A Holographic Triptych at Large N

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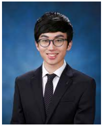
2309.06469, 2312.08909 + to appear

"The Garden of Earthly Delights" Hieronymus Bosch (circa 1490)

The large N team

Anthony Charles (Leuven)



Sunjin Choi (Seoul)

Marina David (Leuven)

Pieter-Jan De Smet (Leuven)

Friðrik Gautason (Southampton)

Junho Hong (Leuven)

Kiril Hristov (Sofia)

Vincent Min (Leuven)

Valentina Puletti (Reykjavik)

Valentin Reys (Saclay)

Jesse van Muiden (Trieste)

Xuao Zhang (Leuven)

Motivation

Supersymmetric localization allows for the exact calculation of physical observables in supersymmetric QFTs.

Apply this tool to SCFTs with holographic duals in string and M-theory.

$$Z_{\rm CFT}[J] = Z_{\rm string/M}[\phi]\,.$$

Focus on subleading terms in the large N expansion to learn about quantum corrections to the supergravity approximation.

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Why?

- Explore precision holography.
- New handle on AdS vacua of string and M-theory with non-trivial fluxes.
- Learn about quantum corrections to black hole thermodynamics.

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This talk

Recent progress on these topics for 3d SCFTs with AdS_4 duals in type IIA string theory and M-theory.

ABJM partition functions

[Aharony,Bergman,Jafferis,Maldacena]; [Kapustin,Willett,Yaakov]; [Hama,Hosomichi,Lee]; [Drukker,Mariño,Putrov]; [Mariño,Putrov]; [Fuji,Hirano,Moriyama]; [Herzog,Klebanov,Pufu,Tesileanu]; [Benini,Zaffaroni]; [Closset,Kim]; [Benini,Hristov,Zaffaroni]; [Liu,Pando Zayas,Rathee,Zhao]; [NPB,Hong,Reys]; [Nosaka]; [Hatsuda]; [Hristov]; [Chester,Kalloor,Sharon]; [Bhattacharya^2,Minwalla,Raju]; [Kim]; [Choi,Hwang,Kim]; [Choi,Hwang]; [Nian,Pando Zayas]; [NPB,Choi,Hong,Reys]; [NPB,De Smet,Hong,Reys,Zhang]

ABJM and holography

The ABJM theory: $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ 3d CS-matter theory with two pairs of bi-fundamental chirals $(A_{1,2}, B_{1,2})$ and superpotential

$$\mathcal{W} = \mathrm{Tr}(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1)\,.$$

For $k > 2$ it has $\mathcal{N} = 6$ supersymmetry and $\mathrm{SU}(4)_R \times \mathrm{U}(1)_b$ global symmetry. Describes N M2-branes on $\mathbb{C}^4/\mathbb{Z}_k$.

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- In the limit of fixed k and large N, the ABJM theory is dual to the M-theory background $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$

$$(L/\ell_{\mathrm{P}})^6 \sim k\,N\,.$$

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- At large k and fixed 't Hooft coupling $\lambda = N/k$ the theory is dual to type IIA string theory on $\mathrm{AdS}_4 \times \mathbb{CP}^3$

$$k\,g_{\mathrm{st}} = L/\ell_{\mathrm{s}} \sim \lambda^{1/4}\,.$$

  Perturbative type IIA string theory at large k and small g_{st}, i.e. fixed λ and large N.

ABJM on S^3 - An Airy Tale

The path integral on a squashed S^3 with real mass deformation can be computed by supersymmetric localization and reduces to a matrix model. At large N and fixed k one finds

$$Z_{S^3}(N,k,m_a,b) = e^{\mathcal{A}(k,m_a,b)} C^{-\frac{1}{3}} \mathrm{Ai}[C^{-\frac{1}{3}}(N-B)] + \mathcal{O}(\mathrm{e}^{-\sqrt{N}}) \,,$$

with

$$C = \frac{2}{\pi^2 k} \frac{(b+b^{-1})^{-4}}{\prod_{a=1}^4 \Delta_a} \,, \quad B = \frac{k}{24} - \frac{1}{12k} \sum_{a=1}^{4} \frac{1}{\Delta_a} + \frac{1 - \frac{1}{4}\sum_a \Delta_a^2}{3k(b+b^{-1})^2 \prod_{a=1}^4 \Delta_a} \,,$$

and

$$\Delta_1 = \frac{1}{2} - \mathrm{i}\, \frac{m_1+m_2+m_3}{b+b^{-1}} \,, \qquad \Delta_2 = \frac{1}{2} - \mathrm{i}\, \frac{m_1-m_2-m_3}{b+b^{-1}} \,,$$

$$\Delta_3 = \frac{1}{2} + \mathrm{i}\, \frac{m_1+m_2-m_3}{b+b^{-1}} \,, \qquad \Delta_4 = \frac{1}{2} + \mathrm{i}\, \frac{m_1-m_2+m_3}{b+b^{-1}} \,.$$

