Back-to-back dijets in DIS at small x

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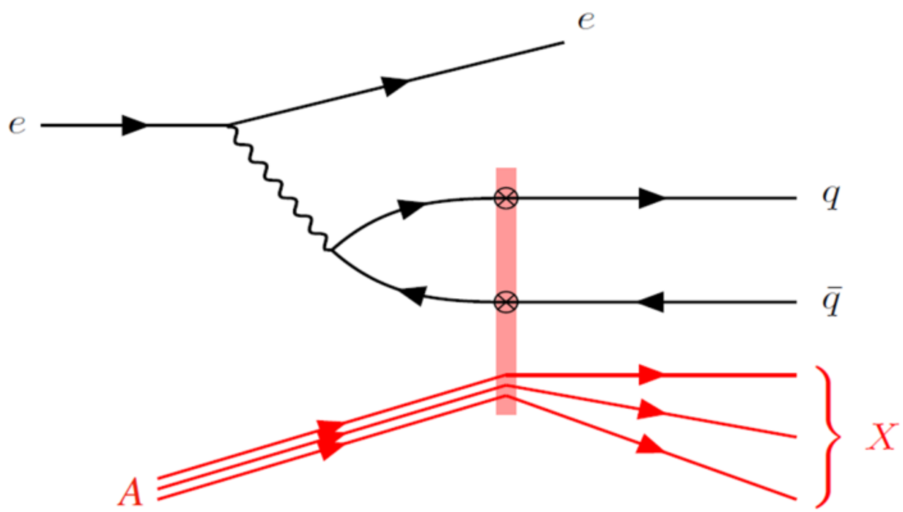
Outline

- Dijet production in DIS
- CGC calculation at LO and NLO
- Back-to-back limit
- Numerical results

Dijet production in DIS

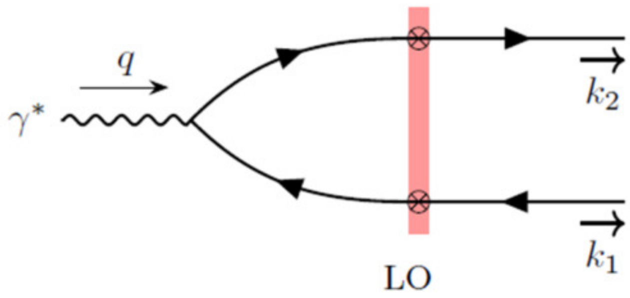


$$\frac{\mathrm{d}\sigma^{e+A\to e'+q\bar{q}+X}}{\mathrm{d}x_{\mathrm{Bj}}\mathrm{d}Q^2\mathrm{d}^2\boldsymbol{k}_{1\perp}\mathrm{d}^2\boldsymbol{k}_{2\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2} = \sum_{\lambda=\mathrm{L,T}} f_\lambda(x_{\mathrm{Bj}}, Q^2)\ \frac{\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{k}_{1\perp}\mathrm{d}^2\boldsymbol{k}_{2\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2}\,.$$

$$f_{\lambda=\mathrm{L}}(x_{\mathrm{Bj}}, Q^2) = \frac{\alpha_{\mathrm{em}}}{\pi Q^2 x_{\mathrm{Bj}}}(1-y)\,,$$

$$f_{\lambda=\mathrm{T}}(x_{\mathrm{Bj}}, Q^2) = \frac{\alpha_{\mathrm{em}}}{2\pi Q^2 x_{\mathrm{Bj}}}[1+(1-y)^2]\,,$$

CGC calculation at LO



$$\left.\frac{\mathrm{d}\sigma^{\gamma_\lambda^\star+A\to q\bar{q}+X}}{\mathrm{d}^2\boldsymbol{k}_{1\perp}\mathrm{d}^2\boldsymbol{k}_{2\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2}\right|_{\mathrm{LO}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}}e^{-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}} \times \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\mathcal{R}^\lambda_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\,.$$

Impact factor:

$$\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'}) = 8z_1^3z_2^3Q^2K_0(\bar{Q}r_{xy})K_0(\bar{Q}r_{x'y'})\,,$$

$$\mathcal{R}^{\mathrm{T}}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'}) = 2z_1z_2\left[z_1^2+z_2^2\right]\frac{\boldsymbol{r}_{xy}\cdot\boldsymbol{r}_{x'y'}}{r_{xy}r_{x'y'}}\bar{Q}^2K_1(\bar{Q}r_{xy})K_1(\bar{Q}r_{x'y'})\,,$$

Color correlator:

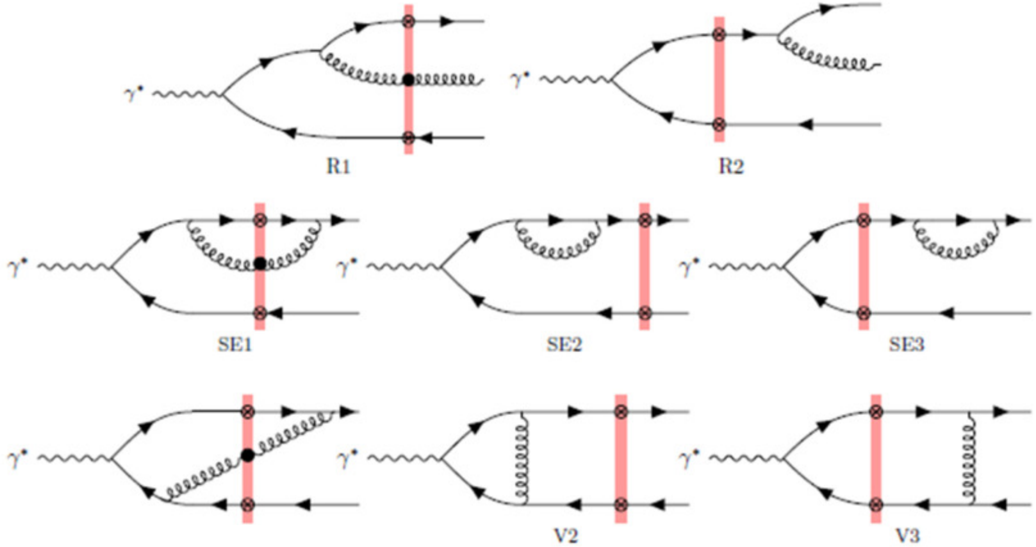
$$\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[\left(V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)-\mathbb{1}\right)\left(V(\boldsymbol{y}'_\perp)V^\dagger(\boldsymbol{x}'_\perp)-\mathbb{1}\right)\right]\right\rangle_Y$$

