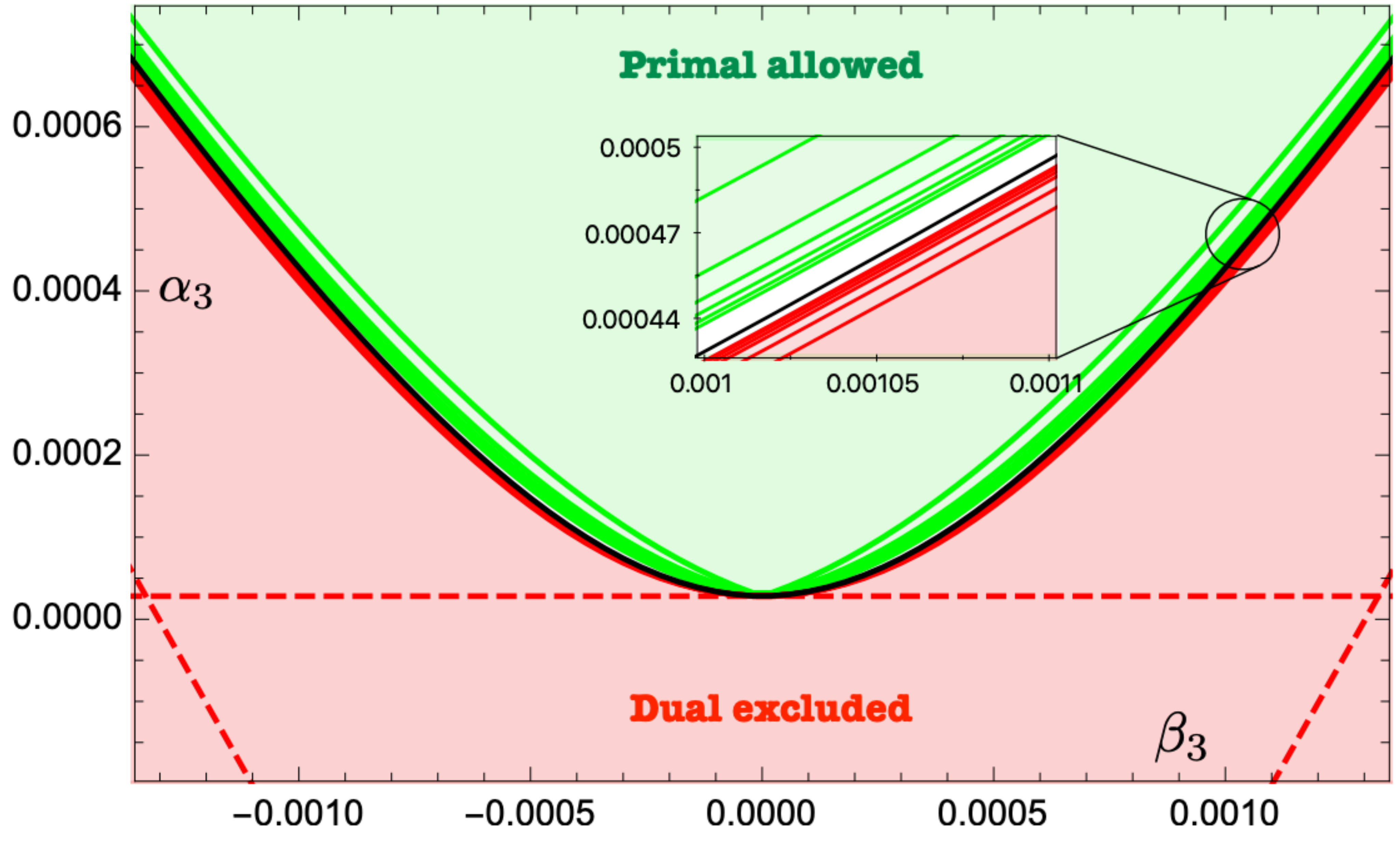
	$SU(3)$	$SU(5)$	$SU(\infty)$
2^{++}			
$m_a^L \ell_s$	$1.85^{+0.02}_{-0.03}$	$1.64^{+0.04}_{-0.04}$	1.5
Q_a^L	$0.380^{+0.006}_{-0.006}$	$0.389^{+0.008}_{-0.008}$	-
2^{+-}			
$m_a^L \ell_s$	$1.85^{+0.02}_{-0.02}$	$1.64^{+0.04}_{-0.04}$	1.5
Q_a^L	$0.358^{+0.004}_{-0.005}$	$0.358^{+0.009}_{-0.009}$	-

