CP violation in $H \to \tau^+ \tau^- \gamma$

An overview of phenomenological analysis

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(Experimental aspects by Nikolai Fomin and Anna Lipniacka)

Conference of Norwegian Financial Mechanism
“Early Universe” project

Department of Physics and Technology, University of Bergen, Norway

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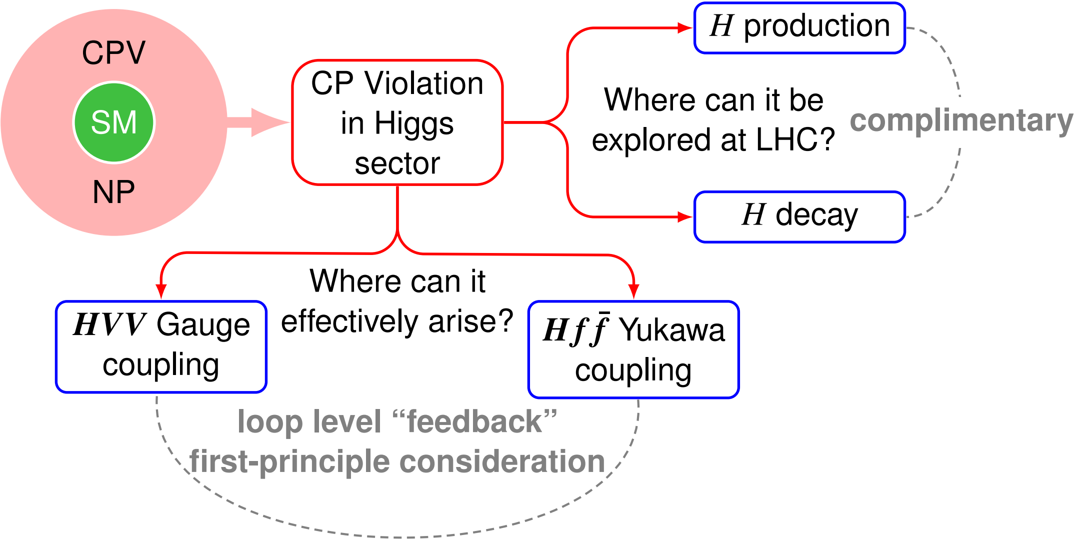
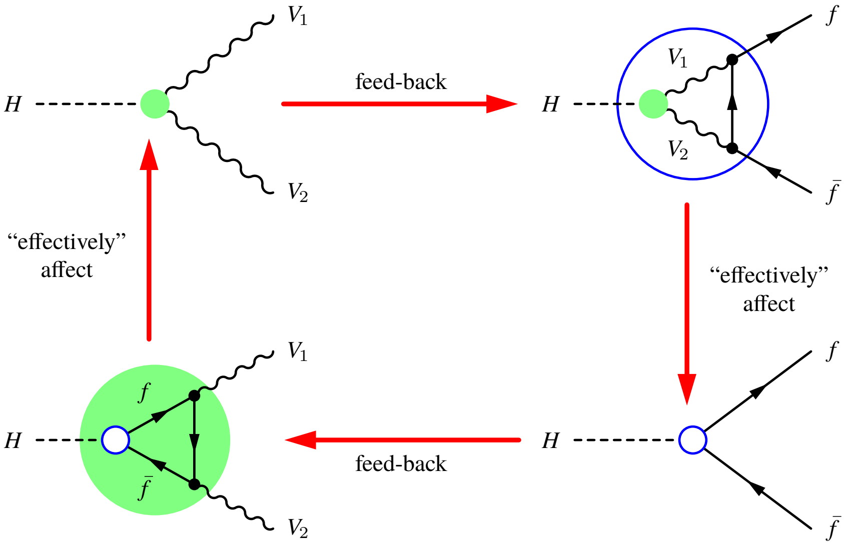
General perspective on CP violation in Higgs



Possible "feedback" of CP violation at loop level

At least in principle



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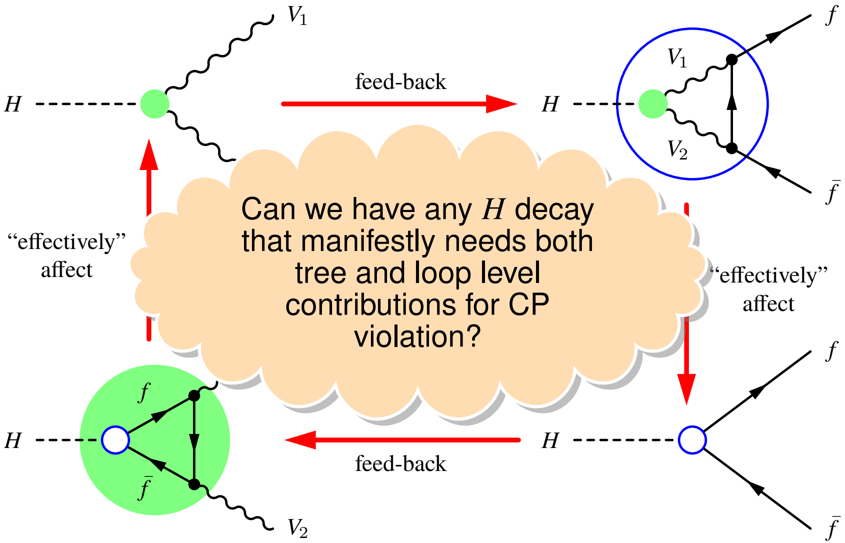
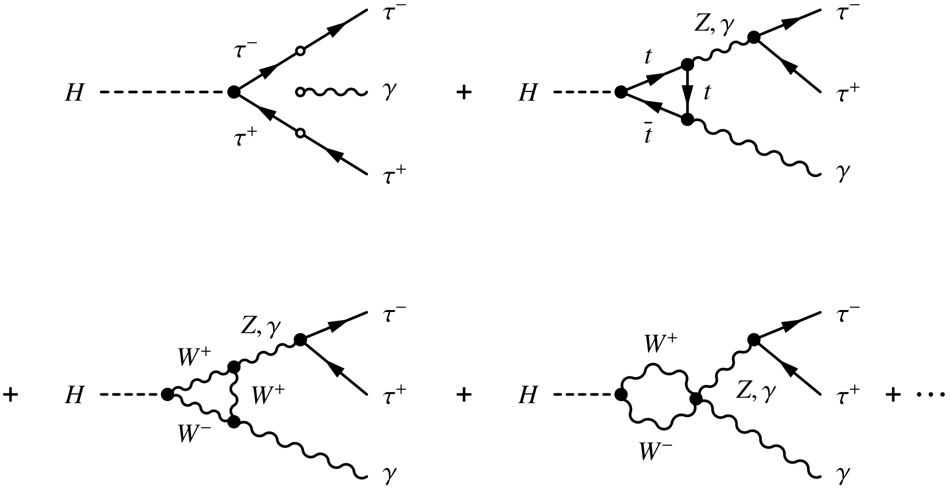


