YFS for Future Lepton Colliders

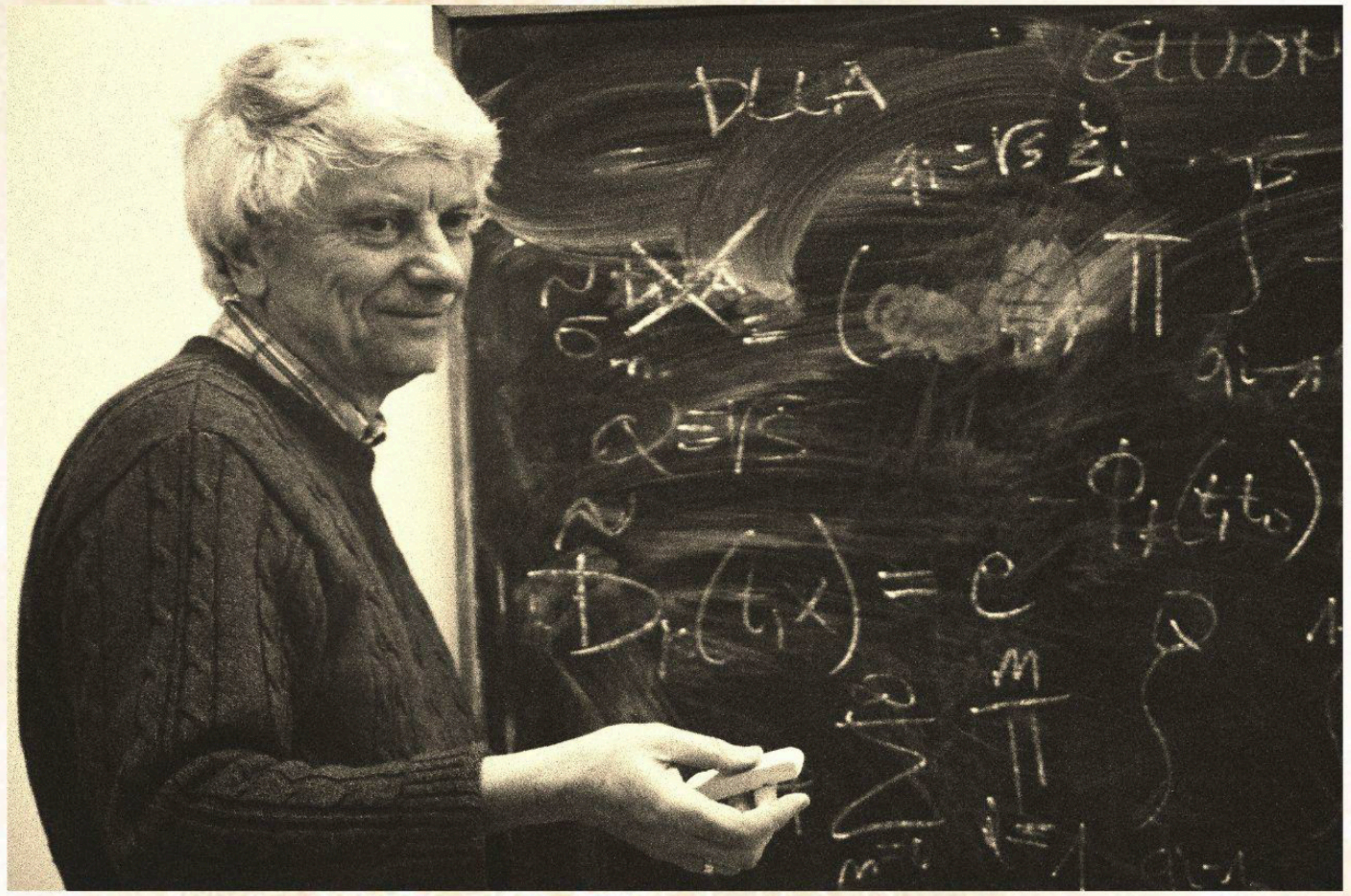
Alan Price on behalf of
the Sherpa Authors

STAR Sensing and Sensibility
Transcending Disciplines for a
Responsible Future

XXX Cracow EPIPHANY Conference

Open scientific and memorial session dedicated to Prof. Stanislaw Jadach

10 January 2024, 14:30-18:30, Auditorium Maximum, ul. Krupnicza 33
Krakow, Poland



We would like to invite everyone to an open scientific and memorial session dedicated to Prof. Stanislaw Jadach, who passed away in 2023

Organizing Committee:

Marcin Chrząszcz	(IFJ PAN)
Iwona Grabowska-Bołd	(AGH)
Marek Jeżabek	(IFJ PAN)
Aleksander Kusina	(IFJ PAN)
Krzysztof Kutak	(IFJ PAN)
Wiesław Płaczek	(Jagiellonian University)
Sebastian Sapeta	(IFJ PAN)
Magdalena Sławińska	(IFJ PAN)
Andrzej Siódmok	(Jagiellonian University)
Maciej Skrzypek	(IFJ PAN)
Zbigniew Wąs	(IFJ PAN)

Invited Speakers:

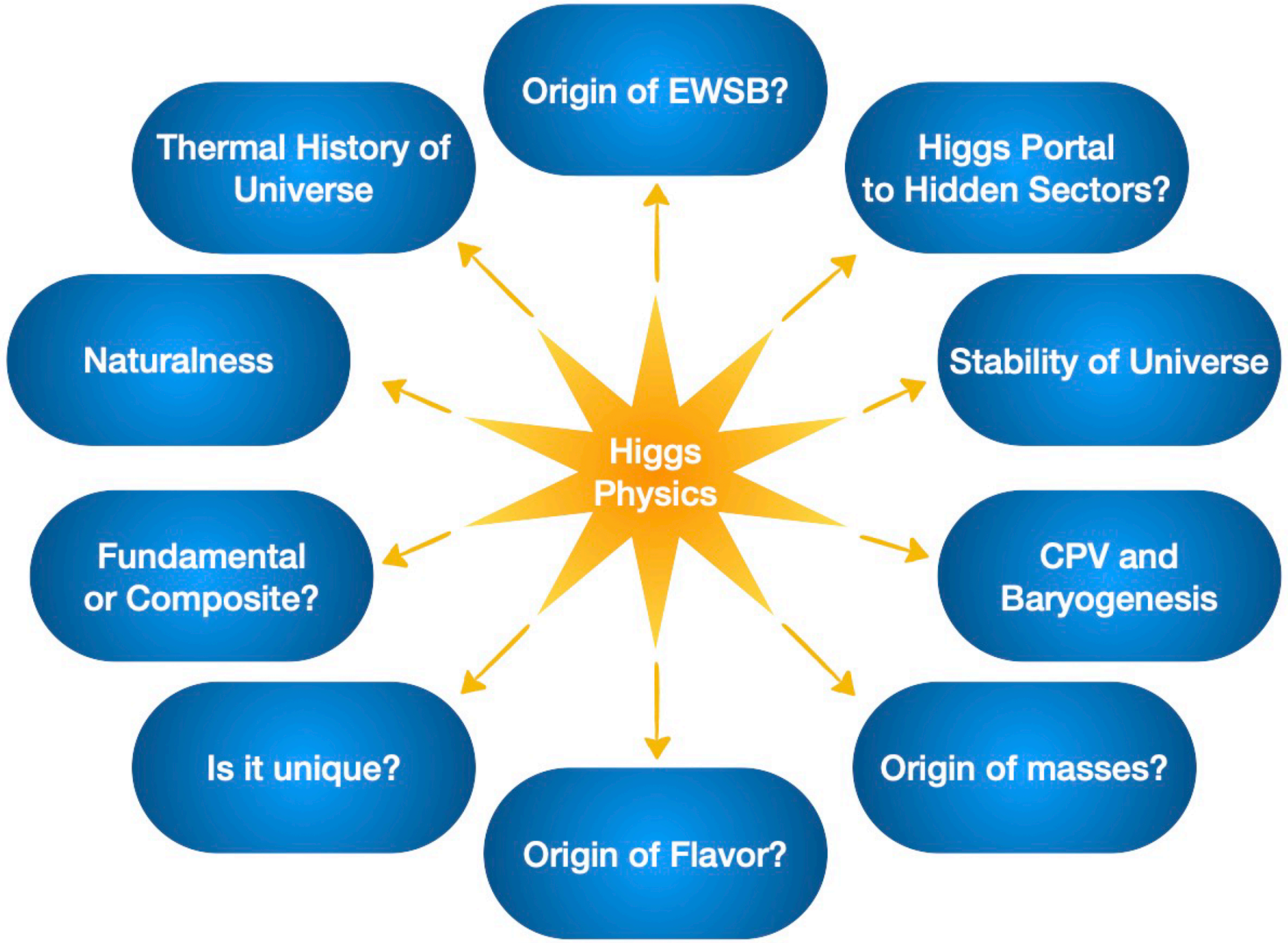
Alain Blondel
Rolf-Dieter Heuer
Patrick Janot
Bennie F.L. Ward
Zbigniew Wąs

http://epiphany.ifj.edu.pl

Open Questions

`**Disclaimer:** I am not a model builder but a phenomenologist`

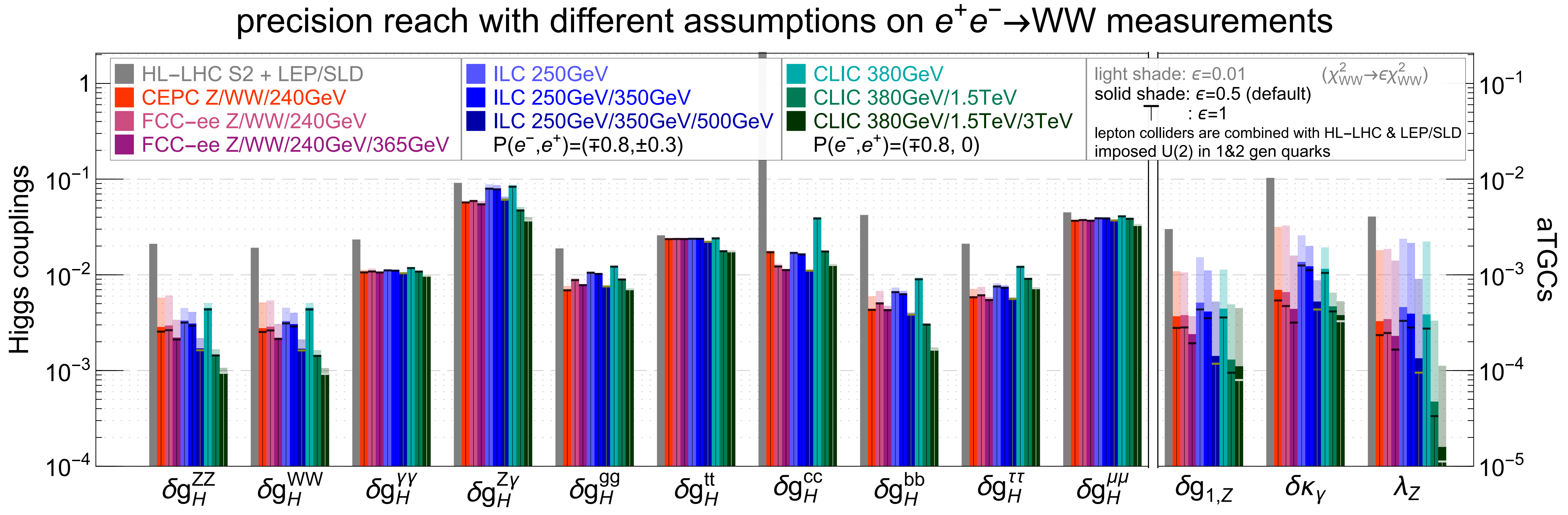
- Dark matter?
- Explanation for the fermion masses
- Matter-antimatter asymmetry
- Nature & properties of neutrinos?
- Limitations of the Standard Model (SM)?



Snowmass 2021 US Community Study on the Future of Particle Physic

Physics Landscape at Higgs Factories

- Higgs couplings measured to a few %
- Self coupling with 50% precision
- Top-quark pole mass uncertainty of 500 MeV
- Flavour physics observables improved by about one order of magnitude compared to today
- Improvement on direct Dark matter limits
- Possible surprises?



