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SEIT 1386

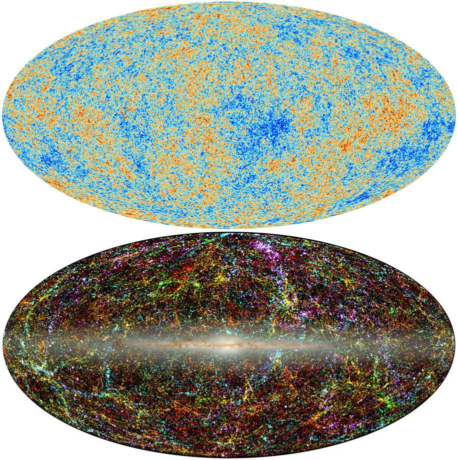
Kinetic Field Theory For Cosmic Structure Formation

Matthias Bartelmann

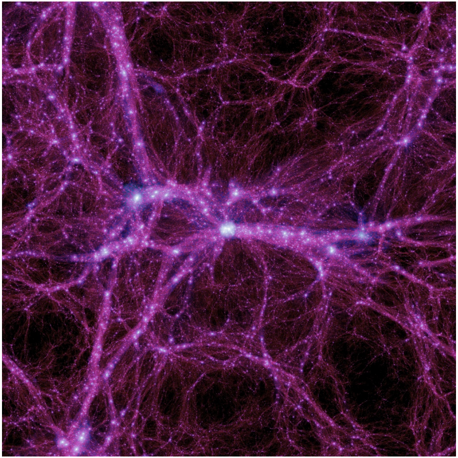
Institute for Theoretical Physics, Heidelberg University

CERN Theory Colloquium
Sep. 27th, 2023

Cosmic Structure Formation



Planck, 2-MASS

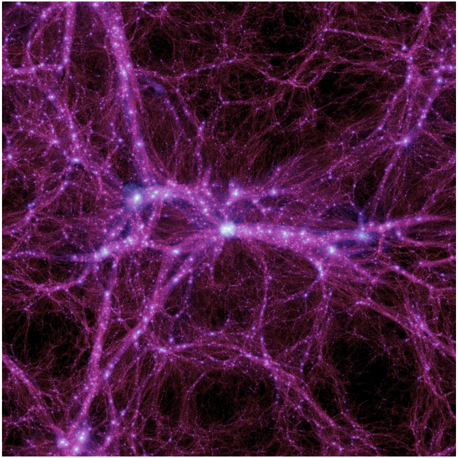


Boylan-Kolchin et al.

Cosmic Structure Formation

Questions:

- How do fundamental principles determine cosmic structures?
- Where do universal properties of cosmic structures come from?



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Cosmic Structure Formation

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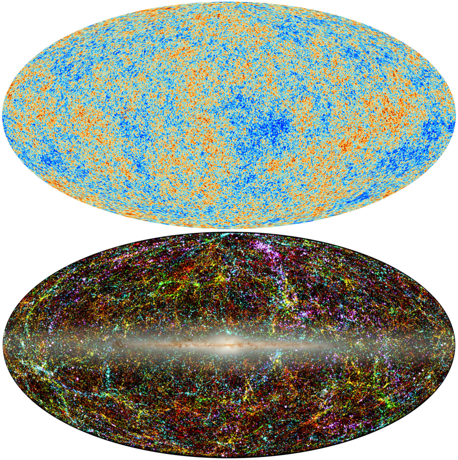

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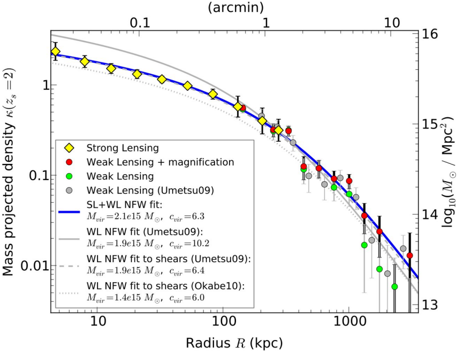
Cosmic Structure Formation