$$= \left\langle Q_{xy,y'x'} - D_{xy} - D_{y'x'} + 1\right\rangle_Y\,,$$

where Wilson line: $V(\boldsymbol{x}_\perp) = P\exp\left(ig\int \mathrm{d}x^- A^+_{\mathrm{cl}}(\boldsymbol{x}_\perp, x^-)\right)$

NLO corrections

P. Caucal, F. Salazar and R. Venugopalan *(2021)*



NLO impact factor:

$$\mathrm{d}\sigma_{\mathrm{R_2\times R_2,sud2}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^{\lambda}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})$$
$$\times C_F\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\times\frac{\alpha_s}{\pi}\int_0^1\frac{\mathrm{d}\xi}{\xi}\left[1-e^{-i\xi\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}}\right]\ln\left(\frac{\boldsymbol{k}_{1\perp}{}^2\boldsymbol{r}_{xx'}^2R^2\xi^2}{c_0^2}\right)$$

$$\mathrm{d}\sigma_{\mathrm{R_2\times R_2',sud2}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^{\lambda}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})$$
$$\times\Xi_{\mathrm{NLO,3}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\times\frac{(-\alpha_s)}{\pi}\int_0^1\frac{\mathrm{d}\xi}{\xi}\left[1-e^{-i\xi\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xy'}}\right]\ln\left(\frac{\boldsymbol{P}_\perp^2\boldsymbol{r}_{xy'}^2\xi^2}{z_2^2c_0^2}\right)$$

$$\mathrm{d}\sigma^{\gamma_L^*+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,LO}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c}{(2\pi)^8}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}(4\alpha_sC_F)\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$$
$$\times\frac{e^{-i\boldsymbol{k}_{g\perp}\cdot\boldsymbol{r}_{zz'}}}{(\boldsymbol{k}_{g\perp}-\frac{z_g}{z_1}\boldsymbol{k}_{1\perp})^2}\left\{8z_1z_2^3(1-z_2)^2Q^2\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)K_0(\bar{Q}_{\mathrm{R2}}r_{xy})K_0(\bar{Q}_{\mathrm{R2}}r_{x'y'})\delta_z^{(3)}\right.$$
$$\left.-\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Theta(z_1-z_g)\delta_z^{(2)}\right\}+(1\leftrightarrow2)$$

$$\mathrm{d}\sigma^{\gamma_L^*+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,NLO_3}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c}{(2\pi)^8}\int \mathrm{d}^2\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}(-4\alpha_s)\Xi_{\mathrm{NLO,3}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$$
$$\times\frac{e^{-i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xy'}}}{\boldsymbol{l}_\perp^2}\left\{8z_1^2z_2^2(1-z_2)(1-z_1)Q^2K_0(Q_{\mathrm{R2}}r_{xy})K_0(Q_{\mathrm{R2'}}r_{x'y'})\left[1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right]\right.$$
$$\left.\times e^{-i\boldsymbol{l}_\perp\cdot\boldsymbol{r}_{xy'}}\frac{\boldsymbol{l}_\perp\cdot(\boldsymbol{l}_\perp+\boldsymbol{K}_\perp)}{(\boldsymbol{l}_\perp+\boldsymbol{K}_\perp)^2}\delta_z^{(3)}-\mathcal{R}^{\mathrm{L}}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Theta\left(\frac{c_0^2}{\boldsymbol{r}_{xy'}^2}\geq\boldsymbol{l}_\perp^2\geq\boldsymbol{K}_\perp^2\right)\Theta(z_1-z_g)\delta_z^{(2)}\right\}$$
$$+(1\leftrightarrow2)$$

$$\mathrm{d}\sigma^{\gamma_L^*+A\to q\bar{q}g+X}_{\mathrm{R,no-sud,other}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(3)}}{(2\pi)^8}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\pi}\frac{\mathrm{d}^2\boldsymbol{z}'_\perp}{\pi}e^{-i\boldsymbol{k}_{g\perp}\cdot\boldsymbol{r}_{zz'}}$$
$$\alpha_s\left\{-\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(\bar{Q}_{\mathrm{R2}}r_{w'y'})\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)\Xi_{\mathrm{NLO,1}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{w}'_\perp,\boldsymbol{y}'_\perp)\right.$$
$$+\frac{\boldsymbol{r}_{zy}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(\bar{Q}_{\mathrm{R2'}}r_{w'y'})\left(1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right)\Xi_{\mathrm{NLO,1}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{w}'_\perp,\boldsymbol{y}'_\perp)$$
$$+\frac{1}{2}\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(QX'_{\mathrm{R}})\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)\Xi_{\mathrm{NLO,4}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp)$$
$$-\frac{1}{2}\frac{\boldsymbol{r}_{zy}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(QX'_{\mathrm{R}})\left(1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right)\Xi_{\mathrm{NLO,4}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp)$$
$$\left.+(1\leftrightarrow2)+\mathrm{c.c.}\right\}-\frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(2)}}{(2\pi)^8}\alpha_s\Theta(z_f-z_g)\times\text{"slow"}$$