The $H \to \tau^+ \tau^- \gamma$ decay proceeds via both tree and loop diagrams



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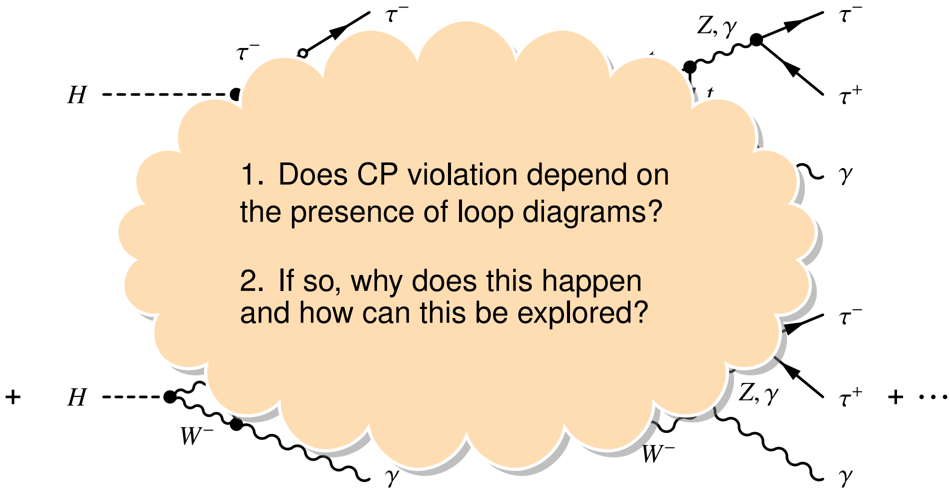
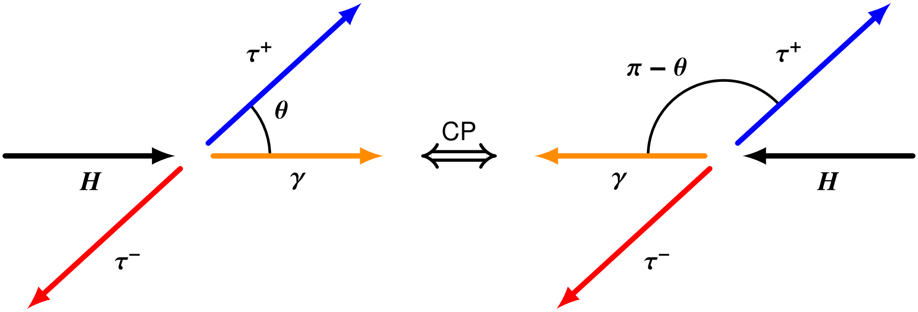


1. Does CP violation depend on the presence of loop diagrams?

2. If so, why does this happen and how can this be explored?

A first-principle analysis of $H \to \tau^+ \tau^- \gamma$

What happens to kinematic configuration under CP in the center-of-momentum frame of $\tau^+ \tau^-$?



CP violation $\Leftrightarrow$ **asymmetry** under $\theta \leftrightarrow \pi - \theta$ ($\equiv \cos\theta \leftrightarrow -\cos\theta$) exchange

A first-principle analysis of $H \to \tau^+ \tau^- \gamma$

What is required for CPV to be observed?

- ❖ No CP violation in the SM Higgs sector $\Longrightarrow$ Observation of CPV requires NP.
- ❖ Amplitude: $\mathscr{M} = \mathscr{M}_{\text{SM}} + \mathscr{M}_{\text{NP}}$.
- ❖ NP contribution comes from higher dimensional operators and is suppressed.
- ❖ Any CP violating observable would depend on amplitude square,

$$|\mathscr{M}|^2 = \underbrace{|\mathscr{M}_{\text{SM}}|^2}_{\text{dominant}} + \underbrace{|\mathscr{M}_{\text{NP}}|^2}_{\text{negligible}} + \underbrace{\mathscr{M}_{\text{SM}}\,\mathscr{M}^*_{\text{NP}} + \mathscr{M}_{\text{NP}}\,\mathscr{M}^*_{\text{SM}}}_{\text{ought to be } \neq 0\text{, sizable and violate CP}}$$

A first-principle analysis of $H \to \tau^+ \tau^- \gamma$

What is required for CPV to be observed?

- CP violating amplitudes, in general, would have a CP-even phase (‘strong’ phase δ) and a CP-odd phase (‘weak’ phase ϕ),

$$\mathscr{M} = |\mathscr{M}_{\mathrm{SM}}|\, e^{i\,(\delta_{\mathrm{SM}}+\phi_{\mathrm{SM}})} + |\mathscr{M}_{\mathrm{NP}}|\, e^{i\,(\delta_{\mathrm{NP}}+\phi_{\mathrm{NP}})},$$

  so that the CP conjugate configuration would be described by the amplitude,

$$\overline{\mathscr{M}} = |\mathscr{M}_{\mathrm{SM}}|\, e^{i\,(\delta_{\mathrm{SM}}-\phi_{\mathrm{SM}})} + |\mathscr{M}_{\mathrm{NP}}|\, e^{i\,(\delta_{\mathrm{NP}}-\phi_{\mathrm{NP}})}.$$

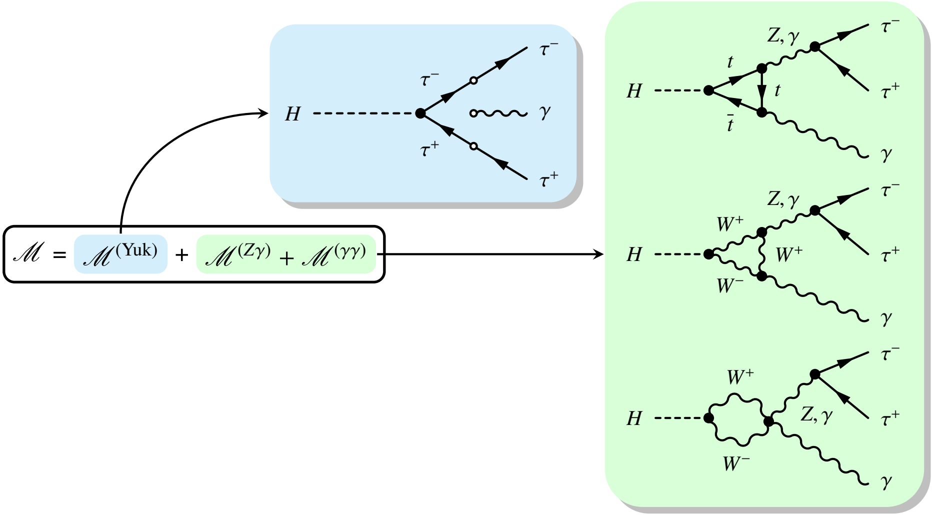
- The observable difference between the two kinematic configurations would then probe the CP asymmetry,

$$\mathscr{A}_{\mathrm{CP}} = \frac{|\mathscr{M}|^2 - |\overline{\mathscr{M}}|^2}{|\mathscr{M}|^2 + |\overline{\mathscr{M}}|^2} \propto |\mathscr{M}_{\mathrm{SM}}|\, |\mathscr{M}_{\mathrm{NP}}|\, \sin\left(\delta_{\mathrm{SM}} - \delta_{\mathrm{NP}}\right)\, \sin\left(\phi_{\mathrm{SM}} - \phi_{\mathrm{NP}}\right),$$

  which is non-zero only when $|\mathscr{M}_{\mathrm{NP}}| \neq 0, \quad \delta_{\mathrm{NP}} \neq \delta_{\mathrm{SM}}, \quad \phi_{\mathrm{NP}} \neq \phi_{\mathrm{SM}}.$ Are these conditions ‘effectively’ met in the actual calculation?

