

# TOWARDS A NLO PARTON SHOWER: GAUGE CHOICE



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# DEDUCTOR

In **DEDUCTOR** we did everything at **LO level** that is humanly possible:

- ✓ **All order definition** of parton shower cross sections
  - ✓ Evolution at **density operator** level (*aka* amplitude level evolution)
  - ✓ Multi-variable evolution (**angular ordering**, ...)
- ✓ **Colour evolution** beyond the leading colour approximation
  - ✓ LC+ base approximation
  - ✓ subleading colour treated perturbatively
  - ✓ Fully exponentiated **Glauber gluon** effect
- ✓ Summation of **threshold logarithms**
  - ✓ **PDF factorisation schemes** vs. shower evolution variable
- ✓ Testing **log accuracy** of parton showers **analytically**
- ✗ We haven't implemented spin dependence

***It is time to move on and go beyond the first order!***

# Shower Kernel

At second order level we are not that lucky. The shower kernel is much more complicated:

$$\begin{aligned} \frac{1}{\mu_s^2} S^{(2)}(\mu^2) = & \mathcal{F}(\mu^2) \left( \frac{\partial \mathcal{D}^{(2)}(\mu^2, \mu_s^2)}{\partial \mu_s^2} - \mathcal{D}^{(1)}(\mu^2) \frac{\partial \mathcal{D}^{(1)}(\mu^2, \mu_s^2)}{\partial \mu_s^2} \right)_{\mu_s^2 = \mu^2} \mathcal{F}^{-1}(\mu^2) \\ & - \left( \frac{\partial \mathcal{V}^{(2)}(\mu^2, \mu_s^2)}{\partial \mu_s^2} - \mathcal{V}^{(1)}(\mu^2) \frac{\partial \mathcal{V}^{(1)}(\mu^2, \mu_s^2)}{\partial \mu_s^2} \right)_{\mu_s^2 = \mu^2} \\ & + \left[ \mathcal{V}^{(1)}(\mu^2), \frac{1}{\mu^2} \mathcal{S}^{(1)}(\mu^2) \right] \end{aligned}$$

This is highly non-trivial operator and cancelation of all the singularities in the first term is rather delicate.