$$\mathrm{d}\sigma_{\mathrm{V,no-sud,LO}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^{\lambda}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)$$
$$\times\frac{\alpha_sC_F}{\pi}\left\{-\frac{3}{4}\ln\left(\frac{\boldsymbol{k}_{1\perp}{}^2\boldsymbol{k}_{2\perp}{}^2\boldsymbol{r}_{xy}^2\boldsymbol{r}_{x'y'}^2}{c_0^4}\right)-3\ln(R)+\frac{1}{2}\ln^2\left(\frac{z_1}{z_2}\right)+\frac{11}{2}+3\ln(2)-\frac{\pi^2}{2}\right\}$$

$$\mathrm{d}\sigma^{\lambda=\mathrm{L}}_{\mathrm{V,no-sud,NLO_3}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^3\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2K_0(\bar{Q}r_{x'y'})$$
$$\times\frac{\alpha_s}{\pi}\int_0^{z_1}\frac{\mathrm{d}z_g}{z_g}\left\{K_0(\bar{Q}_{\mathrm{V3}}r_{xy})\left[\left(1-\frac{z_g}{z_1}\right)^2\left(1+\frac{z_g}{z_2}\right)(1+z_g)e^{i(\boldsymbol{P}_\perp+z_g\boldsymbol{q}_\perp)\cdot\boldsymbol{r}_{xy}}K_0(-i\Delta_{\mathrm{V3}}r_{xy})\right.\right.$$
$$\left.-\left(1-\frac{z_g}{2z_1}+\frac{z_g}{2z_2}-\frac{z_g^2}{2z_1z_2}\right)e^{i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xy}}\mathcal{J}_{\odot}\left(\boldsymbol{r}_{xy},\left(1-\frac{z_g}{z_1}\right)\boldsymbol{P}_\perp,\Delta_{\mathrm{V3}}\right)\right]$$
$$\left.+K_0(\bar{Q}r_{xy})\ln\left(\frac{z_gP_\perp r_{xy}}{c_0z_1z_2}\right)+(1\leftrightarrow2)\right\}\Xi_{\mathrm{NLO,3}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)+\mathrm{c.c.}$$

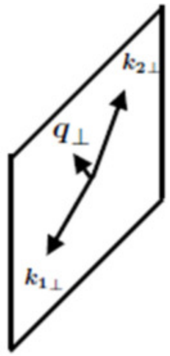
$$\mathrm{d}\sigma^{\lambda=\mathrm{L}}_{\mathrm{V,no-sud,other}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}8z_1^3z_2^3Q^2K_0(\bar{Q}r_{x'y'})\int_0^{z_1}\frac{\mathrm{d}z_g}{z_g}$$
$$\times\frac{\alpha_s}{\pi}\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\pi}\left\{\frac{1}{\boldsymbol{r}_{zx}^2}\left[\left(1-\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)e^{-i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{zx}}K_0(QX_V)-\Theta(z_f-z_g)K_0(\bar{Q}r_{xy})\right]\Xi_{\mathrm{NLO,1}}\right.$$
$$-\frac{1}{\boldsymbol{r}_{zx}^2}\left[\left(1-\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2e^{\gamma_E}}}K_0(\bar{Q}r_{xy})-\Theta(z_f-z_g)e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2e^{\gamma_E}}}K_0(\bar{Q}r_{xy})\right]C_F\Xi_{\mathrm{LO}}$$
$$-\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{zy}^2}\left[\left(1-\frac{z_g}{z_1}\right)\left(1+\frac{z_g}{z_2}\right)\left(1-\frac{z_g}{2z_1}-\frac{z_g}{2(z_2+z_g)}\right)e^{-i\frac{z_g}{z_1}\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{zx}}K_0(QX_V)\right.$$
$$\left.\left.-\Theta(z_f-z_g)K_0(\bar{Q}r_{xy})\right]\Xi_{\mathrm{NLO,1}}+(1\leftrightarrow2)\right\}+\mathrm{c.c.}.$$

$$\mathrm{d}\sigma_{\mathrm{sud1}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c\delta_z^{(2)}}{(2\pi)^6}\int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k}_{1\perp}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k}_{2\perp}\cdot\boldsymbol{r}_{yy'}}\mathcal{R}^{\lambda}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\times\frac{\alpha_s}{\pi}$$
$$\times\left\{C_F\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\left[\ln\left(\frac{z_f}{z_1}\right)\ln\left(\frac{\boldsymbol{r}_{xx'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)+\ln\left(\frac{z_f}{z_2}\right)\ln\left(\frac{\boldsymbol{r}_{yy'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)\right]\right.$$
$$\left.+\Xi_{\mathrm{NLO,3}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\left[\ln\left(\frac{z_1}{z_f}\right)\ln\left(\frac{\boldsymbol{r}_{xy'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)+\ln\left(\frac{z_2}{z_f}\right)\ln\left(\frac{\boldsymbol{r}_{yx'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)\right]\right\}$$

Very hard to do numerics…

F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan (2011)

Back-to-back limit (LO)



$$q_\perp = k_{1\perp} + k_{2\perp}$$
$$P_\perp = z_2 k_{1\perp} - z_1 k_{2\perp}$$

