What a flavour (and CP) symmetry can do for you

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Overview

Introduction

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  Nevertheless, several phenomena are not explained within SM.

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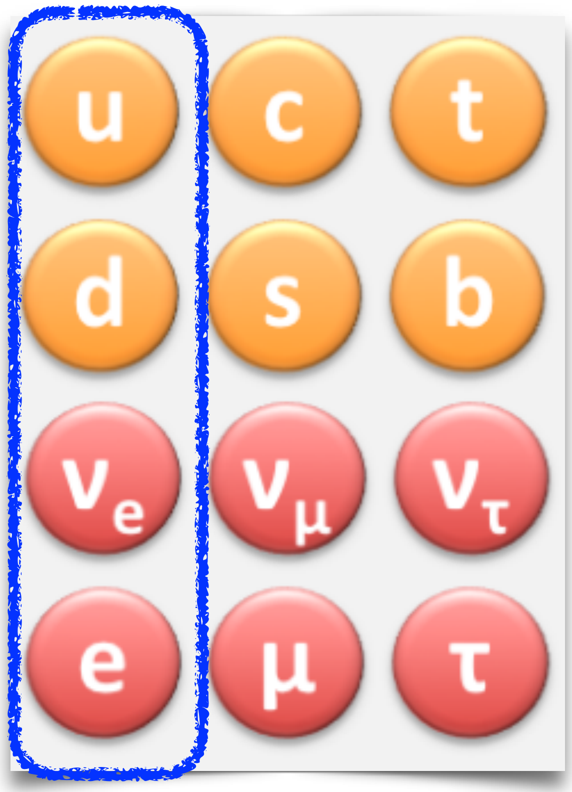
  - **Replication of fermion generations**

1. 2. 3. generation



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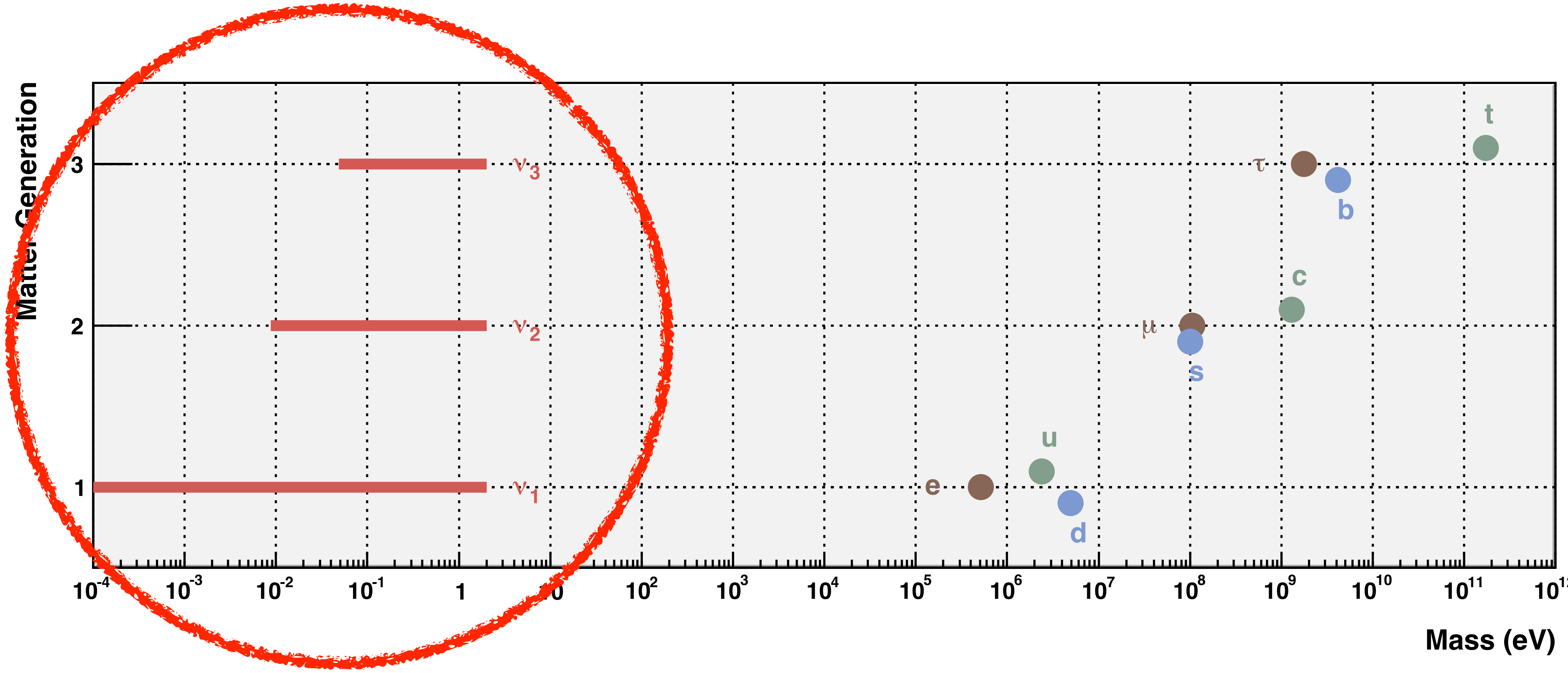
  - Replication of fermion generations
  - **Fermion masses**



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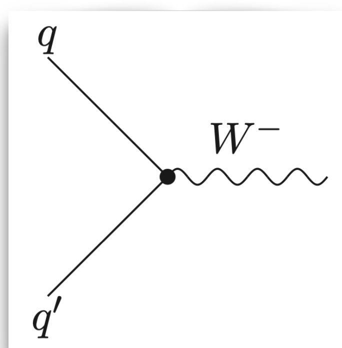
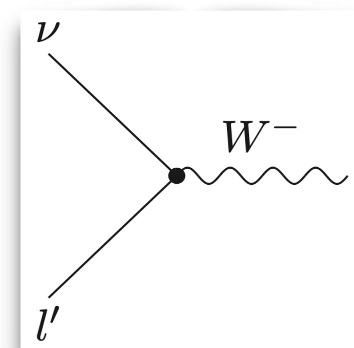
  - Replication of fermion generations
  - Fermion masses
  - **Quark and lepton mixing**



$$\begin{pmatrix} 0.97 & 0.22 & 3.7 \cdot 10^{-3} \\ 0.22 & 0.97 & 0.042 \\ 9.0 \cdot 10^{-3} & 0.041 & 0.999 \end{pmatrix}$$

PDG ('20) Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix



$$\begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.29 & 0.59 & 0.75 \\ 0.49 & 0.59 & 0.64 \end{pmatrix}$$

NuFIT 5.1 ('21)

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix

Introduction

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  Nevertheless, several phenomena are not explained within SM.
  - Replication of fermion generations
  - Fermion masses
  - Quark and lepton mixing
  - **Baryon asymmetry of the Universe** (BAU)

$$Y_B = \frac{n_B - n_{\overline{B}}}{s}\Big|_0 = 8.75 \times 10^{-11}$$

Planck ('18)

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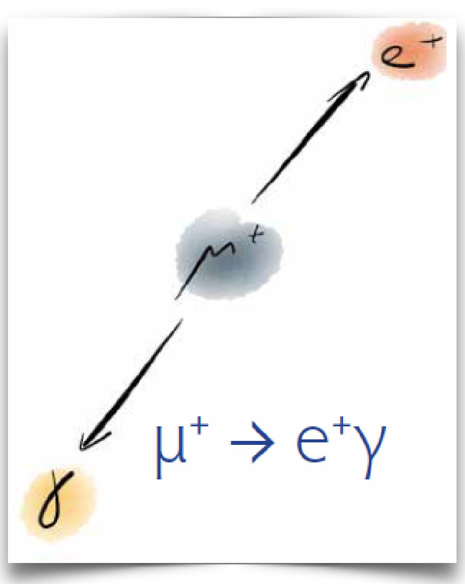
  - **Processes forbidden/highly suppressed in SM can be in reach**



Current experimental limit

$$\mathrm{BR}(\mu \to e\gamma) < 4.2 \cdot 10^{-13}$$

MEG at PSI ('16)

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  - **Flavour and CP violation needs to be kept under control**

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  - Processes forbidden/highly suppressed in SM can be in reach
  - Flavour and CP violation needs to be kept under control
  - **Possible correlations among different signals**

Flavour and CP symmetries

- Let us be inspired by the success of gauge symmetries.
- Assume a **new symmetry, acting on flavour space**, e.g.

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \to \begin{pmatrix} q_2 \\ q_3 \\ q_1 \end{pmatrix}$$

with q_i being the ith quark generation.
This constrains the couplings in the flavour sector, i.e. the quark masses and mixing.

Properties of this new symmetry G_f ?

Flavour and CP symmetries

Properties of this new symmetry G_f ?

G_f could be …

- … abelian or non-abelian
- … continuous or discrete
- … local or global
- … spontaneously broken or explicitly
- … broken arbitrarily or to non-trivial subgroups
- … broken at low or high energies

Its maximal possible size depends on the chosen gauge group.

Flavour and CP symmetries

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Flavour and CP symmetries

Properties of this new symmetry G_f?

G_f could be …

- … abelian or **non-abelian** (**three generations**)
- … continuous or **discrete** (**preferred directions**)
- … local or global
- … spontaneously broken or explicitly
- … broken arbitrarily or to **non-trivial subgroups** (**predictive**)
- … broken at low or high energies

Its maximal possible size depends on the chosen gauge group.

Flavour and CP symmetries

There are many options …

- Dihedral symmetries D_n as well as D'_n
- Symmetric and alternating groups, S_n and A_n
- Discrete subgroups of modular group
- Groups $\Sigma(n\,\varphi)$
- Adding CP symmetries
- Series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$ — also with CP
- …

Altarelli, Antusch, Branco, Calibbi, Centelles Chulia, Chen, Chu, Dasgupta, de Medeiros Varzielas, Ding, Everett, Feruglio, Gavela, Gehrlein, Girardi, Gonzalez Felipe, Grimus, CH, He, Hirsch, Joaquim, King, Lavoura, Luhn, Mahanthappa, Machado, Medina, Melis, Meloni, Merlo, Meroni, Mohapatra, Neder, Nilles, Nishi, Pas, Pascoli, Petcov, Rodejohann, Schumacher, Serodio, Shimizu, Smirnov, Spinrath, Srivastava, Stuart, Tanimoto, Titov, Valle, Vicente, Vien, Vives, Xu, Yamamoto, Ziegler, …

Reviews

Ishimori et al. ('10), King/Luhn ('13), Feruglio/Romanino ('19); Grimus/Ludl ('11)

Flavour and CP symmetries

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Example 1 & 2

Flavour and CP symmetries

There are many options …

Example 3

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- Symmetric and alternating groups, S_n and A_n
- Discrete subgroups of modular group
- Groups $\Sigma(n\,\varphi)$
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- Series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$ — also with CP
- …

Flavour and CP symmetries

Series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$

- Have 3-dim irrep(s)
- Can also offer 1-dim irreps and 2-dim irreps
- Are subgroups of SU(3)

$\Delta(3\,n^2)$

Luhn/Nasri/Ramond ('07)

$$a^3 = e \ , \quad c^n = e \ , \quad d^n = e \ ,$$
$$c\,d = d\,c \ , \quad a\,c\,a^{-1} = c^{-1}d^{-1} \ , \quad a\,d\,a^{-1} = c$$

$$g = a^\alpha c^\gamma d^\delta \quad \text{with} \quad \alpha = 0,1,2 \ , \quad 0 \le \gamma, \delta \le n-1$$

A well-known member is the permutation group A_4

Flavour and CP symmetries

Series of groups $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$

- Have 3-dim irrep(s)
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$\Delta(6\,n^2)$ Add to relations of $\Delta(3\,n^2)$ Escobar / Luhn ('08)

$$b^2 = e \;\;,\;\; (a\,b)^2 = e \;\;,\;\; b\,c\,b^{-1} = d^{-1} \;\;,\;\; b\,d\,b^{-1} = c^{-1}$$

$$g = a^\alpha b^\beta c^\gamma d^\delta \quad \text{with} \quad \alpha = 0,1,2\;,\;\; \beta = 0,1\;,\;\; 0 \leq \gamma, \delta \leq n-1$$

A well-known member is the permutation group S_4

Flavour and CP symmetries

Add CP as further symmetry

Grimus/Rebelo ('95),

Ecker/Grimus/Neufeld (´84,'87,'88)

- Motivation:
  For more than one generation of certain particle species, define CP that also acts on generations of particles, e.g.

$$\Phi_i(x) \quad \rightarrow \quad X_{ij}\,\Phi_j^\dagger(x_P) \;\; \text{with} \;\; (x_P)_\mu = x^\mu$$

  with

$$X\,X^\dagger = X\,X^\star = 1$$

- CP is involution and corresponds to automorphism of flavour symmetry

Feruglio/CH/Ziegler ('12)
Holthausen/Lindner/Schmidt ('12), Chen et al. ('14)

Flavour and CP symmetries

Breaking of symmetries

Feruglio/CH/Ziegler ('12)

Idea: Keep some residual symmetry among charged leptons and neutrinos, G_e and G_ν , with $G_e \neq G_\nu$

Mismatch of symmetries corresponds to lepton mixing

G_f and CP

↙ ↘

charged leptons	neutrinos
G_e	$G_\nu = Z_2 \times$ CP
U_e	$U_\nu = \Omega_\nu R(\theta) K_\nu$

↘ ↙

$$U_{PMNS} = U_e^\dagger \Omega_\nu R(\theta) K_\nu$$

Flavour and CP symmetries

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$\swarrow \qquad \searrow$

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Minimal choice Z_3

3 different masses

Choice Z_2 and CP

Majorana masses

Free parameter

Size of masses not explained

CH / Meroni / Molinaro ('14)

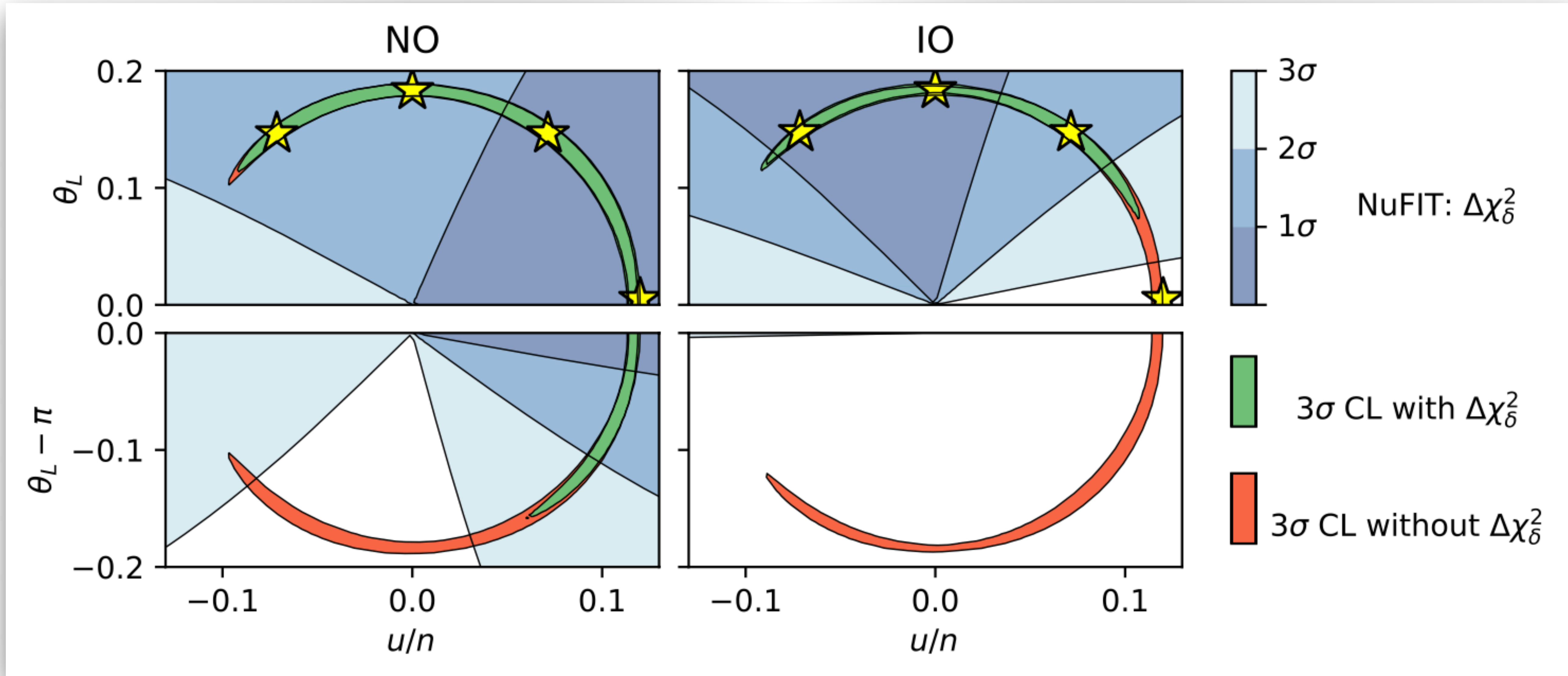
Result: four different types of mixing patterns with different properties

Case 1) **Case 2)** **Case 3 a)** **Case 3 b.1)**

Flavour and CP symmetries

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 2)



$u = 2\,s - t$ fixed by CP symmetry

$v = 3\,t$ relevant mainly for Majorana phase α

Flavour and CP symmetries

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 2)

$n = 14$

u	$u=-1$	$u=0$	$u=+1$
θ_L	0.146 (0.148)	0.184	0.146 (0.148)
$\sin^2\theta_{12}$	0.341	0.341	0.341
$\sin^2\theta_{13}$	0.0222 (0.0224)	0.0222 (0.0224)	0.0222 (0.0224)
$\sin^2\theta_{23}$	0.437	0.5	0.563
$\Delta\chi^2$	9.25 (11.2)	10.8 (12.5)	8.27 (8.62)

$\sin\delta = -1$ for $u = 0$

$\sin\delta \approx -0.811\,(-0.813)$ for $u = \pm 1$

several choices for v admitted

Example 1: Inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

- Consider a scenario of **(3,3) ISS**,
  i.e. 3 generations of LH doublets,
  3 generations of N_i and S_j, all of them gauge singlets

$$-(y_D)_{\alpha i}\,\overline{L^c_\alpha}\,H\,N^c_i - (M_{NS})_{ij}\,\overline{N}_i\,S_j - \frac{1}{2}\,(\mu_S)_{kl}\,\overline{S^c_k}\,S_l + \text{h.c.}$$

Mass matrix of neutral states

$$\mathcal{M}_{\text{Maj}} = \begin{pmatrix} \mathbb{0} & m_D & \mathbb{0} \\ m_D^T & \mathbb{0} & M_{NS} \\ \mathbb{0} & M_{NS}^T & \mu_S \end{pmatrix} \quad \text{with} \;\; m_D = y_D\,\frac{v}{\sqrt{2}}$$

- Light neutrino masses

$|\mu_S| \ll |m_D| \ll |M_{NS}|$:

$$m_\nu = m_D\left(M_{NS}^{-1}\right)^T \mu_S\, M_{NS}^{-1}\, m_D^T$$

Mohapatra / Valle ('86),
Mohapatra (´86),
Bernabeu et al. (´87),
Gonzalez-Garcia / Valle (´89)

Example 1: Inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

- We take

$$\alpha_R \sim 1$$

[detail: use additional Z_3 to distinguish e, μ, τ]

$$L_\alpha \sim 3 \ , N_i \sim 3 \ , S_j \sim 3$$

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irreducible, faithful, complex

Reason:
Fully explore the predictive power of flavour and CP symmetry

CH/Meroni/Molinaro ('14)

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Charged lepton mass matrix

residual symmetry G_e

$$\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

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Reason:
Get m_D and M_{NS} invariant, encode flavour and CP symmetry breaking in μ_S

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Neutrino mass matrix

residual symmetry G_ν

$$\mathcal{M}_{\rm Maj} = \begin{pmatrix} \mathbb{0} & m_D & \mathbb{0} \\ m_D^T & \mathbb{0} & M_{NS} \\ \mathbb{0} & M_{NS}^T & \mu_S \end{pmatrix} \text{ with } m_D = y_D \frac{v}{\sqrt{2}}$$

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Neutrino mass matrix residual symmetry G_ν

$$-(y_D)_{\alpha i}\, \overline{L}^c_\alpha\, H\, N^c_i - (M_{NS})_{ij}\, \overline{N}_i\, S_j - \frac{1}{2}\, (\mu_S)_{kl}\, \overline{S}^c_k\, S_l + \text{h.c.}$$

No symmetry breaking

$$m_D = y_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{v}{\sqrt{2}} \quad \text{with} \;\; y_0 > 0$$

$$M_{NS} = M_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{with} \;\; M_0 > 0$$

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Neutrino mass matrix

residual symmetry G_ν

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Symmetry breaking

$$U_S^T\, \mu_S\, U_S = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}$$

$$U_S = \Omega(\mathbf{3})\, R_{fh}(\theta_S)$$

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Light neutrino mass matrix

$$m_\nu = \frac{y_0^2 \, v^2}{2 \, M_0^2} \, \mu_S = \frac{y_0^2 \, v^2}{2 \, M_0^2} \, U_S^\star \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix} U_S^\dagger$$

Neutrino masses

$$m_i = \frac{y_0^2 \, v^2}{2 \, M_0^2} \, \mu_i \quad \text{for} \quad i = 1, 2, 3$$

Lepton mixing

$$\widetilde{U}_{\mathrm{PMNS}} = \Omega(\mathbf{3}) \, R_{fh}(\theta_S)$$

at leading order

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Heavy states

$$M_{h,i} = M_0 - \frac{\mu_i}{2} \quad \text{and} \quad M_{h,i+3} = M_0 + \frac{\mu_i}{2} \quad \text{with} \quad i = 1, 2, 3 \, .$$

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Back to light neutrinos

… go beyond leading order Hettmansperger/Lindner/Rodejohann ('11)

