UNIVERSITY OF SUSSEX

MARIE CURIE ACTIONS

UK Research and Innovation

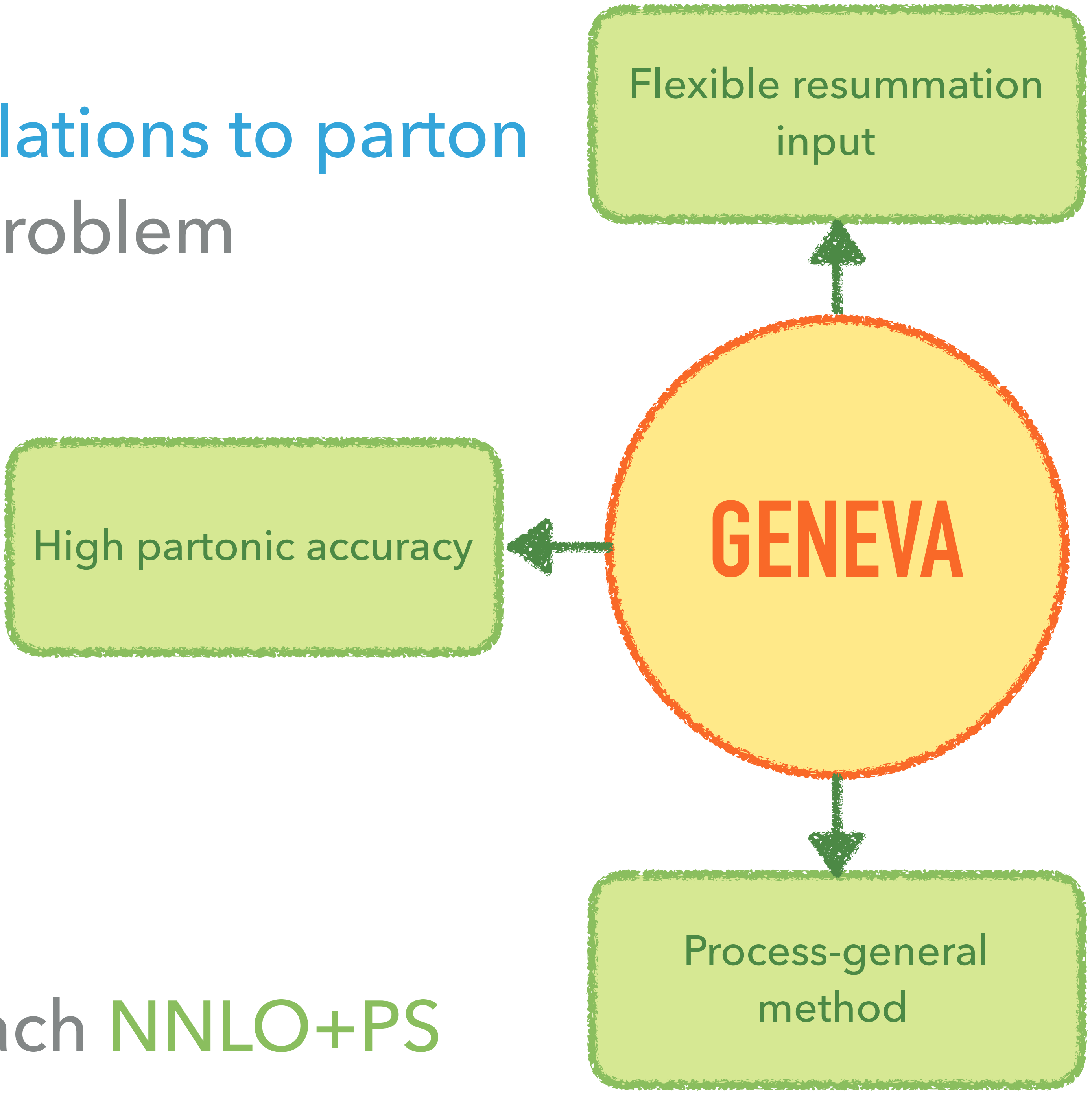
NNLO+PS IN GENEVA: RECENT DEVELOPMENTS

MATTHEW A. LIM

XXX KRAKÓW EPIPHANY CONFERENCE

HIGHER ORDER MONTE CARLO EVENT GENERATORS

- Matching fixed order calculations to parton showers is a well-studied problem
- At NLO, several successful methods available - POWHEG, MC@NLO, KrkNLO, multiplicative-accumulative…
- GENEVA is a method to reach NNLO+PS accuracy



GENEVA

Flexible resummation input

High partonic accuracy

Process-general method

CALCULATIONS BEYOND LEADING ORDER

$$\boxed{\frac{A}{\epsilon^2}+\frac{B}{\epsilon}+C}$$

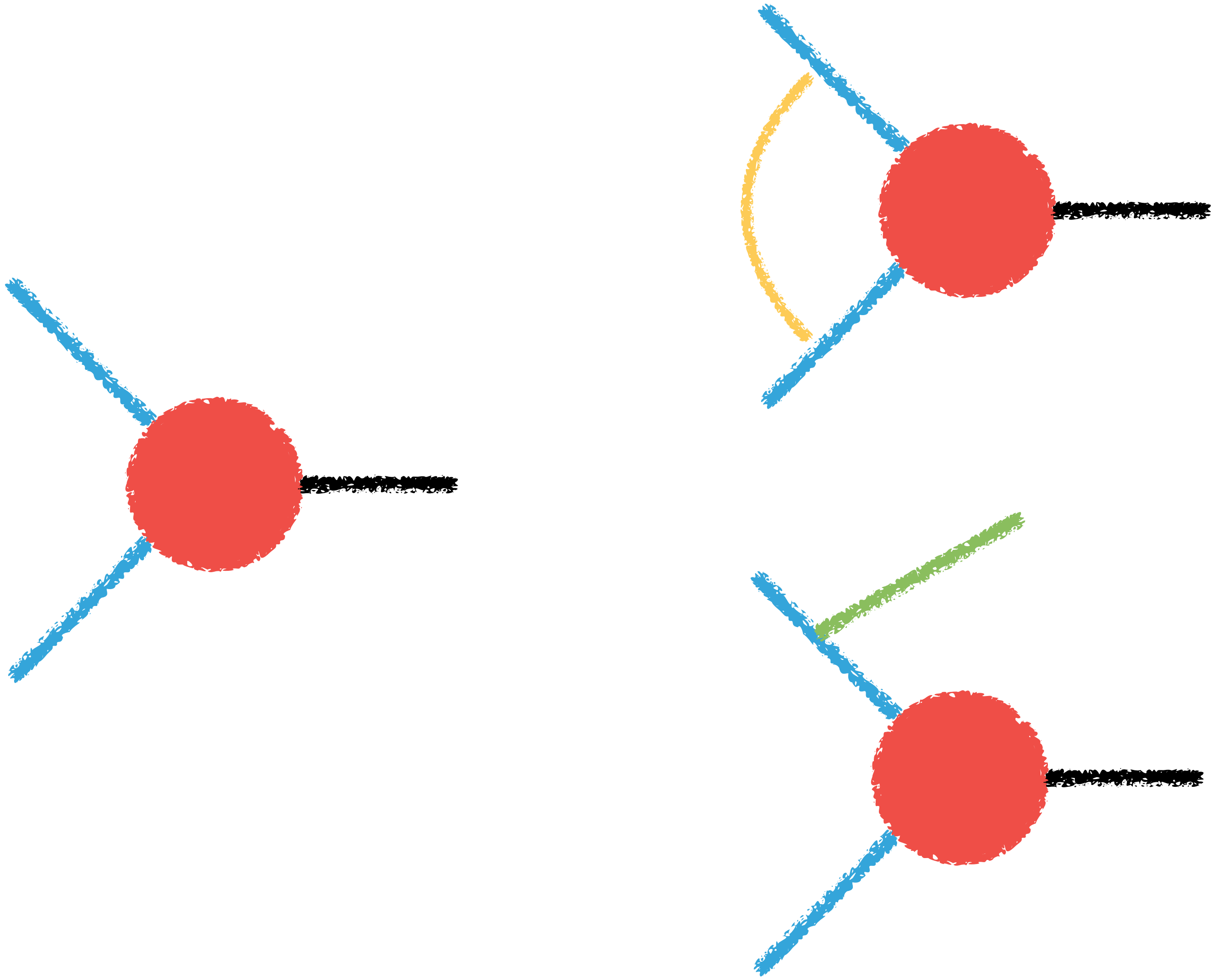
$$\sigma^{\mathrm{NLO}}(X) = \int \mathrm{d}\Phi_N \left[B_N(\Phi_N) + \boxed{V_N^C(\Phi_N)}\right] M_X(\Phi_N)$$

$$\boxed{-\frac{A}{\epsilon^2}-\frac{B}{\epsilon}+D}$$

$$+ \boxed{\int \mathrm{d}\Phi_{N+1} \{ B_{N+1}(\Phi_{N+1})} M_X(\Phi_{N+1}) - \sum_m C^m_{N+1}(\Phi_{N+1})\, M_X[\hat{\Phi}^m_N(\Phi_{N+1})] \}$$

- Calculations beyond leading order suffer from infrared divergences
- Happens when particles become indistinguishable - soft (low-energy) or collinear
- Divergences cancel between matrix elements with different numbers of final-state particles