Back-to-back limit:

$$q_\perp, Q_s \ll P_\perp$$

High energy limit: $P_\perp \ll W$

Factorization at the LO:

$$\mathrm{d}\sigma_{\mathrm{LO}}^{\gamma_\lambda^\star + A \to q\bar{q} + X} = \alpha_{\mathrm{em}} e_f^2 \alpha_s \delta_z^{(2)} \mathcal{H}_{\mathrm{LO}}^{\lambda,ij}(\boldsymbol{P}_\perp) \int \frac{\mathrm{d}^2 \boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot \boldsymbol{r}_{bb'}} \hat{G}_Y^{ij}(\boldsymbol{b}_\perp, \boldsymbol{b}'_\perp) + \mathcal{O}\left(\frac{q_\perp}{P_\perp}, \frac{Q_s}{P_\perp}\right)$$

where the hard factors:

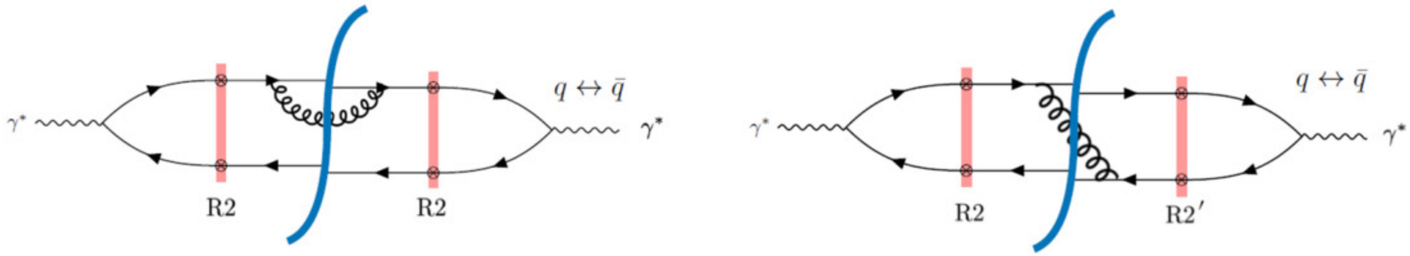
$$\mathcal{H}_{\mathrm{LO}}^{\lambda=\mathrm{L},ij}(\boldsymbol{P}_\perp) = 16 z_1^3 z_2^3 Q^2 \frac{\boldsymbol{P}_\perp^i \boldsymbol{P}_\perp^j}{(\boldsymbol{P}_\perp^2 + \bar{Q}^2)^4},$$
$$\mathcal{H}_{\mathrm{LO}}^{\lambda=\mathrm{T},ij}(\boldsymbol{P}_\perp) = z_1 z_2 (z_1^2 + z_2^2) \left\{ \frac{\delta^{ij}}{(\boldsymbol{P}_\perp^2 + \bar{Q}^2)^2} - \frac{4\bar{Q}^2 \boldsymbol{P}_\perp^i \boldsymbol{P}_\perp^j}{(\boldsymbol{P}_\perp^2 + \bar{Q}^2)^4} \right\}$$

and Weizsäcker-Williams (WW) distribution:

$$\hat{G}_Y^{ij}(\boldsymbol{b}_\perp, \boldsymbol{b}'_\perp) \equiv \frac{-2}{\alpha_s} \left\langle \mathrm{Tr}\left[V(\boldsymbol{b}_\perp) \left(\partial^i V^\dagger(\boldsymbol{b}_\perp) \right) V(\boldsymbol{b}'_\perp) \left(\partial^j V^\dagger(\boldsymbol{b}'_\perp) \right) \right] \right\rangle_Y$$

Caucal, Salazar, Schenke, and Venugopalan (2022)

Back–to–back limit (NLO)



One obtains Sudakov logs:

$$d\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X} \propto \mathcal{H}(Q,\boldsymbol{P}_\perp)\int d^2\mathbf{r}_{bb'}\, e^{-i\boldsymbol{q}_\perp\cdot\mathbf{r}_{bb'}} \left[1+\frac{\alpha_s N_c}{4\pi}\ln^2\left(\frac{\boldsymbol{P}_\perp^2\mathbf{r}_{bb'}^2}{c_0^2}\right)+\cdots+\alpha_s\ln\left(\Lambda_f^-/\Lambda^-\right)\mathcal{K}_{LL}\otimes\right]\widetilde{G}_Y(\mathbf{r}_{bb'})$$

But with the wrong (+) sign.

Should be compared to result by Mueller, Xiao, Yuan (2013) for joint small–x and soft gluon resummation:

$$d\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X} \propto \mathcal{H}(Q,\boldsymbol{P}_\perp)\int \frac{d^2\mathbf{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\mathbf{r}_{bb'}}\,\widetilde{G}_Y^0(\mathbf{r}_{bb'})\,e^{-S_{\mathrm{Sud}}(\mathbf{r}_{bb'},\boldsymbol{P}_\perp)}$$

Sudakov factor: $S_{\mathrm{Sud}}(\mathbf{r}_{bb'},P_\perp)=\frac{\alpha_s N_c}{\pi}\int_{c_0^2/\mathbf{r}_{bb'}^2}^{P_\perp^2}\frac{1}{2}\ln\left(\frac{P_\perp^2}{\mu^2}\right)$

Taels, Altinoluk, Marquet, Beuf, JHEP 10, 184 (2022),
Caucal, Salazar, Schenke, and Venugopalan, JHEP 11 (2022) 169

