On hybrid factorization at NLO from NLO collinear amplitudes

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Goal and outline

1. Talk based on results of Andreas van Hameren, L. Motyka and Grzegorz Ziarko, arXiv:2205.09585 [JHEP]
2. **The goal:** construct a general approach to determine NLO impact factors in k_T-factorization from NLO collinear amplitudes. Applications to KT NLO Monte Carlo [Andreas van Hameren]
3. **Outline:**
   - A bottom-up approach: decomposition of NLO collinear amplitudes with auxillary partons into objects that can be interpreted within the High Energy Factorization (HEF) framework
   - Insight into calculational technique and results
   - Remarks on evolution scheme

Disclaimer: We consider the linear evolution regime.

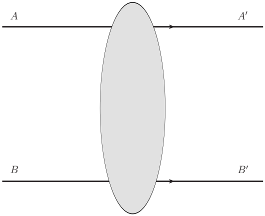
The HE / Regge factorization

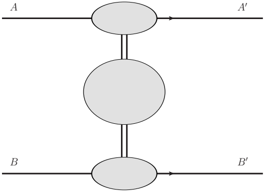
At NLO in QCD Regge Factorization is proven:

1. The QCD amplitudes at high energies may be factorized into projectile and target impact factors and the Green's function that incorporates the evolution (implicitly the BFKL evolution)
2. Convolution of the target impact factor and the Green's function gives the unintegrated (or transverse momentum dependent) gluon density function
3. There is a residual ambiguity in defining the impact factors and the Green's function related to the choice of the energy scale

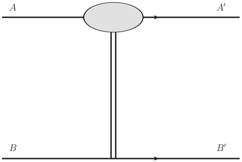
The HE / Regge factorization

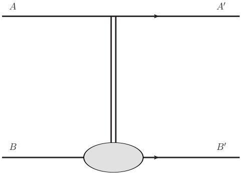
Regge:

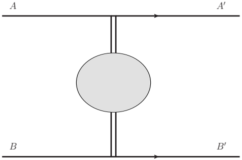




NLO:



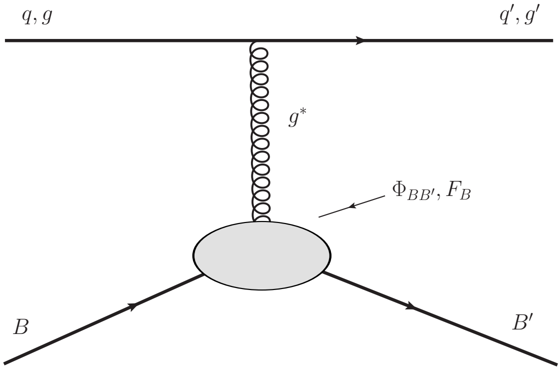




The auxillary parton method at LO

- At LO in QCD the definition of the impact factors and the unintegrated gluon density may be formulated using an auxillary fast parton A — quark or gluon, related to a Wilson line
- The impact factor of such a fast (eikonal) particle introduces no momentum dependence — it is a constant function of g^* transverse momentum
- The prescription to obtain LO scattering amplitudes $B + g^* \to B'$ with an off-shell gluon g^*:
  Start from a collinear amplitude $q + B \to q' + B'$ or $g + B \to g' + B'$ and factor out the quark or gluon constant impact factor
- In result, a universal "partonic" amplitude is found $B + g^* \to B'$ that does not depend on the kind of auxillary parton used