Phenomenological Lagrangians and Amplitudes

Starting point of an 'effective' approach to check conditions for CPV



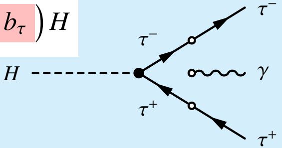
1-loop SM box diagrams negligible

Phenomenological Lagrangians and Amplitudes

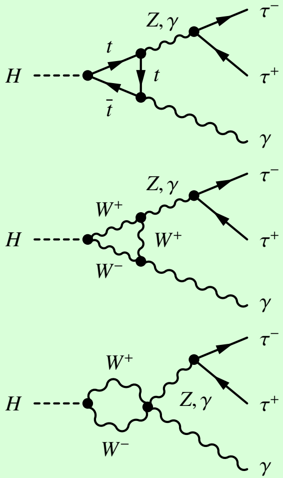
Starting point of an 'effective' approach to check conditions for CPV

$$\mathscr{L}_{H\tau\tau} = -\frac{m_\tau}{v}\,\bar{\tau}\left(a_\tau + i\gamma^5\, b_\tau\right)\tau\, H$$

$$a_\tau^{\text{SM}} = 1,\; b_\tau^{\text{SM}} = 0$$
$$a_\tau^{\text{NP}} \neq 1,\; b_\tau^{\text{NP}} \neq 0$$



$$\mathscr{M} = \mathscr{M}^{(\text{Yuk})} + \mathscr{M}^{(Z\gamma)} + \mathscr{M}^{(\gamma\gamma)}$$



$$\mathscr{L}_{H\mathscr{V}\gamma} = \frac{H}{4\,v}\Big(2\, A_2^{Z\gamma}\, F^{\mu\nu} Z_{\mu\nu} + 2\, A_3^{Z\gamma}\, F^{\mu\nu}\widetilde{Z}_{\mu\nu} + A_2^{\gamma\gamma}\, F^{\mu\nu} F_{\mu\nu} + A_3^{\gamma\gamma}\, F^{\mu\nu}\widetilde{F}_{\mu\nu}\Big),$$

where $\mathscr{V}_{\mu\nu} = \partial_\mu \mathscr{V}_\nu - \partial_\nu \mathscr{V}_\mu, \quad \widetilde{\mathscr{V}}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\mathscr{V}^{\rho\sigma},$

for $\mathscr{V} = Z, \gamma$.

1-loop SM box diagrams negligible

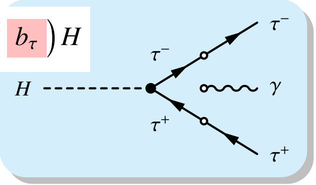
Phenomenological Lagrangians and Amplitudes

Starting point of an ‘effective’ approach to check conditions for CPV

$$\mathscr{L}_{H\tau\tau} = -\frac{m_\tau}{v}\,\bar{\tau}\left(a_\tau + i\gamma^5\, b_\tau\right) H$$

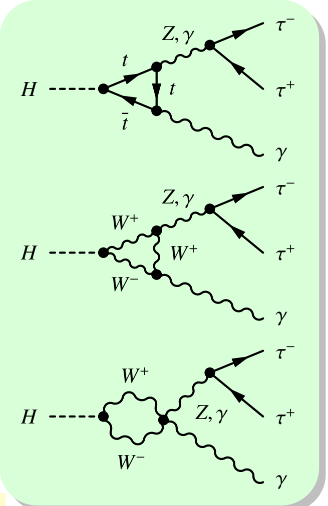
$a_\tau^{\text{SM}} = 1,\ b_\tau^{\text{SM}} = 0$

$a_\tau^{\text{NP}} \neq 1,\ b_\tau^{\text{NP}} \neq 0$



$$\mathscr{M} = \mathscr{M}^{(\text{Yuk})} + \mathscr{M}^{(Z\gamma)} + \mathscr{M}^{(\gamma\gamma)}$$

SM loop effects only



$$\mathscr{L}_{H\mathscr{V}\gamma} = \frac{H}{4\,v}\Big(2\,A_2^{Z\gamma}\,F^{\mu\nu}Z_{\mu\nu} + 2\,A_3^{Z\gamma}\,F^{\mu\nu}\widetilde{Z}_{\mu\nu} + A_2^{\gamma\gamma}\,F^{\mu\nu}F_{\mu\nu} + A_3^{\gamma\gamma}\,F^{\mu\nu}\widetilde{F}_{\mu\nu}\Big),$$

where $\mathscr{V}_{\mu\nu} = \partial_\mu \mathscr{V}_\nu - \partial_\nu \mathscr{V}_\mu$, $\quad \widetilde{\mathscr{V}}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\mathscr{V}^{\rho\sigma}$,

for $\mathscr{V} = Z, \gamma$.

We shall consider $A_3^{\gamma\gamma} = 0 = A_3^{Z\gamma}$.

op SM box diagrams negligible

Kinematics: Only 2 independent variables

- Only 3 Lorentz invariant mass-squares possible,

$$m_{+-}^2 \equiv (p_H - p_0)^2 = (p_+ + p_-)^2 \,, \qquad \Longrightarrow 4\, m_\tau^2 \leqslant m_{+-}^2 \leqslant m_H^2$$
$$m_{+0}^2 \equiv (p_H - p_-)^2 = (p_+ + p_0)^2 \,, \qquad \Longrightarrow m_\tau^2 \leqslant m_{+0}^2 \leqslant (m_H - m_\tau)^2$$
$$m_{-0}^2 \equiv (p_H - p_+)^2 = (p_- + p_0)^2 \,. \qquad \Longrightarrow m_\tau^2 \leqslant m_{-0}^2 \leqslant (m_H - m_\tau)^2$$

Note: $m_{+-}^2 + m_{+0}^2 + m_{-0}^2 = m_H^2 + 2\, m_\tau^2.$ $\Longrightarrow$ Only 2 *independent* mass-squares.

- In the center-of-momentum frame of $\tau^+ \tau^-$ (also called Gottfried-Jackson frame, or GJ frame in short),

$$\left.\begin{aligned} m_{+0}^2 &= M^2 - M'^2 \cos\theta, \\ m_{-0}^2 &= M^2 + M'^2 \cos\theta, \end{aligned}\right\} \Longrightarrow \begin{cases} & \theta \leftrightarrow \pi - \theta \\ \equiv & \cos\theta \leftrightarrow -\cos\theta \\ \equiv & m_{+0}^2 \leftrightarrow m_{-0}^2 \end{cases}$$

where $M^2 = \frac{1}{2}\left(m_H^2 + 2\, m_\tau^2 - m_{+-}^2\right), \quad M'^2 = \frac{1}{2}\left(m_H^2 - m_{+-}^2\right) \sqrt{1 - 4\, m_\tau^2 / m_{+-}^2}.$