DEFINING IR-FINITE EVENTS



VIRTUAL

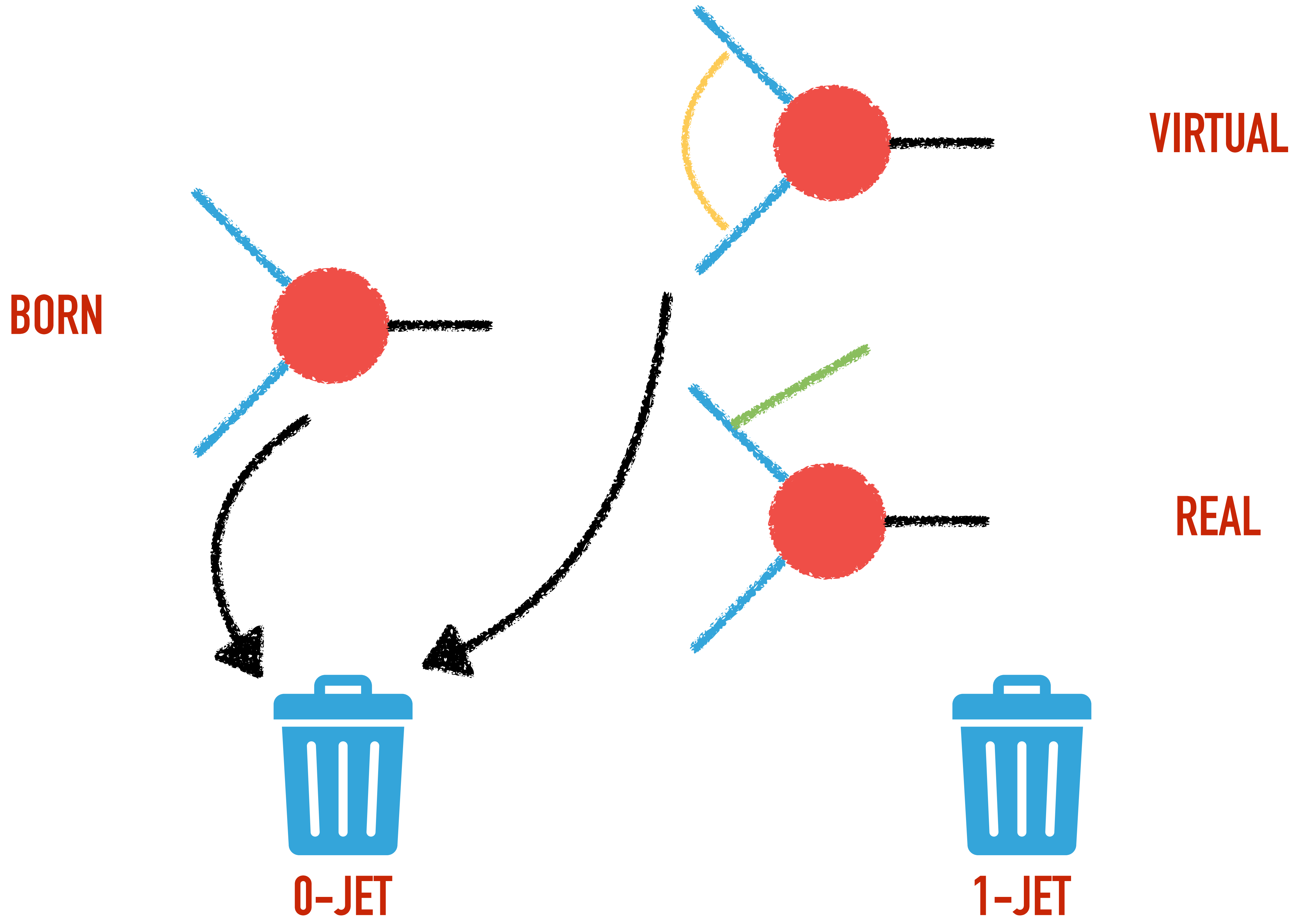
BORN

REAL

0-JET

1-JET

DEFINING IR-FINITE EVENTS



VIRTUAL

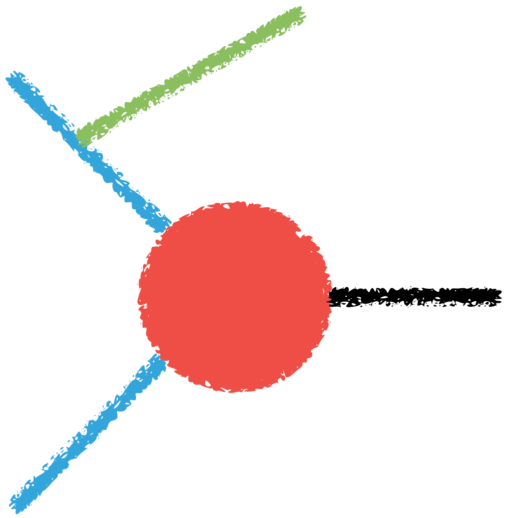
BORN

REAL

0-JET

1-JET

DEFINING IR-FINITE EVENTS



REAL

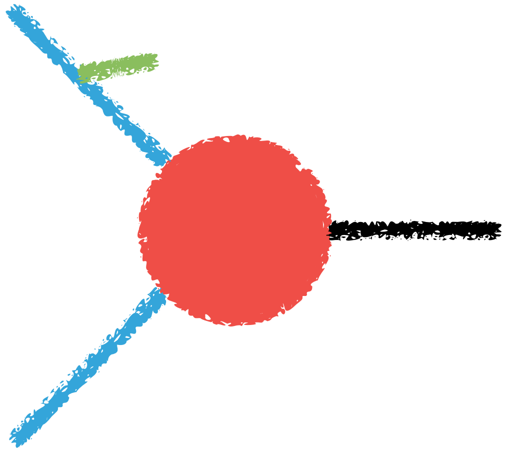
0-JET

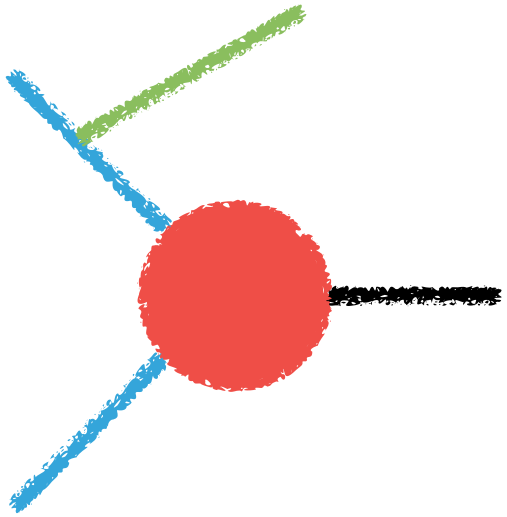
1-JET

DEFINING IR-FINITE EVENTS

SOFT/COLL. REAL

$r_0 < r_0^{\rm cut}$





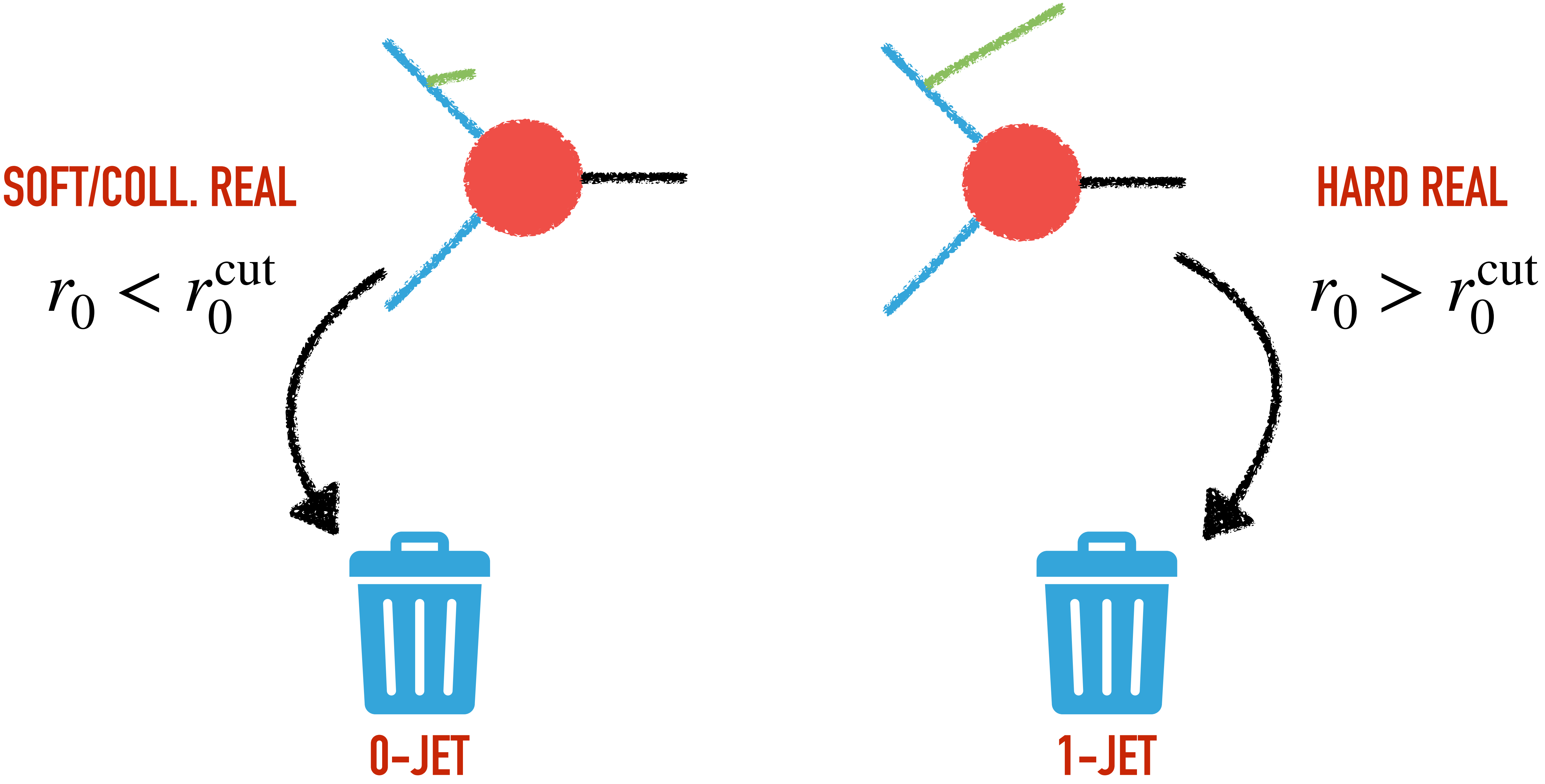
HARD REAL

$r_0 > r_0^{\rm cut}$

0-JET

1-JET

DEFINING IR-FINITE EVENTS



SOFT/COLL. REAL

$r_0 < r_0^{\text{cut}}$

0-JET

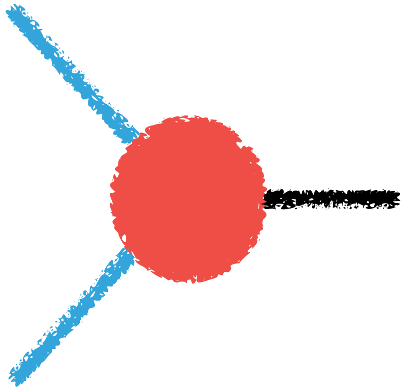
HARD REAL

$r_0 > r_0^{\text{cut}}$

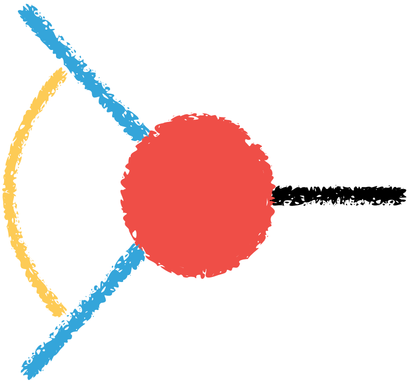
1-JET

DEFINING IR-FINITE EVENTS

BORN



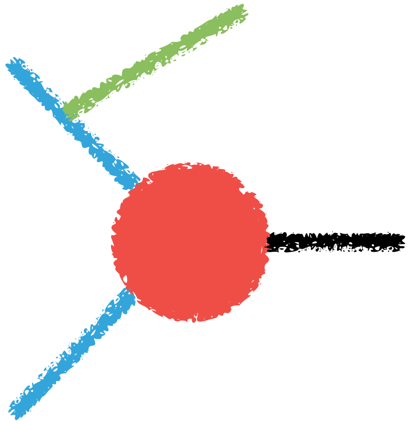
VIRTUAL



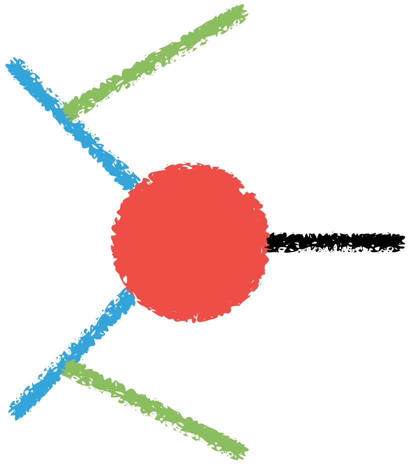
DOUBLE-VIRTUAL

0-JET

REAL



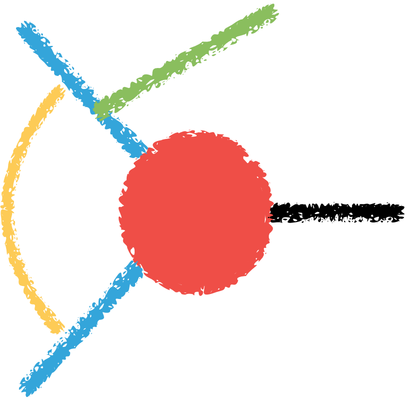
DOUBLE-REAL



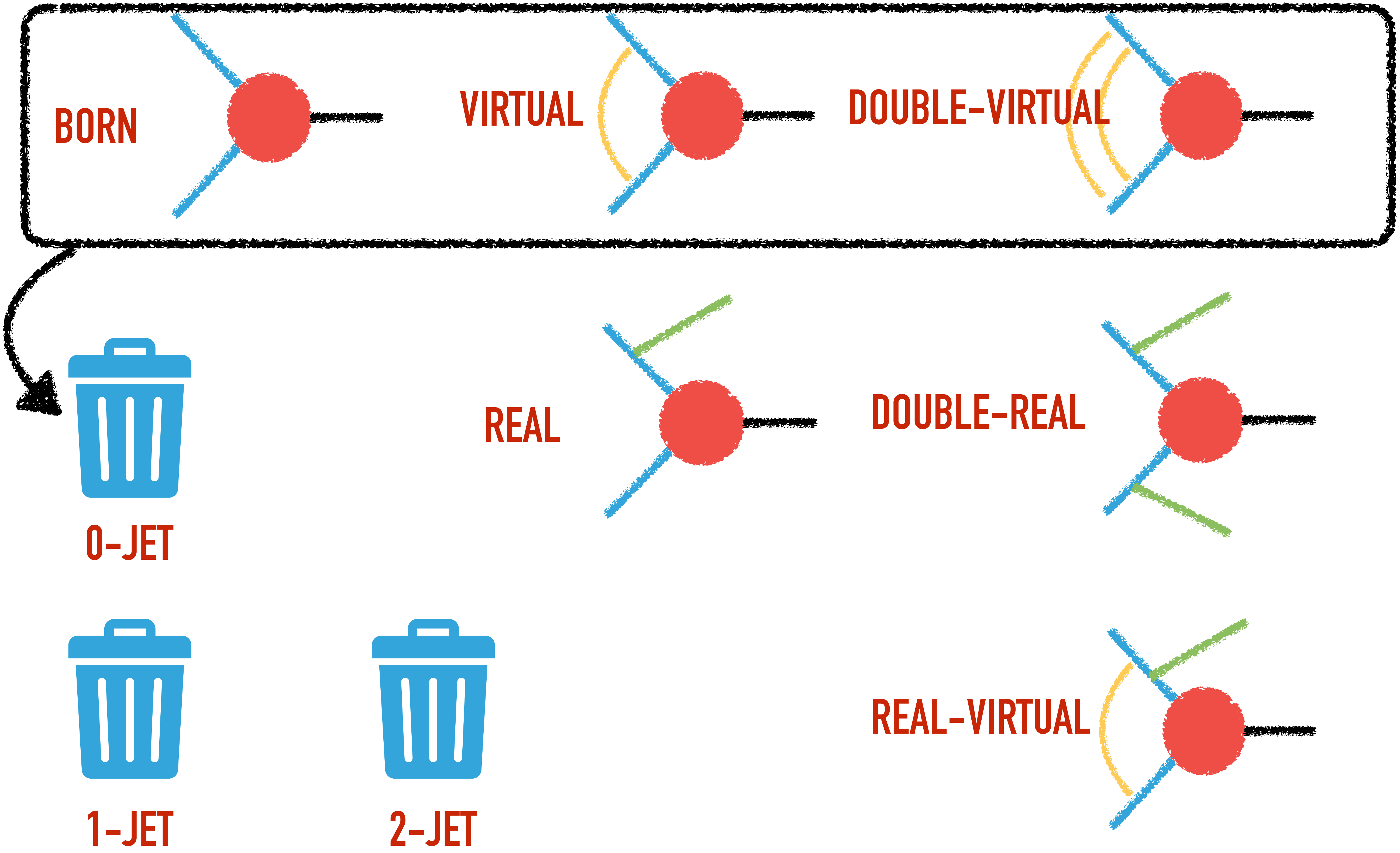
1-JET

2-JET

REAL-VIRTUAL



DEFINING IR-FINITE EVENTS



BORN

VIRTUAL

DOUBLE-VIRTUAL

REAL

DOUBLE-REAL

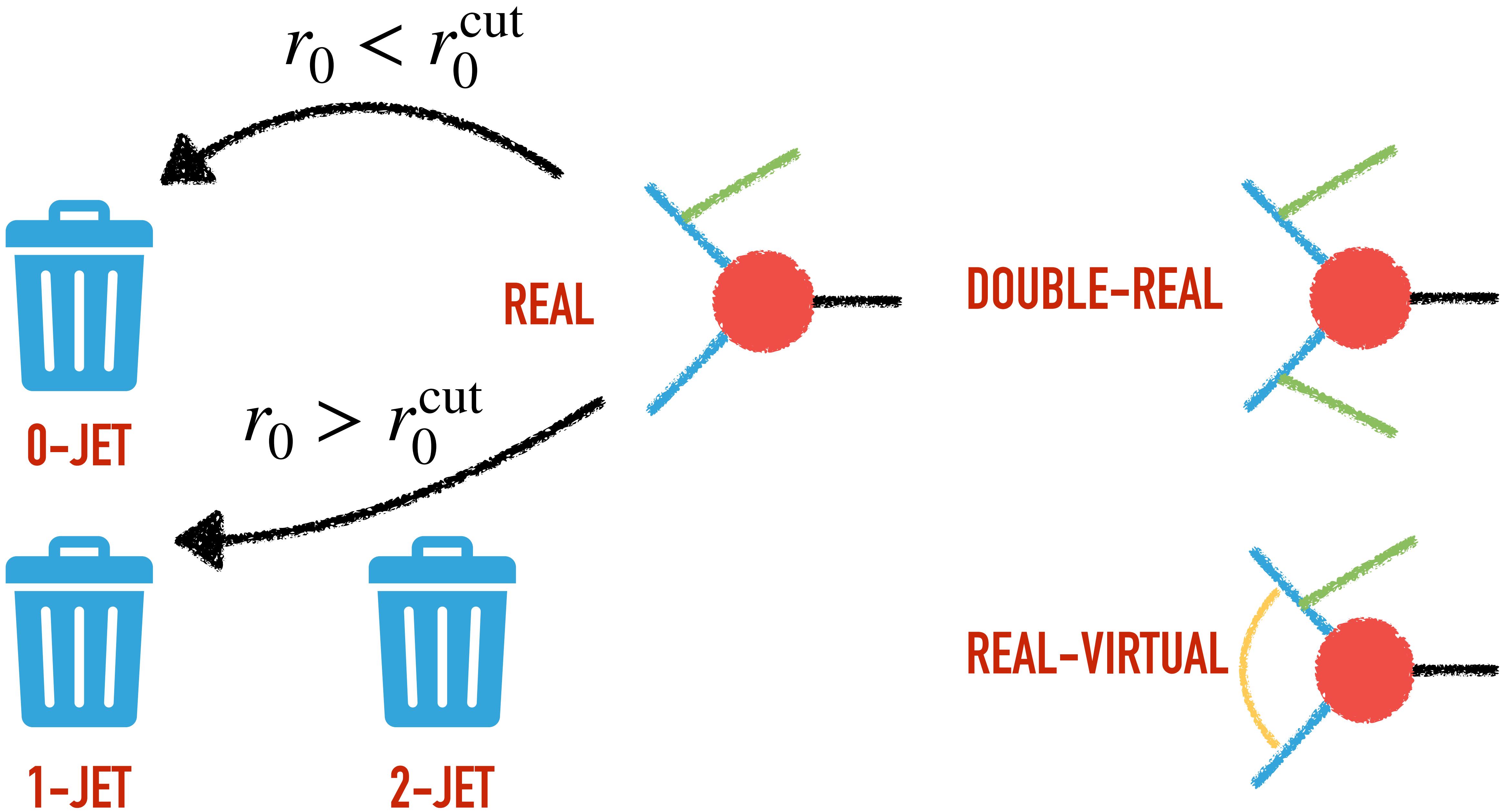
REAL-VIRTUAL

0-JET

1-JET

2-JET

DEFINING IR-FINITE EVENTS



$r_0 < r_0^{\mathrm{cut}}$

REAL

DOUBLE-REAL

0-JET

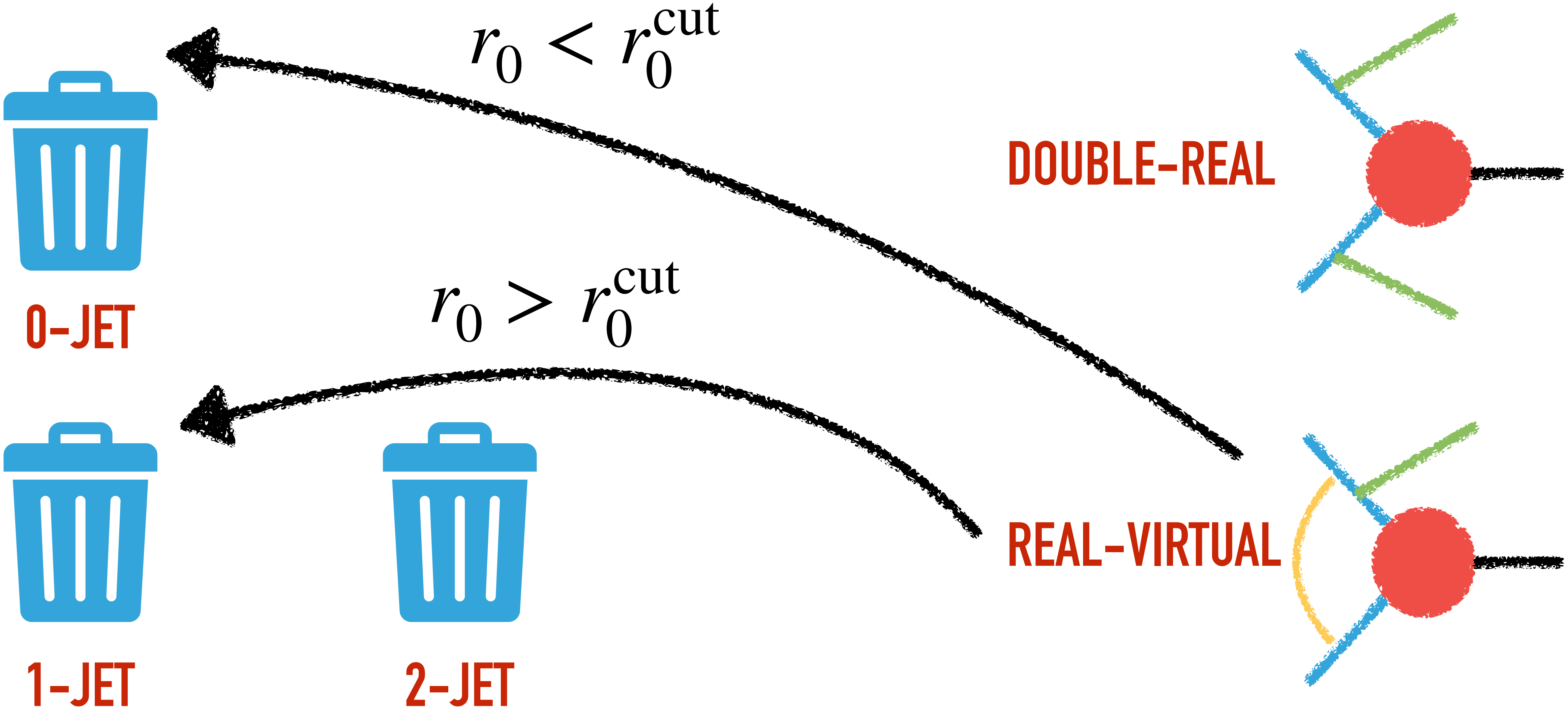
$r_0 > r_0^{\mathrm{cut}}$

REAL-VIRTUAL

1-JET

2-JET

DEFINING IR-FINITE EVENTS



$r_0 < r_0^{\rm cut}$

$r_0 > r_0^{\rm cut}$

DOUBLE-REAL

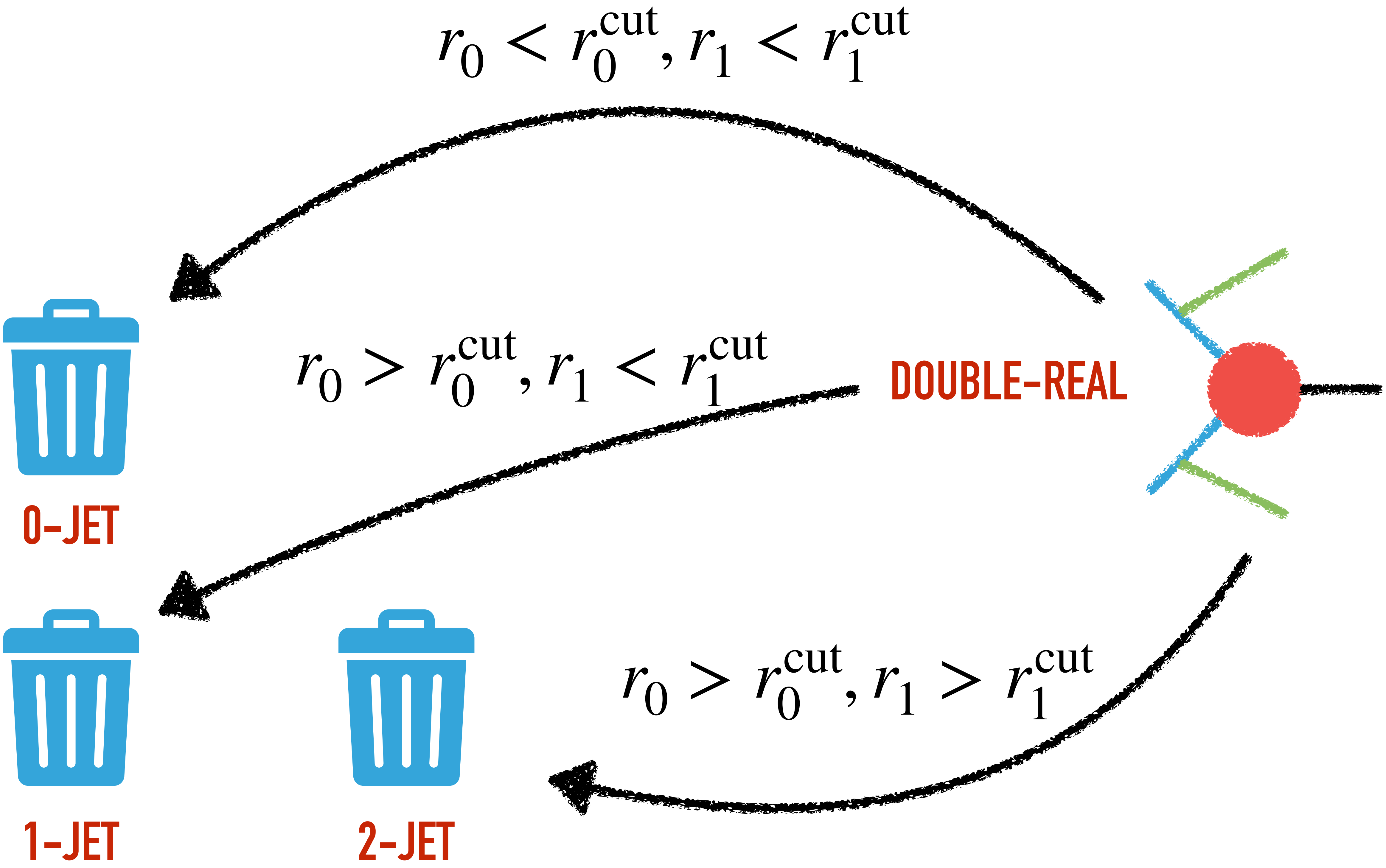
REAL-VIRTUAL

0-JET

1-JET

2-JET

DEFINING IR-FINITE EVENTS



$r_0 < r_0^{\text{cut}}, r_1 < r_1^{\text{cut}}$

$r_0 > r_0^{\text{cut}}, r_1 < r_1^{\text{cut}}$

DOUBLE-REAL

$r_0 > r_0^{\text{cut}}, r_1 > r_1^{\text{cut}}$

0-JET

1-JET

2-JET