The auxillary parton method at LO



The auxillary partons at NLO

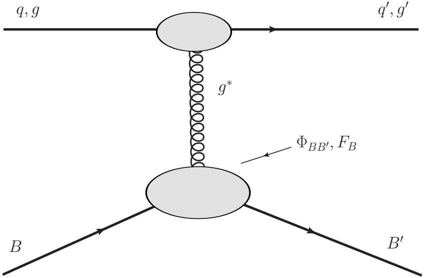
- A naive approach: in analogy to the auxillary parton method at LO — take NLO collinear amplitudes with auxillary parton and factor out constant couplings of auxillary partons
- Clearly, at NLO it does not work. In particular, the impact factors $B + g^* \to B'$ determined in this way depend on the choice of the auxillary parton (q or g)
- An expected solution: the auxillary parton definition has to include the NLO effects. It is necessary to reinterpret the auxillary parton as a physical parton and dress it up in NLO QCD effects

The auxillary partons at NLO

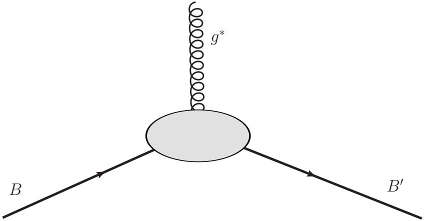
- Regge factorization at NLO guarantees universality of the remaining part of the amplitude: the $B + g^* \longrightarrow B'$ impact factor, the evolution kernel and of the unintegrated gluon density in the target B
- Explicit checks performed for several different processes — with the NLO auxillary parton impact factors the $B + g^* \longrightarrow B'$ amplitudes do not depend on the kind of the auxillary parton
- Jumping ahead: the NLO auxillary parton impact factors agree the NLO quark and gluon inclusive impact factors obtained by M. Ciafaloni and D. Colferai [hep-ph/9806350]

The auxillary partons at NLO

Collinear amplitude with auxillary parton:



Target impact factor / unintegrated gluon in B:



The strategy of NLO calculations in hybrid factorization

1. Starting point: we consider a “collinear embedding” of a $B + g^* \to B'$ process with an auxillary parton: quark or gluon
2. At one loop the virtual corrections obey the Regge factorization principle
   - The auxillary parton BB' part does not depend on the process.
   - The evolution part is well known gluon Regge trajectory.
   - The remaining part goes into the AA' impact factor.
3. The real emission corrections are also computed for the collinear embedding.
   Necessary to perform the power-expansion in x and division of the phase-space into regions, to obtain contributions to the impact factors A, B and to the evolution kernel

Pros and cons

1. Good theoretical controll provided by the hard factorization framework. Non-trivial issues like the definition of the unintegrated parton densities at higher orders get replaced by a separation procedure for the collinear embedding of an off-shell amplitude
2. Readily available results for amplitudes of many interesting processes, both for both virtual and real emission corrections
3. Well known procedures for IR subtraction schemes for non-trivial final states e.g. Catani–Seymour dipoles for multi-jet final states — well suited for MC implementations
4. However: this approach is limited to linear scattering regime. Still, it may serve as a cross-check for CGC calculations in the low density regime.

Technical details

1. Analysis is carried out in the high energy limit. The collision energy squared $S = \Lambda\nu^2 \gg \mu_i^2$. ν^2 is fixed $\sim$ hadronic mass squared and Λ is a large dimensionless parameter. The leading power of Λ is retained

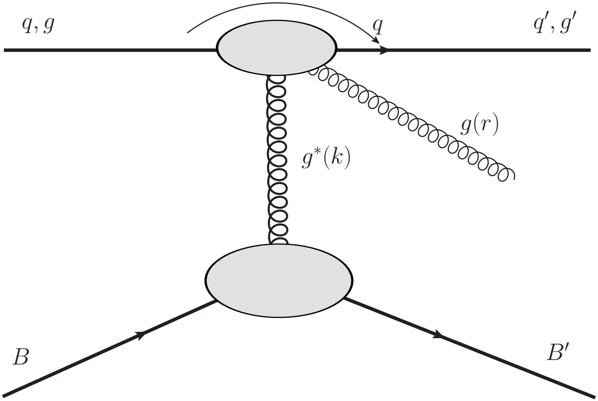
2. We use the dimensional regularization of UV and IR singularities and perform the renormalization of the UV divergencies