Choice of independent variables in $H \to \tau^+ \tau^- \gamma$

	$\left(m^2_{+0}, m^2_{-0}\right)$	$\left(m^2_{+-}, \cos\theta\right)$	$\left(E_+, E_-\right)$
Differential Decay rate	$\frac{d^2\Gamma_{\tau\tau\gamma}}{dm^2_{+0}\, dm^2_{-0}}$	$\frac{d^2\Gamma_{\tau\tau\gamma}}{dm^2_{+-}\, d\cos\theta}$	$\frac{d^2\Gamma_{\tau\tau\gamma}}{dE_+\, dE_-}$
Frame of reference	Any frame	GJ frame	H rest frame

- $E_\pm$ = energy of $\tau^\pm$ in H rest frame. $m^2_{\pm 0} = m^2_H - 2\, m_H\, E_\pm$ & $m^2_{+0} \leftrightarrow m^2_{-0} \equiv E_+ \leftrightarrow E_-$
- Differential decay rate is frame dependent:

$$\left(\frac{d^2\Gamma_{\tau\tau\gamma}}{dm^2_{+0}\, dm^2_{-0}}\right)_{\text{H rest}} = \frac{\left|\mathscr{M}_{\tau\tau\gamma}\right|^2}{256\, \pi^3\, m^3_H},$$

$$\left(\frac{d^2\Gamma_{\tau\tau\gamma}}{dm^2_{+-}\, d\cos\theta}\right)_{\text{H rest}} = \frac{m^2_H - m^2_{+-}}{512\, \pi^3\, m^3_H} \sqrt{1 - \frac{4\, m^2_\tau}{m^2_{+-}}}\, \left|\mathscr{M}_{\tau\tau\gamma}\right|^2,$$

$$\left(\frac{d^2\Gamma_{\tau\tau\gamma}}{dm^2_{+-}\, d\cos\theta}\right)_{\text{GJ}} = \frac{m_{+-}\left(m^2_H - m^2_{+-}\right)}{256\, \pi^3\, m^2_H \left(m^2_H + m^2_{+-}\right)} \sqrt{1 - \frac{4\, m^2_\tau}{m^2_{+-}}}\, \left|\mathscr{M}_{\tau\tau\gamma}\right|^2.$$

Source of CP asymmetry in the amplitude square

$$|\mathscr{M}|^2 = \left|\mathscr{M}^{(\mathrm{Yuk})}\right|^2 + \left|\mathscr{M}^{(Z\gamma)}\right|^2 + \left|\mathscr{M}^{(\gamma\gamma)}\right|^2 + 2\,\mathrm{Re}\left(\mathscr{M}^{(\gamma\gamma)}\,\mathscr{M}^{(Z\gamma)*}\right)$$
$$+ 2\,\mathrm{Re}\left(\mathscr{M}^{(\mathrm{Yuk})}\,\mathscr{M}^{(Z\gamma)*}\right) + 2\,\mathrm{Re}\left(\mathscr{M}^{(\mathrm{Yuk})}\,\mathscr{M}^{(\gamma\gamma)*}\right),$$

where if we focus only on the CP-even and CP-odd couplings, m_{+-}^2 and the $\cos\theta$ dependence, we notice that

$$\left|\mathscr{M}^{(\mathrm{Yuk})}\right|^2 \propto \frac{\left(a_\tau^2 + b_\tau^2\right) m_\tau^2 \left(m_H^4 + m_{+-}^4\right)}{\left(m_H^2 - m_{+-}^2\right)^2 \ \sin^2\theta},$$

$$\left|\mathscr{M}^{(Z\gamma)}\right|^2 \propto \frac{\left(\left(A_2^{Z\gamma}\right)^2 + \left(A_3^{Z\gamma}\right)^2\right) m_{+-}^2 \left(m_H^2 - m_{+-}^2\right)^2}{\left(\left(m_{+-}^2 - m_Z^2\right)^2 + \Gamma_Z^2 m_Z^2\right)} \left(1 + \cos^2\theta\right),$$

$$\left|\mathscr{M}^{(\gamma\gamma)}\right|^2 \propto \frac{\left(\left(A_2^{\gamma\gamma}\right)^2 + \left(A_3^{\gamma\gamma}\right)^2\right)\left(m_H^2 - m_{+-}^2\right)^2}{m_{+-}^2} \left(1 + \cos^2\theta\right),$$

Source of CP asymmetry in the amplitude square

$$\mathrm{Re}\left(\mathscr{M}^{(\gamma\gamma)}\,\mathscr{M}^{(Z\gamma)*}\right) \propto \frac{\left(m_H^2 - m_{+-}^2\right)^2}{\left(\left(m_{+-}^2 - m_Z^2\right)^2 + \Gamma_Z^2 m_Z^2\right)} \left(2\,c_A^\tau\,\left(A_2^{\gamma\gamma}A_3^{Z\gamma} - A_2^{Z\gamma}A_3^{\gamma\gamma}\right) m_Z\,\Gamma_Z\,\cos\theta\right.$$

$$\left. + \, c_V^\tau\,\left(A_2^{\gamma\gamma}A_2^{Z\gamma} + A_3^{\gamma\gamma}A_3^{Z\gamma}\right)\left(m_{+-}^2 - m_Z^2\right)\left(1 + \cos^2\theta\right)\right),$$

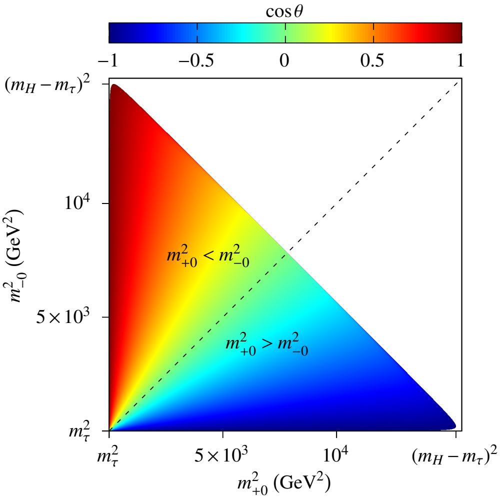
$$\mathrm{Re}\left(\mathscr{M}^{(\mathrm{Yuk})}\,\mathscr{M}^{(\gamma\gamma)*}\right) \propto \frac{m_\tau^2}{m_{+-}^2\,\sin^2\theta}\left(A_2^{\gamma\gamma}a_\tau\left(m_H^2 - m_{+-}^2\,\cos^2\theta\right) + A_3^{\gamma\gamma}b_\tau\left(m_H^2 - m_{+-}^2\right)\right),$$

Source of CP asymmetry in the amplitude square

$$
\begin{aligned}
\mathrm{Re}\left(\mathscr{M}^{(\mathrm{Yuk})}\,\mathscr{M}^{(Z\gamma)*}\right) \propto\ & \frac{m_\tau^2}{\left(\left(m_{+-}^2 - m_Z^2\right)^2 + \Gamma_Z^2 m_Z^2\right)\ \sin^2\theta} \\
& \times\left(c_A^\tau\ \left(A_3^{Z\gamma} a_\tau - A_2^{Z\gamma} b_\tau\right)\ m_Z\,\Gamma_Z\ \left(m_H^2 - m_{+-}^2\right)\ \cos\theta\right. \\
& \left. + c_V^\tau\left(m_{+-}^2 - m_Z^2\right)\left(A_2^{Z\gamma} a_\tau \left(m_H^2 - m_{+-}^2\ \cos^2\theta\right) + A_3^{Z\gamma} b_\tau \left(m_H^2 - m_{+-}^2\right)\right)\right).
\end{aligned}
$$

The $\cos\theta$ term in $\mathrm{Re}\left(\mathscr{M}^{(\mathrm{Yuk})}\,\mathscr{M}^{(Z\gamma)*}\right)$ gives rise to the asymmetry under $\theta \leftrightarrow \pi - \theta \equiv \cos\theta \leftrightarrow -\cos\theta$ exchange.

