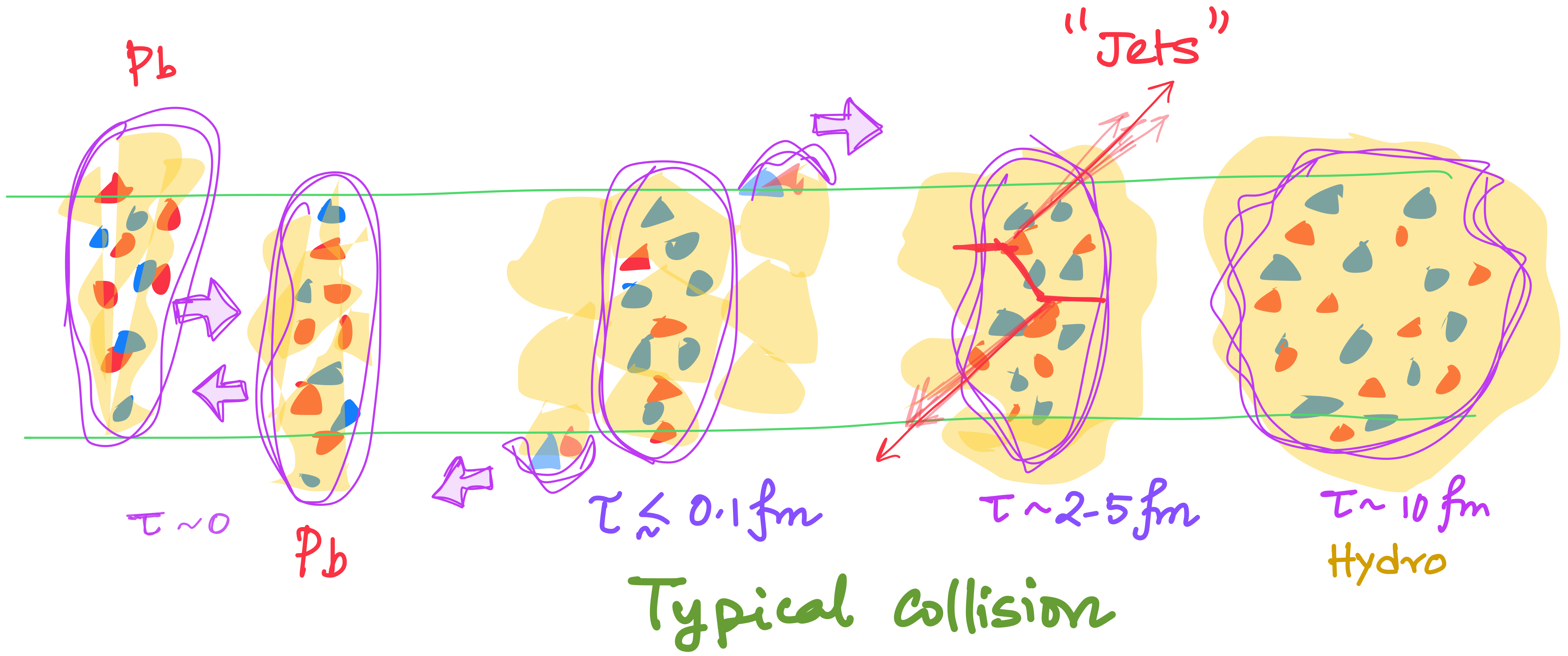
Exploring the equilibration time of the QGP with jet quenching



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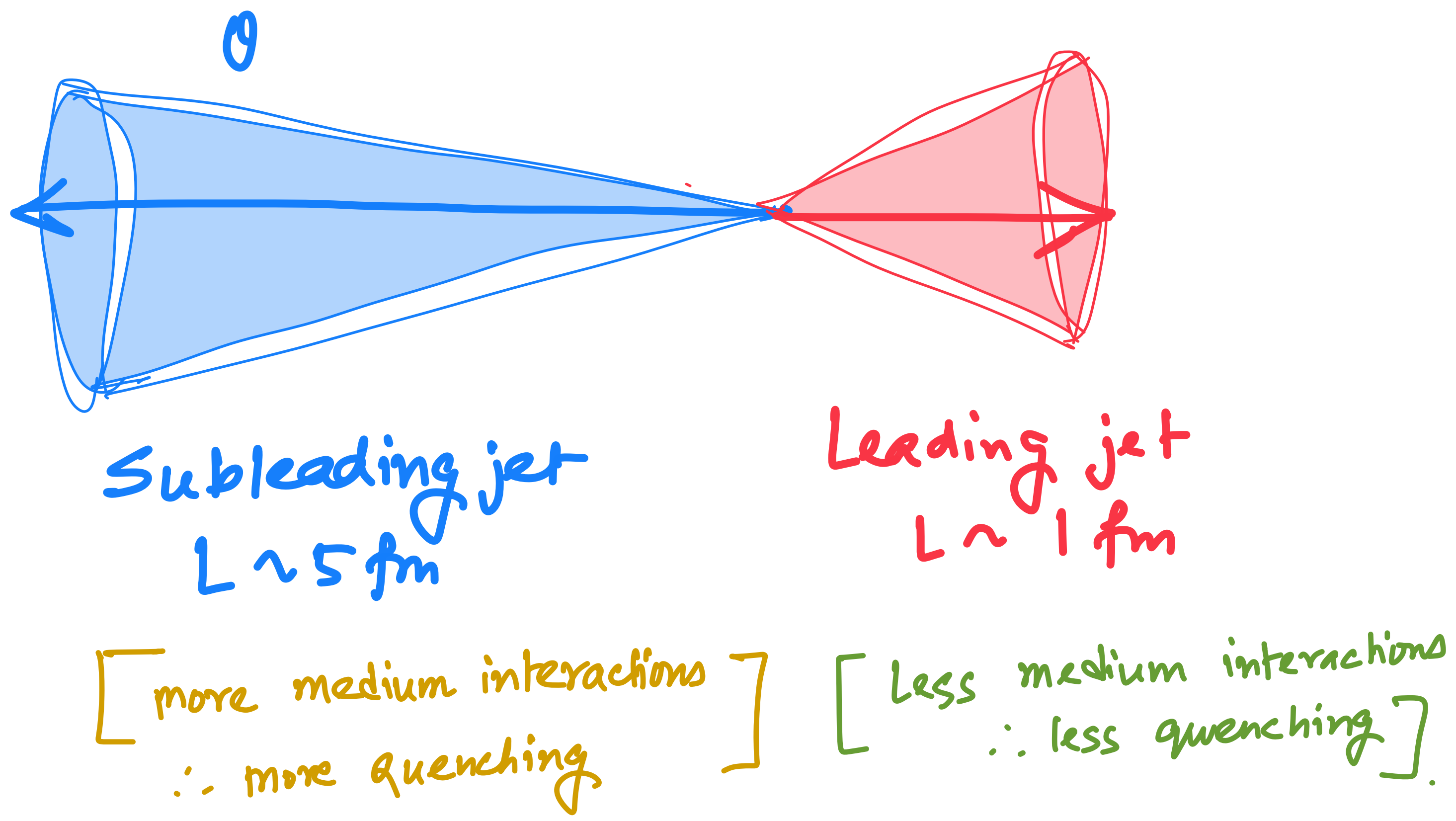
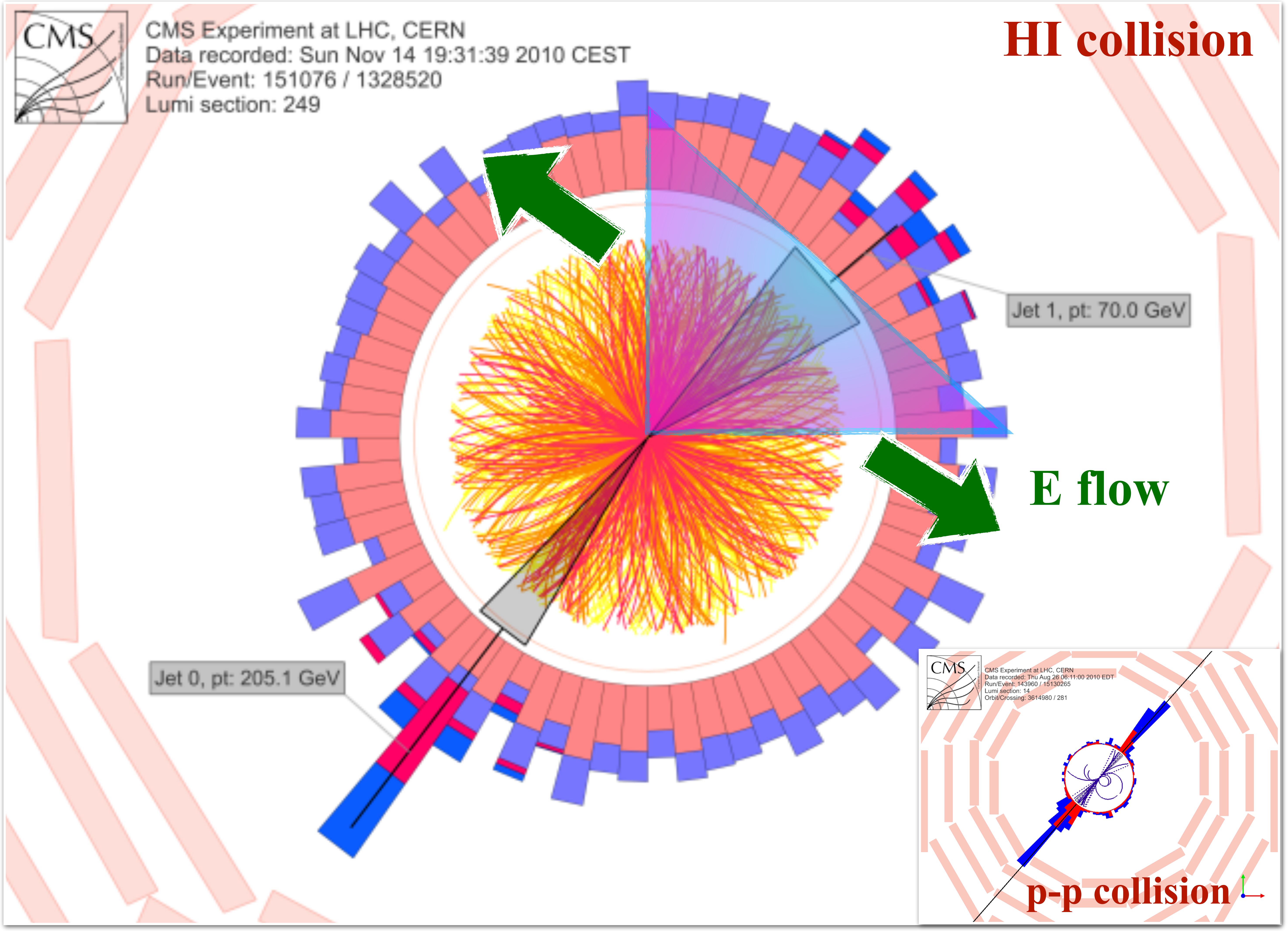
XXX Cracow EPIPHANY Conference
on Precision Physics at High Energy Colliders
dedicated to the memory of Staszek Jadach
8-12 January 2024

Fellowship Agreement No: PAN.BFD.S.BDN.612.022.2021, PASIFIC Program, Horizon 2020 within the Marie Skłodowska Curie Actions.