3. Scattering amplitudes are obtained using the formalism of helicity amplitudes at NLO, including some general identities

4. Not discussed here: we performed a general analysis of NLO virtual corrections showing the Regge factorization in virtual 1-loop contributions

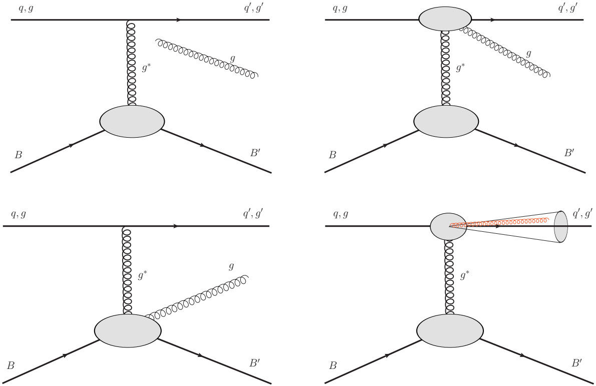
The real radiation: kinematics

Kinematics of the real gluon radiation:



Real gluon radiation: regions

Emitted gluon: energy $\sim x_r$ and $r_\perp$
Coherence of the emission: separation into evolution and impact factors subtle



Guiding principle: separate regions so that the real and virtual IR divergencies match within the impact factors and in the kernel

Treatment of the real radiation

The emission from the auxillary parton is obtained in a well-known form

$$\mathcal{Q}_{\rm aux}(x_q, q_\perp, x_r, r_\perp) = x_q x_r \mathcal{P}_{\rm aux}(x_q, x_r)\, |k_\perp|^2 \Bigg(\frac{c_{\bar{q}}}{|r_\perp|^2 |r_\perp + k_\perp|^2} + \frac{c_q\,(1-x_r)^2}{|r_\perp + k_\perp|^2 |r_\perp + x_r k_\perp|^2} + \frac{c_r\, x_r^2}{|r_\perp|^2 |r_\perp + x_r k_\perp|^2} \Bigg)$$

$$\mathcal{R}_{\rm aux} = \frac{a_\epsilon}{2\pi_\epsilon \mu^{\bar{\epsilon}}} \int_0^u dx\, \mathcal{P}_{\rm aux}(1-x, x) \int d^{2+\bar{\epsilon}} r_\perp\, \theta\big(0 < |r_\perp|^2 < \nu^2 x \bar{x}_{\overline{\rm in}} \Lambda'\big) \times |k_\perp|^2 \Bigg(\frac{c_{\bar{q}}}{|r_\perp|^2 |r_\perp + k_\perp|^2} + \frac{c_q\,(1-x)^2}{|r_\perp + k_\perp|^2 |r_\perp + x k_\perp|^2} + \frac{c_r\, x^2}{|r_\perp|^2 |r_\perp + x k_\perp|^2} \Bigg)$$

$$\mathcal{P}_{\rm aux}(1-x, x) = \frac{\mathcal{P}_{\rm aux}^{\rm pole}}{x} + \mathcal{P}_{\rm aux}^{\rm rest}(x)$$

Treatment of real radiation

1. Separation between the impact factor and the real part of the evolution kernel is necessary: the term in the gluon splitting function $1/z$ generates logs of energy after the z-integration — so it is part of the evolution kernel and is not included in the impact factor

2. When the z-integration extends to 1, the splitting function develops soft singularities, besides collinear singularities appear for $r_\perp \longrightarrow 0$.