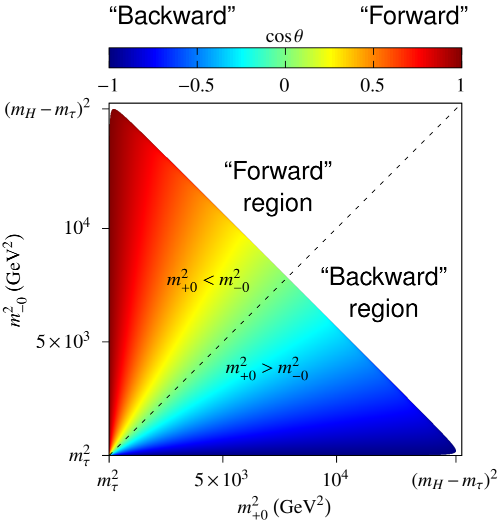
Dalitz Plot: Notations, Regions & Expectations



- Let $\mathscr{D}\left(m_{+0}^2, m_{-0}^2\right) \equiv \dfrac{\mathrm{d}^2\Gamma_{\tau\tau\gamma}}{\mathrm{d}m_{+0}^2\,\mathrm{d}m_{-0}^2}$ denote **distribution of events in the m_{+0}^2** vs. m_{-0}^2 **Dalitz plot.**

- Area of the Dalitz plot $\propto$ Available phase space

Dalitz Plot: Notations, Regions & Expectations



Notation:

Region	"Forward"	"Backward"
$\cos\theta$	$[0, 1]$	$[-1, 0]$
$m_{\pm0}^2$	$m_{+0}^2 < m_{-0}^2$	$m_{+0}^2 > m_{-0}^2$
Distribution	$\mathscr{D}\left(m_{+0}^2 < m_{-0}^2\right)$	$\mathscr{D}\left(m_{+0}^2 > m_{-0}^2\right)$
No. of events	N_F	N_B

Expectation: CP violation $(b_\tau \neq 0) \implies$

- ❖ $\mathscr{D}\left(m_{+0}^2 < m_{-0}^2\right) \neq \mathscr{D}\left(m_{+0}^2 > m_{-0}^2\right)$
- ❖ $N_F \neq N_B$

Dalitz Plot Asymmetries: Quantify CP violation

- **Non-integrated or distribution asymmetry:** Compare the distribution of events *across* the Dalitz plot in the "forward" and "backward" regions.

$$\mathscr{A}\left(m_{+0}^2, m_{-0}^2\right) = \frac{\left|\mathscr{D}\left(m_{+0}^2 < m_{-0}^2\right) - \mathscr{D}\left(m_{+0}^2 > m_{-0}^2\right)\right|}{\mathscr{D}\left(m_{+0}^2 < m_{-0}^2\right) + \mathscr{D}\left(m_{+0}^2 > m_{-0}^2\right)}.$$

- **Integrated asymmetry:** Count and compare the number of events *contained inside* the Dalitz plot in the "forward" and "backward" regions.

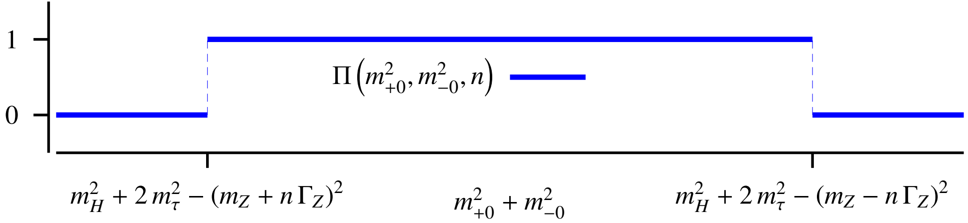
$$a_{\mathrm{FB}} = \frac{\left|\iint_{m_\tau^2}^{(m_H - m_\tau)^2} \left[\mathscr{D}\left(m_{+0}^2 < m_{-0}^2\right) - \mathscr{D}\left(m_{+0}^2 > m_{-0}^2\right)\right] \mathrm{d}m_{+0}^2 \, \mathrm{d}m_{-0}^2\right|}{\iint_{m_\tau^2}^{(m_H - m_\tau)^2} \mathscr{D}\left(m_{+0}^2, m_{-0}^2\right) \mathrm{d}m_{+0}^2 \, \mathrm{d}m_{-0}^2} = \frac{|N_F - N_B|}{N_F + N_B} = a_{\mathrm{DP}}.$$

Dalitz Plot Asymmetries: Quantify CP violation

- **Regional integrated asymmetries:** Count and compare the number of events in certain '*islands*' of the Dalitz plot, e.g. a'_{FB} which specifically probes region around Z-pole,

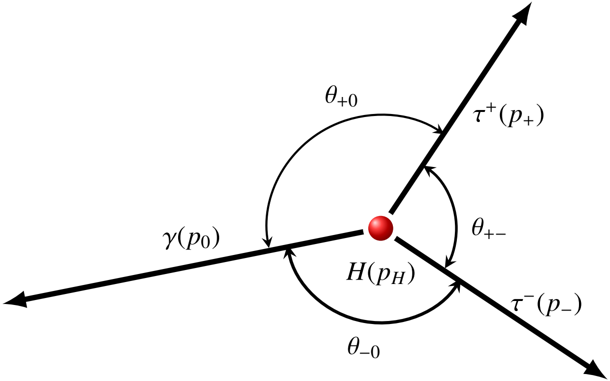
$$a'_{\text{FB}}(n) = \frac{\left| \displaystyle\iint_{m_\tau^2}^{(m_H - m_\tau)^2} \left[\mathscr{D}\left(m_{+0}^2 < m_{-0}^2\right) - \mathscr{D}\left(m_{+0}^2 > m_{-0}^2\right) \right] \Pi\left(m_{+0}^2, m_{-0}^2, n\right) \, \mathrm{d}m_{+0}^2 \, \mathrm{d}m_{-0}^2 \right|}{\displaystyle\iint_{m_\tau^2}^{(m_H - m_\tau)^2} \mathscr{D}\left(m_{+0}^2, m_{-0}^2\right) \Pi\left(m_{+0}^2, m_{-0}^2, n\right) \, \mathrm{d}m_{+0}^2 \, \mathrm{d}m_{-0}^2}.$$

where the rectangular function $\Pi\left(m_{+0}^2, m_{-0}^2, n\right)$ is graphically shown below.



