Spin effects in τ-lepton pair induced by anomalous magnetic and electric dipole moments

A. Yu. Korchin

Institute of Nuclear Physics PAN, Cracow, Poland
&
Kharkiv Institute of Physics and Technology
V.N. Karazin Kharkiv National University, Ukraine

XXX Cracow EPIPHANY Conference
on Precision Physics at High Energy Colliders,
8 – 12 January 2024

Outline of the talk

- Anomalous magnetic dipole moment (AMDM) and electric dipole moment (EDM) of a fermion
- τ-lepton AMDM and EDM: theory and experiment
- Weak magnetic and electric form-factors/moments
- Formalism of spin correlations in $e^-e^+ \to \tau^-\tau^+$ and $q\,\bar{q} \to \tau^-\tau^+$ reactions
- Spin correlations in two-photon production of $\tau^-\tau^+$ pair
- Examples of spin effects in the τ-pair production

This work is done in collaboration with Swagato Banerjee, Elzbieta Richter-Was and Zbigniew Was, arXiv: 2307.03526 [hep-ph], Phys. Rev. D, to be published

AMDM and EDM of a fermion

Definitions of the electromagnetic dipole moments of a fermion f with magnetic field $\vec{\mathcal{B}}$ and electric field $\vec{\mathcal{E}}$:

$$H_M = -\vec{\mu}_f\,\vec{\mathcal{B}} = -\frac{g_f}{2}\mu_B\,\vec{\sigma}\,\vec{\mathcal{B}} \equiv -(1+a_f)\,\mu_B\,\vec{\sigma}\,\vec{\mathcal{B}}$$

$$H_E = -\vec{d}_f\,\vec{\mathcal{E}} = -\big(\frac{2m_f}{e}\,d_f\big)\mu_B\,\vec{\sigma}\,\vec{\mathcal{E}}$$

where $\mu_B = e/(2m_f)$ is the Bohr magneton in units $\hbar = c = 1$, g_f is gyromagnetic factor, a_f is AMDM and d_f is EDM.
The Hamiltonian H_E is P, T and also CP violating.

The electromagnetic vertex γff in a covariant form is

$$\Gamma^\mu = -ieQ_f\Big\{F_1(s)\gamma^\mu + \frac{\sigma^{\mu\nu}q_\nu}{2m_f}\big[i\,F_2(s) + \gamma_5\,F_3(s)\big]\Big\}$$

in terms of the Dirac ($F_1(s)$), Pauli ($F_2(s)$) and electric ($F_3(s)$) form factors ($s = q^2$, q is the photon four-momentum and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$).

At the real-photon point the form factors are normalized as:

$$F_1(0) = 1, \quad F_2(0) = \frac{1}{2}(g_f - 2) \equiv a_f, \quad F_3(0) = \frac{2m_f}{eQ_f}d_f$$

Motivations to study AMDM of τ lepton

In the Standard Model (SM) a_τ is calculated with a very high accuracy [e.g., S. Eidelman, M. Passera, 2007]:

$$a_{QED}(3\,loops) = 117324(2) \times 10^{-8},$$
$$a_{EW}(1\,loop) = 47.4(5) \times 10^{-8},$$
$$a_{hadron+Light-by-Light} = 350.1(4.8) \times 10^{-8}$$
$$a_\tau^{(SM)} = a_{QED} + a_{EW} + a_{hadron+Light-by-Light} = 0.00117721(5)$$

The largest theoretical uncertainty comes from the **hadron contribution**.

Effects of New Physics (NP), which arise due to new heavy particles in the loops, are expected to be

$$a_\tau^{(NP)} = \mathcal{C}\,\frac{m_\tau^2}{\Lambda^2}$$

[e.g. W. Marciano, 1994, 1995], where Λ is the scale of NP and $\mathcal{C}$ is a constant. Depending on an assumption, $\mathcal{C} \sim \mathcal{O}(1)$, $\sim \mathcal{O}(\alpha/\pi)$, or even $\sim \mathcal{O}((\alpha/\pi)^2)$.

Compared to the muon, effects of NP for τ lepton can be enhanced by a factor of $m_\tau^2/m_\mu^2 \approx 280$.

Magnetic dipole moment of the τ: experiment

No direct measurement for the τ has been performed because of its very short lifetime $\tau = 2.903(5) \times 10^{-13}\,\mathrm{s}$. This does not allow applying methods used in the electron and muon "$g-2$" experiments.

The limit on a_τ was obtained by DELPHI collaboration in 2003 from the $e^+e^- \to e^+e^-\tau^+\tau^-$ total cross section at LEP2. At 95 % CL

$$a_\tau = (-0.052, +0.013), \quad or \quad a_\tau = -0.018(17)$$

Recently ATLAS Collaboration presented new constraints [PRL 131, 151802 (2023)] from the $Pb + Pb \to Pb\,(\gamma\gamma \to \tau^-\tau^+)\,Pb$ collisions at $\sqrt{s_{NN}} = 5.02$ TeV:

$$-0.057 < a_\tau < 0.024$$

and also CMS Collaboration [PRL 131, 151803 (2023)] has presented the value:

$$a_\tau = 0.001^{+0.055}_{-0.089}$$

The experimental uncertainties are much larger than the SM prediction $a_\tau^{SM} = 0.00117721(5)$.