Gaikwad, Gorbenko, ALG ’23

Flux-Tube S-matrix Bootstrap in 4D

$$\mathcal{A}_{EFF} = \int d^2\sigma\sqrt{-h}\left(\frac{1}{\ell_s^2} + \ldots + \alpha_3 \ell_s^6 K^4 + \beta_3 \ell_s^6 R^2 + \ldots\right)$$

$$\gamma_3 = \alpha_3 - \beta_3$$

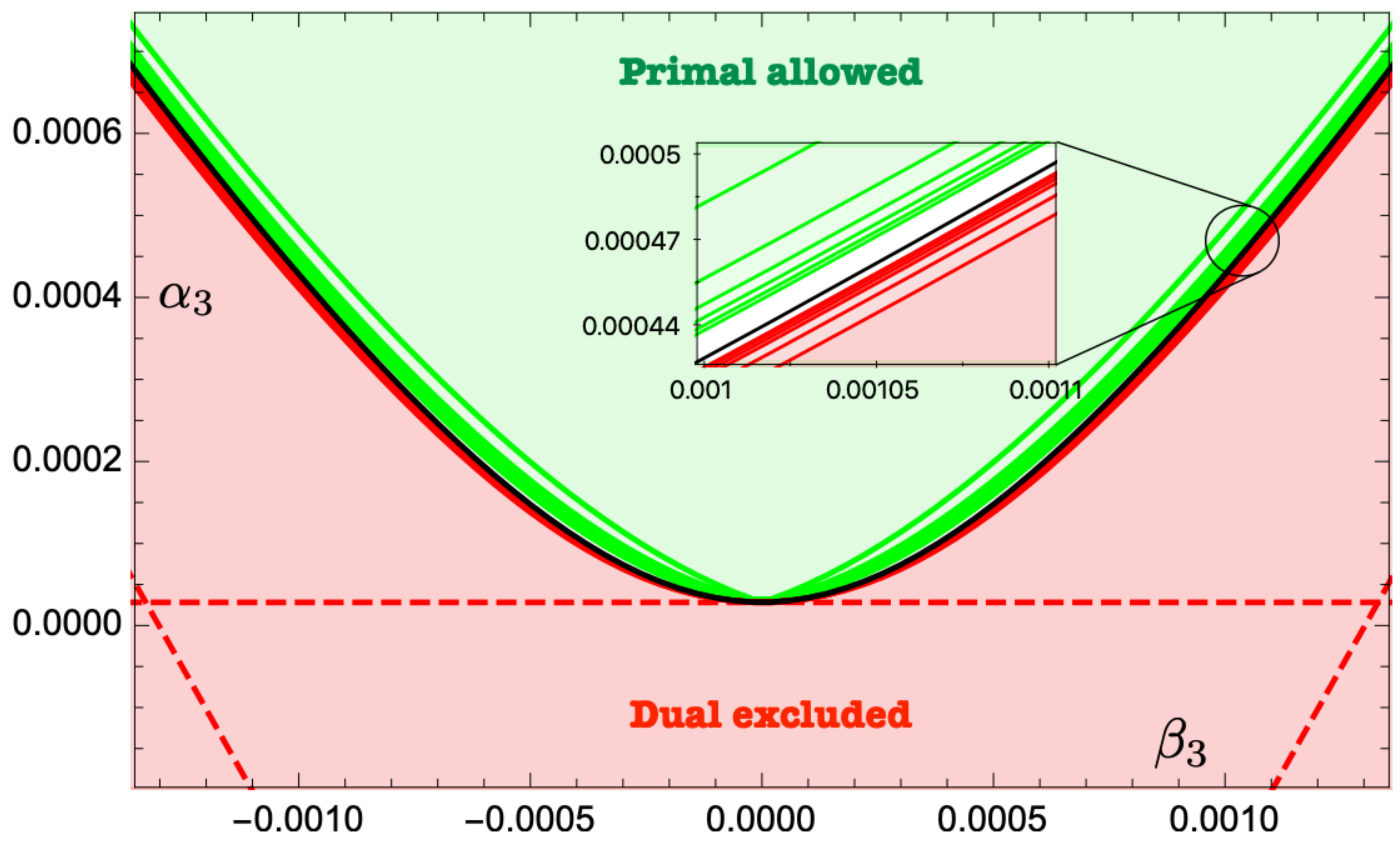


Elias-Miró, ALG, Hebbar, Penedones, Vieira 1906.08098
Elias-Miró, ALG 2106.07957

Flux-Tube S-matrix Bootstrap in 4D

