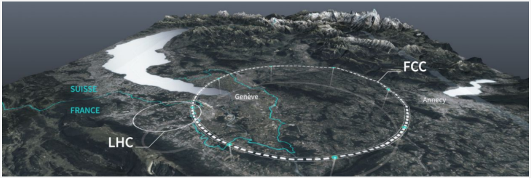
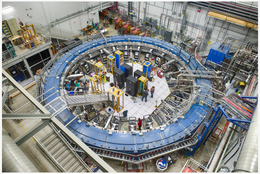
Precision Physics at High Energy Colliders and Low Energy Connections

Janusz Gluza

Epiphany 2024

8 January 2024, Cracow





In memoriam and honour of Staszek Jadach (06.08.1947 – 23.02.2023)

CERN 2019, photo by Tord Riemann

Eur. Phys. J. C 28, 261–278 (2003)
Digital Object Identifier (DOI) 10.1140/epjc/s2003-01146-0

THE EUROPEAN PHYSICAL JOURNAL C

Measuring the FSR-inclusive $\pi^+\pi^-$ cross section

J. Gluza[1,3], A. Hoefer[2], S. Jadach[2], F. Jegerlehner[3]

CERN Yellow Reports: Monographs

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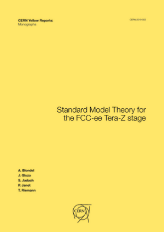
Vol. 3 (2019): Standard Model Theory for the FCC-ee Tera-Z stage



Report on the Mini Workshop Precision EW and QCD Calculations for the FCC Studies: Methods and Tools, 12–13 January 2018, CERN, Geneva.

DOI: https://doi.org/10.23731/CYRM-2019-003

"FCC is a HEP project of the XXI century" (Staszek, ∼2014)

This table on slide 17 is scary!!! greetings, Staszek

From Staszek Jadach
To Janusz Gluza
Date 2016-12-08 19:04

2016_0601_Santander_FCCee_physics_dEnteria.pdf (4.9 MB)

Przerażająca ta tabelka na slajdzie 17 !!! pozdrawiam, staszek

High-precision W, Z, top: FCC-ee uncertainties

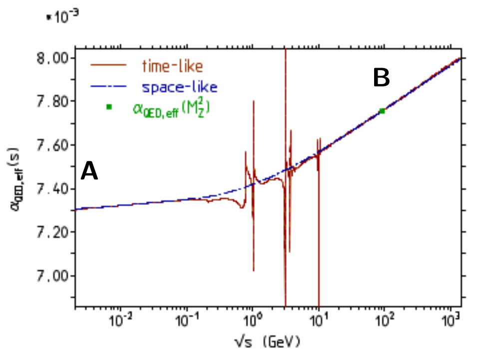
[D.d'E., arXiv:1602.05043]

- Exp. uncertainties (stat. uncert. ~negligible) improved by factors ×3–100:

Observable	Measurement	Current precision	FCC-ee stat.	Possible syst.	Challenge
m_Z (MeV)	Z lineshape	91187.5 ± 2.1	0.005	< 0.1	QED corr.
Γ_Z (MeV)	Z lineshape	2495.2 ± 2.3	0.008	< 0.1	QED corr.
R_ℓ	Z peak	20.767 ± 0.025	0.0001	< 0.001	QED corr.
R_b	Z peak	0.21629 ± 0.00066	0.000003	< 0.00006	$g \to b\bar{b}$
N_ν	Z peak	2.984 ± 0.008	0.00004	0.004	Lumi meas.
N_ν	$e^+e^- \to \gamma Z(\text{inv.})$	2.92 ± 0.05	0.0008	< 0.001	–
$A_{FB}^{\mu\mu}$	Z peak	0.0171 ± 0.0010	0.000004	< 0.00001	E_{beam} meas.
$\alpha_s(m_Z)$	$R_\ell, \sigma_{had}, \Gamma_Z$	0.1190 ± 0.0025	0.000001	0.00015	New physics
$1/\alpha_{QED}(m_Z)$	$A_{FB}^{\mu\mu}$ around Z peak	128.952 ± 0.014	0.004	0.002	EW corr.
m_W (MeV)	WW threshold scan	80385 ± 15	0.3	< 1	QED corr.
$\alpha_s(m_W)$	Γ_W, B_{had}^W	$B_{had}^W = 67.41 \pm 0.27$	0.00018	0.00015	CKM matrix
m_t (MeV)	$t\bar{t}$ threshold scan	173200 ± 900	10	10	QCD
Γ_t (MeV)	$t\bar{t}$ threshold scan	1410^{+290}_{-150}	12	?	$\alpha_s(m_Z)$
y_t	$t\bar{t}$ threshold scan	$\mu = 2.5 \pm 1.05$	13%	?	$\alpha_s(m_Z)$
$F_{1V,2V,1A}^{\gamma t, Z t}$	$d\sigma^{t\bar{t}}/dx\, d\cos(\theta)$	4%–20% (LHC-14 TeV)	(0.1–2.2)%	(0.01–100)%	–

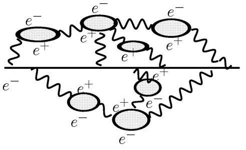
- Theoretical developments needed to match expected experimental uncertainties

$\alpha_{QED}(s)$, vacuum polarisation



A: $\alpha_{QED}(0) \simeq 1/137$

B: $\alpha_{QED}(M_Z^2) \simeq 1/128$

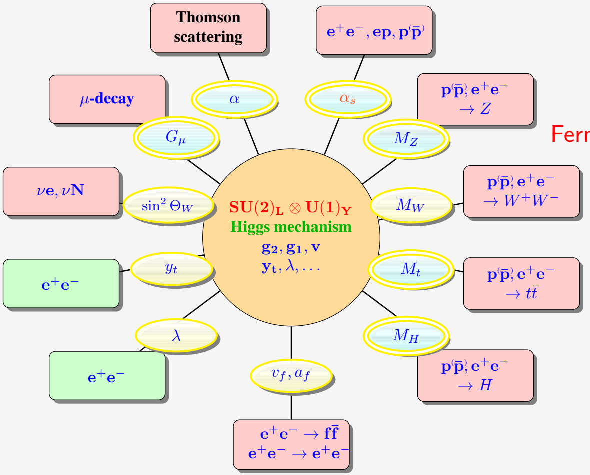


F. Jegerlehner, http://dx.doi.org/10.23731/CYRM-2020-003.9

The effective $\alpha(s)$ in terms of the photon vacuum polarization (VP) self-energy correction $\Delta\alpha(s)$ by

$$\alpha(s) = \frac{\alpha}{1 - \Delta\alpha(s)} \; ; \quad \Delta\alpha(s) = \Delta\alpha_{\text{lep}}(s) + \Delta\alpha^{(5)}_{\text{had}}(s) + \Delta\alpha_{\text{top}}(s)\,.$$

Input and calculated/measured parameters



Fermilab, backup

Fig. from the FCC-ee report 'α_{QED}' by F. Jegerlehner in 1905.05078

Input and calculated/measured parameters

$$\frac{\delta\alpha}{\alpha} \sim 3.6 \times 10^{-9}$$

$$\frac{\delta \mathbf{G}_\mu}{\mathbf{G}_\mu} \sim 8.6 \times 10^{-6}$$

$$\frac{\delta \mathbf{M_Z}}{\mathbf{M_Z}} \sim 2.4 \times 10^{-5}$$

e

$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 0.9 \div 1.6 \times 10^{-4}$$

$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 5 \times 10^{-5} \quad (\texttt{FCC}-\texttt{ee/ILC requirement})$$

$$\longrightarrow \frac{\delta M_W}{M_W} \sim 1.5 \times 10^{-4}, \ \frac{\delta M_H}{M_H} \sim 1.3 \times 10^{-3}, \ \frac{\delta M_t}{M_t} \sim 2.3 \times 10^{-3},$$