Electric dipole moment of the τ: theory and experiment

EDM of a lepton can take nonzero values only if the parity P, time reversal T, and CP symmetries are violated. This induces great interest to EDM of the τ lepton.

In SM, the lepton EDM is extremely small because it originates from the 4-loop diagrams [I. Khriplovich, M. Pospelov, 1991], and additionally due to the smallness of CP violation in the CKM matrix. It remains very small even in minimal extensions of SM: Majorana neutrinos, $m_\nu \neq 0$, mixing in lepton sector, ...

One can obtain estimation for tau lepton:

$$d_\tau^{(SM)} \lesssim 3.5 \cdot 10^{-35}\, e \cdot cm$$

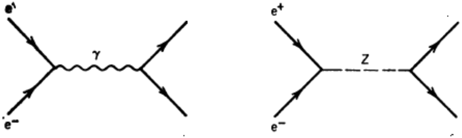
The experimental constraints are obtained at the KEKB e^+e^- collider [Belle Collaboration, JHEP 04 (2022) 110], after collecting 833 fb^{-1} of data at the $\Upsilon(4S)$, $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$ and $\Upsilon(5S)$ resonances:

$$\mathrm{Re}\, d_\tau = (-0.62 \pm 0.63) \times 10^{-17} \mathrm{e} \cdot \mathrm{cm}, \quad \mathrm{Im}\, d_\tau = (-0.40 \pm 0.32) \times 10^{-17} \mathrm{e} \cdot \mathrm{cm}$$

which are 17 orders of magnitude (!) off the SM estimate.

In any case extremely small SM value of d_τ is hardly reachable in experiments. Therefore observation of τ EDM in experiment will be indication of CP violation beyond the SM.

At high energies, Z-boson interaction with fermions becomes important



The $Z\tau^-\tau^+$ vertex has the structure: SM + Dipole moments

$$\Gamma^\mu_Z(q) = -i\frac{g_Z}{2}\left\{\gamma^\mu(v_\tau - \gamma_5 a_\tau) + \frac{\sigma^{\mu\nu}q_\nu}{2m_\tau}\left[iX(s) + Y(s)\gamma_5\right]\right\},$$

where $g_Z = e/(s_W c_W) = 2M_Z(\sqrt{2}G_F)^{1/2}$, $s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$, θ_W is the weak mixing angle and G_F is Fermi constant, $v_\tau = -1/2 + 2s_W^2$ and $a_\tau = -1/2$.

$X(s)$ is weak anomalous magnetic (CP conserving) form-factor, while $Y(s)$ is weak electric (CP violating) form-factor.
On the Z mass shell, i.e. $s = M_Z^2$, they are related to the weak dipole moments

$$X(M_Z^2) = a_w, \quad Y(M_Z^2) = \frac{2m_\tau}{e}d_w$$

Spin effects in e^-e^+ and $q\bar{q}$ production of τ-lepton pair

Consider electron-positron or quark-antiquark annihilation to a pair of **polarized** τ **leptons**

$$f(k_1) + \bar{f}(k_2) \to \tau^-(p_-) + \tau^+(p_+), \qquad f = (\ell,\, q)$$

with the polarization four-vectors of the τ^- and τ^+ in their rest frames:

$$s^-_{rest} = (0,\, \vec{s}^{\,-}), \quad \textit{and} \quad s^+_{rest} = (0,\, \vec{s}^{\,+}), \qquad \textit{where} \quad \vec{s} \equiv \frac{2}{\hbar}\langle \vec{S} \rangle \textit{ is averaged double spin}$$

In the center-of-mass (CM) frame the coordinate system is defined with OZ axis parallel to the τ^- momentum $\vec{p}$, and plane XOZ is spanned on $\vec{p}$ and e^- momentum $\vec{k}$, and OY axis is along $\vec{p} \times \vec{k}$. Then

$$p_- = (E, \vec{p}), \qquad p_+ = (E,\, -\vec{p}), \qquad \vec{p} = (0,\, 0,\, p),$$
$$k_1 = (E,\, \vec{k}), \qquad k_2 = (E,\, -\vec{k}), \qquad \vec{k} = (E\,\sin(\theta),\, 0,\, E\,\cos(\theta))$$

where θ is the scattering angle.