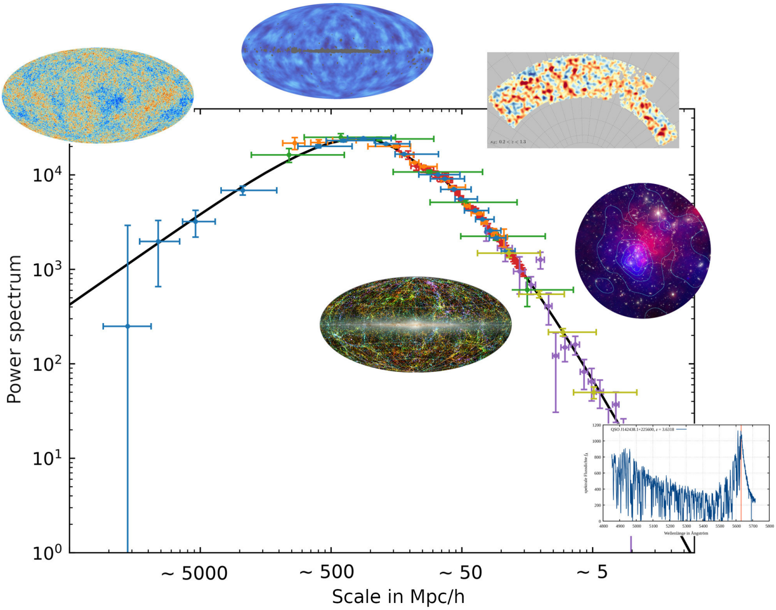
Abell 2261, CLASH Project



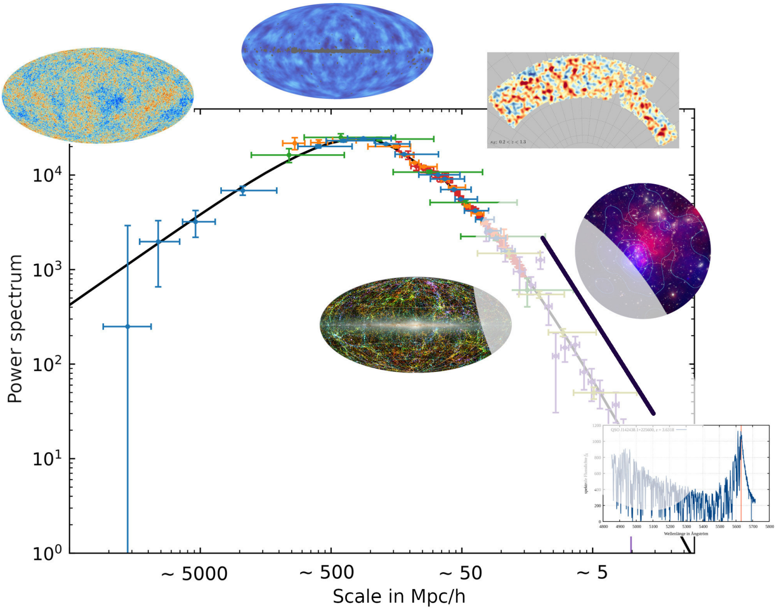
Coe et al. 2012

universal density profile, $\rho(r) = \frac{\rho_0}{x(1+x)^2}$, $\quad r = x r_s$

Universality In Cosmic Structures



Universality In Cosmic Structures



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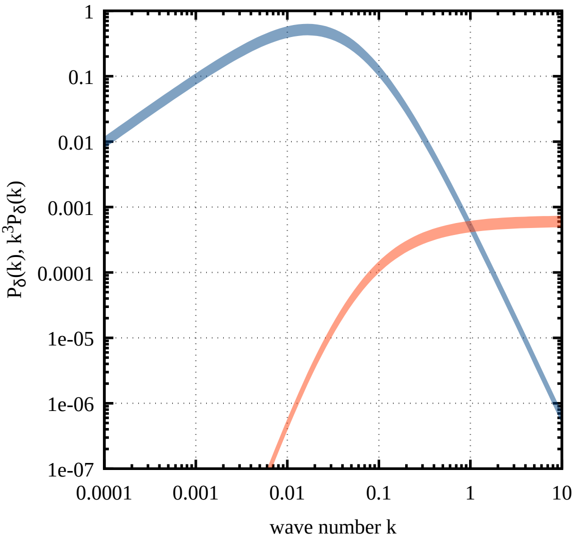

power $k^3 P_\delta(k) \rightarrow \mathrm{const}$ at small scales conventional:

- hydrodynamical equations:

$$\dot{\delta} + \vec{\nabla} \cdot \vec{u} = 0$$
$$\dot{\vec{u}} + 2H\vec{u} = -\vec{\nabla}\phi$$
$$\vec{\nabla}^2\phi = 4\pi G\bar{\rho}\delta$$

- however: dark matter is no fluid
- multiple streams, where shocks would form in a fluid