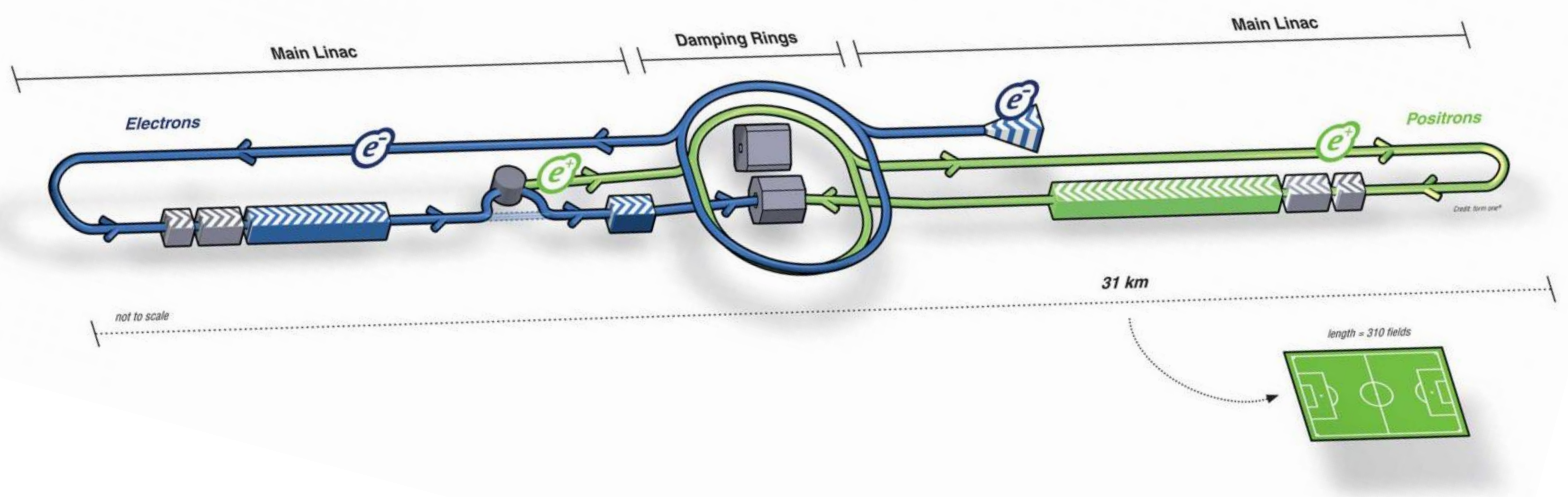
J. De Blas *et al* JHEP 12 (2019) 117

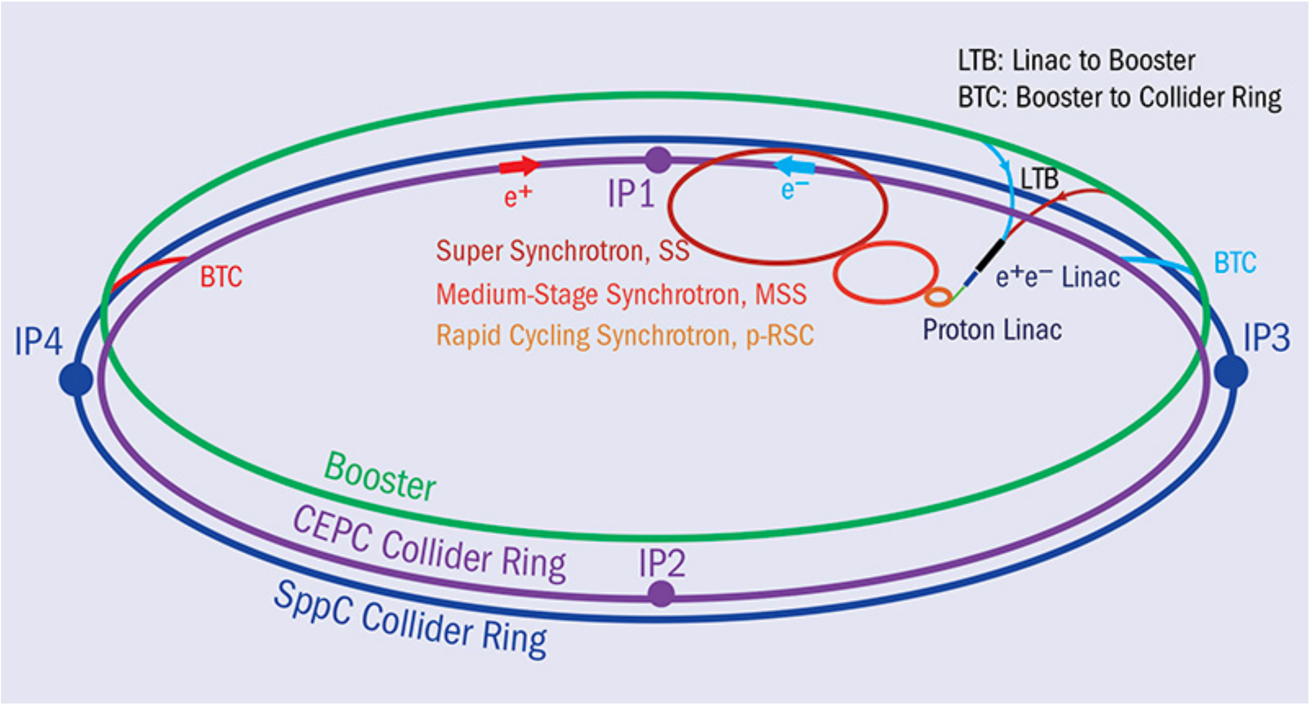
An electron-positron Higgs factory is the highest-priority next collider. –EUROPEAN STRATEGY FOR PARTICLE PHYSICS