DEFINING IR-FINITE EVENTS

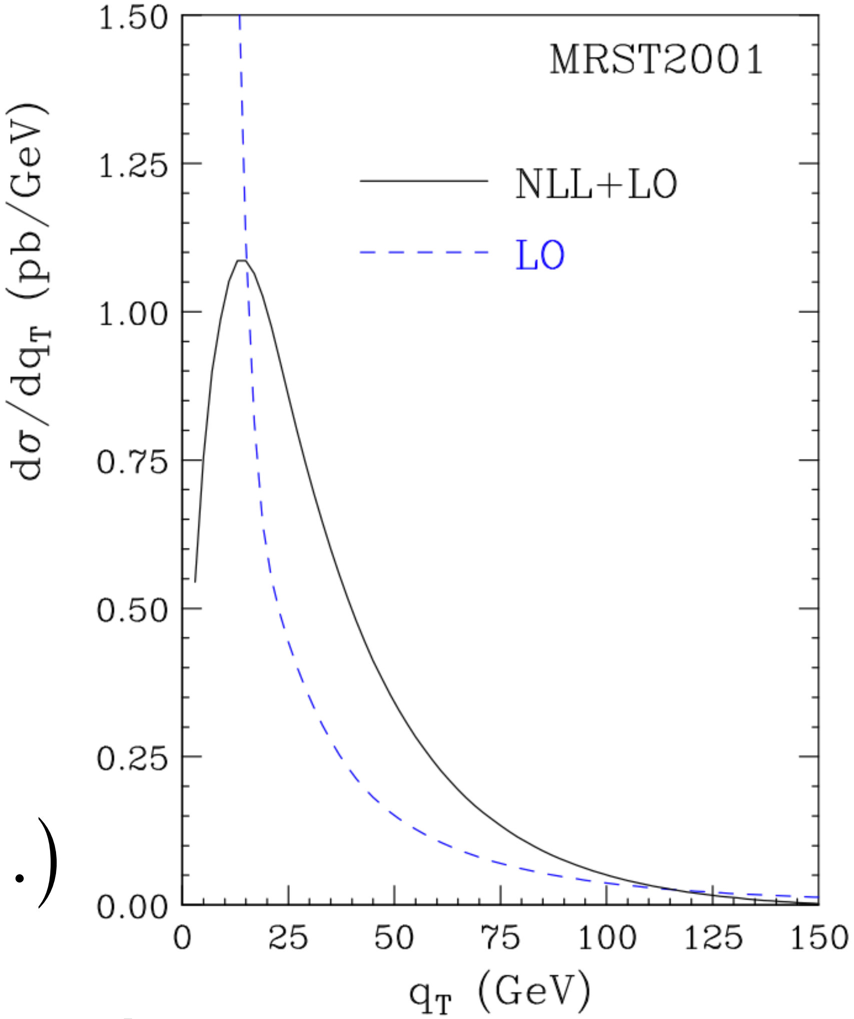
- Defining events this way introduced a projection from a higher multiplicity to a lower multiplicity phase space
- Results are only (N)NLO accurate up to power corrections in r_0^{cut} - as $r_0^{\mathrm{cut}} \to 0$, exact fixed order result is recovered
- Causes large logarithms to appear which spoil perturbative convergence!

$L = \log(Q/r_0^{\mathrm{cut}})$ becomes large…

RESUMMATION – THE CURE FOR LARGE LOGS

- Large logs signal the breakdown of the perturbative series in the coupling, leading term $\alpha L^2 \sim 1 \Rightarrow \alpha L \ll 1$
- Reordering the series to expand in a genuinely small parameter cures behaviour

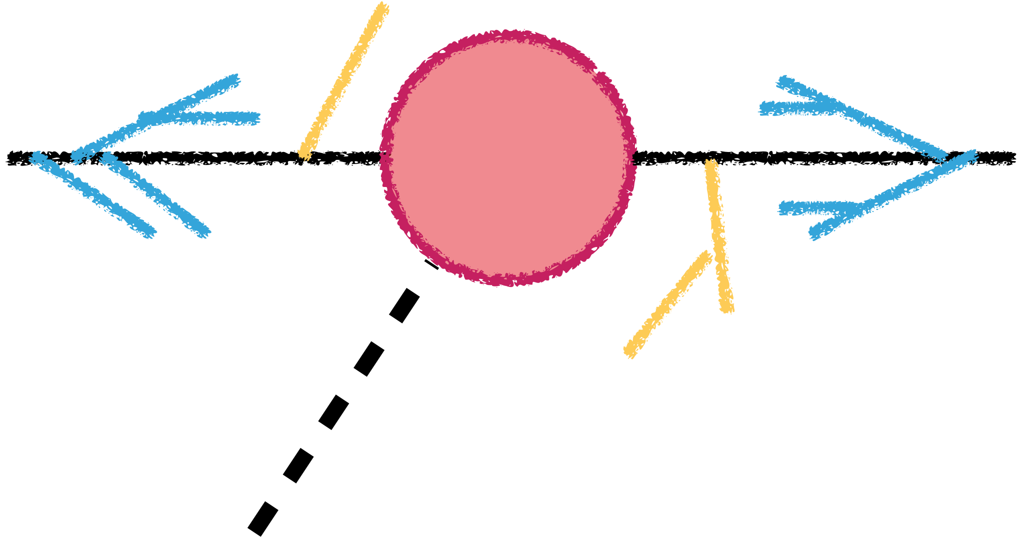
$$d\sigma = C(\alpha_s)\,\exp\left(L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right)$$



- Different formalisms available to achieve this: parton branching, soft-collinear effective theory

RESUMMATION FROM EFFECTIVE FIELD THEORIES

- Soft-Collinear effective theory formalism - an EFT with QCD as its UV limit
- QCD Lagrangian split into low-energy modes
- For $r_0 \ll Q$, the partonic cross section typically factorises:



$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}r_0} \approx H(Q,\mu_H)\, B_i(r_0,\mu_B) \otimes B_j(r_0,\mu_B) \otimes S(r_0,\mu_S)$$

SOFT-COLLINEAR EFFECTIVE THEORY

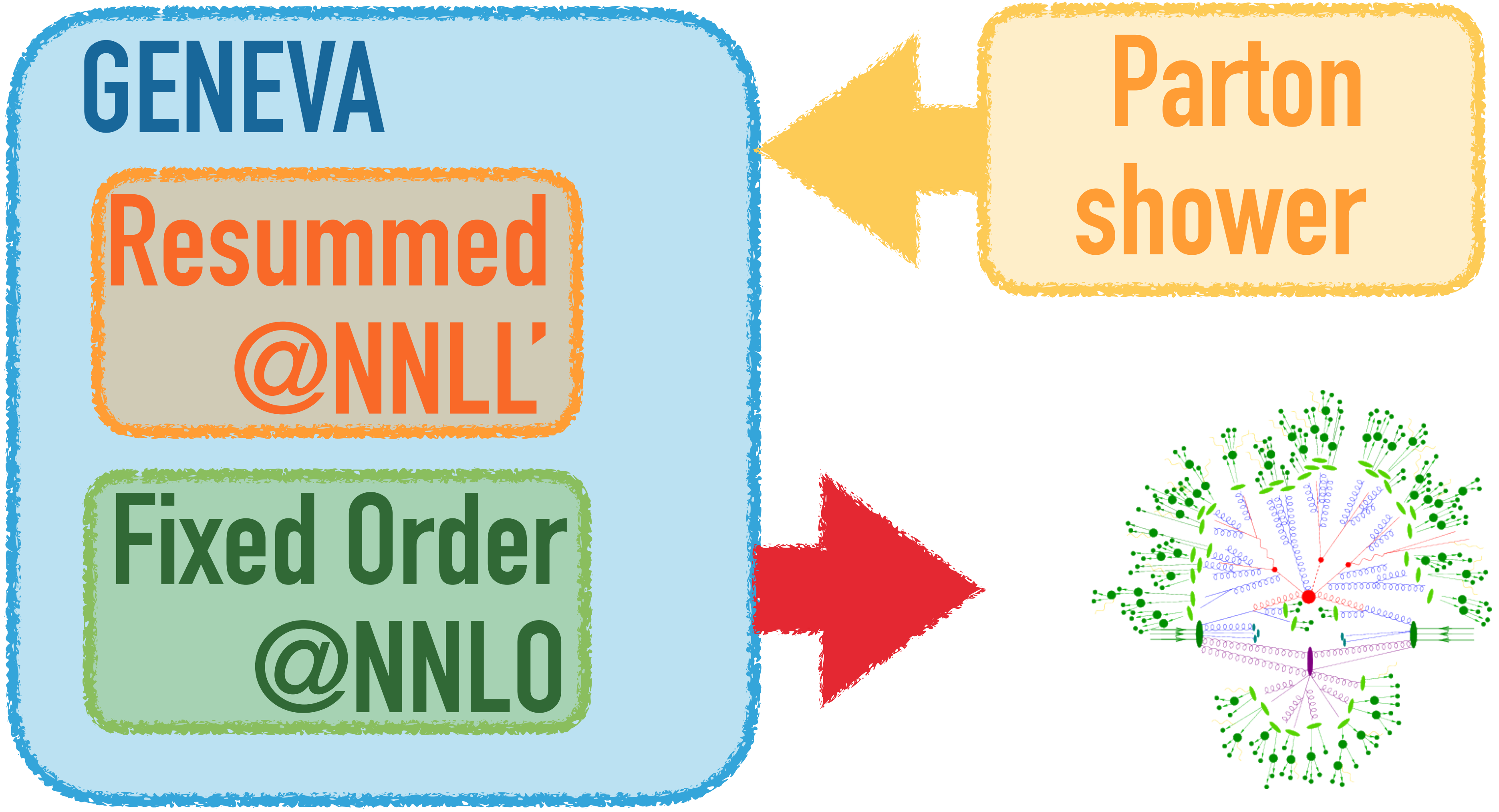
- Beam and soft functions correspond to matrix elements of collinear/soft SCET modes, hard function gives matching onto full QCD (Wilson coefficient)

- Each component of factorisation theorem is evaluated at its own scale $\Longrightarrow$ no large logs! Evolution to common scale via double-log RGE resums large log terms.