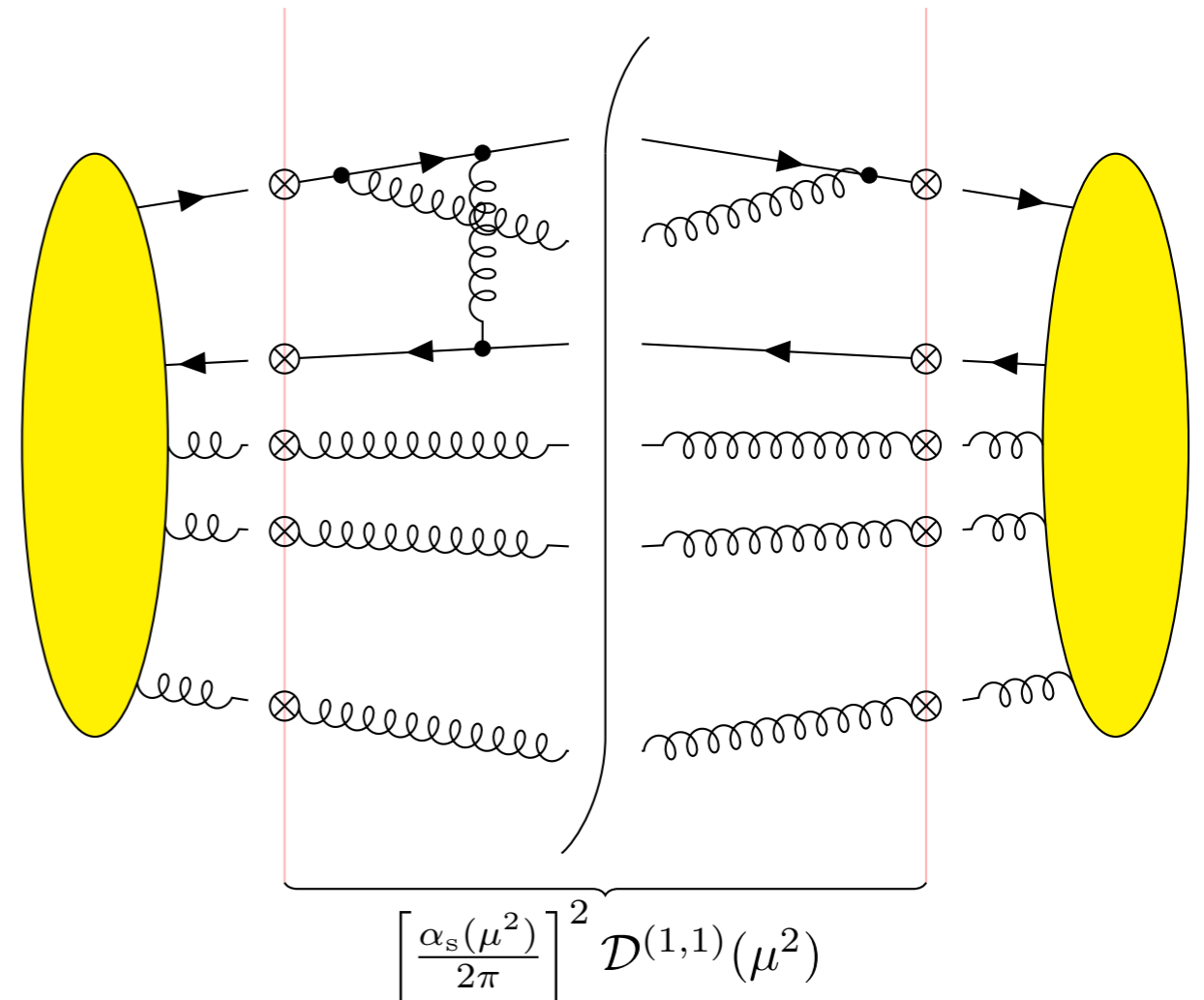
$$\mathcal{D}^{(2)}(\mu^2, \mu_s^2) = \overbrace{\mathcal{D}^{(2,0)}(\mu^2, \mu_s^2)}^{\text{Double real}} + \underbrace{\mathcal{D}^{(1,1)}(\mu^2, \mu_s^2)}_{\text{Real-virtual}} + \overbrace{\mathcal{D}^{(0,2)}(\mu^2, \mu_s^2)}^{\text{Double virtual}}$$

and

$$\mathcal{D}^{(1)}(\mu^2, \mu_s^2) = \overbrace{\mathcal{D}^{(1,0)}(\mu^2, \mu_s^2)}^{\text{Single real}} + \underbrace{\mathcal{D}^{(0,1)}(\mu^2, \mu_s^2)}_{\text{Single virtual}}$$

# Infrared sensitive operator

Consider the momenta coming from the hard part as fixed and on shell.