Among the basic input parameters $\alpha(M_Z), G_\mu, M_Z$, $\alpha(M_Z)$ *is the least precise and requires a major effort of improvement.*

Input, theoretical and parametric errors,

A. Freitas et al., "Theoretical uncertainties for electroweak and Higgs-boson precision measurements at FCC-ee", https://arxiv.org/abs/1906.05379

Quantity	FCC-ee	Current intrinsic error		Projected intrinsic error (at start of FCC-ee)
M_W [MeV]	0.5–1 ‡	4	$(\alpha^3, \alpha^2\alpha_s)$	1
$\sin^2\theta^\ell_{\rm eff}$ $[10^{-5}]$	0.6	4.5	$(\alpha^3, \alpha^2\alpha_s)$	1.5
Γ_Z [MeV]	0.1	0.4	$(\alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$	0.15
R_b $[10^{-5}]$	6	11	$(\alpha^3, \alpha^2\alpha_s)$	5
R_l $[10^{-3}]$	1	6	$(\alpha^3, \alpha^2\alpha_s)$	1.5

‡ The pure experimental precision on M_W is $\sim 0.5\,\mathrm{MeV}$.

Quantity	FCC-ee	future parametric unc.	Main source
M_W [MeV]	$0.5 - 1$	1 (0.6)	$\delta(\Delta\alpha)$
$\sin^2\theta^\ell_{\rm eff}$ $[10^{-5}]$	0.6	2 (1)	$\delta(\Delta\alpha)$
Γ_Z [MeV]	0.1	0.1 (0.06)	$\delta\alpha_s$
R_b $[10^{-5}]$	6	< 1	$\delta\alpha_s$
R_ℓ $[10^{-3}]$	1	1.3 (0.7)	$\delta\alpha_s$

$$\frac{\delta M_W}{M_W} \sim \frac{1}{2}\frac{\sin^2\Theta_W}{\cos^2\Theta_W - \sin^2\Theta_W}\,\delta\Delta\alpha \sim 0.23\,\delta\Delta\alpha\,,$$

$$\frac{\delta\sin^2\Theta_f}{\sin^2\Theta_f} \sim \frac{\cos^2\Theta_f}{\cos^2\Theta_f - \sin^2\Theta_f}\,\delta\Delta\alpha \sim 1.54\,\delta\Delta\alpha\,.$$

W-boson mass measurements vs. prediction from μ decay

ILC: Baak et al., 1310.6708 FCC-ee: Freitas et al., 1906.05379

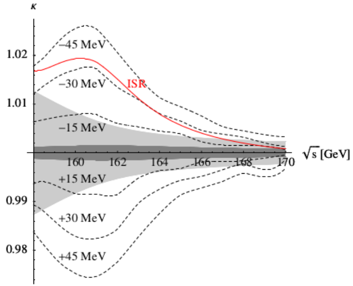
	experimental accuracy				theory uncertainty				
		$\sigma_{\rm WW}$ @ threshold			intrinsic			parametric	
	current	LEP2	ILC	FCC-ee	current	source	prospect	prospect	source
$\Delta M_{\rm W}$[MeV]	13	200	3–6	0.5–1	3	$\alpha^3, \alpha^2\alpha_s$	1	1(0.6)	$\Delta\alpha_{\rm had}$

complicated reconstructions — basically counting experiments — $M_{\rm W}$ calculated from μ decay

Amoroso et al., 2308.09417

Sensitivity of $\sigma_{\rm WW}$ to $M_{\rm W}$:

Beneke et al. '07



$$\kappa = \frac{\sigma_{\rm WW}(s, M_{\rm W} + \delta M_{\rm W})}{\sigma_{\rm WW}(s, M_{\rm W})}$$

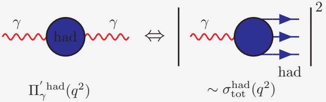
$\Delta\kappa = 0.1\%\,(0.02\%) \leftrightarrow \delta M_{\rm W} = 1.5\,(0.3)\,\mathrm{MeV}$ for $\sqrt{s} = 161\,\mathrm{GeV}$

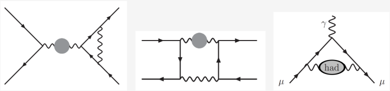
$\Rightarrow$ FCC-ee requires $\Delta_{\rm TH} \sim 0.01{-}0.04\%$ in $\sigma_{\rm WW}$

Shaded areas / ISR curve:
some uncertainties of NLO(EFT) calculation,
improveable via full NLO(ee$\to 4f$) and NNLO(EFT)

R-data evaluation of $\alpha(M_Z^2)$

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}\ ; \quad \Delta\alpha(s) = \Delta\alpha_{\text{lep}}(s) + \Delta\alpha^{(5)}_{\text{had}}(s) + \Delta\alpha_{\text{top}}(s)\,.$$





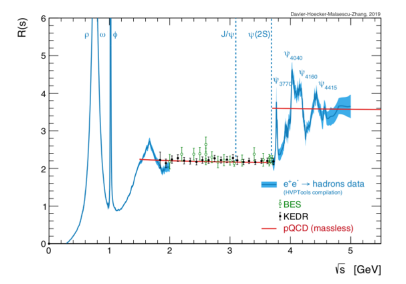
The non-perturbative hadronic piece from the five light quarks $\Delta\alpha^{(5)}_{\text{had}}(s) = -\left(\Pi'_\gamma(s) - \Pi'_\gamma(0)\right)^{(5)}_{\text{had}}$ can be evaluated in terms of $\sigma(e^+e^- \to \text{hadrons})$ data via the dispersion integral **(s can be any, also negative!)**

$$\Delta\alpha^{(5)}_{\text{had}}(s) = -\frac{\alpha\, s}{3\pi}\left(\int_{m^2_{\pi^0}}^{E^2_{\text{cut}}} \mathrm{ds}'\, \frac{\mathrm{R}^{\text{data}}_\gamma(\mathrm{s}')}{\mathrm{s}'(\mathrm{s}'-\mathrm{s})} + \int_{\mathrm{E}^2_{\text{cut}}}^{\infty} \mathrm{ds}'\, \frac{\mathrm{R}^{\text{pQCD}}_\gamma(\mathrm{s}')}{\mathrm{s}'(\mathrm{s}'-\mathrm{s})}\right),$$

$$a^{\text{had}}_\mu = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds\, \frac{R(s)\ \hat{K}(s)}{s^2}\ , \hat{K}(s) \in 0.63 \div 1.$$

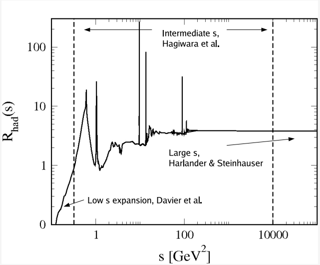
$$R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons}) / \left(\frac{4\pi\alpha^2}{3s}\right)$$

Davier et al

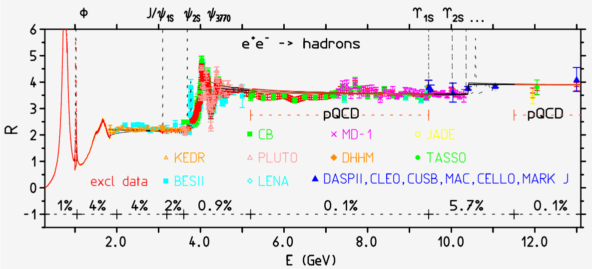


E.g. Parametrization used for Bhabha

Phys.Rev.D 78 (2008) 085019

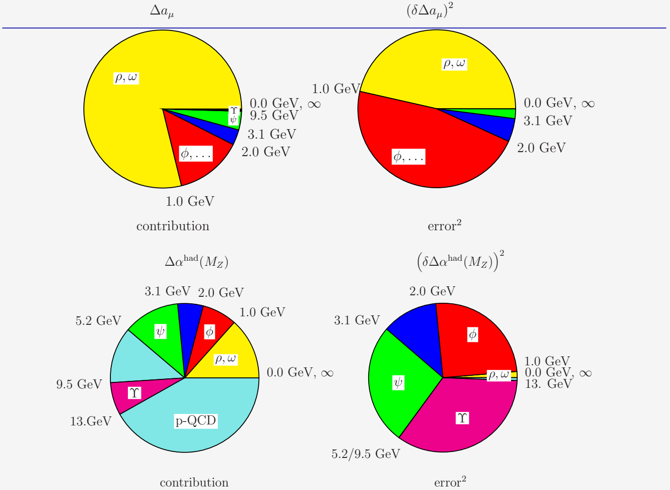


The compilation of $R(s)$-data utilized by F. Jegerlehner for $\Delta\alpha_{\rm had}$.