Introduction to jet quenching

- A Jet is an energetic and collimated bunch of particles produced in a high-energy collision.
- Jets are extended objects, ideal to study space time evolution.
- energy is lost in soft particles at large angles.





Vacuum vs medium picture

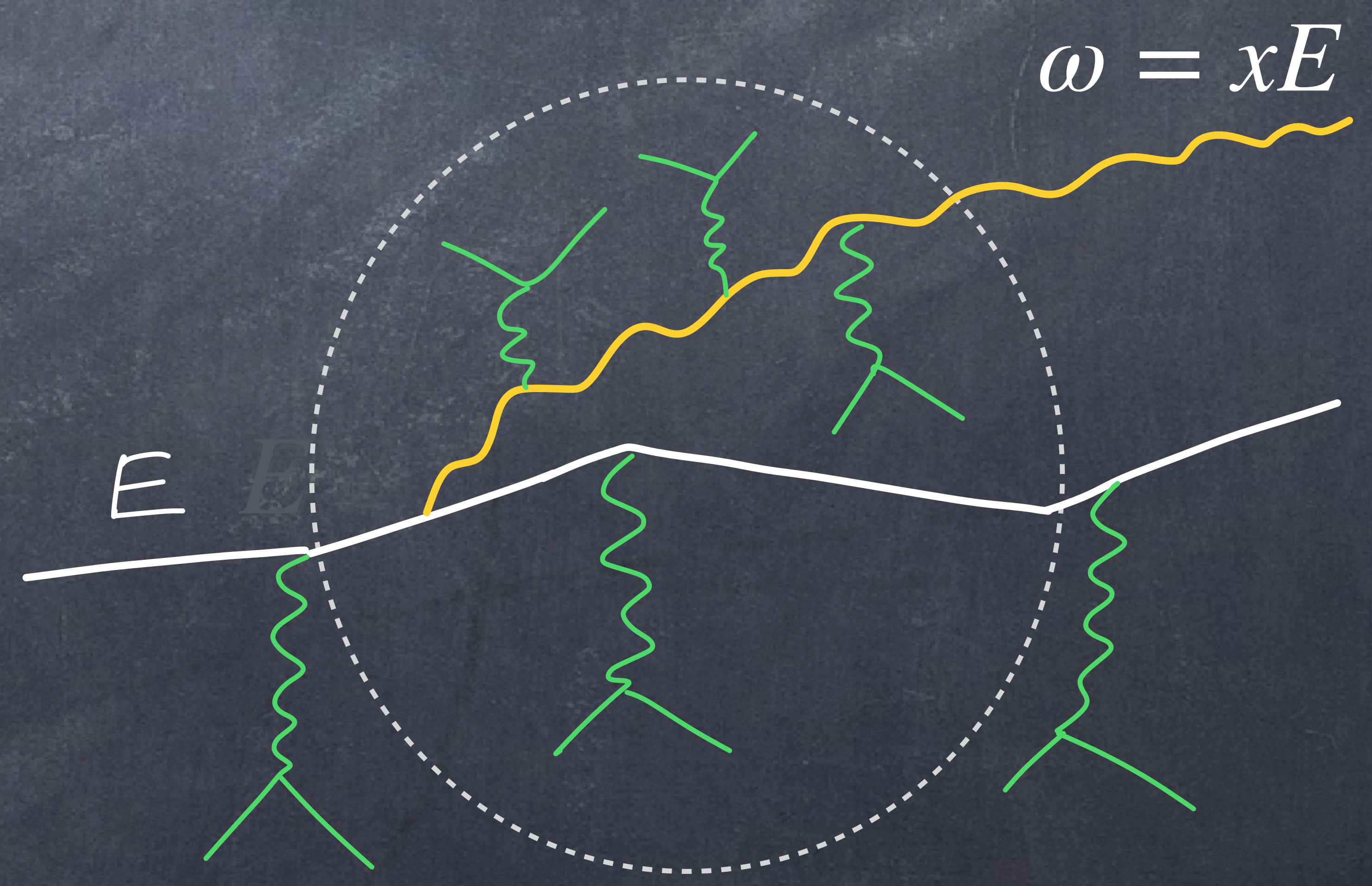
- ## Vacuum picture :

- 1. Jets originate from energetic partons (quarks and gluons) that successively branch (similarly to an accelerated electron that radiates photons).
- 3. Diverges for ***soft*** and ***collinear*** radiation.
- 4. Successive branchings are ordered from larger to smaller angles.

$$dP = \alpha_s C_R \frac{d\omega}{\omega}\frac{d\theta}{\theta}$$

Jets in the QGP: Multi-scale problem !

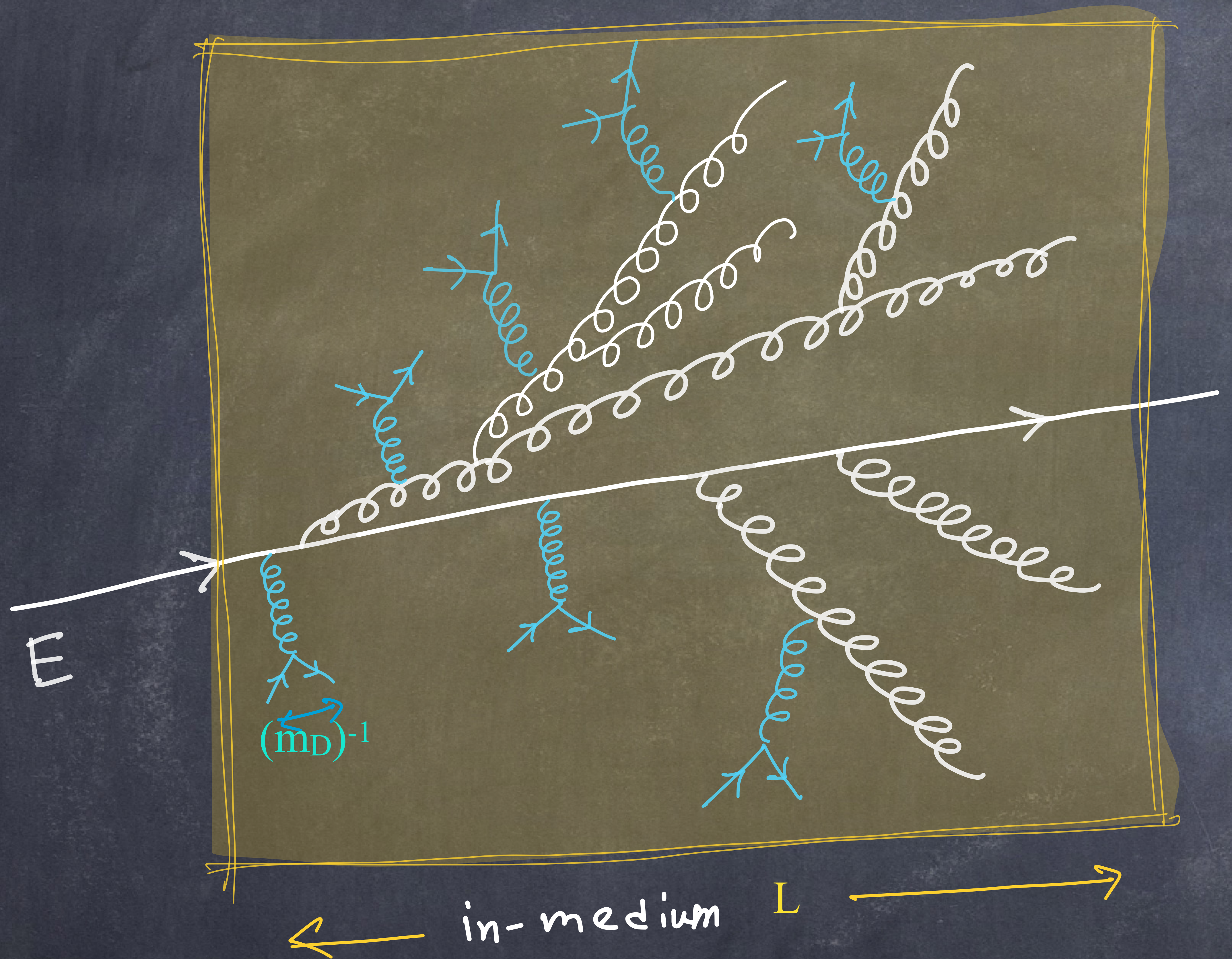
- ## Medium picture :



- Many interactions occur during the formation of a soft gluon.
  - Suppression of small angle radiation.
- No collinear divergence!
- In QCD: formation time of gluons decrease with energy decrease!

$$\longleftarrow \quad t_f = \frac{\omega}{k_\perp^2} \sim \sqrt{\frac{\omega}{\hat{q}}} \quad \longrightarrow$$