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$$\Delta_3 = \frac{1}{2} + \mathrm{i}\,\frac{m_1+m_2-m_3}{b+b^{-1}} \,, \qquad \Delta_4 = \frac{1}{2} + \mathrm{i}\,\frac{m_1-m_2+m_3}{b+b^{-1}} \,.$$

The large N expansion takes the explicit form

$$-\log Z_{S^3} = \frac{2}{3\sqrt{C}} N^{\frac{3}{2}} - \frac{B}{\sqrt{C}} N^{\frac{1}{2}} + \frac{1}{4}\log N - \mathcal{A} + \frac{1}{4}\log\frac{32}{k} + \mathcal{O}(N^{-\frac{1}{2}}) \,.$$

The Triptych: The pieces in red above!

ABJM on S^3 - An Airy tale

This can be rewritten *à la 't Hooft* into a type IIA string expansion ($\lambda = N/k$)

$$F_{S^3} = -\log Z_{S^3} = -\sum_{\mathrm{g}\geq 0} (2\pi\mathrm{i}\lambda)^{2\mathrm{g}-2} F_{\mathrm{g}}(\lambda)\, N^{2-2\mathrm{g}}\,.$$

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The genus g type IIA free energies can be computed systematically (up to $\mathrm{e}^{-\sqrt{\lambda}}$ corrections) and read (for $m_a = 0$ and $b = 1$)

$$\begin{aligned}
F_0(\lambda) &= \frac{4\pi^3\sqrt{2}}{3}\,\hat{\lambda}^{\frac{3}{2}} + \frac{\zeta(3)}{2}\,,\\
F_1(\lambda) &= \frac{\pi}{3\sqrt{2}}\,\hat{\lambda}^{\frac{1}{2}} - \frac{1}{4}\log\hat{\lambda} + \frac{1}{6}\log\lambda + \frac{1}{12}\log\frac{\pi^2}{32} + 2\zeta'(-1) - \frac{1}{2}\log 2\,,\\
F_2(\lambda) &= \frac{5\,\hat{\lambda}^{-\frac{3}{2}}}{96\pi^3\sqrt{2}} - \frac{\hat{\lambda}^{-1}}{48\pi^2} + \frac{\hat{\lambda}^{-\frac{1}{2}}}{144\pi\sqrt{2}} - \frac{1}{360}\,,\\
F_3(\lambda) &= \frac{5\,\hat{\lambda}^{-3}}{512\pi^6} - \frac{5\,\hat{\lambda}^{-\frac{5}{2}}}{768\pi^5\sqrt{2}} + \frac{\hat{\lambda}^{-2}}{1152\pi^4} - \frac{\hat{\lambda}^{-\frac{3}{2}}}{10368\pi^3\sqrt{2}} - \frac{1}{22680}\,,
\end{aligned}$$

where

$$\hat{\lambda} = \lambda - \frac{1}{24}\,.$$

Derive this from type IIA string theory?

The topologically twisted index

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Using supersymmetric localization the path integral can be reduced to a matrix integral and computed at large N and fixed k. The free energy is:

$$F_{S^1\times\Sigma_{\mathfrak{g}}} = \frac{\pi\sqrt{2k\Delta_1\Delta_2\Delta_3\Delta_4}}{3}\sum_{a=1}^{4}\frac{\mathfrak{n}_a}{\Delta_a}\Big(\hat{N}_\Delta^{\frac{3}{2}} - \frac{\mathfrak{c}_a}{k}\hat{N}_\Delta^{\frac{1}{2}}\Big) + \frac{1-\mathfrak{g}}{2}\log\hat{N}_\Delta - \hat{f}_0(k,\Delta,\mathfrak{n}) + \mathcal{O}(\mathrm{e}^{-\sqrt{N}})\,,$$

where $\sum_{a=1}^4 \Delta_a = 2$, $\sum_{a=1}^4 \mathfrak{n}_a = 2(1-\mathfrak{g})$, and

$$\hat{N}_\Delta \equiv N - \frac{k}{24} + \frac{1}{12k}\sum_{a=1}^{4}\frac{1}{\Delta_a}\,, \qquad \mathfrak{c}_a = \frac{\prod_{b\neq a}(\Delta_a+\Delta_b)}{8\Delta_1\Delta_2\Delta_3\Delta_4}\sum_{b\neq a}\Delta_b\,.$$

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The holographic dual is given by (Euclidean) supersymmetric static Reissner-Nordström BHs in AdS_4. The TTI computes the entropy of these BHs.