Some additional considerations in H rest frame

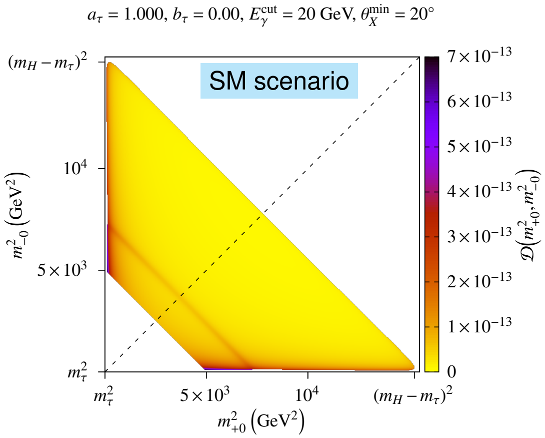
- ❖ Photon energy, $E_\gamma > E_\gamma^{\rm cut} = 5$ GeV, or 20 GeV.

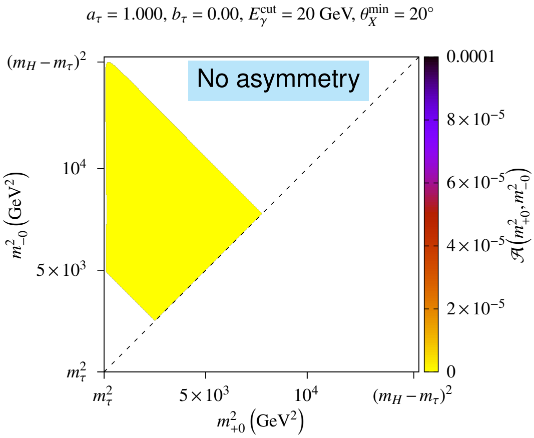


- ❖ Angles between outgoing particles, θ_X with $X \in \{+-, +0, -0\}$, be such that

$$\theta_X > 5^\circ, \text{ or } 10^\circ, \text{ or } 15^\circ \text{ etc.}$$

Analytical Dalitz plot distributions and asymmetry





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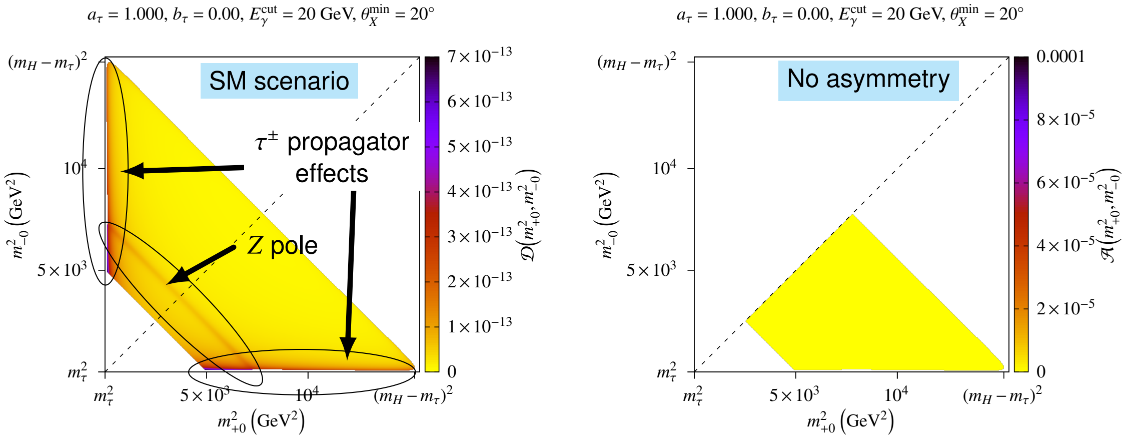

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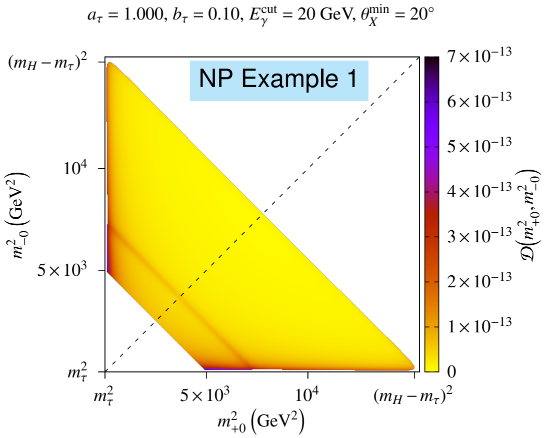

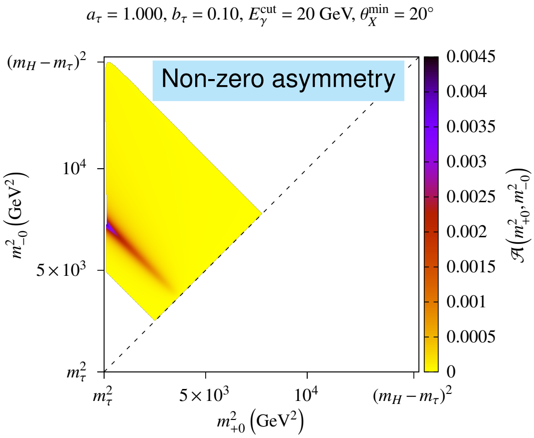

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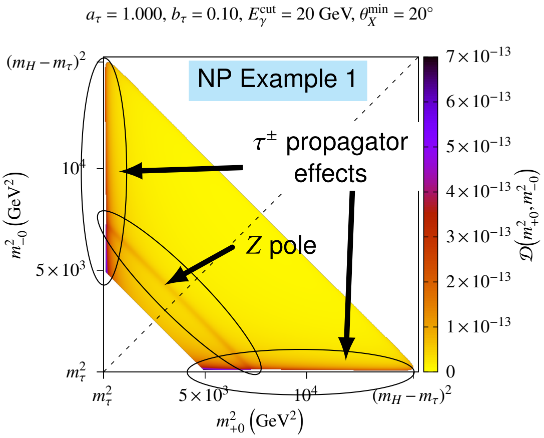

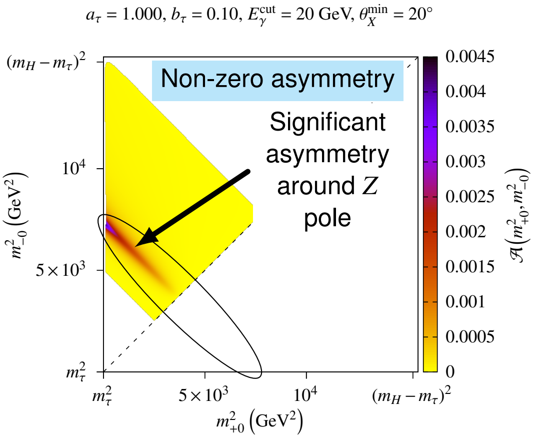

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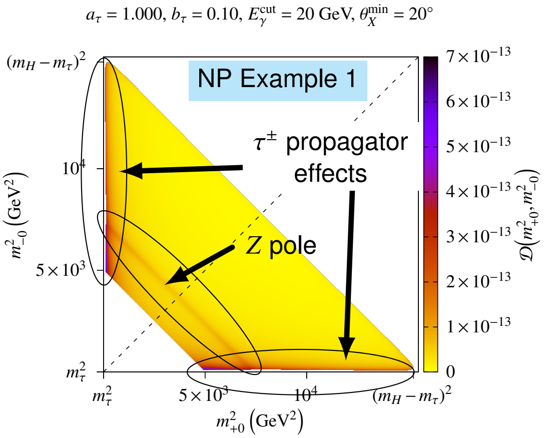