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\ln H(\Phi_0,\mu) = \Gamma_{\mathrm{cusp}}\ln\frac{\mu_H^2}{\mu^2} + \gamma_H$$

- Accuracy improvable by going to higher loop orders

COMBINING RESUMMED AND FIXED ORDER CALCULATIONS IN GENEVA



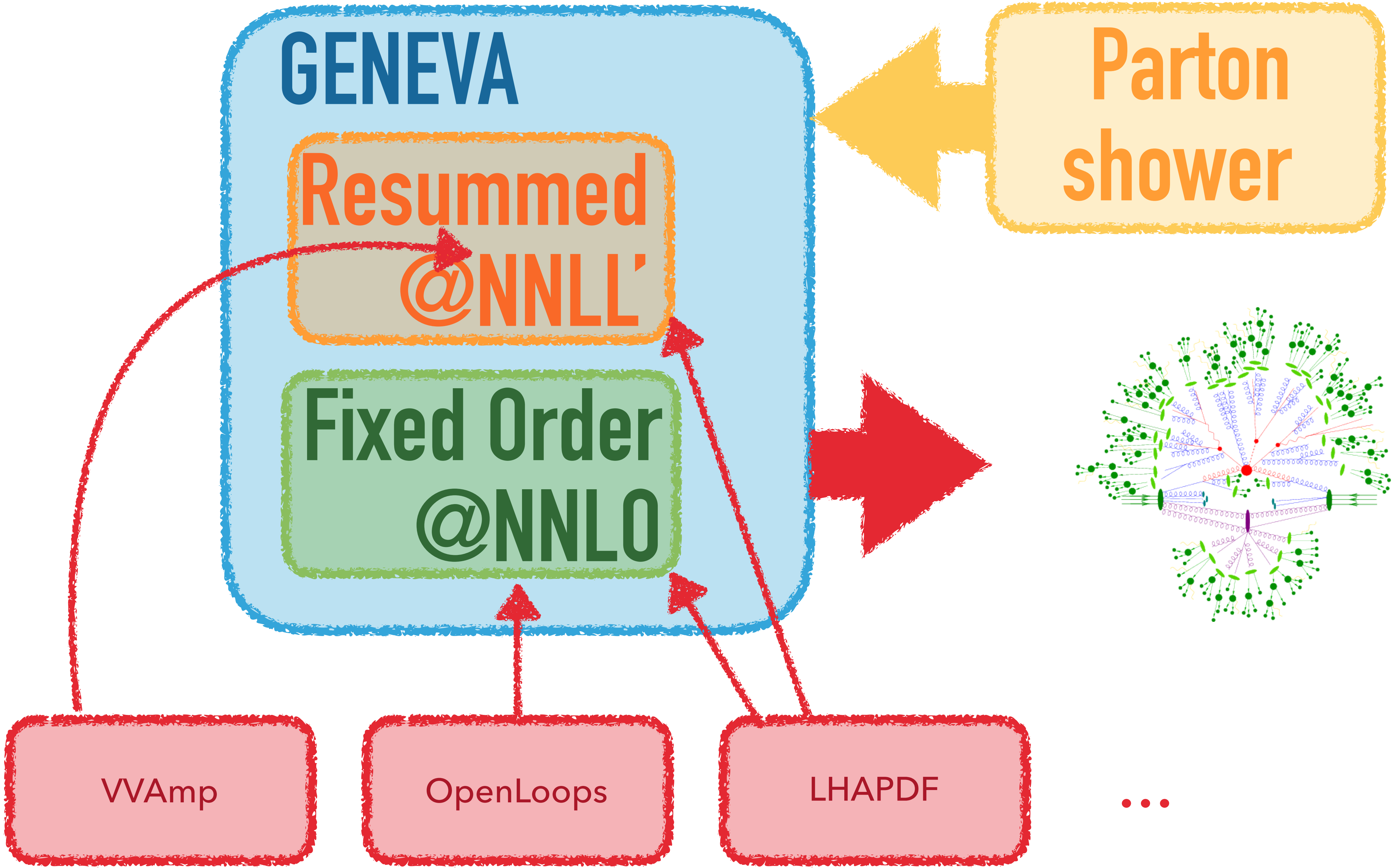
GENEVA

Resummed @NNLL′

Fixed Order @NNLO

Parton shower

COMBINING RESUMMED AND FIXED ORDER CALCULATIONS IN GENEVA



GENEVA

Resummed @NNLL′

Fixed Order @NNLO

Parton shower

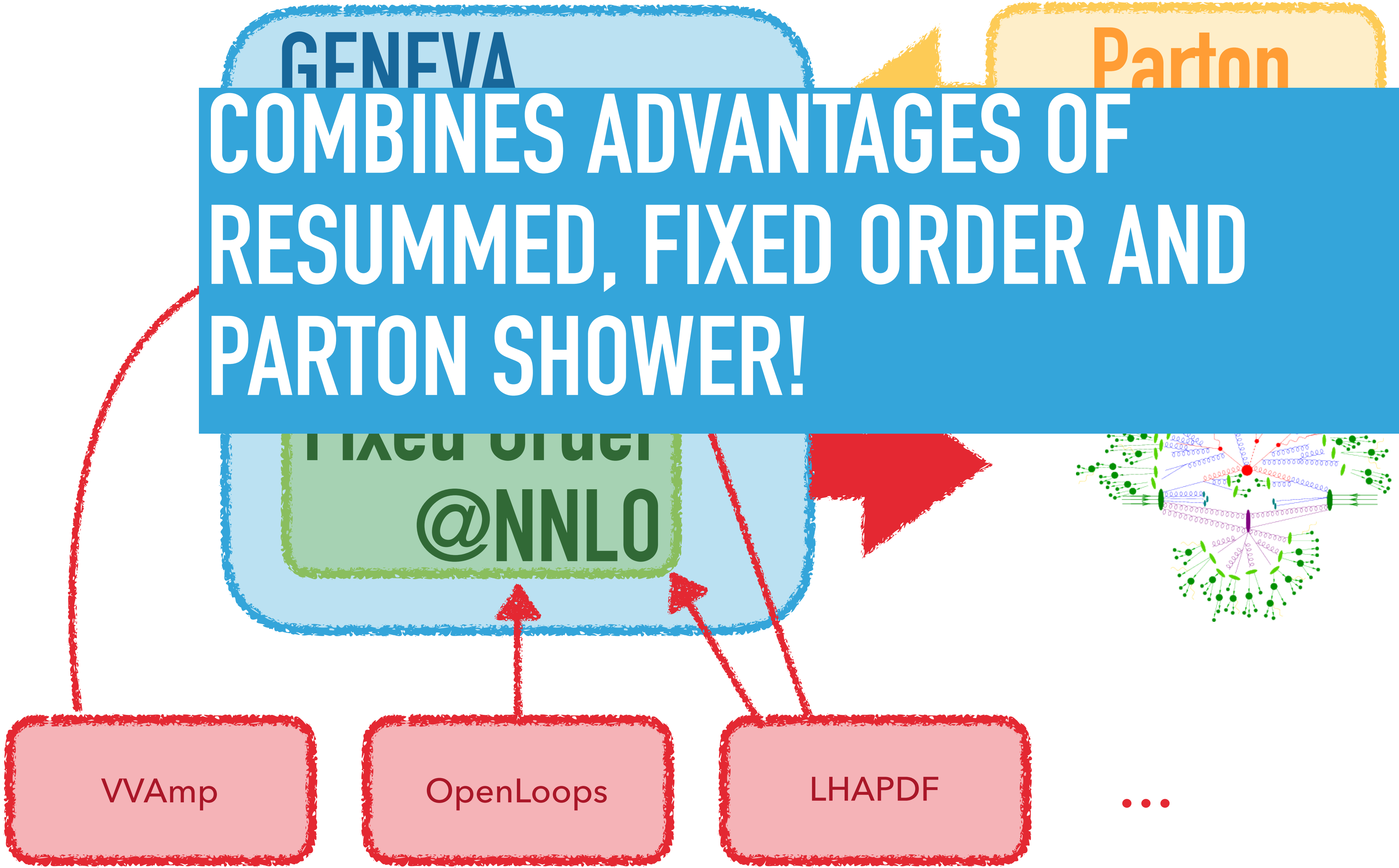
VVAmp

OpenLoops

LHAPDF

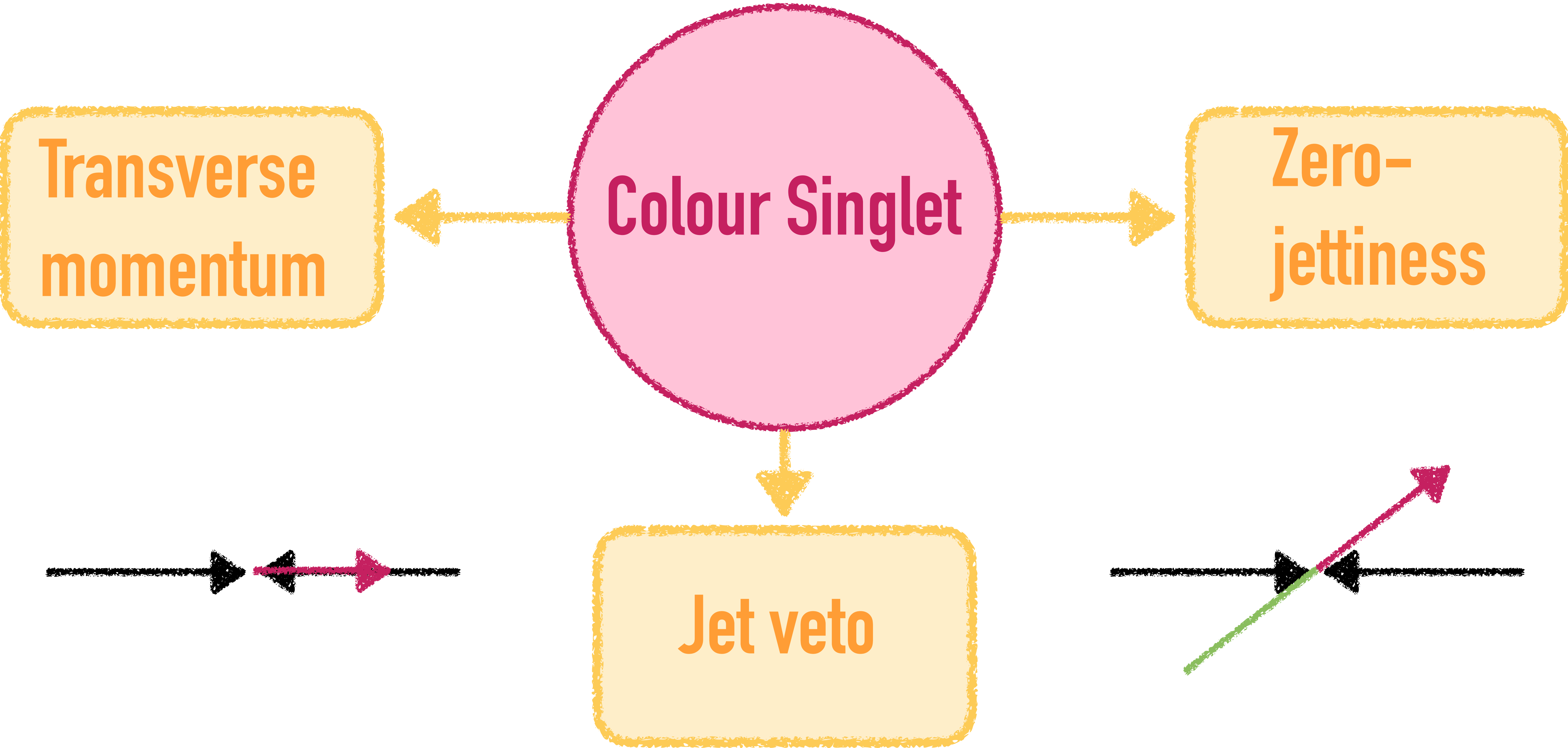
…

COMBINING RESUMMED AND FIXED ORDER CALCULATIONS IN GENEVA



COMBINES ADVANTAGES OF RESUMMED, FIXED ORDER AND PARTON SHOWER!

RESOLUTION VARIABLES



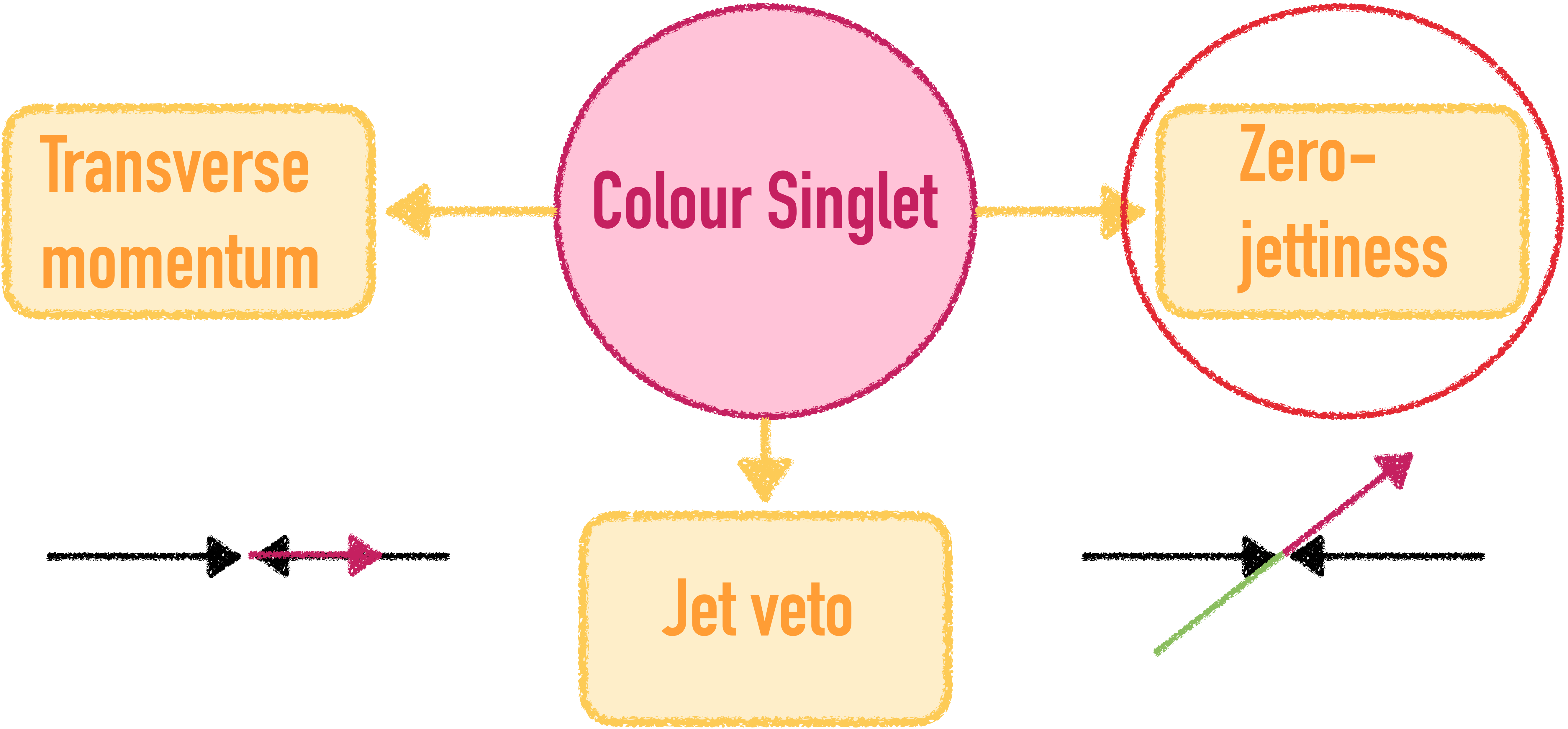
Colour Singlet

Transverse momentum

Zero-jettiness

Jet veto

RESOLUTION VARIABLES



Colour Singlet

Transverse momentum

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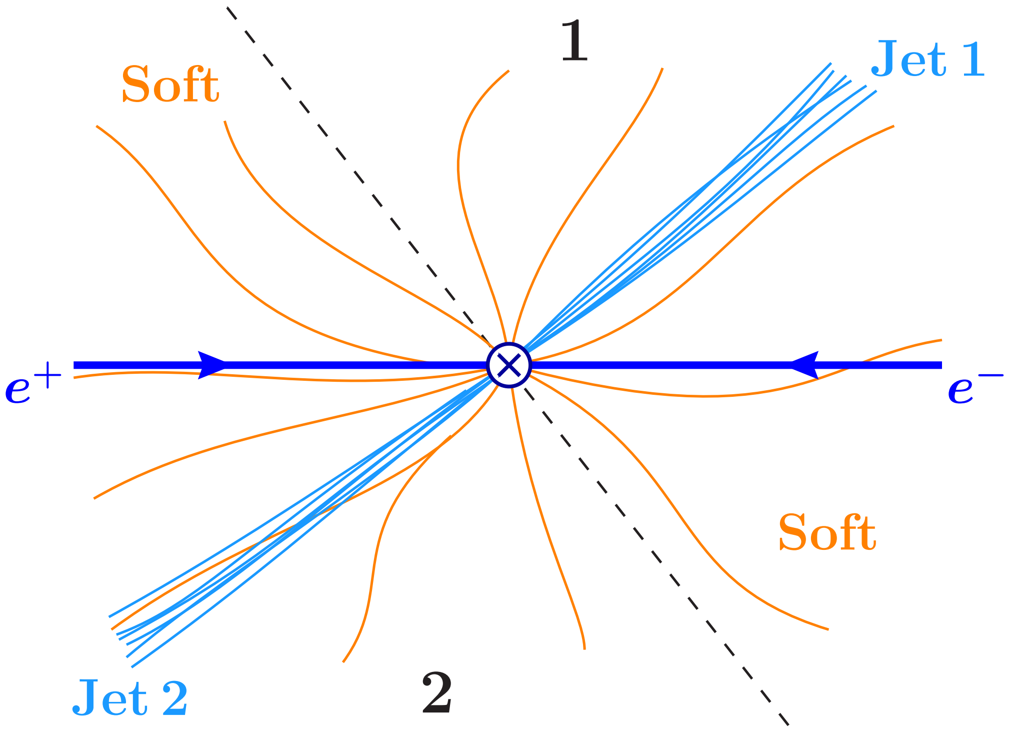
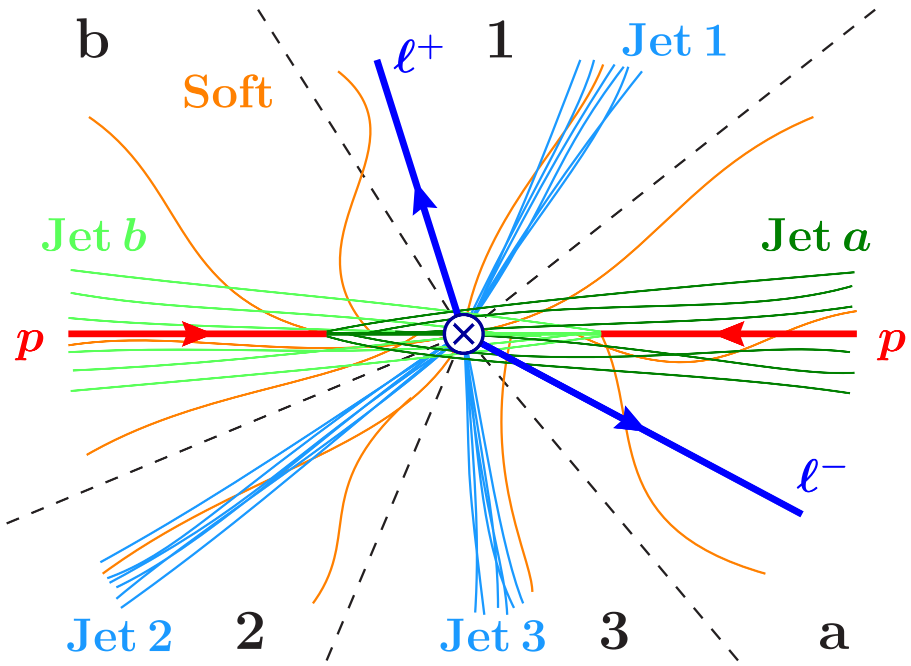
THE N-JETTINESS OBSERVABLE