Setting up the picture



Propagation of a fast parton in dense medium

Branching

Scattering

Dynamical picture

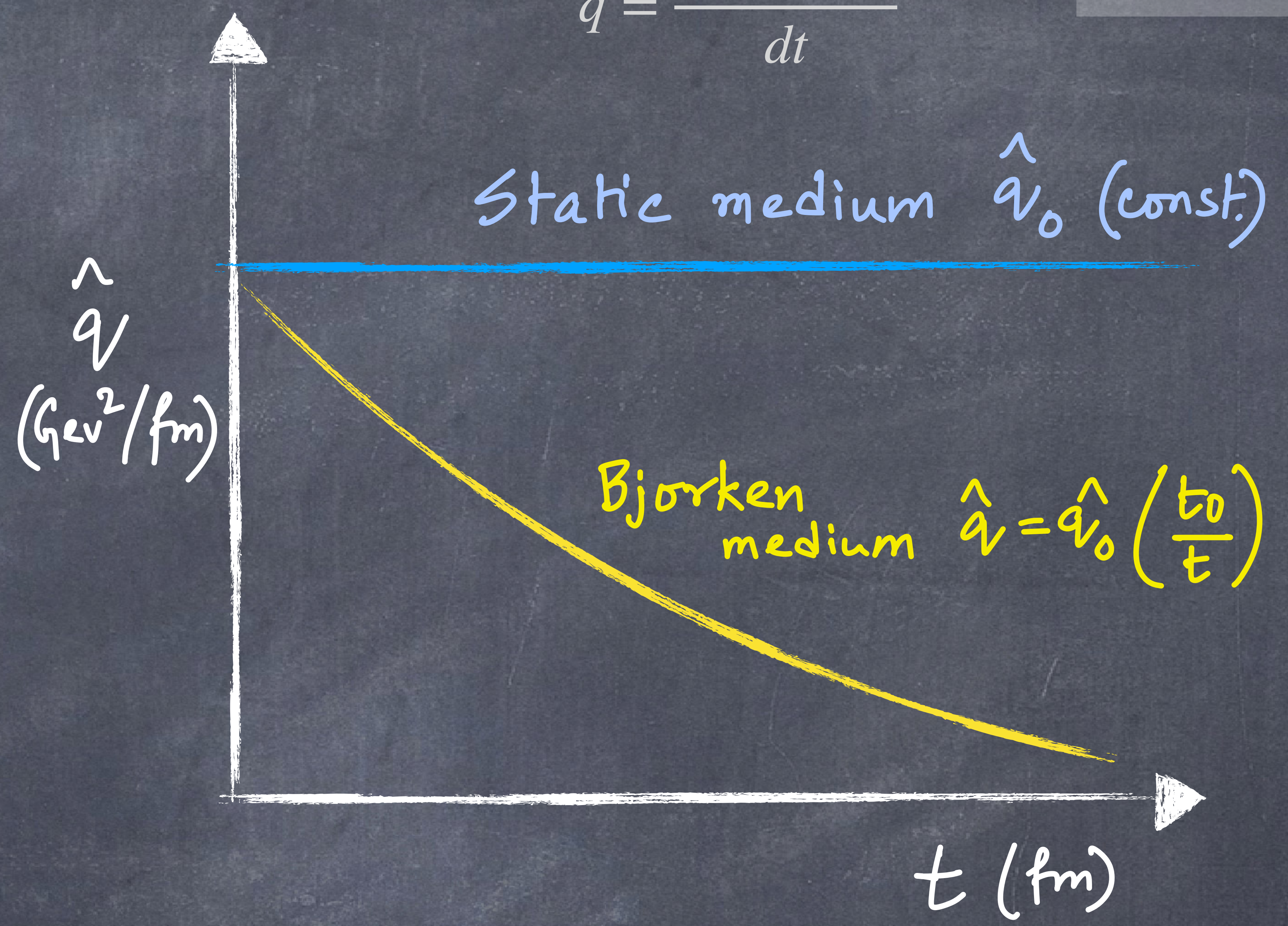
Information on "soft" and "hard" gluons in angular space?

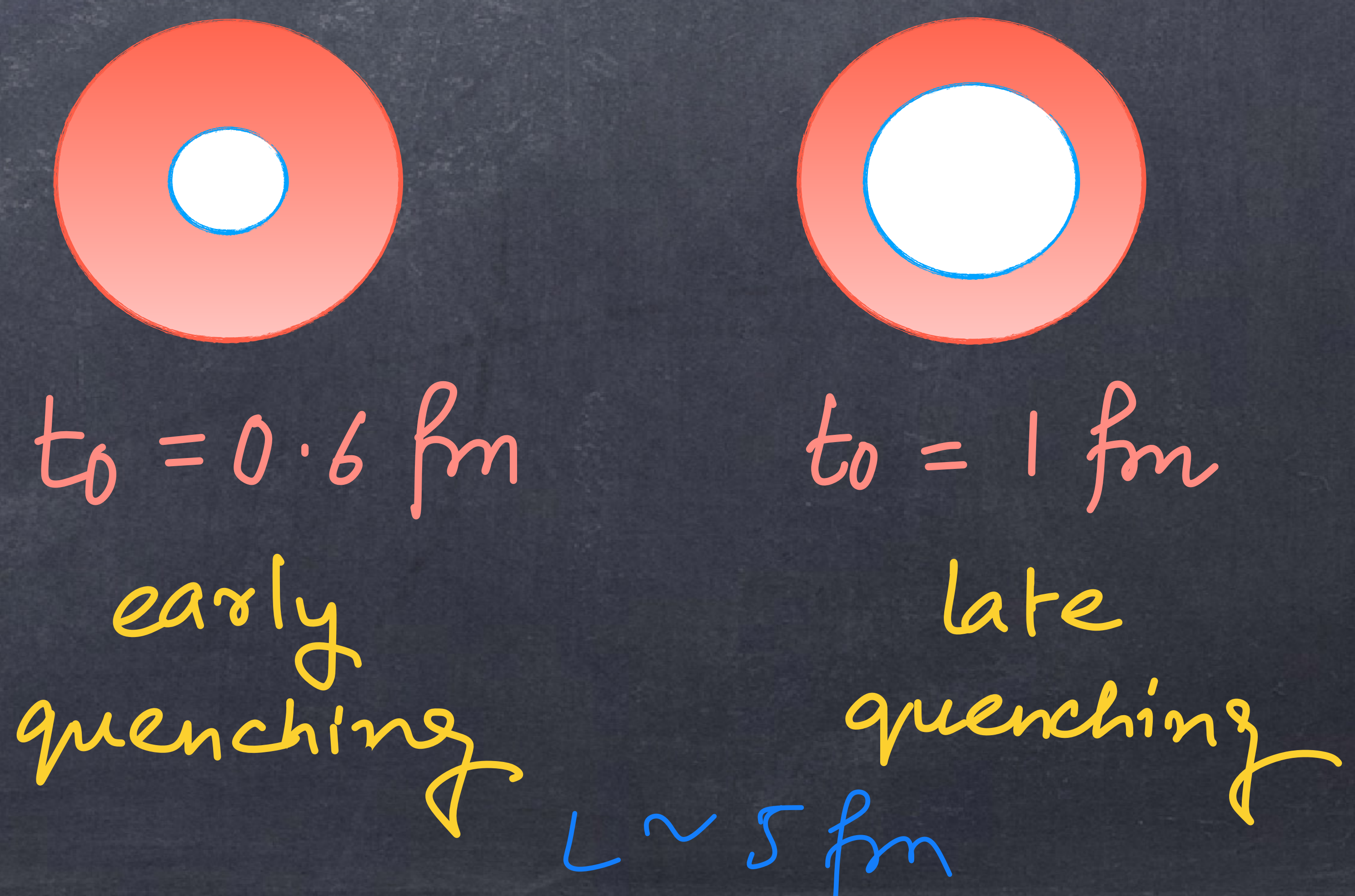
Souvik Priyam Adhya

Modelling the medium

- Inclusion of finite medium size effects.
- Expanding medium with varying time for the onset of the quenching (equilibration time).
- Scaling relations in effective lengths between expanding and static medium profiles, successful in describing R_{AA} *and* v_2 *of jets* with *sensitivity* to medium expansions recently.
- The QGP favours an early quenching time/equilibration time
- Exploratory study of hard and soft jets in angular regions through the equilibration time.

$$\hat{q} = \frac{d < k_T^2 >}{dt}$$





Adhya, Kutak, Placzek, Rohrmoser, Tywoniuk, EPJC, 2022
Adhya, Salgado, Spousta, Tywoniuk, EPJC, 2022.
Adhya, Salgado, Spousta, Tywoniuk, JHEP, 2020.

What are we aiming to explore ?

- QUESTION : Possible to have an analytical formula of the spectra across all gluon frequencies ?

- The static medium has already been explored recently, is it enough ?

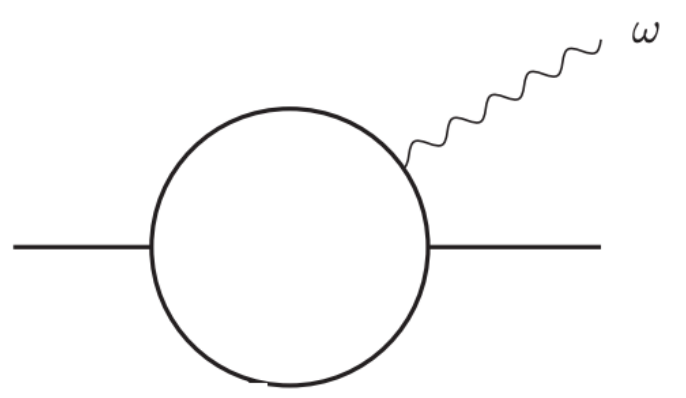
[STATIC medium IOE]Y. Mehtar-Tani, K.Tywoniuk and others (2019 onwards)

- Finite medium size effects :
  - include realistic medium scenarios relevant for inclusion in phenomenological in-medium parton shower models.
  - validity of the soft multiple and hard scattering not only as a function of energy but also as a function of the initial quenching time of the medium.

  - ANSWER : Are multiple scatterings important for radiative in-medium parton showers ?

Medium induced gluon radiation spectra

- Various MC in-medium parton showers use two analytical approaches :
  - DILUTE medium: Single-hard scattering approximation (**Opacity expansion**).
  - **DENSE** medium: multiple-soft scattering. All order re-summation w/o Coulomb logarithm; Harmonic oscillator (HO) approach [BDMPS-Z (1996), C. A. Salgado, U. Weidemann (2006), K. Tywoniuk, S. P. Adhya (2022) …].
- Also full numerical solutions [Caron-Huot and Gale (2010), Ke , Xu, Bass (2018) …]



LO (N=0): vacuum radiation

Opacity $\chi \equiv L/\lambda$ => denseness of the medium.