The four-vectors of τ polarizations are boosted to the $\tau^-\tau^+$ CM frame:

$$s^-_{CM} = \left(\frac{\vec{p}\vec{s}^{\,-}}{m_\tau},\, \vec{s}^{\,-} + \frac{\vec{p}(\vec{p}\vec{s}^{\,-})}{m_\tau(m_\tau + E)}\right), \qquad s^+_{CM} = \left(-\frac{\vec{p}\vec{s}^{\,+}}{m_\tau},\, \vec{s}^{\,+} + \frac{\vec{p}(\vec{p}\vec{s}^{\,+})}{m_\tau(m_\tau + E)}\right)$$

Spin effects in e^-e^+ and $q\bar{q}$ production of τ pairs

Then the cross section in the CM frame can be expressed through the $\tau^\mp$ polarizations:

$$\frac{d\sigma}{d\Omega}(f\,\bar{f}\to\tau^-\tau^+) = \frac{d\sigma}{d\Omega}(f\,\bar{f}\to\tau^-\tau^+)\Big|_{\rm unpol}$$
$$\times\frac{1}{4}\Big(1+\sum_{i=1}^{3} r_{i,4}\,s_i^- + \sum_{j=1}^{3} r_{4,j}\,s_j^+ + \sum_{i,j=1}^{3} r_{i,j}\,s_i^-\,s_j^+\Big), \qquad (i,j=1,\,2,\,3=x,\,y,\,z)$$

16 coefficients can carry information on dipole moments of the τ lepton:
(i) polarizations of τ leptons $\mathcal{P}_i(\tau^-)=r_{i,4}$ and $\mathcal{P}_j(\tau^+)=r_{4,j}$,
(ii) 9 elements $r_{i,j}$ called spin-correlation elements,
(iii) element R_{44}
Here $r_{i,j}\equiv R_{i,j}/R_{44}$, $r_{i,4}\equiv R_{i,4}/R_{44}$, $r_{4,j}\equiv R_{4,j}/R_{44}$, where R_{44} determines the unpolarized cross section:

$$\frac{d\sigma}{d\Omega}(f\,\bar{f}\to\tau^-\tau^+)\Big|_{\rm unpol} = \frac{\beta}{16\pi^2 s}\,R_{44},$$

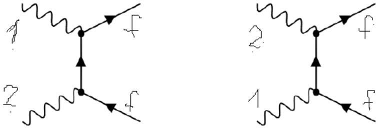
where $\beta=\sqrt{1-1/\gamma^2}$ is velocity of τ and $\gamma=E/m_\tau$ is Lorentz factor.

On the next stage, the τ decays are included, and polarization vectors s_i^- and s_j^+ are replaced by the so-called polarimetric vectors h_i^- and h_j^+, which depend on the τ-decay matrix elements.

Spin effects in two-photon production of polarized τ leptons

Similar approach can be used for production of τ pairs in photon-photon collisions

$$\gamma(k_1) + \gamma(k_2) \to \tau^-(p_-) + \tau^+(p_+), \qquad (\textit{for almost real photons } k_1^2 \approx 0,\ k_2^2 \approx 0)$$



$$\frac{d\sigma}{d\Omega}(\gamma\,\gamma \to \tau^-\tau^+) = \frac{d\sigma}{d\Omega}(\gamma\gamma \to \tau^-\tau^+)\Big|_{\rm unpol}\,\frac{1}{4}\Big(1 + \sum_{i,j=1}^{3} r_{i,j}^{\gamma\gamma}\, s_i^-\, s_j^+\Big)$$

with spin correlation matrix $r_{i,j}^{\gamma\gamma} \equiv R_{i,j}^{\gamma\gamma}/R_{44}^{\gamma\gamma}$ and unpolarized cross section

$$\frac{d\sigma}{d\Omega}(\gamma\gamma \to \tau^-\tau^+)\Big|_{\rm unpol} = \frac{\beta}{16\pi^2 s}\, R_{44}^{\gamma\gamma}.$$

There are no single τ polarizations, i.e. $\vec{\mathcal{P}}(\tau^\mp) = 0$ as long as $k^2 = 0$ and DM are real.

Spin correlations are expected to be more sensitive to the dipole moments than the cross section for unpolarized tau's, and can possibly reduce limits on AMDM a_τ.

EDM contribution to two-photon production of τ leptons

As an example, we show EDM contribution $\sim d_\tau$ to the spin-spin correlation matrix. This contribution is completely separated from the rest of the terms.
It is of transverse – normal-to-reaction-plane $\hat{x}\hat{y}$, and normal-to-reaction-plane – longitudinal $\hat{y}\hat{z}$ type:

$$
\begin{aligned}
|\mathcal{M}|^2_{\mathrm{EDM}} &= \frac{e^4\,\beta\,F_3(0)}{4(1-\beta^2\cos^2\theta)^2}\Big[(s_x^- s_y^+ - s_y^- s_x^+)(\beta^2\cos(4\theta)+4\cos(2\theta)+15\beta^2-20) \\
&\quad +2(s_y^- s_z^+ - s_z^- s_y^+)\,\gamma(\beta^2\cos(2\theta)-3\beta^2+2)\,\sin(2\theta)\Big].
\end{aligned}
$$

This can be convenient for studying observables sensitive to EDM of the τ in the $\gamma\gamma \to \tau^-\tau^+$ reaction.