This gives us an operator as

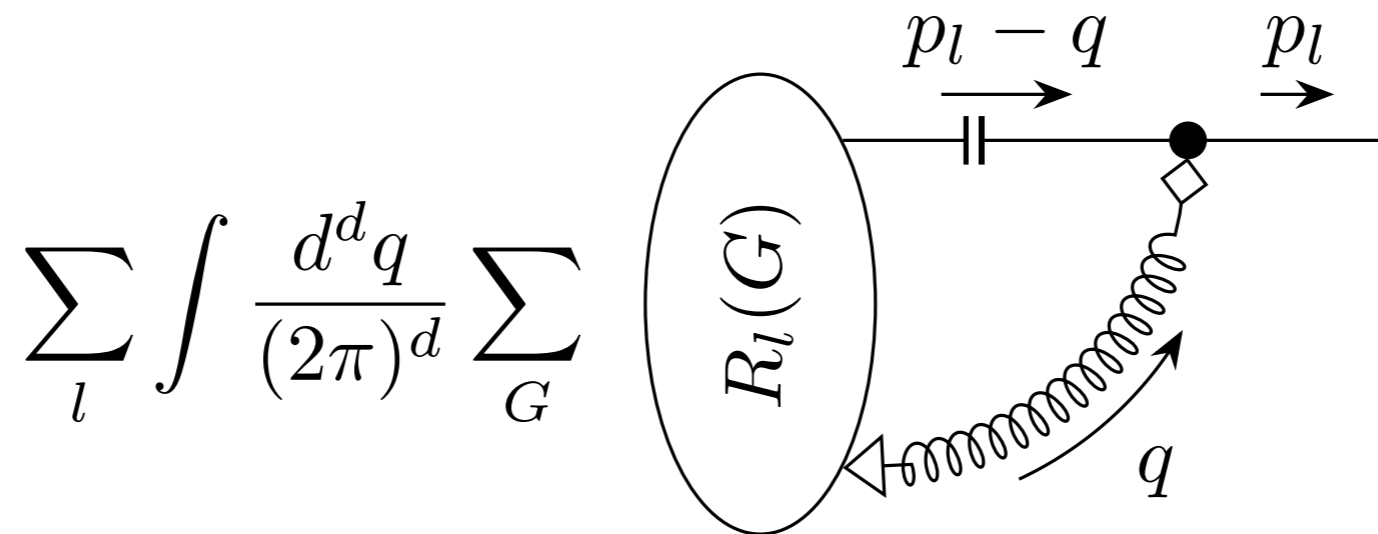
$$\begin{aligned}
 & \left( \{\hat{p}, \hat{f}, \hat{s}, \hat{s}', \hat{c}, \hat{c}'\}_{m+n_R} \mid \rho(\mu^2) \right) \\
 & \sim \frac{1}{m!} \int [d\{p\}_m] \sum_{\{f\}_m} \sum_{\{s, s', c, c'\}_m} \\
 & \quad \times \left( \{\hat{p}, \hat{f}, \hat{s}, \hat{s}', \hat{c}, \hat{c}'\}_{m+n_R} \mid \mathcal{D}(\mu^2) \mid \{p, f, s, s', c, c'\}_m \right) \\
 & \quad \times \left( \{p, f, s, s', c, c'\}_m \mid \rho_{\text{hard}}(\mu^2) \right)
 \end{aligned}$$

# Interpolating Gauge

- ❖ I will describe the **interpolating gauge** invented by Doust (1987) and probably reinvented by Baulieu and Zwanziger (1999) and we also reinvented (2022) before we found it in the literature.
  - Doust wanted to fix the ambiguities of the Coulomb gauge Feynman integrals.
  - Baulieu and Zwanziger wanted to understand confinement.
- ❖ Our interest in simplifying the soft and collinear singularities of QCD amplitudes.
- ❖ This may be useful to **define the infrared sensitive operator** that is need for subtraction terms for fixed order calculation and for splitting operators in parton showers.
- ❖ Our particular interest in defining the splitting operators for a **parton shower at order  $\alpha_s^2$  level**.
- ❖ Interpolating gauge interpolates between **Feynman gauge** (or Lorenz gauge) and **Coulomb gauge**.
- ❖ With our different goal we adopt a different notation and emphasise different features of the gauge.
- ❖ We also explore technical issues in some detail.

# Why not Feynman gauge?

The gluon propagator is very simple in Feynman gauge, but consider the graphs with a virtual gluon that couples to an external leg



- There are **collinear singularities** that give a **logarithmic divergence** from  $q \rightarrow xp_l$ .
- The collinear divergences appear even when the gluon connects to an off-shell internal line in the graph.
- The unphysical polarisations of the gluon causes the problem and we can get rid of the by using the Ward identity.
- This is rather simple at 1-loop level, but higher order level it gets extremely complicated very quickly.
- It might be better if these **unphysical collinear singularities don't occur**.