- ($L << \lambda$) : Medium DILUTE, or weakly interacting
- ($L >> \lambda$) : Medium DENSE, or strongly interacting

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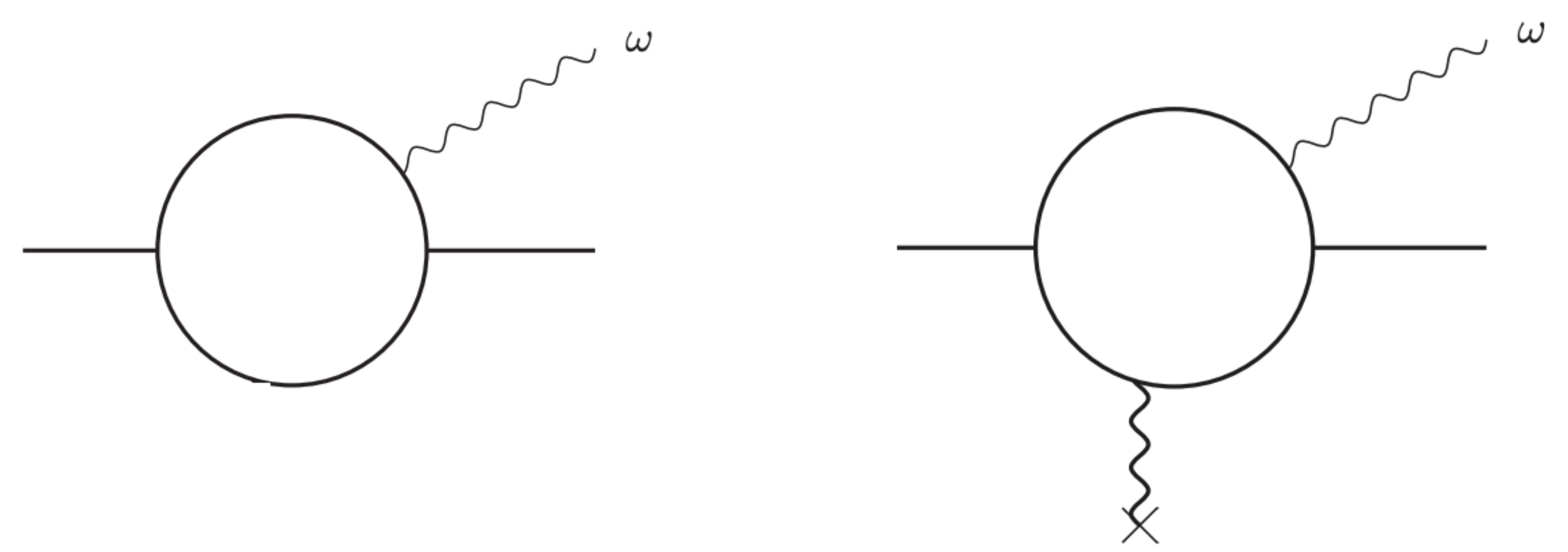

LO (N=0): vacuum radiation

NLO (N=1): In medium Single scattering

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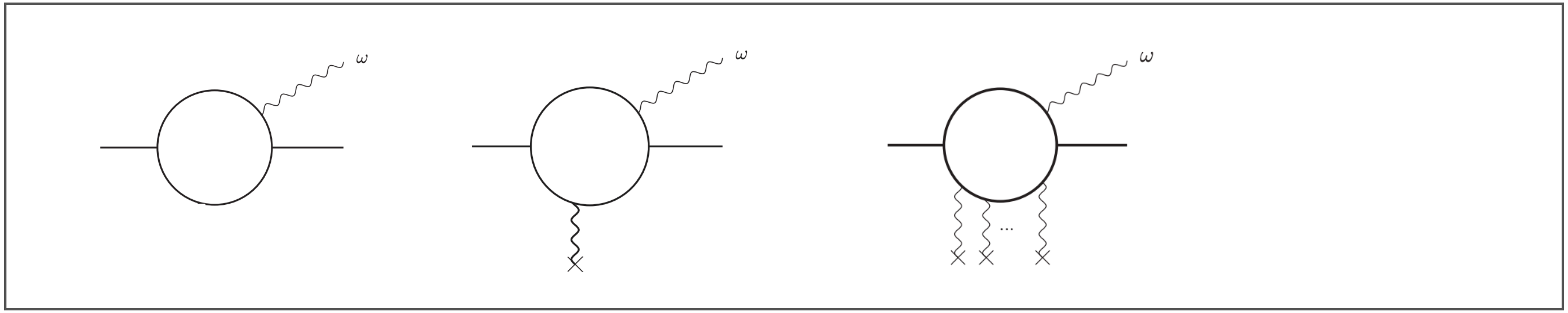

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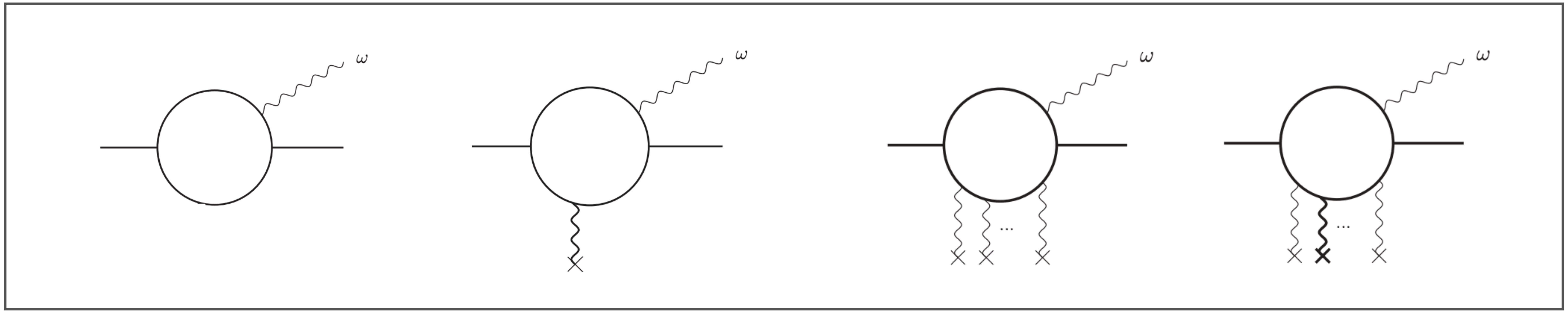

LO (N=0): vacuum radiation

NLO (N=1): In medium Single scattering

LO (HO): Multiple soft scatterings (wavy vertical lines)

NLO : One hard scattering included (*thick wavy line*)
+
Multiple soft scattering re-summed to all orders

Opacity $\chi \equiv L/\lambda$ => denseness of the medium.

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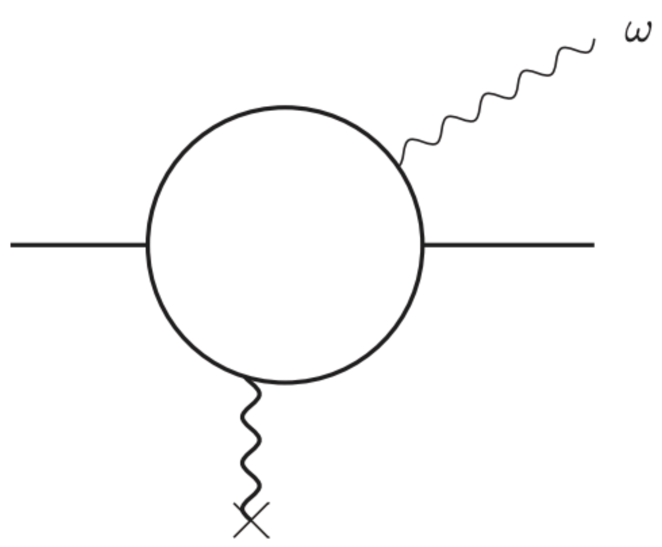
Opacity expansion (N = 1)/GLV

- In high-energy regime $\omega \gg \omega_c$, where OE formally valid.

The spectrum in a STATIC medium reads :

$$\omega\frac{dI^{N=1}}{d\omega} \simeq 2\bar{\alpha}\frac{\pi}{4}\chi\frac{\bar{\omega}_c}{\omega} \qquad \chi = \frac{L}{\lambda}$$



NLO (N=1): In medium Single scattering

[STATIC medium] J. Isaksen, A. Takacs, K. Tywoniuk (2023)

The spectrum in a GENERIC EXPANDING medium ,

$$\omega\frac{dI^{N=1}}{d\omega} \simeq 2\bar{\alpha}\frac{\pi}{2}\chi\frac{\bar{\omega}_c}{\omega}g_\alpha(x_m) \approx 2\bar{\alpha}\frac{\pi}{2(2-\alpha)}\chi\left(\frac{t_m}{L}\right)^{\alpha}\frac{\bar{\omega}_c}{\omega}$$

$$g_\alpha(x_m) \approx x_m^\alpha/(2-\alpha)$$

$$\hat{q}_0(t) = \begin{cases} \hat{q}_0\left(\frac{t_m}{t+t_m}\right)^{\alpha} & \text{for } t < L \\ 0 & \text{for } t > L \end{cases}$$

$$\bar{\omega}_c = \frac{1}{2}\mu^2 L$$

$$\mu^2 \sim m_D^2 = (1 + N_f/6)g^2T^2.$$

Question: Re-definition of scales by introducing expanding medium ? But first, lets have a look at ROE ..

Wiedemann (2000); Gyulassy, Levai, Vitev (2001)

Pushing to re-summed opacity expansion (ROE)

- Soft emissions with short formation times, a single scattering still gives the leading contribution to the spectrum (Bethe-Heitler regime).

Sudakov FF = probability of no elastic scattering b/w two times

- Expansion of finite transverse mom. exchange (*real*) + all-order re-summation of zero transverse mom. exchange (*virtual, through Sudakov*) in scattering potential.

$$\Sigma(\boldsymbol{k}^2, t)^{HTL} = \frac{\hat{q}_0(t)}{\boldsymbol{k}^2 + m_D(t)^2}$$

- The elastic Sudakov form factor (GENERIC EXPANDING medium)

$$\Delta(t,t_0)^{HTL} \equiv \mathrm{e}^{-\frac{3}{c_1(\alpha-3)}\left[t\left\{\hat{q}_0\left(\frac{t_m}{t+t_m}\right)^{\alpha}\right\}^{1/3} - t_0\left\{\hat{q}_0\left(\frac{t_m}{t_0+t_m}\right)^{\alpha}\right\}^{1/3} + t_m\left\{\left(\hat{q}_0\left(\frac{t_m}{t+t_m}\right)^{\alpha}\right)^{1/3} - \left(\hat{q}_0\left(\frac{t_m}{t_0+t_m}\right)^{\alpha}\right)^{1/3}\right\}\right]}$$

- The ROE spectrum in a GENERIC EXPANDING medium ,

$$\omega\frac{\mathrm{d}I^{N_r=1}}{\mathrm{d}\omega} = \frac{4\alpha_s C_R}{\omega}\int_0^L \mathrm{d}t_2 \int_0^{t_2} \mathrm{d}t_1 \int_{\boldsymbol{p}} \Sigma(\boldsymbol{p}^2, t_2)\Delta(t_2, t_1)\sin\left[\frac{\boldsymbol{p}^2}{2\omega}(t_2 - t_1)\right].$$