The superconformal index

The superconformal index (SCI), or $S^1 \times_\omega S^2$ partition function, counts $\frac{1}{16}$-BPS operators in 3d $\mathcal{N} = 2$ SCFTs. It can be computed by supersymmetric localization.

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It is useful to consider the Cardy-like limit $\omega \to 0$. The SCI can then be analyzed with similar tools as the TTI.

For the ABJM theory at fixed k and large N we find the following ω^{-1} and ω^0 results (for $\Delta_a = 1/2$)

$$
\begin{aligned}
&\log Z_{S^1 \times_\omega S^2}(N,k,\omega) \\
&\quad = -\frac{\pi\sqrt{2k}}{3}\left[\left(\frac{1}{2\omega}+1\right)\left(N-\frac{k}{24}+\frac{2}{3k}\right)^{\frac{3}{2}} - \frac{3}{k}\left(N-\frac{k}{24}+\frac{2}{3k}\right)^{\frac{1}{2}}\right] \\
&\qquad - \frac{2}{\omega}\hat{g}_0(k) - \frac{1}{2}\log\left(N-\frac{k}{24}+\frac{2}{3k}\right) + \hat{f}_0(k) + \mathcal{O}(\mathrm{e}^{-\sqrt{N}}) + \mathcal{O}(\omega)\,.
\end{aligned}
$$

This index captures the entropy of supersymmetric AdS$_4$ Kerr-Newman black holes.

Higher-derivative corrections

[Lauria,Van Proeyen]; [Bergshoeff,de Roo,de Wit]; [Butter,de Wit,Kuzenko,Lodato]; [Myers];
[Camanho,Edelstein,Maldacena,Zhiboedov]; [NPB,Charles,Hristov,Reys]

Higher-derivative supergravity

The complete set of 8-derivative corrections to 11d supergravity is not known.
Use 4d $\mathcal{N} = 2$ **minimal** gauged supergravity instead!

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Employ conformal supergravity to show that the leading 4-der correction is

$$\mathcal{L}_{4\partial} = -(16\pi\, G_N)^{-1}\Big[R + 6\, L^{-2} - \tfrac{1}{4}\, F_{\mu\nu}F^{\mu\nu}\Big] + (c_1 - c_2)\, \mathcal{L}_{\mathrm{W}^2} + c_2\, \mathcal{L}_{\mathsf{GB}}\,.$$

Two undetermined constants c_1 and c_2. They should encode information about the 8-der terms in 11d and the internal manifold X^7.

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Two undetermined constants c_1 and c_2. They should encode information about the 8-der terms in 11d and the internal manifold X^7.

The regularized on-shell action is related to the "free energy" in the dual QFT. For **all** 2-der solutions (including non-susy ones) one finds

$$\boxed{I_{4\partial} = \left[1 + \frac{64\pi G_N}{L^2}(c_2 - c_1)\right]\frac{\pi L^2}{2G_N}\mathcal{F} + 32\pi^2 c_1\,\chi\,.}$$

$\mathcal{F} = \frac{2G_N}{\pi L^2}(I_{2\partial} + I_{2\partial}^{\mathsf{CT}})$: regularized on-shell action of the 2-der theory.

$\chi = \frac{1}{32\pi^2}(I_{\mathsf{GB}} + I_{\mathsf{GB}}^{\mathsf{CT}})$: Euler characteristic of the 4-manifold.

Upshot: $I_{4\partial}$ can be computed explicitly for all known 2-der solutions of 4d minimal gauged supergravity.