- potentially new contributions to m_ν
- effects of non-unitarity

$$\widetilde{U}_{\rm PMNS} = \left(\mathbb{1} - \eta\right) U_0$$

$$\eta = \frac{y_0^2\, v^2}{4\, M_0^2}\, \mathbb{1} \equiv \eta_0\, \mathbb{1}$$

flavour-diagonal and flavour-universal

Example 1: Inverse seesaw mechanism

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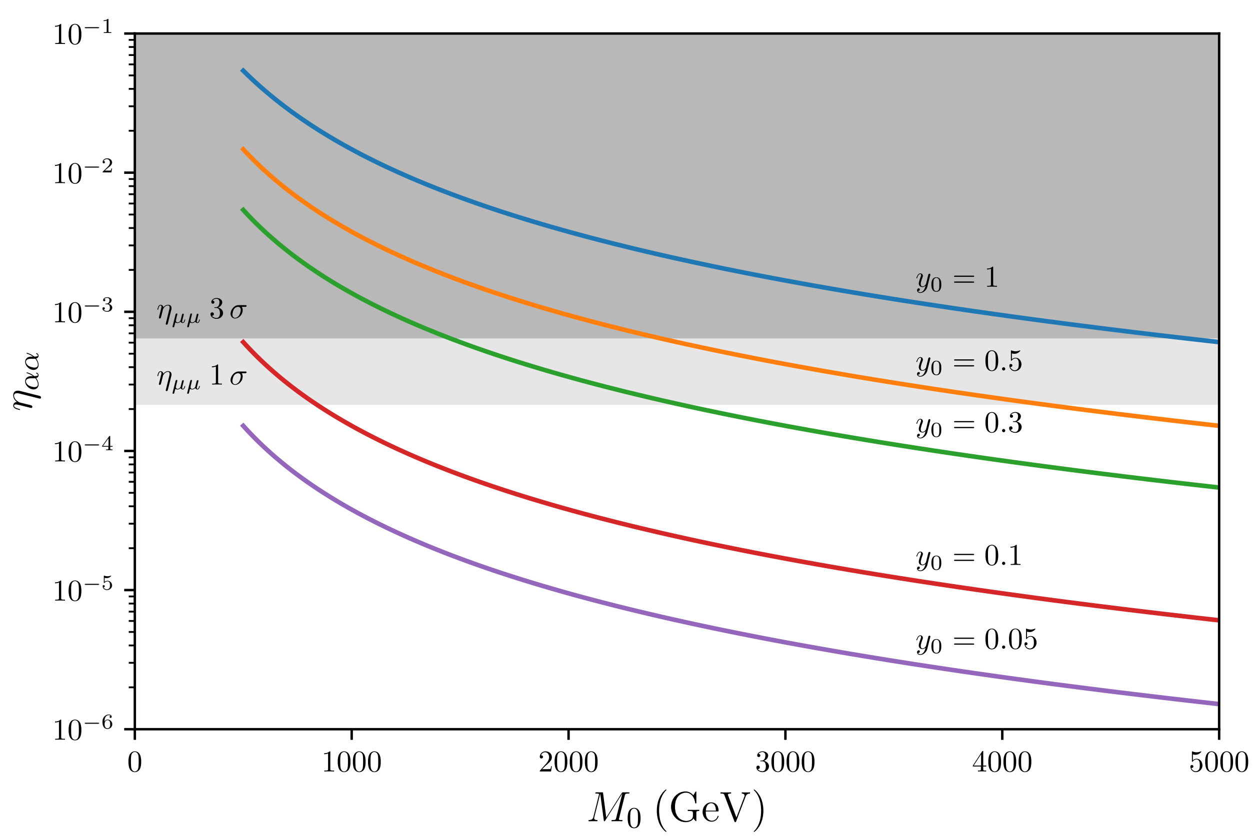
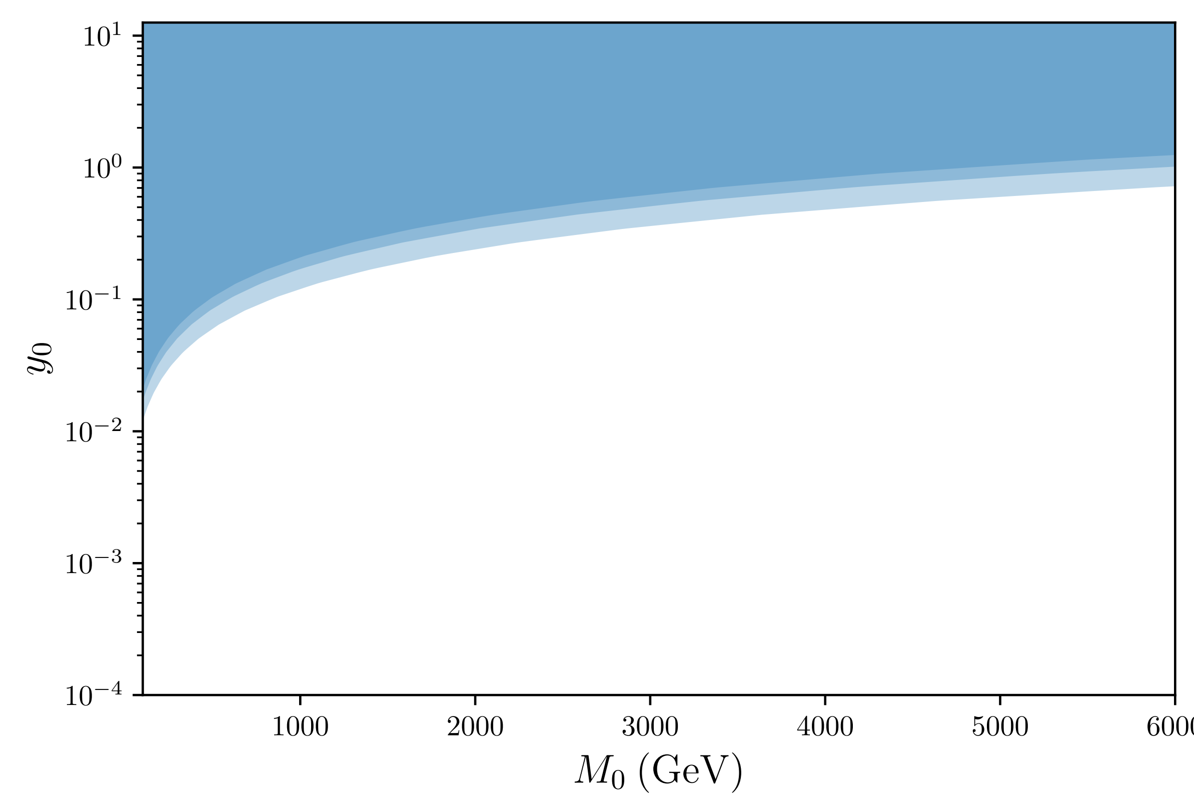
Constraints from non-unitarity



Strongest bound comes from $\eta_{\mu\mu}$

$$|\eta_{\alpha\beta}| \leq \begin{pmatrix} 1.3 \cdot 10^{-3} & 1.2 \cdot 10^{-5} & 1.4 \cdot 10^{-3} \\ 1.2 \cdot 10^{-5} & 2.2 \cdot 10^{-4} & 6.0 \cdot 10^{-4} \\ 1.4 \cdot 10^{-3} & 6.0 \cdot 10^{-4} & 2.8 \cdot 10^{-3} \end{pmatrix}$$

Fernandez-Martinez et al. ('16)



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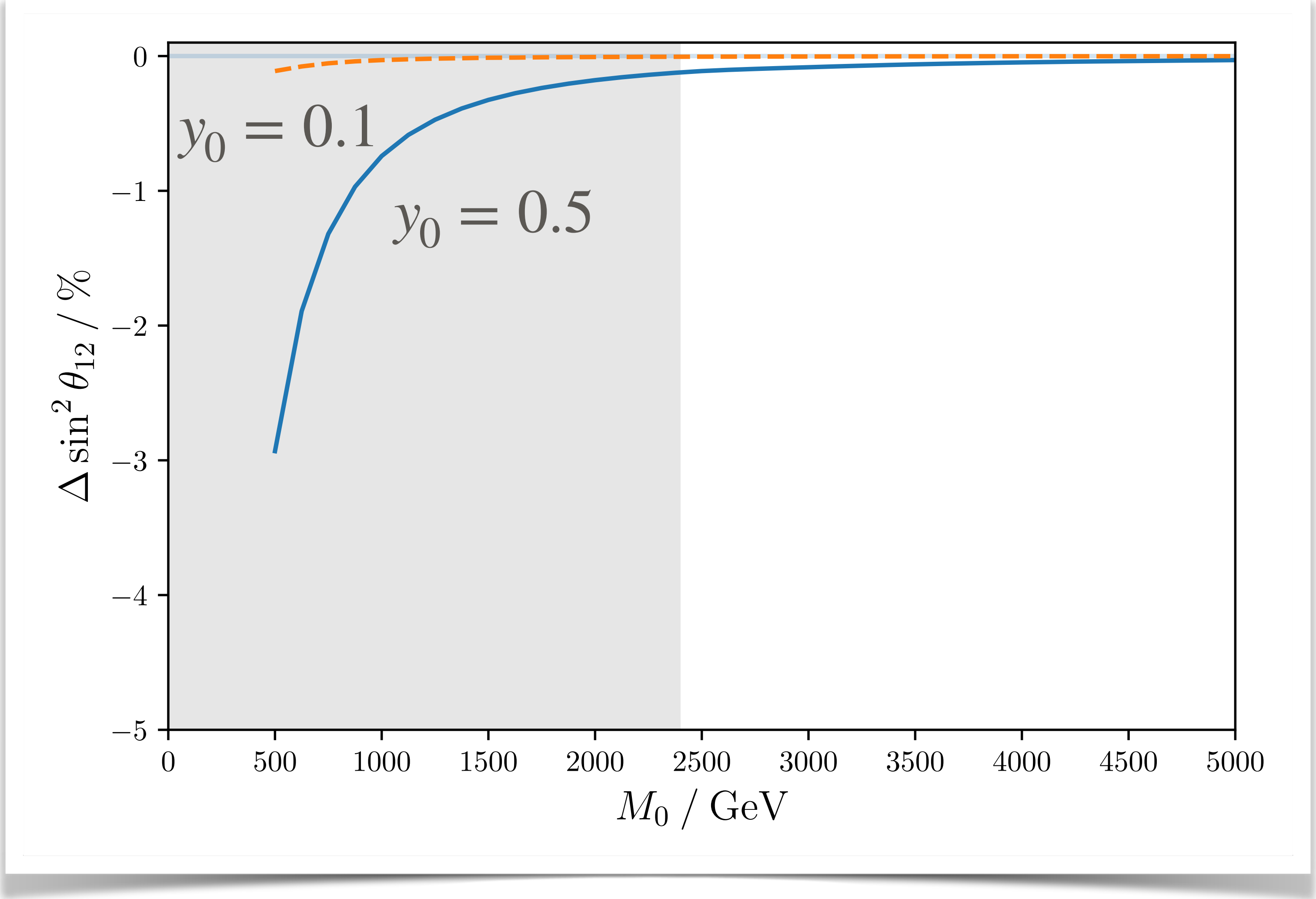
Effect on lepton mixing

Case 1)

$$\sin^2\theta_{12} = \frac{1}{2+\cos 2\theta}$$

$$\theta \approx 0.18$$



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Charged lepton flavour violation

Relevant points

- Lepton number and flavour breaking are **both** encoded in the matrix

$$U_S^T\,\mu_S\,U_S = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}$$

$$U_S = \Omega(\mathbf{3})\,R_{fh}(\theta_S)$$

- Non-unitarity effects are **flavour-diagonal and flavour-universal**

$$\eta = \frac{y_0^2\,v^2}{4\,M_0^2}\,\mathbb{1} \equiv \eta_0\,\mathbb{1}$$

- Mass spectrum of heavy states is peculiar:
  they form **pseudo-Dirac pairs** with very small mass splitting
  and all three such pairs have a **common mass scale**

$$M_{h,i} = M_0 - \frac{\mu_i}{2} \quad \text{and} \quad M_{h,i+3} = M_0 + \frac{\mu_i}{2} \quad \text{with} \quad i = 1,2,3\,.$$

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$$U_S^T \, \mu_S \, U_S = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \cdots \end{pmatrix}$$

Conclusion

Rates of charged lepton flavour violating processes

$\ell_\beta \to \ell_\alpha \gamma \qquad \ell_\beta \to 3\,\ell_\alpha \qquad \mu - e$ conversion

are very suppressed!

for general formulae see Alonso et al. ('12), Ilakovac/Pilaftsis ('95)

...ular:

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Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

- Consider a scenario of **type I seesaw with 3 RH neutrinos**, i.e. 3 generations of LH doublets and 3 generations of gauge singlets $\nu_{R\,i}$

$$\mathcal{L} \supset \mathrm{i}\,\overline{\nu_R}\,\partial\!\!\!/\,\nu_R - \frac{1}{2}\overline{\nu_R^c}\,M_R\,\nu_R - \overline{l_L}\,Y_D\,\varepsilon H^*\,\nu_R + \mathrm{h.c.}$$

- Light neutrino masses

$$m_\nu = -\,m_D\,M_R^{-1}\,m_D^T$$ with $$m_D = Y_D\,\langle H\rangle$$

Minkowski ('77), Glashow ('80), Gell-Mann/Ramond/Slansky ('79), Mohapatra/Senjanovic ('80), Yanagida ('80), Schechter/Valle ('80)

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- We take

$$\alpha_R \sim 1$$

[detail: use additional Z_3 to distinguish e, μ, τ]

$$l_{L\alpha} \sim 3 \, , \nu_{R\,i} \sim 3'$$

see also Dev/CH/Molinaro ('18); Chauhan/Dev ('22)

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Charged lepton mass matrix

residual symmetry G_e

$$\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

- We take

$$\alpha_R \sim 1$$

[detail: use additional Z_3 to distinguish e, μ, τ]

$$l_{L\alpha} \sim 3 \, , \nu_{R\,i} \sim 3'$$

irreducible, in general unfaithful, real

Reason:
(flavour-universal) mass term for $\nu_{R\,i}$ w/o breaking flavour and CP symmetry, breaking encoded in Y_D

Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

- We take

$\alpha_R \sim 1$ $\qquad$ $l_{L\alpha} \sim 3 \,, \nu_{R\,i} \sim 3'$

[detail: use additional Z_3 to distinguish e, μ, τ]

Neutral lepton sector $\qquad$ residual symmetry G_ν

$$\mathcal{L} \supset \mathrm{i}\,\overline{\nu_R}\,\partial\!\!\!/\,\nu_R - \frac{1}{2}\overline{\nu_R^c}\,M_R\,\nu_R - \overline{l_L}\,Y_D\,\varepsilon H^*\,\nu_R + \text{h.c.}$$

No symmetry breaking

$$M_R = M_R^0 = M \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

RH neutrino masses are degenerate

Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

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Symmetry breaking

$$Y_D = \Omega(\mathbf{3})\,R_{ij}(\theta_L)\,\mathrm{diag}(y_1, y_2, y_3)\,P^{ij}_{kl}\,R_{kl}(-\theta_R)\,\Omega(\mathbf{3}')^\dagger$$

CH/Molinaro ('16)

Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Neutral lepton sector

$$Y_D = \Omega(\mathbf{3}) R_{ij}(\theta_L) \, \mathrm{diag}(y_1, y_2, y_3) P_{kl}^{ij} R_{kl}(-\theta_R) \Omega(\mathbf{3}')^\dagger$$

In total five free real parameters corresponding to three light neutrino masses, one free parameter for lepton mixing and one free parameter related to RH neutrinos

Possible small symmetry breaking for RH neutrino masses

$$\delta M_R = \kappa \, M \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

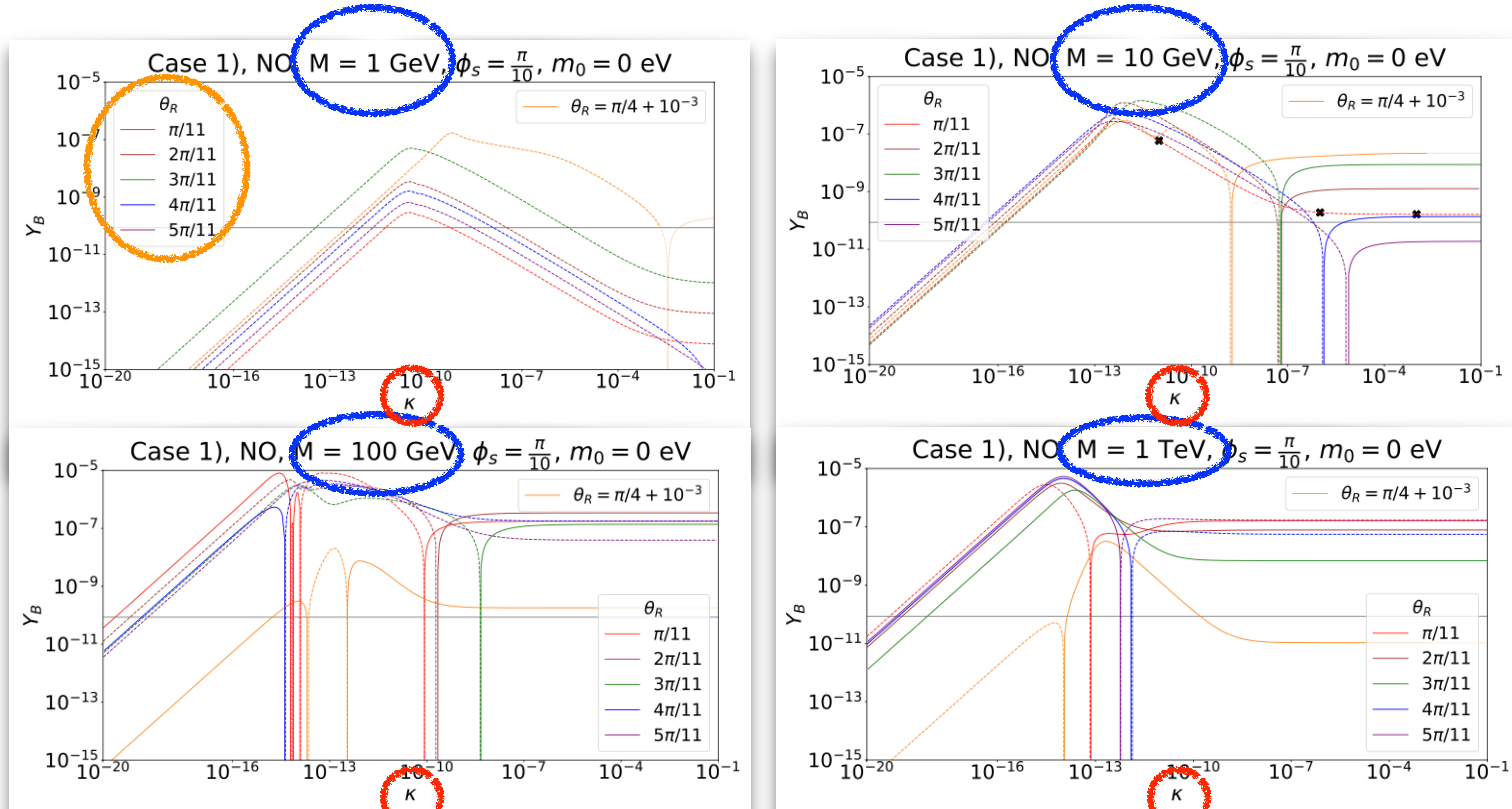
$$M_1 = M\,(1 + 2\,\kappa) \;\; \text{and} \;\; M_2 = M_3 = M\,(1-\kappa)$$

Often needed for generating correct amount of BAU.

Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)



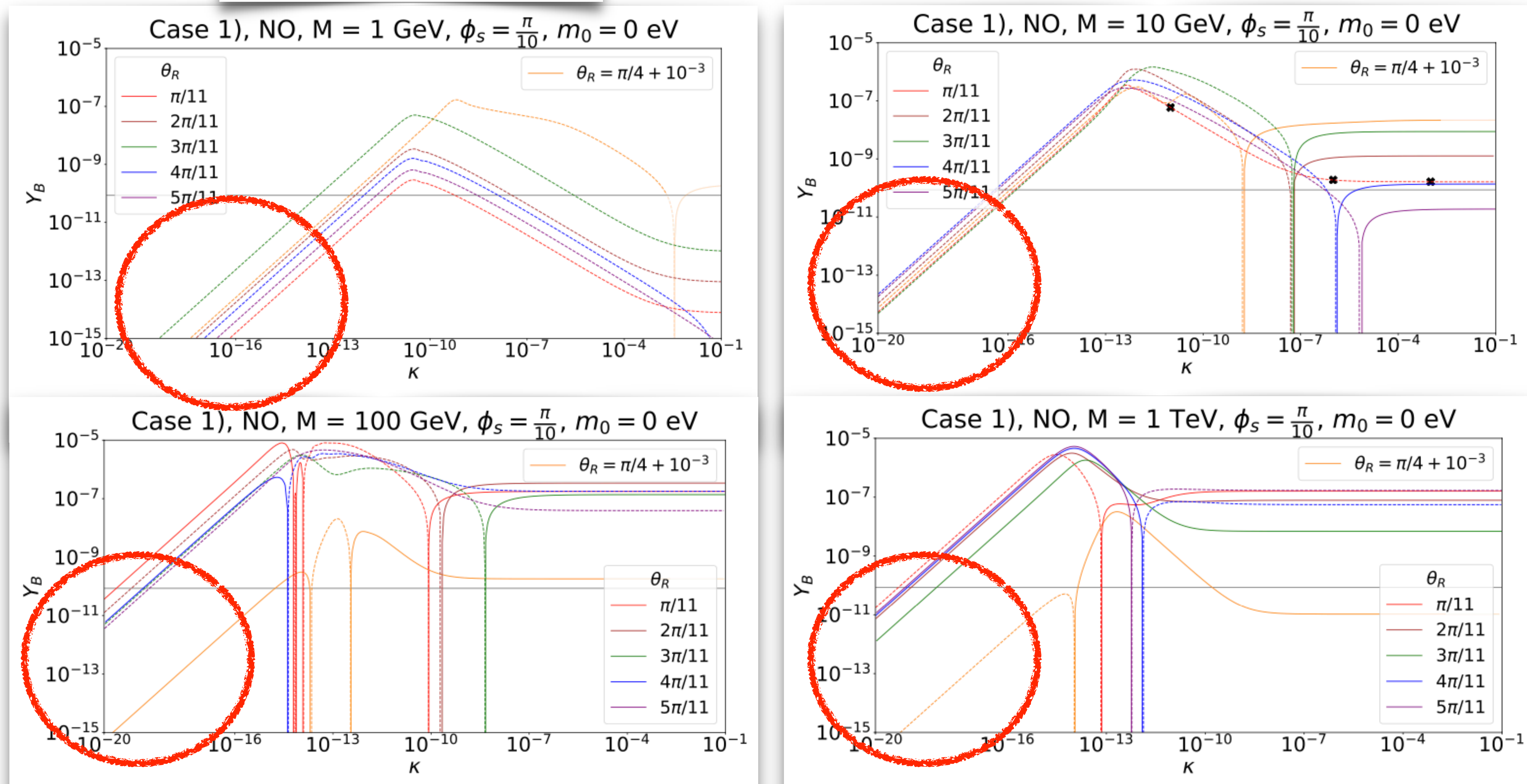
(e) Vanishing initial conditions.