Analytic Approaches To Cosmic Structures

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kinetic field theory:

- non-equilibrium statistics of N classical particle trajectories
- description of particle ensembles by generating functional (partition sum) Z
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kinetic field theory:

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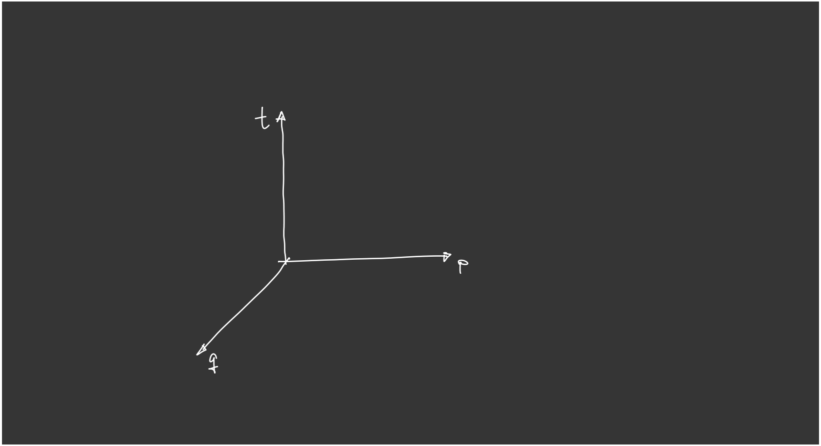
notorious problems of conventional approaches are avoided by construction

Kinetic Field Theory (1)



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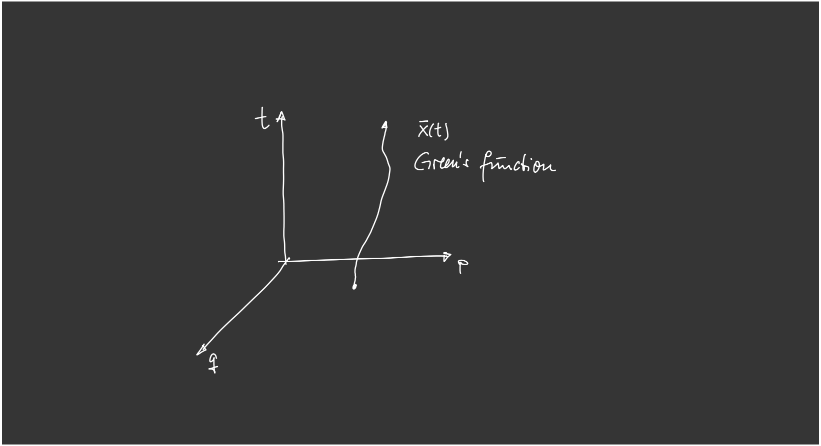

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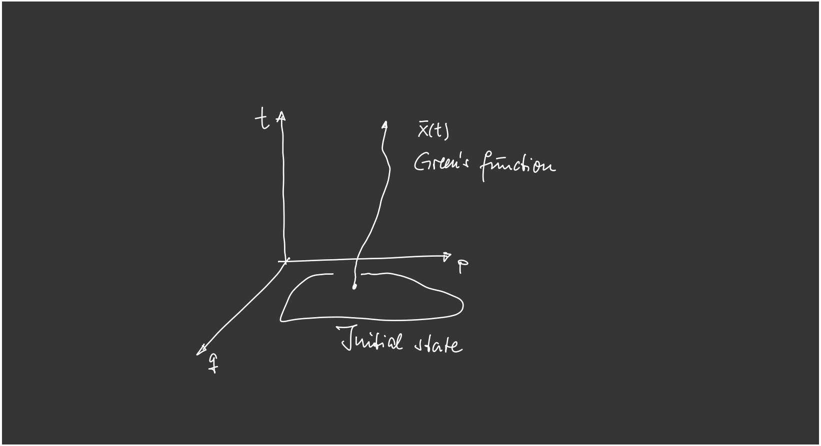