M2-branes at large N

General arguments about HD terms in holography combined with the 2-der structure of 11d supergravity imply the following large N behavior

$$\frac{L^2}{2G_N} = A\,N^{\frac{3}{2}} + a\,N^{\frac{1}{2}}\,, \quad c_1 = v_1\,\frac{N^{\frac{1}{2}}}{32\pi}\,, \quad c_2 = v_2\,\frac{N^{\frac{1}{2}}}{32\pi}\,.$$

With this at hand the 4-der on-shell action becomes

$$I_{4\partial} = \pi\,\mathcal{F}\,\left[A\,N^{\frac{3}{2}} + (a + v_2)\,N^{\frac{1}{2}}\right] - \pi\,(\mathcal{F} - \chi)\,v_1\,N^{\frac{1}{2}}\,.$$

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Idea: Fix the unknown constants $(A, a + v_2, v_1)$ by using supersymmetric localization results for C_T and the round S^3 free energy.

This allows us to fix $I_{4\partial}$ to order $N^{\frac{1}{2}}$ for ABJM!

$$A = \frac{\sqrt{2k}}{3}\,, \qquad a + v_2 = -\frac{k^2 + 8}{24\sqrt{2k}}\,, \qquad v_1 = -\frac{1}{\sqrt{2k}}\,.$$

Consistency checks using the $N^{\frac{1}{2}}$ terms in the TTI, SCI, and squashed S^3 partition functions. Non-trivial predictions for other partition functions!

Log corrections

[Kundera]; [Fradkin,Tseytlin]; [Gibbons,Nicolai]; [Camporesi,Higuchi]; [Vassilevich]; [Sen]; [Bhattacharyya,Grassi,Mariño,Sen]; [Liu,Pando Zayas,Rathee,Zhao]; [Pando Zayas,Xin]; [Hristov,Reys]; [David,Godet,Liu,Pando Zayas]; [NPB,David,Hong,Reys,Zhang]

Log corrections

There are log corrections to the BH entropy

$$S_{\rm BH} = \frac{\rm Area}{4G_{\rm N}} + s_0 \log \frac{\rm Area}{G_{\rm N}} + \ldots$$

Ashoke Sen: s_0 can be computed via 1-loop contributions of all “light” fields in the BH background. Agreement with microscopic string theory calculations for BPS black holes. “IR window into UV physics!”

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Here: Log corrections in AdS_4, i.e. $\log \frac{L^2}{G_{\mathrm{N}}} \sim \log N$.

$$F_{S^3}(b,\Delta) = f_{\frac{3}{2}}(b,\Delta) N^{\frac{3}{2}} + f_{\frac{1}{2}}(b,\Delta) N^{\frac{1}{2}} + \frac{1}{4}\log N + \dots$$

$$F_{S^1\times\Sigma_{\mathfrak{g}}}(\mathfrak{n},\Delta) = g_{\frac{3}{2}}(\mathfrak{n},\Delta) N^{\frac{3}{2}} + g_{\frac{1}{2}}(\mathfrak{n},\Delta) N^{\frac{1}{2}} + \frac{1-\mathfrak{g}}{2}\log N + \dots$$

$$F_{S^1\times_\omega S^2}(\omega,\Delta) = h_{\frac{3}{2}}(\omega,\Delta) N^{\frac{3}{2}} + h_{\frac{1}{2}}(\omega,\Delta) N^{\frac{1}{2}} + \frac{1}{2}\log N + \dots$$

The coefficient of $\log N$ does NOT depend on continuous parameters!

Heat kernel in 4d

Study the log term in the (Euclidean) path integral of GR+EFT in AdS$_4$ with cutoff scale Λ

$$-\log Z_{\mathrm{GR+EFT}} = \frac{1}{16\pi G_{\mathrm{N}}} S_{\mathrm{cl}}(\phi) + {\color{red}\mathcal{C}} \log L\Lambda + \ldots .$$

All fields ϕ with $\mathrm{mass}_\phi < \Lambda$ contribute to $\mathcal{C}$. Use the heat kernel method to compute $\mathcal{C}$.

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Input: The kinetic operator $\mathcal{Q}_\phi$ and the number of zero modes

$$\mathcal{C} = \sum_\phi \int d^4x \sqrt{g}\, a_4(x, \mathcal{Q}_\phi) + \mathcal{C}_{\mathrm{ZM}} \,.$$

The Seeley-de Witt coefficient $a_4(x, \mathcal{Q}_\phi)$ depends on the background fields

$$16\pi^2 a_4(x, \mathcal{Q}_\phi) = a_E E_4 + c W^2 + b_1 R^2 + b_2 R F_{\mu\nu} F^{\mu\nu} \,.$$

Possible (tedious) to calculate $a_4(x, \mathcal{Q}_\phi)$ for massive fields of spin ≤ 2.