Nontrivial contributions from different energy regions for $a_\mu^{\rm had}$ and $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$

and square uncertainties



In contrast to the low energy dominated $a_\mu^{\rm had}$, $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ is sensitive to data from much higher energies. *In order to achieve the required factor 5 improvement alternative methods to determine $\Delta\alpha^{(5)}_{\rm had}(s)$ at high energies have to be developed.*

Improvements towards smaller uncertainties: Adler function controlled pQCD

Euclidean split trick:

$$\alpha(M_Z^2) = \alpha^{\mathrm{data}}(-M_0^2) + \Big[\alpha(-M_Z^2) - \alpha(-M_0^2)\Big]^{\mathrm{pQCD}} + \Big[\alpha(M_Z^2) - \alpha(-M_Z^2)\Big]^{\mathrm{pQCD}} .$$

The non-perturbative offset $\alpha^{\mathrm{data}}(-M_0^2)$ may be obtained integrating $R(s)$ data, by choosing $s = -M_0^2$ in DR $\Delta\alpha^{(5)}_{\mathrm{had}}(s)$.
The Adler function is defined as the derivative of the VP function:

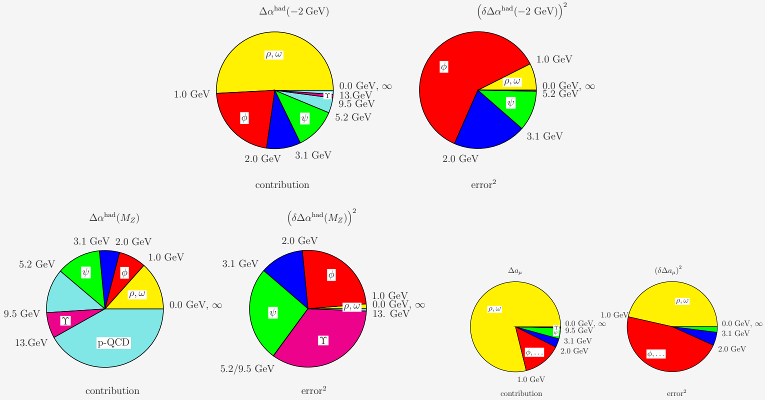
$$R(s) \longrightarrow D(-s) \quad \equiv \quad \frac{3\pi}{\alpha}\, s\, \frac{\mathrm{d}}{\mathrm{ds}} \Delta\alpha_{\mathrm{had}}(s) = -\left(12\pi^2\right)\, s\, \frac{\mathrm{d}\Pi'_\gamma(\mathrm{s})}{\mathrm{ds}}$$

and can be evaluated in terms of -annihilation data by the dispersion integral

$$D(Q^2) \quad = \quad Q^2 \left(\int_{4m_\pi^2}^{E^2_{\mathrm{cut}}} \mathrm{ds}\, \frac{\mathrm{R(s)}^{\mathrm{data}}}{(\mathrm{s}+\mathrm{Q}^2)^2} + \int_{\mathrm{E}^2_{\mathrm{cut}}}^{\infty} \mathrm{ds}\, \frac{\mathrm{R}^{\mathrm{pQCD}}(\mathrm{s})}{(\mathrm{s}+\mathrm{Q}^2)^2} \right) .$$

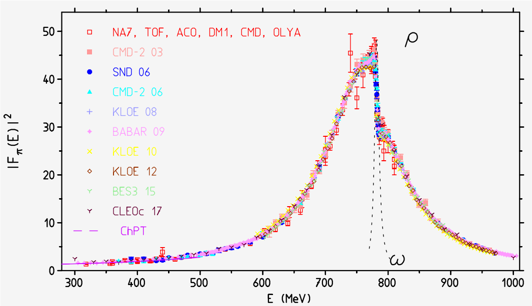
It is a finite object not subject to renormalization and it tends to a constant in the high energies limit, where it is perfectly perturbative.

Contributions and square errors from e^+e^- data ranges and form pQCD to $\Delta\alpha^{(5)}_{\text{had}}(-M_0^2)$ vs. $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$.



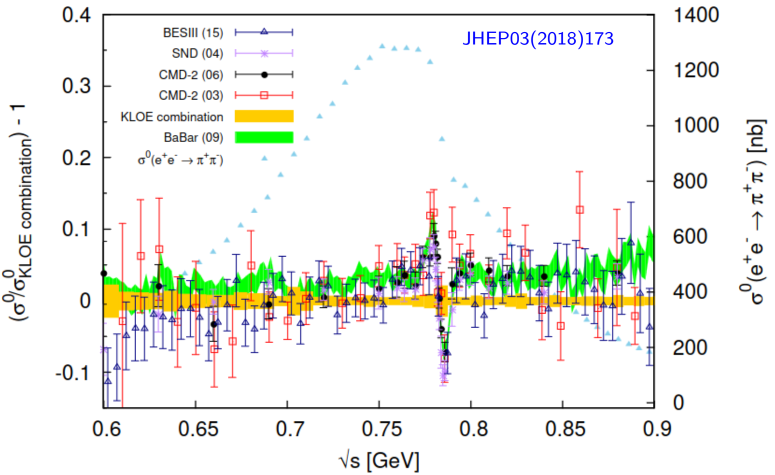
- *Profit gained from the Euclidean split trick:* The profile of the offset α at M_0 for α much similar to the profile found for the hadronic contribution to a_μ.
- Thus, *improving a_μ^{had} automatically lead to an improvement of $\Delta\alpha^{(5)}_{\text{had}}(-M_0^2)$.*
- *$\sigma(e^+e^- \to \pi^+\pi^-)$ cross section gives more than 50% to total HVP contribution to a_μ*

A piece of low energy mess: $\rho - \omega$



The low energy tail of R is provided by $\pi^+\pi^-$ production data.

$$R(s) = \frac{1}{4}\,\beta_\pi^3\,|F_\pi^{(0)}(s)|^2\,,\ \beta_\pi = (1-4m_\pi^2/s)^{1/2}$$



(a) KLOE combination vs. other experiments

The biggest difference between KLOE and BABAR measurements, amounts there to about 2%. It goes even up to 10% around the narrow ω resonance For higher $\pi^+\pi^-$ invariant masses (at 0.9 GeV) the difference raises to 5%.

PHOKHARA MC generator

EVA: $e^+e^- \to \pi^+\pi^-\gamma$

- tagged photon ($\theta_\gamma > \theta_{cut}$)
- ISR at LO + Structure Function
- FSR: point-like pions

[Binner et al.]