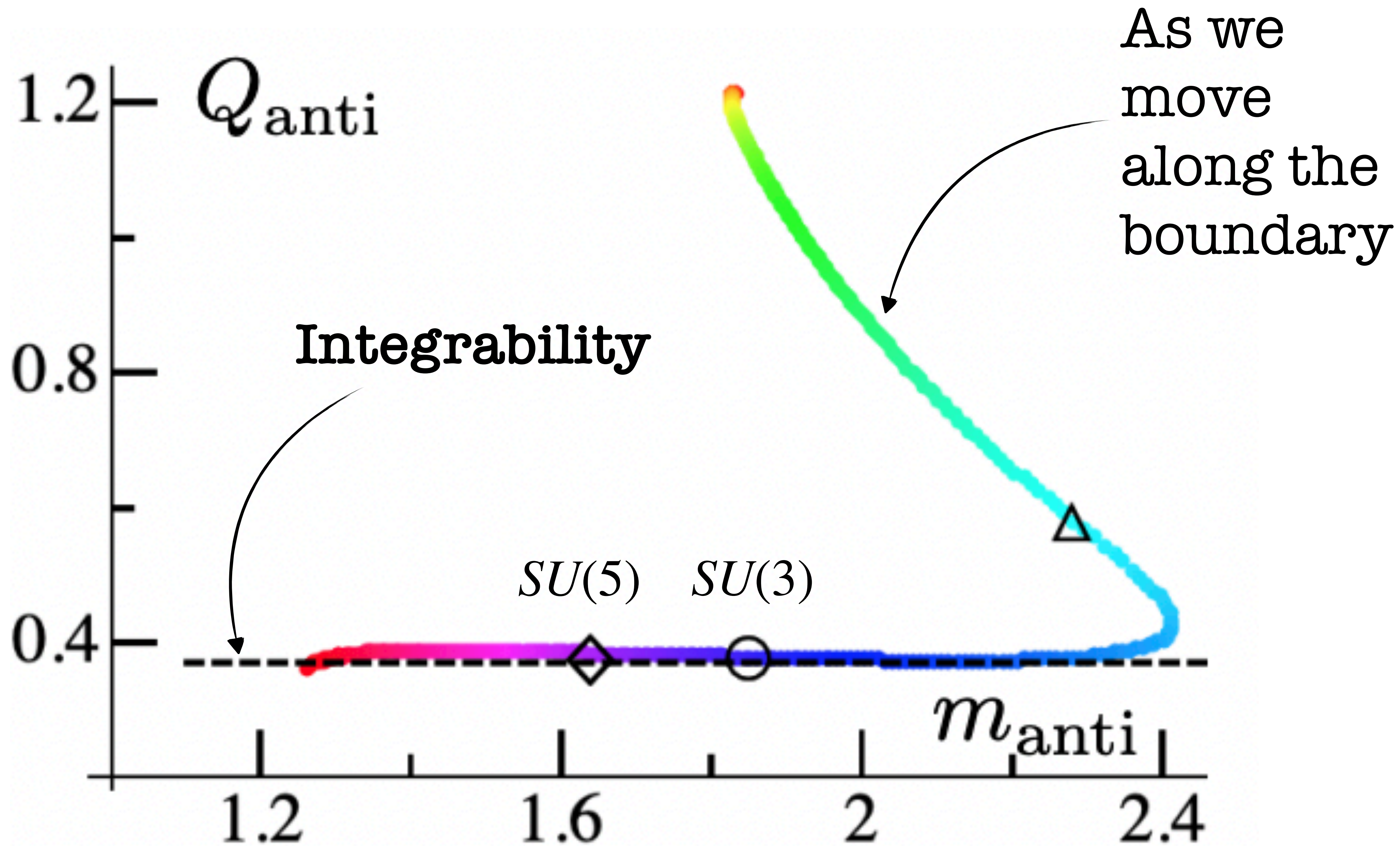
$$\mathscr{A}_{EFF} = \int d^2\sigma\sqrt{-h}\left(\frac{1}{\ell_s^2} + \ldots + \alpha_3 \ell_s^6 K^4 + \beta_3 \ell_s^6 R^2 + \ldots\right)$$

$$\gamma_3 = \alpha_3 - \beta_3$$



Elias-Miró, ALG, Hebbar, Penedones, Vieira 1906.08098
Elias-Miró, ALG 2106.07957

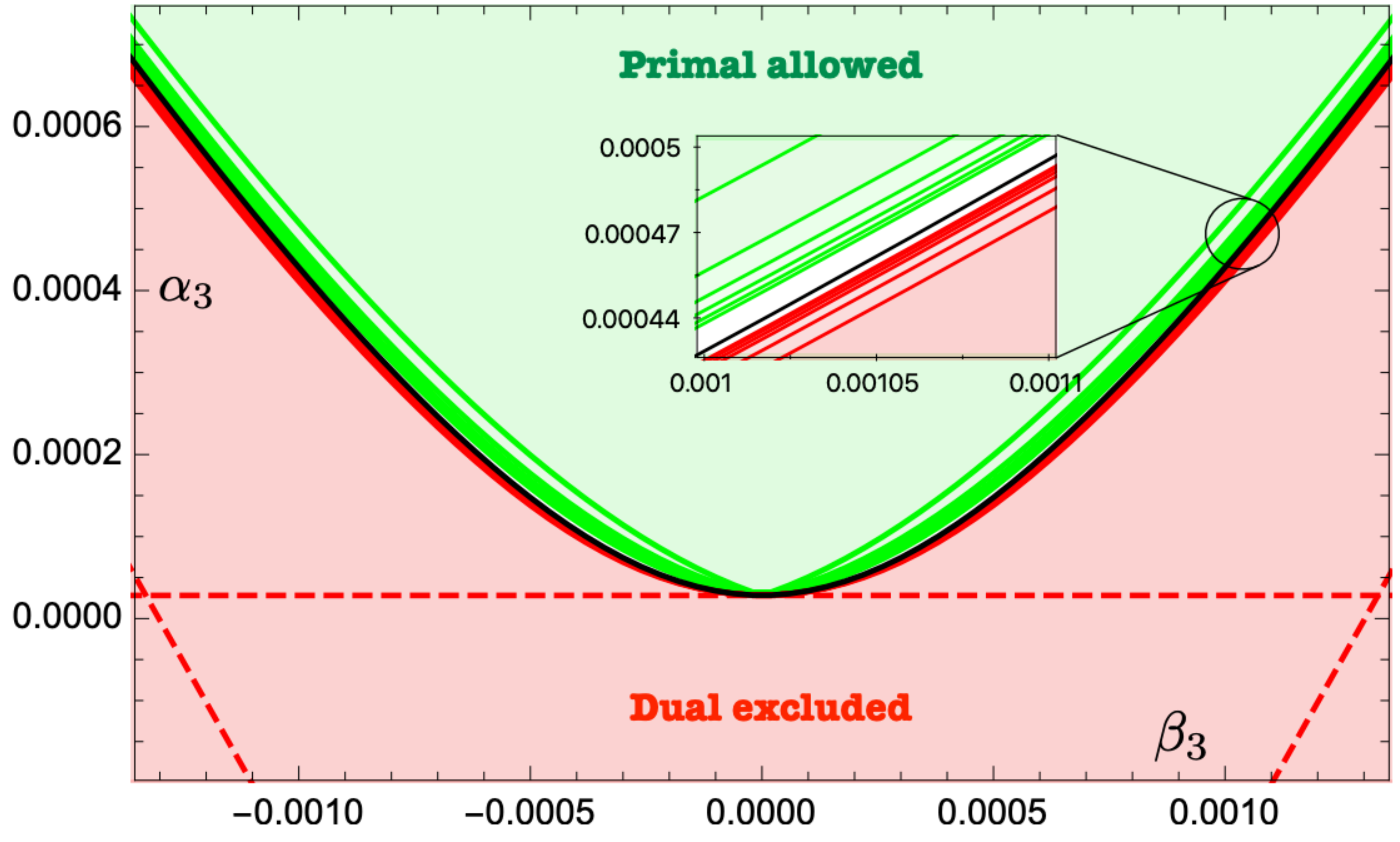
Surprise!
Extremal Bootstrap amplitudes contain an axion with integrable coupling!