(g) Vanishing initial conditions.

Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1) $C_{\mathrm{DEG},\alpha}$ is zero.



(e) Vanishing initial conditions.

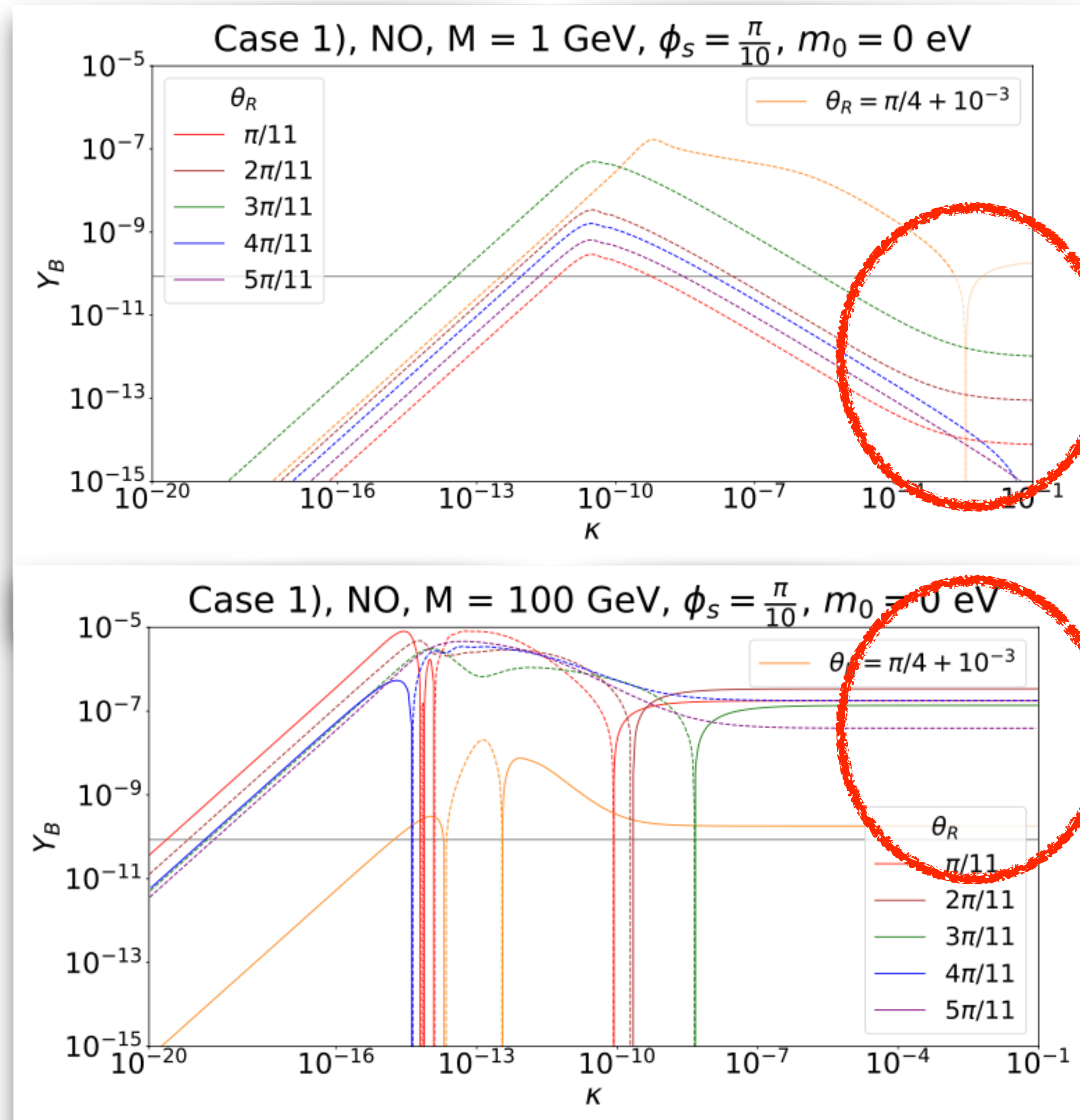
(g) Vanishing initial conditions.

Example 2: Low-scale seesaw mechanism

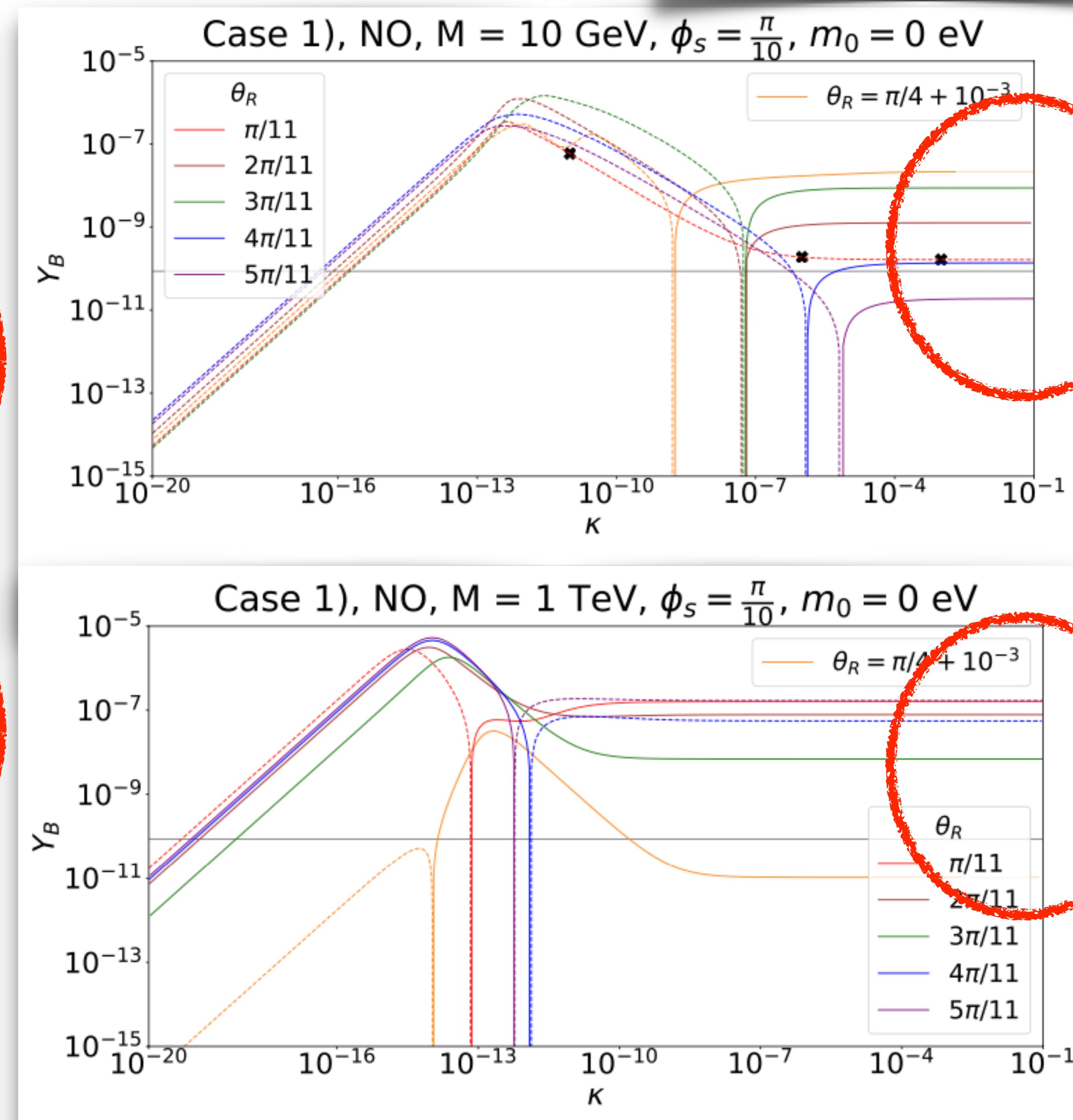
[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)

$C^{(23)}_{\mathrm{DEG},\alpha}$ is non-zero



(e) Vanishing initial conditions.

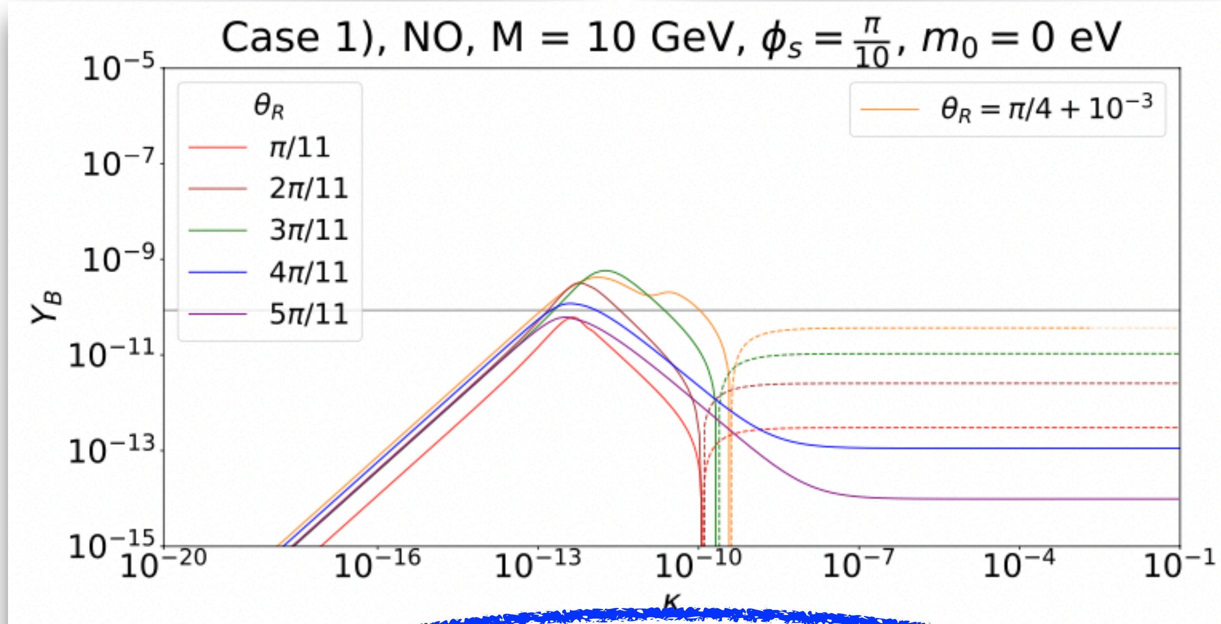


(g) Vanishing initial conditions.

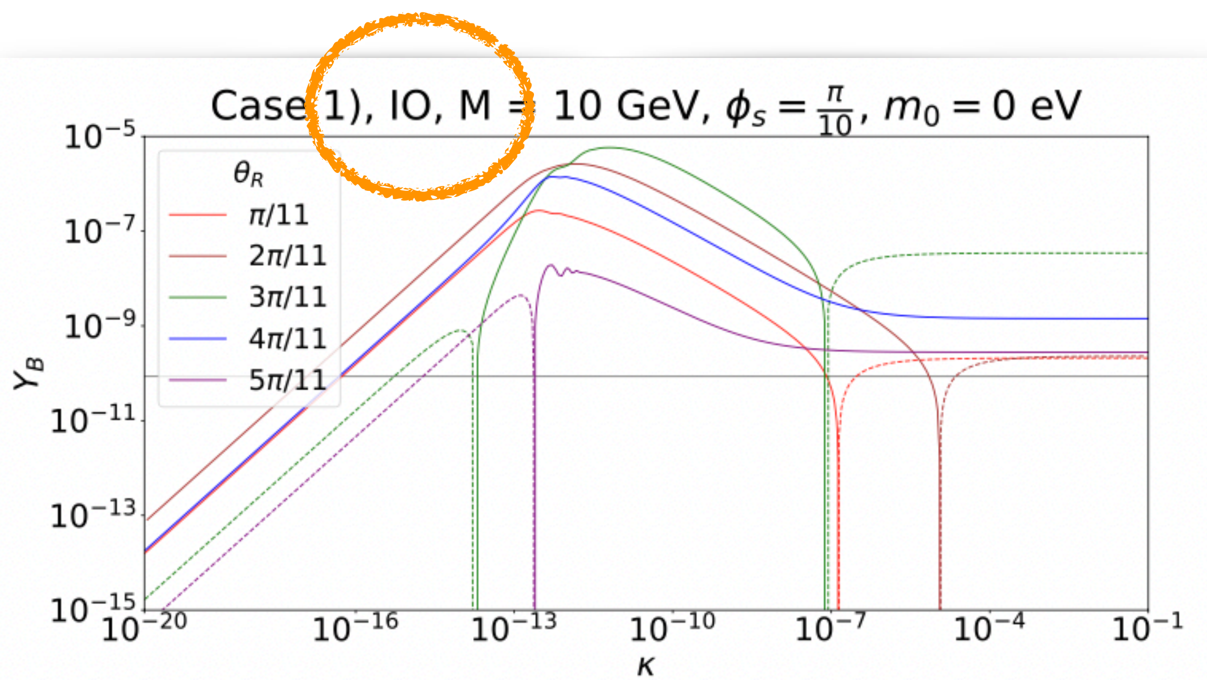
Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

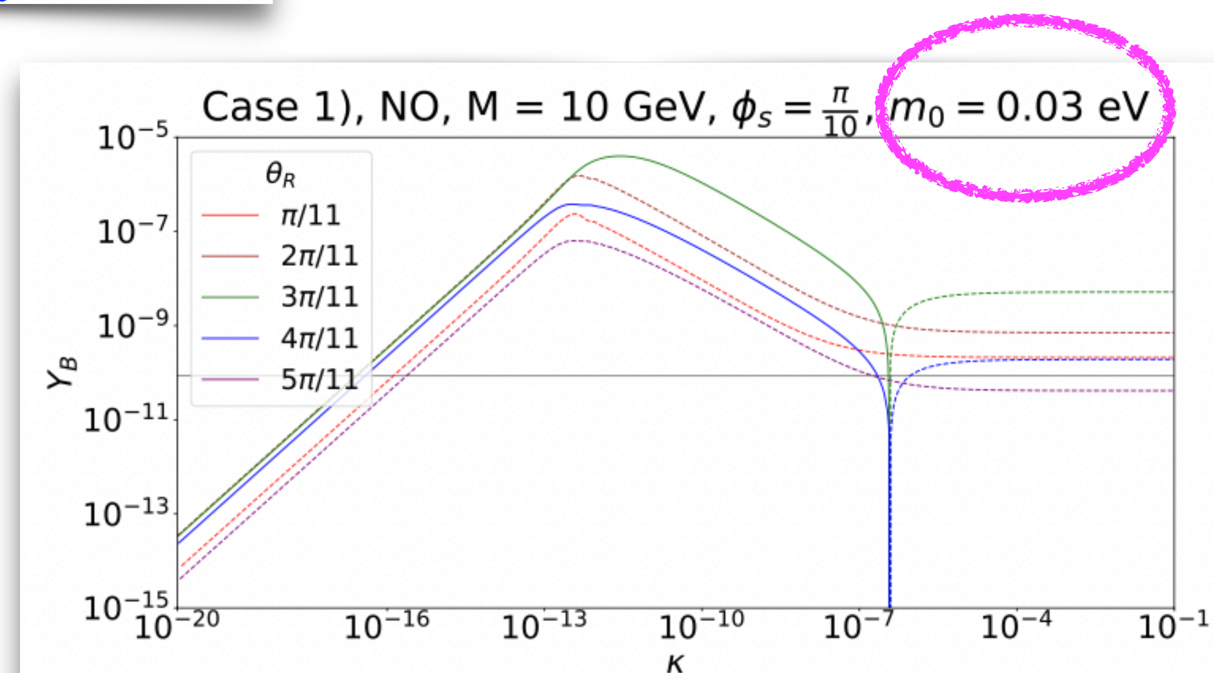
Case 1)



(d) Thermal initial conditions.



(a) Vanishing initial conditions.

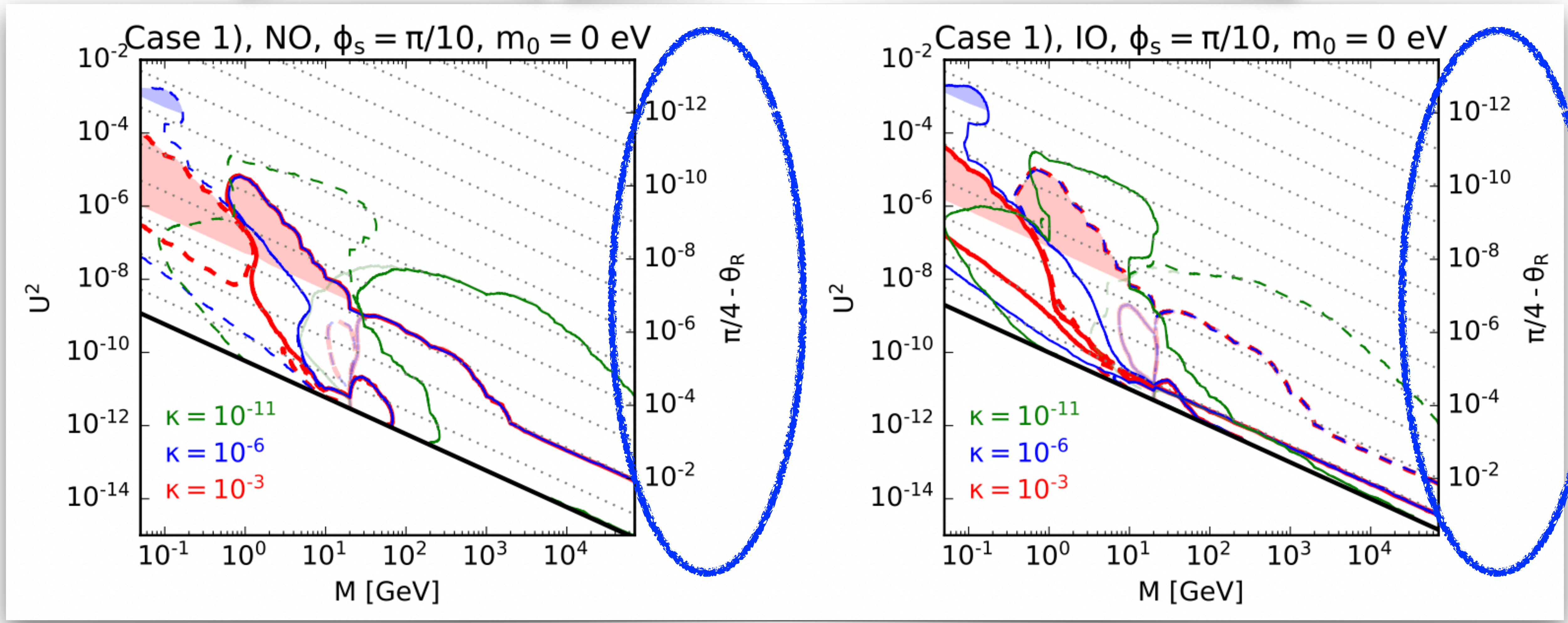


(b) Vanishing initial conditions.

Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1) $m_2 = \frac{y_2^2 \langle H \rangle^2}{M}$ $m_1 = 0$ and $m_3 = \frac{y_3^2 \langle H \rangle^2}{M} |\cos 2\theta_R|$ (strong NO)

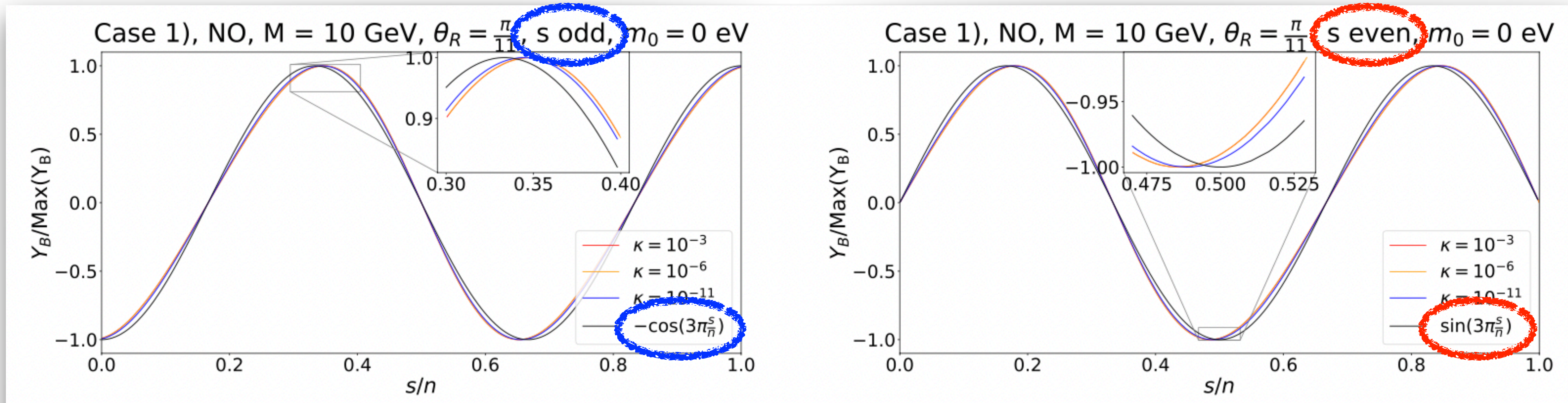


Values of θ_R so close to $\frac{\pi}{4}$ are not (always) a tuning, but related to enhanced residual symmetry, i.e. check $Y_D^\dagger Y_D$

Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)



(a) Vanishing initial conditions.　(b) Vanishing initial conditions.