See talks P. Janot & A. Blonde

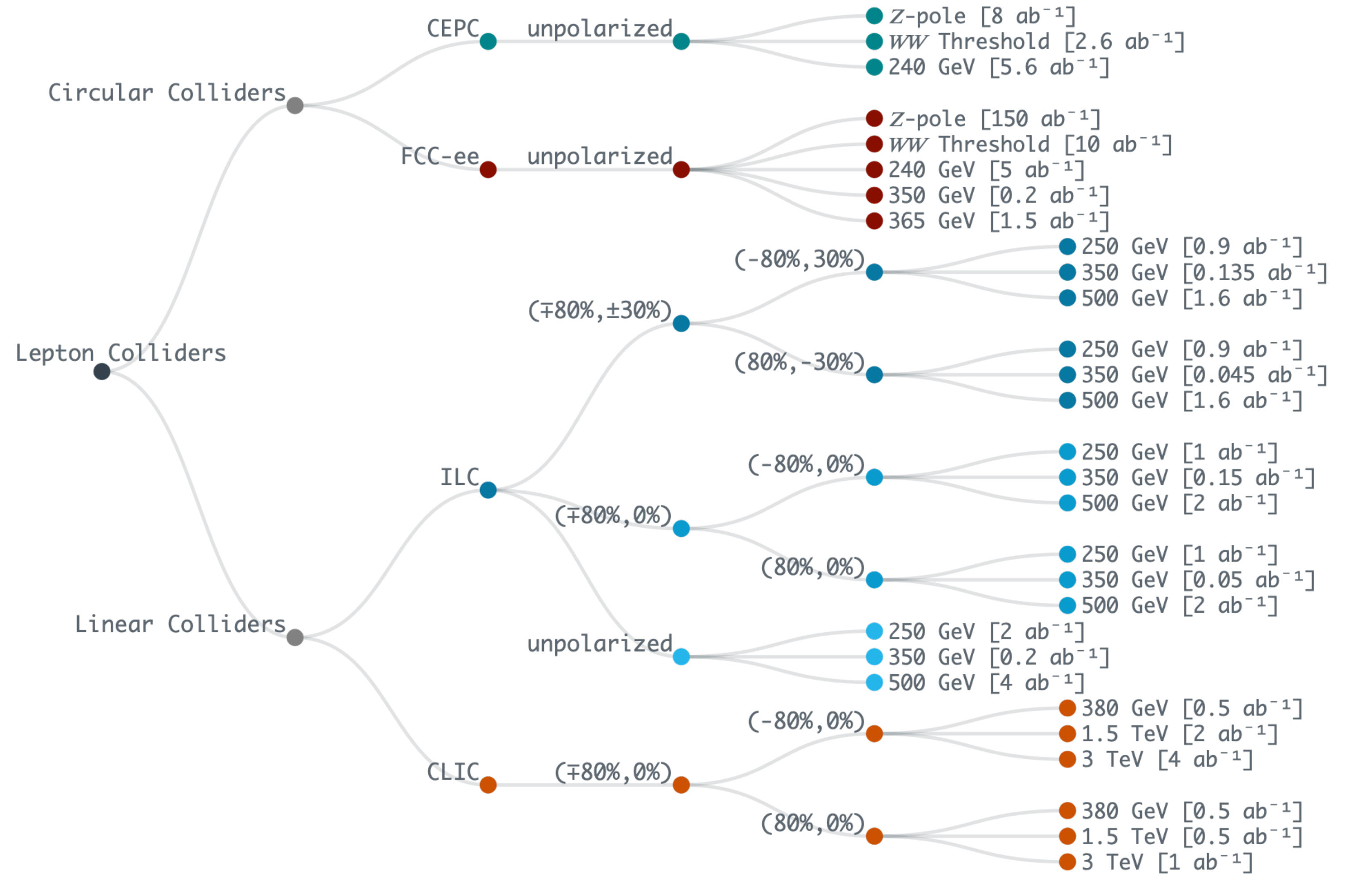
Possible Colliders

ILC



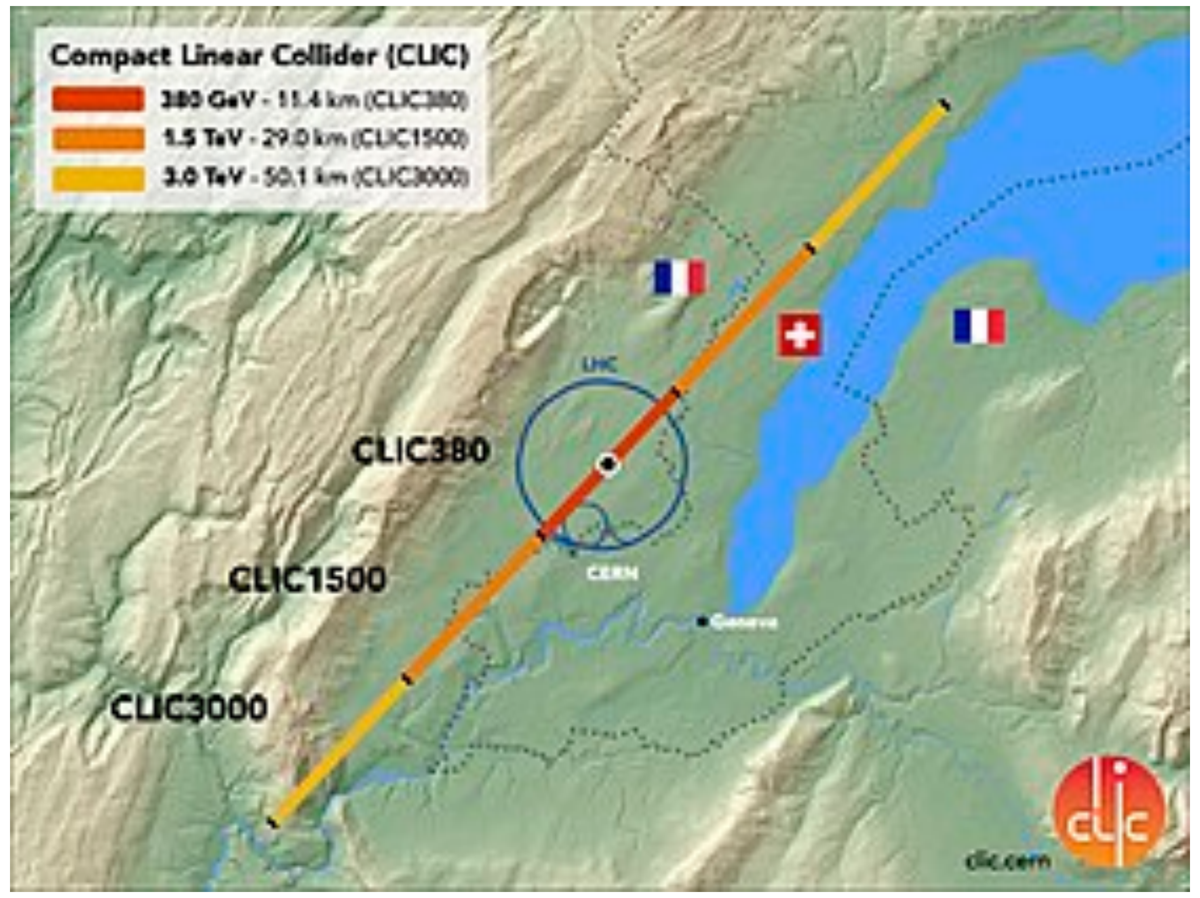


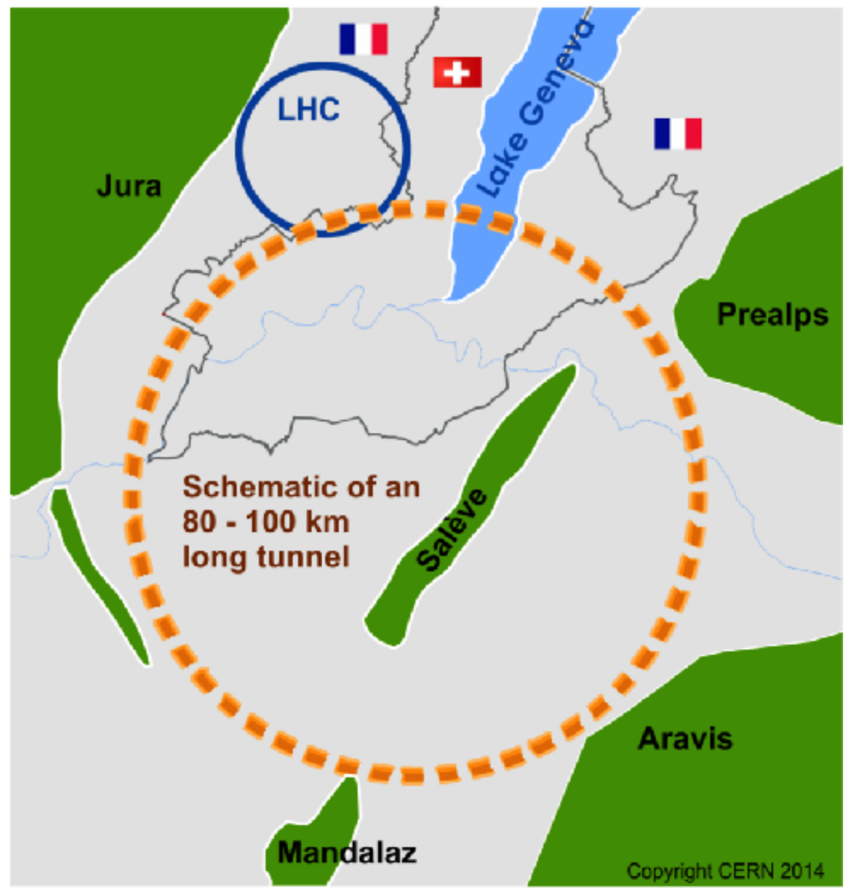
CEPC



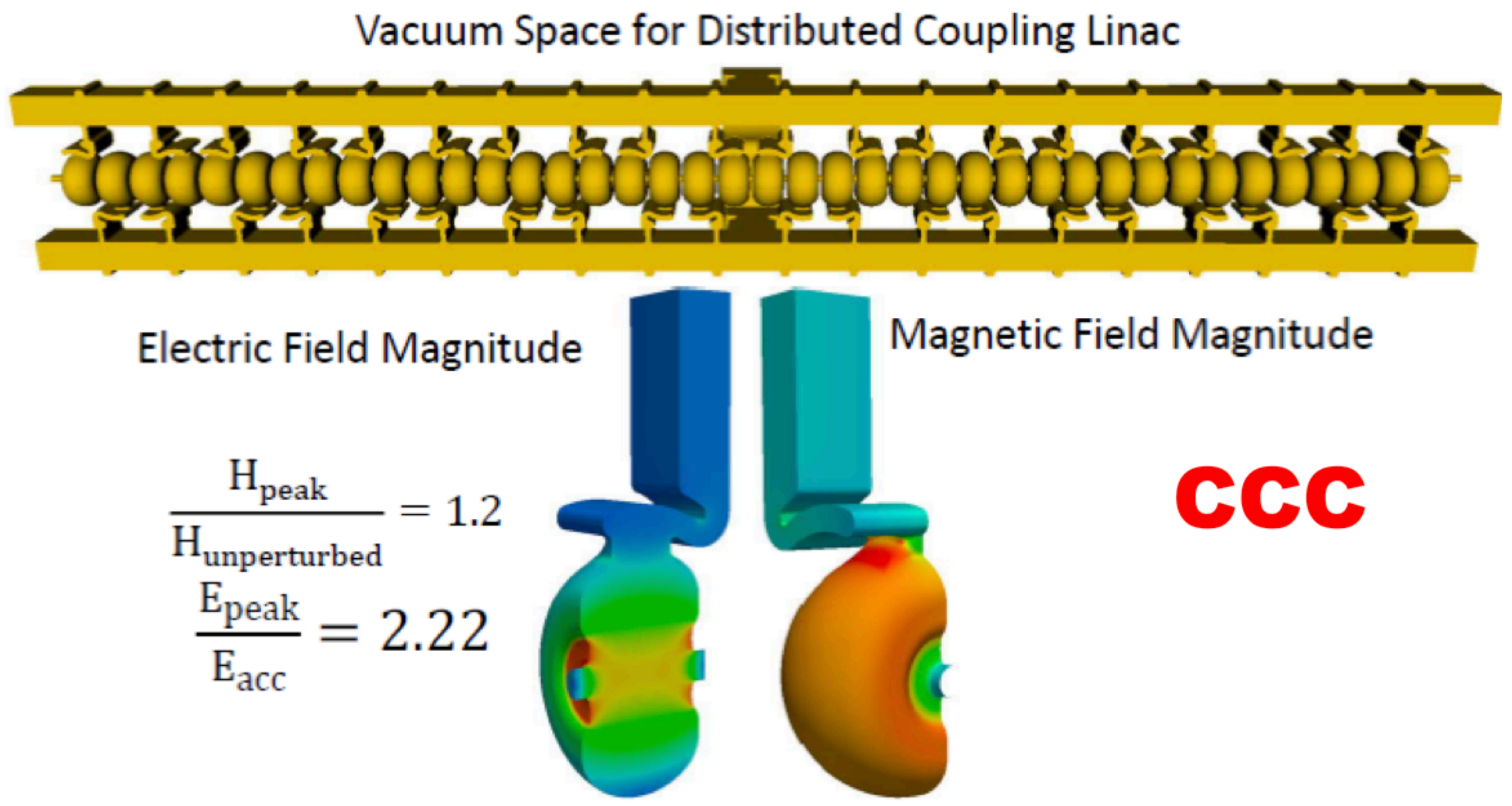
J. De Blas *et al* JHEP 12 (2019) 117

CLIC





FCC



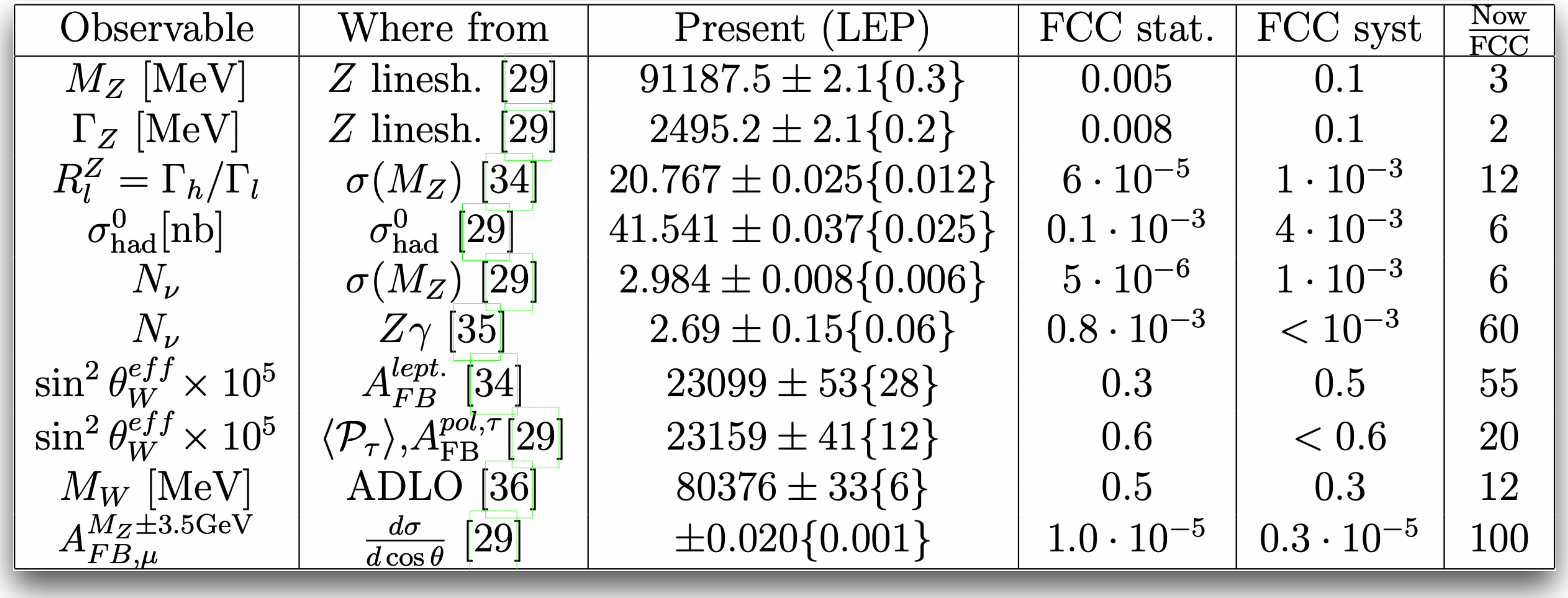
$$\frac{H_{peak}}{H_{unperturbed}} = 1.2$$

$$\frac{E_{peak}}{E_{acc}} = 2.22$$

CCC

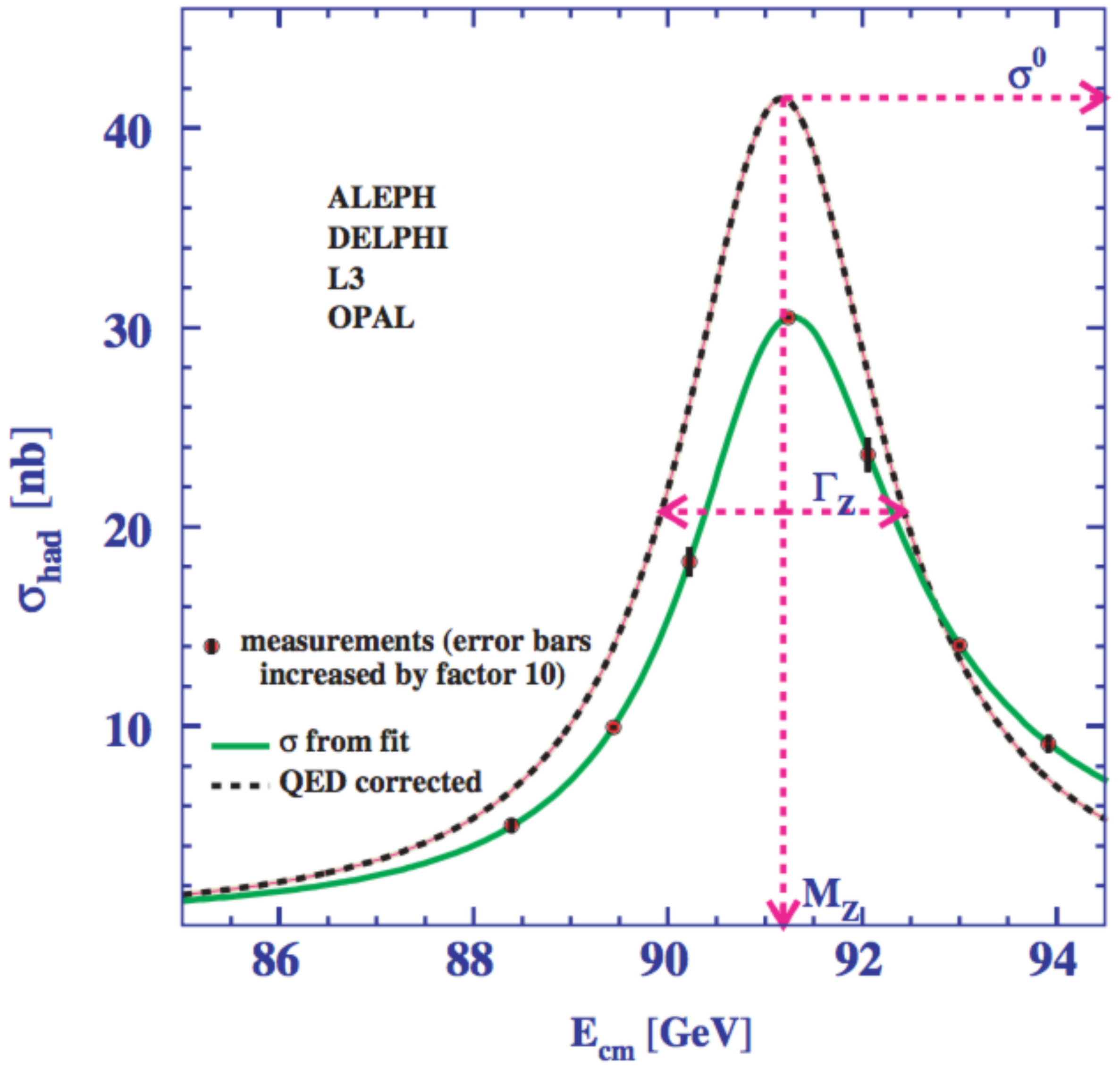
Theory Requirements

- Factor 5-200 reduction of experimental error
- QED effects of 0.1% could be included in LEP error budget
- Future colliders will deliver full LEP Statistics in minutes



Observable	Where from	Present (LEP)	FCC stat.	FCC syst	$\frac{\text{Now}}{\text{FCC}}$
M_Z [MeV]	Z linesh. [29]	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3
Γ_Z [MeV]	Z linesh. [29]	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$ [34]	$20.767 \pm 0.025\{0.012\}$	$6 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	12
$\sigma^0_{\rm had}$[nb]	$\sigma^0_{\rm had}$ [29]	$41.541 \pm 0.037\{0.025\}$	$0.1 \cdot 10^{-3}$	$4 \cdot 10^{-3}$	6
N_ν	$\sigma(M_Z)$ [29]	$2.984 \pm 0.008\{0.006\}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	6
N_ν	$Z\gamma$ [35]	$2.69 \pm 0.15\{0.06\}$	$0.8 \cdot 10^{-3}$	$< 10^{-3}$	60
$\sin^2\theta_W^{eff} \times 10^5$	$A_{FB}^{lept.}$ [34]	$23099 \pm 53\{28\}$	0.3	0.5	55
$\sin^2\theta_W^{eff} \times 10^5$	$\langle\mathcal{P}_\tau\rangle$,$A_{\rm FB}^{pol,\tau}$ [29]	$23159 \pm 41\{12\}$	0.6	< 0.6	20
M_W [MeV]	ADLO [36]	$80376 \pm 33\{6\}$	0.5	0.3	12
$A_{FB,\mu}^{M_Z \pm 3.5{\rm GeV}}$	$\frac{d\sigma}{d\cos\theta}$ [29]	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100