Kinematical constraint

One needs to impose kinematical constraint for small-x evolution and get correct Sudakov double log:

$$\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X} \propto \mathcal{H}(Q,\boldsymbol{P}_\perp)\int \mathrm{d}^2\mathbf{r}_{bb'} e^{-i\boldsymbol{q}_\perp\cdot\mathbf{r}_{bb'}}$$
$$\left[1 - \frac{\alpha_s N_c}{4\pi}\ln^2\left(\frac{\boldsymbol{P}_\perp^2\mathbf{r}_{bb'}^2}{c_0^2}\right) - \frac{\alpha_s}{\pi}s_L\ln\left(\frac{\boldsymbol{P}_\perp^2\mathbf{r}_{bb'}^2}{c_0^2}\right) + \alpha_s\mathcal{K}_{LL,\mathrm{coll}}\otimes\right]\widetilde{G}_Y(\mathbf{r}_{bb'}) + \mathcal{O}(\alpha_s)$$

Correct Sudakov double log

Kinematically improved small-x evolution

where kinematically constrained equation for TMD:

$$\frac{\partial\hat{G}^{(0)}_{Y_f}(\boldsymbol{r}_{bb'})}{\partial Y_f} = -\frac{\alpha_s N_c}{\pi}\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{2\pi}\frac{\boldsymbol{r}_{bb'}^2}{\boldsymbol{r}_{zb}^2\boldsymbol{r}_{zb'}^2}\Theta\left(-Y_f - \ln(\min(\boldsymbol{r}_{zb}^2,\boldsymbol{r}_{zb'}^2)\mu_\perp^2)\right)$$
$$\times\left\{\hat{G}^{(0)}_{Y_f}(\boldsymbol{r}_{bb'}) + \frac{2}{\boldsymbol{r}_{bb'}^2}\left[1 - \frac{2(\boldsymbol{r}_{zb}\cdot\boldsymbol{r}_{zb'})^2}{\boldsymbol{r}_{zb}^2\boldsymbol{r}_{zb'}^2}\right]\hat{G}^{(2)}_{Y_f}(\boldsymbol{r}_{zb'},\boldsymbol{r}_{zb}) + \left[\frac{\boldsymbol{r}_{bb'}^i}{\boldsymbol{r}_{bb'}^2} + \frac{\boldsymbol{r}_{zb}^i}{\boldsymbol{r}_{zb}^2}\right]\hat{G}^{(1),i}_{Y_f}(\boldsymbol{r}_{zb},\boldsymbol{r}_{zb'}) + \left[\frac{\boldsymbol{r}_{b'b}^i}{\boldsymbol{r}_{b'b}^2} + \frac{\boldsymbol{r}_{zb'}^i}{\boldsymbol{r}_{zb'}^2}\right]\hat{G}^{(1),i}_{Y_f}(\boldsymbol{r}_{zb'},\boldsymbol{r}_{zb})\right\}$$

Caucal, Salazar, Schenke, TS and Venugopalan JHEP 08 (2023) 062 and 2308.00022

Back-to-back limit (NLO)

$$\left\langle \mathrm{d}\sigma_{\mathrm{LO}}^{(0),\lambda} + \alpha_s \mathrm{d}\sigma_{\mathrm{NLO}}^{(0),\lambda} \right\rangle_{\eta_c} = \frac{1}{2}\mathcal{H}_{\mathrm{LO}}^{\lambda,ii} \int \frac{\mathrm{d}^2\boldsymbol{B}_\perp}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{G}^0_{\eta_c}(\boldsymbol{r}_{bb'},\mu_0) \Bigg\{ 1 + \frac{\alpha_s(\mu_R)}{\pi} \Bigg[\underbrace{-\frac{N_c}{4}\ln^2\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right)}_{\text{Sudakov double log}}$$

$$\underbrace{-s_L \ln\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) + \pi\beta_0 \ln\left(\frac{\mu_R^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right)}_{\text{Sudakov single logs}} + \frac{N_c}{2} f_1^\lambda(\chi, z_1, R) + \frac{1}{2N_c} f_2^\lambda(\chi, z_1, R) \Bigg] \Bigg\}$$

$$+ \frac{\alpha_s(\mu_R)}{2\pi}\mathcal{H}_{\mathrm{LO}}^{\lambda,ii} \int \frac{\mathrm{d}^2\boldsymbol{B}_\perp}{(2\pi)^2} \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{h}^0_{\eta_c}(\boldsymbol{r}_{bb'},\mu_0) \left\{ \frac{N_c}{2}\left[1 + \ln(R^2)\right] - \frac{1}{2N_c}\ln(z_1 z_2 R^2) \right\}.$$

- Factorized expression even at NLO!
- Single log calculated: coefficient,

$$\mathrm{C_F} \log\left(\frac{1}{z_1 z_2 R^2}\right) + N_c \log\left(1 + \frac{Q^2}{M_{q\bar{q}}^2}\right) - \beta_0$$

agrees with result obtained in the collinear Collins, Soper, Sterman (CSS) resummation.

Hatta, Xiao, Yuan, Zhou (2021)

Back-to-back limit (NLO)

$$\left\langle \mathrm{d}\sigma_{\mathrm{LO}}^{(0),\lambda} + \alpha_s \mathrm{d}\sigma_{\mathrm{NLO}}^{(0),\lambda} \right\rangle_{\eta_f} = \frac{1}{2}\mathcal{H}_{\mathrm{LO}}^{\lambda,ii} \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{G}_{\eta_f}^0(\boldsymbol{r}_{bb'},\mu_0) \left\{ 1 + \frac{\alpha_s(\mu_R)}{\pi} \left[-\frac{N_c}{4}\ln^2\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) - s_L \ln\left(\frac{\boldsymbol{P}_\perp^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) + \pi\beta_0 \ln\left(\frac{\mu_R^2 \boldsymbol{r}_{bb'}^2}{c_0^2}\right) + \frac{N_c}{2} f_1^\lambda(\chi, z_1, R) + \frac{1}{2N_c} f_2^\lambda(\chi, z_1, R) \right] \right\}$$

$$+ \frac{\alpha_s(\mu_R)}{2\pi}\mathcal{H}_{\mathrm{LO}}^{\lambda,ii} \int \frac{\mathrm{d}^2\boldsymbol{r}_{bb'}}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{bb'}} \hat{h}_{\eta_f}^0(\boldsymbol{r}_{bb'},\mu_0) \left\{ \frac{N_c}{2}\left[1+\ln(R^2)\right] - \frac{1}{2N_c}\ln(z_1 z_2 R^2) \right\}$$