- $\mathcal{T}_N = 0$ implies there are exactly N pencil-like jets
- Large $\mathcal{T}_N$ implies a spherical distribution of radiation

$$\mathcal{T}_N = \frac{2}{Q}\sum_k \min\{q_a \cdot p_k, q_b \cdot p_k, q_1 \cdot p_k, \ \ldots, q_N \cdot p_k\}$$





ZERO-JETTINESS RESUMMATION FOR COLOUR SINGLET

SCET allows us to write a factorisation formula as

$$\frac{\mathrm{d}\sigma^{\text{resum}}}{\mathrm{d}\Phi_0\mathrm{d}\mathcal{T}_0} = \sum_{ij}\int \mathrm{d}t_a\mathrm{d}t_b\, \underbrace{\boxed{B_i(t_a, x_a, \mu_B)\; B_j(t_b, x_b, \mu_B)}}_{\text{Beams}}\, \underbrace{\boxed{H_{ij}(\Phi_0, \mu_H)}}_{\text{Hard}}\, \underbrace{\boxed{S\Big(\mathcal{T}_0 - \frac{t_a + t_b}{Q}, \mu_S\Big)}}_{\text{Soft}}$$

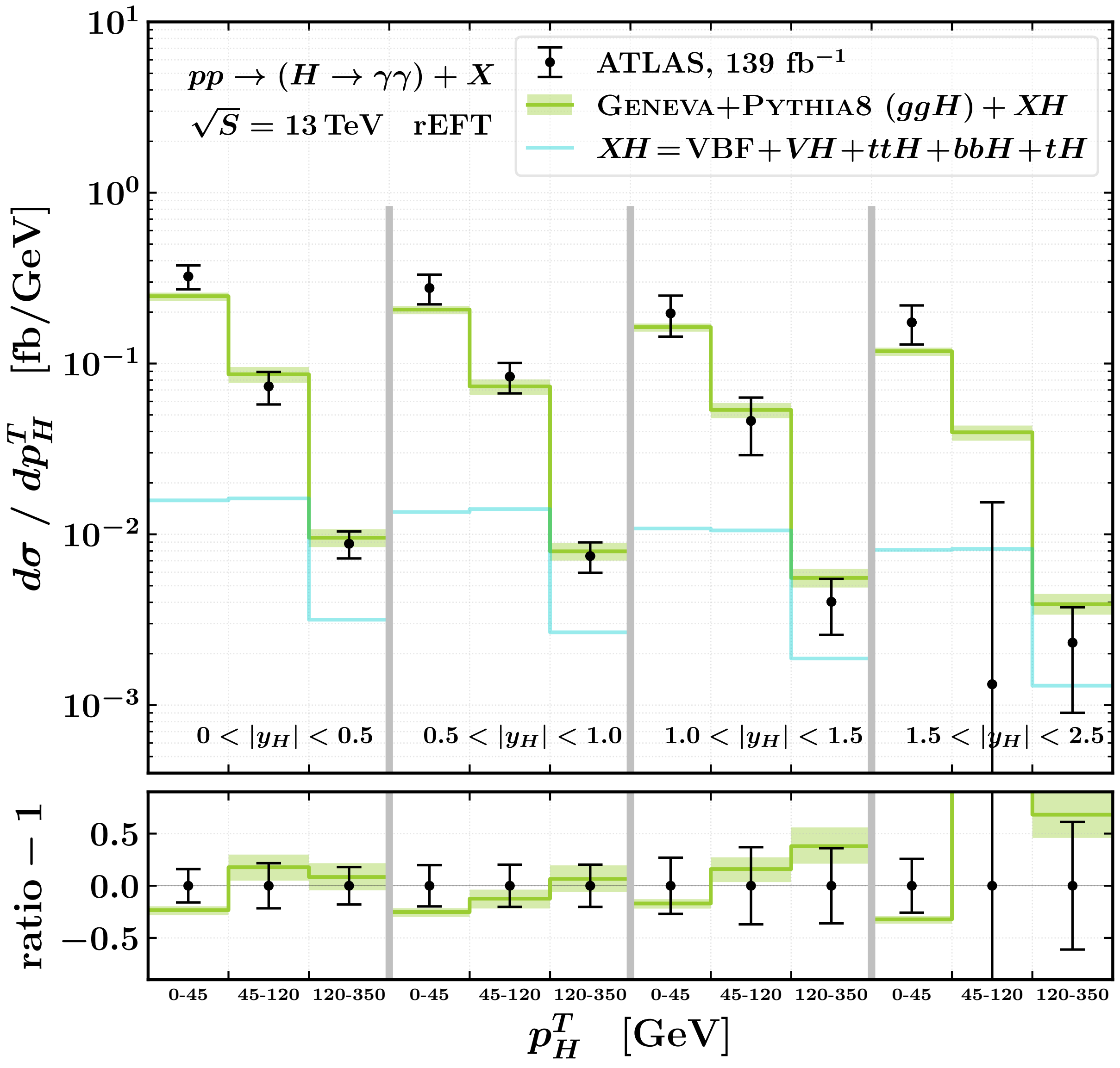
All single-scale objects!

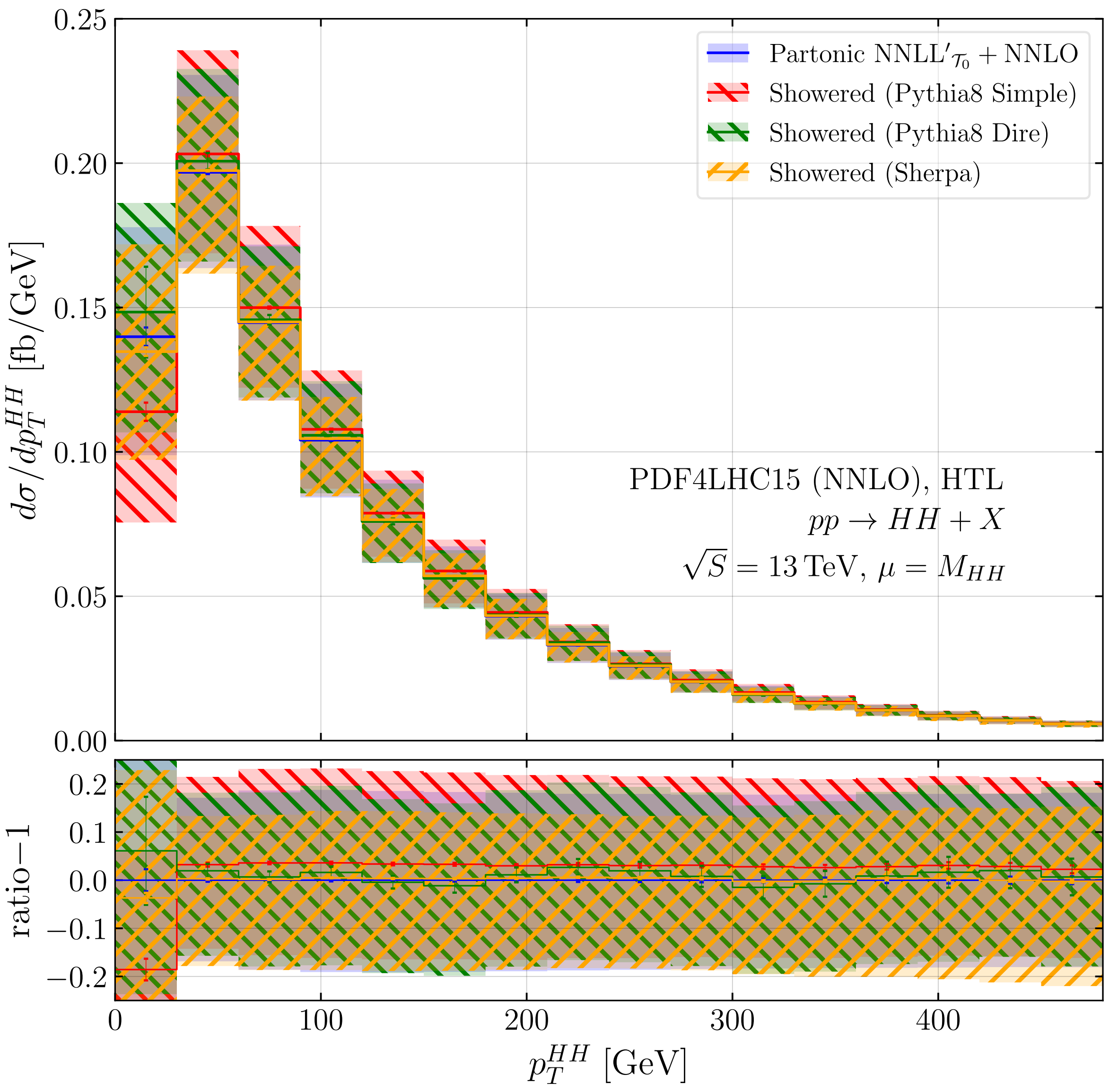
Resummation via RGE running to common scale:

$$B_i(t_a, x_a, \mu) = B_i(t_a, x_a, \mu_B) \otimes \underbrace{\boxed{U_B(\mu, \mu_B)}}_{\text{Resums logs of } \mu/\mu_B}$$

0910.0467, I. Stewart, F. Tackmann, W. Waalewijn

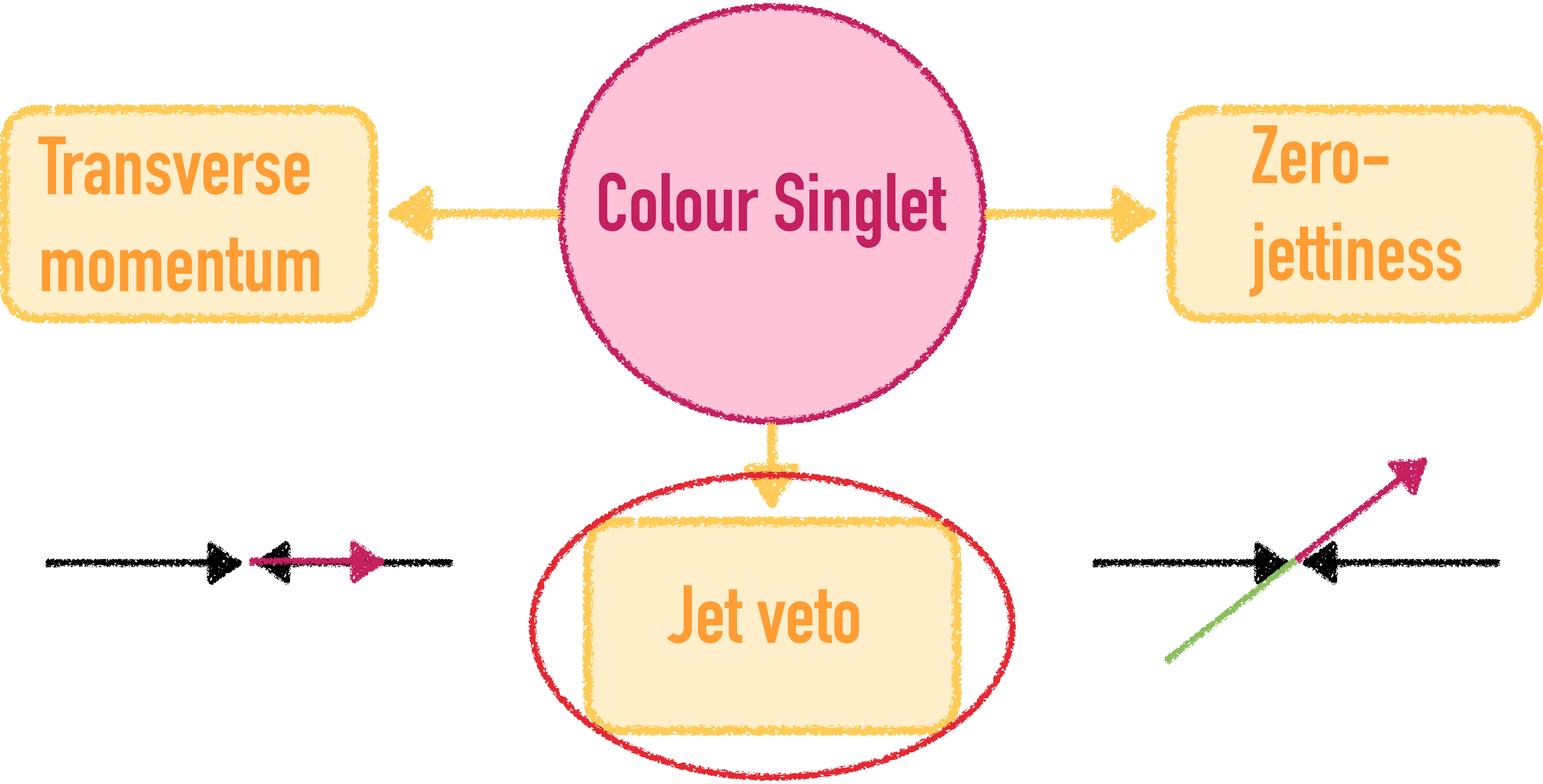
ZERO-JETTINESS RESUMMATION IN GENEVA





2212.10489, 2301.11875, S. Alioli, G. Billis, A. Broggio, A. Gavardi, S. Kallweit, MAL, G. Marinelli, R. Nagar, D. Napoletano

RESOLUTION VARIABLES



Colour Singlet

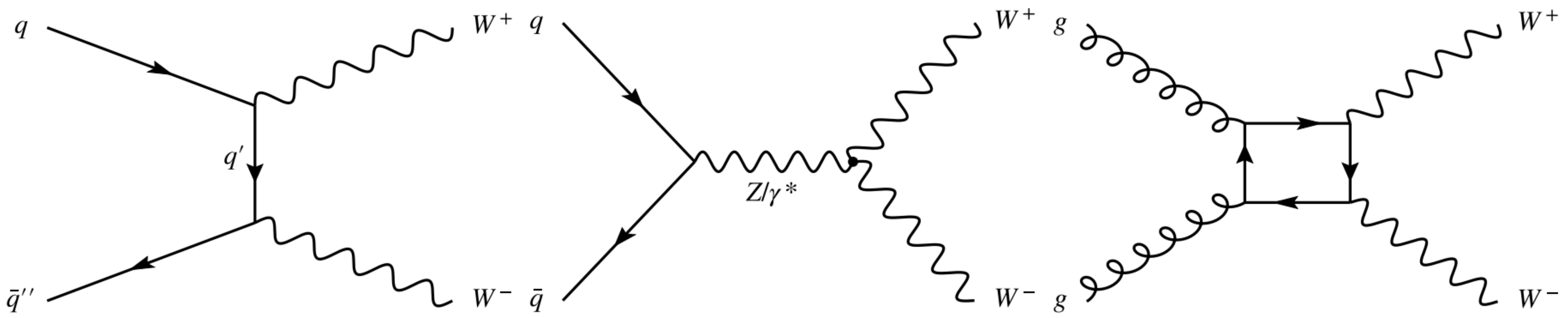
Transverse momentum

Zero-jettiness

Jet veto

GENEVA USING JET VETO RESUMMATION

- W^+W^- production an interesting case study - jet vetoes used in analyses to reject $t\bar{t}$ background
- Aim to improve description of jet-vetoed cross section within an NNLO+PS event generator
- Combine NNLL′ resummation for WW + 0 jets with NLL′ resummation for WW + 1 jet to define events at NNLO