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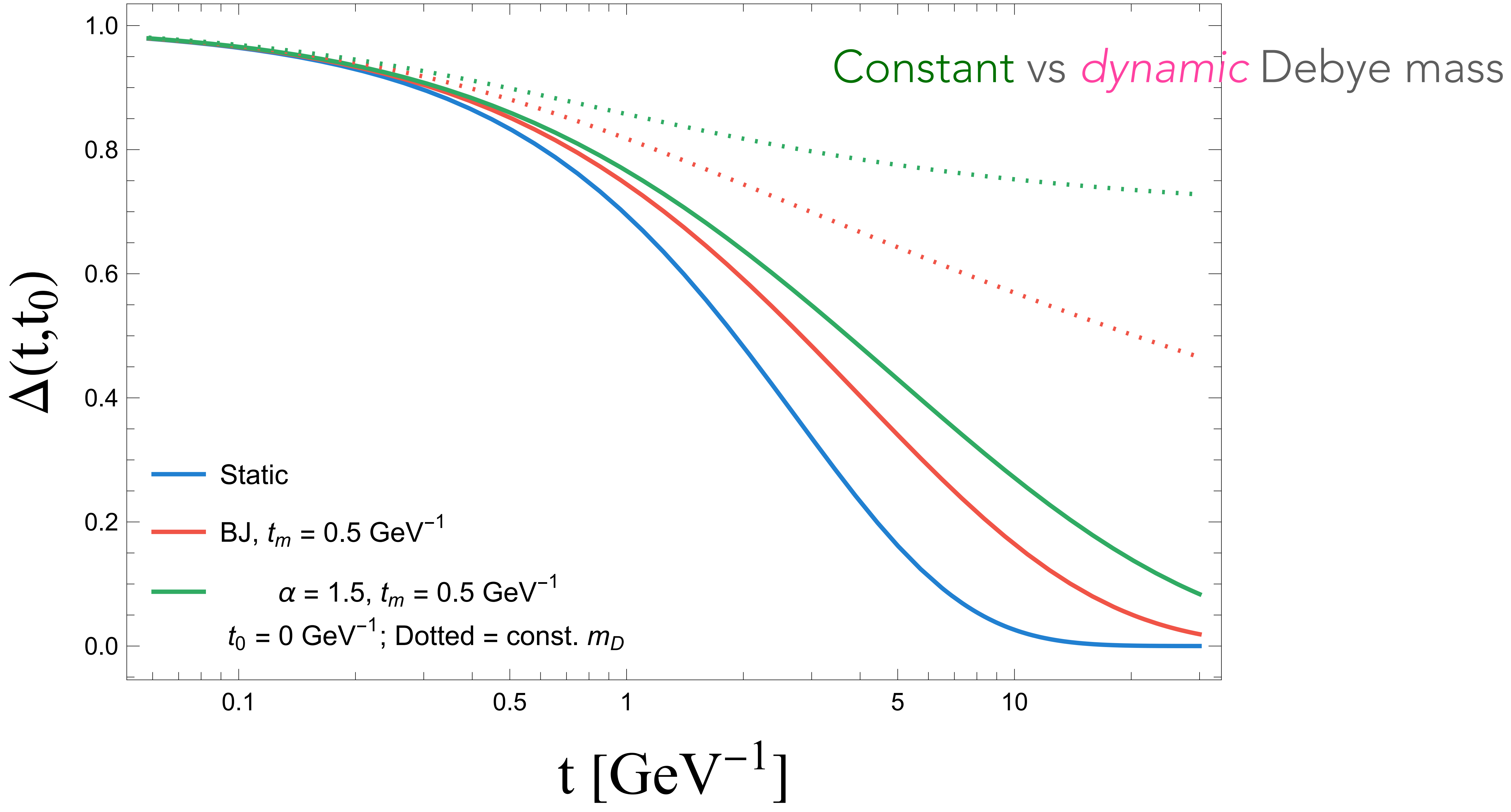
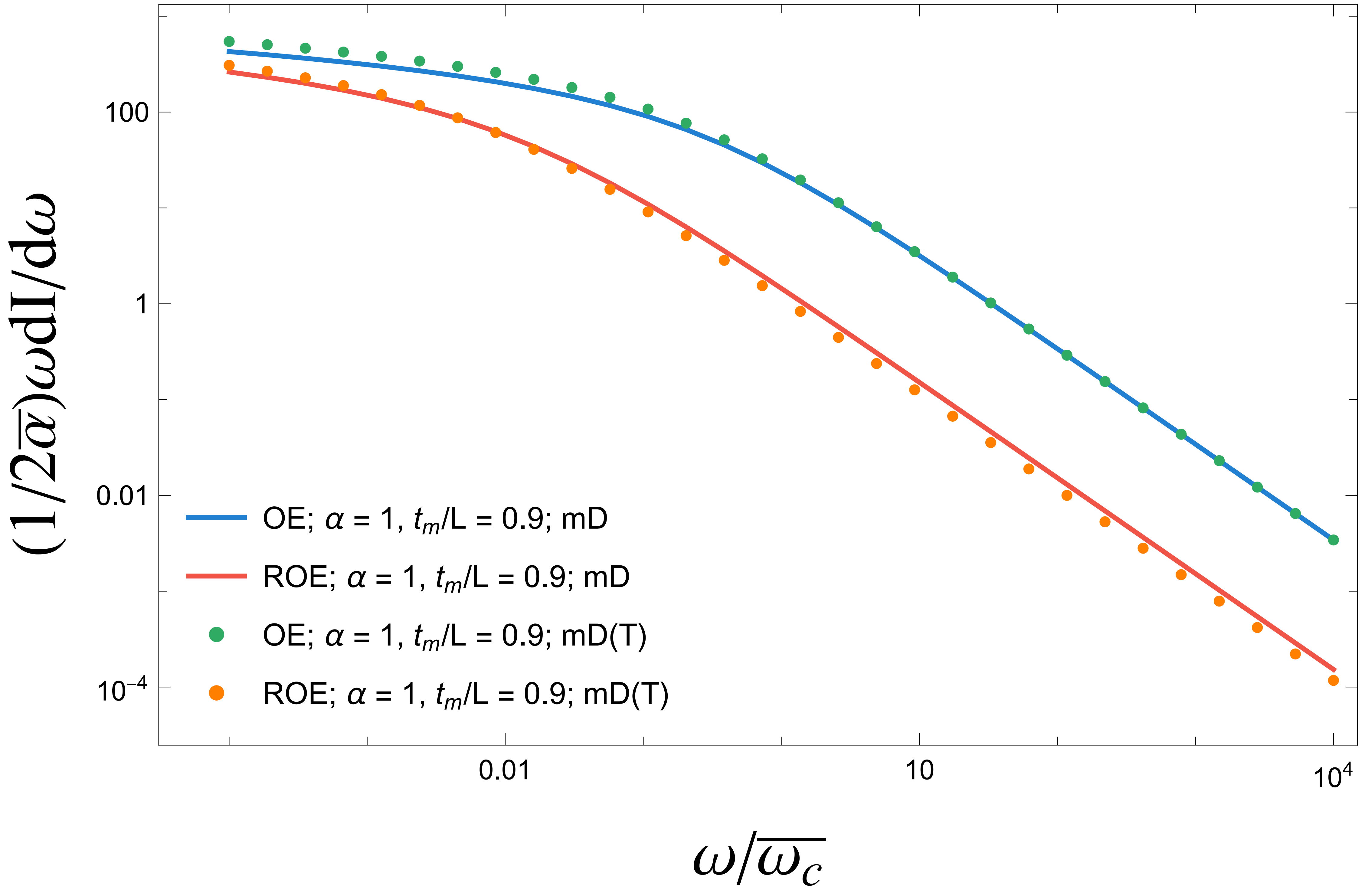
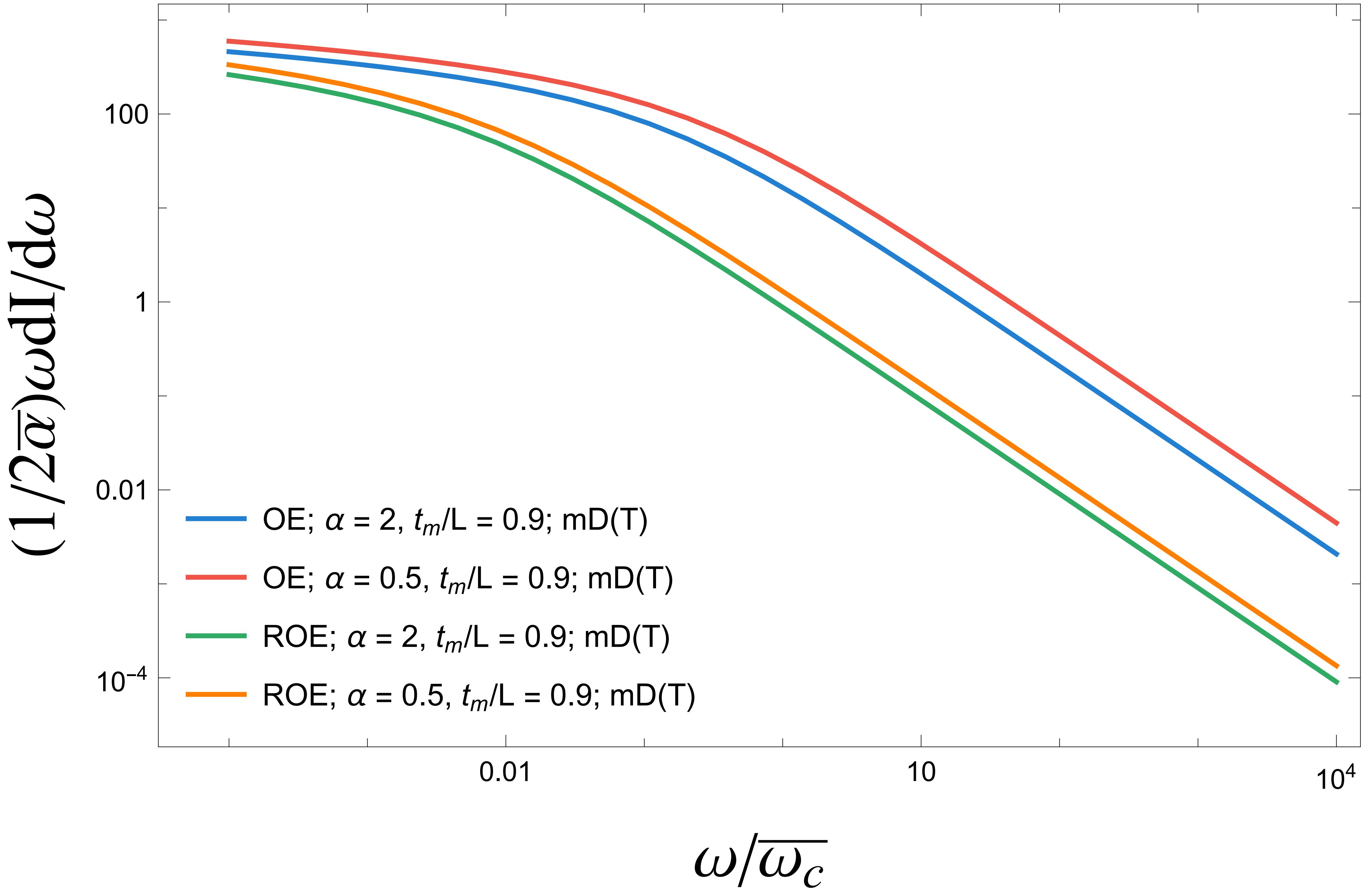