Assumption: we can resum the large logs by exponentiation:

$$\mathcal{S} = \exp\left(-\int_{\frac{c_0^2}{\boldsymbol{r}_{bb'}^2}}^{\mu_h^2} \frac{\mathrm{d}\mu^2}{\mu^2} \frac{\alpha_s N_c}{\pi} \left[\frac{1}{2}\ln\left(\frac{\mu_h^2}{\mu^2}\right) + \frac{s_L - \beta_0}{N_c} \right] \right),$$

NLO hard coefficient functions

Longitudinal polarization of photon:

$$
\begin{aligned}
f_1^{\lambda=\mathrm{L}}(\chi, z_1, R, \eta_f) = 9 &- \frac{3\pi^2}{2} - \frac{3}{2}\ln\left(\frac{z_1 z_2 R^2}{\chi^2}\right) - \ln(z_1)\ln(z_2) - \ln(1+\chi^2)\ln\left(\frac{1+\chi^2}{z_1 z_2}\right) \\
&+ \left\{ \mathrm{Li}_2\left(\frac{z_2 - z_1\chi^2}{z_2(1+\chi^2)}\right) - \frac{1}{4(z_2 - z_1\chi^2)} \right. \\
&\left. + \frac{(1+\chi^2)(z_2(2z_2 - z_1) + z_1(2z_1 - z_2)\chi^2)}{4(z_2 - z_1\chi^2)^2}\ln\left(\frac{z_2(1+\chi^2)}{\chi^2}\right) + (1 \leftrightarrow 2) \right\} \\
&+ \ln^2\left(\frac{x_c}{x_f}\right) + 2\ln\left(\frac{x_c}{x_f}\right) - \mathcal{I}_{\mathrm{kc}}\left(\sqrt{\frac{x_c}{x_f}}\right), \\
f_2^{\lambda=\mathrm{L}}(\chi, z_1, R) = -8 &+ \frac{19\pi^2}{12} + \frac{3}{2}\ln(z_1 z_2 R^2) - \frac{3}{4}\ln^2\left(\frac{z_1}{z_2}\right) - \ln(\chi), \\
&+ \left\{ \frac{1}{4(z_2 - z_1\chi^2)} + \frac{(1+\chi^2)z_1(z_2 - (1+z_1)\chi^2)}{4(z_2 - z_1\chi^2)^2}\ln\left(\frac{z_2(1+\chi^2)}{\chi^2}\right) \right. \\
&\left. + \frac{1}{2}\mathrm{Li}_2(z_2 - z_1\chi^2) - \frac{1}{2}\mathrm{Li}_2\left(\frac{z_2 - z_1\chi^2}{z_2}\right) + (1 \leftrightarrow 2) \right\}.
\end{aligned}
$$

where

$$\chi = \frac{Q}{M_{q\bar{q}}}$$

Tranverse polarization: similar expressions.

WW TMD's evolution

TMD satisfies kinematically constrained evolution equation which is not closed (involves other than WW-type correlators).

For numerical evaluation we assume Gaussian approximation:

$$\hat{G}^{ij}(\boldsymbol{r}_{bb'}) = \frac{2C_F S_\perp}{\alpha_s} \frac{\partial^i \partial^j \Gamma(\boldsymbol{r}_{bb'})}{\Gamma(\boldsymbol{r}_{bb'})} \left[1 - \exp\left(-\frac{C_A}{C_F}\Gamma(\boldsymbol{r}_{bb'})\right)\right]$$

WW TMD

$$\Gamma(\boldsymbol{r}_{bb'}) = -\ln\left(S(\boldsymbol{r}_{bb'})\right) \qquad S = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]\right\rangle_Y$$

S satisfies kinematically constrained BK equation (written in terms of target rapidity η):

$$\frac{\partial \overline{S}_{\eta_f}(\boldsymbol{r}_{bb'})}{\partial \eta_f} = \frac{\alpha_s N_c}{\pi} \int \frac{\mathrm{d}^2 \boldsymbol{z}_\perp}{2\pi} \Theta\left(\ln\left(\frac{1}{x_c}\right) + \ln\left(\frac{\boldsymbol{r}_{bb'}^2}{r_<^2}\right) - \eta_f\right) \Theta\left(\eta_f - \ln\left(\frac{\boldsymbol{r}_{bb'}^2}{r_<^2}\right) - \ln\left(\frac{1}{x_0}\right)\right) \frac{\boldsymbol{r}_{bb'}^2}{\boldsymbol{r}_{zb}^2 \boldsymbol{r}_{zb'}^2}$$
$$\times \left[\overline{S}_{\eta_f - \ln(\boldsymbol{r}_{bb'}^2/\boldsymbol{r}_{zb}^2)}(\boldsymbol{r}_{zb}) \overline{S}_{\eta_f - \ln(\boldsymbol{r}_{bb'}^2/\boldsymbol{r}_{zb'}^2)}(\boldsymbol{r}_{zb'}) - \overline{S}_{\eta_f}(\boldsymbol{r}_{bb'})\right].$$

Iancu, Mueller, Soyez, Triantafyllopolous (2019)