FACTORISATION WITH A JET VETO FOR COLOUR SINGLET

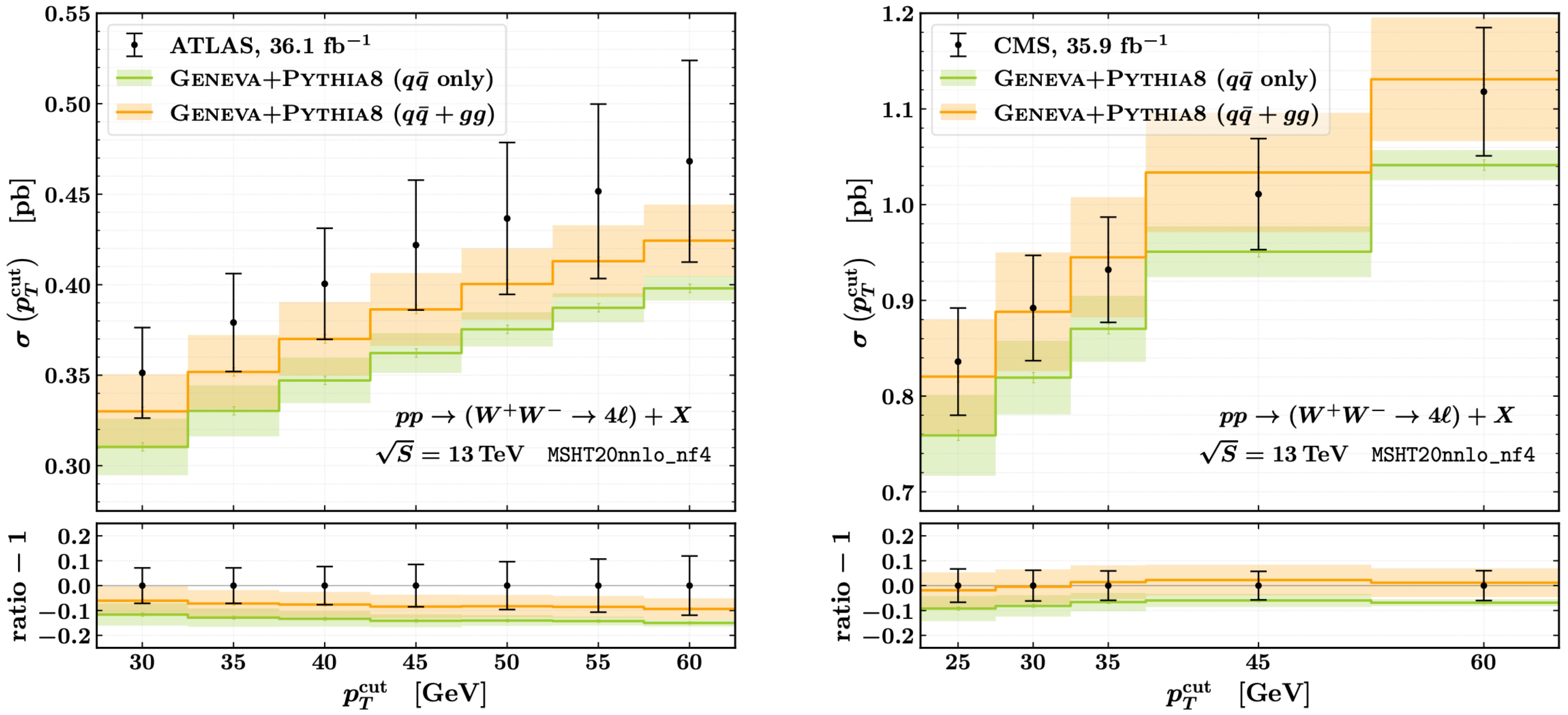
- Consider colour singlet production, vetoing all jets with $p_T > p_T^{\mathrm{veto}}$. Resummation has been studied in both QCD and SCET. T. Becher, M. Neubert, 1205.3806, F. Tackmann, J. Walsh, S. Zuberi, 1206.4312, A. Banfi, G. Salam, G. Zanderighi, 1203.5773, I. Stewart, F. Tackmann, J. Walsh, S. Zuberi, 1307.1808, T. Becher, M. Neubert, L. Rothen, 1307.0025
- Factorisation into hard, beam and soft functions

$$\frac{\mathrm{d}\sigma(p_T^{\mathrm{veto}})}{\mathrm{d}\Phi_0} = H(\Phi_0,\mu)\;[B_a \times B_b](p_T^{\mathrm{veto}}, R, x_a, x_b, \mu, \nu)\; S_{ab}(p_T^{\mathrm{veto}}, R, \mu, \nu)$$

- Radius of vetoed jets R
- Additional scale ν necessary to separate soft/collinear modes (SCET II)

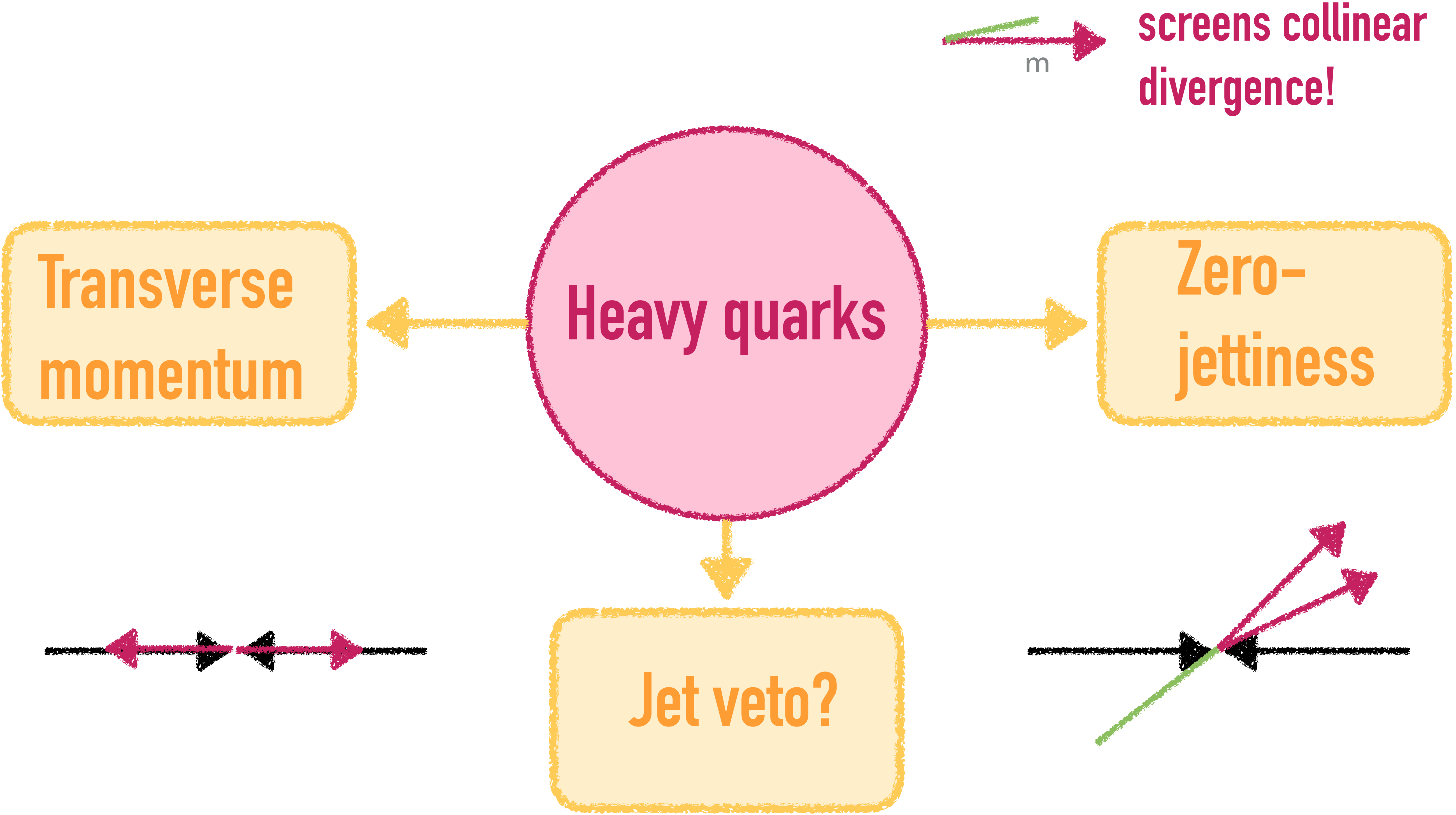
COMPARISON TO ATLAS/CMS

- Vetoed cross section measurements



2308.11577 A. Gavardi, MAL, S. Alioli, F. Tackmann

RESOLUTION VARIABLES



Quark mass screens collinear divergence!

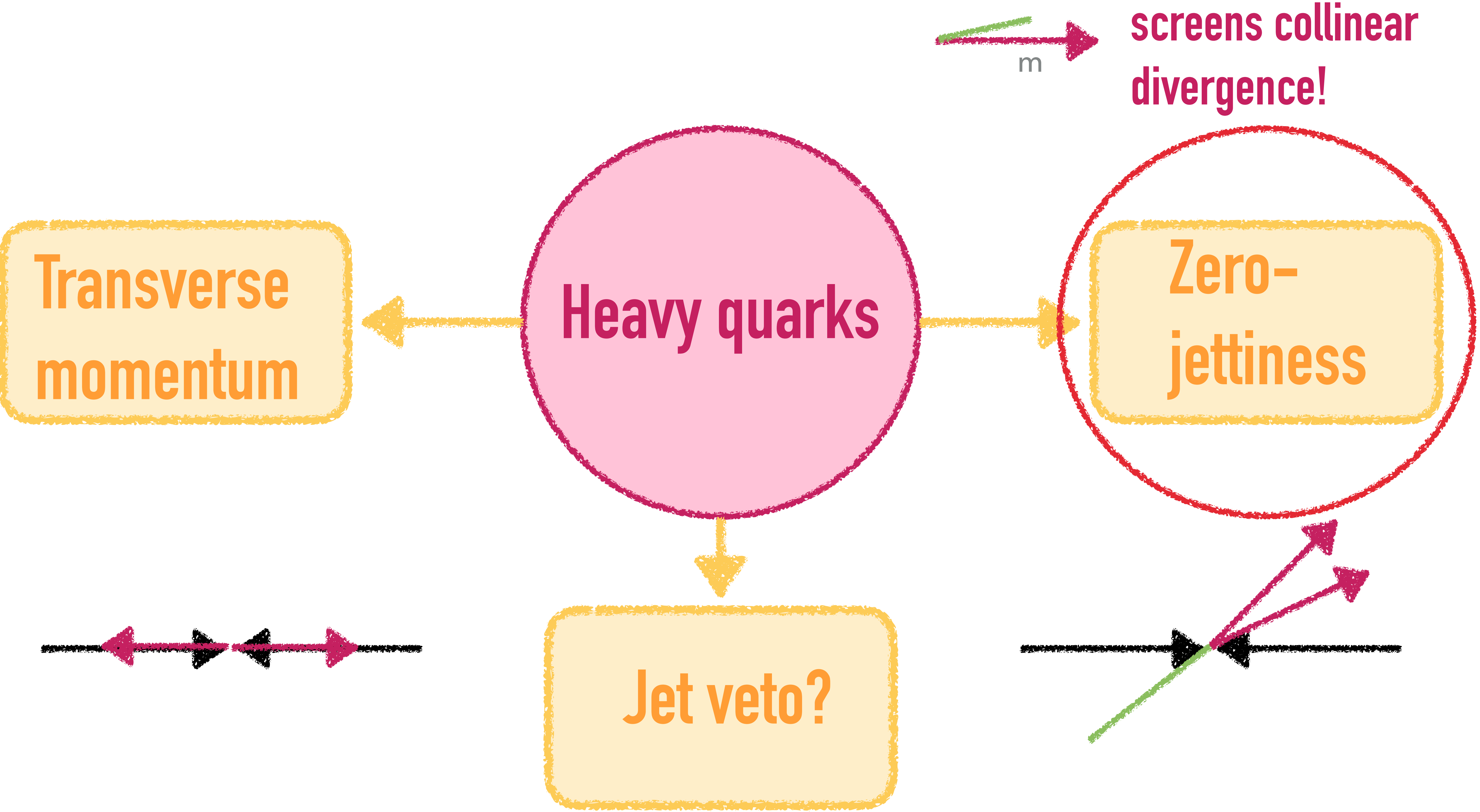
Heavy quarks

Transverse momentum

Zero-jettiness

Jet veto?

RESOLUTION VARIABLES



Quark mass screens collinear divergence!

ZERO-JETTINESS RESUMMATION FOR HEAVY QUARK PAIRS

SCET allows us to write a factorisation formula as

$$\frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\Phi_0 \mathrm{d}\mathcal{T}_0} = \sum_{ij} \int \mathrm{d}t_a \mathrm{d}t_b \; \boxed{B_i(t_a, x_a, \mu_B)\; B_j(t_b, x_b, \mu_B)} \; \mathrm{Tr}\left\{ \boxed{\mathbf{H}_{ij}(\Phi_0, \mu_H)} \boxed{\mathbf{S}\left(\mathcal{T}_0 - \frac{t_a + t_b}{Q}, \Phi_0, \mu_S\right)} \right\}$$

Same as before

Matrices in colour space!

Arises from exchange of soft gluons from heavy quark lines. Evolution equations more complicated:

$$\mathbf{H}(\Phi_0, \mu) = \mathbf{U}(\Phi_0, \mu, \mu_H)\mathbf{H}(\Phi_0, \mu_H)\mathbf{U}^\dagger(\Phi_0, \mu, \mu_H)$$

2111.03632, S. Alioli, A. Broggio, MAL

ZERO-JETTINESS RESUMMATION FOR HEAVY QUARK PAIRS

Derived for the first time! Ingredients partially unknown.

$$\frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\Phi_0\mathrm{d}\mathcal{T}_0} = \sum_{ij}\int \mathrm{d}t_a\mathrm{d}t_b\, \boxed{B_i(t_a, x_a, \mu_B)\; B_j(t_b, x_b, \mu_B)}\; \mathrm{Tr}\left\{\boxed{\mathbf{H}_{ij}(\Phi_0, \mu_H)}\,\boxed{\mathbf{S}\left(\mathcal{T}_0 - \frac{t_a + t_b}{Q}, \Phi_0, \mu_S\right)}\right\}$$

Known up to 3-loops

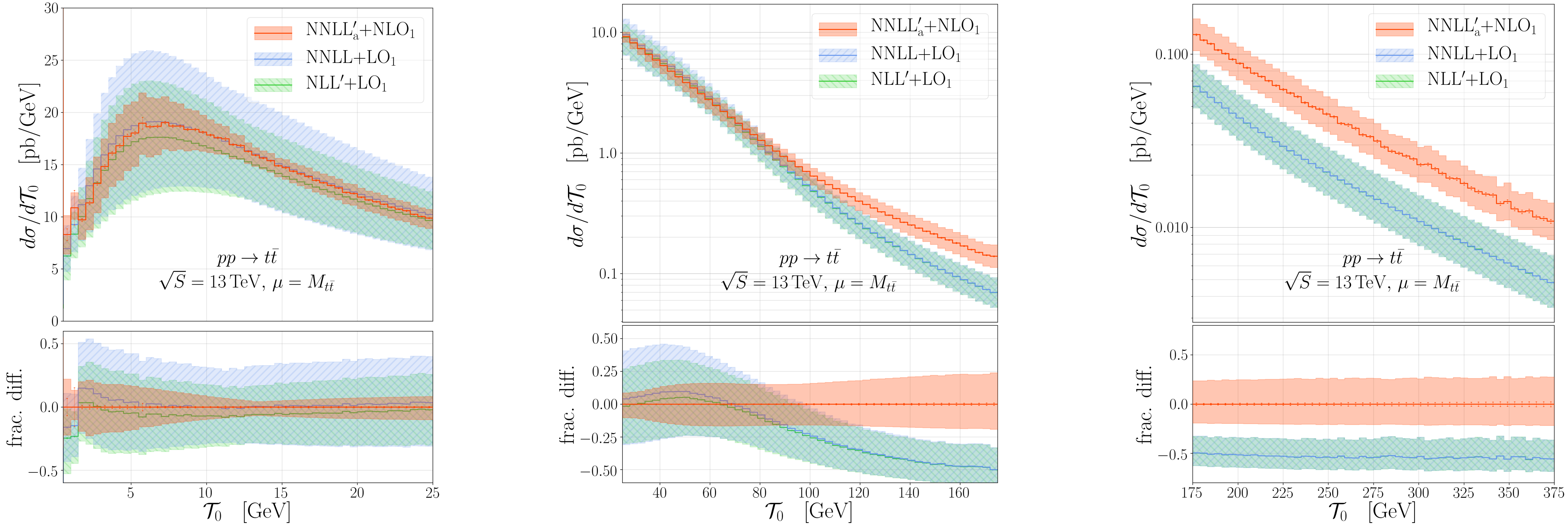
Known up to 2-loops (in principle)

Unknown!

We computed the soft function up to 1-loop. Some 2-loop terms can be obtained via RGE.