S.Jadach and M.Skrzypek, Eur. Phys. J.C 79, no.9, 756 (2019)



$$d\sigma(L,\hat{L}) = \alpha^k \sum_n^{\infty} \alpha^n \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \hat{\sigma}_{n,i,j} L^i \hat{L}^j$$

$$\hat{L} = \log\left(\frac{Q^2}{E_\gamma^2}\right) \qquad L = \log\left(\frac{Q^2}{m_e^2}\right)$$

Soft Logarithms **Collinear Logarithms**

Perturbative Frameworks

Collinear Factorization:

See talks Z. Nagy, R. Poncelet, J. Whitehead, M. Ubiali, M.Lim….

$$d\sigma_{e^+e^-\to X} = \int dx_1 dx_2 f_{e^+}(x_1, Q^2)\, d\hat{\sigma}_{e^+e^-\to X}(x_1x_2s) f_{e^-}(x_2, Q^2)$$

$f_{e^{+/-}}(x, Q^2)$

Process independent parton distribution function which resums **collinear** log

LO/LL structure function M.Skrzypek, **Stanislaw Jadach** Z.Phys.C 49 (1991) 577-584

NLO/NLL PDF S.Frixone et.al *JHEP* 03 (2020)

$d\hat{\sigma}_{e^+e^-\to X}(x_1x_2s)$

Short distance cross-section: LO, NLO, NNLO

Exclusive Photons: Combined with a QED shower. Non-trivial for NLO

Perturbative Frameworks

Yenni-Frautschi-Suura Theorem:

$$d\sigma_{e^+e^-\to X} = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!}\, d\Phi_Q \left[\prod_{i=1}^{n_\gamma} d\Phi_i^\gamma\, S(k_i)\, \Theta(k_i,\Omega) \right] \left(\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(k_j)}{S(k_j)} + \sum_{\substack{j,k=1\\ j<k}}^{n_\gamma} \frac{\tilde{\beta}_2(k_j,k_k)}{S(k_j)S(k_k)} + \cdots \right),$$

**This formula is very familiar to many in the room,
for others this may new**

Perturbative Frameworks

YFS Theorem:

$$d\sigma_{e^+e^-\to X} = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!}\, d\Phi_Q \left[\prod_{i=1}^{n_\gamma} d\Phi_i^\gamma\, S(k_i)\, \Theta(k_i,\Omega)\right] \left(\tilde\beta_0 + \sum_{j=1}^{n_\gamma}\frac{\tilde\beta_1(k_j)}{S(k_j)} + \sum_{\substack{j,k=1\\ j<k}}^{n_\gamma} \frac{\tilde\beta_2(k_j,k_k)}{S(k_j)S(k_k)} + \cdots\right),$$

YFS Form-Factor:

All order resummation of real and virtual IR divergences

Gives a MC algorithm to construct multi-photon emissions analytically
Exclusive photons for free

$$Y(\Omega) = 2\alpha \sum_{i<j}\left(\mathscr{Re}\, B(p_i,p_j) + \tilde B(p_i,p_j,\Omega)\right),$$

$$B(p_i,p_j) = -\frac{i}{8\pi^3} Z_iZ_j\theta_i\theta_j \int \frac{d^4k}{k^2}\left(\frac{2p_i\theta_i - k}{k^2-2(k\cdot p_i)\theta_i} + \frac{2p_j\theta_j+k}{k^2+2(k\cdot p_j)\theta_j}\right)^2,$$

$$\tilde B(p_i,p_j,\Omega) = \frac{1}{4\pi^2} Z_iZ_j\theta_i\theta_j \int d^4k\,\delta(k^2)\big(1-\Theta(k,\Omega)\big)\left(\frac{p_i}{(p_i\cdot k)} - \frac{p_j}{(p_j\cdot k)}\right)^2$$

Perturbative Frameworks

YFS Theorem:

$$d\sigma_{e^+e^-\to X} = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!} d\Phi_Q \left[\prod_{i=1}^{n_\gamma} d\Phi_i^\gamma \, S(k_i) \, \Theta(k_i, \Omega) \right] \left(\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(k_j)}{S(k_j)} + \sum_{\substack{j,k=1 \\ j<k}}^{n_\gamma} \frac{\tilde{\beta}_2(k_j, k_k)}{S(k_j)S(k_k)} + \cdots \right),$$

$$\tilde{\beta}_{n_\gamma} = \sum_{\bar{n}_\gamma=0}^{\infty} \tilde{\beta}_{n_\gamma}^{\bar{n}_\gamma+n_\gamma}$$

$$S_{ij}(k) = \frac{\alpha}{4\pi^2} Z_i Z_j \theta_i \theta_j \left(\frac{p_i}{p_i \cdot k} - \frac{p_j}{p_j \cdot k} \right)^2$$

IR finite corrections for $\bar{n}_\gamma$ virtual and n_γ real photon corrections

Each β is individually IR finite and can be defined to arbitrary precision

Essentially define an **EW subtraction Scheme**

YFS Monte-Carlo Tools

See talks by Andrzej, Zbigniew, and Bennie

Process Specific:

KKMC

YFSWW/KoralW

BHLUMI/BHWIDE

The MC implementation of YFS has been championed by the Krakow group lead by Stanislaw

Comput.Phys.Commun. 56 (1990) 351-384

Eur.Phys.J.C 80 (2020) 6, 499

Process Independent:

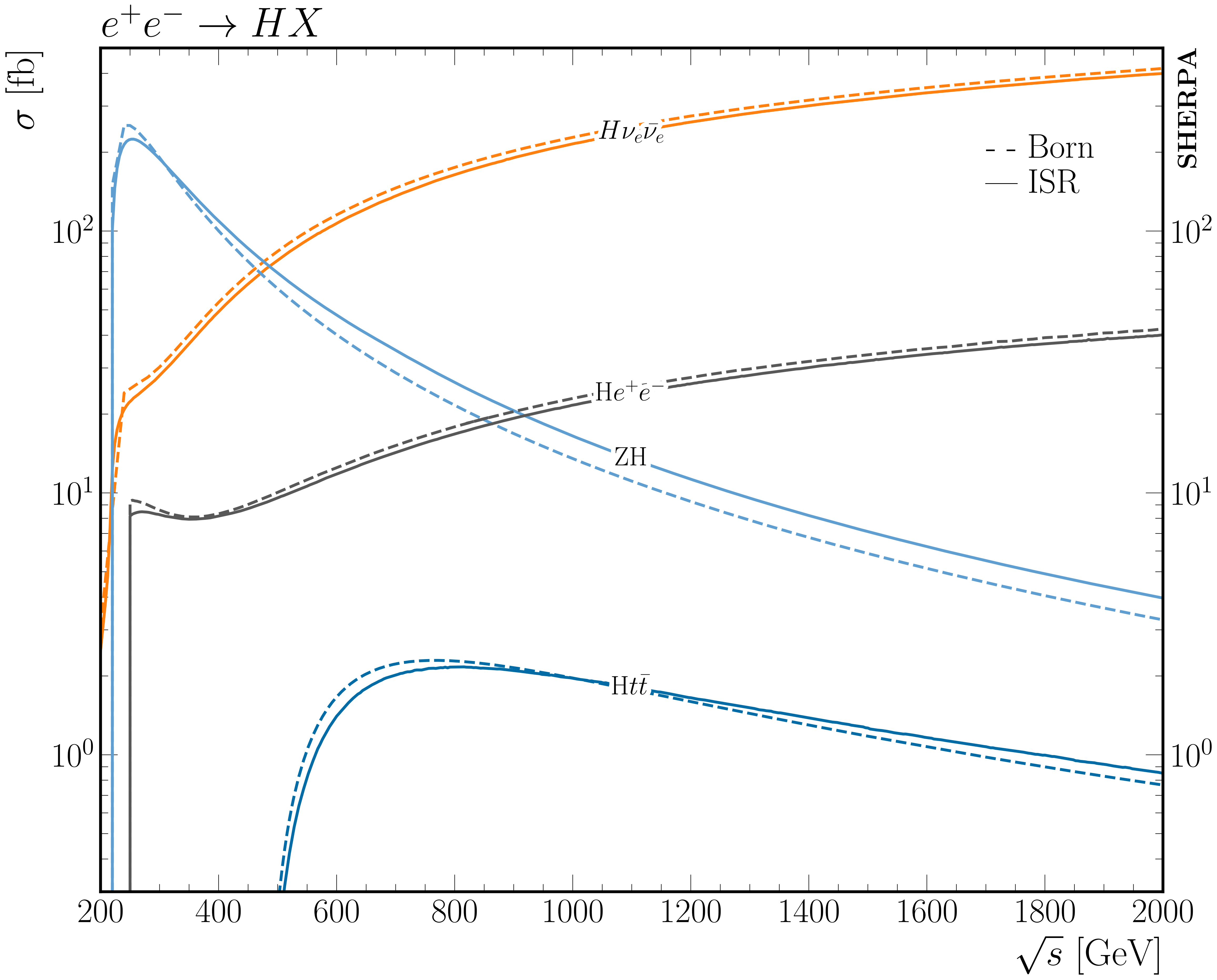
Sherpa

Process Independent YFS

$$\tilde{\beta}_0^0(\Phi_n) = \left| \mathscr{M}_0^0(\Phi_n) \right|^2$$

LO amplitudes are full automated within Sherpa using internal ME tools

YFS is **not** applied to final state colored cartons: Cannot be interleaved with PS



SciPost Phys. 13 (2022) 2, 026, SciPost Phys. 13 (2022) 026

YFS @NLO: Virtual Corrections

$$\tilde{\beta}_0^1(\Phi_n) = \mathscr{V}(\Phi_n) - \sum_{ij} \mathscr{D}_{ij}(\Phi_{ij})$$

Full One-loop amplitude, including IR divergence

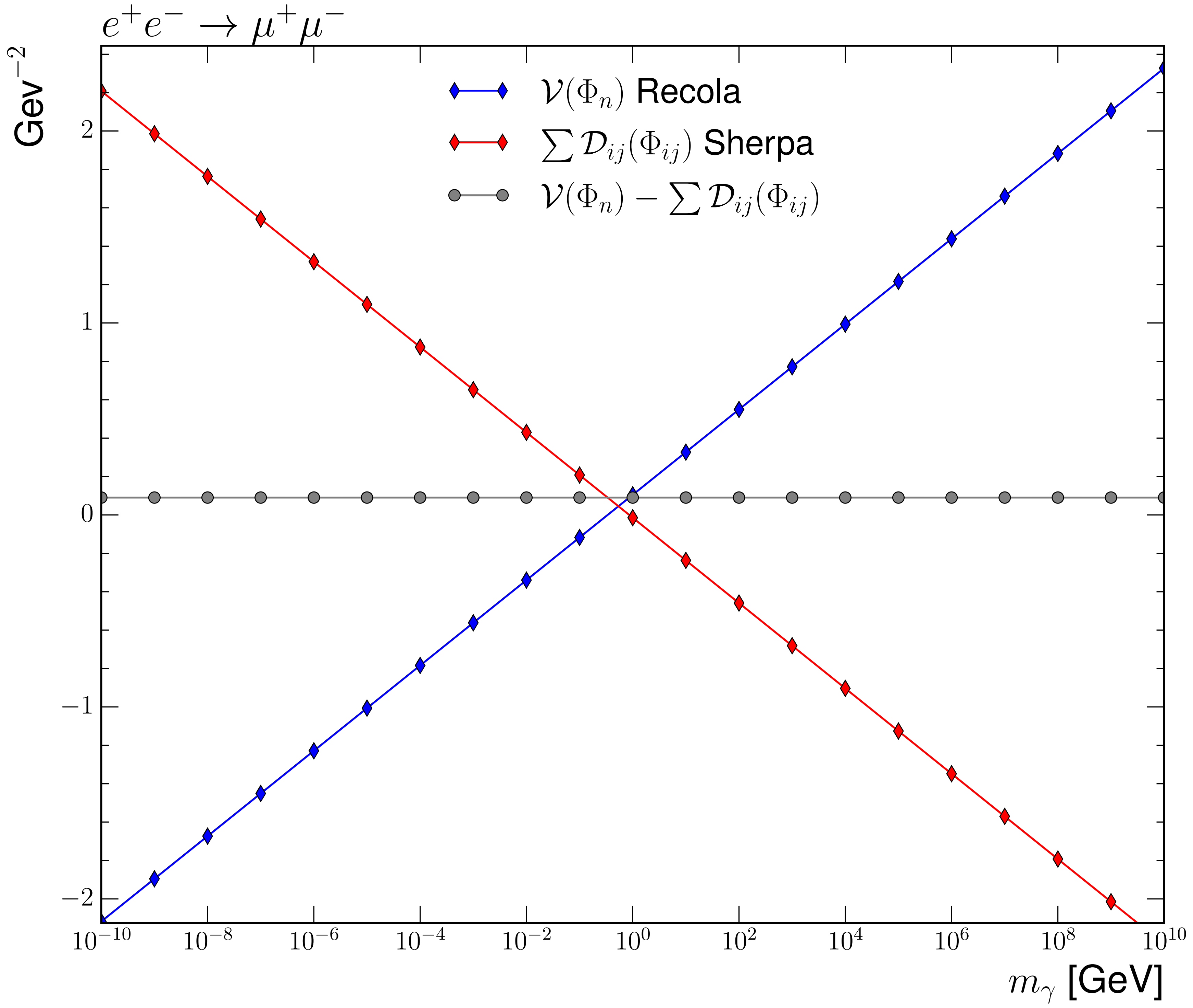
Taken from one-loop providers e.g Recola