$e^+e^- \to 4\pi + \gamma$

- ISR at LO + Structure Function

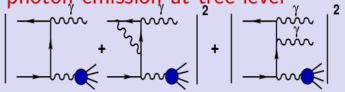
[Czyż, Kühn,2000]

F. Campanario, H.C., J. Gluza,
A. Grzelińska, M. Gunia, P. Kisza,
J. H. Kühn, E. Nowak-Kubat, T. Riemann,
G. Rodrigo, Sz. Tracz, A. Wapienik,
V. Yundin, D. Zhuridov

PHOKHARA 10.0: $\pi^+\pi^-, \mu^+\mu^-,$ 4π, $\bar{N}N$, 3π, KK, $\Lambda\bar{\Lambda}$, $P\gamma$

$J/\psi, \psi(2S), \chi_{c_1}, \chi_{c_2}$

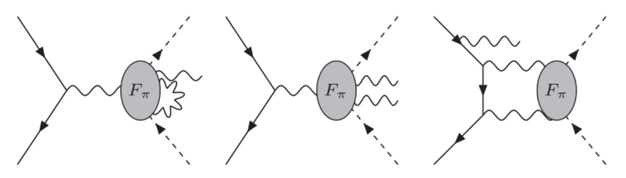
- **ISR at NLO:** virtual corrections to one photon events and two photon emission at tree level

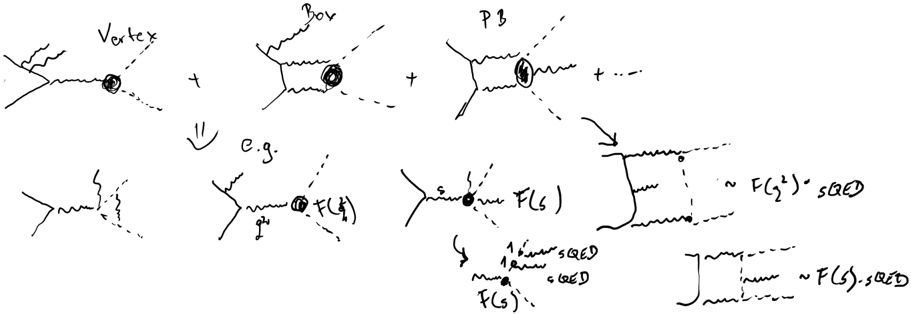


- FSR at NLO: $\pi^+\pi^-$, $\mu^+\mu^-$, K^+K^-, $\bar{p}p$
- tagged or untagged photons
- $e^+e^- \to hadrons$ ($muons$) ISR at NNLO
- Modular structure

http://ific.uv.es/∼rodrigo/phokhara

"Standard model radiative corrections in the pion form factor measurements do not explain the a_μ anomaly",
F. Campanario et al, PRD100,076004(2019)

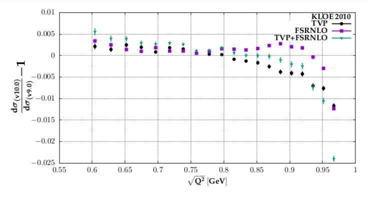


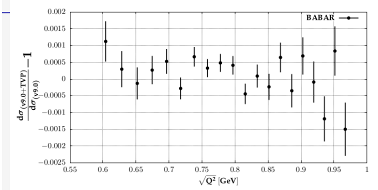


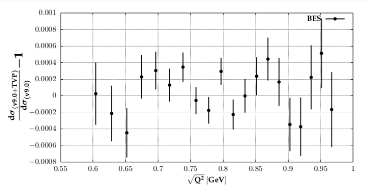
sQED + form factors: FSR at NLO and pentaboxes tested and implemented to Phokhara10.0

http://ific.uv.es/∼rodrigo/phokhara

NLO pentabox corrections, results for KLOE, BABAR and BESS







- Missing NLO radiative corrections cannot be the source of the discrepancies between the different extractions of the pion form factor performed by BaBar, BES and KLOE.
- They cannot be the origin of the discrepancy between the experimental measurement and the SM prediction of a_μ (too small).

Phokhara, status

PHOKHARA

radiative return at flavour factories

Physics	Electron--positron annihilation into hadrons plus an energetic photon from initial state radiation (ISR) allows the hadronic cross-section to be measured over a wide range of energies at high luminosity flavour factories [DAPHNE, CESR, PEP-II, KEK-B, Super-KEKB, BESIII].
Content	**PHOKHARA** is a Monte Carlo event generator which simulates this process at the next-to-leading order (NLO) accuracy. This includes virtual and soft photon corrections to one photon emission events and the emission of two real hard photons.
Downloads	**VERSION 10.0 (October 2020):** Includes complete NLO radiative corrections for the extraction of the **pion form factor**. The new implementation is described in detail in Phys. Rev. D100 (2019) no.7, 076004 [arXiv:1903.10197 hep-ph]. • manual [PDF], source [.tar.gz]

Forthcoming features	• Further updates are not expected.

Comment on BaBar arXiv:2308.05233

"Measurement of additional radiation in the initial-state-radiation processes $e^+e^- \to \mu^+\mu^-\gamma$ and $e^+e^- \to \pi^+\pi^-\gamma$ at BABAR"

Category	$\mu\mu$ $m_{\pi\pi} < 1.4\,\mathrm{GeV}/c^2$	$\pi\pi$ $0.6 < m_{\pi\pi} < 0.9\,\mathrm{GeV}/c^2$
LO	0.7716(4)(14)	0.7839(5)(12)
NLO SA-ISR	0.1469(3)(36)	0.1401(2)(16)
NLO LA-ISR	0.0340(2)(9)	0.0338(2)(9)
NLO ISR	0.1809(4)(35)	0.1739(3)(20)
NLO FSR	0.0137(2)(7)	0.0100(1)(16)
NNLO ISR [a]	0.0309(2)(38)	0.0310(2)(39)
NNLO FSR [b]	0.00275(6)(9)	0.00194(12)(50)
NNLO 2LA [c]	0.00103(3)(1)	0.00066(4)(4)

[a] NNLO ISR = 2SA-ISR or SA-ISR + LA-ISR
[b] NNLO FSR = SA-ISR + LA-FSR
[c] NNLO 2LA = 2LA-ISR, LA-ISR + LA-FSR or 2LA-FSR

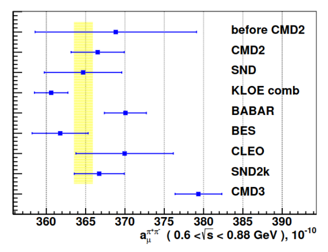
NNLO effects visible.
Comparisons with Phokhara, however

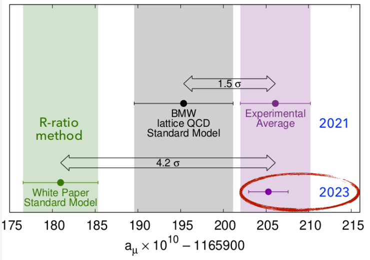
- The event selections used in arXiv:2308.05233 require to have at least 2 hard photons in the final state
- The matrix elements in Phokhara for $e^+e^- \to \pi^+\pi^-\gamma\gamma$ and $e^+e^- \to \mu^+\mu^-\gamma\gamma$ are calculated at LO , so no surprise the accuracy is not high

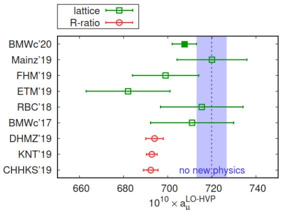
CMD3, new $\pi^+\pi^-$ results, latice QCD, smaller tensions

CMD3: https://arxiv.org/abs/2302.08834

"The CMD-3 result reduces the tension between the experimental value of the a_μ and its Standard Model prediction."







KKMC-ee, Phokhara, ...