Radiative corrections

For $e^- e^+ \to \tau^- \tau^+$ and for $q\bar{q} \to \tau^- \tau^+$ parton-level processes at the LHC, at high energies about $\sqrt{s} \sim 160$ GeV, the contributions from WW- and ZZ-box diagrams need to be taken into account. These contributions are the part of radiative corrections.

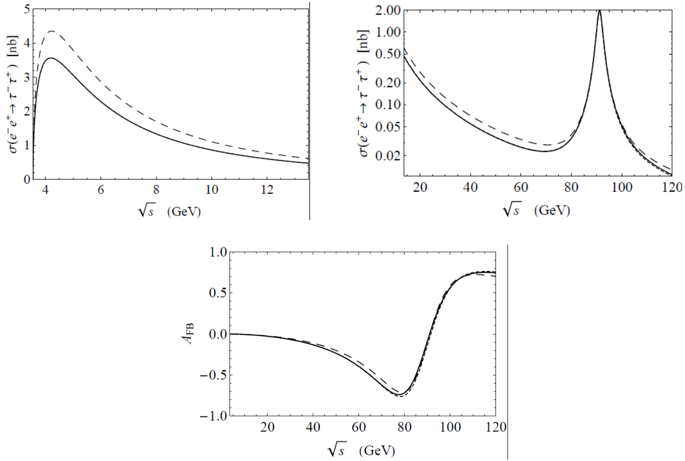
These corrections are included in the Improved Born Approximation (IBA) [D. Bardin et al. Comp. Phys. Comm. 133, 229 (2001); E. Richter-Was, Z. Was. Eur. Phys. J. C74, 3177 (2014); A. Arbuzov, S. Jadach, Z. Was et al. Comp. Phys. Comm., 260, 107734 (2021)]

The IBA takes into account:

- corrections to the photon propagator from vacuum-polarization loops,
- corrections to Z-boson propagator and couplings,
- contributions from WW- and ZZ-box diagrams,
- mixed $\mathcal{O}(\alpha\alpha_s,\, \alpha\alpha_s^2, \ldots)$ corrections originating from gluon insertions in the self-energy loop diagrams.

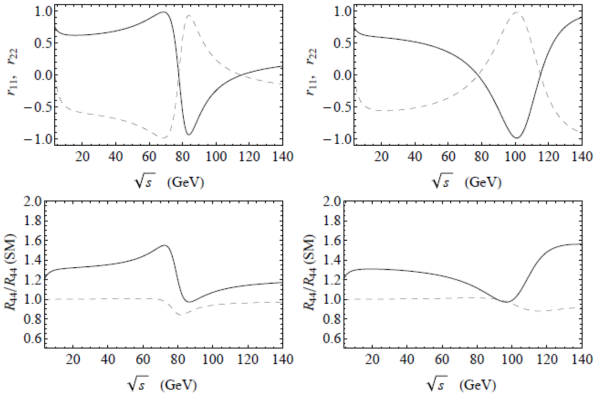
All necessary form-factors are available in `Dizet` library [D. Bardin et al. Comp. Phys. Comm. 133, 229 (2001)].

Cross section and FB asymmetry in $e^-e^+ \to \tau^-\tau^+$

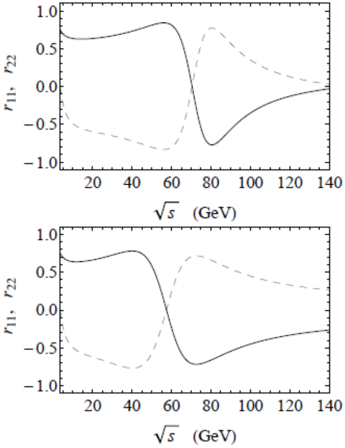


Cross section and forward-backward asymmetry in $e^-e^+ \to \tau^-\tau^+$ near threshold (for Belle) and at large $\sqrt{s}$ (for LHC)
Solid lines - Standard Model, dashed - magnetic FF $Re(F_2) = 0.1$, dotted - weak magnetic FF $Re(X) = 0.1$