The ROE and OE gluon spectra

Sensitivity to time dependent Debye mass



Sensitivity to medium expansion parameter



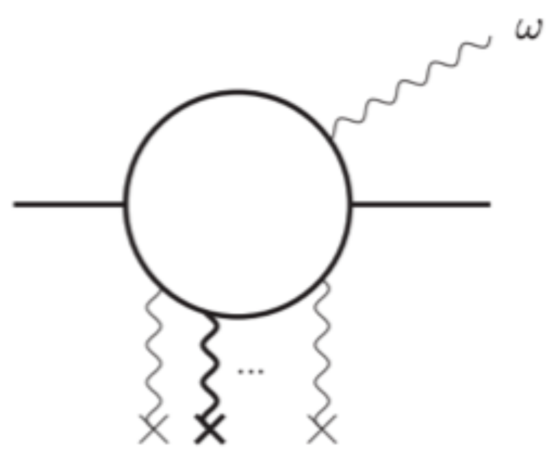
Question:

- Impact of time dependent Debye mass
  - Another level of complexity/ completeness ?

$$T(t) = T_0 \left(\frac{t_m}{t+t_m}\right)^{\alpha/3}$$

SPA, K. Tywoniuk (in preparation)

Improved opacity expansion in generic medium



- Achieved by expanding the leading log potential around the harmonic oscillator.
- We have to match the spectra in the soft ω -> 0 limit.
  - Need a matching scale Q^2 (chosen as typical transverse mom. generated during splitting).

$$\nu = \frac{1}{2-\alpha}$$

- The LO spectra (0) spectra reads (BDMPS-Z) :

$$\lim_{\omega\to 0} \omega \frac{\mathrm{d}I^{(0)}}{\mathrm{d}\omega} = \bar{\alpha}\, 2\nu \sqrt{\frac{\hat{q} t_m^{\alpha}}{\omega}} \left[(L+t_m)^{\frac{1}{2\nu}} - t_m^{\frac{1}{2\nu}} \right]$$

- The NLO (1) spectra reads :

$$\lim_{\omega\to 0} \omega \frac{\mathrm{d}I^{(1)}}{\mathrm{d}\omega} = \left(\frac{\hat{q}_0}{\hat{q}}\right) \frac{\bar{\alpha}}{2} 2\nu \sqrt{\frac{\hat{q} t_m^{\alpha}}{\omega}} \left[(L+t_m)^{\frac{1}{2\nu}} \Xi(L) - t_m^{\frac{1}{2\nu}} \Xi(0) \right]$$

$$\Xi(s) = \gamma_E + \frac{\pi}{4} + 2\nu - 1 + \log\left[\frac{\sqrt{\hat{q}\omega}}{\sqrt{2}Q^2} \left(\frac{t_m}{s+t_m} \right)^{\frac{2\nu-1}{2\nu}} \right]$$

- **Features :**
  - Impossible to choose matching Q scale for BOTH **LOGS** to vanish.
  - The spectra *includes large frequency limit for the OE too.*
  - The spectral structure retains "memory" of medium evolution.

SPA, K.Tywoniuk (in preparation)

Improved opacity expansion in generic medium

$L^{\frac{1}{2\nu}} \gg t_m^{\frac{1}{2\nu}}$ approximation

(USEFUL analytical insight to choose matching scale)

- Ratio of radiative spectrum to NLO in expansion around Harmonic oscillator (LO) gives matching scale.

$$\lim_{\omega\to 0} \frac{\mathrm{d}I^{(1)}/\mathrm{d}\omega}{\mathrm{d}I^{(0)}/\mathrm{d}\omega} \approx \left(\frac{\hat{q}_0}{\hat{q}}\right)\frac{1}{2}\left\{\gamma_E + \frac{\pi}{4} + 2\nu - 1 + \log\left[\frac{\sqrt{\hat{q}\omega}}{\sqrt{2}Q^2}\left(\frac{t_m}{L}\right)^{\frac{2\nu-1}{2\nu}}\right]\right\}$$

$$\lambda = \mu^2/\hat{q}_0$$

$$Q^2 = \sqrt{\hat{q}\omega\left(\frac{t_m}{L}\right)^{\alpha}} = \sqrt{\hat{q}_0\omega\left(\frac{t_m}{L}\right)^{\alpha}\ln\frac{Q^2}{\mu_*^2}}$$

$$\hat{q}_0(t) = \begin{cases} \hat{q}_0\left(\frac{t_m}{t+t_m}\right)^{\alpha} & \text{for } t < L \\ 0 & \text{for } t > L \end{cases}$$

- Re-definition of the scales of the problem (**$\omega_c \gg \omega_{BH}$**); ω_{BH} = Bethe-Heitler frequency

$$\mu^2\lambda\left(\frac{L}{t_m}\right)^{\alpha} \ll \omega \ll \hat{q}t_m^{\alpha}L^{2-\alpha}$$

$$\omega_{\mathrm{BH}} \sim \mu^2\lambda\left(\frac{L}{t_m}\right)^{\alpha}$$

$$1 \ll \frac{L}{\lambda}\left(\frac{t_m}{L}\right)^{\alpha/3}$$

Strict conditions on Equilibration time and MFP

Fixing matching scale on level of rate

- A *more* correct way of dealing with the non-local nature of the emission spectrum ==> fix the scale at the level of the parton splitting rate.

$$\lim_{\omega\to 0}\frac{\mathrm{d}I^{(1)}/(\mathrm{d}\omega\mathrm{d}t)}{\mathrm{d}I^{(0)}/(\mathrm{d}\omega\mathrm{d}t)} = \left(\frac{\hat{q}_0}{\hat{q}}\right)\frac{1}{2}\left\{\gamma_E + \frac{\pi}{4} + \log\left[\frac{\sqrt{\hat{q}\omega}}{\sqrt{2}Q^2}\left(\frac{t_m}{t+t_m}\right)^{\alpha/2}\right]\right\}$$

- The effective jet transport parameter can be written as,

$$Q^2(t) = \sqrt{\hat{q}\omega\left(\frac{t_m}{t+t_m}\right)^{\alpha}} = \sqrt{\hat{q}(t)\omega}$$

$$\hat{q}_{\mathrm{eff}}(t) = \hat{q}_0(t)\ln\frac{Q^2(t)}{\mu_*^2}\left(1+\frac{1.016}{\ln\frac{Q^2(t)}{\mu_*^2}}\right)$$

- Re-definition of the scales of the problem,

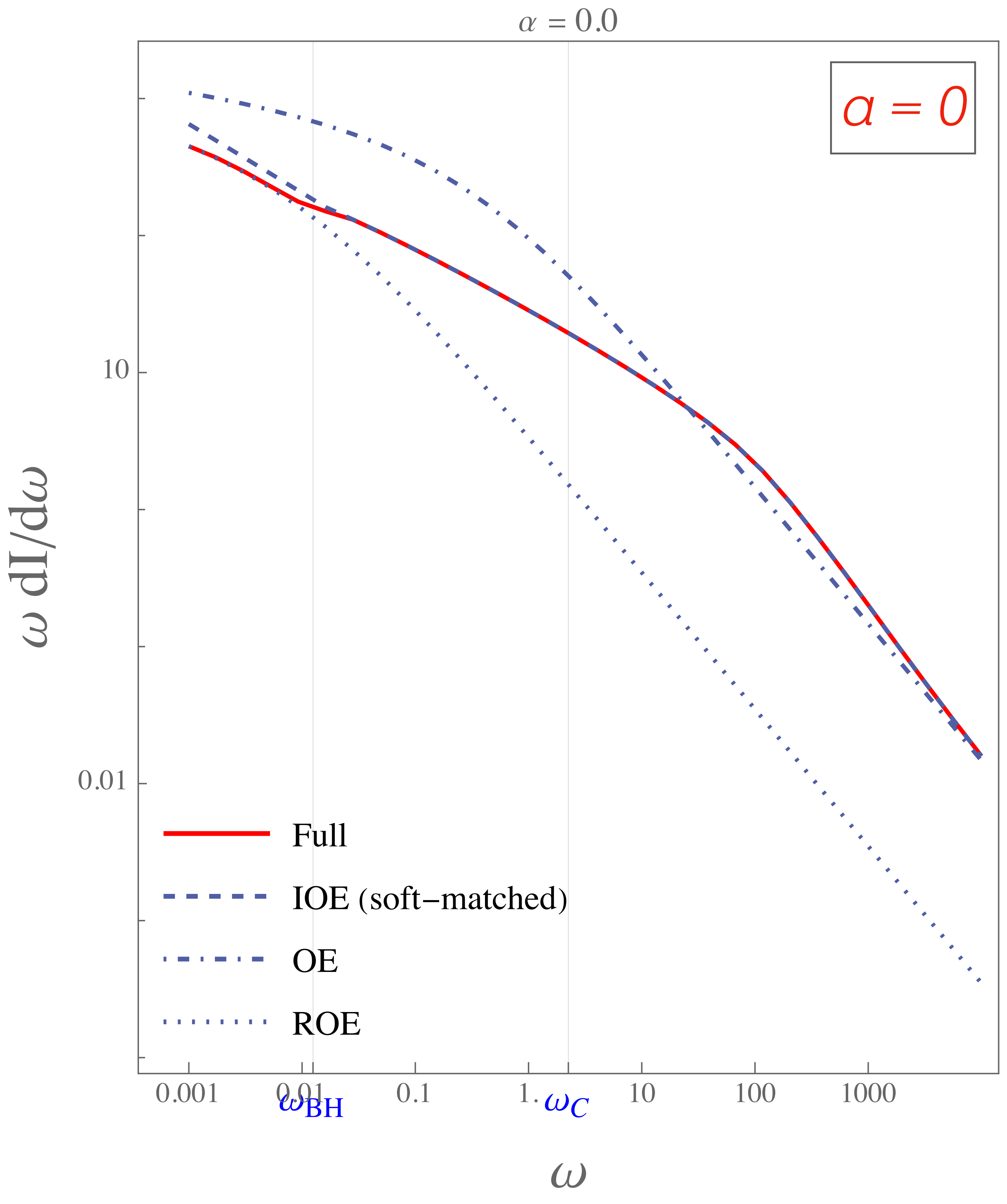
$$\omega_c^{(\alpha)}(s) = \frac{1}{2}\hat{q}s^2 f_\alpha^2(t_m/s)$$

$$\omega_{\mathrm{BH}}^{(\alpha)}(s) = \frac{2\mu_*^4\,\mathrm{e}}{\hat{q}_0}\left(\frac{s+t_m}{t_m}\right)^{\alpha}$$

where $f_\alpha(x) = x^{\alpha/2}[(1+x)^{1-\alpha/2} - x^{1-\alpha/2}]/(1-\alpha/2)$

Shrinking phase space for multiple emissions

SPA, K. Tywoniuk (in preparation)



- In static medium (α = 0), arrived at a nice matching spectra over a broad kinematical range.