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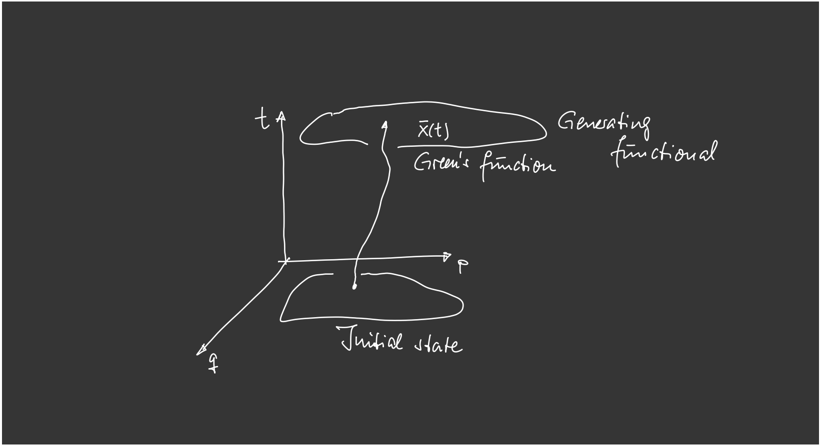

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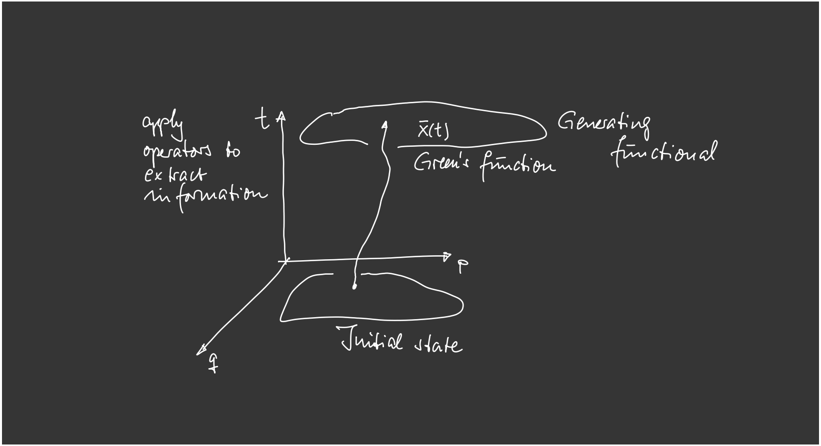

Kinetic Field Theory (2)

Generating functional:

$$Z[J] = \int P(x,t)\, e^{i(x,J)}$$

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Kinetic Field Theory (2)

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Generating functional:

$$Z[J] = \int P(x,t)\, e^{i(x,J)}$$

$$= \int\int \underbrace{P(x,t\,|\,x^{(i)})}_{\delta_D(x-\phi(x^{(i)}))}\, \underbrace{P(x^{(i)})\, dx^{(i)}}_{d\Gamma}\, e^{i(x,J)}$$

Hamilton

Kinetic Field Theory (2)

Generating functional:

$$Z[J] = \int P(x,t)\, e^{i(x,J)}$$

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$$= \int d\Gamma\, e^{i(\bar{x},J)}$$

Kinetic Field Theory (3)

Trajectories:

$$\bar{x}(t) = \underbrace{G(t,0)\,x^{(i)}}_{\text{free},\ \bar{x}_0} + \underbrace{\int_0^t G(t,t')\,F(t')\,dt'}_{\text{interacting},\ \bar{x}_I}$$

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$$Z = \int d\Gamma\, e^{i(J,\bar{x})} = \int d\Gamma\, e^{i(J,\bar{x}_0) + i\underbrace{(J,\bar{x}_I)}_{}}$$

operator

$$= e^{i\hat{S}_I} \underbrace{\int d\Gamma\, e^{i(J,\bar{x}_0)}}_{\text{free},\ Z_0} = e^{i\hat{S}_I}\, Z_0[J]$$

Kinetic Field Theory (4)

Interactions:

$$e^{i\hat{S}_I} Z_0[J] \;\nearrow\; \sum_{n=0}^{\infty} \frac{(i\hat{S}_I)^n}{n!} Z_0[J]$$

perturbation theory

$\searrow$

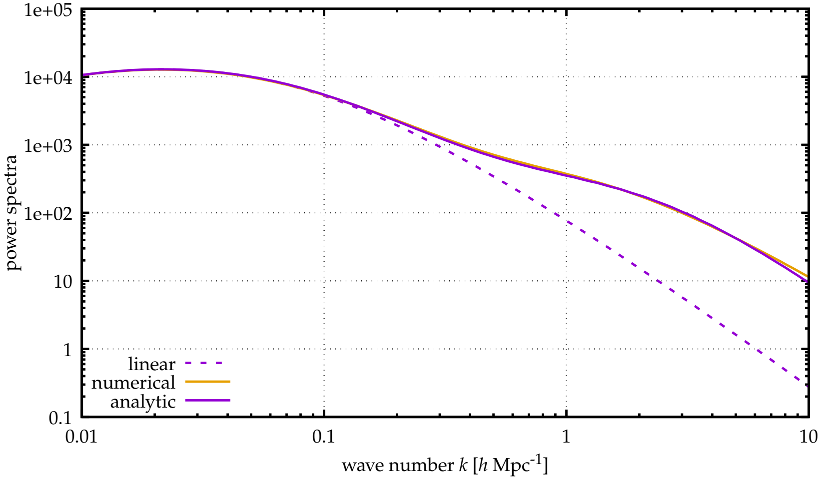
Kinetic Field Theory (4)