Majorana phase α fulfils $\quad |\sin\alpha| = |\sin\left(\frac{6\pi s}{n}\right)|$

[Remember $\quad \sin\left(\frac{6\pi s}{n}\right) = 2\cos\left(\frac{3\pi s}{n}\right)\sin\left(\frac{3\pi s}{n}\right)$]

Majorana phase β and CP phase δ are both trivial, $\sin\beta = 0$ and $\sin\delta = 0$.

Flavour and CP symmetries

Dihedral symmetries D_n

- Have 1-dim and 2-dim irrep(s)
- Are subgroups of SO(3)
- Generators and relations

$$a^n = e \,, \quad b^2 = e \,, \quad a\, b\, a = b$$

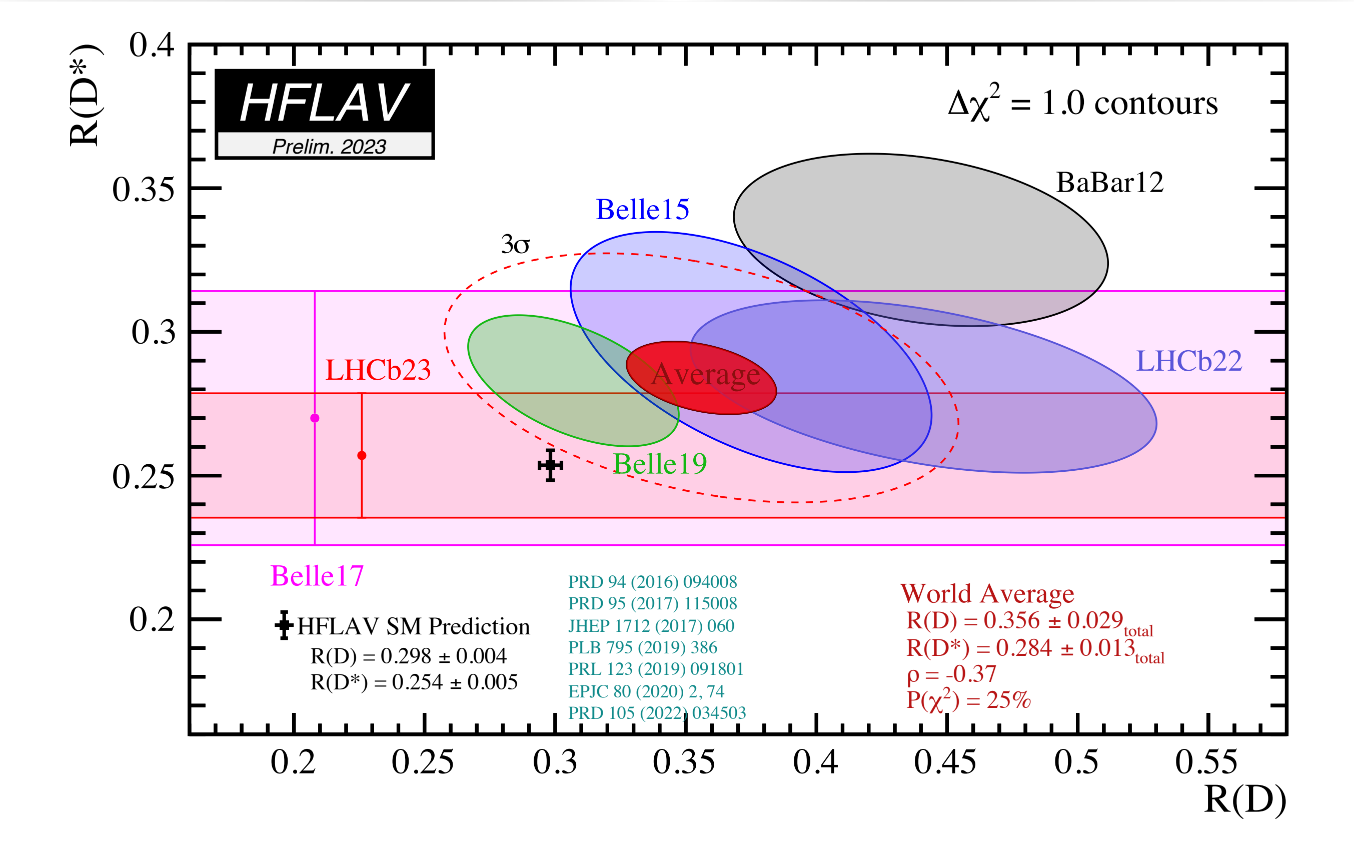
Well-known members are the dihedral group D_4 and the permutation group $S_3 \simeq D_3$

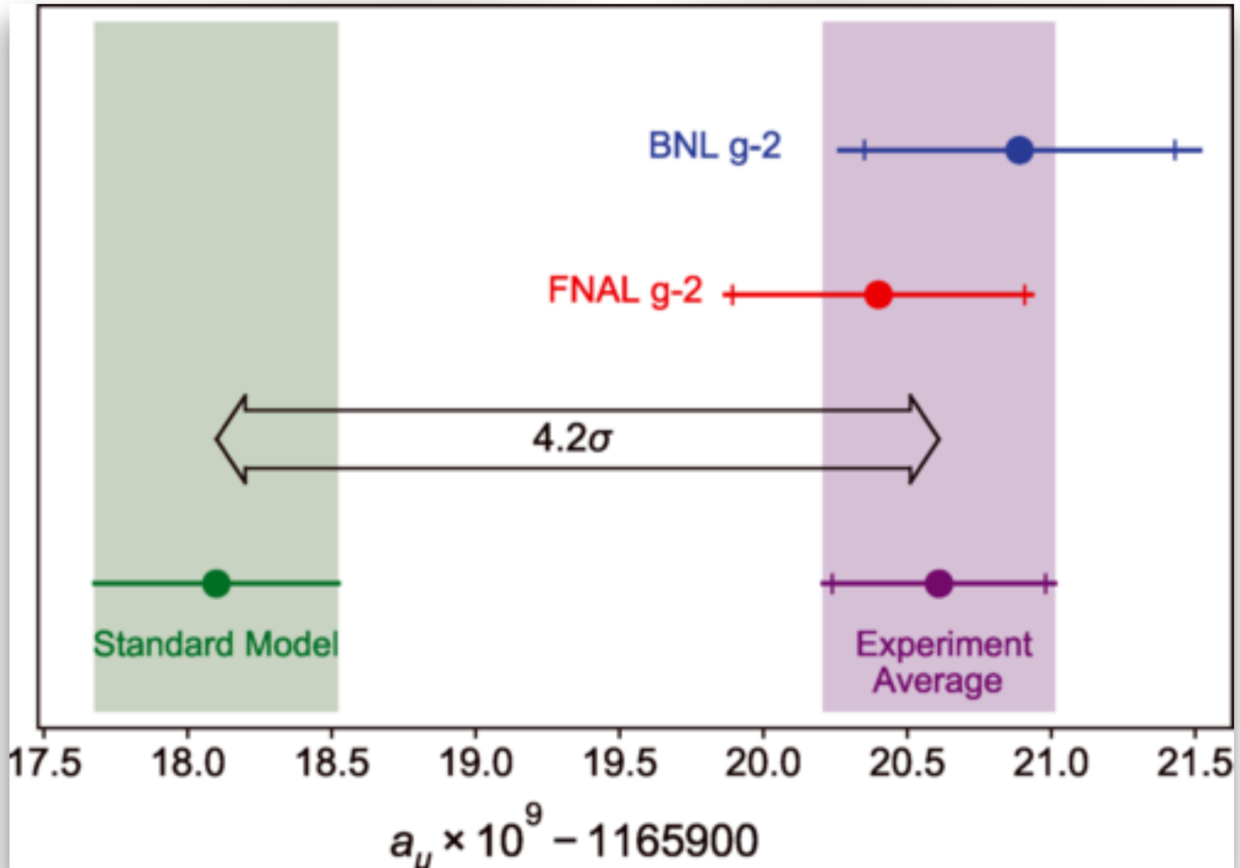
Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

- There are some experimental anomalies in certain flavour observables —- here looked at $R(D), R(D^\star)$ and anomalous magnetic moment of muon

$$R(D^{(\star)}) = \frac{\Gamma(B \to D^{(\star)}\tau\nu)}{\Gamma(B \to D^{(\star)}\ell\nu)}$$





- Consider an **extension of the SM with a leptoquark (LQ)**

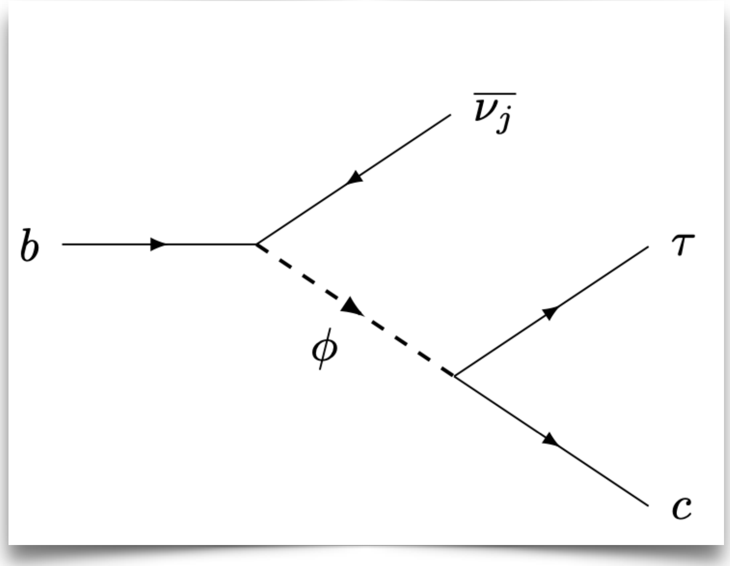
Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

- Consider an **extension of the SM with a leptoquark (LQ)**

$$\phi \sim (3, 1, -\tfrac{1}{3})$$

- It couples simultaneously to leptons and quarks, e.g.



meaning in the Lagrangian we have couplings

$$\mathscr{L}_{\rm LQ}^{\rm int} = \hat{x}_{ij}\, \overline{L_i^c}\, \phi^\dagger\, Q_j + \hat{y}_{ij}\, \overline{e^c_{Ri}}\, \phi^\dagger\, u_{Rj} + \text{h.c.}$$

For review on LQs see Dorsner et al. ('16)

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

- Take as ``aim" for textures of LQ couplings the following

$$\mathbf{x} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & x_{22} & x_{23} \\ 0 & x_{32} & x_{33} \end{pmatrix} \text{ and } \mathbf{y} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_{23} \\ 0 & y_{32} & 0 \end{pmatrix}$$

in particular use

$$\mathbf{x} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda^3 & \lambda \\ 0 & \lambda^2 & 1 \end{pmatrix} \text{ and } \mathbf{y} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \lambda^3 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda \approx 0.2$$

following the analysis in Cai et al. ('17)

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

- Note the assumed form of the charged fermion mass matrices is

$$M_e \sim \begin{pmatrix} \lambda^4 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \langle H_d^0 \rangle \ , \quad M_d \sim \begin{pmatrix} \lambda^4 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \langle H_d^0 \rangle$$

$$M_u \sim \begin{pmatrix} \lambda^8 & \lambda^5 & \lesssim \lambda^3 \\ 0 & \lambda^4 & \lambda^2 \\ 0 & 0 & 1 \end{pmatrix} \langle H_u^0 \rangle \, .$$

$$\lambda \approx 0.2$$

- No discussion of neutrino masses

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

Field	SU(3)	SU(2)	U(1)	D_{17}	Z_{17}
$Q = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$	3	2	$\frac{1}{6}$	$\mathbf{2_2}$	1
Q_3	3	2	$\frac{1}{6}$	$\mathbf{1_1}$	16
u_{R1}	3	1	$\frac{2}{3}$	$\mathbf{1_2}$	13
u_{R2}	3	1	$\frac{2}{3}$	$\mathbf{1_1}$	8
u_{R3}	3	1	$\frac{2}{3}$	$\mathbf{1_1}$	1
$d_R = \begin{pmatrix} d_{R1} \\ d_{R2} \end{pmatrix}$	3	1	$-\frac{1}{3}$	$\mathbf{2_4}$	1
d_{R3}	3	1	$-\frac{1}{3}$	$\mathbf{1_1}$	7
$L = \begin{pmatrix} L_1 \\ L_2 \end{pmatrix}$	1	2	$-\frac{1}{2}$	$\mathbf{2_1}$	2
L_3	1	2	$-\frac{1}{2}$	$\mathbf{1_1}$	1
$e_R = \begin{pmatrix} e_{R1} \\ e_{R2} \end{pmatrix}$	1	1	-1	$\mathbf{2_3}$	2
e_{R3}	1	1	-1	$\mathbf{1_1}$	9

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

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e_{R3}	1	1	-1	$\mathbf{1_1}$	9

Product of dihedral group and cyclic one

Reasons: 1-dim & 2-dim irreps; number of inequivalent 2-dim irreps; residual subgroup to protect form of LQ couplings

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

Field	SU(3)	SU(2)	U(1)	D_{17}	Z_{17}
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e_{R3}	1	1	-1	$\mathbf{1_1}$	9

2+1 structure for most charged fermions

Reasons:
3rd generation much more massive;
Cabibbo angle

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

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$Q = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$	3	2	$\frac{1}{6}$	$\mathbf{2_2}$	1
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e_{R3}	1	1	-1	$\mathbf{1_1}$	9

1+1+1 structure for RH up-type quarks

Reasons: stronger mass hierarchy among up-type quarks; achieve appropriate form of LQ coupling **y**

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

H_u	1	2	$-\frac{1}{2}$	$\mathbf{1_1}$	15
H_d	1	2	$\frac{1}{2}$	$\mathbf{1_1}$	9
ϕ	3	1	$-\frac{1}{3}$	$\mathbf{1_1}$	0

Reason: minimality

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

H_u	1	2	$-\frac{1}{2}$	$\mathbf{1_1}$	15
H_d	1	2	$\frac{1}{2}$	$\mathbf{1_1}$	9
ϕ	3	1	$-\frac{1}{3}$	$\mathbf{1_1}$	0

Reason: simplicity

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

H_u	1	2	$-\frac{1}{2}$	$\mathbf{1}_1$	15
H_d	1	2	$\frac{1}{2}$	$\mathbf{1}_1$	9
ϕ	3	1	$-\frac{1}{3}$	$\mathbf{1}_1$	0

Reason:
generate masses
of all 3rd generation
fermions at tree level

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

$S = \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}$	1	1	0	$\mathbf{2_1}$	16
$T = \begin{pmatrix} T_1 \\ T_2 \end{pmatrix}$	1	1	0	$\mathbf{2_2}$	8
$U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$	1	1	0	$\mathbf{2_2}$	8
$W = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}$	1	1	0	$\mathbf{2_2}$	12

Introduce spurions to break G_f

Reason: easier than with multiple Higgs doublets; simplification not to consider their potential, corrections to it, etc.

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

$S = \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}$	1	1	0	$\mathbf{2_1}$	16
$T = \begin{pmatrix} T_1 \\ T_2 \end{pmatrix}$	1	1	0	$\mathbf{2_2}$	8
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Introduce spurions to break G_f

Role of S:

- spurion for LQ couplings
- preserves residual symmetry

$$\langle S \rangle = \begin{pmatrix} \lambda \\ 0 \end{pmatrix}$$

$$\lambda \approx 0.2$$

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

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- spurion for LQ couplings
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$$\langle S \rangle = \begin{pmatrix} \lambda \\ 0 \end{pmatrix}$$

$$\lambda \approx 0.2$$

$$\mathbf{x} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda^3 & \lambda \\ 0 & \lambda^2 & 1 \end{pmatrix} \text{ and } \mathbf{y} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \lambda^3 \\ 0 & 1 & 0 \end{pmatrix}$$

Field	Z_{17}^{diag}	Field	Z_{17}^{diag}	Field	Z_{17}^{diag}	Field	Z_{17}^{diag}	Field	Z_{17}^{diag}
Q_1	3	d_{R1}	5	e_{R1}	5	S_1	0	W_1	14
Q_2	16	d_{R2}	14	e_{R2}	16	S_2	15	W_2	10
Q_3	16	d_{R3}	7	e_{R3}	9	T_1	10		
u_{R1}	13	L_1	3	H_u	15	T_2	6		
u_{R2}	8	L_2	1	H_d	9	U_1	10		
u_{R3}	1	L_3	1	ϕ	0	U_2	6		

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

$S = \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}$	1	1	0	$\mathbf{2_1}$	16
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Introduce spurions to break G_f

Role of T:

- spurion for mass of second generation of down-type quarks and charged leptons

$$\langle T \rangle = \begin{pmatrix} \lambda^2 \\ 0 \end{pmatrix}$$

$$\lambda \approx 0.2$$

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

$$\langle T \rangle = \begin{pmatrix} \lambda^2 \\ 0 \end{pmatrix} \qquad \lambda \approx 0.2$$

Role of T:

- spurion for mass of second generation of down-type quarks and charged leptons

$$\mathscr{L}^d_{\text{Yuk,LO}} = \alpha^d_1 \, \overline{Q_3} \, H_d \, d_{R3} + \alpha^d_2 \, \overline{Q} \, H_d \, d_R \, T + \alpha^d_3 \, \overline{Q} \, H_d \, d_R \, U$$

$$\mathscr{L}^e_{\text{Yuk,LO}} = \alpha^e_1 \, \overline{L_3} \, H_d \, e_{R3} + \alpha^e_2 \, \overline{L} \, H_d \, e_R \, T + \alpha^e_3 \, \overline{L} \, H_d \, e_R \, U$$

Note VEV of T breaks residual symmetry

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

$S = \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}$	1	1	0	$\mathbf{2_1}$	16
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$U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$	1	1	0	$\mathbf{2_2}$	8
$W = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}$	1	1	0	$\mathbf{2_2}$	12

Introduce spurions to break G_f

$$\langle U \rangle = \begin{pmatrix} 0 \\ \lambda^4 \end{pmatrix}$$

$$\lambda \approx 0.2$$

Role of U:

- spurion for mass of first generation of down-type quarks and charged leptons

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

$$\langle U \rangle = \begin{pmatrix} 0 \\ \lambda^4 \end{pmatrix}$$

$$\lambda \approx 0.2$$

Role of U:

- spurion for mass of first generation of down-type quarks and charged leptons

$$\mathscr{L}^d_{\text{Yuk,LO}} = \alpha^d_1 \overline{Q_3} H_d d_{R3} + \alpha^d_2 \overline{Q} H_d d_R T + \alpha^d_3 \overline{Q} H_d d_R U$$

$$\mathscr{L}^e_{\text{Yuk,LO}} = \alpha^e_1 \overline{L_3} H_d e_{R3} + \alpha^e_2 \overline{L} H_d e_R T + \alpha^e_3 \overline{L} H_d e_R U$$

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Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

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$$\mathcal{L}^e_{\text{Yuk,LO}} = \alpha^e_1 \, \overline{L_3} \, H_d \, e_{R3} + \alpha^e_2 \, \overline{L} \, H_d \, e_R \, T + \alpha^e_3 \, \overline{L} \, H_d \, e_R \, U$$

Note VEV of U breaks residual symmetry

Why not just VEV for second component of T?

Reasons:

- problem with order one factor between m_d and m_e
- with T and U and their VEV structure higher-order operators are better under control

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

$S = \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}$	1	1	0	$\mathbf{2_1}$	16
$T = \begin{pmatrix} T_1 \\ T_2 \end{pmatrix}$	1	1	0	$\mathbf{2_2}$	8
$U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$	1	1	0	$\mathbf{2_2}$	8
$W = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}$	1	1	0	$\mathbf{2_2}$	12

Introduce spurions to break G_f

Role of W:

- spurion for mass of charm quark
- generation of Cabibbo angle

$$\langle W \rangle = \begin{pmatrix} \lambda^5 \\ \lambda^4 \end{pmatrix}$$

$$\lambda \approx 0.2$$

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

$$\langle W \rangle = \begin{pmatrix} \lambda^5 \\ \lambda^4 \end{pmatrix} \qquad \lambda \approx 0.2$$

Role of *W*:

- spurion for mass of charm quark
- generation of Cabibbo angle

$$\mathscr{L}^u_{\text{Yuk,LO}} = \alpha^u_1 \overline{Q_3}\, H_u\, u_{R3} + \alpha^u_2 \overline{Q}\, H_u\, u_{R2}\, W + \alpha^u_3 \overline{Q}\, H_u\, u_{R3}\, (S^\dagger)^2 + \alpha^u_4 \overline{Q}\, H_u\, u_{R1}\, T^2\, U.$$

Note VEV of *W* breaks residual symmetry

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

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Note VEV of W breaks residual symmetry

What is missing?

- mass for up quark
- smaller quark mixing angles

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

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Note VEV of *W* breaks residual symmetry

What is missing?