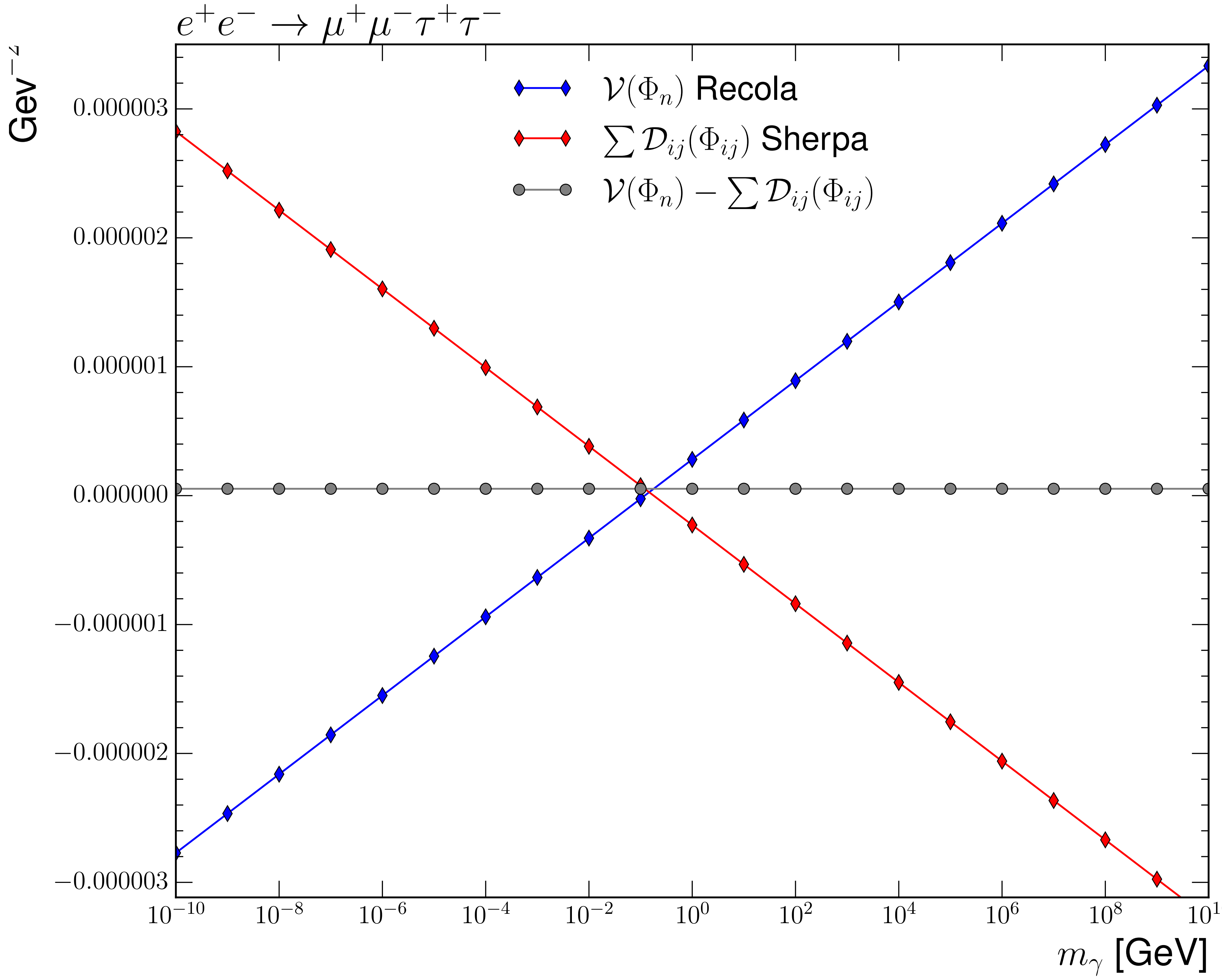
YFS dipole subtraction term taken over all possible charged dipoles

Calculated automatically in Sherpa

YFS @NLO: Virtual Corrections

$$\tilde{\beta}_0^1(\Phi_n) = \mathscr{V}(\Phi_n) - \sum_{ij} \mathscr{D}_{ij}(\Phi_{ij}) \qquad \log(m_\gamma^2) \rightarrow \frac{\Gamma(1+\epsilon)}{\epsilon}(4\pi\mu^2)^\epsilon$$





`IR cancellation in action`

YFS @NLO: Real Corrections

$$S_{ij}(k) = \frac{\alpha}{4\pi^2} Z_i Z_j \theta_i \theta_j \left(\frac{p_i}{p_i \cdot k} - \frac{p_j}{p_j \cdot k} \right)^2$$

$$\tilde{\beta}_1^1\left(\Phi_{n+1}; k\right) = \mathscr{R}\left(\Phi_{n+1}\right) - \tilde{\beta}_0^0\left(\Phi_n\right) \sum_{ij} S_{ij}(k)$$

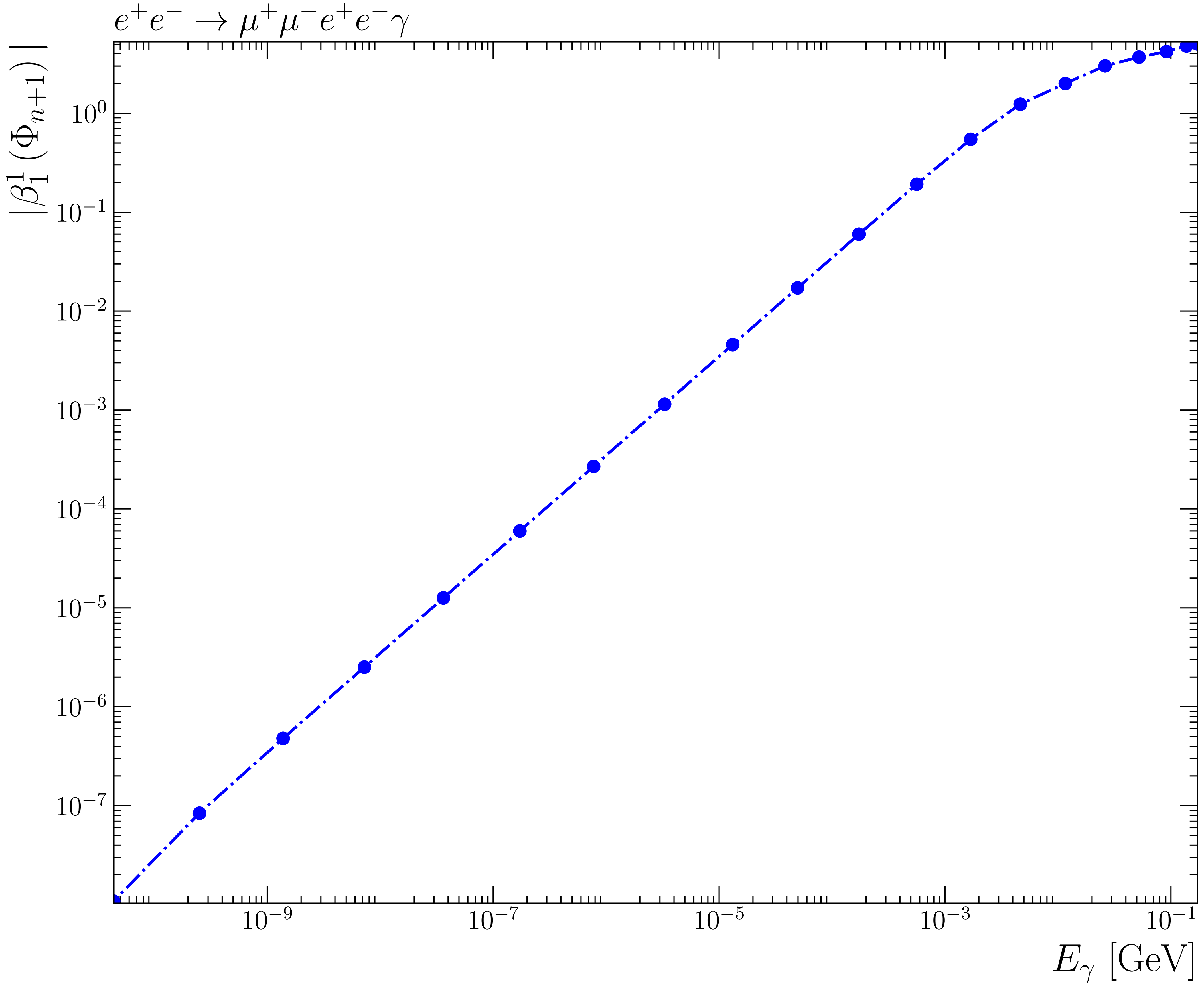
Full One-Real amplitude, including IR divergence

Calculated automatically with Sherpa’s ME Generators

YFS dipole subtraction term taken over all possible charged dipoles

Calculated automatically in Sherpa

YFS @NLO: Real Corrections



$$\tilde{\beta}_1^1\left(\Phi_{n+1};k\right) = \mathscr{R}\left(\Phi_{n+1}\right) - \tilde{\beta}_0^0\left(\Phi_n\right)\sum_{ij} S_{ij}(k)$$

$$k \to 0, \quad \mathscr{R}\left(\Phi_{n+1}\right) \approx \tilde{\beta}_0^0\left(\Phi_n\right)\sum_{ij} S_{ij}(k)$$

`IR` **`finite`** `and numerically`
`stable` `for soft emissions`

YFS@NNLO?