Flux-Tube S-matrix Bootstrap in 4D

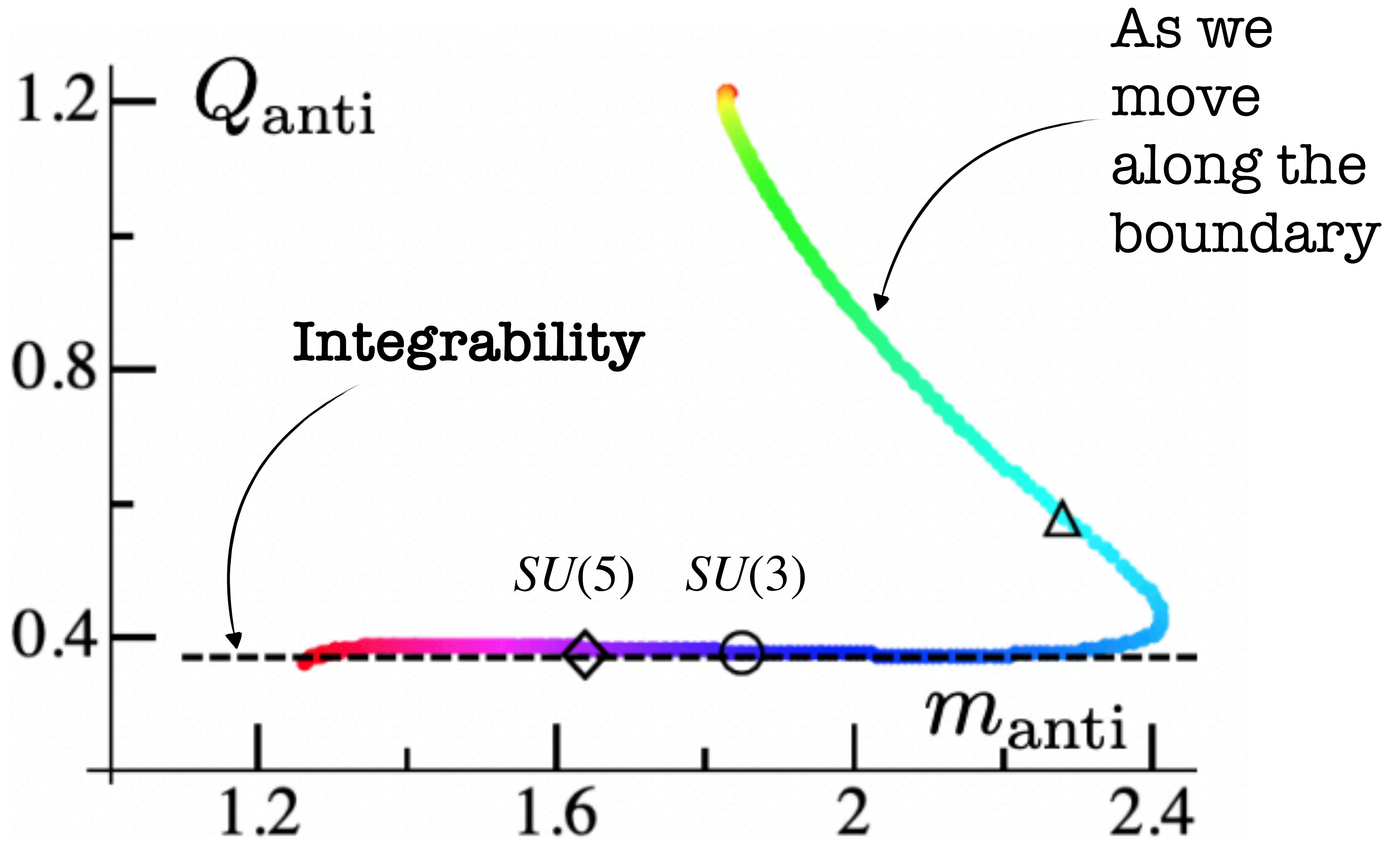
$$\mathscr{A}_{EFF} = \int d^2\sigma\sqrt{-h}\left(\frac{1}{\ell_s^2} + \ldots + \alpha_3 \ell_s^6 K^4 + \beta_3 \ell_s^6 R^2 + \ldots\right)$$