- mass for up quark
- smaller quark mixing angles

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

Result for charged fermion mass matrices

$$M_u = \begin{pmatrix} f_{11}\,\lambda^8 & f_{12}\,\lambda^5 & f_{13}\,\lambda^8 \\ f_{21}\,\lambda^{10} & f_{22}\,\lambda^4 & f_{23}\,\lambda^2 \\ f_{31}\,\lambda^{12} & f_{32}\,\lambda^4 & f_{33} \end{pmatrix} \langle H_u^0 \rangle$$

$$M_d = \begin{pmatrix} d_{11}\,\lambda^4 & d_{12}\,\lambda^8 & d_{13}\,\lambda^8 \\ d_{21}\,\lambda^{10} & d_{22}\,\lambda^2 & d_{23}\,\lambda^2 \\ d_{31}\,\lambda^{12} & d_{32}\,\lambda^4 & d_{33} \end{pmatrix} \langle H_d^0 \rangle$$

$$M_e = \begin{pmatrix} e_{11}\,\lambda^4 & e_{12}\,\lambda^{12} & \mathcal{O}(\lambda^{12}) \\ e_{21}\,\lambda^8 & e_{22}\,\lambda^2 & e_{23}\,\lambda \\ e_{31}\,\lambda^9 & e_{32}\,\lambda^3 & e_{33} \end{pmatrix} \langle H_d^0 \rangle$$

- Charged fermion masses are reproduced well
- Quark mixing is reproduced well

apart from …

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

Result for charged fermion mass matrices

$$M_u = \begin{pmatrix} f_{11}\,\lambda^8 & f_{12}\,\lambda^5 & f_{13}\,\lambda^8 \\ f_{21}\,\lambda^{10} & f_{22}\,\lambda^4 & f_{23}\,\lambda^2 \\ f_{31}\,\lambda^{12} & f_{32}\,\lambda^4 & f_{33} \end{pmatrix} \langle H_u^0 \rangle$$

… this element should be enhanced

$$M_d = \begin{pmatrix} d_{11}\,\lambda^4 & d_{12}\,\lambda^8 & d_{13}\,\lambda^8 \\ d_{21}\,\lambda^{10} & d_{22}\,\lambda^2 & d_{23}\,\lambda^2 \\ d_{31}\,\lambda^{12} & d_{32}\,\lambda^4 & d_{33} \end{pmatrix} \langle H_d^0 \rangle$$

$$M_e = \begin{pmatrix} e_{11}\,\lambda^4 & e_{12}\,\lambda^{12} & \mathcal{O}(\lambda^{12}) \\ e_{21}\,\lambda^8 & e_{22}\,\lambda^2 & e_{23}\,\lambda \\ e_{31}\,\lambda^9 & e_{32}\,\lambda^3 & e_{33} \end{pmatrix} \langle H_d^0 \rangle$$

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

Leading to LQ couplings **x**, **y** and **z**

$$\mathbf{x} = L_e^T \, \hat{\mathbf{x}} \, L_d = \begin{pmatrix} a_{11}\,\lambda^9 & a_{12}\,\lambda^{11} & a_{13}\,\lambda^9 \\ a_{21}\,\lambda^8 & a_{22}\,\lambda^3 & a_{23}\,\lambda \\ a_{31}\,\lambda^8 & a_{32}\,\lambda^2 & a_{33} \end{pmatrix}$$

$$\mathbf{y} = R_e^T \, \hat{\mathbf{y}} \, R_u = \begin{pmatrix} b_{11}\,\lambda^9 & b_{12}\,\lambda^9 & b_{13}\,\lambda^9 \\ b_{21}\,\lambda^8 & b_{22}\,\lambda^3 & b_{23}\,\lambda^3 \\ b_{31}\,\lambda^5 & b_{32} & b_{33}\,\lambda^4 \end{pmatrix}$$

$$\mathbf{z} = L_e^T \, \hat{\mathbf{x}} \, L_u = \begin{pmatrix} c_{11}\,\lambda^9 & c_{12}\,\lambda^{10} & c_{13}\,\lambda^9 \\ c_{21}\,\lambda^4 & c_{22}\,\lambda^3 & c_{23}\,\lambda \\ c_{31}\,\lambda^3 & c_{32}\,\lambda^2 & c_{33} \end{pmatrix}$$

Dominant entries of LQ couplings achieved

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

Leading to LQ couplings **x**, **y** and **z**

Efficient suppression of LQ couplings to SM fermions of first generation

$$\mathbf{x} = L_e^T \, \hat{\mathbf{x}} \, L_d = \begin{pmatrix} a_{11}\,\lambda^9 & a_{12}\,\lambda^{11} & a_{13}\,\lambda^9 \\ a_{21}\,\lambda^8 & a_{22}\,\lambda^3 & a_{23}\,\lambda \\ a_{31}\,\lambda^8 & a_{32}\,\lambda^2 & a_{33} \end{pmatrix}$$

$$\mathbf{y} = R_e^T \, \hat{\mathbf{y}} \, R_u = \begin{pmatrix} b_{11}\,\lambda^9 & b_{12}\,\lambda^9 & b_{13}\,\lambda^9 \\ b_{21}\,\lambda^8 & b_{22}\,\lambda^3 & b_{23}\,\lambda^3 \\ b_{31}\,\lambda^5 & b_{32} & b_{33}\,\lambda^4 \end{pmatrix}$$

$$\mathbf{z} = L_e^T \, \hat{\mathbf{x}} \, L_u = \begin{pmatrix} c_{11}\,\lambda^9 & c_{12}\,\lambda^{10} & c_{13}\,\lambda^9 \\ c_{21}\,\lambda^4 & c_{22}\,\lambda^3 & c_{23}\,\lambda \\ c_{31}\,\lambda^3 & c_{32}\,\lambda^2 & c_{33} \end{pmatrix}$$

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

Leading to LQ couplings **x**, **y** and **z**

$$\mathbf{x} = L_e^T \, \hat{\mathbf{x}} \, L_d = \begin{pmatrix} a_{11}\,\lambda^9 & a_{12}\,\lambda^{11} & a_{13}\,\lambda^9 \\ a_{21}\,\lambda^8 & a_{22}\,\lambda^3 & a_{23}\,\lambda \\ a_{31}\,\lambda^8 & a_{32}\,\lambda^2 & a_{33} \end{pmatrix}$$

$$\mathbf{y} = R_e^T \, \hat{\mathbf{y}} \, R_u = \begin{pmatrix} b_{11}\,\lambda^9 & b_{12}\,\lambda^9 & b_{13}\,\lambda^9 \\ b_{21}\,\lambda^8 & b_{22}\,\lambda^3 & b_{23}\,\lambda^3 \\ b_{31}\,\lambda^5 & b_{32} & b_{33}\,\lambda^4 \end{pmatrix}$$

$$\mathbf{z} = L_e^T \, \hat{\mathbf{x}} \, L_u = \begin{pmatrix} c_{11}\,\lambda^9 & c_{12}\,\lambda^{10} & c_{13}\,\lambda^9 \\ c_{21}\,\lambda^4 & c_{22}\,\lambda^3 & c_{23}\,\lambda \\ c_{31}\,\lambda^3 & c_{32}\,\lambda^2 & c_{33} \end{pmatrix}$$

Potentially relevant LQ couplings

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

List of Primary Observables

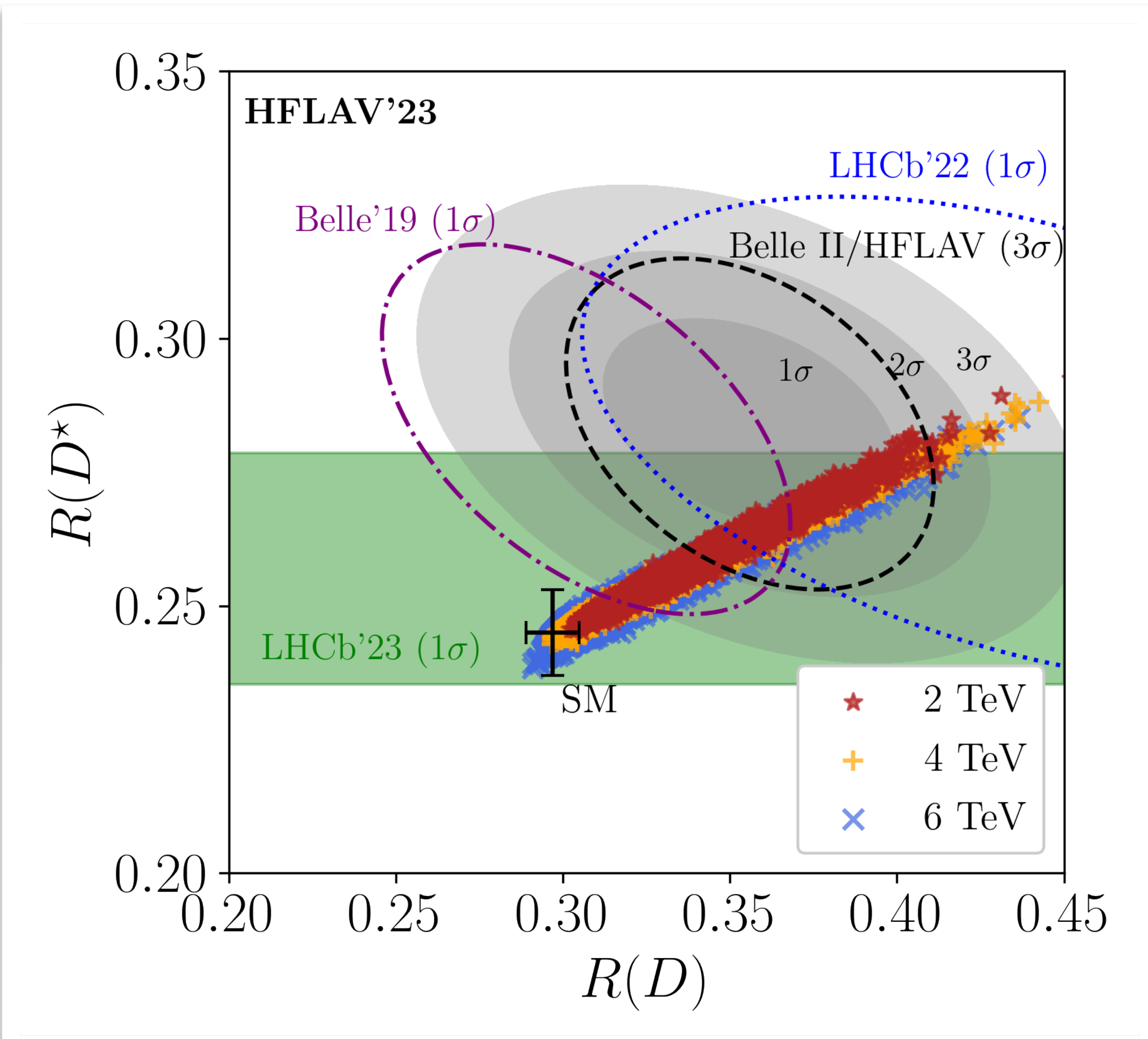
Observable	Experiment					
	Current constraint/measurement			Future reach	
$R(D)$	$0.339 \pm 0.026 \pm 0.014$	at $1\,\sigma$ level	[19]	$\pm 0.016\,(0.008)$ for $5\,(50)$ ab^{-1}	[98]
$R(D^\star)$	$0.295 \pm 0.010 \pm 0.010$	at $1\,\sigma$ level	[19]	$\pm 0.009\,(0.0045)$ for $5\,(50)$ ab^{-1}	[98]
Δa_μ	$(2.51 \pm 0.59) \times 10^{-9}$	at $1\,\sigma$ level	[21; 57]	$\pm 0.4 \times 10^{-9}$	[99]
$\mathrm{BR}(\tau \to \mu\gamma)$	4.2×10^{-8}	at 90% C.L.	[100]	6.9×10^{-9}	[101]
$\mathrm{BR}(\mu \to e\gamma)$	4.2×10^{-13}	at 90% C.L.	[102]	6×10^{-14}	[103]
$\mathrm{BR}(\tau \to 3\,\mu)$	2.1×10^{-8}	at 90% C.L.	[104]	3.6×10^{-10}	[101]
$\mathrm{BR}(\tau \to \mu e \bar{e})$	1.8×10^{-8}	at 90% C.L.	[104]	2.9×10^{-10}	[101]
$\mathrm{BR}(\mu \to 3\,e)$	1.0×10^{-12}	at 90% C.L.	[105]	$20\,(1) \times 10^{-16}$	[106]
$\mathrm{CR}(\mu \to e;\,\mathrm{Al})$				$2.6\,(2.9) \times 10^{-17}$	[107; 108]
$R^\nu_{K^\star}$	2.7	at 90% C.L.	[109]	$1.0 \pm 0.25\,(0.1)$ for $5\,(50)$ ab^{-1}	[110]
$g_{\tau_A}/g_A^{\mathrm{SM}}$	1.00154 ± 0.00128	at $1\,\sigma$ level	[111; 112]	$\pm 7.5\,(0.75) \times 10^{-5}$	[112–114]
$\tau_{B_c}^{\mathrm{SM}}$	$0.52^{+0.18}_{-0.12}$ ps	at $1\,\sigma$ level	[115]		
$c\bar{c} \to \tau\bar{\tau}$	$\lvert b_{32}\rvert < 2.6$ $(\hat{m}_\phi = 2)$		[116; 117]		

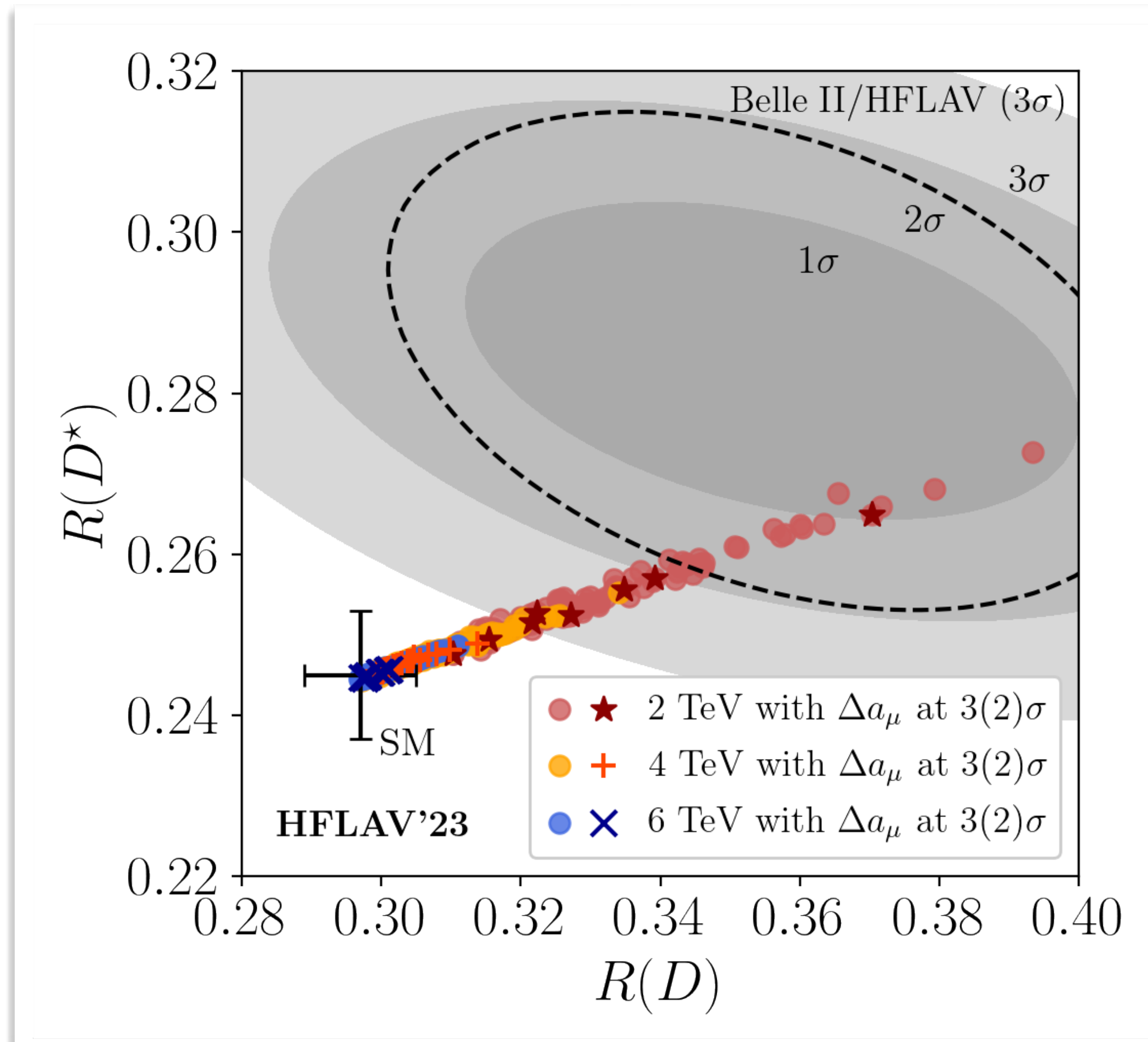
Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

- Perform numerical scan in interaction basis for all observables, also fitting charged fermion masses and quark mixing

Flavour anomalies



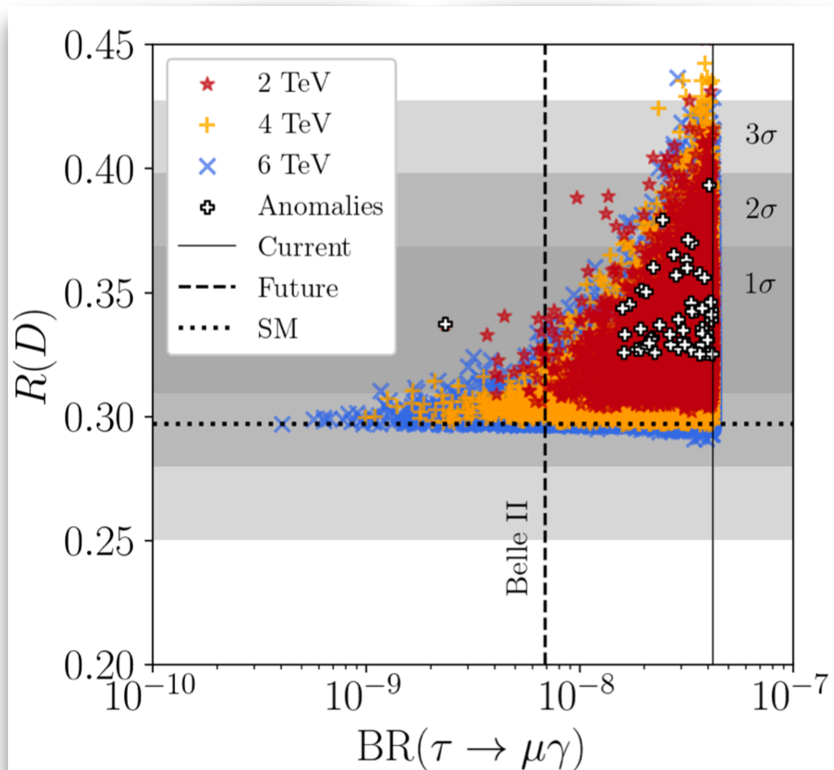


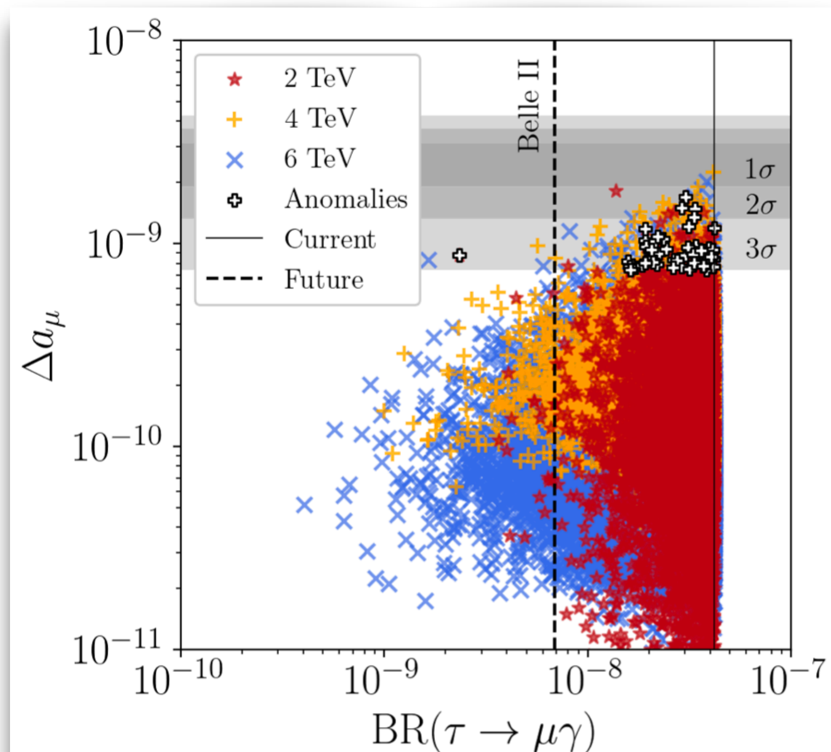
Example 3: Model with leptoquark

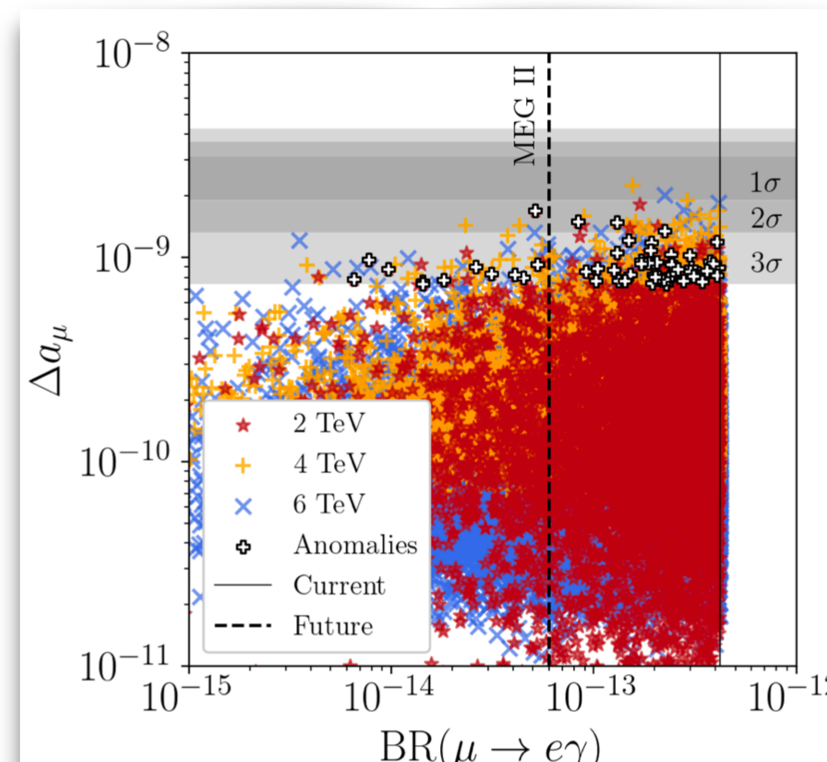
[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

- Perform numerical scan in interaction basis for all observables, also fitting charged fermion masses and quark mixing