2111.03632, S. Alioli, A. Broggio, MAL

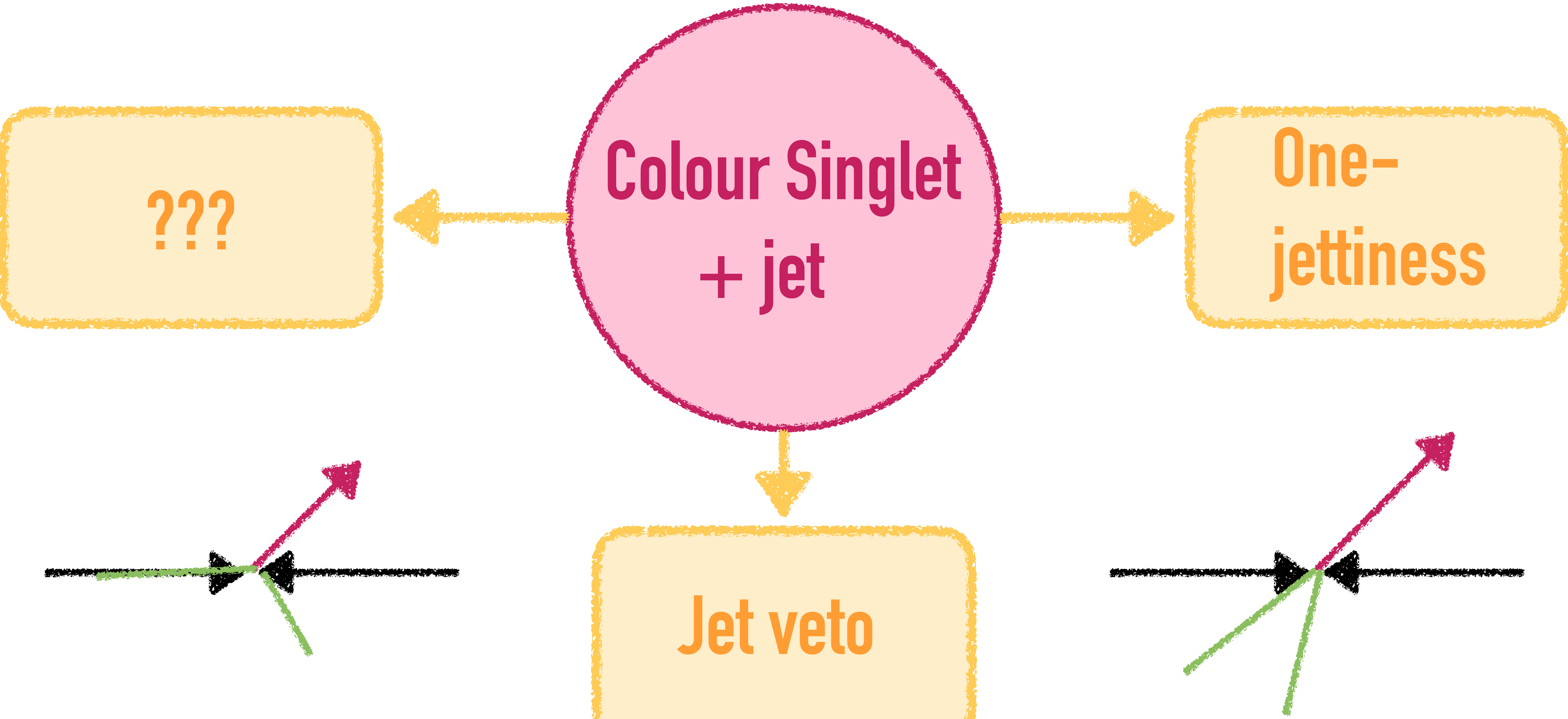
ZERO-JETTINESS RESUMMATION FOR TOP-QUARK PAIRS



- Still missing - two-loop hard (not included here) and one piece of the two-loop soft.
- Allows approximate NNLL’ accuracy.

2111.03632, S. Alioli, A. Broggio, MAL

RESOLUTION VARIABLES



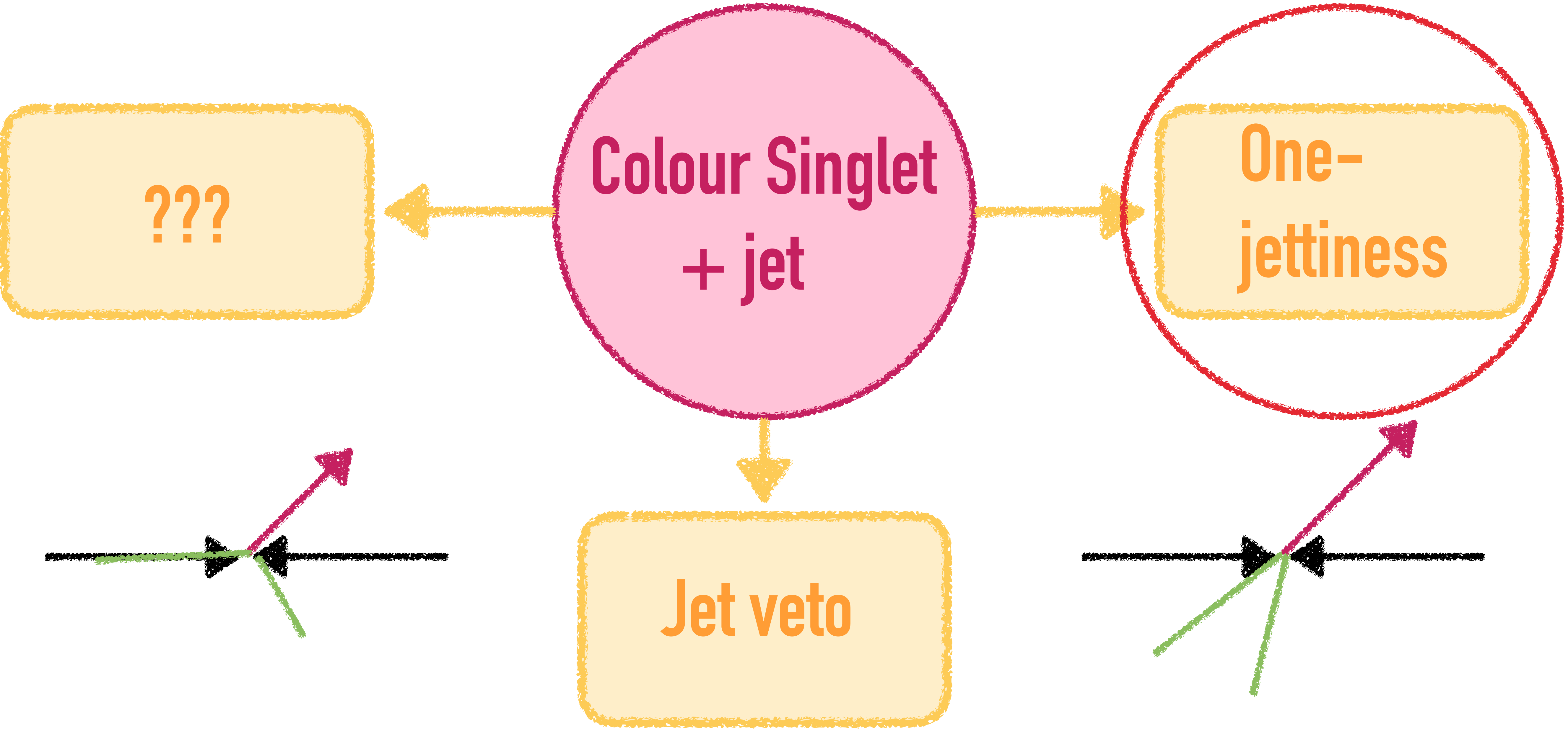
Colour Singlet + jet

???

One-jettiness

Jet veto

RESOLUTION VARIABLES



Colour Singlet + jet

???

One-jettiness

Jet veto

ONE-JETTINESS RESUMMATION FOR COLOUR SINGLET + JET

Similar factorisation to zero-jet case:

$$\frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\Phi_1 \mathrm{d}\mathcal{T}_1} = \sum_{ijk} \int \mathrm{d}t_a \mathrm{d}t_b \mathrm{d}s_J \;\; B_i(t_a, x_a, \mu_B)\, B_j(t_b, x_b, \mu_B)\, \boxed{J_k(s_J, \mu_J)}\, \mathrm{Tr}\left\{ \mathbf{H}_{ij}(\Phi_1, \mu_H) \;\; \mathbf{S}\left(\mathcal{T}_1 - \frac{t_a}{Q_a} - \frac{t_b}{Q_b} - \frac{s_J}{Q_J}, \Phi_1, \mu_S \right) \right\}$$

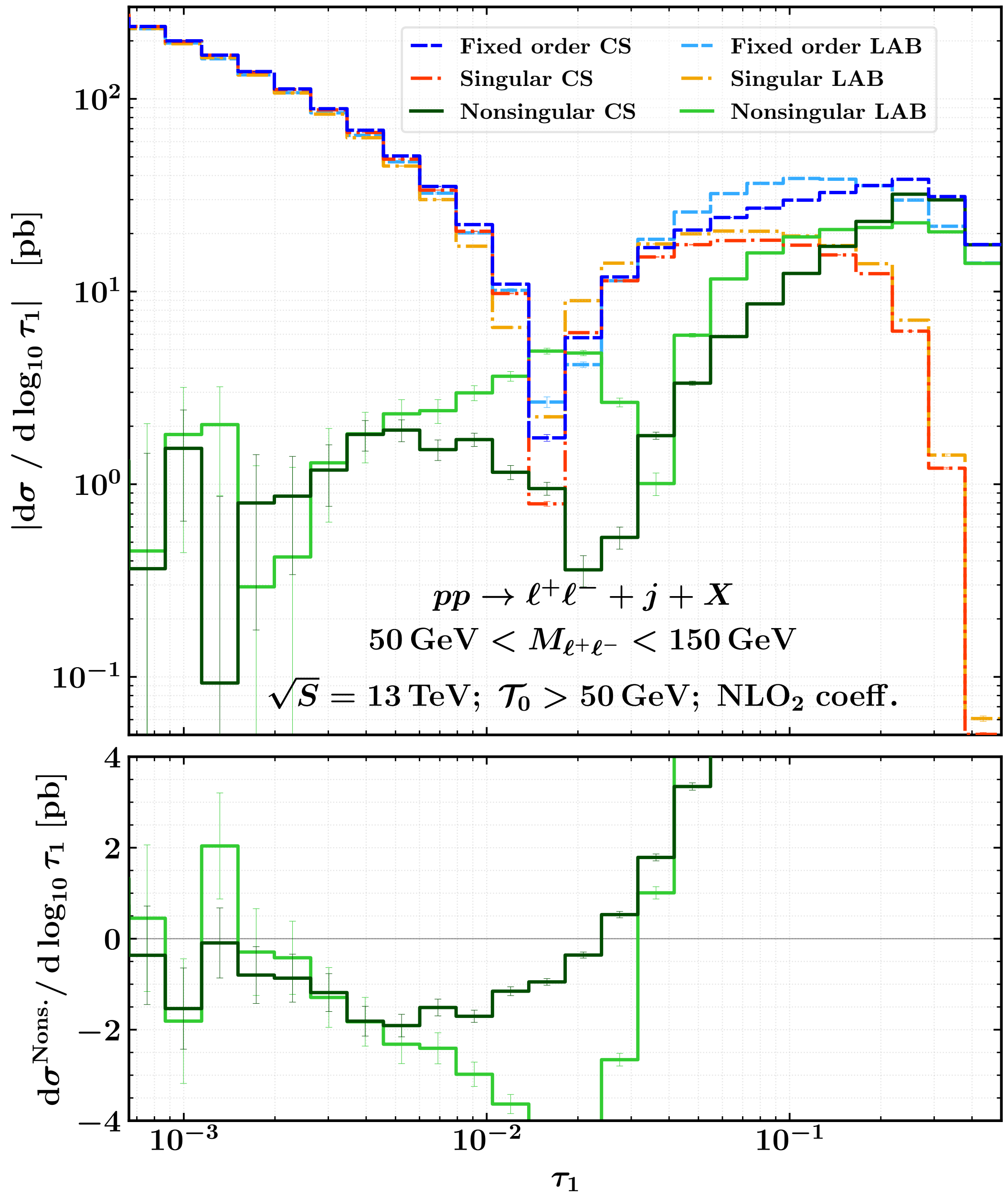
New jet function

- Only three coloured legs - colour algebra is diagonal
- Ingredients for N^{3}LL all known, we use new numerical of two-loop soft function from SoftSERVE
- One-jettiness definition requires choice of frame - can evaluate energies in lab or in CS centre-of-mass

0910.0467, 1302.0846, T. Jouttenus, I. Stewart, F. Tackmann, W. Waalewijn

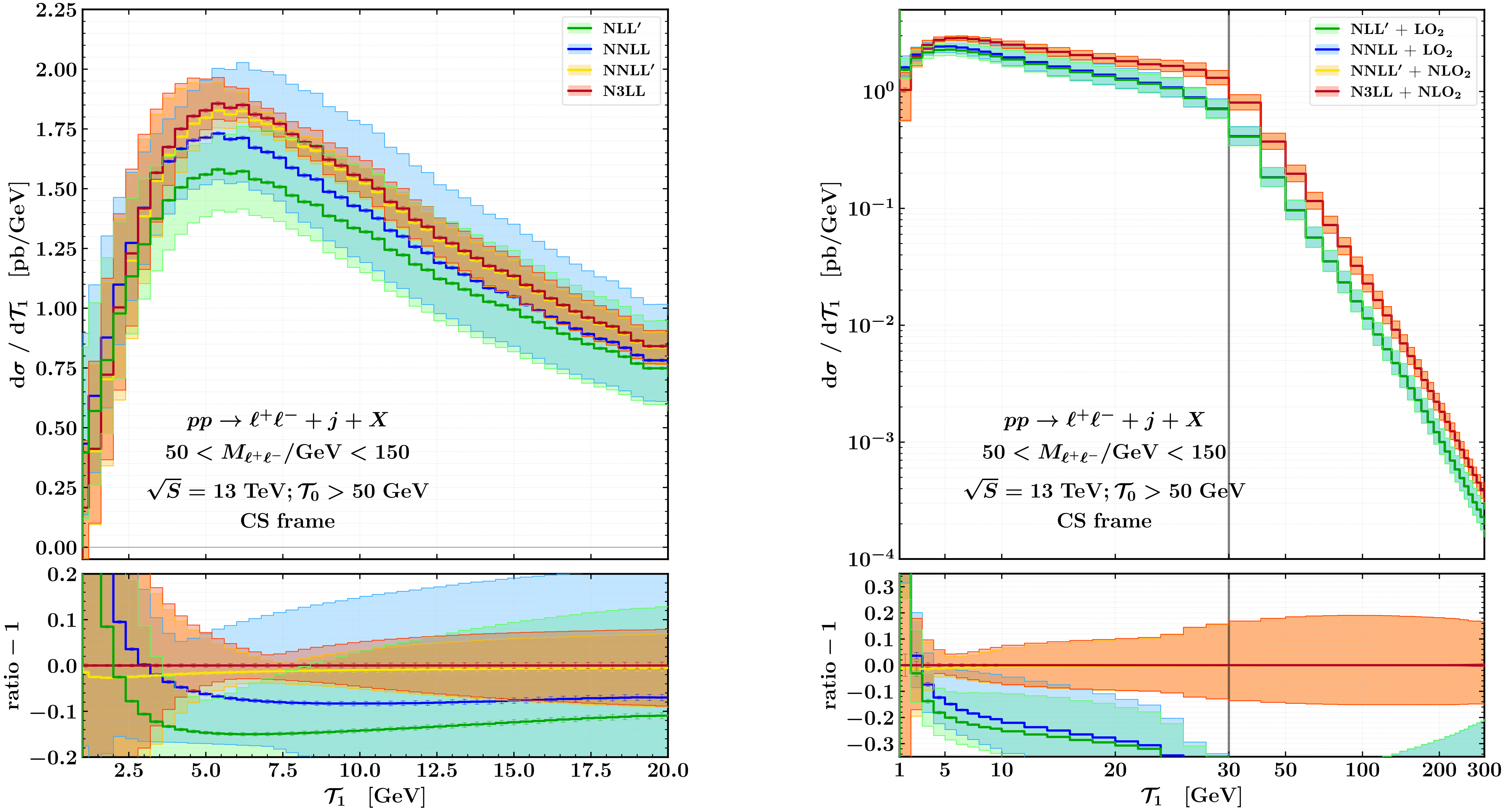
FIXED-ORDER VALIDATION OF ONE-JETTINESS FACTORISATION

- Factorisation theorem must reproduce result of fixed order in the small $\tau_1 = \mathcal{T}_1/Q$ limit
- Size of nonsingular difference has implications for numerical accuracy of slicing calculations



2312.06496, S. Alioli, G. Bell, G. Billis, A. Broggio, B. Dehnadi, MAL, G. Marinelli, R. Nagar, D. Napoletano, R. Rahn

RESUMMED AND MATCHED ONE-JETTINESS SPECTRA



2312.06496, S. Alioli, G. Bell, G. Billis, A. Broggio, B. Dehnadi, MAL, G. Marinelli, R. Nagar, D. Napoletano, R. Rahn

CONCLUSIONS

- GENEVA allows matching of NNLO calculations to parton shower algorithms for a range of colour singlet production processes
- Ongoing work aims to extend this to heavy quark production and processes with jets
- Main limitation is availability of suitable resummed calculation - SCET allows these to be obtained in a systematic way, different resolution variables to be explored

CONCLUSIONS

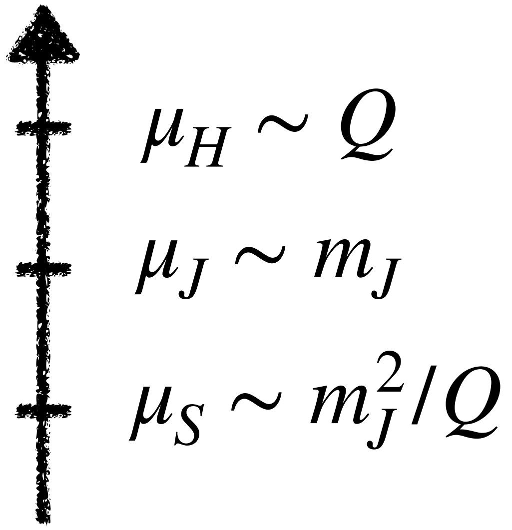
- Recent colour singlet results include single and double Higgs production using zero-jettiness, WW production using jet transverse momentum
- Zero-jettiness for top-quark pair production also studied
- Recent work pushes one-jettiness resummation to N^{3}LL for $Z+\mathrm{jet}$, full NNLO+PS generator is work in progress

Thanks for your attention!