$$\gamma_3 = \alpha_3 - \beta_3$$



Elias-Miró, ALG, Hebbar, Penedones, Vieira 1906.08098
Elias-Miró, ALG 2106.07957

Surprise!
Extremal Bootstrap amplitudes contain an axion with integrable coupling!



$$Q_a^L \approx Q_a^c \approx Q_a^b$$

Can we explain this **triple** coincidence?

String Theory and M-theory Expectations

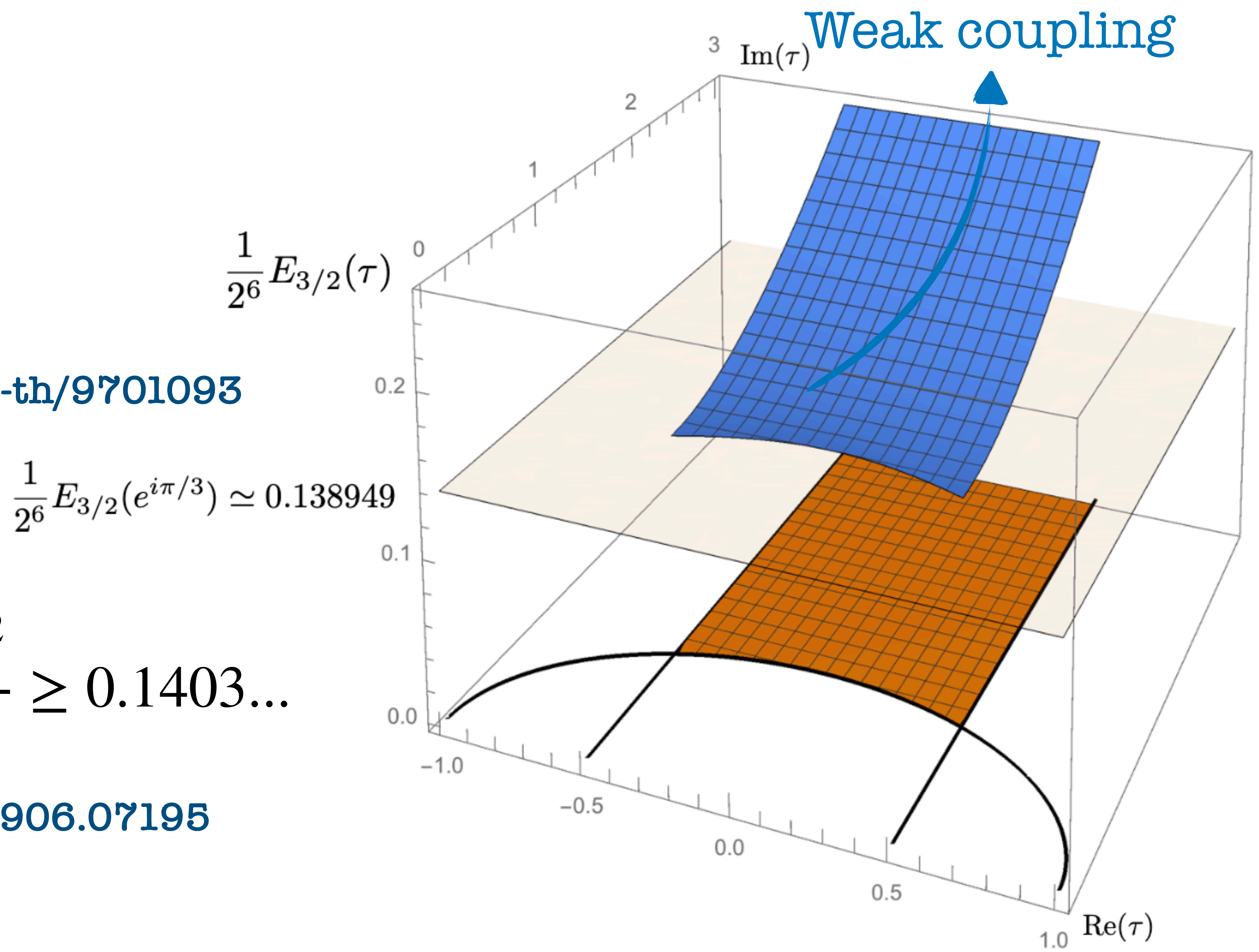
α_D is one-loop exact up to non-perturbative corrections

D=10 type IIB: $\alpha_{10}^{IIB} = \frac{1}{2^6} E_{3/2}(\tau,\bar\tau) = \frac{1}{2^6} \sum_{n,m\in\mathbb{Z}'} \frac{(Im\tau)^{3/2}}{|m\tau+n|^3}$