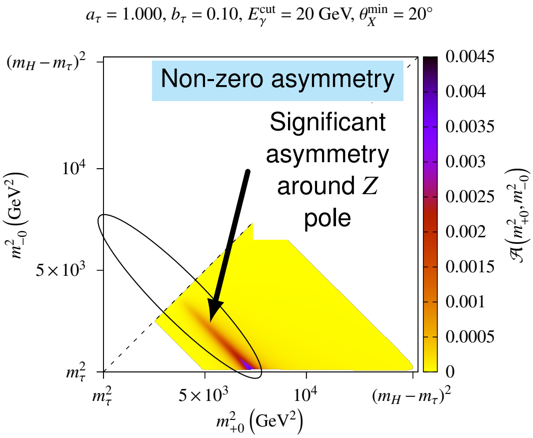

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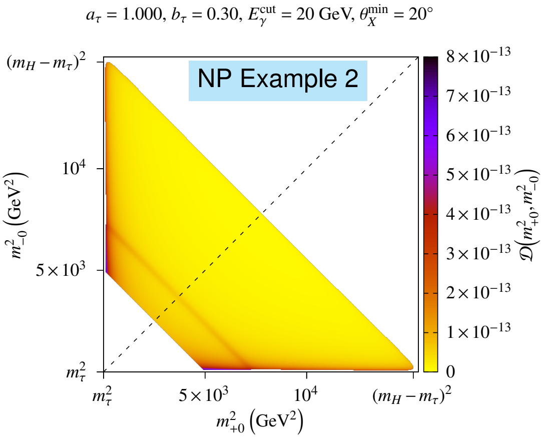

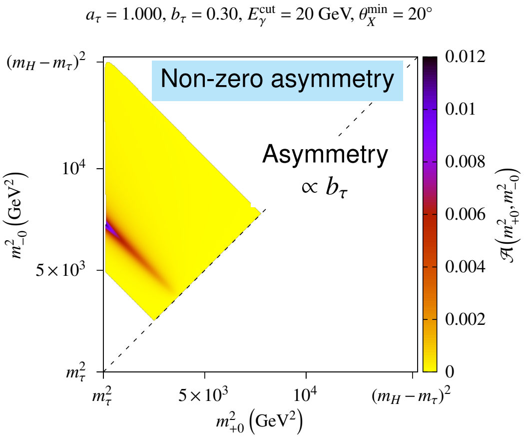

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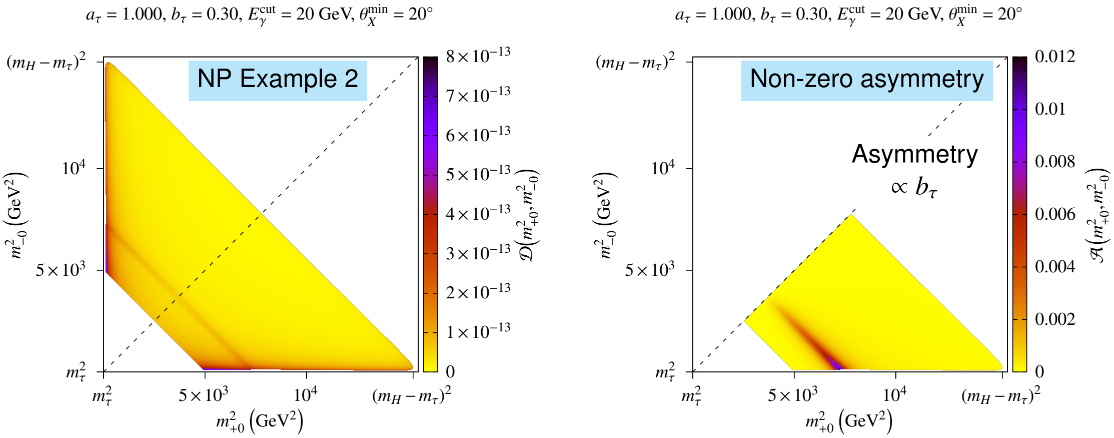
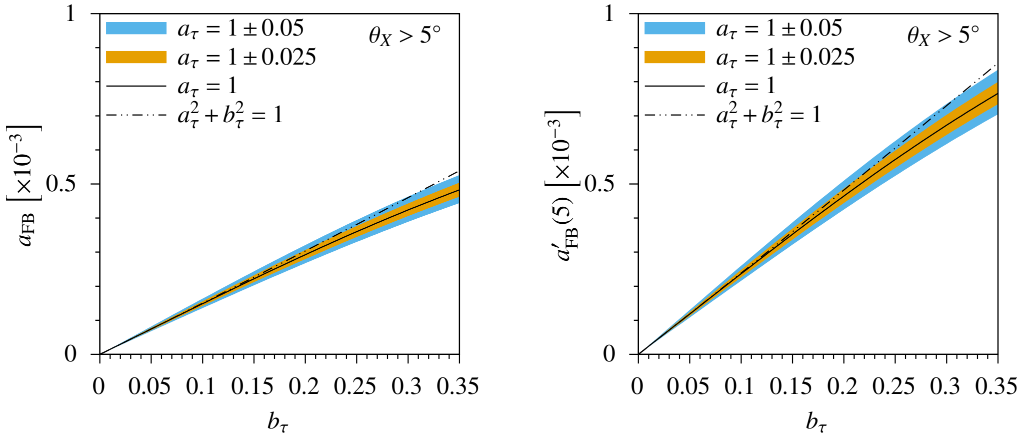


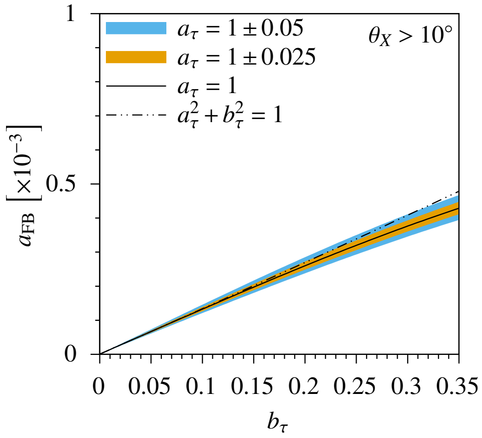
Integrated Asymmetry and Regional Asymmetry