BK evolution

$$\frac{\partial \bar{S}_{\eta_f}(\boldsymbol{r}_{bb'})}{\partial \eta_f} = \frac{\alpha_s N_c}{\pi} \int \frac{\mathrm{d}^2 \boldsymbol{z}_\perp}{2\pi} \Theta\left(\ln\left(\frac{1}{x_c}\right) + \ln\left(\frac{\boldsymbol{r}_{bb'}^2}{r_<^2}\right) - \eta_f\right) \Theta\left(\eta_f - \ln\left(\frac{\boldsymbol{r}_{bb'}^2}{r_<^2}\right) - \ln\left(\frac{1}{x_0}\right)\right) \frac{\boldsymbol{r}_{bb'}^2}{\boldsymbol{r}_{zb}^2 \boldsymbol{r}_{zb'}^2}$$
$$\times \left[\bar{S}_{\eta_f - \ln(\boldsymbol{r}_{bb'}^2/\boldsymbol{r}_{zb}^2)}(\boldsymbol{r}_{zb}) \bar{S}_{\eta_f - \ln(\boldsymbol{r}_{bb'}^2/\boldsymbol{r}_{zb'}^2)}(\boldsymbol{r}_{zb'}) - \bar{S}_{\eta_f}(\boldsymbol{r}_{bb'})\right].$$

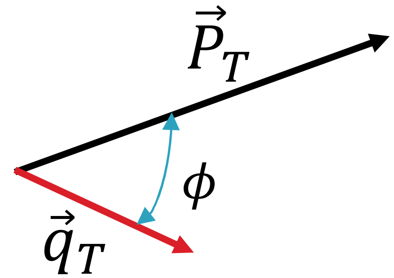
Initial condition at $x_0 = 0.01$ ($\eta_0 = 0$) is MV model:

$$\bar{S}_{\eta_0}(r_\perp) = \exp\left[-\frac{r_\perp^2 Q_{s0}^2}{4} \ln\left(\frac{1}{r_\perp \Lambda} + e\right)\right]$$

where initial saturation scales:

$$Q_{s0}^2 = \begin{cases} 0.1\ \mathrm{GeV}^2 & \text{for proton} \\ \mathrm{A}^{1/3} \times 0.1\ \mathrm{GeV}^2 & \text{for nucleus} \end{cases}$$

Observables for $\gamma^* + p$ or $\gamma^* + A$



$$\frac{d\sigma^{\gamma^*_\lambda + A \to \text{dijet} + X}}{d^2\boldsymbol{P}_\perp d^2\boldsymbol{q}_\perp d\eta_1 d\eta_2} = d\sigma^{(0),\lambda}(P_\perp, q_\perp, \eta_1, \eta_2) + 2\sum_{n=1}^{\infty} d\sigma^{(n),\lambda}(P_\perp, q_\perp, \eta_1, \eta_2)\cos(n\phi)\,,$$

Azimuthally averaged cross-section

$v_2 = \frac{d\sigma^{(2)}}{d\sigma^{(0)}}$ anisotropy

Yield:

$$dN_n = \frac{d\sigma^{(n)}}{S_\perp}$$

transverse size of target

Nuclear modification factor:

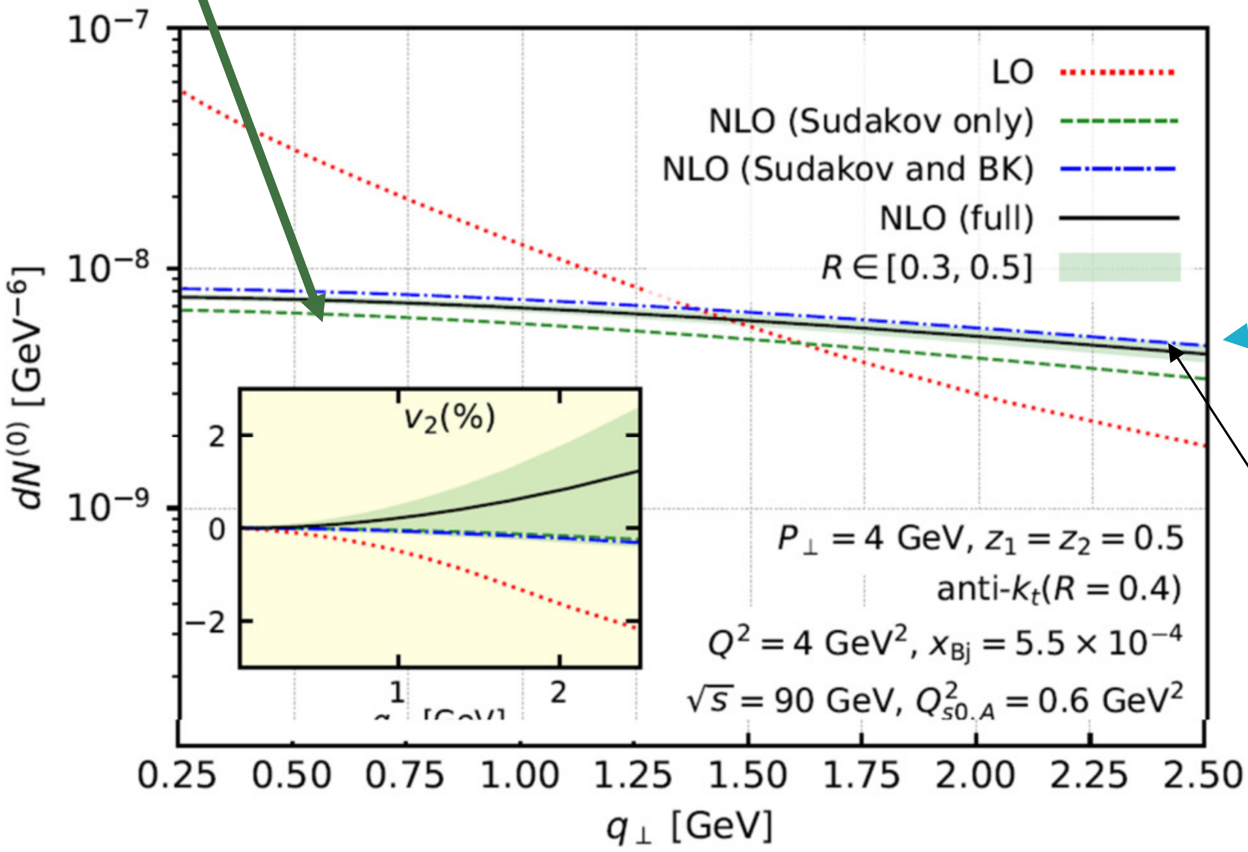
$$R_{eA} = \frac{1}{\mathrm{A}^{1/3}} \frac{\mathrm{d}N_0^{e+A}}{\mathrm{d}N_0^{e+p}}$$