Green, Gutperle hep-th/9701093

$\alpha_{10} \geq 3^{1/4}\zeta(\frac{3}{2})(\zeta(\frac{3}{2},\frac{1}{2}) - \zeta(\frac{3}{2},\frac{2}{3}))/\sqrt{2} \simeq 0.1389...$

Alday, Bissi 1311.3215



D=10 type IIA: $\alpha_{10}^{IIA} = \alpha_{10}^{IIB} - (instantons) = \frac{\zeta_3}{32 g_s^{3/2}} + \frac{g_s^{1/2}\pi^2}{96} \geq 0.1403...$

Binder, Chester, Pufu 1906.07195

D=11: $\alpha_{11} = \frac{(2\pi)^2}{3\times 2^7} = 0.1028...$ Obtained from the IIA decompacifying one dimension $g_s = (R/l_{11})^{3/2}$

Green, Vanhove hep-th/9701093

D=9: $\alpha_9(\tau,\nu) = \frac{1}{2^6}\left(\nu^{-3/7}E_{3/2}(\tau,\bar\tau) + \frac{2\pi^2}{3}\nu^{4/7}\right) \geq 0.2417...$ $\nu = \left(\frac{r}{\ell_s}\right)^{7/4}\sqrt{g_9} = \left(\frac{\ell_P}{\tilde r}\right)^{7/4}$

Green, Russo, Vanhove hep-th/0610299

D $\leq$ 8 α_D mixes with the one-loop non analytic terms and can be even negative!

Bossard, Loty 2308.02847

Bounds on Glueball couplings

SU(3) Yang-Mills glueballs in 3+1 D

Cutoff $\Lambda \sim m$, no small parameters!

Stable Glueballs spectrum

	J^{PC}	Mass
G	0^{++}	1
H	2^{++}	1.437 ± 0.006
G^*	0^{++}	1.72 ± 0.01
H^*	2^{++}	1.99 ± 0.01

Athenodorou, Teper 2007.06422, 2106.00364

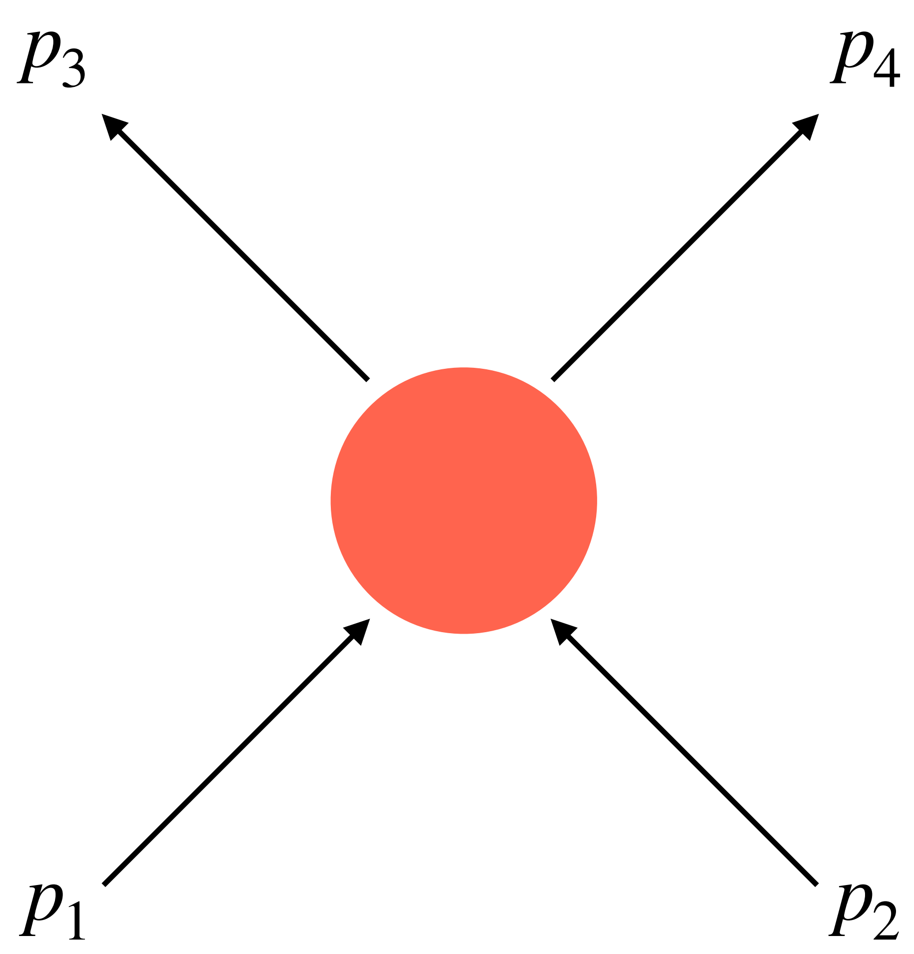
Bounds on Glueball couplings

SU(3) Yang-Mills glueballs in 3+1 D

Cutoff $\Lambda \sim m$, no small parameters!