Strongest constraints





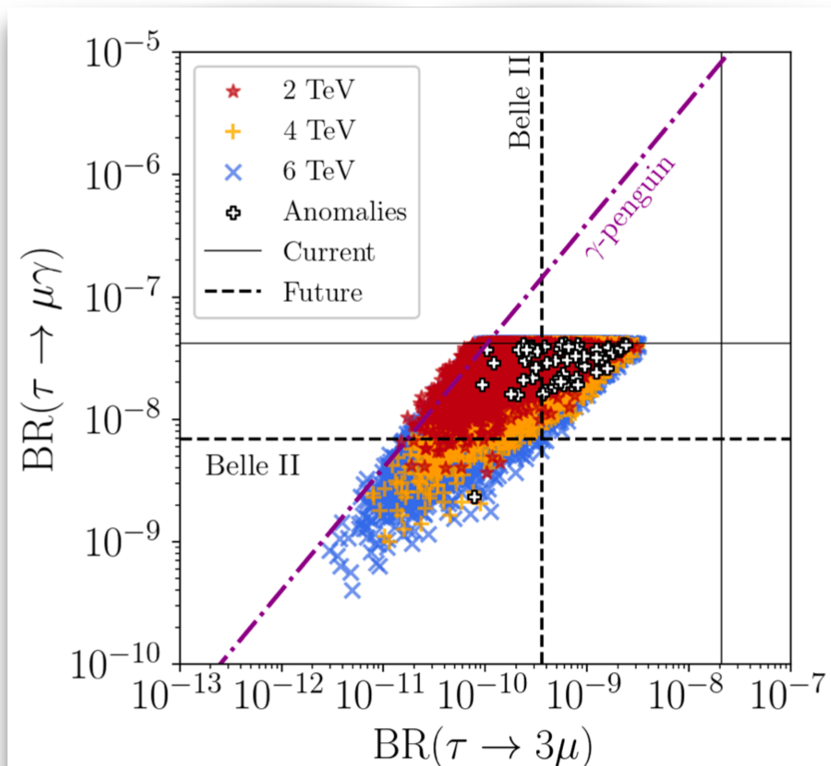


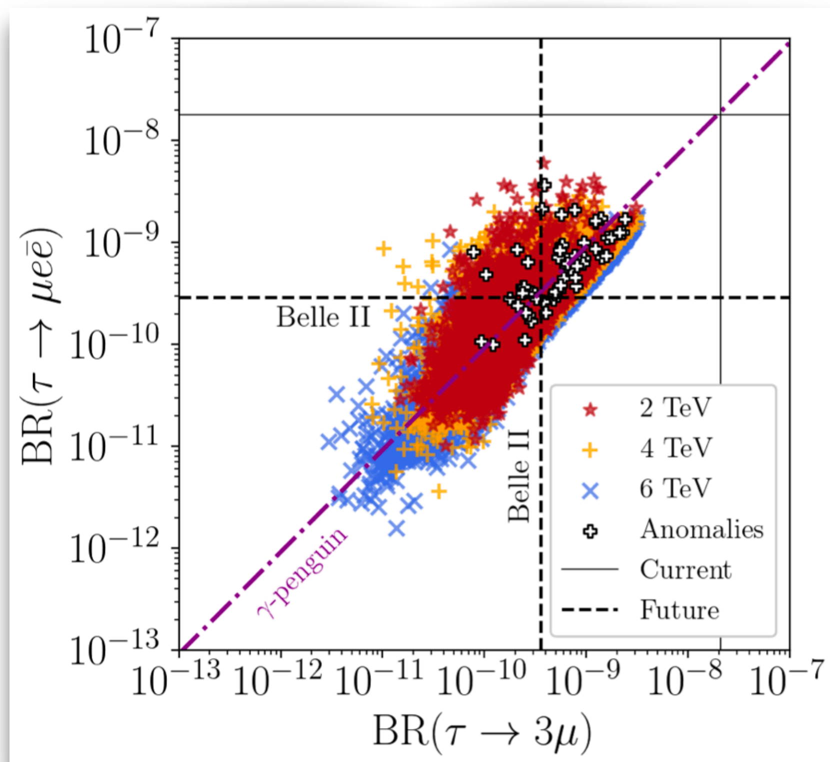
Example 3: Model with leptoquark

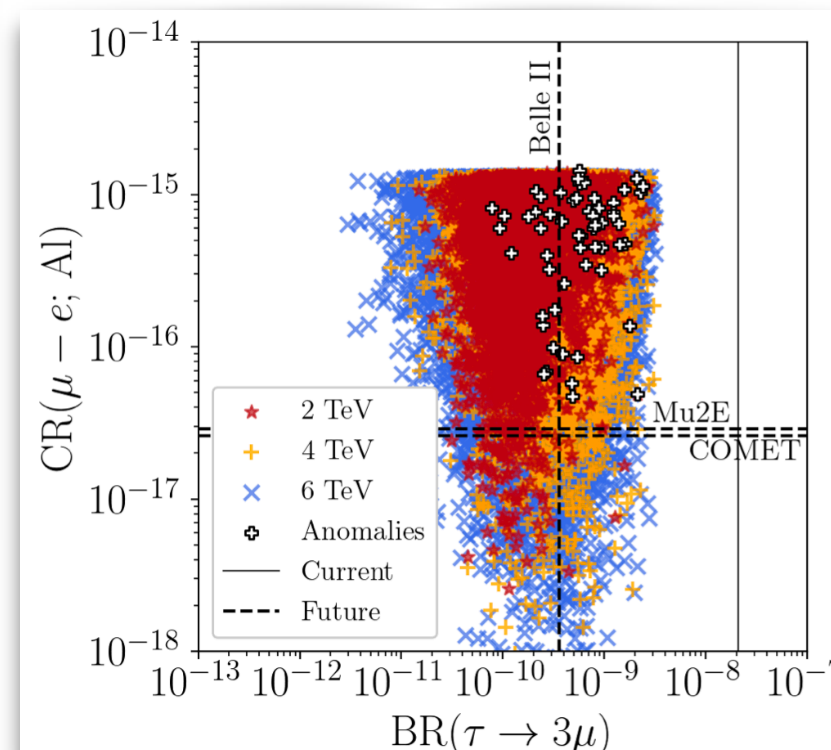
[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

- Perform numerical scan in interaction basis for all observables, also fitting charged fermion masses and quark mixing

Possible correlations







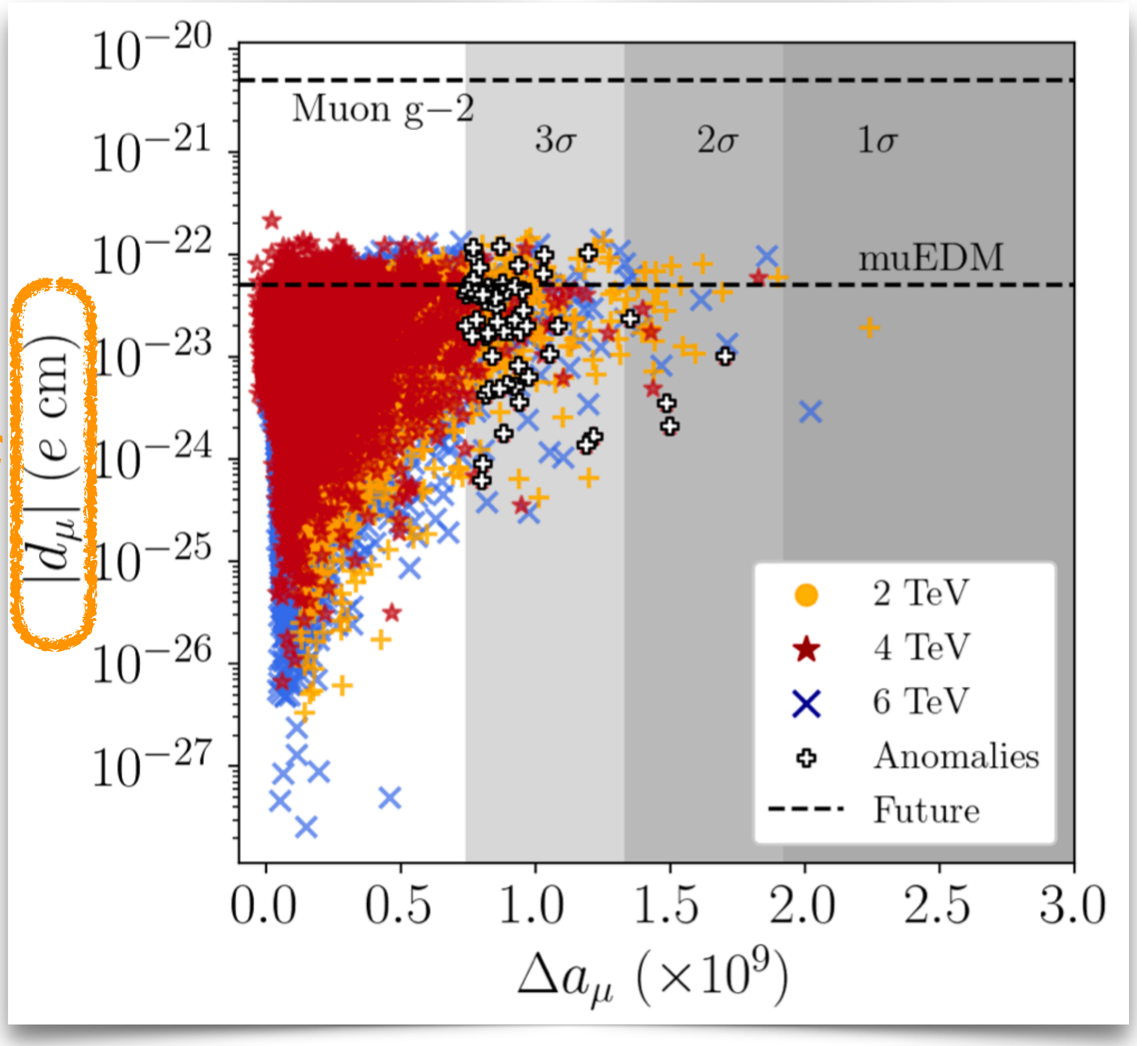
Example 3: Model with leptoquark

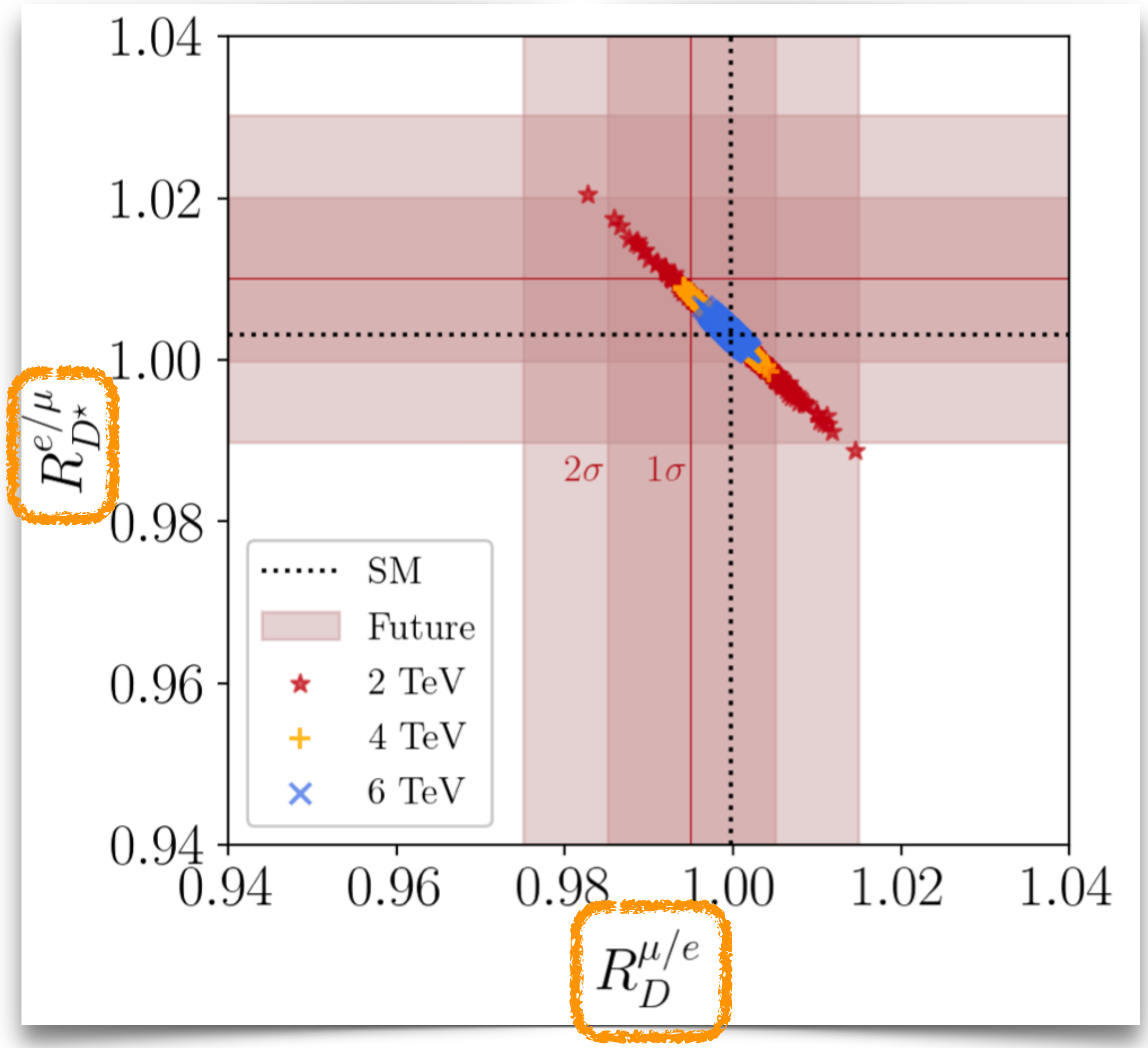
[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

- Perform numerical scan in interaction basis for all observables, also fitting charged fermion masses and quark mixing

Predictions for further observables

EDM of muon





$$R_D^{\mu/e} = \frac{\Gamma(B \to D\mu\nu)}{\Gamma(B \to De\nu)} \quad \text{and} \quad R_{D^\star}^{e/\mu} = \frac{\Gamma(B \to D^\star e\nu)}{\Gamma(B \to D^\star \mu\nu)}$$

Summary and Outlook

- Flavour and CP symmetries are useful for understanding fermion mixing and potentially also fermion masses
- They also have considerable effect on other observables in extensions of the SM
- Three examples with interesting applications
  - Example 1: Inverse seesaw mechanism with **strong suppression** of charged lepton flavour violating processes
  - Example 2: Low-scale seesaw mechanism with **strongly degenerate** RH neutrino masses and generation of BAU possibly **correlated with** low energy CP phases
  - Example 3: Model with leptoquark for **explanation of** $R(D)$**,** $R(D^\star)$ **and the anomalous magnetic moment of the muon**, while **passing all other experimental bounds** and making testable predictions

Summary and Outlook

- Example 1: Explore other versions of symmetry breaking and variants of the inverse seesaw mechanism
- Example 2: Explore phenomenology of heavy neutral leptons and variants of the low-scale type I seesaw mechanism
- Example 3: Extend in order to include neutrino masses and lepton mixing

- One can think about embedding the examples in larger frameworks
- And obviously flavour and CP symmetries can be applied to many more extensions of the SM
- …

Many thanks for your attention!

Back-up slides

Example 1: Inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Back to light neutrinos

… go beyond leading order

- potentially new contributions to m_ν
- effects of non-unitarity

$$m_\nu^1 = -\frac{1}{2}\, m_D \left(M_{NS}^{-1}\right)^T \left[\mu_S\, M_{NS}^{-1}\, m_D^T\, m_D^\star \left(M_{NS}^{-1}\right)^\dagger + \left(M_{NS}^{-1}\right)^\star m_D^\dagger\, m_D \left(M_{NS}^{-1}\right)^T \mu_S\right] M_{NS}^{-1}\, m_D^T$$

$$m_\nu^1 = -\frac{y_0^4\, v^4}{4\, M_0^4}\, \mu_S = -\frac{y_0^4\, v^4}{4\, M_0^4}\, U_S^\star \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix} U_S^\dagger$$

Compare to

$$m_\nu = \frac{y_0^2\, v^2}{2\, M_0^2}\, \mu_S = \frac{y_0^2\, v^2}{2\, M_0^2}\, U_S^\star \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix} U_S^\dagger$$

Example 1: Inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Back to light neutrinos

… go beyond leading order

- potentially new contributions to m_ν
- effects of non-unitarity

$$\widetilde{U}_{\mathrm{PMNS}} = \left(\mathbb{1} - \eta\right) U_0$$

$$\eta = \frac{1}{2}\, m_D^\star \left(M_{NS}^{-1}\right)^\dagger M_{NS}^{-1}\, m_D^T$$

$$\eta = \frac{y_0^2\, v^2}{4\, M_0^2}\, \mathbb{1} \equiv \eta_0\, \mathbb{1}$$

Universal effect in flavour α and for different patterns **Case 1) Case 2) Case 3 a) Case 3 b.1)**

Compare to

$$\widetilde{U}_{\mathrm{PMNS}} = \Omega(\mathbf{3})\, R_{fh}(\theta_S)$$

Example 1: Inverse seesaw mechanism

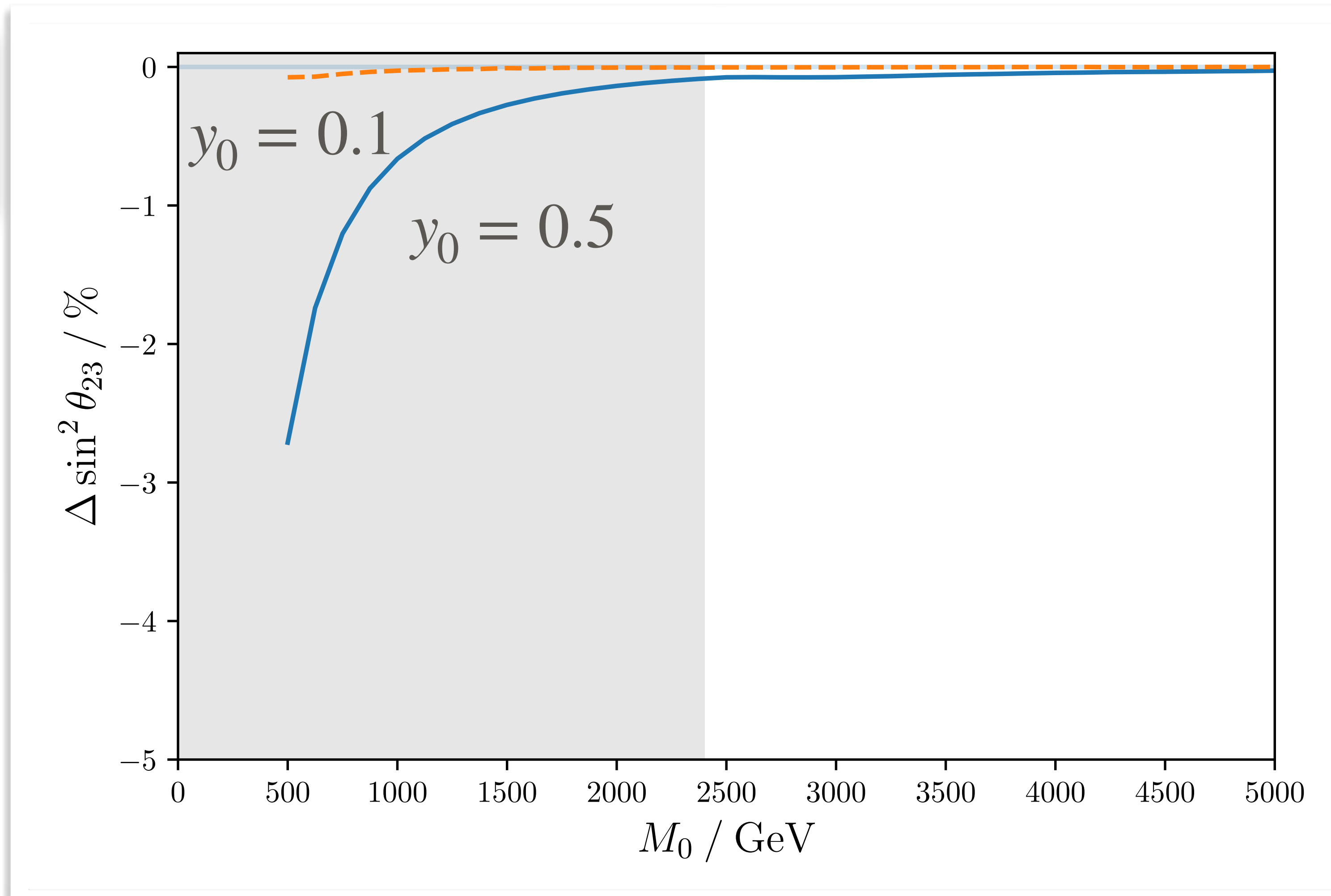
[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Effect on lepton mixing