Staszek Jadach e-Print: hep-ph/0506180 [hep-ph]

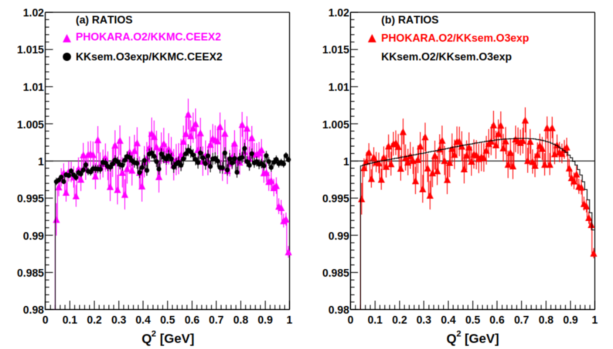
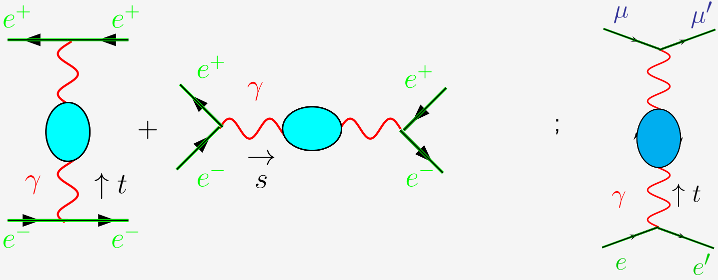


Fig. 1. Muon pair mass (square) spectrum in case of ISR only. $\sqrt{s} = 1.01942$ GeV.

- Agreement to within 0.3% with KKMC.
- Phokhara has no exponentiation, difference for high Q^2

Improvements: $\Delta\alpha_{\rm had}(-Q^2)$ and the low energy $\alpha(t)$



- independent $\Delta\alpha_{\rm had}(-Q^2)$ and $\alpha(-Q^2)$ determination (the number at $Q \sim 2.5$ GeV);
- $\alpha(-Q^2)$ via $\mu^- e^-$-scattering in the `MUonE` *project at CERN*
- NNLO corrections mandatory (good progress)

Summary on α

Precision in $\alpha(M_Z^2)$:

present	direct	1.7×10^{-4}	
	Adler	1.2×10^{-4}	
future	Adler QCD 0.2%	5.4×10^{-5}	
	Adler QCD 0.1%	3.9×10^{-5}	
future	via $A_{\mathrm{FB}}^{\mu\mu}$ off Z	$3 \quad \times 10^{-5}$	*Janot:2015gjr*

Hadronic uncertainty $\delta\Delta\alpha_{\mathrm{had}}(\sqrt{t})$ †

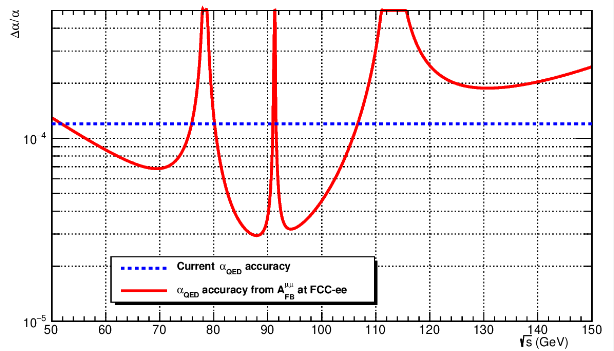
$\sqrt{s}$	$\sqrt{t}$	1996*	present	FCC–ee expected**
M_Z	3.5 GeV	0.040%	0.013%	$0.6 10^{-4}$
350 GeV	13 GeV		$1.2 10^{-4}$	$2.4 10^{-4}$

* *Jadach:1996gu,Arbuzov:1996eq*, ** *Jadach:2018jjo*

† The estimates are based on expected improvements possible for $\Delta\alpha_{\mathrm{had}}(-Q^2)$ in the appropriate energy ranges, centered at $\sqrt{t}$,

F. Jegerlehner, 1905.05078.

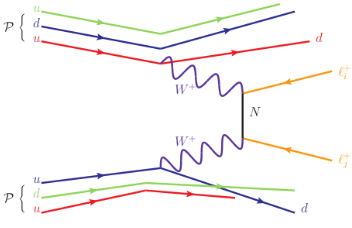
$e^+e^- \to \mu^+\mu^-$, $A_{FB}^{\mu\mu}$ and $\delta\alpha(s)$, *Janot:2015gjr*



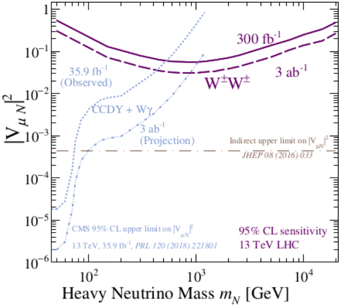
The best accuracy is obtained for one year of running either just below or just above the Z pole, at 87.9 and 94.3 GeV, respectively.

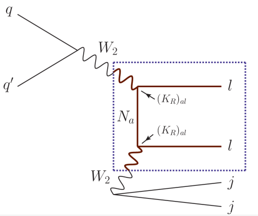
Another aspects of high-low energy connections: rare processes

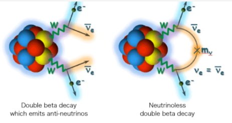
R. Ruiz et al JHEP02(2015)072

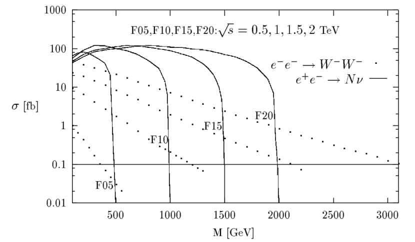


(a)



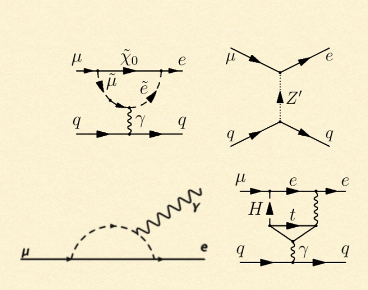






JG et al Phys.Rev.D52(1995)6238 JHEP10(2019)093

Complementarity: low energies, LFV: $\mu \to e\gamma$, conversion $\mu \to e$



$m_\mu \sim 200\ m_e$

$R^{\mu\to e} < 7 \cdot 10^{-13}$, expected precision improvement by four (!) orders

Sensitivity to new effects: $\sim$ 10 000 TeV!

Another aspects of high-low energy connections: rare processes

Process	Present Limits	Expected Limits	Experiments
$\mu^+ \to e^+\gamma$	$< 4.2 \times 10^{-13}$	5×10^{-14}	MEG II
$\mu^+ \to e^+e^-e^+$	$< 1.0 \times 10^{-12}$	10^{-16}	Mu3e
$\mu^-\mathrm{Al} \to e^-\mathrm{Al}$	$< 6.1 \times 10^{-13}$	10^{-17}	Mu2e, COMET
$\mu^-\mathrm{Si/C} \to e^-\mathrm{Si/C}$	$-$	5×10^{-14}	DeeMe
$\tau \to e\gamma$	$< 3.3 \times 10^{-8}$	5×10^{-9}	Belle II, FC
$\tau \to \mu\gamma$	$< 4.4 \times 10^{-8}$	10^{-9}	Belle II, FC
$\tau \to eee$	$< 2.7 \times 10^{-8}$	5×10^{-10}	Belle II, FC
$\tau \to \mu\mu\mu$	$< 2.1 \times 10^{-8}$	5×10^{-10}	Belle II, FC
$\tau \to e$ had	$< 1.8 \times 10^{-8}$	3×10^{-10}	Belle II, FC
had $\to \mu e$	$< 4.7 \times 10^{-12}$	10^{-12}	NA62
$h \to e\mu$	$< 3.5 \times 10^{-4}$	3×10^{-5}	HL-LHC, FC
$h \to \tau\mu$	$< 2.5 \times 10^{-3}$	3×10^{-4}	HL-LHC, FC
$h \to \tau e$	$< 6.1 \times 10^{-3}$	3×10^{-4}	HL-LHC, FC

Summary

From a bird's view:

- Precision goals for SM theory high-energy studies at present and future colliders need progress in precision low energy input.
- *Improved input parameters by roughly one order of magnitude needed* ($\alpha, \alpha_{\mathrm{s}}$, $m_W, m_H, m_t, \Delta\alpha_{\mathrm{had}}$, ...), e.g. $\alpha_{\mathrm{QED}}(M_Z)$ roughly five times, apparently $\Delta\alpha_{\mathrm{had}}$ (pions etc.).
- Such an approach, based on the SM theory, is independent of other efforts based on global BSM studies and, in my opinion, is crucial for spotting BSM effects.