Focusing around Z pole yields larger asymmetry

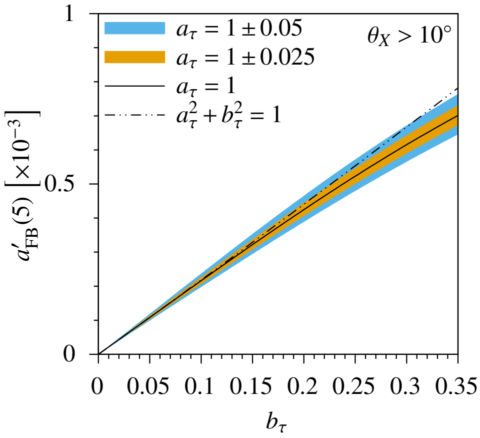


Integrated Asymmetry and Regional Asymmetry

Larger angular cuts reduce observable asymmetry

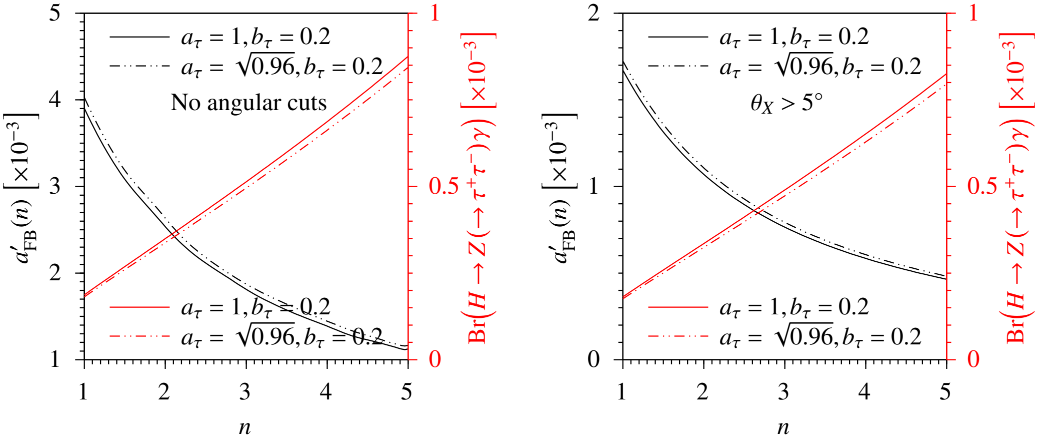


Focusing around Z pole yields larger asymmetry



Asymmetry around Z pole & corresponding branching ratio

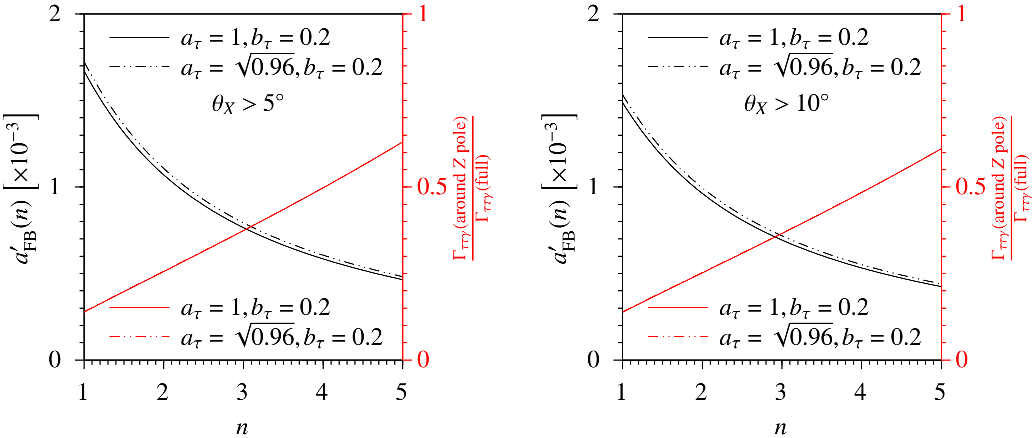
Angular cut $\implies$ phase space reduction → Decreased branching ratio



Angular cut $\implies$ Regions with highest asymmetry get sliced → Smaller asymmetry

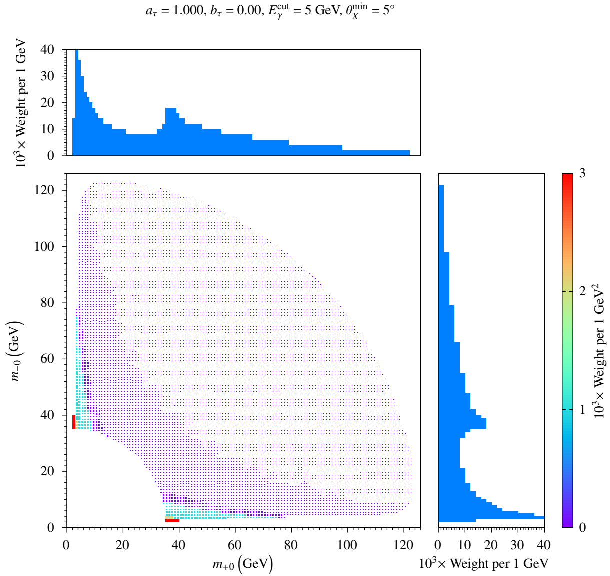
Asymmetry around Z pole vis-á-vis reduction in decay rate

Considering only those events surrounding Z pole $\Longrightarrow$ drastic reduction in phase space

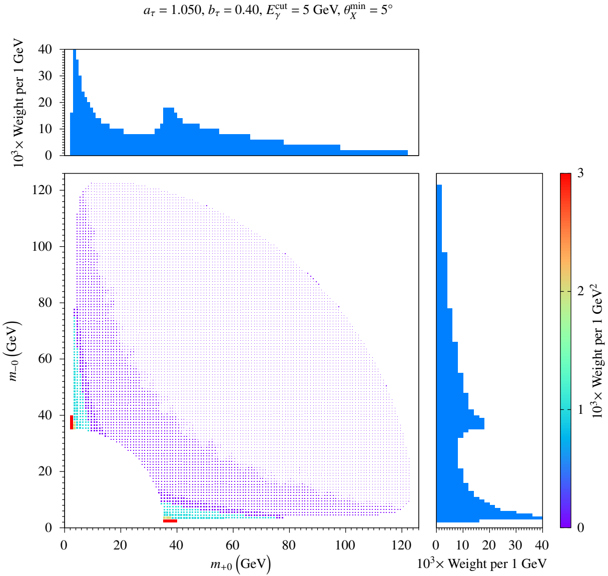


There is also some compensation due to presence of Z pole.

Weight distributions for more realistic studies



Weight distributions for more realistic studies



Feasibility study for experimental prospect...

❖ **In summary**

(1) CP violation $(b_\tau \neq 0) \implies$ Forward-Backward asymmetry in Gottfried-Jackson frame

(2) Forward-Backward asymmetry ≡ Asymmetry in m^2_{+0} vs. m^2_{-0} Dalitz plot under $m^2_{+0} \leftrightarrow m^2_{-0}$:

$$\underbrace{\mathscr{A}\left(m^2_{+0}, m^2_{-0}\right) \neq 0,}_{\text{full distribution asymmetry}} \quad \underbrace{a_{\rm FB} \neq 0,}_{\text{full integrated asymmetry}} \quad \underbrace{a'_{\rm FB}(n) \neq 0.}_{\text{asymmetry around } Z \text{ pole}} \quad \left[\text{All asymmetries } \sim \mathcal{O}\left(10^{-3}\right)\right]$$

(3) m^2_{+0} vs. m^2_{-0} Dalitz plot: can be obtained in *any frame of reference*

(4) Asymmetry is prominent surrounding the Z pole

❖ **Feasibility: Can these asymmetries be probed in ongoing or future experiments?**

❖ **Prospect: What range of b_τ would get constrained from such experimental studies?**


Thank you

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Understanding the Early Universe:
interplay of theory and collider experiments

Joint research project between the University of Warsaw & University of Bergen