Consider the $GG \to GG$ scattering

$$T(s,t,u) =$$



$$s = (p_1 + p_2)^2, t = (p_1 - p_4)^2, s + t + u = 4m^2$$

Stable Glueballs spectrum

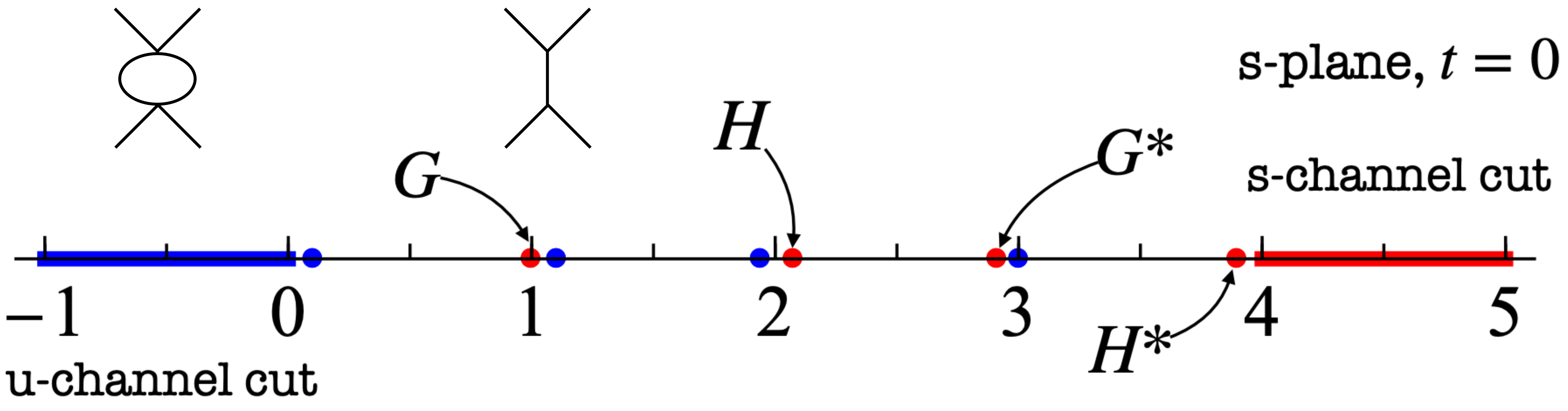
	J^{PC}	Mass
G	0^{++}	1
H	2^{++}	1.437 ± 0.006
G^*	0^{++}	1.72 ± 0.01
H^*	2^{++}	1.99 ± 0.01

Athenodorou, Teper 2007.06422, 2106.00364

Bounds on Glueball couplings

SU(3) Yang-Mills glueballs in 3+1 D

Cutoff $\Lambda \sim m$, no small parameters!



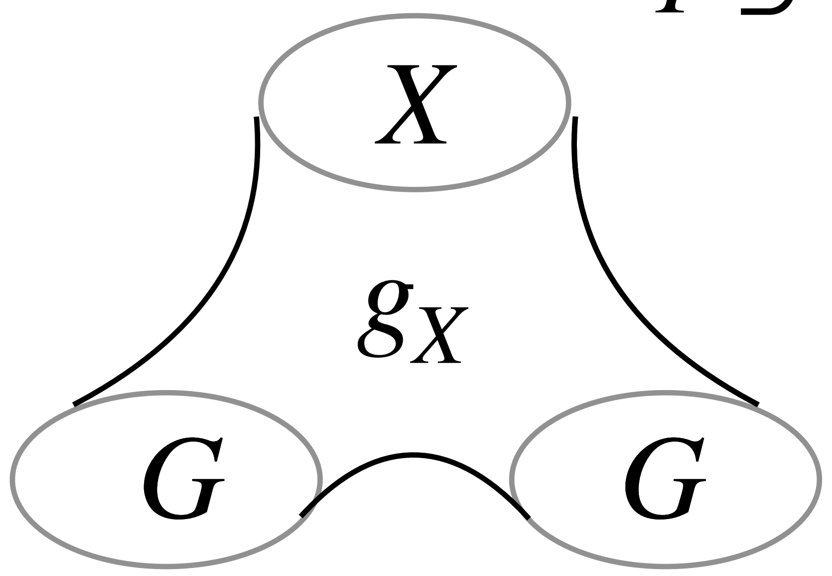
Stable Glueballs spectrum

	J^{PC}	Mass
G	0^{++}	1
H	2^{++}	1.437 ± 0.006
G^*	0^{++}	1.72 ± 0.01
H^*	2^{++}	1.99 ± 0.01

Athenodorou, Teper 2007.06422, 2106.00364

To each stable Glueball we associate a pole in the amplitude

$$T \supset \frac{-g_X^2}{s - m_X^2} P_\ell \left(1 + \frac{2t}{m_X^2 - 4} \right) + (t - channel) + (u - channel)\dots$$