Subtlety: It is in general hard to compute $\mathcal{C}_{\mathrm{ZM}}$. Rigorous results only for AdS$_4$ and AdS$_2 \times \Sigma_{\mathfrak{g}}$.

KK supergravity

"Log-Bootstrap": Study various 4d supergravity backgrounds and impose that $\mathcal{C}$ does not depend on continuous parameters. Leads to the strong constraint

$$\boxed{c^{\text{tot}} = b_1^{\text{tot}} = b_2^{\text{tot}} = 0}$$

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Top-down: 11d sugra on S^7. The resulting 4d $\mathcal{N} = 8$ gauged sugra is not a standard EFT, it has infinitely many fields!

Organize the KK modes into $\mathcal{N} = 8$ multiplets and compute the SdW coefficients. At each KK level n one has $c(n) = b_1(n) = 0$.

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For the total a_E coefficient one finds the divergent sum

$$a_E = \frac{1}{72} \sum_{n=0}^{\infty} (n+1)(n+2)(n+3)^2(n+4)(n+5)\,.$$

Unclear how to regulate this sum. If we postulate $a_E = 1/3$ then we find

$$\boxed{\mathcal{C}(\mathcal{M}) = \frac{1}{4}\chi(\mathcal{M})}\,.$$

Perfect agreement with all susy localization results in the ABJM theory!

The unbearable lightness of the KK scale

Assumption: The UV completion of GR+EFT in AdS_4 is holographic, i.e. there is a dual sequence of 3d CFTs with a suitable large N limit.

Consider such a GR+EFT with **finitely** many fields and a cutoff Λ such that $L\Lambda \sim N^{\alpha}$ with $\alpha > 0$ (or $L\Lambda \sim \lambda^{\beta}$ for a marginal coupling λ).

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The free energy of the 3d CFT on a compact Euclidean manifold M_3 is

$$\log Z_{\mathrm{CFT}}(M_3) = F_0 + \mathcal{C}_{\log} \log N \,,$$

where F_0 contains all positive powers of N.

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Assumption: The UV completion of GR+EFT in AdS_4 is holographic, i.e. there is a dual sequence of 3d CFTs with a suitable large N limit.

Consider such a GR+EFT with **finitely** many fields and a cutoff Λ such that $L\Lambda \sim N^\alpha$ with $\alpha > 0$ (or $L\Lambda \sim \lambda^\beta$ for a marginal coupling λ).

The free energy of the 3d CFT on a compact Euclidean manifold M_3 is

$$\log Z_{\rm CFT}(M_3) = F_0 + \mathcal{C}_{\log} \log N \,,$$

where F_0 contains all positive powers of N.

If $\mathcal{C}_{\log}$ does **not** depend on continuous parameters (mass, squashing, angular velocity) then the SdW coefficients of the 4d bulk theory are constrained

$$c^{\rm tot} = b_1^{\rm tot} = b_2^{\rm tot} = 0\,.$$

This is a strong constraint for the UV consistency of EFTs in AdS_4!
Obeyed for many AdS_4 vacua in IIA, IIB, and 11d supergravity.

A new tool to delineate the landscape of scale separated AdS_4 vacua?

Non-perturbative corrections

[Gautason,Puletti,van Muiden]; [Beccaria,Giombi,Tseytlin]; [NPB,Hong,Reys];
[NPB,Gautason,Hong,Puletti,Reys,van Muiden]

Non-perturbative effects

Consider the IIA limit, i.e. fixed $\lambda = N/k$ with both N and k large.

For S^3 the leading non-perturbative correction to the free energy is

$$F_{\rm np}^{\rm CFT} = \frac{k^2}{4\pi^2}\,\mathrm{e}^{-2\pi\sqrt{2\lambda}} + \ldots\,.$$

For the TTI ($S^1 \times \Sigma_{\mathfrak{g}}$) the result is

$$F_{\rm np}^{\rm CFT} = 4k^2\lambda\,\mathrm{e}^{-2\pi\sqrt{2\lambda}} + \ldots\,.$$

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Agreement for the S^3 setup after a careful calculation of $Z_{1-{\rm loop}}$.