Bottle neck for NNLO implementation is two-loop amplitudes

Have to take it process by process

Test Process $e^+e^- \rightarrow \mu^+\mu^-$ at 91.2GeV

Born	YFS LO	YFS NLO	YFS NNLO
2114.5 pb	1463.09 pb	1494.7(8) pb	1497.5(7) pb

```
============================================================

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============================================================
                     version 1.0
            Lisong Chen and Ayres Freitas
           https://arxiv.org/abs/2211.16272
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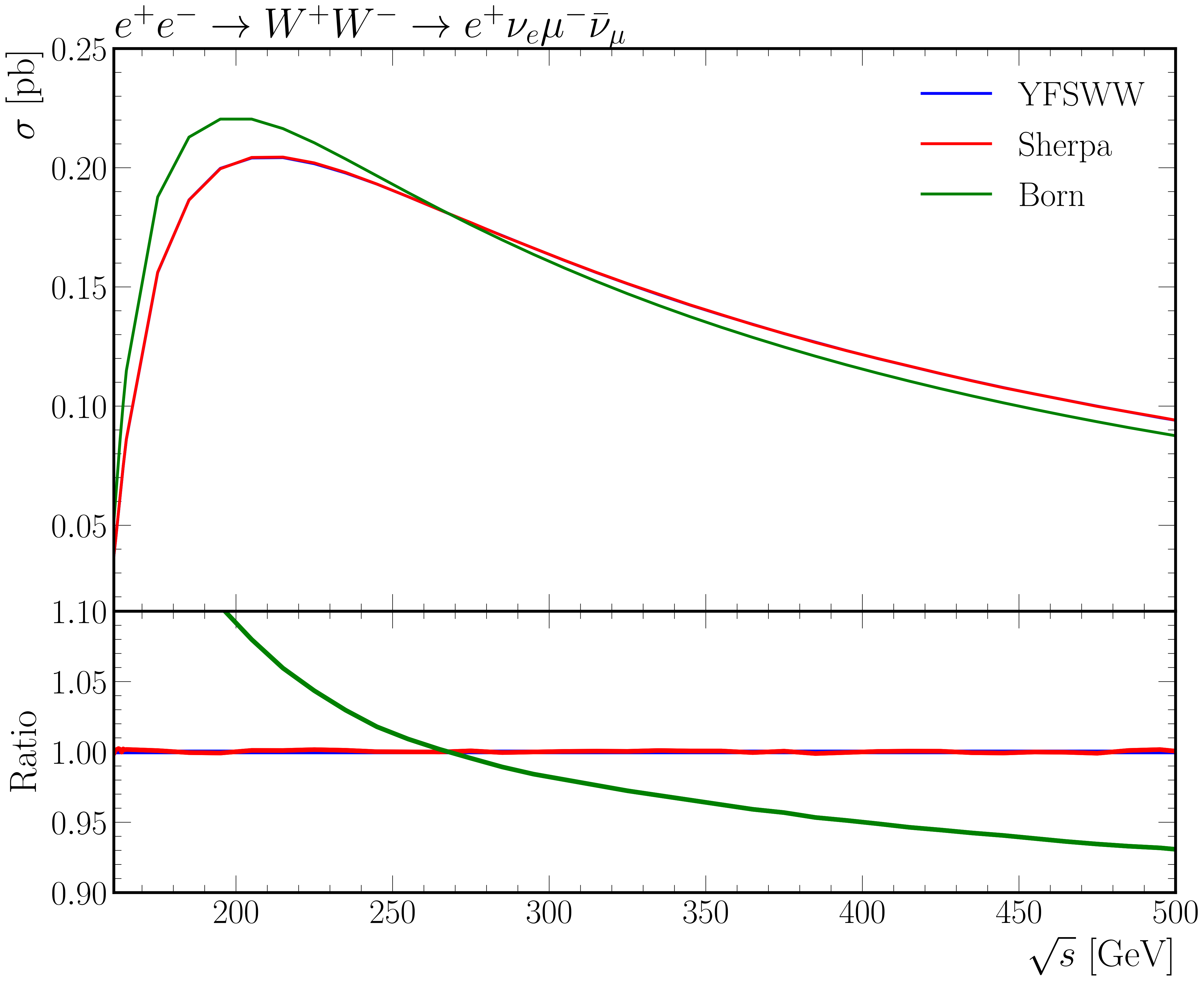
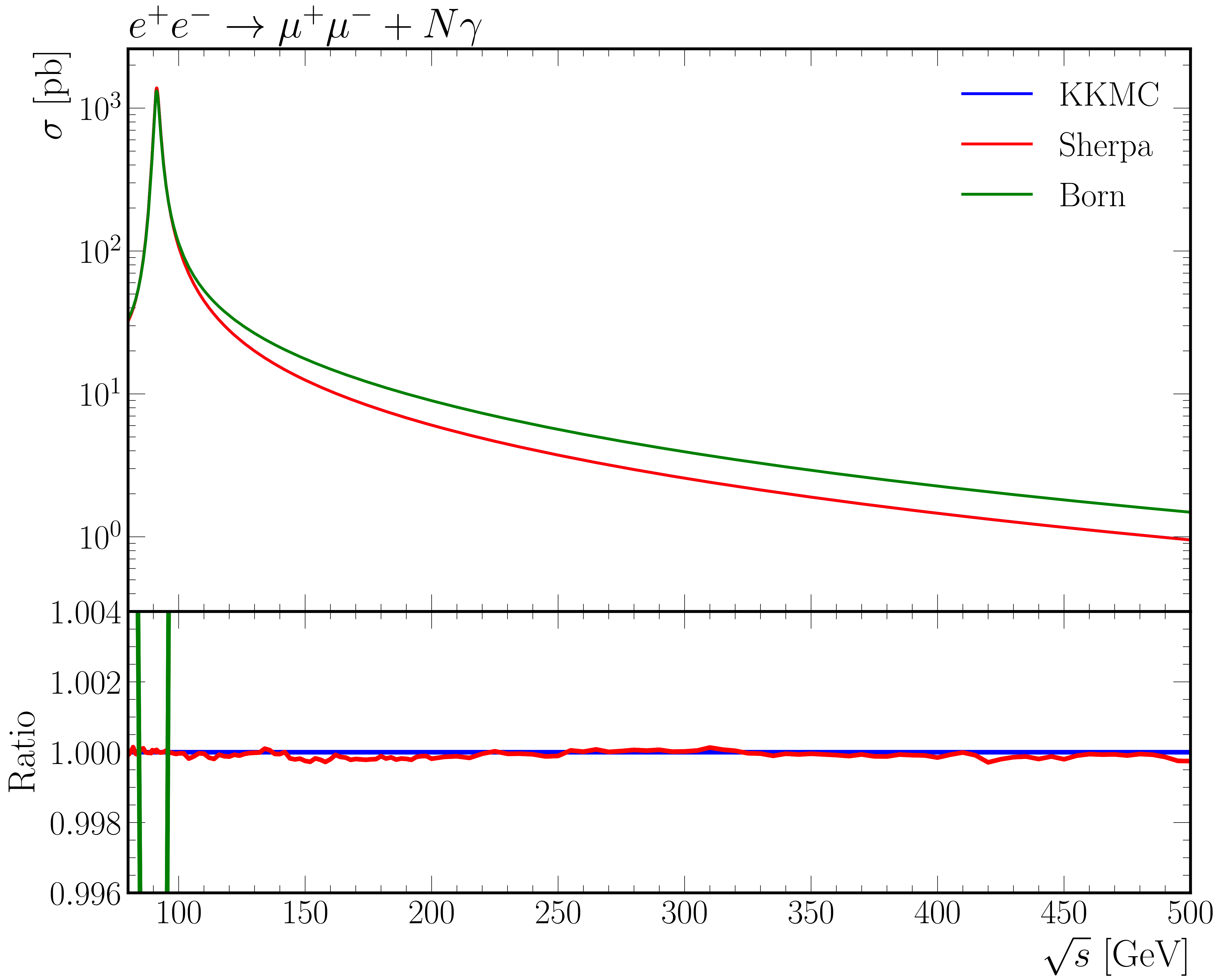
GRIFFIN: A C++ library for EW radiative corrections [2211.16272](https://arxiv.org/abs/2211.16272)

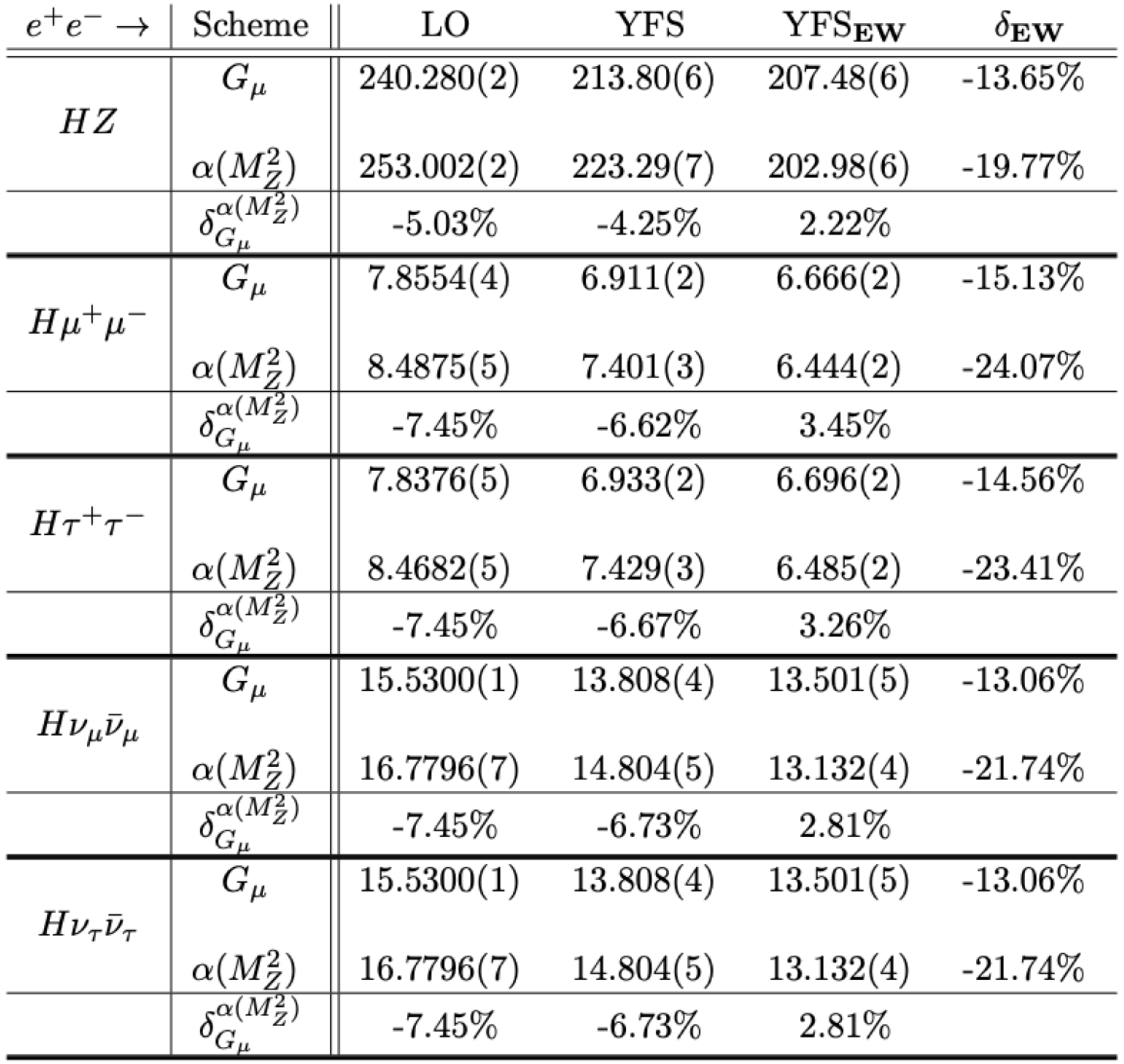
Developed by A. Freitas and L.Chen

YFS Validation





YFS for HZ



$e^+e^- \to$	Scheme	LO	YFS	$\text{YFS}_{\textbf{EW}}$	$\delta_{\textbf{EW}}$
HZ	G_μ	240.280(2)	213.80(6)	207.48(6)	-13.65%
	$\alpha(M_Z^2)$	253.002(2)	223.29(7)	202.98(6)	-19.77%
	$\delta_{G_\mu}^{\alpha(M_Z^2)}$	-5.03%	-4.25%	2.22%	
$H\mu^+\mu^-$	G_μ	7.8554(4)	6.911(2)	6.666(2)	-15.13%
	$\alpha(M_Z^2)$	8.4875(5)	7.401(3)	6.444(2)	-24.07%
	$\delta_{G_\mu}^{\alpha(M_Z^2)}$	-7.45%	-6.62%	3.45%	
$H\tau^+\tau^-$	G_μ	7.8376(5)	6.933(2)	6.696(2)	-14.56%
	$\alpha(M_Z^2)$	8.4682(5)	7.429(3)	6.485(2)	-23.41%
	$\delta_{G_\mu}^{\alpha(M_Z^2)}$	-7.45%	-6.67%	3.26%	
$H\nu_\mu\bar{\nu}_\mu$	G_μ	15.5300(1)	13.808(4)	13.501(5)	-13.06%
	$\alpha(M_Z^2)$	16.7796(7)	14.804(5)	13.132(4)	-21.74%
	$\delta_{G_\mu}^{\alpha(M_Z^2)}$	-7.45%	-6.73%	2.81%	
$H\nu_\tau\bar{\nu}_\tau$	G_μ	15.5300(1)	13.808(4)	13.501(5)	-13.06%
	$\alpha(M_Z^2)$	16.7796(7)	14.804(5)	13.132(4)	-21.74%
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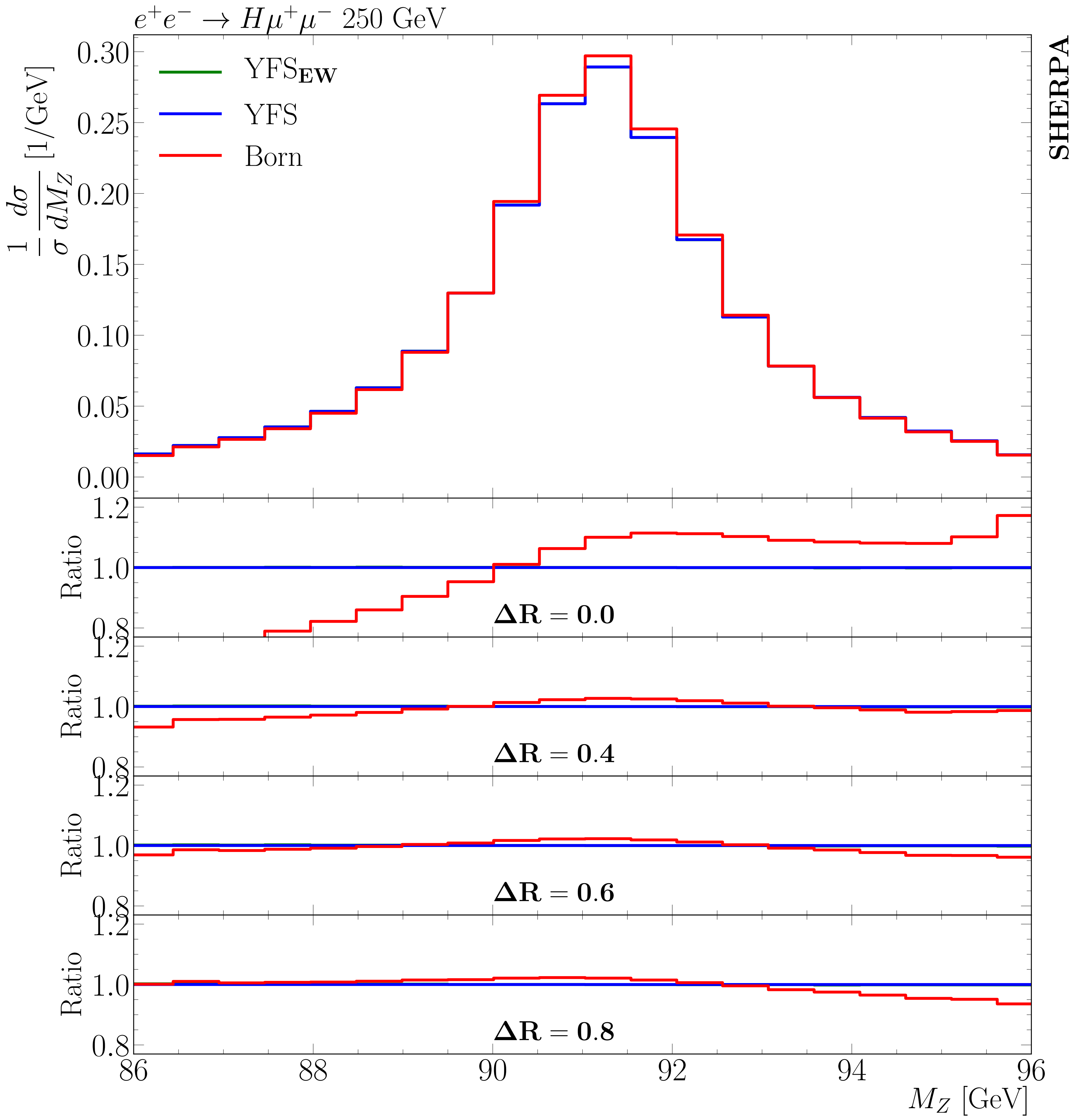
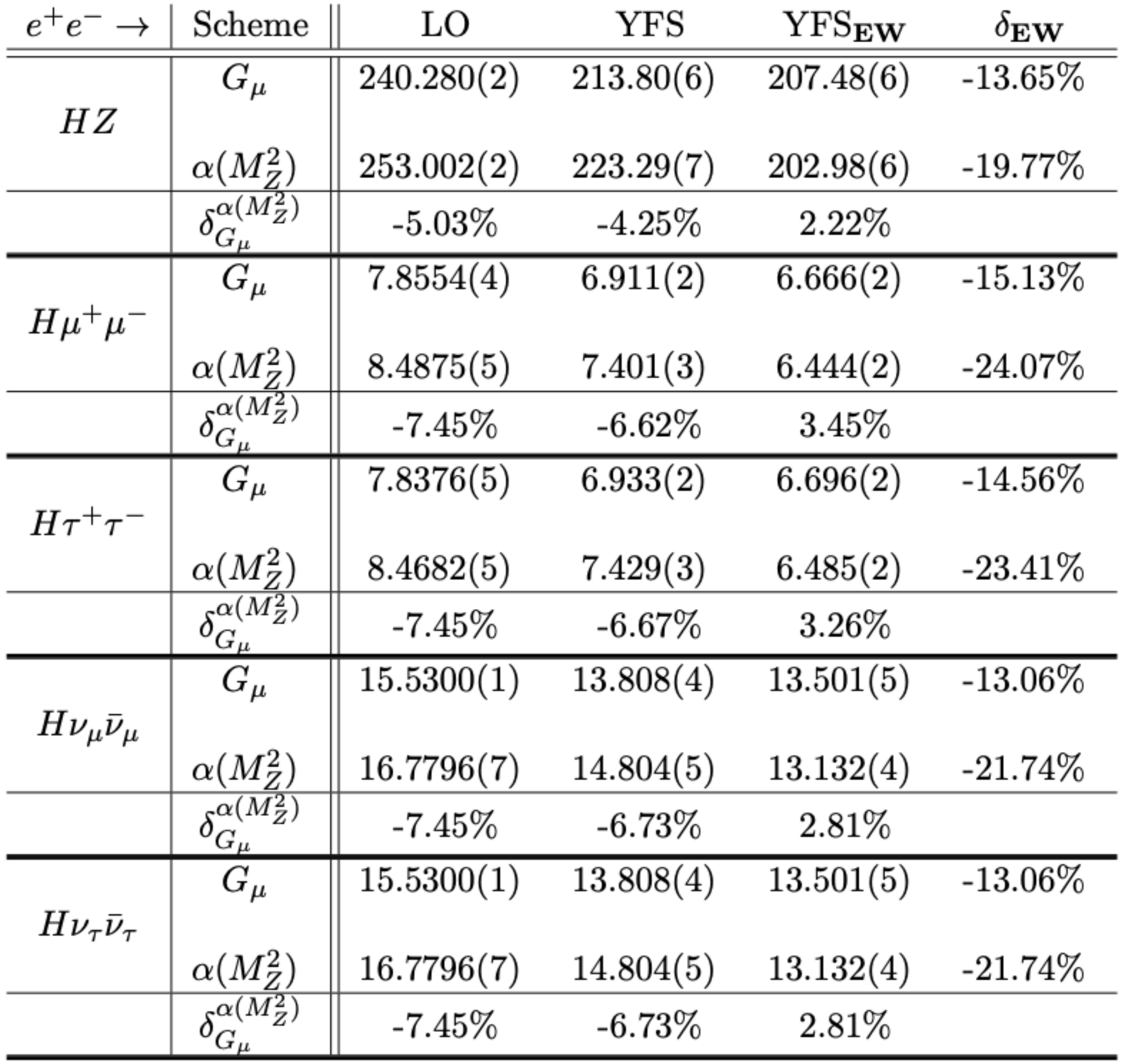


YFS for HZ



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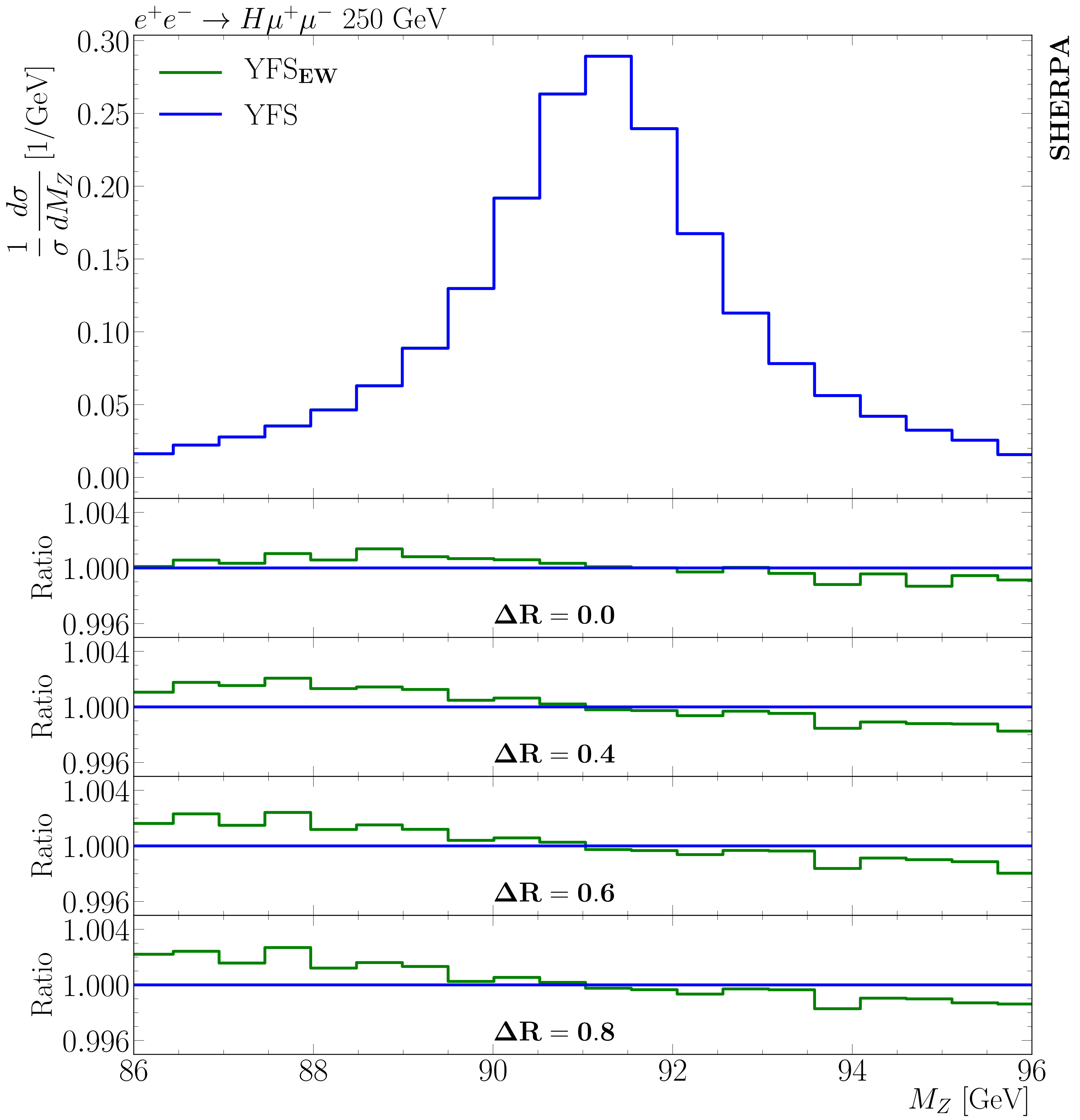
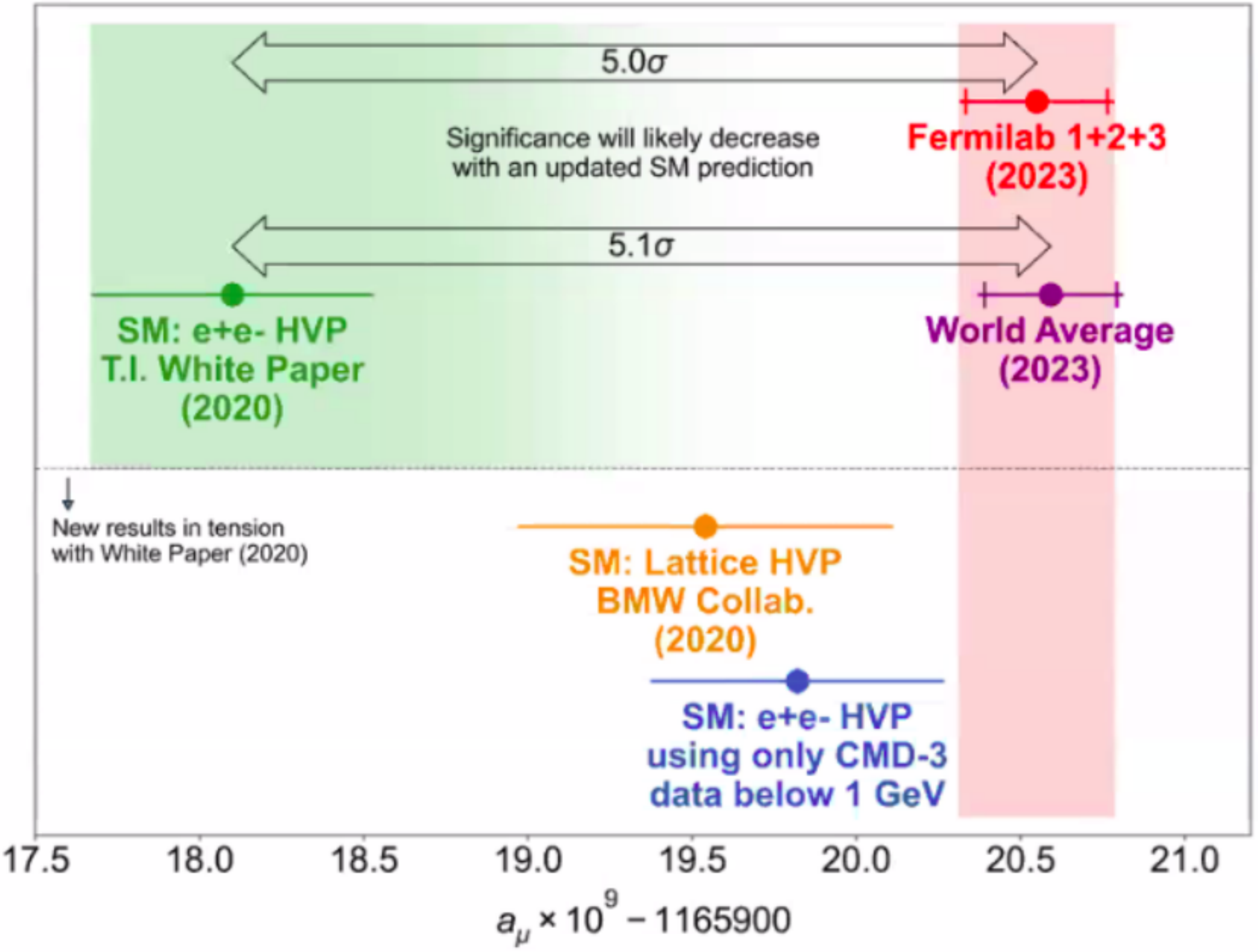


YFS for the Muon Magnetic Moment

See talk by J.Gluza



White Paper [T. Aoyama et al., Phys. Rept. 887 (2020) 1-166]

Uncertainty in SM prediction of a_μ is dominated by hadronic contributions

$$a_\mu^{\text{had,LO}} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\text{th}}}^{\infty} ds \frac{1}{s}\hat{K}(s)\sigma_{\text{had}}(s)$$