The progress is great![1]

Thank you for your attention.

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[1] *'At each meeting it always seems to me that very little progress is made. Nevertheless, if you look ever any reasonable length of time, a few years say, you find a fantastic progress and it is hard to understand how that can happen at the same time that nothing is happening in anyone moment (Zeno's paradox).'* - R.P. Feynman

Backup slides

$\delta\alpha_{QED}(0)$: 81 parts per trillion

REPORT

Measurement of the fine-structure constant as a test of the Standard Model

Richard H. Parker[1,*], Chenghui Yu[1,*], Weicheng Zhong[1], Brian Estey[1], Holger Müller[1,2,†]
+ See all authors and affiliations

Science 13 Apr 2018:
Vol. 360, Issue 6385, pp. 191-195
DOI: 10.1126/science.aap7706

Article | Published: 02 December 2020

Determination of the fine-structure constant with an accuracy of 81 parts per trillion

Léo Morel, Zhibin Yao, Pierre Cladé & Saïda Guellati-Khélifa

Nature **588**, 61–65(2020) | Cite this article

$\alpha^{-1}(Cs)$= 137.035 999 046(27)

$\alpha^{-1}(Rb)$= 137.035 999 206(11)

$\alpha^{-1}(a_e)$= 137.035 999 139(31)

Remarks:
(i) new result - deviation from SM in the same direction as in $(g-2)_\mu$,
(ii) substantial disagreement with Cs ($\sim 5.4\sigma$).

Over 2 decades of improvements
https://www.nature.com/articles/s41586-020-2964-7 [02 December 2020]

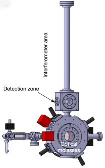
$\alpha_{QED}(0)$ and BSM

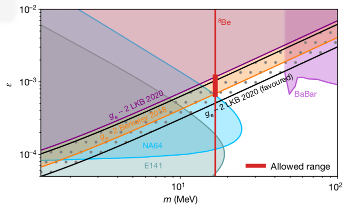
Table 1 | Error budget on α

Source	Correction ($\times 10^{-11}$)	Relative uncertainty ($\times 10^{-11}$)
Gravity gradient	−0.6	0.1
Alignment of the beams	0.5	0.5
Coriolis acceleration		1.2
Frequencies of the lasers		0.3
Wave-front curvature	0.6	0.3
Wave-front distortion	3.9	1.9
Gouy phase	108.2	5.4
Residual Raman light shift	2.3	2.3
Index of refraction	0	<0.1
Internal interaction	0	<0.1
Light shift (two-photon transition)	−11.0	2.3
Second-order Zeeman effect		0.1
Phase shifts in Raman phase-lock loop	−39.8	0.6
Global systematic effects	**64.2**	**6.8**
Statistical uncertainty		**2.4**
Relative mass of ^{87}Rb[a]: 86.9091805310(60)		3.5
Relative mass of the electron[b]: 5.48579909065(16) × 10^{-4}		1.5
Rydberg constant[b]: 10,973,731.568160(21) m^{-1}		0.1
Total: α^{-1} = 137.035999206(11)		**8.1**

For each systematic effect, more discussion can be found in Methods.
[a]From ref. 13.
[b]From https://pml.nist.gov/cuu/Constants/.

Morel, L., Yao, Z., Cladé, P. et al. Determination of the fine-structure constant with an accuracy of 81 parts per trillion. Nature 588, 61–65 (2020).



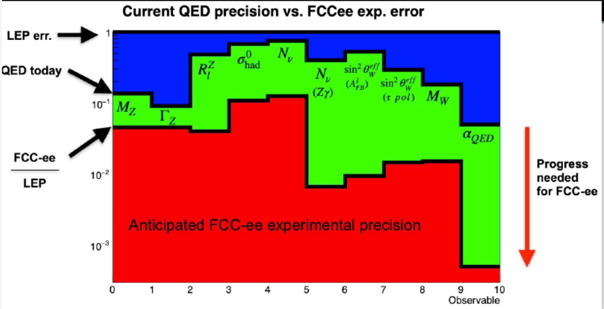


Substructure: $\alpha_{QED}(0) \longrightarrow$ modification of $\delta a_e \simeq m_e/m^*$
Excluded (light, states, weakly coupled):

$$m^* < 520 \text{ GeV}.$$

Future δa_e improvement by an order of magnitude in next years, sensitivity similar as for $(g-2)_\mu$.

QED challenges beyond LEP: the FCC-ee example



The present precision of QED theoretical predictions would severely limit the analysis of precise measurements at FCC-ee.

To properly confront the data with theoretical predictions of similar accuracy demands a huge progress in precision calculations!

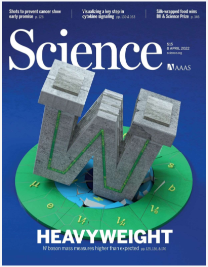
Needed factor 6-200 improvement with respect to LEP.

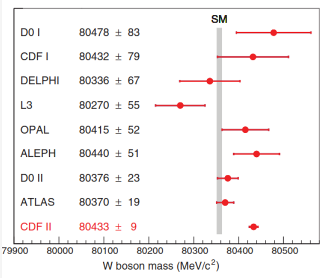
arXiv:1903.09895

(Jadach&Skrzypek)

EW pseudo-observables EWPOs

Observable EWPO	Source LEP	Err.{QED} LEP	Stat[Syst] FCC-ee	LEP / FCC-ee	main development to be done
M_Z [MeV]	Z linesh.	2.1{0.3}	0.005[0.1]	$3\times3^\star$	light fermion pairs
Γ_Z [MeV]	Z linesh.	2.1{0.2}	0.008[0.1]	$2\times3^\star$	fermion pairs
$\sigma^0_{\rm had}$ [pb]	$\sigma^0_{\rm had}$	37{25}	0.1[4.0]	$6\times3^\star$	better lumi MC
$R_l^Z\times10^3$	$\sigma(M_Z)$	25{12}	0.06[1.0]	$12\times3^{\star\star}$	better FSR
$N_\nu\times10^3$	$\sigma(M_Z)$	8{6}	0.005[1.0]	$6\times3^{\star\star}$	CEEX in lumi MC
$N_\nu\times10^3$	$Z\gamma$	150{60}	0.8[< 1]	$60\times3^{\star\star}$	$\mathcal{O}(\alpha^2)$ for $Z\gamma$
$\sin^2\theta_W^{eff}\times10^5$	$A_{FB}^{lept.}$	53{28}	0.3[0.5]	$55\times3^{\star\star}$	h.o. and EWPOs
$\sin^2\theta_W^{eff}\times10^5$	$\langle\mathcal{P}_\tau\rangle, A_{FB}^{pol,\tau}$	41{12}	0.6[< 0.6]	$20\times3^{\star\star}$	better τ decay MC
M_W [MeV]	mass rec.	33{6}	0.3[?.?]	$20\times3^{\star\star}$	$\mathcal{O}(\alpha)$, FSR_{exp}
M_W [MeV]	threshold	200{30}	0.5[0.3]	$100\times3^{\star\star\star}$	$\mathcal{O}(\alpha^2)$ at thresh.
$A_{FB,\mu}^{M_Z\pm3.5\text{GeV}}\times10^5$	$\frac{d\sigma}{d\cos\theta}$	2000{100}	1.0[0.3]	$100\times3^{\star\star\star}$	improved IFI