3. When the radiation becomes collinear to the initial state auxillary parton and soft, it is associated with this parton and it is not part of the kernel

4. The **angular ordering** condition on gluon emission ensures that the soft-collinear divergencies are absent in the kernel — in agreement with classical results of M. Ciafaloni [hep-ph/9801322] and M. Ciafaloni, D. Colferai [hep-ph/9806350]

Angular ordering

- Analysis of jet fragmentation: importance of color coherence
- Coherent region: wide angle emissions, where a system of partons acts as a coherent antenna
- Fragmentation region: at small emission angles coherence supressed emissions
- Color coherence $\longrightarrow$ angular ordering of gluon emissions

$$\frac{|r_\perp|}{x_r} > \frac{|k_\perp + r_\perp|}{1 - x_r} \simeq |k_\perp + r_\perp|$$

  The phase space for the kernel emission is reduced:

$$|r_\perp|/x_r < \sqrt{s} \longrightarrow |r_\perp|/x_r < \sqrt{s},\ |r_\perp|/x_r > |k_\perp|$$

- The double soft–collinear pole $1/\epsilon^2$ is eliminated from the kernel part

Main result: factorization formula at NLO

A hybrid factorization formula at NLO for parton A on target B scattering

$$d\sigma^{(1)} = \int dx_{\rm in}\, d^2k_\perp\, d\bar{x}_{\overline{\rm in}} \Big\{$$

$$F(x_{\rm in},|k_\perp|)\, f(\bar{x}_{\overline{\rm in}}) \Big[d\mathrm{V}^\star(x_{\rm in},k_\perp,\bar{x}_{\overline{\rm in}}) + d\mathrm{R}^\star(x_{\rm in},k_\perp,\bar{x}_{\overline{\rm in}}) \Big]_{\rm cancelling}$$

$$+\Big[F^{(1)}(x_{\rm in},|k_\perp|) + F(x_{\rm in},|k_\perp|)\Delta_A + \Delta^\star_{\rm coll} \Big] f(\bar{x}_{\overline{\rm in}})\, d\mathrm{B}^\star(x_{\rm in},k_\perp,\bar{x}_{\overline{\rm in}})$$

$$+F(x_{\rm in},|k_\perp|) \Big[f^{(1)}(\bar{x}_{\overline{\rm in}}) + \Delta_{\overline{\rm coll}} \Big] d\mathrm{B}^\star(x_{\rm in},k_\perp,\bar{x}_{\overline{\rm in}}) \Big\}$$

with f and F — the collinear and k_T-dependent PDFs resp.
and $f^{(1)}$ and $F^{(1)}$ being the NLO corrections to PDFs

B is for Born, and V – virtual, R – real corrections matrix elements and the related part of the phase space. This formula describes the cross section with auxillary parton, which enters as Δ_A

Inclusive NLO quark/gluon impact factors

$$\Delta_A = \frac{a_\epsilon N_c}{\epsilon}\left(\frac{\mu^2}{|k_\perp|^2}\right)^\epsilon \left[2\mathcal{J}_{\rm univ} + \mathcal{J}_{\rm aux} - 2\ln\frac{s_{\rm in}}{|k_\perp|^2}\right]$$

with

$$\mathcal{J}_{\rm univ} = \frac{11}{6} - \frac{n_f}{3N_c} - \frac{\mathcal{K}}{N_c}(-\epsilon) \quad \text{where} \quad \mathcal{K} = N_c\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5n_f}{9}$$

and

$$\mathcal{J}_{\text{aux-q}} = \frac{3}{2} - \frac{1}{2}(-\epsilon) \quad , \quad \mathcal{J}_{\text{aux-g}} = \frac{11}{6} + \frac{n_f}{3N_c^3} + \frac{n_f}{6N_c^3}(-\epsilon)$$

$\mathcal{J}_{\rm univ} + \mathcal{J}_{\rm aux}$ equals to the NLO quark and gluon impact factors found by M. Ciafaloni and D. Colferai. $\mathcal{J}_{\rm univ}$ is related to running coupling effects.