Transverse spin correlations in $e^-e^+ \to \tau^-\tau^+$

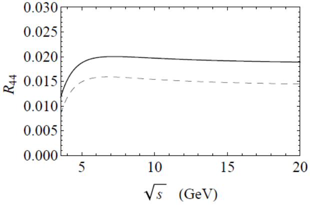


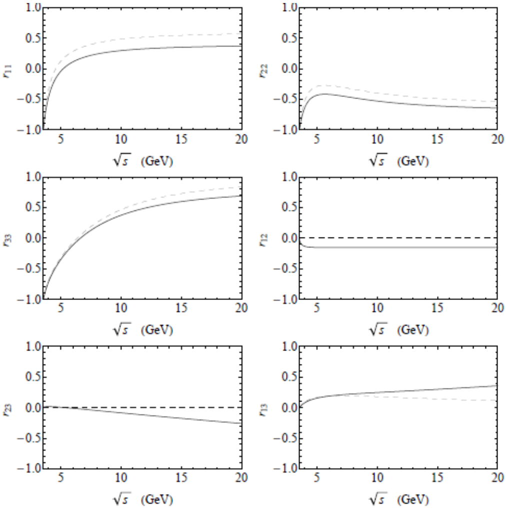
Upper plots: **transverse spin-correlation elements: r_{11} (solid) and r_{22} (dashed) for e^-e^+ initial state, in the Standard Model.**
Angle θ is $\pi/3$ – for the left plot and $2\pi/3$ – for the right plot.
Lower plots: $R_{44}(SM + DM)/R_{44}(SM)$ at $\theta = \pi/3$ (left), and $\theta = 2\pi/3$ (right). Solid lines: form-factor $\mathrm{Re}(F_2(s)) = 0.1$, dashed lines: form-factor $\mathrm{Re}(X(s)) = 0.1$, the other form-factors are zero.



Transverse spin-correlation components r_{11} (solid lines) and r_{22} (dashed lines) for the $u\,\bar{u}$ (top plots) and $d\,\bar{d}$ (bottom plots) initial states in the Standard Model.
The angle θ of quark vs τ^- is $\pi/3$ in the left plots and $2\pi/3$ in the right plots.

Spin correlations for $\gamma\gamma \to \tau^-\tau^+$: effect of dipole moments





Energy dependence of $R_{44}^{\gamma\gamma}$ and $r_{i,j}^{\gamma\gamma}$. Dashed lines – SM, solid lines – SM+dipole moments: $F_2(0) = 0.1$ and $F_3(0) = 0.1$. The angle $\theta = \pi/3$.

Observables sensitive to dipole moments for $e^-e^+ \to \tau^-\tau^+$ at Z-boson peak

At the Z-boson peak, the spin-correlation pattern is different from that at low energies relevant for the Belle II experiments. The largest spin-correlation components are of longitudinal- transverse type:

$$\sim \mathrm{Im}(Y)(s_x^- s_z^+ - s_z^- s_x^+), \quad \sim \mathrm{Im}(X)(s_y^- s_z^+ + s_z^- s_y^+)$$

and the $\tau^\mp$ polarizations are:

$$\sim \mathrm{Re}(X)(s_x^- + s_x^+), \quad \sim \mathrm{Re}(Y)(s_y^- - s_y^+)$$

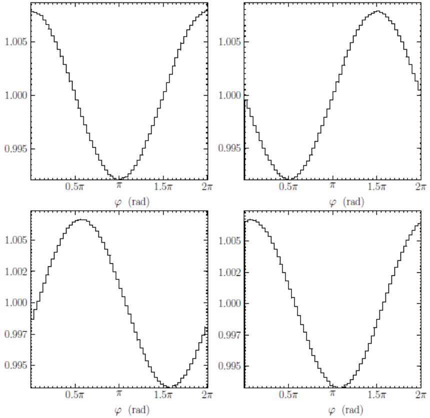
(Here X is weak magnetic and Y is weak electric dipole moments at $s = M_Z^2$).
Let us choose the two-step process

$$e^- + e^+ \to \tau^- + \tau^+ \to \pi^- + \pi^+ + \nu_\tau + \bar{\nu}_\tau$$

and construct observable which is the acoplanarity angle between the two planes: one is $e^-\tau^-$ reaction plane and another is plane built on $\pi^-\tau^-$. This can be done if momenta of e^-, τ^- and π^- are determined.

The number of events with and without dipole moments vs acoplanarity angle is calculated for the energy, corresponding to the Z-boson peak. Then Z-boson exchange plays the dominant role, and γZ interference gives a minor contribution.

Weak dipole moments contribution at Z-boson pole



Ratio of number of events with/without weak DMs: N_{SM+DM}/N_{SM}**, as a function of acoplanarity angle** φ (backward angles θ are selected).

Values of weak form-factors: top left – $\mathrm{Re}(X) = 0.0004$, top right – $\mathrm{Re}(Y) = 0.0004$, bottom left – $\mathrm{Im}(X) = 0.0004$, and bottom right – $\mathrm{Im}(Y) = 0.0004$ (other form-factors, $F_2(M_Z^2)$ and $F_3(M_Z^2)$, are zero).