# Interpolating gauge

⇒ Use a **special reference** frame defined by a light like vector  $n^\mu$ , with  $n^2 = 1$ .

⇒ Define a **tensor**  $h^{\mu\nu}$  as

$$h^{\mu\nu} = g^{\mu\nu} - \left(1 - \frac{1}{v^2}\right) n^\mu n^\nu$$

In the  $\vec{n} = 0$  frame this tensor is

$$h^{\mu\nu} = \begin{pmatrix} 1/v^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

⇒ For any vector  $q^\mu$  we define an **associated vector**  $\tilde{q}^\mu$  by

$$\tilde{q}^\mu = h^{\mu\nu} q_\nu \quad \tilde{q}^\mu = \left(\frac{1}{v^2} E, \vec{q}\right)$$

⇒ Note, in the  $v^2 \rightarrow 0$  limit  $h^{\mu\nu} \rightarrow g^{\mu\nu}$

⇒ Use the gauge fixing condition  $G[A] = 0$  with

$$G[A]_c(x) = \tilde{\partial}_\mu A_c^\mu(x) - \omega_c(x)$$

Compare this to

$$G[A]_c(x) = \partial_\mu A_c^\mu(x) - \omega_c(x)$$

for covariant gauge

# Interpolating gauge

⇒ The gauge fixing Lagrangian with two gauge parameters ( $v, \xi$ ) is

$$\mathcal{L}_{\text{GF}}(x) = -\frac{v^2}{2\xi} (\tilde{\partial}_\mu A_a^\mu(x)) (\tilde{\partial}_\nu A_a^\nu(x))$$

⇒ With this the **gluon propagator** is

$$D^{\mu\nu}(q) = \frac{1}{q^2 + i0} \left[ -g^{\mu\nu} + \frac{q^\mu \tilde{q}^\nu + \tilde{q}^\mu q^\nu}{q \cdot \tilde{q} + i0} - \left(1 + \frac{1}{v^2}\right) \frac{q^\mu q^\nu}{q \cdot \tilde{q} + i0} \right] - \frac{\xi - 1}{v^2} \frac{q^\mu q^\nu}{(q \cdot \tilde{q} + i0)^2}$$

⇒ Usually we choose  $\xi = 1$ .

⇒ The **ghost propagator** is

$$\tilde{D}(q) = \frac{1}{q \cdot \tilde{q} + i0}$$



# Gluon propagator

Introducing the polarisation vectors in the usual way, the gluon propagator can be decomposed as

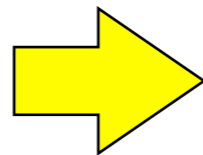
Describes the propagation of the **transversely polarised** gluons  
(T-gluons)

$$D^{\mu\nu}(q) = \overbrace{\frac{1}{q^2 + i0} \sum_{\lambda=1,2} \varepsilon_{\lambda}^{\mu}(q) \varepsilon_{\lambda}^{\nu}(q)^*} + \underbrace{\frac{-\varepsilon_0^{\mu}(q) \varepsilon_0^{\nu}(q)^* + \varepsilon_3^{\mu}(q) \varepsilon_3^{\nu}(q)^*}{q \cdot \tilde{q} + i0}}_{\text{Describes the propagation of the longitudinally polarised gluons (L-gluons)}}$$

Describes the propagation of the **longitudinally polarised** gluons  
(L-gluons)

Note, that  $q \cdot \tilde{q} = \frac{1}{v^2} E^2 - \vec{q}^2$ , and the condition for on-shell propagation is

$$E = \pm v |\vec{q}|$$



L-gluons propagate with **speed  $v$**  in the  $\vec{n} = 0$  frame.

We usually take  $v > 1$ , this means the **L-gluons are tachyonic states**. This is also true for the ghost fields.