Can be extracted from e+e- -> hadrons at low energy experiments with data-driven methods.

This method suffers from uncertainties coming from the experimental error as well a multiple hadronic resonance present

MUonE Experiment: $\mu e \to \mu e$

- Scattering muons on low Z fixed targets seems to be optimal for extracting $\Delta\alpha_{\text{had}}$
- Purely t-channel process at LO
- With M2 muon beam at CERN, we have access to ~150 GeV beam
- This will allow us to measure $\Delta\alpha_{\text{had}}$ with ~ 0.3% accuracy in the range –0.153 < t < 0 GeV^2
- Will require accurate Monte Carlo tools

Abbiendi et al., EPJC 77 (2017) 3, 139

Abbiendi et al., Letter of Intent: the MUonE project, CERN-SPSC-2019-026, SPSC-I-252 (2019)

$$\frac{d\sigma_{\text{data}}(\Delta\alpha_{\text{had}})}{d\sigma_{\text{MC}}(\Delta\alpha_{\text{had}}=0)} \sim 1 + 2\Delta\alpha_{\text{had}}(t)$$

Tools for MUonE

McMule

mule-tools.gitlab.io

Fixed-order integrator NNLO QED framework for 2->2 leptonic processes [McMule 20]

MESMER

JHEP 11 (2021) 098

MESMER is a Monte Carlo event generator for high-precision simulation of muon-electron scattering at low energies

Tools for MUonE

“is particularly amicable to a YFS parton shower”

mule-tools.gitlab.io

“within MCMULE an effort is ongoing to include a YFS parton shower

processes [McMule 20]

MESMER

“a YFS approach allows to approximate the missing NNLO virtual amplitudes”

MESM r for

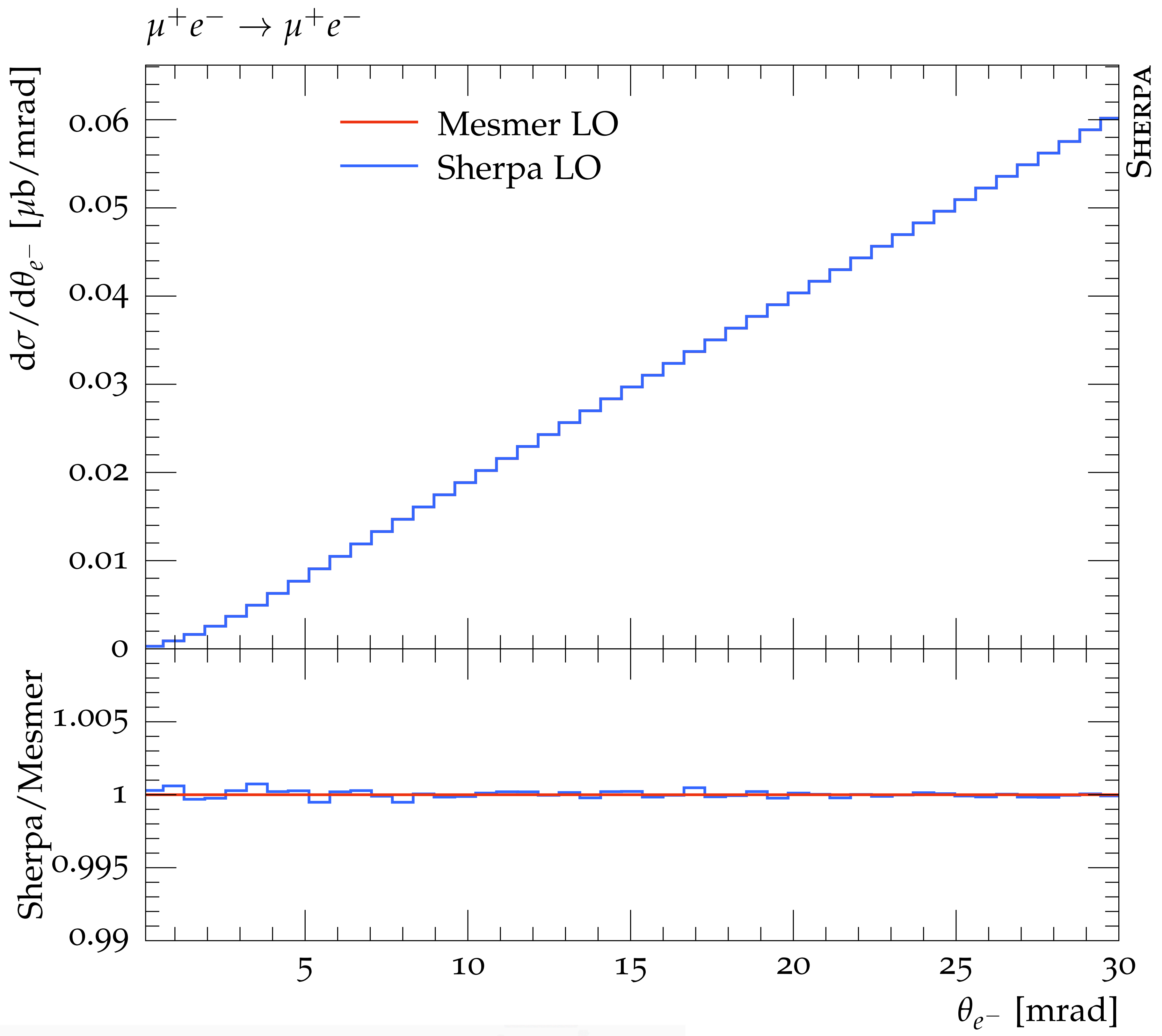
high tron

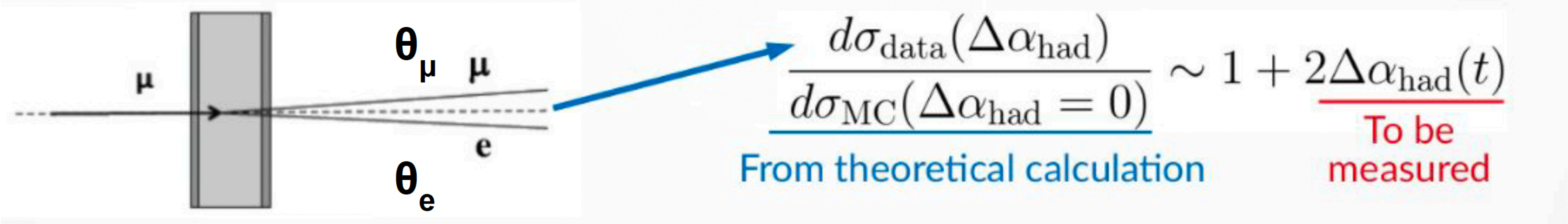
scattering at low energies

“YFS inspired..”

YFS for MUonE Experiment

- New fixed target mode allowing Sherpa to calculate $\mu^{\pm}e^{-} \rightarrow \mu^{\pm}e^{-}$ for MUonE
- Excellent agreement at LO with Mesmer. Predictions correspond to setup 1 in **JHEP 11 (2020) 028**
- YFS certainly feasible to achieve sub-permille precision

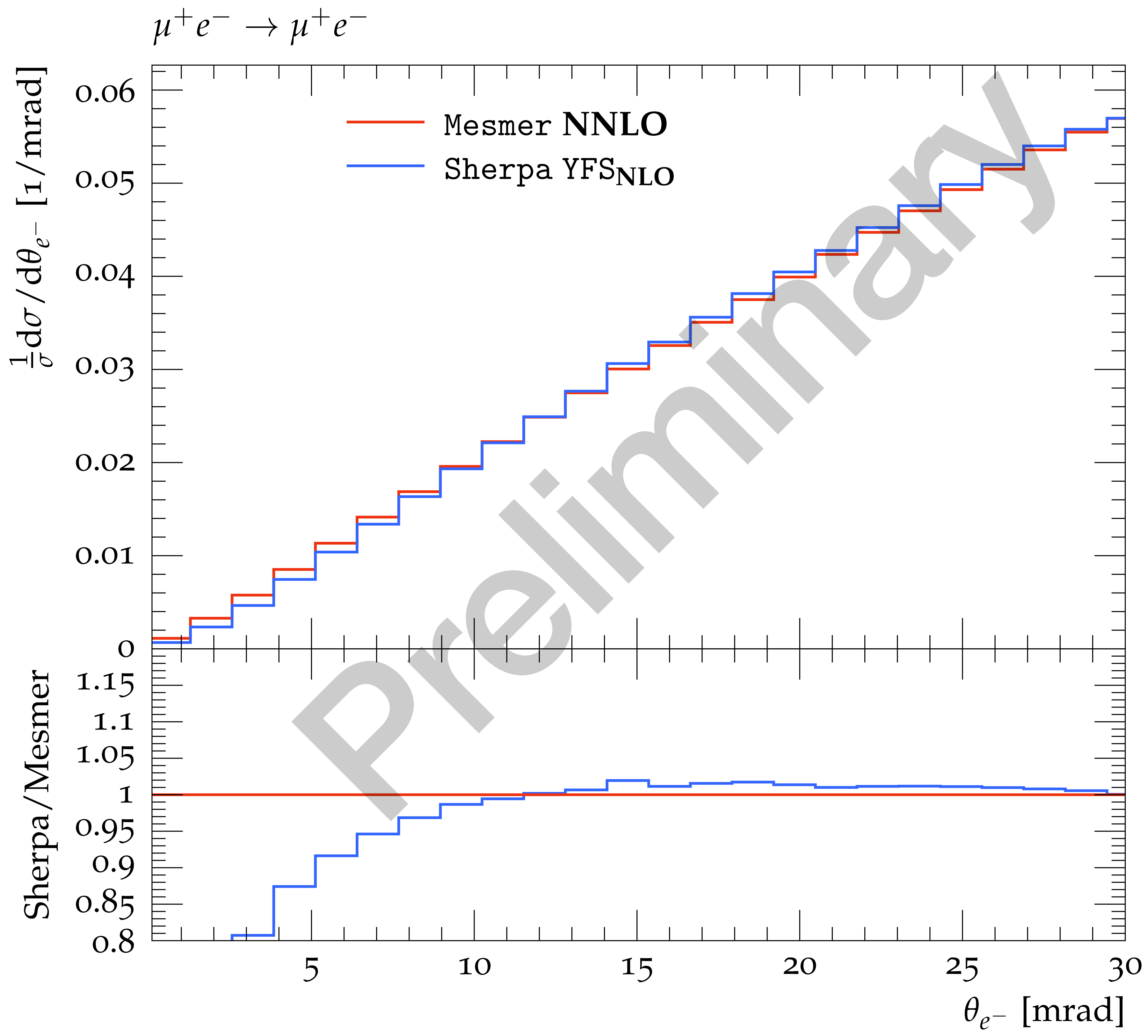




$$\underbrace{\frac{d\sigma_{\text{data}}(\Delta\alpha_{\text{had}})}{d\sigma_{\text{MC}}(\Delta\alpha_{\text{had}}=0)}}_{\text{From theoretical calculation}} \sim 1 + \underbrace{2\Delta\alpha_{\text{had}}(t)}_{\text{To be measured}}$$

YFS for MUonE Experiment