Counter-terms

In the cross sections at NLO we find contributions that have IR poles

$$\Delta_{\overline{\mathrm{coll}}} = -\frac{a_\epsilon}{\epsilon}\int_{\bar{x}_{\overline{\imath\mathrm{n}}}}^{1} dz\,\left[\mathcal{P}_{\overline{\imath\mathrm{n}}}^{\mathrm{reg}}(z) + \gamma_{\overline{\imath\mathrm{n}}}\delta(1-z)\right]\frac{1}{z} f\left(\frac{\bar{x}_{\overline{\imath\mathrm{n}}}}{z}\right)$$

$$\Delta^{\star}_{\mathrm{coll}} = -\frac{a_\epsilon}{\epsilon}\int_{x_{\mathrm{in}}}^{1} dz\left[\frac{2N_{\mathrm{c}}}{[1-z]_+} + \gamma_{\mathrm{g}}\delta(1-z)\right]\frac{1}{z} F\left(\frac{x_{\mathrm{in}}}{z}, |k_\perp|\right)$$

- They appear because the amplitudes describe the collinear partons at NLO, and they should be used with NLO collinear parton densities. Then the IR poles of the amplitudes and the PDF-s cancel
- In $\Delta^{\star}_{\mathrm{coll}}$ the original IR singularity of the auxillary parton propagates ino the emission of a gluon g^*

Hint on evolution — BFKL and soft logs

1. We analyze the cross sections at fixed order — NLO. Therefore we have no means to derive the evolution equation that necessarily requires an all order nested (ladder) structure.
2. The obtained NLO amplitude, however, should contain a one step of the evolution equation. In particular, the real gluon radiation leads to a following change of the unintegrated gluon density:

$$\tilde{F}(x_{\rm in}, k_\perp) = \frac{2a_\epsilon N_c}{\pi_\epsilon \mu^{\bar{\epsilon}}} \int_{x_{\rm in}}^1 \frac{dz}{z(1-z)} \int \frac{d^{2+\bar{\epsilon}} r_\perp}{|r_\perp|^2} \frac{|k_\perp|^2}{|k_\perp + r_\perp|^2} \theta\Big(|r_\perp| < |k_\perp|(1-z)\Big) F\left(\frac{x_{\rm in}}{z}, k_\perp + r_\perp\right)$$

3. This expression generates the BFKL log (the $1/z$ factor) and the soft log (the $1/(1-z)$ factor). It closely resembles the real emission part of the firstly proposed CCFM equation. Also consistent with recent proposal of M. Nefedov [2003.02194]

Towards a general approach to NLO impact factor

The obtained results aim to provide a general framework to compute NLO k_T dependent impact factors $B + g^* \to B'$ from NLO collinear 1-loop amplitudes with an auxillary parton AA':

1. Start from NLO $A + B \to A' + B'$ amplitude in high energy limit
2. Subtract the NLO auxillary parton contribution from the NLO parton-level cross section
3. Subtract one step of the LO evolution kernel with appropriate phase space cut (issue of the soft logs!)
4. Compute the contribution of the resolved real radiation at the $B \to B'$ side
5. Integrate over the radiated gluon phase space, if necessary with appropriate IR subtraction terms
6. First applications: extraction of virtual 1-loop corrections to $gg^* \to H$, $qg^* \to qe^+e^-$, $\gamma^* g^* \to q\bar{q}$
   E. Blanco, A. Giachino, A. v. Hameren, P. Kotko, arXiv:2212.03572
   — Treatment of the real radiation: [A. Giachino, A. v. Hameren, G. Ziarko,2312.02808]

Conclusions and outlook

1. We implement an approach to k_T-factorization at NLO based on embedding the off-shell amplitudes into collinear ones
2. NLO formula in hybrid factorization is proposed
3. A scheme is developed to extract the NLO k_T dependent impact factors from NLO collinear amplitudes
4. The framework works in momentum space and for linear evolution. It may be applied to compute processes at high Q^2 at EIC and provide checks for dipole–saturation approach
5. In turn, a possible definition of an unitegrated gluon using an enveloping collinear amplitude and the NLO parton impact factors

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THANKS!