Weak dipole moments contribution at Z-boson pole

The real parts of weak magnetic X and electric Y form-factors generate event distributions:

$$\mathrm{Re}(X): \quad \sim 1 + 0.008\cos(\varphi)$$
$$\mathrm{Re}(Y): \quad \sim 1 - 0.008\sin(\varphi)$$

These observables are sensitive to the transverse τ spin components $S_x^\pm$, and normal to the reaction plane τ spin component $S_y^\pm$.

The imaginary parts of weak magnetic X and electric Y form-factors generate event distributions:

$$\mathrm{Im}(X): \quad \sim 1 + 0.008\sin(\varphi - \delta)$$
$$\mathrm{Im}(Y): \quad \sim 1 + 0.008\cos(\varphi - \delta')$$

with small shifts δ and δ'. To increase the magnitude of the distributions, we impose the condition $E_{\pi^+} > E_{\bar{\nu}_\tau}$ on the energies of π^+ and $\bar{\nu}_\tau$.

The magnitude of the dipole moments effects in all distributions is about 0.008, which is much larger than the chosen very small value 0.0004 of the dipole moments.
This enhancement is due to the large Lorentz factor $\gamma = M_Z/(2m_\tau) \approx 25.7$.

Reweighting procedure in Monte Carlo programs

The analytical formulas for the spin-correlation matrix are implemented into reweighting algorithms in the Monte Carlo programs `KKMC` for $e^- e^+ \to \tau^- \tau^+$ and `TauSpinner` for $q\bar{q} \to \tau^- \tau^+$ and $\gamma\gamma \to \tau^- \tau^+$.

The `KKMC` MC for $e^- e^+ \to \tau^- \tau^+$ was used earlier for Belle II kinematics and now is extended for the higher energies up to and above Z boson.

As for `TauSpinner` MC, it was developed earlier for the proton-proton collisions [Z. Czyczula, T. Przedzinski, Z. Was. Eur. Phys. J. C, 72, 1988 (2012)].

$$d\sigma = \sum_{flavors} \int dx_1 \int dx_2 \, f(x_1, \ldots)\, f(x_2, \ldots)\, d\Omega_{prod}^{parton\ level}\, d\Omega_{\tau^-}\, d\Omega_{\tau^+}$$
$$\times \left(\sum_{\lambda_1, \lambda_2} |\mathcal{M}_{parton\ level}^{prod}|^2 \right) \left(\sum_{\lambda_1} |\mathcal{M}^{\tau^+}|^2 \right) \left(\sum_{\lambda_2} |\mathcal{M}^{\tau^-}|^2 \right) wt_{spin},$$

with the spin weight

$$wt_{spin} = \sum_{i,j=1,2,3,4} r_{i,j}\, h_i^{\tau^-}\, h_j^{\tau^+}$$

which depends on kinematics of $\tau^\pm$ production and decay. Here $f(x_{1,2}, \ldots)$ are parton distribution functions.

Reweighting procedure in Monte Carlo programs

Starting with the $q\bar{q} \to \tau^-\tau^+$ parton level processes, the functions calculating the spin correlations and an overall event weight have been updated in `TauSpinner`.

As each parton process contributes incoherently to the final state, the process $\gamma\gamma \to \tau^-\tau^+$ for (nearly) on-mass-shell photons can be included together with $q\bar{q} \to \tau^-\tau^+$ process. For this one needs the corresponding PDF for the photon as an additional parton.

Note that in some cases, e.g. at very large impact parameters (experiments of ATLAS and CMS with heavy ions), the contribution from $\gamma\gamma \to \tau^-\tau^+$ becomes dominant, and usually large Drell-Yan contributions $q\bar{q} \to \tau^-\tau^+$ may be small.

Conclusions

- An algorithm for calculation of effects of electromagnetic and weak dipole moments of the τ lepton in simulations of events in the processes: $e^-e^+ \to \tau^-\tau^+$, $q\bar{q} \to \tau^-\tau^+$ and $\gamma\gamma \to \tau^-\tau^+$, with the τ decays, is developed.
  This algorithm is prepared to work with `KKMC` and `TauSpinner` Monte Carlo generators, and can be used at the Belle II energies ($\sqrt{s} = 10.58$ GeV) and at higher energies of the LHC and the Future Circular Collider.
- Effect of dipole moments is demonstrated for the process $e^- + e^+ \to \tau^- + \tau^+ \to \pi^- + \pi^+ + \nu_\tau + \bar{\nu}_\tau$ at Z-boson peak. Distribution of events vs acoplanarity angle between the reaction plane and $\tau^- \to \pi^- + \nu_\tau$ plane is shown to be sensitive to the τ weak dipole moments.
- Additional attention will be required at high energies, because of presence of initial-state bremsstrahlung with high-p_T photons. This is not a constraint at Belle II energies, however, for example, at the FCC, the photons lost in the beam pipe can have p_T comparable to the τ mass.

Sw. Banerjee, A.Yu. Korchin, Z. Was. Phys. Rev. D 106 (2022) 11, 113010,
Sw. Banerjee, A.Yu. Korchin, E. Richter-Was, Z. Was. Phys. Rev. D (to be published); e-Print: 2307.03526 [hep-ph]

Thank you for attention!