Interactions:

$$e^{i\hat{S}_I} Z_0[J] \;\rightarrow\; \sum_{n=0}^{\infty} \frac{(i\hat{S}_I)^n}{n!} Z_0[J] \qquad \text{perturbation theory}$$

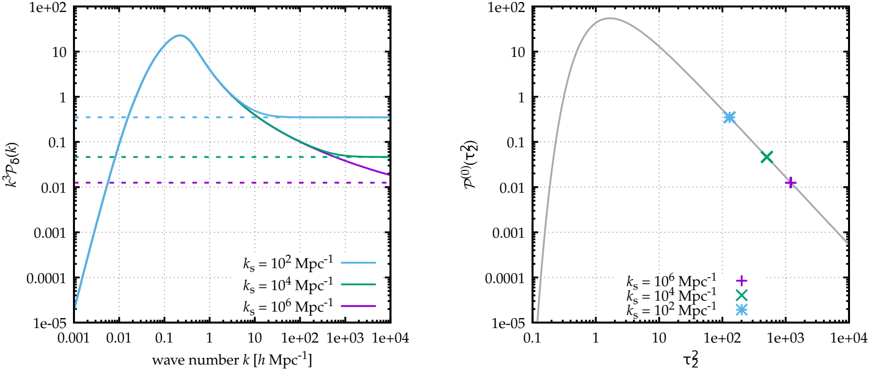
$$e^{i\hat{S}_I} Z_0[J] \;\rightarrow\; e^{i\langle \hat{S}_I \rangle} Z_0[J] \qquad \text{mean field}$$