- For **$\omega_c >> \omega_{BH}$**, we have to demand

$$1 \ll \frac{L}{\lambda}\left(\frac{t_m}{L}\right)^{\alpha/3}$$

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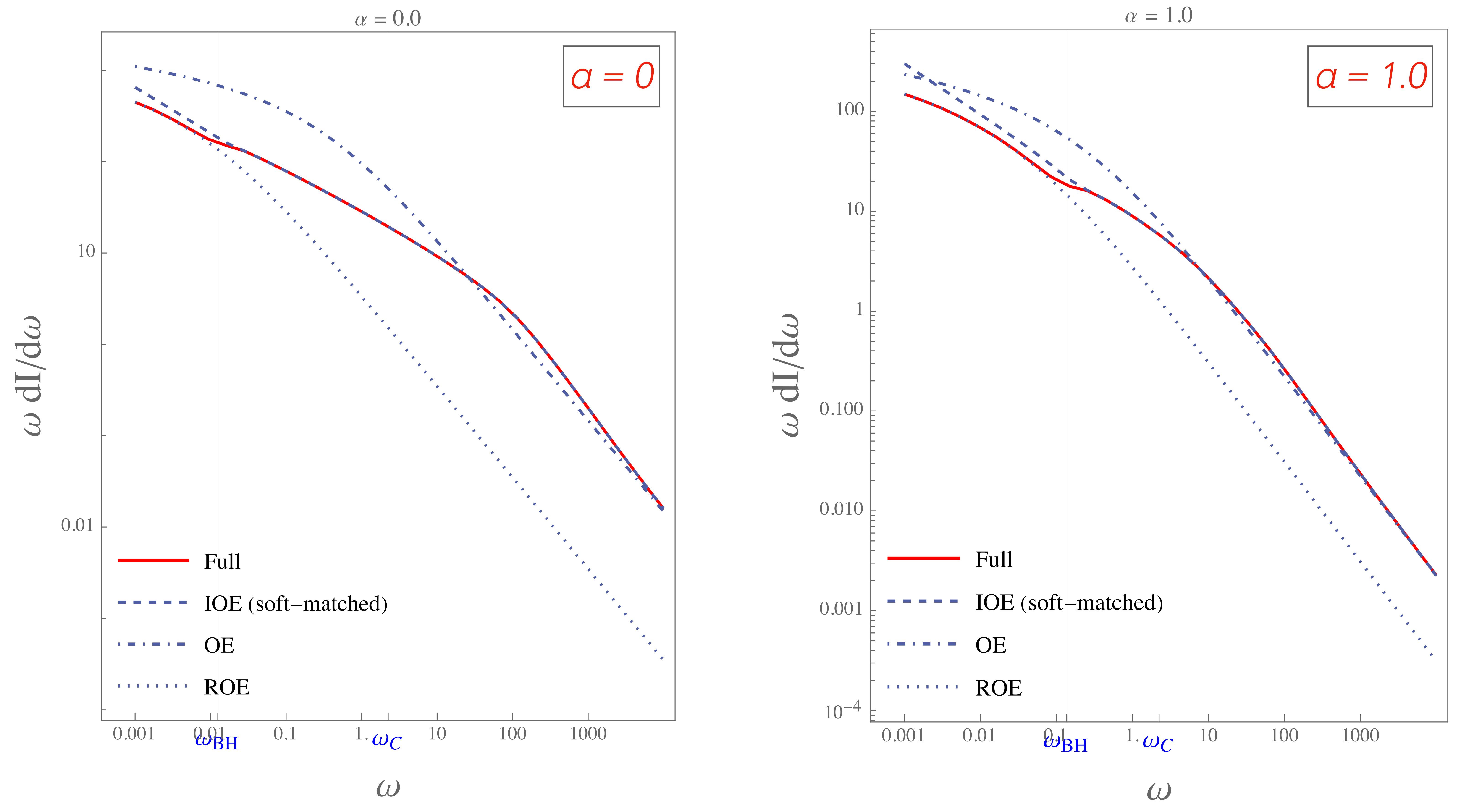

- In Bjorken medium ($\alpha = 1.0$), the medium "hydrodynamization" time should be much bigger than the mean-free-path ($t_m \gg \lambda$) in order to get contributions from the leading-order IOE terms.

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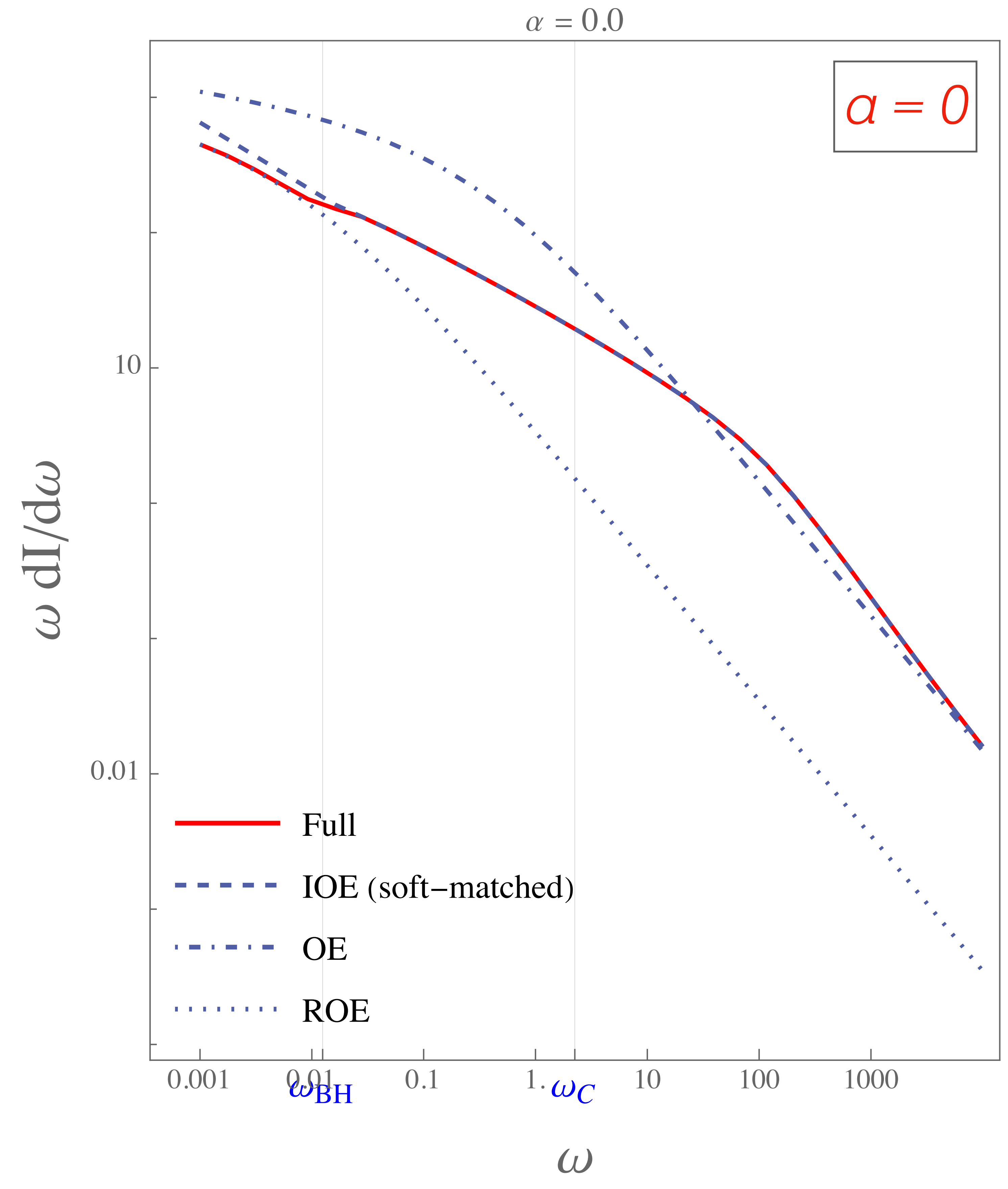