ADDITIONAL SLIDES

Electric dipole moment of the τ

It is important to note that these values are not the EDM d_τ, but rather the form factor $F_3(s)$ at the energy of KEKB collider $\sqrt{s} = 10.58$ GeV. The difference between $F_3(s)$ and $F_3(0)$, in general, can be sizable in models of NP.

For example, variation of form factors with s can be seen for magnetic moment by comparing values of $F_2(0)$ and $F_2(s)$. In QED in the 1st order in $\alpha \approx 1/137$:

$$F_2(s)^{(QED)} = \frac{\alpha m_\tau^2}{\pi\, s\, \beta(s)} \left(\log\frac{1-\beta(s)}{1+\beta(s)} + i\,\pi\right) = -0.000244 + i\,0.000219$$

while

$$F_2(0)^{(QED)} = a_\tau^{(QED)} = \frac{\alpha}{2\pi} = 0.00116141 \qquad (Schwinger's\ result)$$

Here $\beta(s) = \sqrt{1 - 4m_\tau^2/s} = 0.94$ is the tau-lepton velocity.

It is seen that the values and the signs are different, and also an imaginary part appears at $\sqrt{s} \geq 2m_\tau$, which is of the same order as the real part.

In any case, the SM value of d_τ is hardly reachable in experiments. Therefore observation of τ EDM in experiment will be indication of CP violation beyond the SM.

This is important as any additional sources of CP violation can help in solution of the problem of matter-antimatter asymmetry in the Universe.

Summary of the τ dipole moments/ form-factors

	Standard Model predictions	**Experiments [PDG 2022]**
a	$1.17721(5) \times 10^{-3}$ [Eidelman:2007]	$-0.052 < a < 0.013$ [DELPHI:2003]
a_w	$-(2.10 + i\,0.61) \times 10^{-6}$ [Bernabeu:1994]	$\mathrm{Re}(a_w) < 1.14 \times 10^{-3}$ [ALEPH:2002] $\mathrm{Im}(a_w) < 2.65 \times 10^{-3}$ [ALEPH:2002]
d	-7.32×10^{-38} e·cm [Yamaguchi:2020]	$(-1.85 < \mathrm{Re}(d) < 0.61) \times 10^{-17}$ e·cm [Belle:2021] $(-1.03 < \mathrm{Im}(d) < 0.23) \times 10^{-17}$ e·cm [Belle:2021]
d_w	–	$\mathrm{Re}(d_w) < 5.01 \times 10^{-18}$ e·cm [ALEPH:2002] $\mathrm{Im}(d_w) < 11.15 \times 10^{-18}$ e·cm [ALEPH:2002]

Table: Current status of predictions and measurements of anomalous magnetic (a), weak anomalous magnetic (a_w), electric (d) and weak electric (d_w) dipole moments/form-factors, of the τ lepton, quoted at 95% confidence level.

γ-exchange contribution to spin-correlation matrix

The simplest contribution $R^{(\gamma)}_{i,j}$ has the form (below $A(s) \equiv F_2(s)$ and $B(s) \equiv F_3(s)$):

$$R^{(\gamma)}_{11} = \frac{e^4\, Q_f^2}{4\gamma^2\, N_f} \left[4\gamma^2\, \mathrm{Re}(A(s)) + \gamma^2 + 1\right] \sin^2(\theta),$$

$$R^{(\gamma)}_{12} = -R^{(\gamma)}_{21} = \frac{e^4\, Q_f^2}{2\, N_f} \beta \sin^2(\theta)\, \mathrm{Re}(B(s)),$$

$$R^{(\gamma)}_{13} = R^{(\gamma)}_{31} = \frac{e^4\, Q_f^2}{4\gamma\, N_f} \left[(\gamma^2 + 1)\mathrm{Re}(A(s)) + 1\right] \sin(2\theta),$$

$$R^{(\gamma)}_{22} = -\frac{e^4\, Q_f^2}{4\, N_f} \beta^2 \sin^2(\theta),$$

$$R^{(\gamma)}_{23} = -R^{(\gamma)}_{32} = -\frac{e^4\, Q_f^2}{4\, N_f} \beta\, \gamma \sin(2\theta)\, \mathrm{Re}(B(s)),$$

$$R^{(\gamma)}_{33} = \frac{e^4\, Q_f^2}{4\gamma^2 N_f} \left\{\left[4\gamma^2\, \mathrm{Re}(A(s)) + \gamma^2 + 1\right] \cos^2(\theta) + \beta^2\gamma^2\right\},$$

$$R^{(\gamma)}_{14} = -R^{(\gamma)}_{41} = \frac{e^4\, Q_f^2}{4\, N_f} \beta\, \gamma \sin(2\theta)\, \mathrm{Im}(B(s)),$$