Science 376 (2022) 6589, 170-176

$$
\begin{aligned}
\mathrm{SM}: M_W &= 80357 \pm 6\ \mathrm{MeV},\ (\mathrm{PDG2020}) \\
\mathrm{Global}: M_W &= 80379 \pm 12\ \mathrm{MeV},\ (\mathrm{PDG2020}) \\
\mathrm{CDFII}: M_W &= 80433.5 \pm 9.4\ \mathrm{MeV} \\
\mathrm{ATLAS\ 2023}^*: M_W &= 80360 \pm 16\ \mathrm{MeV} \\
\texttt{FCC-ee}\ \mathrm{forecast}: M_W &= X \pm \mathbf{0.4 - 1}\ \mathrm{MeV!}
\end{aligned}
$$

* CDF-II data on W mass are in contradiction with global electroweak e^+e^- fits and recent ATLAS LHC analysis, with systematic uncertainty improved by 15% ATLAS:2023fsi and optimised reconstruction of the W-boson transverse momentum ATLAS:2023llf.

Input and calculated/measured parameters

Experimental values:

$$\begin{aligned}
\hat{\alpha} &= 1/137.0359895(61),\ \gamma^* \to e^+e^- \\
\hat{G}_F &= 1.16639(1)\times 10^{-5}\,\mathrm{GeV}^{-2}\ \text{muon decay} \\
\hat{m}_Z &= 91.1875 \pm 0.0021\,\mathrm{GeV} \\
\hat{m}_W &= 80.426 \pm 0.034\,\mathrm{GeV} \\
\hat{s}^2_{\rm eff} &= 0.23150 \pm 0.00016, \text{effective} \sin^2\theta_{\rm W}, A_{LR} \equiv \frac{(1/2-\hat{s}^2_{\rm eff})^2 - \hat{s}^4_{\rm eff}}{(1/2-\hat{s}^2_{\rm eff})^2 + \hat{s}^4_{\rm eff}} \\
\hat{\Gamma}_{l^+l^-} &= 83.984 \pm 0.086\,\mathrm{MeV}
\end{aligned}$$

$$\left\{\begin{array}{l}
\mathbf{g}(= e/s_W)\ SU(2) \\
\mathbf{g'}(e/c_W)\ U(1)_Y \\
\mathbf{v}\ \text{VEV},
\end{array}\right. \longrightarrow \left\{\begin{array}{l}
\hat{\alpha} = \frac{e^2}{4\pi} \\
\hat{G}_F = \frac{1}{\sqrt{2}v^2} \\
\hat{m}_Z^2 = \frac{e^2v^2}{4s^2c^2} \\
\hat{m}_W^2 = \frac{e^2v^2}{4s^2} \\
\hat{s}^2_{\rm eff} = s^2 \\
\hat{\Gamma}_{l^+l^-} = \frac{v}{96\pi}\frac{e^3}{s^3c^3}\left[\left(-\frac{1}{2}+2s^2\right)^2 + \frac{1}{4}\right]
\end{array}\right.$$

Introduction to Precision Electroweak Analysis by J. Welss, 0512342

Shaping the SM, tree level estimates

In terms of $\hat{\alpha}, \hat{G}_F$ and $\hat{m}_Z$

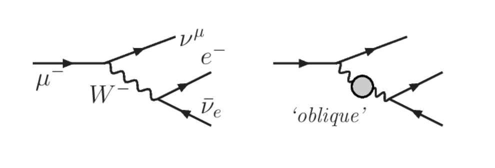
$$\hat{m}_W^2 \quad = \quad \pi\sqrt{2}\hat{G}_F^{-1}\hat{\alpha}\left(1-\sqrt{1-\frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}}\right)^{-1}$$

$$\hat{s}_{\rm eff}^2\hat{c}_{\rm eff}^2 = \frac{\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2} \quad \equiv \quad \hat{s}_{\rm eff}^2 = \frac{1}{2} - \frac{1}{2}\sqrt{1-\frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}}$$

$$\hat{\Gamma}_{l^+l^-} \quad = \quad \frac{\sqrt{2}\hat{G}_F\hat{m}_Z^3}{12\pi}\left\{\left(\frac{1}{2}-\sqrt{1-\frac{4\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_Z^2}}\right)^2+\frac{1}{4}\right\}$$

$$\begin{aligned} Prediction: \hat{m}_W &= 80.939 \pm 0.003\,\mathrm{GeV}\ 15\sigma\ \text{away} \\ Prediction: \hat{s}_{\rm eff}^2 &= 0.21215 \pm 0.00003\ 120\sigma\ \text{away} \\ Prediction: \hat{\Gamma}_{l^+l^-} &= 84.843 \pm 0.012\,\mathrm{MeV}\ 10\sigma\ \text{away} \end{aligned}$$

Shaping SM, oblique corrections also not sufficient



$$\tau_\mu^{-1} = \frac{\hat{G}_F^2 m_\mu^5}{192\pi^3} K(\alpha, m_e, m_\mu, m_W)$$

$$\begin{aligned} \frac{(\hat{G}_F)^{\mathrm{th}}}{\sqrt{2}} &= \frac{g^2}{8m_W^2}\left[1 + i\Pi_{WW}(q^2)\left(\frac{-i}{q^2 - m_W^2}\right)\right]_{q\to 0} \\ &= \frac{1}{2v^2}\left[1 - \frac{\Pi_{WW}(0)}{m_W^2}\right]. \end{aligned}$$

Primary role of SM radiative corrections, F. Jegerlehner, in 1905.05078

$$\sin^2\Theta_i\,\cos^2\Theta_i = \frac{\pi\,\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}\,\frac{1}{1-\Delta r_i} \quad \Delta r_i = \Delta r_i(\alpha, G_\mu, M_Z, m_H, m_{f\neq t}, m_t)\,,$$

$$\Delta r_i = \Delta\alpha - \frac{c_W^2}{s_W^2}\,\Delta\rho + \Delta r_{i\,\mathrm{reminder}}\,,$$

$$\Delta\rho = \frac{3\,m_t^2\,\sqrt{2}G_\mu}{16\,\pi^2}$$

$$\hat{\alpha}(m_Z) = \frac{\hat{\alpha}}{1-\Delta\alpha(m_Z)} = \frac{e^2}{4\pi}\left[1+\frac{\Pi_{\gamma\gamma}(m_Z)}{m_Z^2}\right] \sim 128 \text{ (137 at the Thomson limit)}$$

Still, well visible disagreement between SM prediction and experiment for EWPOs without subleading SM corrections, and only with the leading corrections $\Delta\alpha(m_Z)$ and $\Delta\rho$.