- New fixed target mode allowing Sherpa to calculate $\mu^{\pm}e^{-} \rightarrow \mu^{\pm}e^{-}$ for MUonE
- At low angles the effects of soft photon become important
- The YFS corrections are comparable with state of the art Fixed-Order calculations



$\mu^+e^- \to \mu^+e^-$	LO	YFS_{Born}	YFS_{EEX}
SHERPA	245.034(3)	261.296(9)	256.315(8)
	LO	NLO	NNLO
Mesmer	245.038910(1)	255.8437(5)	256.092(1)

Table 1: Total cross-sections for $\mu^{\pm}e^{-} \rightarrow \mu^{\pm}e^{-}$in μb.

Conclusion

- YFS provides a robust framework for perturbative calculation at lepton colliders
- Can be combined with modern automated tools which gives hope for reaching precision goals of future Higgs factories
- This work would not be possible without decades of hard work from Stanislaw and Krakow group

YFS@NNLO?

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                  version 1.0
           Lisong Chen and Ayres Freitas
         https://arxiv.org/abs/2211.16272
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GRIFFIN: A C++ library for EW radiative corrections 2211.16272

Developed by A. Freitas and L.Chen

Framework for $f\bar{f} \to Z^*/\gamma* \to f'\bar{f}'$:

- Laurent expansion about Z-pole + regular matrix element off-resonance

$$M_{ij} = M_{ij}^{\text{exp},s_0} + M_{ij}^{\text{noexp}} - M_{ij}^{\text{exp},M_Z^2},$$

@NLO

avoid double counting

$$M_{ij}^{\text{exp},s_0} = \frac{R_{ij}}{s - s_0} + S_{ij} + (s - s_0)S'_{ij} + \ldots \qquad s_0 \equiv M_Z^2 - iM_Z\Gamma_Z$$

@NNLO @NLO

Stuart ’91; Veltman ’94

From **A.Freitas**

With YFS inspired Subtraction!

$$\gamma\gamma \text{ box:} \qquad B_{\text{VV}(1)} = B^{\text{tot}}_{\text{VV}(1)} - S^{(0)}_{\text{VV}} \frac{\alpha}{\pi} Q_e Q_f\, f_{\text{IR}}(m_\gamma, t, u),$$

$$\gamma Z \text{ box:} \qquad B_{\gamma\text{Z},ij(1)} = B^{\text{tot}}_{\gamma\text{Z},ij(1)} - \frac{R^{(0)}_{ij}}{s - s_0} \frac{\alpha}{\pi} Q_e Q_f \left[f_{\text{IR}}(m_\gamma, t, u) + \delta_G(s, t, u)\right],$$

$$f_{\text{IR}}(m_\gamma, t, u) = \ln\left(\frac{1 - c_\theta}{1 + c_\theta}\right)\left[\ln\left(\frac{2m_\gamma^2}{s\sqrt{1 - c_\theta^2}}\right) + \frac{1}{2}\right],$$

$$\delta_G(s, t, u) = -2\ln\left(\frac{1 - c_\theta}{1 + c_\theta}\right)\ln\left(\frac{s_0 - s}{s_0}\right).$$

GRIFFIN=**G**auge-**R**esonance-**I**n-**F**our-**F**ermion-**IN**teraction