Case 1)

$$\sin^2\theta_{23} = \frac{1}{2}\left(1 + \frac{\sqrt{3}\,\sin 2\theta}{2+\cos 2\theta}\right)$$

$$\theta \approx 0.18$$



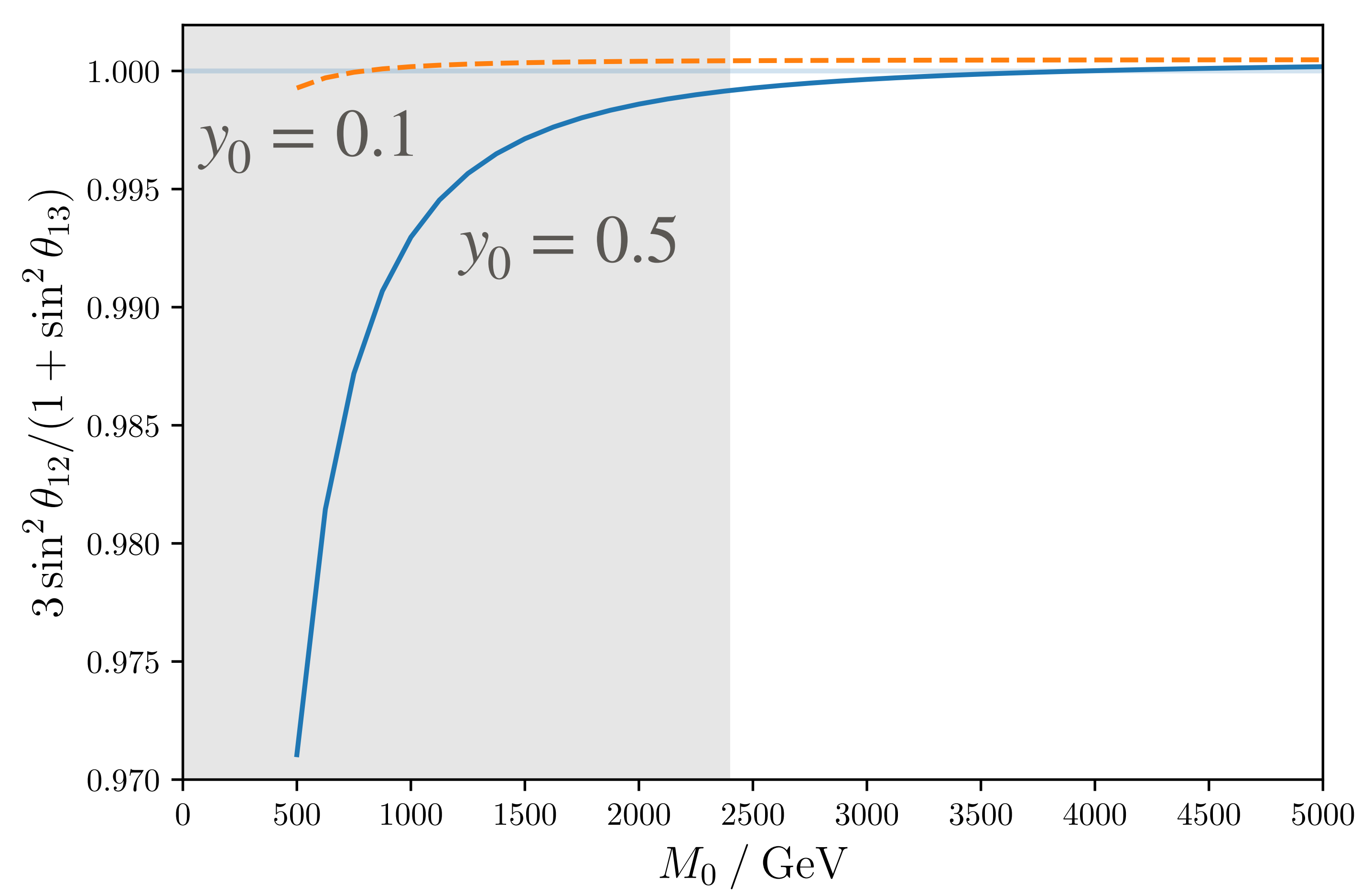
Example 1: Inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Effect on lepton mixing

Case 1)

$$\sin^2\theta_{12} \approx \frac{1}{3}\left(1+\sin^2\theta_{13}\right)$$



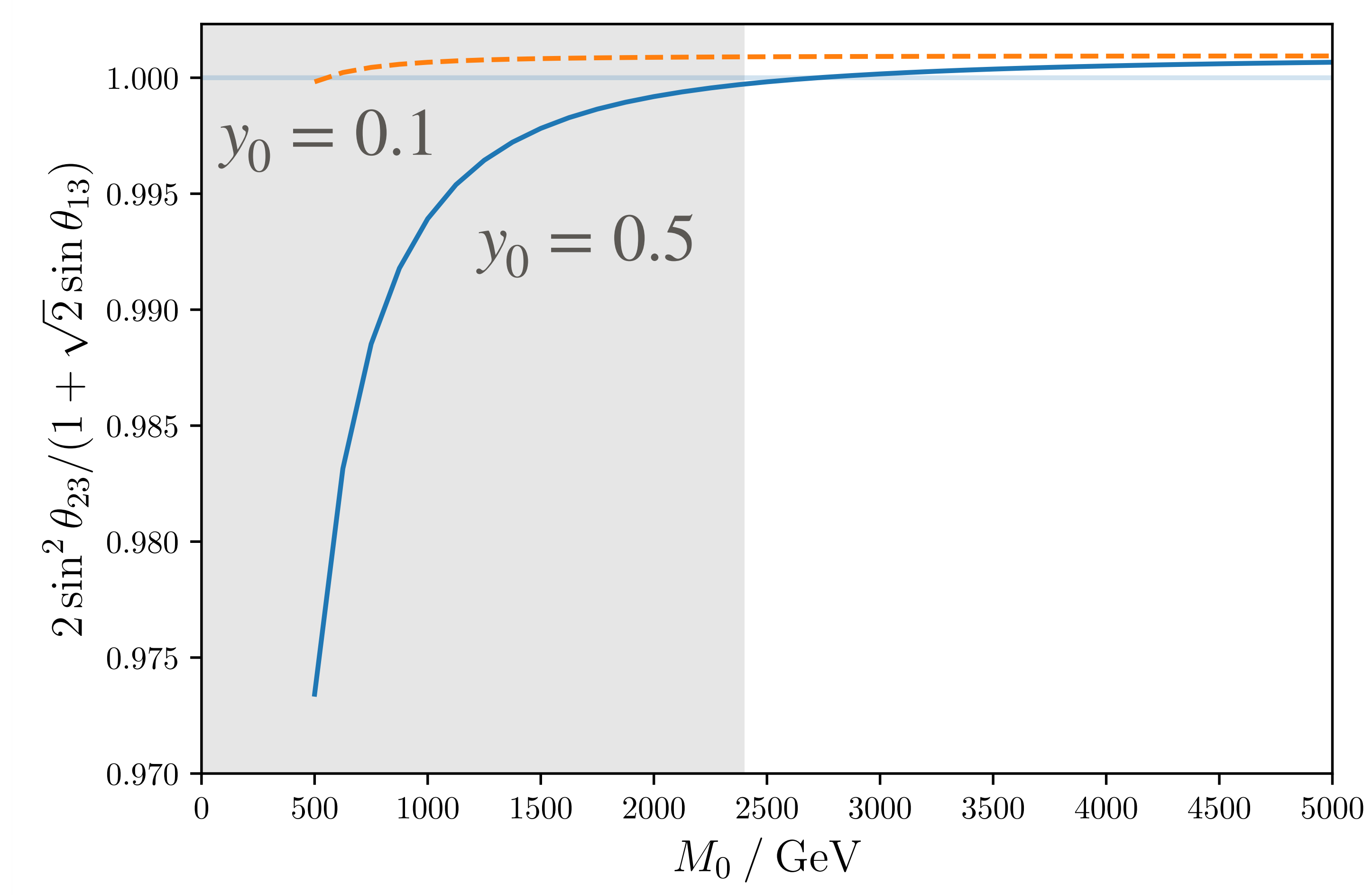
Example 1: Inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Effect on lepton mixing

Case 1)

$$\sin^2\theta_{23} \approx \frac{1}{2}\left(1 \pm \sqrt{2}\,\sin\theta_{13}\right)$$



Example 1: Inverse seesaw mechanism

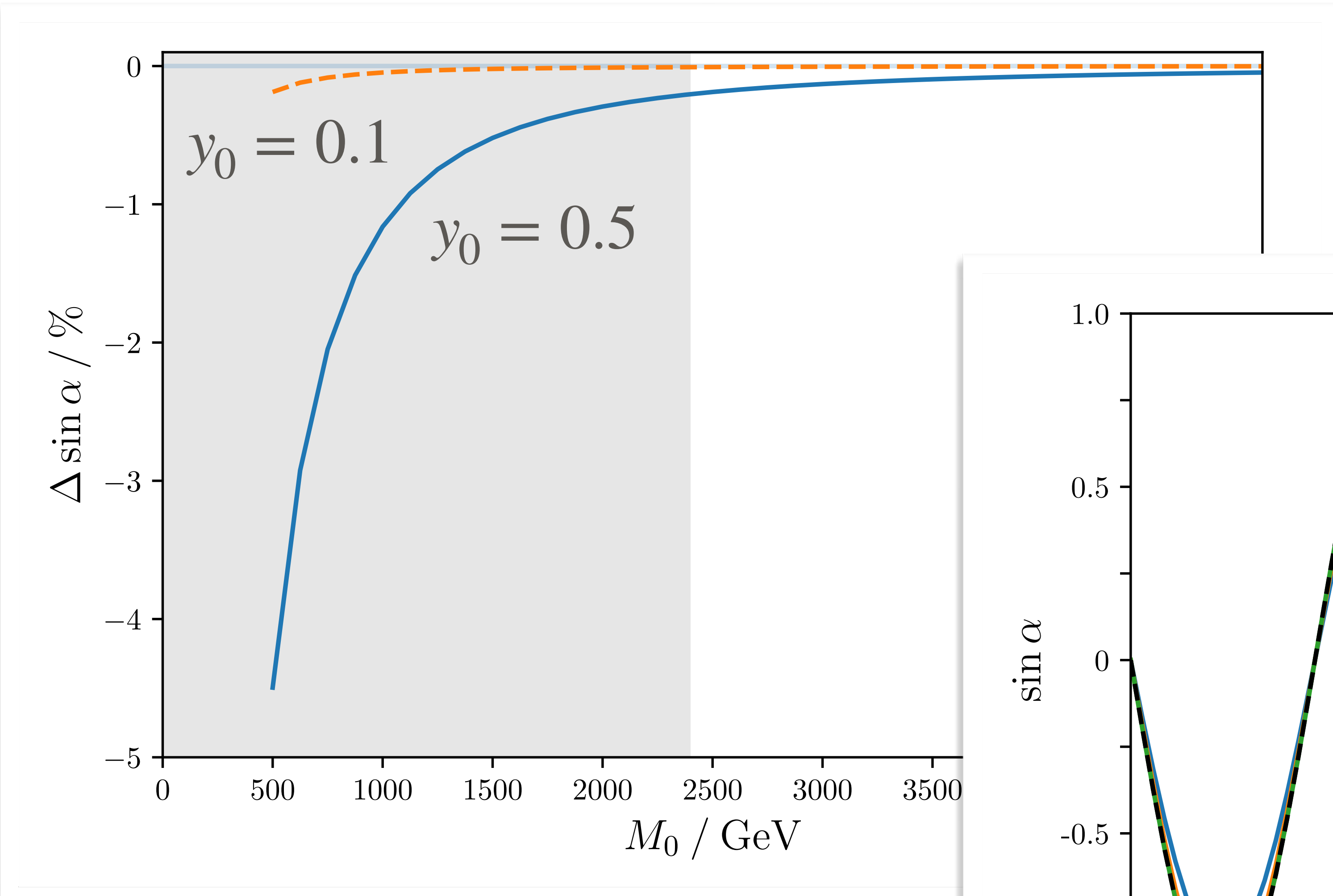
[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

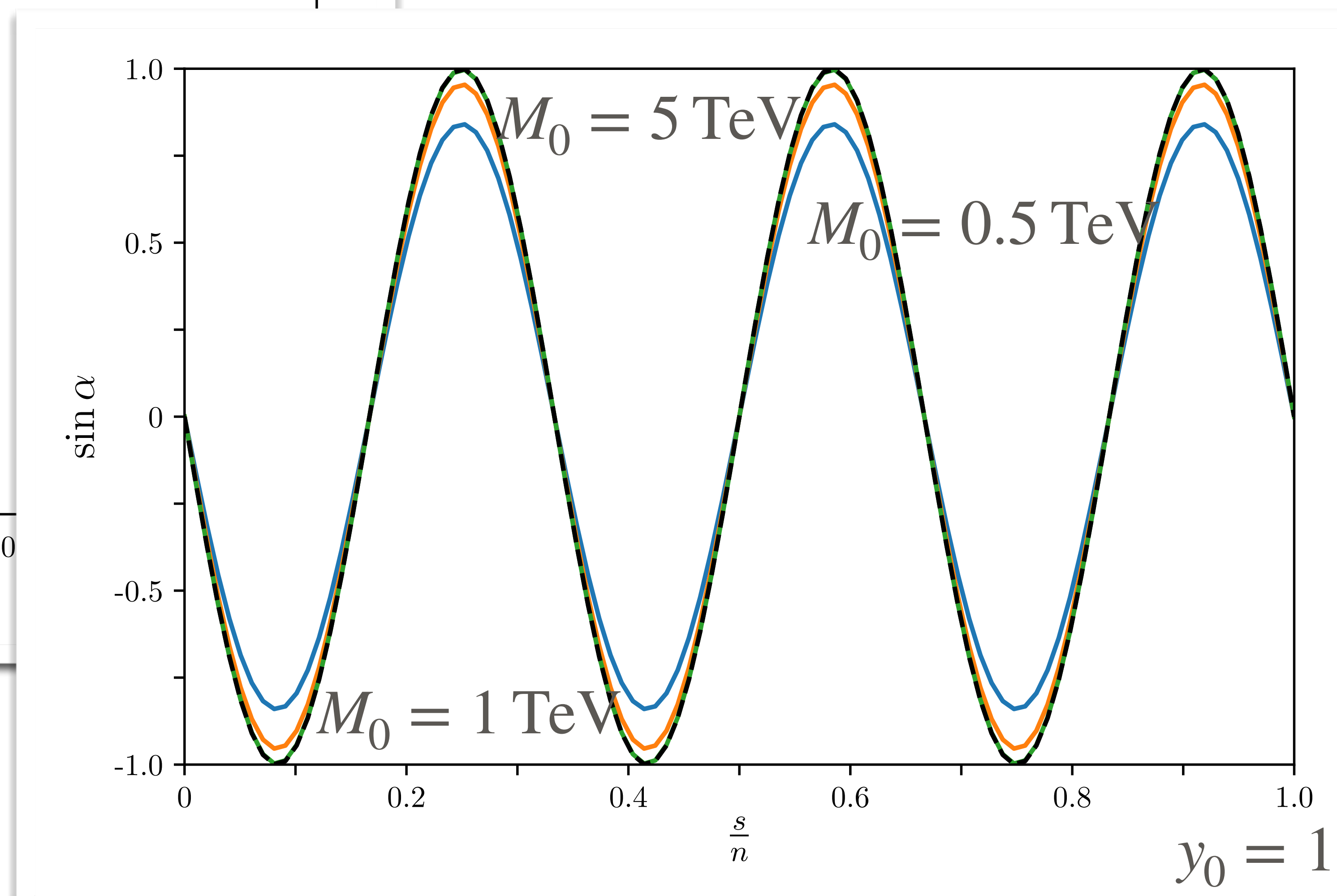
Effect on lepton mixing

Case 1)

$$\sin\alpha = -\sin 6\,\phi_s$$

$$\phi_s = \frac{\pi\, s}{n}$$



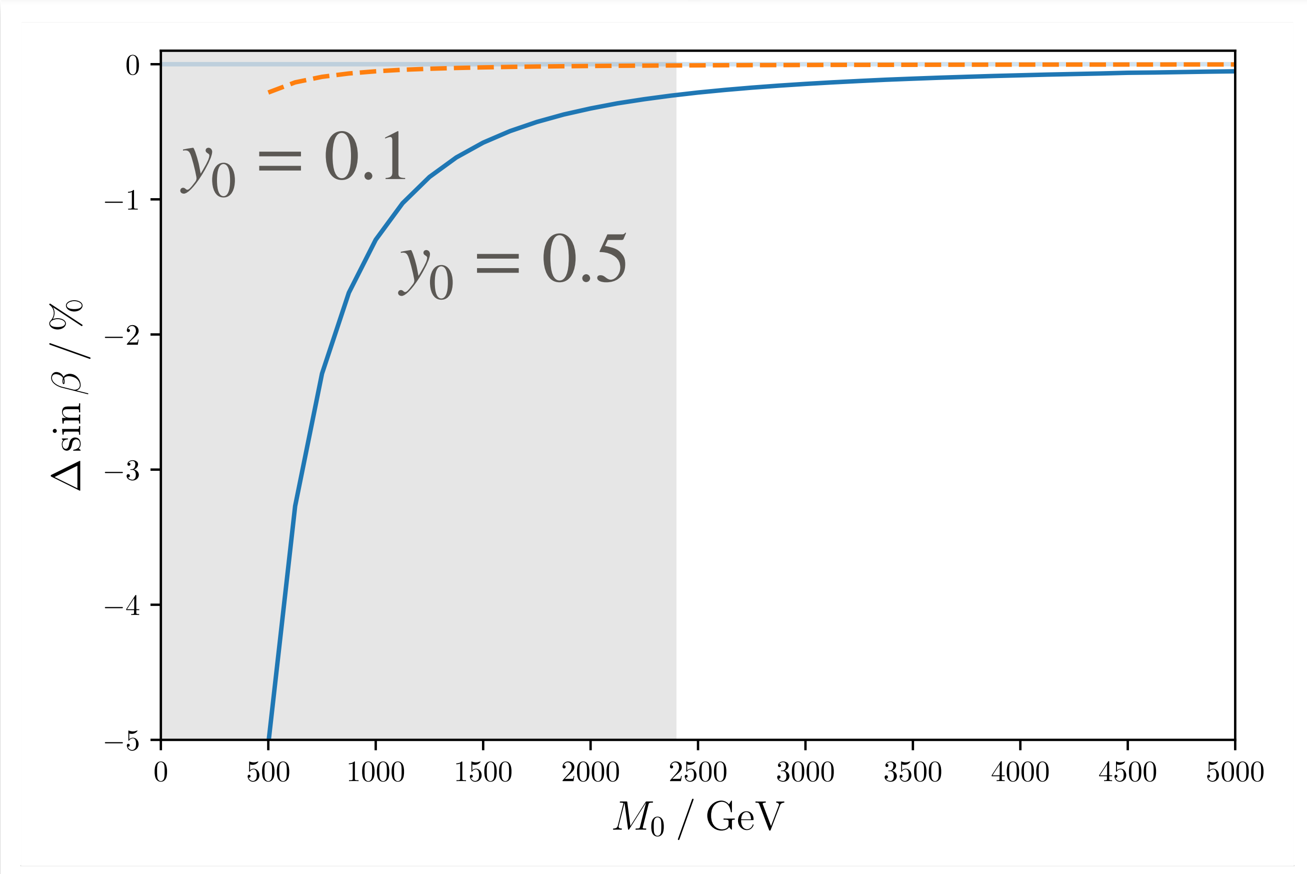


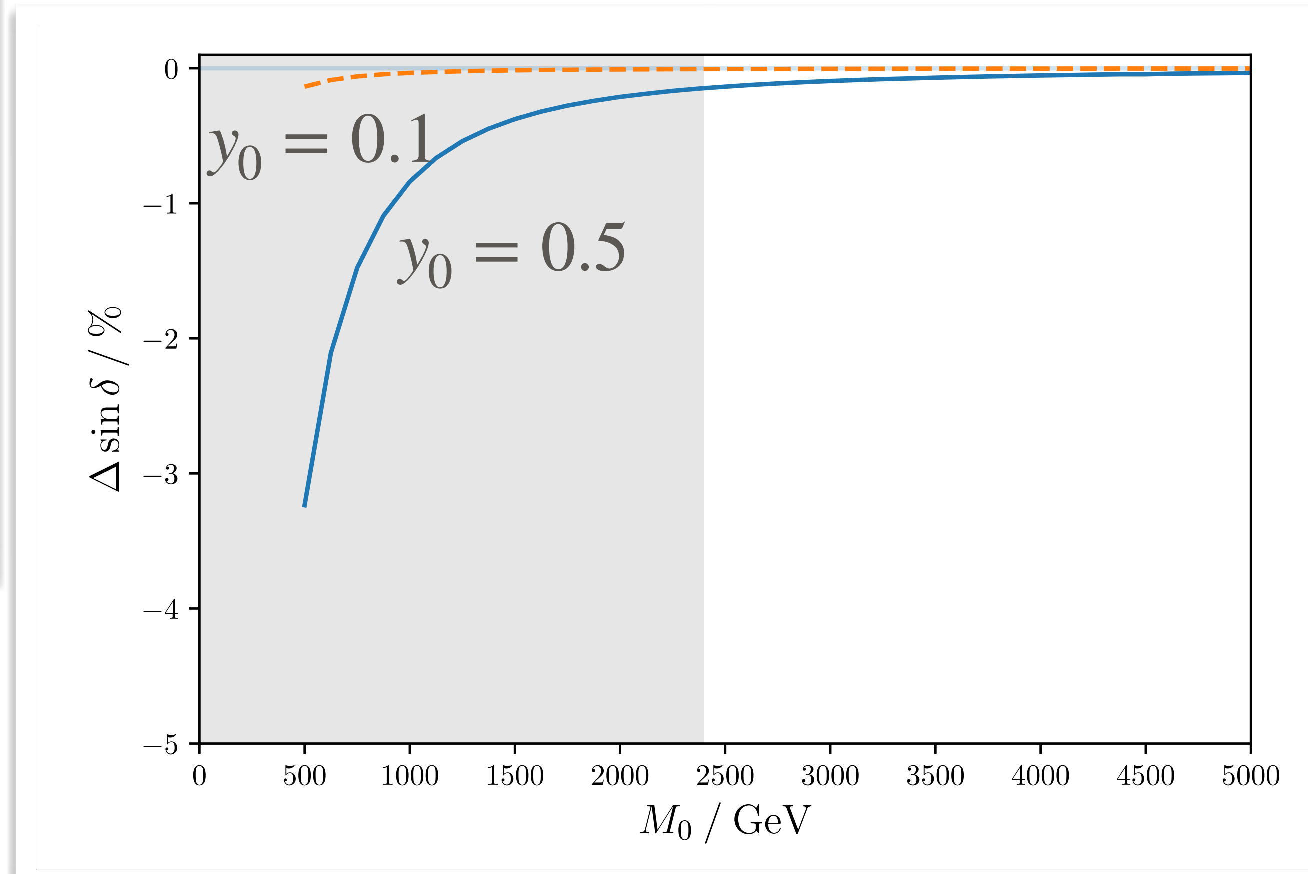
Example 1: Inverse seesaw mechanism

[CH, J. Kriewald, J. Orloff, A.M. Teixeira ('21)]

Effect on lepton mixing

Case 2)