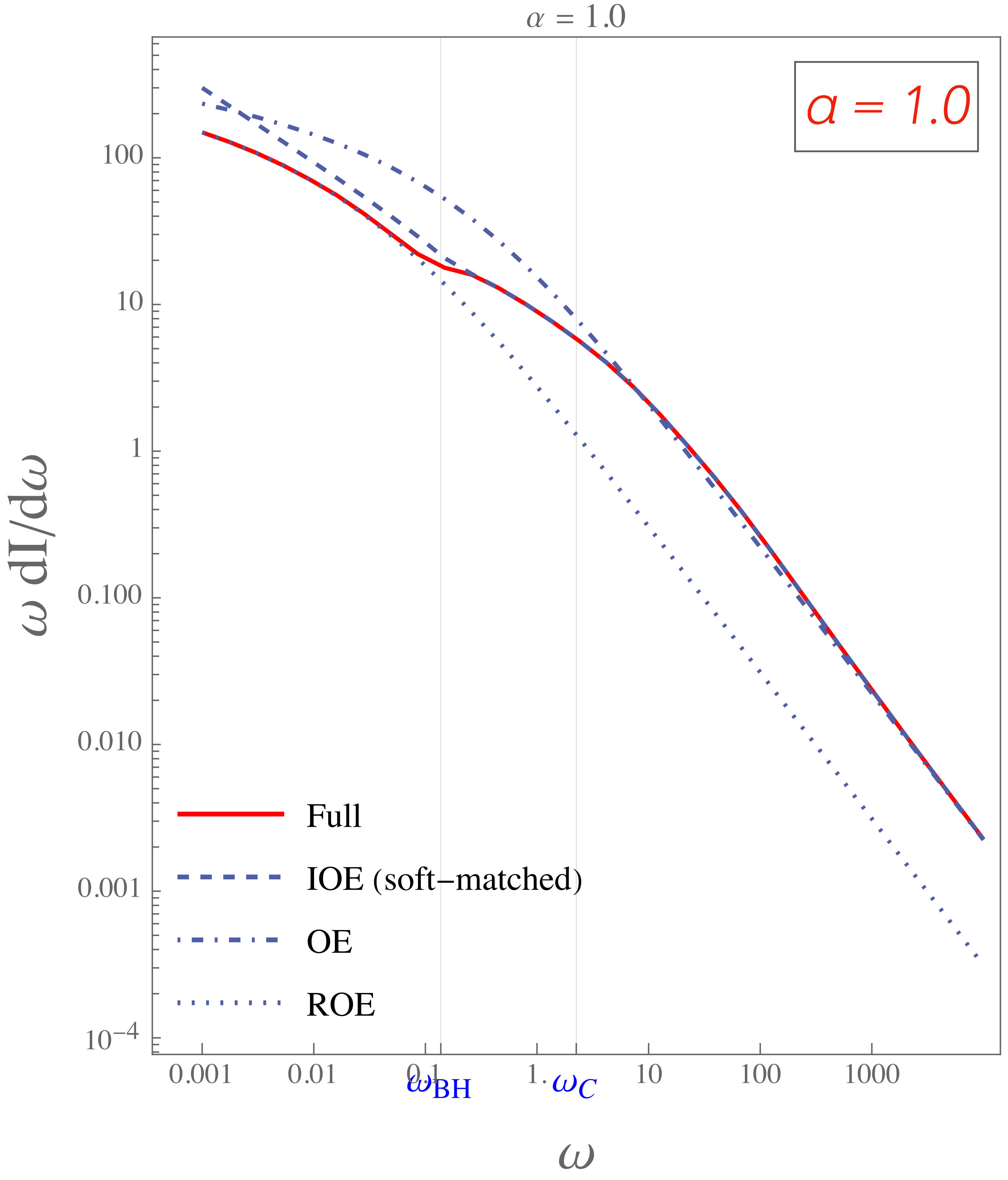

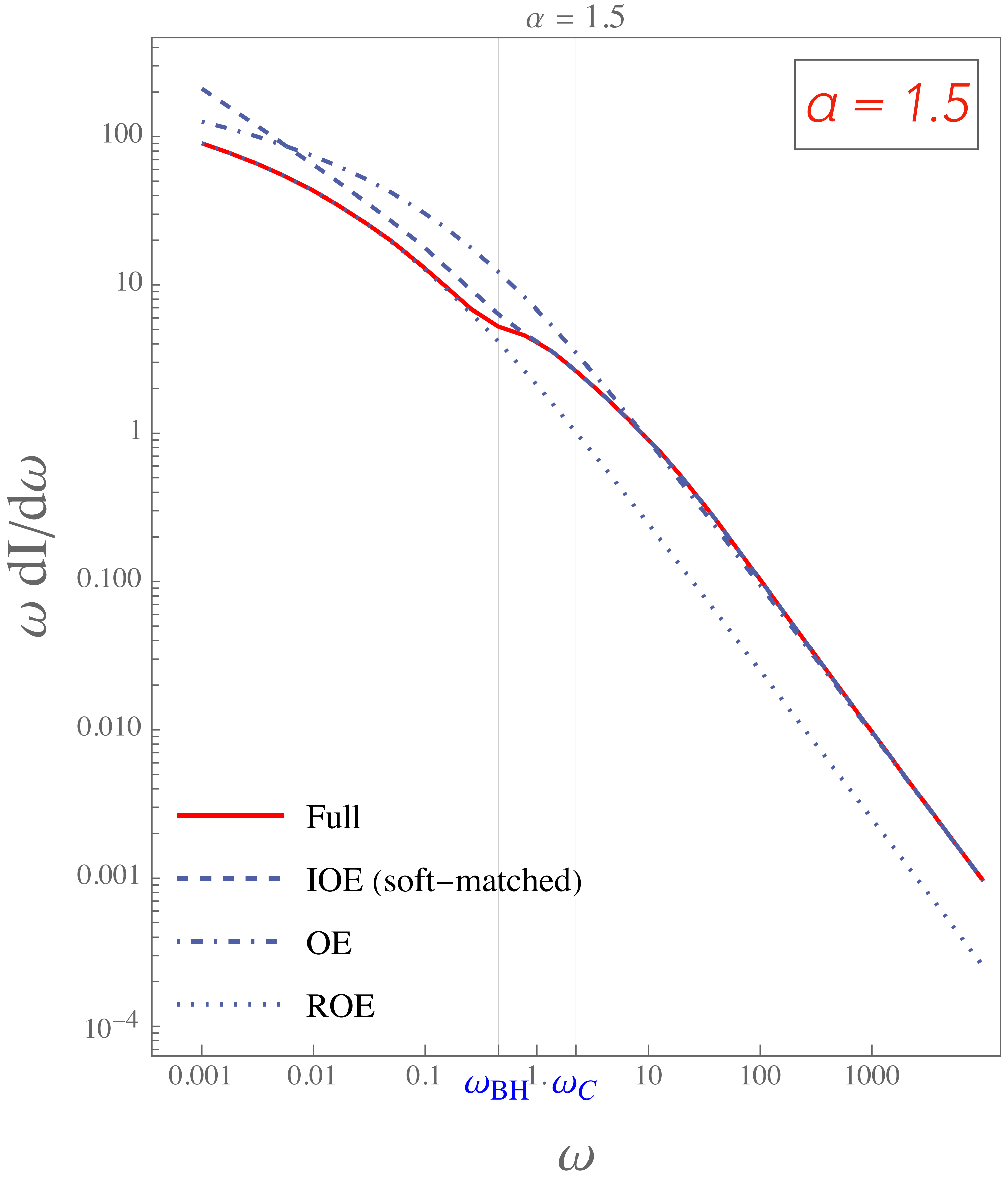

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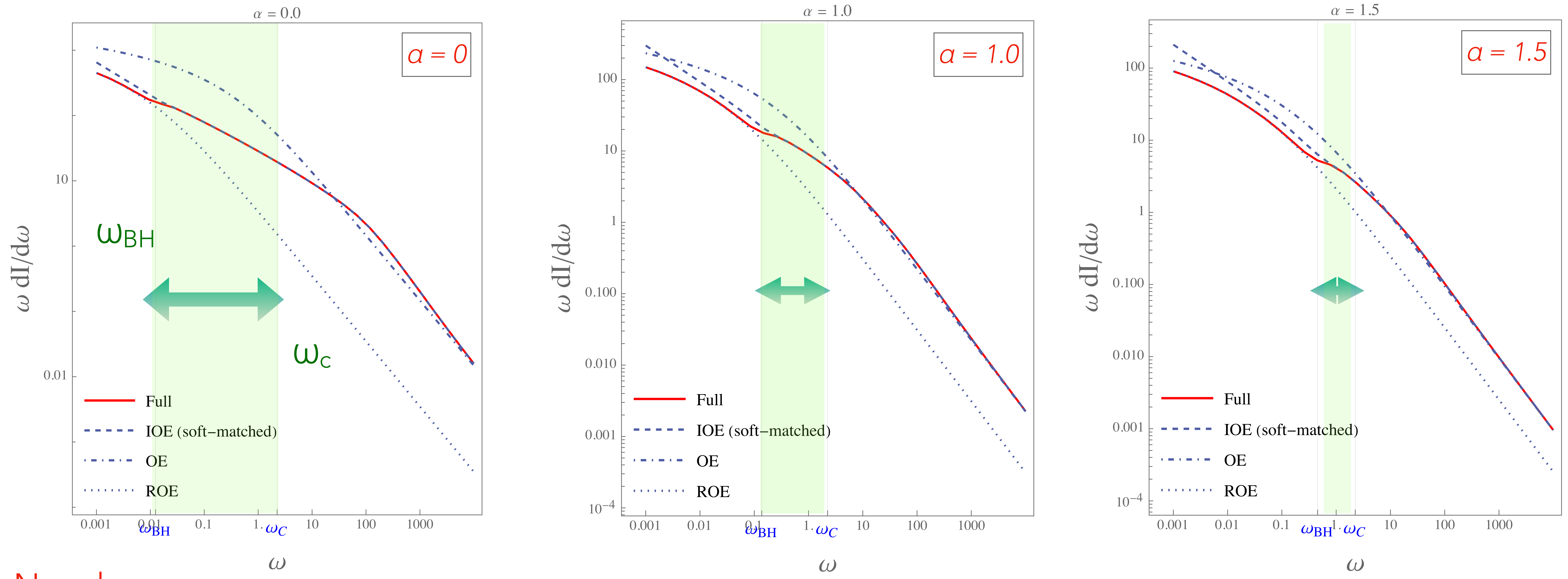


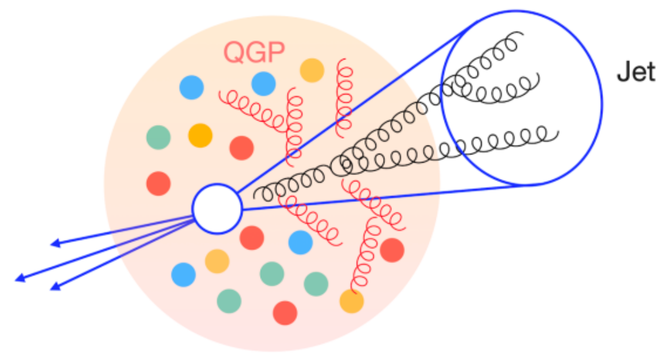
Novel:

- Newer bounds on the maximum and Bethe-Heitler gluon frequencies.
- The expanding media FAILS to see the multiple scattering regime !

- For $\omega_c >> \omega_{BH}$, we have to demand

$$1 \ll \frac{L}{\lambda}\left(\frac{t_m}{L}\right)^{\alpha/3}$$

Summary, prospects and outlook



- Identifying the expansion structure in the different regimes opens for the possibility of studying the accuracy of re-summations in the medium through the OE, IOE and ROE.
- NOVELTY : Extended the formalism to include finite size realistic medium effects.
- IMPACT : Re-definition of scales to trace the phase space of allowed emissions for expansion parameter of the medium and/or equilibration time.
- OUTLOOK : Implementation in Monte- Carlo codes for parton showers (faster, precise). Phenomenology comparisons.

- Also working on :
  - Exploring *gluon saturation* in jet quenching for upcoming Forward calorimeters in RHIC and LHC (with K. Kutak, W. Placzek, M. Rohrmoser and K. Tywoniuk (Bergen, Norway)).
  - In depth analysis of Vacuum like emissions, dipole and antenna picture (projected with E. Iancu and G. Soyez, IPhT, Paris).

If you find any of the works interesting, join us, will be happy to collaborate !