$r_{i\,\mathrm{reminder}}$ **matters!** (see a backup slide)

F. Jegerlehner, in 1905.05078

Example: the W and Z mass from $\alpha(M_Z)$, G_μ and $\sin^2\Theta_{\ell\,\mathrm{eff}}$:
(i) $\sin^2\Theta_W = 1 - M_W^2/M_Z^2$,
$\sin^2\theta_{\ell,\mathrm{eff}}(M_Z) = \left(1 + \frac{\cos^2\Theta_W}{\sin^2\Theta_W}\,\Delta\rho\right)\sin^2\Theta_W$,

$$\Delta\rho = \frac{3\,M_t^2\,\sqrt{2}G_\mu}{16\,\pi^2}\ ;\ \ M_t = 173 \pm 0.4\ GeV$$

The iterative solution with input $\sin^2\theta_{\ell,\mathrm{eff}}(M_Z) \equiv (1 - v_\ell/a_\ell)/4 = 0.23148$ (EXP!) is $\sin^2\Theta_W = 0.22426$.
(ii) $M_W^{\mathrm{exp}} = 80.379 \pm 0.012$ GeV ; $M_Z^{\mathrm{exp}} = 91.1876 \pm 0.0021\ \mathrm{GeV}$,
$\longrightarrow 1 - \mathrm{M_W^2/M_Z^2} = 0.22263.$
Predicting then the masses we have

$$M_W = \frac{A_0}{\sin^2\Theta_W}\ ;\ \ A_0 = \sqrt{\frac{\pi\alpha}{\sqrt{2}G_\mu}}\ ;\ \ M_Z = \frac{M_W}{\cos\Theta_W}$$

where, including photon VP correction $\alpha^{-1}(M_Z) = 128.953 \pm 0.016$. For the W, Z mass we then get

$$M_W^{\mathrm{the}} = 81.1636 \pm 0.0346\ \text{GeV}\ ;\ \ M_Z^{\mathrm{the}} = 92.1484 \pm 0.0264\ \text{GeV}\,.$$

Deviations (errors added in quadrature): $W:\ \ 23\,\sigma\ ;\ \ Z:\ \ 36\,\sigma$

A few sample precision quantities of interest for the FCC-ee program

Quantity	Current precision	FCC-ee target precision	Required theory input	Available calc.	Needed theory improvement*
m_Z	2.1 MeV	0.1 MeV **0.1 MeV**	non-resonant $e^+e^- \to f\bar{f}$, initial-state radiation (ISR)	NLO, ISR logs up to 6th order	NNLO for $e^+e^- \to f\bar{f}$
Γ_Z	2.3 MeV	0.1 MeV **0.4** MeV			
$\sin^2\theta^{\ell}_{\rm eff}$	1.6×10^{-4}	0.6×10^{-5} $\mathbf{4.5} \times 10^{-5}$			
m_W	12 MeV	0.4 MeV **4** MeV	lineshape of $e^+e^- \to WW$ near threshold	NLO ($ee \to 4f$ or EFT framework)	NNLO for $ee \to WW$, $W \to ff$ in EFT setup
HZZ coupling	—	0.2% **3 %**	cross-sect. for $e^+e^- \to HZ$	NLO + NNLO QCD	NNLO electroweak
m_t	>100 MeV	17 MeV **50** MeV	threshold scan $e^+e^- \to t\bar{t}$	N^{3}LO QCD, NNLO EW, resummations up to NNLL	Matching fixed orders with resummations, merging with MC, α_s (input)

Theory: 1906.05379, 2106.11802

1956, 1-loop, Behrends, Finkelstein Sirlin

43 y

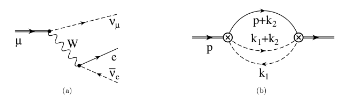
1999, 2-loops, van Ritbergen, Stuart

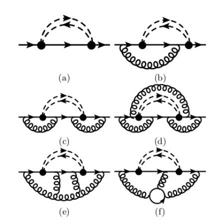
22 y

2021, 3-loops

?

Quantity	Required theory input	Available calc.	Needed theory improvement*
Γ_Z $\sin^2\theta^\ell_{\rm eff}$	vertex corrections for $Z \to f\bar{f}$	NNLO + partial higher orders	N^{3}LO EW + partial higher orders
m_W	SM corrections to the muon decay rate	NNLO + partial higher orders	N^{3}LO EW + partial higher orders





Important (3-loop) step since 1999 (van Ritbergen & Stuart).

$$\Delta\tau_\mu(\alpha^3) \quad = \quad (9 \pm 1) \times 10^8 \;\; \mu s,$$

$$\tau_\mu^{exp} \quad = \quad 2.1969811 \pm 0.0000022 \; \mu s.$$

M. Fael, K. Schönwald, and M. Steinhauser, Third order corrections to the semileptonic $b \to c$ and the muon decays, PRD'2021, arXiv:2011.13654
M. Czakon, A. Czarnecki, and M. Dowling, Three-loop corrections to the muon and heavy quark decay rates, PRD'2021, arXiv:2104.05804

A. Blondel, P. Janot, 2106.13885

Table 3 Measurement of selected precision measurements at FCC-ee, compared with present precision. Statistical errors are indicated in boed phase. The systematic uncertainties are initial estimates, aim is to improve down to statistical errors. This set of measurements, together with those of the Higgs properties, achieves indirect sensitivity to new physics up to a scale Λ of 70 TeV in a description with dim 6 operators, and possibly much higher in specific new physics (non-decoupling) models

Observable	Present value ± error	FCC-ee stat.	FCC-ee syst.	Comment and leading exp. error
m_Z (keV)	91186700 ± 2200	**4**	100	From Z line shape scan
				Beam energy calibration
Γ_Z (keV)	2495200 ± 2300	**4**	25	From Z line shape scan
				Beam energy calibration
$\sin^2\theta_W^{\rm eff} (\times 10^6)$	231480 ± 160	**2**	2.4	from $A_{FB}^{\mu\mu}$ at Z peak
				Beam energy calibration
$1/\alpha_{\rm QED}(m_Z^2)(\times 10^3)$	128952 ± 14	**3**	Small	From $A_{FB}^{\mu\mu}$ off peak
				QED&EW errors dominate
R_ℓ^Z $(\times 10^3)$	20767 ± 25	**0.06**	0.2–1	Ratio of hadrons to leptons
				Acceptance for leptons
$\alpha_s(m_Z^2)$ $(\times 10^4)$	1196 ± 30	**0.1**	0.4–1.6	From R_ℓ^Z above
$\sigma_{\rm had}^0$ $(\times 10^3)$ (nb)	41541 ± 37	**0.1**	4	Peak hadronic cross section
				Luminosity measurement
$N_\nu (\times 10^3)$	2996 ± 7	**0.005**	1	Z peak cross sections
				Luminosity measurement
R_b $(\times 10^6)$	216290 ± 660	**0.3**	< 60	Ratio of $b\bar{b}$ to hadrons

Gluza

Future: W, t, H

- $e^+e^- \to W^+W^-$ at 161 GeV: $\delta m_W^{exp} = 0.5 \div 1$ MeV.
  Challenge to get the same TH error:
  $$\text{NNLO } e^+e^- \to 4f.$$

- $e^+e^- \to t\bar{t}$ at 350 GeV: $\delta {m_t}^{exp} =$ 17 MeV
  Big challenge for theory, today > 100 MeV, future projection $\leq$ 50 MeV:
  $\sim$ 10 MeV unc. from mass def.;
  $\sim$ 15 MeV from α_s unc. to threshold mass def.;
  ~ 30 MeV - h. orders resummation

- $e^+e^- \to HZ$ at 240 GeV: Kinematic constraint fits with $Z \to ll$ and $H \to bb$, ...,
  $m_H = 125.35$ GeV ± 150 MeV [link CMS], $\Gamma_H = 4.1^{5.1}_{4.0}$ MeV, $\Gamma_H <$ 13 MeV at 95 % C.L., 1901.00174
  $\delta m_H^{exp} =$ 10 MeV; Theory errors subdominant.

Monte Carlo generators (**not discussed!**) 'QED challenges at FCC-ee precision measurements', S. Jadach and M. Skrzypek, Eur.Phys.J.C 79 (2019) 9, 756 1903.09895