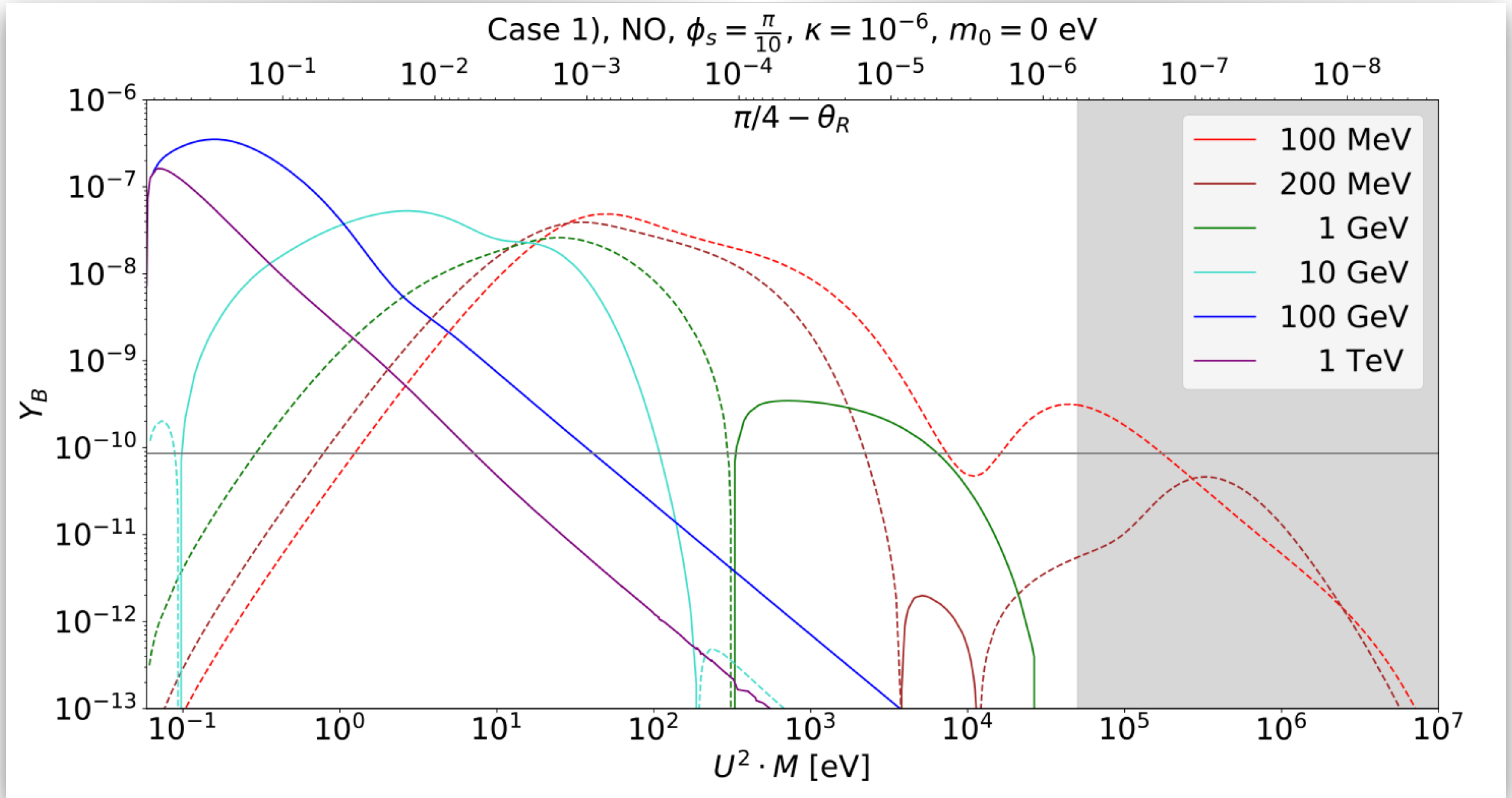


Analysis also performed for **Case 3 a)** **Case 3 b.1)**

Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Case 1)



Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Way towards capturing main dependencies analytically
- CP-violating combinations
- washout parameter

CP-violating combinations: see for related work Hernandez et al. ('15)

Perturbatively solve quantum kinetic equations in H_N and Γ

Leading term for lepton asymmetries

$$\mathrm{Tr}\left[\tilde{\Gamma}_\alpha(\bar{\rho}_N - \rho_N)\right] \propto \mathrm{Tr}\left(\tilde{\Gamma}_\alpha\,[H_N, \Gamma]\right)$$ with $\alpha = e, \mu, \tau$.

Three types of CP-violating combinations are found

$$\begin{aligned} C_{\mathrm{LFV},\alpha} &= i\,\mathrm{Tr}\Big(\left[\hat{M}_R^2, \hat{Y}_D^\dagger \hat{Y}_D\right] \hat{Y}_D^\dagger P_\alpha \hat{Y}_D\Big)\,, \\ C_{\mathrm{LNV},\alpha} &= i\,\mathrm{Tr}\Big(\left[\hat{M}_R^2, \hat{Y}_D^\dagger \hat{Y}_D\right] \hat{Y}_D^T P_\alpha \hat{Y}_D^*\Big)\,, \\ C_{\mathrm{DEG},\alpha} &= i\,\mathrm{Tr}\Big(\left[\hat{Y}_D^T \hat{Y}_D^*, \hat{Y}_D^\dagger \hat{Y}_D\right] \hat{Y}_D^T P_\alpha \hat{Y}_D^*\Big) \end{aligned}$$

Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

$$
\begin{aligned}
C_{\mathrm{LFV},\alpha} &= i\,\mathrm{Tr}\Big(\Big[\hat{M}_R^2, \hat{Y}_D^\dagger \hat{Y}_D\Big]\ \hat{Y}_D^\dagger P_\alpha \hat{Y}_D\Big)\,, \\
C_{\mathrm{LNV},\alpha} &= i\,\mathrm{Tr}\Big(\Big[\hat{M}_R^2, \hat{Y}_D^\dagger \hat{Y}_D\Big]\ \hat{Y}_D^T P_\alpha \hat{Y}_D^*\Big)\,, \\
C_{\mathrm{DEG},\alpha} &= i\,\mathrm{Tr}\Big(\Big[\hat{Y}_D^T \hat{Y}_D^*, \hat{Y}_D^\dagger \hat{Y}_D\Big]\ \hat{Y}_D^T P_\alpha \hat{Y}_D^*\Big)
\end{aligned}
$$

with

$$
P_e = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\ ,\ P_\mu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\ ,\ P_\tau = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

and in mass basis of heavy states, i.e.

$$
\hat{Y}_D = Y_D\, U_R
$$

$$
U_R = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & i \\ 0 & 1 & -i \end{pmatrix}
$$

Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

$$C_{\mathrm{LFV},\alpha} = i\,\mathrm{Tr}\Big(\big[\hat{M}_R^2, \hat{Y}_D^\dagger \hat{Y}_D\big]\,\hat{Y}_D^\dagger P_\alpha \hat{Y}_D\Big)$$

Note the following

- Dominant combination when N_i are in relativistic regime
- Only leads to lepton flavour asymmetry, since

$$\sum_\alpha C_{\mathrm{LFV},\alpha} = 0.$$

- Crucially depends on a flavoured washout

Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

$$C_{\mathrm{LFV},\alpha} = i\,\mathrm{Tr}\left(\left[\hat{M}_R^2, \hat{Y}_D^\dagger \hat{Y}_D\right] \hat{Y}_D^\dagger P_\alpha \hat{Y}_D\right)$$

Note the following

- Dominant combination when N_i are in relativistic regime
- Only leads to lepton flavour asymmetry, since

$$\sum_\alpha C_{\mathrm{LFV},\alpha} = 0.$$

- Crucially depends on a flavoured washout

$$C_{\mathrm{LNV},\alpha} = i\,\mathrm{Tr}\left(\left[\hat{M}_R^2, \hat{Y}_D^\dagger \hat{Y}_D\right] \hat{Y}_D^T P_\alpha \hat{Y}_D^*\right)$$

Note the following

- Sizeable for intermediate/larger masses of N_i
- Directly violates lepton number with

$$C_{\mathrm{LNV}} = \sum_\alpha C_{\mathrm{LNV},\alpha} \neq 0$$

compare to flavoured decay asymmetries $\epsilon_{i\alpha}$ see Dev et al. ('17)

Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

$$C_{\mathrm{DEG},\alpha} = i\,\mathrm{Tr}\Big(\Big[\hat{Y}_D^T\,\hat{Y}_D^*, \hat{Y}_D^\dagger\,\hat{Y}_D\Big]\,\hat{Y}_D^T\,P_\alpha\,\hat{Y}_D^*\Big)$$

Note the following

- Only this CP-violating combination could be non-zero for zero splitting
- Only possible at intermediate temperatures $M/T \sim 1$
- Only leads to lepton flavour asymmetry, since $\sum_\alpha C_{\mathrm{DEG},\alpha} = 0.$

Furthermore, for the limit $\lambda \ll \kappa \lesssim 1$ consider subset of two mass-degenerate states. Define $(\hat{Y}_{(23)})_{\alpha i} = (\hat{Y}_D)_{\alpha i} \quad \text{for} \quad i \in \{2,3\}$

For $\lambda = 0$ we only need

$$C^{(23)}_{\mathrm{DEG},\alpha} = i\,\mathrm{Tr}\Big(\Big[\hat{Y}_{(23)}^T\,\hat{Y}_{(23)}^*, \hat{Y}_{(23)}^\dagger\,\hat{Y}_{(23)}\Big]\,\hat{Y}_{(23)}^T\,P_\alpha\,\hat{Y}_{(23)}^*\Big)$$

Clearly, $\sum_\alpha C^{(23)}_{\mathrm{DEG},\alpha} = 0.$

Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Way towards capturing main dependencies analytically

- CP-violating combinations
- washout parameter

Flavoured washout parameter:

$$f_\alpha = \frac{(\hat{Y}_D \hat{Y}_D^\dagger)_{\alpha\alpha}}{\mathrm{Tr}\left(\hat{Y}_D \hat{Y}_D^\dagger\right)} .$$

Example 2: Low-scale seesaw mechanism

[M. Drewes, Y. Georis, CH, J. Klaric ('22)]

Overview over results

Type of mixing pattern	BAU non-zero for $\kappa = 0$?	BAU non-zero for large κ?	Large total mixing angle U^2 possible?
Case 1)	No, see Fig. 4	Yes, see Fig. 4	Yes, for $\cos 2\,\theta_R \approx 0$ see Fig. 9
Case 2), t even	No, see Fig. 12	No, see Fig. 12	No
Case 2), t odd	Yes, for $m_0 \neq 0$ see Fig. 17, plot (a)	Yes, see Fig. 16	Yes, for $\sin 2\,\theta_R \approx 0$ see Fig. 19
Case 3 b.1), m and s even	No, see Fig. 20	No, see Fig. 20	No
Case 3 b.1), m even, s odd	Yes, see Fig. 22 except for strong IO	No, see Fig. 22	Yes, for $\cos 2\,\theta_R \approx 0$ see Fig. 25
Case 3 b.1), m odd, s even	Yes, see Fig. 26 except for strong IO	Yes, see Fig. 26	Yes, for $\cos 2\,\theta_R \approx 0$
Case 3 b.1), m and s odd	No	No	No

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

Leading to charged fermion masses … all consistent with data

$$
\begin{aligned}
m_u &= \left| f_{11}\,\lambda^8 + \mathcal{O}(\lambda^{10}) \right| \langle H_u^0 \rangle \;, \\
m_c &= \left| f_{22}\,\lambda^4 + \left(\frac{f_{12}^2}{2\,f_{22}} - \frac{f_{23} f_{32}}{f_{33}} \right) \lambda^6 + \mathcal{O}(\lambda^8) \right| \langle H_u^0 \rangle \\
m_t &= \left| f_{33} + \frac{f_{23}^2}{2\,f_{33}}\,\lambda^4 + \mathcal{O}(\lambda^8) \right| \langle H_u^0 \rangle \;.
\end{aligned}
$$

$$
\begin{aligned}
m_d &= \left| d_{11}\,\lambda^4 + \mathcal{O}(\lambda^{12}) \right| \langle H_d^0 \rangle \;, \\
m_s &= \left| d_{22}\,\lambda^2 - \frac{d_{23}(d_{22}d_{23} + 2\,d_{32}d_{33})}{2\,d_{33}^2}\,\lambda^6 + \mathcal{O}(\lambda^{10}) \right| \langle H_d^0 \rangle \\
m_b &= \left| d_{33} + \frac{d_{23}^2}{2\,d_{33}}\,\lambda^4 + \mathcal{O}(\lambda^8) \right| \langle H_d^0 \rangle \;,
\end{aligned}
$$

$$
\begin{aligned}
m_e &= \left| e_{11}\,\lambda^4 + o(\lambda^{12}) \right| \langle H_d^0 \rangle \;, \\
m_\mu &= \left| e_{22}\,\lambda^2 - \frac{e_{23}(e_{22}e_{23} + 2\,e_{32}e_{33})}{2\,e_{33}^2}\,\lambda^4 + \mathcal{O}(\lambda^6) \right| \langle H_d^0 \rangle \\
m_\tau &= \left| e_{33} + \frac{e_{23}^2}{2\,e_{33}}\,\lambda^2 + \mathcal{O}(\lambda^4) \right| \langle H_d^0 \rangle \;.
\end{aligned}
$$

Potentially relevant contribution from LQ

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

Leading to quark mixing

$$V_{\rm CKM} = L_u^\dagger L_d =$$

$$\begin{pmatrix} 1 - \frac{f_{12}^2}{2 f_{22}^2} \lambda^2 + \mathcal{O}(\lambda^4) & -\frac{f_{12}}{f_{22}} \lambda + \mathcal{O}(\lambda^3) & \frac{f_{12}(d_{33} f_{23} - d_{23} f_{33})}{d_{33} f_{22} f_{33}} \lambda^3 + \mathcal{O}(\lambda^5) \\ \frac{f_{12}}{f_{22}} \lambda + \mathcal{O}(\lambda^3) & 1 - \frac{f_{12}^2}{2 f_{22}^2} \lambda^2 + \mathcal{O}(\lambda^4) & \left(\frac{d_{23}}{d_{33}} - \frac{f_{23}}{f_{33}}\right) \lambda^2 + \mathcal{O}(\lambda^4) \\ \frac{d_{22} d_{33} f_{13} - d_{12} d_{33} f_{23} - d_{13} d_{22} f_{33} + d_{12} d_{23} f_{33}}{d_{22} d_{33} f_{33}} \lambda^8 + \mathcal{O}(\lambda^9) & -\left(\frac{d_{23}}{d_{33}} - \frac{f_{23}}{f_{33}}\right) \lambda^2 + \mathcal{O}(\lambda^6) & 1 - \frac{(d_{33} f_{23} - d_{23} f_{33})^2}{2 d_{33}^2 f_{33}^2} \lambda^4 + \mathcal{O}(\lambda^8) \end{pmatrix}$$

V_{td} too small

Consequently, also J_{CP} suppressed

Note also strong correlation

$$|V_{ub}| \approx \left| \frac{f_{12}}{f_{22}} \left(\frac{d_{23}}{d_{33}} - \frac{f_{23}}{f_{33}} \right) \right| \lambda^3 \approx |V_{us}|\,|V_{cb}|$$

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

Leading to quark mixing

$$V_{\rm CKM} = L_u^\dagger L_d =$$

$$\begin{pmatrix} 1 - \frac{f_{12}^2}{2 f_{22}^2}\lambda^2 + \mathcal{O}(\lambda^4) & -\frac{f_{12}}{f_{22}}\lambda + \mathcal{O}(\lambda^3) & \frac{f_{12}(d_{33}f_{23}-d_{23}f_{33})}{d_{33}f_{22}f_{33}}\lambda^3 + \mathcal{O}(\lambda^5) \\ \frac{f_{12}}{f_{22}}\lambda + \mathcal{O}(\lambda^3) & 1 - \frac{f_{12}^2}{2 f_{22}^2}\lambda^2 + \mathcal{O}(\lambda^4) & \left(\frac{d_{23}}{d_{33}} - \frac{f_{23}}{f_{33}}\right)\lambda^2 + \mathcal{O}(\lambda^4) \\ \frac{d_{22}d_{33}f_{13}-d_{12}d_{33}f_{23}-d_{13}d_{22}f_{33}+d_{12}d_{23}f_{33}}{d_{22}d_{33}f_{33}}\lambda^8 + \mathcal{O}(\lambda^9) & -\left(\frac{d_{23}}{d_{33}} - \frac{f_{23}}{f_{33}}\right)\lambda^2 + \mathcal{O}(\lambda^6) & 1 - \frac{(d_{33}f_{23}-d_{23}f_{33})^2}{2 d_{33}^2 f_{33}^2}\lambda^4 + \mathcal{O}(\lambda^8) \end{pmatrix}$$

V_{td} too small

Consequently, also $J_{\rm CP}$ suppressed

Note also strong correlation

$$|V_{ub}| \approx \left| \frac{f_{12}}{f_{22}} \left(\frac{d_{23}}{d_{33}} - \frac{f_{23}}{f_{33}}\right)\right| \lambda^3 \approx |V_{us}|\,|V_{cb}|$$

Source of problem

$$M_u = \begin{pmatrix} f_{11}\lambda^8 & f_{12}\lambda^5 & f_{13}\lambda^8 \\ f_{21}\lambda^{10} & f_{22}\lambda^4 & f_{23}\lambda^2 \\ f_{31}\lambda^{12} & f_{32}\lambda^4 & f_{33} \end{pmatrix} \langle H_u^0 \rangle$$

Scenario A

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

Leading to quark mixing

$$V_{\rm CKM} = L_u^\dagger L_d =$$

$$\begin{pmatrix} 1 - \frac{f_{12}^2}{2 f_{22}^2}\lambda^2 + \mathcal{O}(\lambda^4) & -\frac{f_{12}}{f_{22}}\lambda + \mathcal{O}(\lambda^3) & \frac{f_{12}(d_{33}f_{23}-d_{23}f_{33})}{d_{33}f_{22}f_{33}}\lambda^3 + \mathcal{O}(\lambda^5) \\ \frac{f_{12}}{f_{22}}\lambda + \mathcal{O}(\lambda^3) & 1 - \frac{f_{12}^2}{2 f_{22}^2}\lambda^2 + \mathcal{O}(\lambda^4) & \left(\frac{d_{23}}{d_{33}} - \frac{f_{23}}{f_{33}}\right)\lambda^2 + \mathcal{O}(\lambda^4) \\ \frac{d_{22}d_{33}f_{13}-d_{12}d_{33}f_{23}-d_{13}d_{22}f_{33}+d_{12}d_{23}f_{33}}{d_{22}d_{33}f_{33}}\lambda^8 + \mathcal{O}(\lambda^9) & -\left(\frac{d_{23}}{d_{33}} - \frac{f_{23}}{f_{33}}\right)\lambda^2 + \mathcal{O}(\lambda^6) & 1 - \frac{(d_{33}f_{23}-d_{23}f_{33})^2}{2 d_{33}^2 f_{33}^2}\lambda^4 + \mathcal{O}(\lambda^8) \end{pmatrix}$$

V_{td} too small

Consequently, also $J_{\rm CP}$ suppressed

Note also strong correlation

$$|V_{ub}| \approx \left|\frac{f_{12}}{f_{22}}\left(\frac{d_{23}}{d_{33}} - \frac{f_{23}}{f_{33}}\right)\right| \lambda^3 \approx |V_{us}|\,|V_{cb}|$$

Cure — ad hoc contribution … difficult to achieve alone here

$$M_u = \begin{pmatrix} f_{11}\lambda^8 & f_{12}\lambda^5 & \tilde{f}_{13}\lambda^3 \\ f_{21}\lambda^{10} & f_{22}\lambda^4 & f_{23}\lambda^2 \\ f_{31}\lambda^{12} & f_{32}\lambda^4 & f_{33} \end{pmatrix} \langle H_u^0 \rangle$$

Scenario B

Example 3: Model with leptoquark

[I. Bigaran, T. Felkl, CH, M.A. Schmidt ('22)]

Leading to quark mixing

$$V_{\rm CKM} = \begin{pmatrix} 1 - \frac{f_{12}^2}{2\, f_{22}^2}\,\lambda^2 + \mathcal{O}(\lambda^4) & -\frac{f_{12}}{f_{22}}\,\lambda + \mathcal{O}(\lambda^3) & \left(\frac{f_{12}(d_{33}f_{23}-d_{23}f_{33})}{d_{33}f_{22}f_{33}} - \frac{\tilde{f}_{13}}{f_{33}}\right)\lambda^3 + \mathcal{O}(\lambda^5) \\ \frac{f_{12}}{f_{22}}\,\lambda + \mathcal{O}(\lambda^3) & 1 - \frac{f_{12}^2}{2\, f_{22}^2}\,\lambda^2 + \mathcal{O}(\lambda^4) & \left(\frac{d_{23}}{d_{33}} - \frac{f_{23}}{f_{33}}\right)\lambda^2 + \mathcal{O}(\lambda^4) \\ \frac{\tilde{f}_{13}}{f_{33}}\,\lambda^3 + \mathcal{O}(\lambda^7) & -\left(\frac{d_{23}}{d_{33}} - \frac{f_{23}}{f_{33}}\right)\lambda^2 + \mathcal{O}(\lambda^6) & 1 - \frac{(d_{33}f_{23}-d_{23}f_{33})^2}{2\, d_{33}^2 f_{33}^2}\,\lambda^4 + \mathcal{O}(\lambda^6) \end{pmatrix}$$

V_{td} OK, also $J_{\rm CP}$ OK

Also the correlation is relaxed

$$|V_{ub}| \approx \left|\frac{f_{12}}{f_{22}}\left(\frac{d_{23}}{d_{33}} - \frac{f_{23}}{f_{33}}\right) + \frac{\tilde{f}_{13}}{f_{33}}\right|\lambda^3$$

Cure — ad hoc contribution … difficult to achieve alone here

$$M_u = \begin{pmatrix} f_{11}\,\lambda^8 & f_{12}\,\lambda^5 & \tilde{f}_{13}\,\lambda^3 \\ f_{21}\,\lambda^{10} & f_{22}\,\lambda^4 & f_{23}\,\lambda^2 \\ f_{31}\,\lambda^{12} & f_{32}\,\lambda^4 & f_{33} \end{pmatrix}\langle H_u^0\rangle$$