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For a probe Euclidean string in the RN black hole dual to the TTI we find

$$F_{\rm np}^{\rm bulk} = \mathcal{B}\,k^2\lambda\,{\rm e}^{-2\pi\sqrt{2\lambda}} + \dots .$$

Subtle to fix the numerical factor $\mathcal{B}$. [work in progress]

Black holes and thermal observables

[Witten]; [Horowitz,Myers]; [NPB,Charles,Hristov,Reys]; [NPB,Hong,Reys];
[Iliesiu,Kologlu,Mahajan,Perlmutter,Simmons-Duffin]; [Luo,Wang]; [Benjamin,Lee,Ooguri,Simmons-Duffin]

BHs and thermal observables

Using the results above we can compute the leading corrections to the entropy of **any** large asymptotically AdS$_4 \times S^7/\mathbb{Z}_k$ black hole.

Example: AdS$_4$-Schwarzschild black hole

$$\boxed{S_{\text{Sch}}^{\text{ABJM}} = \frac{2\pi r_+^2}{L^2}\frac{\sqrt{2k}}{3}\left(N^{\frac{3}{2}} + \frac{16-k^2}{16k}N^{\frac{1}{2}}\right) + \frac{2\pi}{\sqrt{2k}}N^{\frac{1}{2}} - \frac{1}{2}\log N + \dots}$$

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Consider a 3d CFT on $S^1_\beta \times \mathbb{R}^2$. The vev of the stress-energy tensor and the thermal free energy are

$$\langle \mathcal{T}^{00}\rangle = \frac{2}{3}\frac{b_{\mathcal{T}}}{\beta^3}\,, \qquad F_{S^1_\beta\times\mathbb{R}^2} = \frac{f_{\mathcal{T}}}{\beta^3}\,, \qquad 3f_{\mathcal{T}} = b_{\mathcal{T}}\,.$$

To compute $f_{\mathcal{T}}$ in the bulk use the "AdS$_4$ soliton". For the ABJM theory we find

$$\boxed{b_{\mathcal{T}} = -\frac{8\pi^2\sqrt{2k}}{27}N^{\frac{3}{2}} + \frac{\pi^2(k^2-16)}{27\sqrt{2k}}N^{\frac{1}{2}} + 0\times\log N + \dots\,.}$$

Somewhat surprisingly to this order at large N $b_{\mathcal{T}} = -\frac{\pi^3}{72}C_{\mathcal{T}}$!

Summary

- Exact results for the large N partition function of the ABJM theory on S^3, $S^1 \times \Sigma_{\mathfrak{g}}$, and $S^1 \times_\omega S^2$.
- Discussed how some of these results can be reproduced by supergravity and string/M-theory via AdS/CFT.
- All order microscopic prediction for the entropy of the supersymmetric AdS_4 Reissner-Nordström and Kerr-Newman black holes.
- New constraints on gravity + EFT in AdS_4?
- Application of these results to non-supersymmetric black hole thermodynamics and CFT thermal observables.

Outlook

Results I did not discuss

- All order large N supersymmetric partition functions for other 3d $\mathcal{N}=2$ holographic SCFTs arising from M2- and D2-branes.
- Similar higher-derivative and logarithmic correction results for the holographically dual AdS_4 backgrounds in string/M-theory.
- Large N and holographic results for 3d $\mathcal{N}=2$ SCFTs arising from M5-branes (class $\mathcal{R}$ SCFTs).

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Some open questions

- Analytic derivation of the TTI, SCI, and deformed S^3 results/conjectures?
- Supersymmetric localization in 4d/11d supergravity?
- Derivation from (and lessons for) type IIA string theory and M-theory?
- OSV-type conjecture for AdS black holes?
- Application of the "unbearable lightness" constraint to candidate scale separated AdS$_4$ vacua?


thank you
danke
謝謝
ngiyabonga
рахмат
teşekkür ederim
gracias
merci
obrigado
bedankt
спасибо
Баярлалаа
dziękuję
go raibh maith agat
mochchakkeram
sukriya
kop khun krap
terima kasih
arigatô
dakujem
grazie
tapadh leat
dank je
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kia ora
barka
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vinaka
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misaotra
matondo
paldies
grazzi
mahalo
хвала
asante
manana
obrigada
tenki
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chokrane
mamnun
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dhanyavad
bayarlalaa
gràçie
köszönöm
enkosi
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gràcies
sulpáy
sobodi
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mèsi
didi madloba
najis tuke
kam sah hamnida
rahmat
ありがとう
tanemirt
rahmet
diolch
dhanyavadagalu
shukriya
mercé
мерси
trugarez
তোমাকে ধন্যবাদ
감사합니다
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ευχαριστώ