$$R^{(\gamma)}_{24} = R^{(\gamma)}_{42} = \frac{e^4\, Q_f^2}{4\, N_f} \beta^2\, \gamma \sin(2\theta)\, \mathrm{Im}(A(s)),$$

$$R^{(\gamma)}_{34} = -R^{(\gamma)}_{43} = -\frac{e^4\, Q_f^2}{2\, N_f} \beta \sin^2(\theta)\, \mathrm{Im}(B(s)),$$

$$R^{(\gamma)}_{44} = \frac{e^4\, Q_f^2}{4\gamma^2\, N_f} \left[4\gamma^2\, \mathrm{Re}(A(s)) + \beta^2\gamma^2 \cos^2(\theta) + \gamma^2 + 1\right].$$

Spin correlations in $\gamma\gamma \to \tau^-\tau^+$

Explicitly the elements of matrix $R^{\gamma\gamma}_{i,j}$ are (below $A = F_2(0)$ and $B = F_3(0)$):

$$
\begin{aligned}
R^{\gamma\gamma}_{11} &= \frac{e^4}{8D^2}\big[-\beta^2(\beta^2-4A-2)\cos(4\theta)+4\beta^2(\beta^2-2)\cos(2\theta) \\
&\quad +4A\,(7\beta^2-8)-11\beta^4+22\beta^2-8\big], \\
R^{\gamma\gamma}_{12} &= -R^{\gamma\gamma}_{21} = \frac{e^4 B}{4D^2}\,\beta\,\big(\beta^2\cos(4\theta)+4\cos(2\theta)+15\beta^2-20\big), \\
R^{\gamma\gamma}_{13} &= R^{\gamma\gamma}_{31} = \frac{e^4}{2D^2}\,\gamma\,\beta^2\big[(\beta^2+A\,(\beta^2-2)-1)\cos(2\theta)+A\beta^2-\beta^2+1\big]\,\sin(2\theta), \\
R^{\gamma\gamma}_{22} &= \frac{e^4}{8D^2}\big[-\beta^4\cos(4\theta)+4\beta^2(\beta^2+4A)\cos(2\theta)+16A\,(\beta^2-2) \\
&\quad -11\beta^4+16\beta^2-8\big], \\
R^{\gamma\gamma}_{23} &= -R^{\gamma\gamma}_{32} = \frac{e^4 B}{2D^2}\,\gamma\,\beta\,(\beta^2\cos(2\theta)-3\beta^2+2)\,\sin(2\theta), \\
R^{\gamma\gamma}_{33} &= \frac{e^4}{8D^2}\big[\beta^2(\beta^2-4A-2)\cos(4\theta)-4\beta^4\cos(2\theta)+4A\,(9\beta^2-8) \\
&\quad +11\beta^4+2\beta^2-8\big], \\
R^{\gamma\gamma}_{44} &= \frac{e^4}{8D^2}\big[-\beta^4\cos(4\theta)+4\beta^2(\beta^2-4A-2)\cos(2\theta)-16A\,(\beta^2-2)-11\beta^4+8\beta^2+8\big].
\end{aligned}
$$

Test of spin correlations

If we switch off e.m. dipole moments we obtain

$$\begin{aligned} |\mathcal{M}|^2_{a=b=0} &= \frac{e^4}{4\gamma^2}\Big\{\gamma^2 + 1 + \beta^2\gamma^2\cos^2(\theta) \\ &+ s_3^- s_3^+\left[\beta^2\gamma^2 + (\gamma^2+1)\cos^2(\theta)\right] \\ &+ \left[s_1^- s_1^+(\gamma^2+1) - s_2^- s_2^+\beta^2\gamma^2\right]\sin^2(\theta) \\ &+ (s_1^- s_3^+ + s_3^- s_1^+)\,\gamma\sin(2\theta)\Big\}, \end{aligned}$$

which agrees with well-known result of Tsai, Yung-Su (1971). Only the spin-spin correlations between components $z-z$, $x-x$, $y-y$, $x-z$, $z-x$ remain.

Now check if conventions of our code for calculation of weights, and orientation of reference frame, match what is used in `KKMC` and `TAUOLA`. For this we calculate the following weights for events generated with `KKMC+TAUOLA`:

$$\begin{aligned} wt^{SM}_{spin} &= R^{SM}_{ij}\, h_i^- h_j^+ \,/ R^{SM}_{44}, \\ wt_{spin} &= R_{ij} h_i^- h_j^+ \,/ R_{44} \,/ wt^{SM}_{spin}, \\ wt &= R_{44} \,/ R^{SM}_{44} \end{aligned}$$

Here $R^{SM}_{ij} \equiv R_{ij}|_{a=b=0}$ is defined.

The so-called polarimetric vectors h_i^- and h_j^+ depend on the τ^-- and τ^+-decay products, and are known for the main decays.