# Interpolating gauge

## Why is it called interpolating gauge?

- ❖ If we take  $\nu = 1$ , we get Feynman gauge for  $\xi = 1$  and Lorenz gauge for  $\xi = 0$ .
- ❖ If we take  $\nu \rightarrow \infty$ , we get Coulomb gauge.
- ❖ The L-gluons give the **Coulomb force**, which propagates with **infinite speed** in the  $\vec{n} = 0$  frame.
- ❖ Now the Coulomb gauge is defined as a limit. This way the loop integrals are defined unambiguously.
- ❖ We don't need  $\nu \rightarrow \infty$ ,  $\nu = \sqrt{2}$  is **perfectly fine**.

# Collinear singularities

$$\sum_l \int \frac{d^d q}{(2\pi)^d} \sum_G$$

The diagram illustrates a collinear singularity. It shows a horizontal line representing a fermion or scalar with momentum  $p_l$ . This line is split into two segments: one with momentum  $p_l - q$  and another with momentum  $p_l$ . A wavy gluon line with momentum  $q$  connects the vertex where the line splits to a blob labeled  $R_l(G)$ .

- ❖ **T-gluons do not** give collider singularities except for self-energy insertion on external legs, that is because  $q \cdot \varepsilon_\lambda(q) = 0$ .
- ❖ **L-gluon do not** give collinear singularities because if  $p_l^2 = 0$  and  $q \rightarrow xp_l$ , then  $(p_l - q)^2 \rightarrow 0$ , but  $q \cdot \tilde{q} \neq 0$ .
- ❖ Thus interpolating gauge is **like physical gauge** respect to collinear divergences.

# Soft singularities

$$\sum_{l < k} \int \frac{d^d q}{(2\pi)^d} \text{ (T) }$$

$$\sum_{l < k} \int \frac{d^d q}{(2\pi)^d} \text{ (L) }$$

- ❖ Both **T-gluons** and **L-gluons** create soft divergences ( $q^\mu \rightarrow 0$ ) when they couple to two external legs.
- ❖ These are pure soft divergences, but without collinear divergences.
- ❖ The interpolating gauge looks a powerful tool to disentangle the soft and collinear divergences.

# Renormalization

- ❖ Renormalization works, we did it at 1-loop level and calculated all the renormalisation constants,

$$Z_\psi, Z_A^{\mu\nu}, Z_\eta, Z_g, Z_v, Z_\xi$$

- ❖ The BRST invariance shows that the S-matrix **independent** of the gauge parameters,  $v, \xi, n^\mu$ .

$$Z_A^{\mu\nu}(\alpha_s, v, \xi) = g^{\mu\nu} + \frac{\alpha_s}{4\pi} \frac{1}{\epsilon_{\overline{\text{MS}}}} \left( \left[ \frac{22v^3 + 35v^2 + 20v - 1}{6v(1+v)^2} - \frac{\xi}{2v} \right] C_A - \frac{4}{3} T_R n_f \right) g^{\mu\nu} \\ - \frac{\alpha_s}{4\pi} \frac{1}{\epsilon_{\overline{\text{MS}}}} \frac{4v(1+2v)}{3(1+v)^2} C_A h^{\mu\nu} + \mathcal{O}(\alpha_s^2)$$

$$Z_g(\alpha_s, v, \xi) = 1 - \frac{\alpha_s}{4\pi} \frac{1}{\epsilon_{\overline{\text{MS}}}} \left( \frac{11}{6} C_A - \frac{4}{6} T_R n_f \right) + \mathcal{O}(\alpha_s^2)$$

# Conclusion

- ❖ Interpolating gauge is not useful to calculate exact matrix element, for that purpose I still recommend Feynman gauge.
- ❖ But for defining subtraction terms for fixed order calculations or splitting operators for parton showers or for studying IR structures, it might be a very useful gauge.
- ❖ Our goal is to define a NLO level parton shower. *Stay tuned!*