BACKUP SLIDES

SCET I VS SCET II

- 'Simple' SCET problems can be either two- or three-scale, depending on nature of observable
- Three-scale case: $\mu_S \ll \mu_J \ll \mu_H$, covered by SCET I

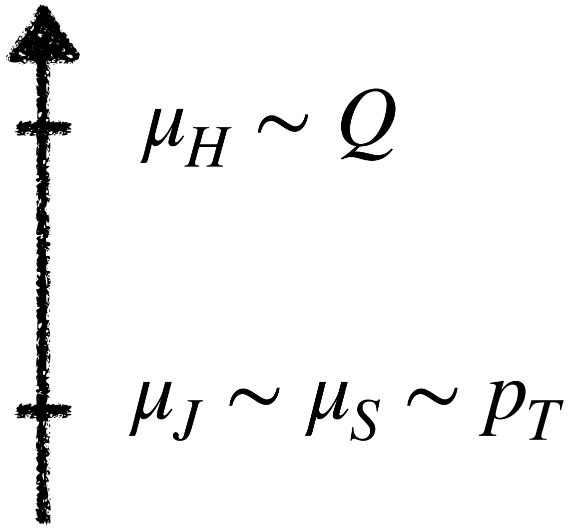


$$\mathrm{d}\hat{\sigma} \approx H(Q,\mu)\, J(m_J,\mu) \otimes S(m_J^2/Q,\mu)$$

$$\ln^2 \frac{Q^2}{m_J^2} = \frac{1}{2} \ln^2 \frac{Q^2}{\mu^2} - \ln^2 \frac{m_J^2}{\mu^2} + \frac{1}{2} \ln^2 \frac{m_J^4}{Q^2 \mu^2}$$

SCET I VS SCET II

- 'Simple' SCET problems can be either two- or three-scale, depending on nature of observable
- Two-scale case: $\mu_S \sim \mu_J \ll \mu_H$, covered by SCET II



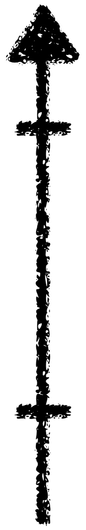
$$\mathrm{d}\hat{\sigma} \approx H(Q,\mu)\, J(p_T,\mu) \otimes S(p_T,\mu)$$

$$\ln^2 \frac{Q^2}{p_T^2} = \ln^2 \frac{Q^2}{\mu^2} - \ln^2 \frac{p_T^2}{\mu^2} + ?$$

- Jet and soft functions ill-defined in dimensional regularisation

SCET I VS SCET II

- 'Simple' SCET problems can be either two- or three-scale, depending on nature of observable
- Two-scale case: $\mu_S \sim \mu_J \ll \mu_H$, covered by SCET II



$\mu_H \sim Q$

$\mu_J \sim \mu_S \sim p_T$

$$\mathrm{d}\hat{\sigma} \approx H(Q, \mu)\ J(p_T, \mu, Q, \nu) \otimes S(p_T, \mu, Q, \nu)$$

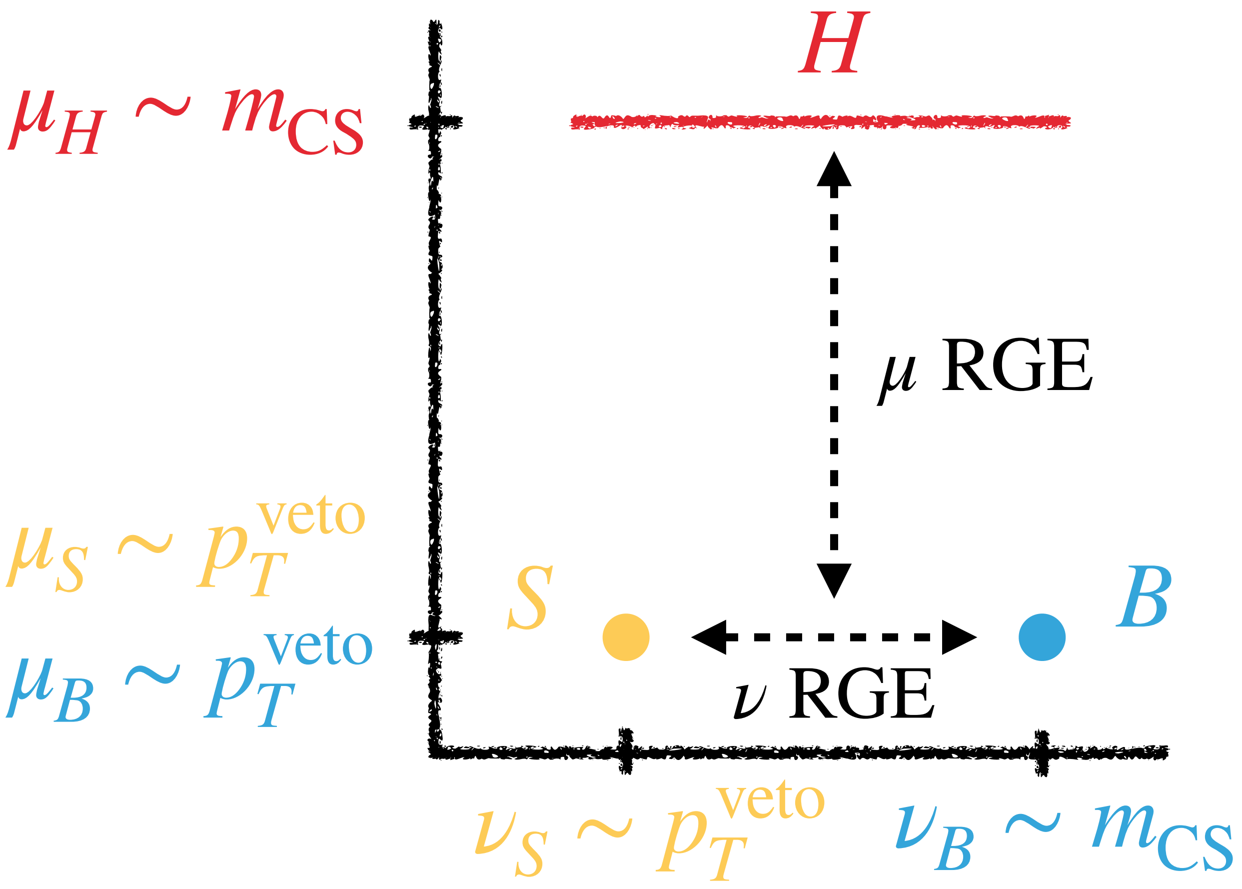
$$\ln^2 \frac{Q^2}{p_T^2} = \ln^2 \frac{Q^2}{\mu^2} - \ln^2 \frac{p_T^2}{\mu^2} - 2 \ln \frac{p_T^2}{mu^2} \ln \frac{Q^2}{\nu^2} - 2 \ln \frac{p_T^2}{\mu^2} \ln \frac{\nu^2}{p_T^2}$$

- Introduction of new rapidity scale ν separates soft and collinear modes

RESUMMATION OF JET VETO LOGS FOR COLOUR SINGLET

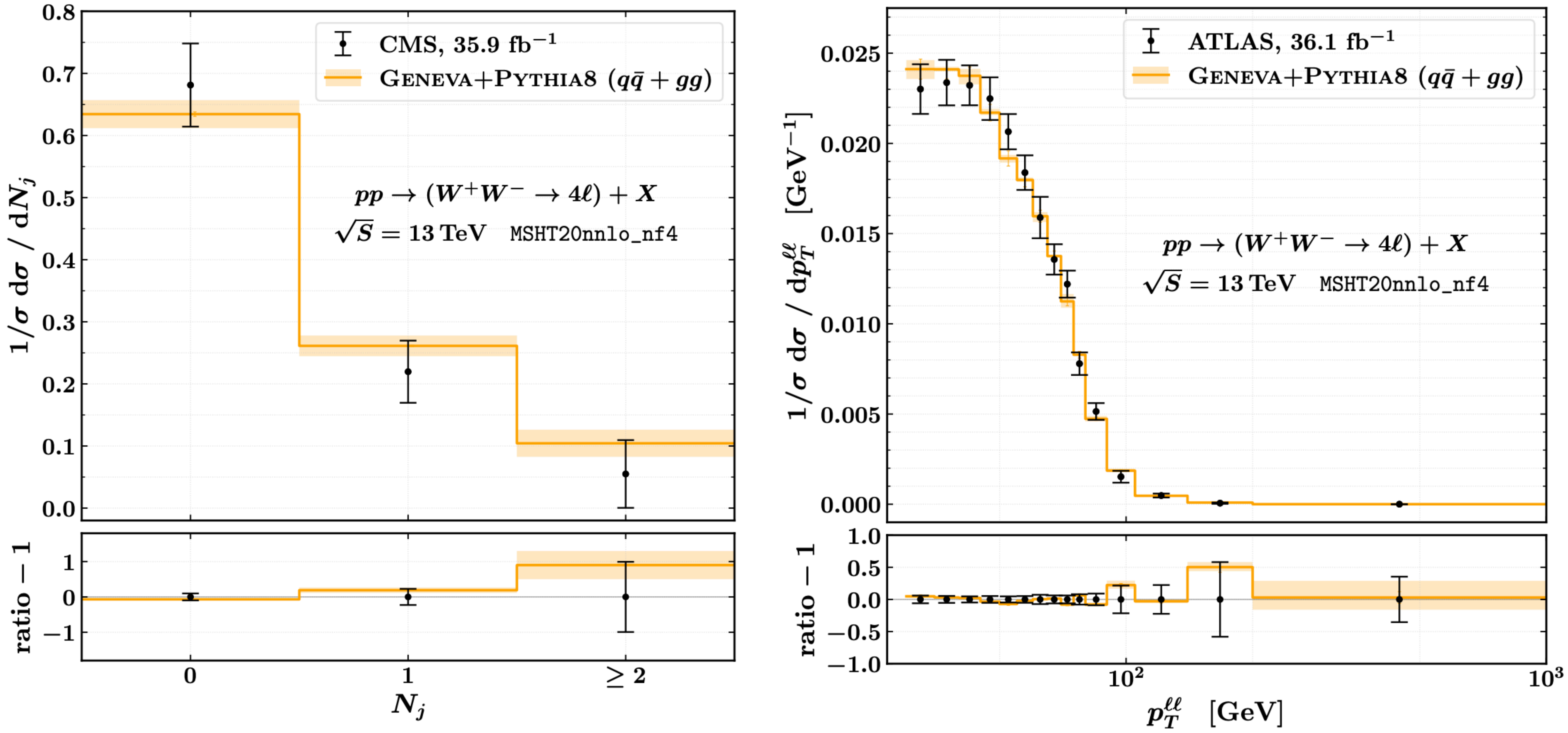
- Rapidity scale ν requires two-dimensional evolution

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\ln S(p_T^{\mathrm{cut}}, R, \mu, \nu) = 4\Gamma_{\mathrm{cusp}}\ln\frac{\mu}{\nu} + \gamma_S \qquad \frac{\mathrm{d}}{\mathrm{d}\ln\nu}\ln S(p_T^{\mathrm{cut}}, R, \mu, \nu) = \gamma_\nu$$



COMPARISON TO ATLAS/CMS

- Compared with ATLAS/CMS measurements



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GENERALISED N-JETTINESS

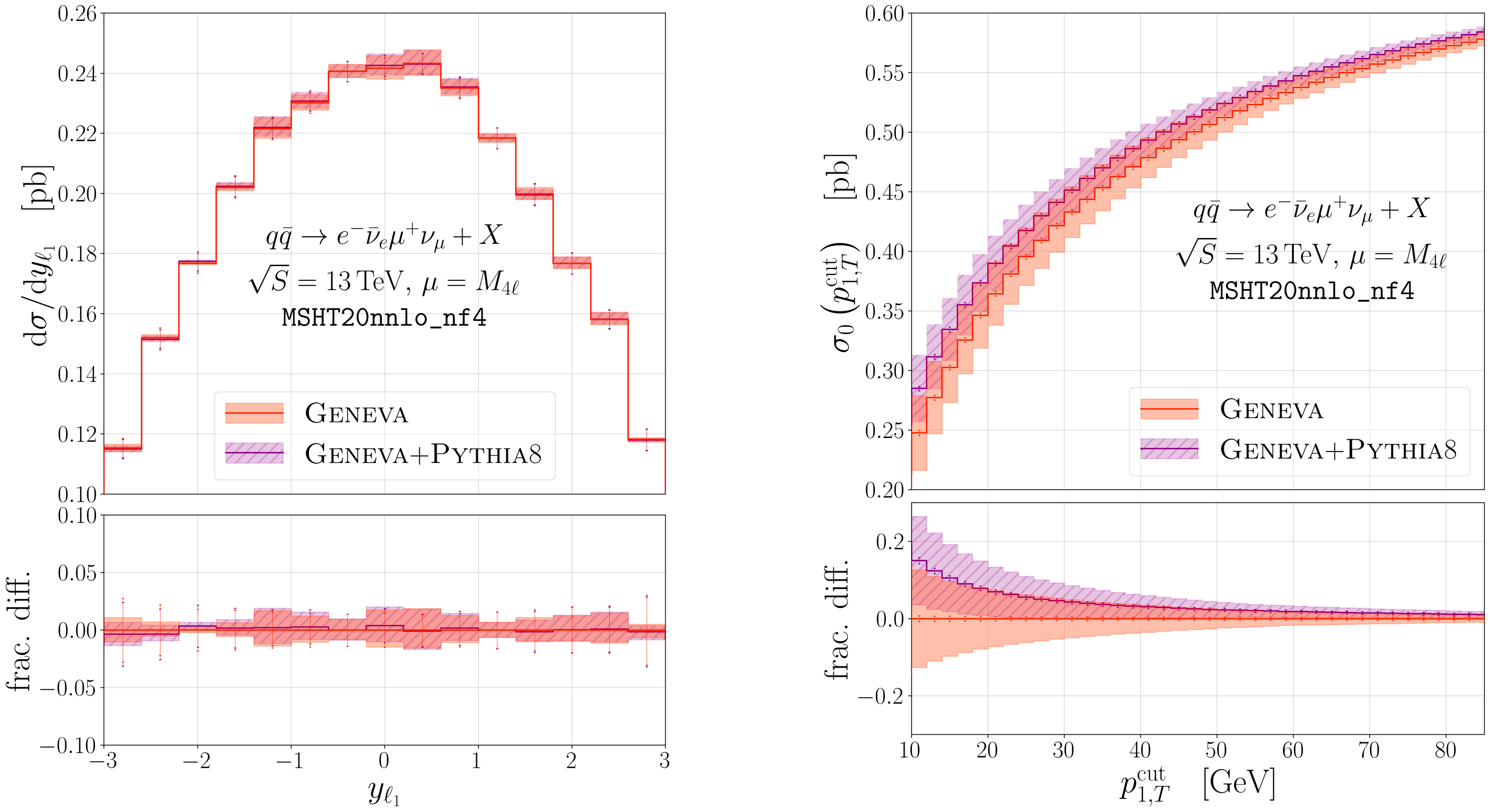
- The $\mathcal{T}_N$ metric need not measure just the invariant mass
- In jet/beam region m, define

$$\mathcal{T}^{(m)} = \sum_{i \in m} f_m(\eta_i, \phi_i) p_{Ti}$$

- Generic form of $\mathcal{T}_N$ can be invariant mass-like or transverse momentum-like (latter used in jet substructure)
- Requires different resummation (SCET-I vs SCET-II)

SHOWERED RESULTS

- Numerically examine effect of shower on accuracy



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