Mean-Field Approximation



Konrad, S. & MB 2022; MB et al. 2021

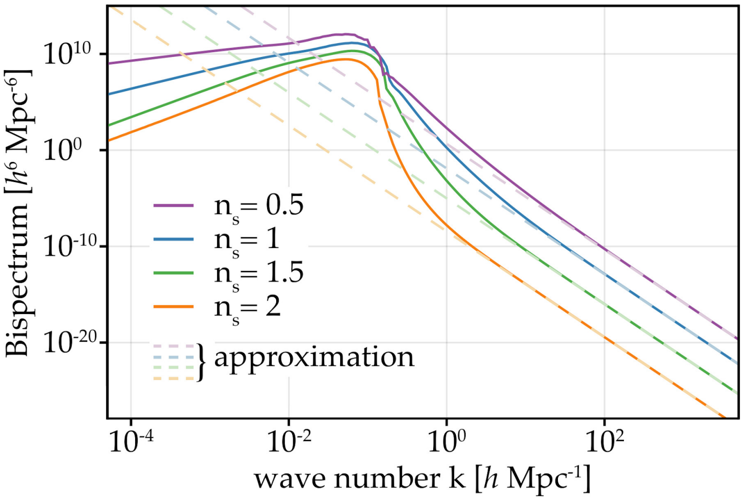
Asymptotic Behaviour



Konrad, S. & MB 2022; Konrad, S. et al. 2022; Konrad, S. & MB 2022

free power spectrum $\sim k^{-3}$

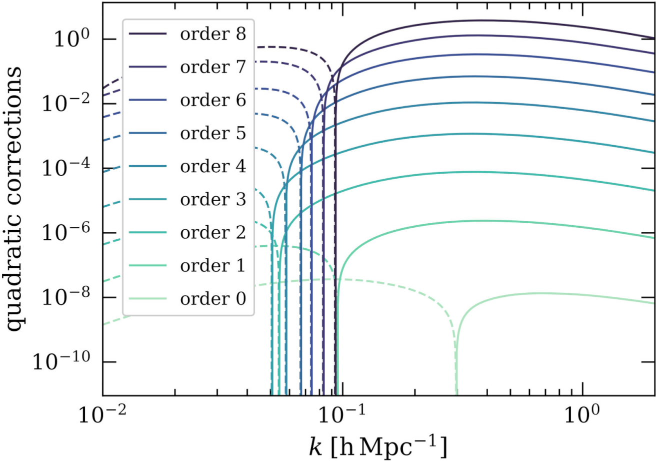
Asymptotic Behaviour



Waibel, R., MSc thesis, unpublished

free bispectrum $\sim k^{-11/2}$

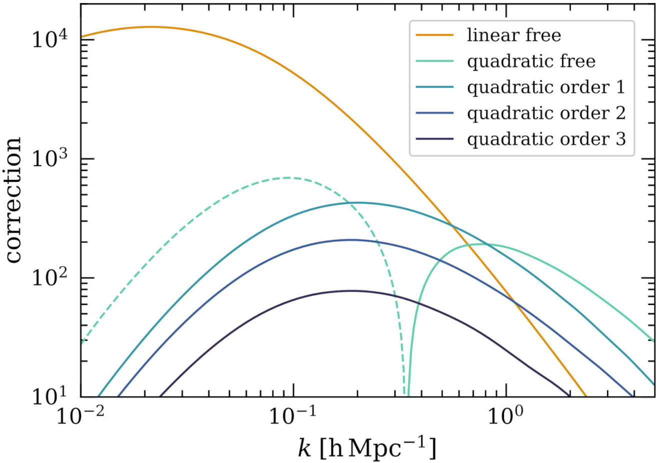
Perturbation Theory



Pixius, C., PhD thesis; Pixius, C. et al. 2022

8th-order perturbation theory with Newtonian trajectories

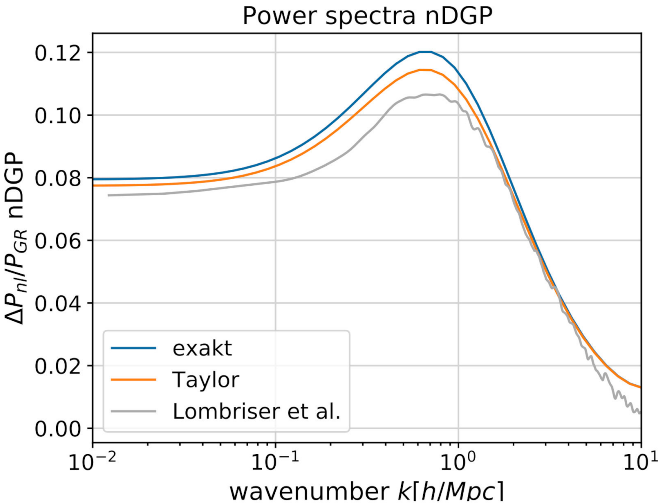
Perturbation Theory



Pixius, C., PhD thesis

3rd-order perturbation theory with Zel'dovich trajectories

Modified Gravity Theories



Heisenberg, L. & MB 2019

Oestreicher, A. et al. 2023, Reinhardt, N., MSc thesis

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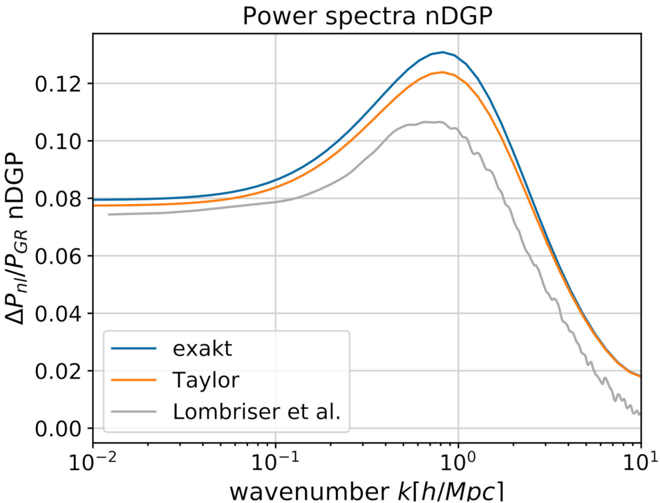

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Summary

- KFT: new statistical approach to classical, non-equilibrium systems
- avoids shell-crossing problem by construction
- mean-field approach successful in recovering non-linear power spectrum
- rigorous statements on asymptotic behaviour of cosmic structures
- perturbation theory can be driven to high orders
- no free parameters
- generalization to different cosmologies, dark-matter models, gravity theories easily possible