where $\mathrm{A}^{1/3} = 6$ and both polarizations of photon are included.

Caucal, Salazar, Schenke, TS and Venugopalan, 2308.00022

Numerical results: $e + A$ cross-section at EIC

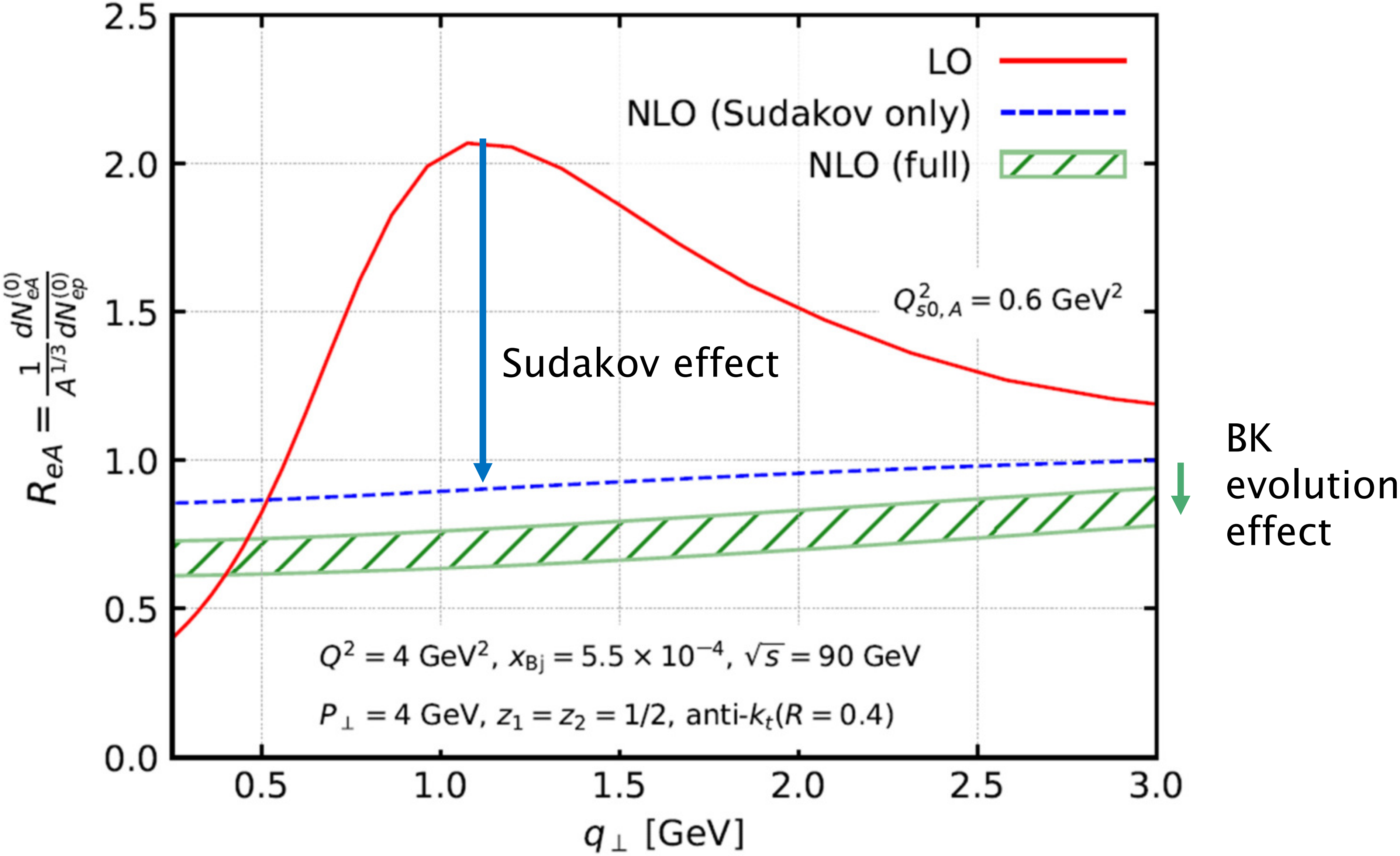
Sudakov effect



BK evolution effect ($\eta_c \approx 1.3$)

NLO coefficient functions effect (very small)

Numerical results: nuclear modification factor



Summary

- We calculated back-to-back inclusive dijets cross-section up to NLO accuracy:

  - identify large Sudakov log (both double and single),
  - hard coefficient functions are given by analytic expressions.
  - kinematical constraint on small-x evolution (BK/JIMWLK).

- Numerical results for EIC:

  - Large effect of Sudakov resummation but effect of small-x evolution also visible.
  - R_{eA} is below 1 due to the small-x evolution (saturation).

Thank you