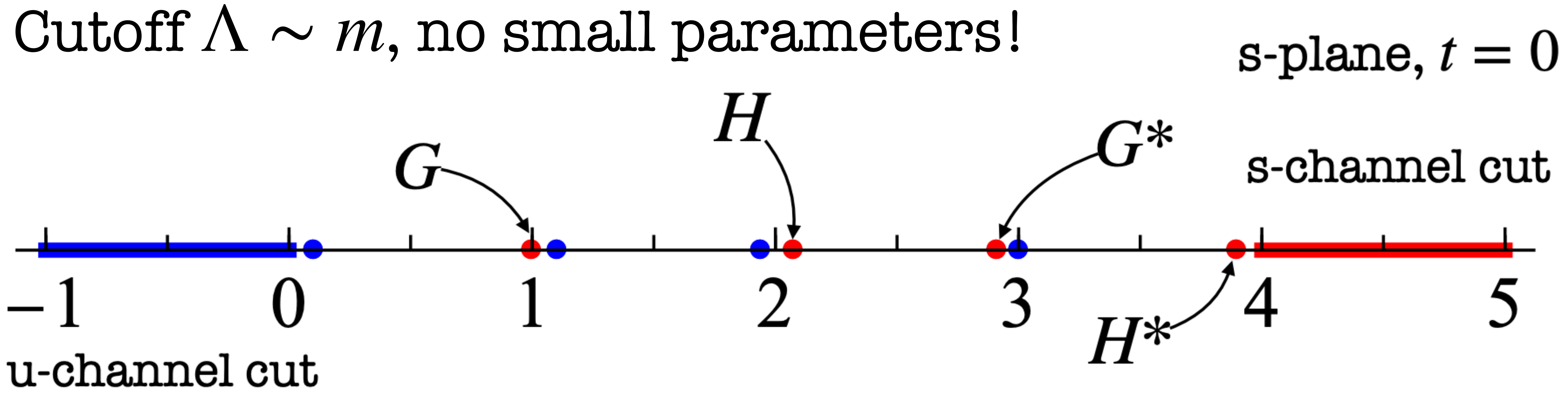


Given the spectrum, how big couplings can be?

Bounds on Glueball couplings

SU(3) Yang-Mills glueballs in 3+1 D

Cutoff $\Lambda \sim m$, no small parameters!



Bootstrap answer

max $\lvert g_G \rvert$	max $\lvert g_H \rvert$	max $\lvert g_{G^*} \rvert$	max $\lvert g_{H^*} \rvert$
213	158	224	2.15
206	156	217	–

AG, Hebbar, van Rees 2312.00127

Stable Glueballs spectrum

	J^{PC}	Mass
G	0^{++}	1
H	2^{++}	1.437 ± 0.006
G^*	0^{++}	1.72 ± 0.01
H^*	2^{++}	1.99 ± 0.01

Athenodorou, Teper 2007.06422, 2106.00364

Bound from first principles

$$0 \leq g_G \leq 213.0$$

SU(3) YM Lattice $g_G \approx 50 \pm 7$

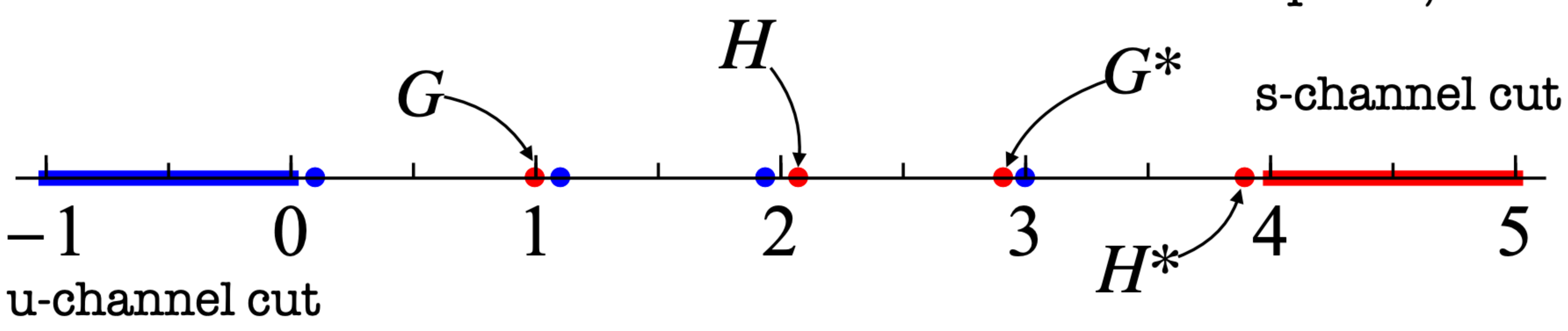
De Forcrand, Schierloz, Schneider, Teper ‘85

Bounds on Glueball couplings

SU(3) Yang-Mills glueballs in 3+1 D

Cutoff $\Lambda \sim m$, no small parameters!

s-plane, $t = 0$



Stable Glueballs spectrum

	J^{PC}	Mass
G	0^{++}	1
H	2^{++}	1.437 ± 0.006
G^*	0^{++}	1.72 ± 0.01
H^*	2^{++}	1.99 ± 0.01

Athenodorou, Teper 2007.06422, 2106.00364

Bootstrap answer

max $\|g_G\|$	max $\|g_H\|$	max $\|g_{G^*}\|$	max $\|g_{H^*}\|$
213	158	224	2.15
206	156	217	–

AG, Hebbar, van Rees 2312.00127

Bound from first principles

$$0 \leq g_G \leq 213.0$$

SU(3) YM Lattice $g_G \approx 50 \pm 7$

De Forcrand, Schierloz, Schneider, Teper '85