

Exclusive bremsstrahlung of one and two photons in pp collisions

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Introduction

- The **inclusive** differential production cross-section of forward photons in pp collisions was measured at $\sqrt{s} = 510$ GeV with the RHICf detector (**Adriani et al**) and at $\sqrt{s} = 0.9, 7$ and 13 TeV with the **LHCf detector**.
- In the ATLAS-LHCf combined analysis (ATLAS 2017) the forward-photon spectra were measured by the LHCf detector, while the ATLAS inner tracker system is used to suppress non-diffractive events. In this method, the preliminary photon energy spectrum has been obtained in two regions of photon rapidity ($8.81 < y < 8.99$ and $y > 10.94$), for events with no charged particles having $p_t > 100$ MeV and $|\eta| < 2.5$.
- At high energies, the $pp \rightarrow pp\gamma$ reaction **has not yet been measured**.
- **Feasibility studies** of the measurement of the exclusive diffractive bremsstrahlung cross sections were performed for RHIC energies (**Chwastowski et al.**) and for LHC energies using the ATLAS forward detectors (**Chwastowski et al.**). This was done within old **Lebiedowicz, Szczurek** approach.

Introduction

- We discuss **exclusive diffractive bremsstrahlung of one and two photons** in pp collisions for the LHC energy $\sqrt{s} = 13$ TeV and at very-forward photon rapidities. We shall work within the **tensor-pomeron model** as proposed by **Ewerz, Maniatis and Nachtmann 2014** for soft hadronic high-energy reactions.
- The diffractive exclusive single-photon bremsstrahlung was discussed recently:
 - $\pi\pi \rightarrow \pi\pi\gamma$ (**Lebiedowicz, Nachtmann, Szczurek Phys. Rev. D105 014022 (2022)**).
 - $pp \rightarrow pp\gamma$ (**Lebiedowicz, Nachtmann, Szczurek Phys. Rev. D 106 (2022) 034023**).
- We consider a new possibility to measure exclusive diffractive process with **LHCf (photons)** and **AFP (protons)**.
- In order to answer the question whether such a measurement is possible one needs to consider other processes that can be **misidentified as bremsstrahlung**. One of the processes which is potentially important in this context is the $pp \rightarrow pp\pi^0$ reaction. The decaying neutral pion is a source of unwanted photons that can hinder the identification of bremsstrahlung photons of interest.

Theoretical formalism, $pp \rightarrow pp\gamma$

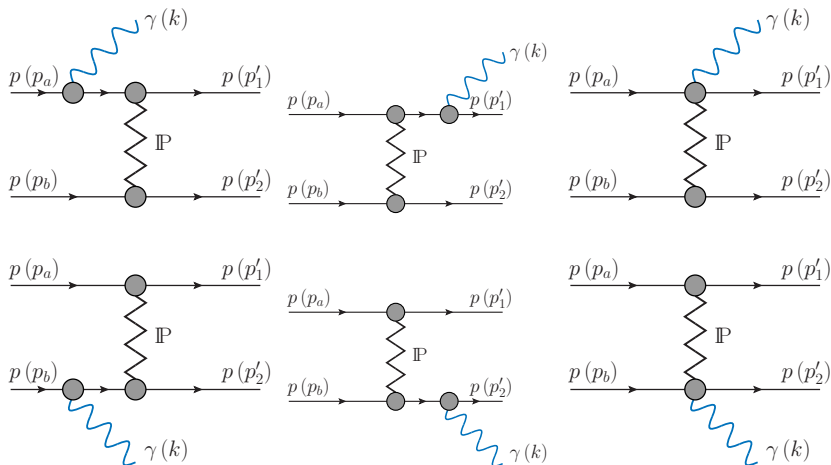


Figure: Diagrams for single photon exclusive bremsstrahlung.

Theoretical formalism, $pp \rightarrow pp\gamma$

We consider the reaction:

$$p(p_a, \lambda_a) + p(p_b, \lambda_b) \rightarrow p(p'_1, \lambda_1) + p(p'_2, \lambda_2) + \gamma(k, \epsilon) \quad (1)$$

The energy-momentum conservation in (1) requires

$$p_a + p_b = p'_1 + p'_2 + k. \quad (2)$$

The kinematic variables are

$$\begin{aligned} s &= (p_a + p_b)^2 = (p'_1 + p'_2 + k)^2, \\ q_1 &= p_a - p'_1, \quad t_1 = q_1^2, \\ q_2 &= p_b - p'_2, \quad t_2 = q_2^2, \\ s_1 &= W_1^2 = (p'_1 + k)^2 = (p_a + q_2)^2, \\ s_2 &= W_2^2 = (p'_2 + k)^2 = (p_b + q_1)^2. \end{aligned} \quad (3)$$

In the following we work in the overall c.m. system where we choose the 3 axis in the direction of \mathbf{p}_a (the beam direction). The rapidity of the photon is then

$$y = \frac{1}{2} \ln \frac{k^0 + k^3}{k^0 - k^3} = \tanh^{-1} \left(\frac{k^3}{k^0} \right) = -\ln \tan \frac{\theta}{2}, \quad (4)$$

where θ is the polar angle of \mathbf{k} , $\cos \theta = k^3/|\mathbf{k}|$.

Theoretical formalism, $pp \rightarrow pp\gamma$

We introduce the variables ξ_1 and ξ_2 which, to a very good approximation, describe the **fractional energy losses** of the protons $p(p_a)$ and $p(p_b)$

$$\xi_1 = \frac{p_b \cdot q_1}{p_b \cdot p_a} = \frac{p_a^0 - p_1'^0}{p_a^0} + \mathcal{O}\left(\frac{M^2}{s}\right), \quad \xi_2 = \frac{p_a \cdot q_2}{p_a \cdot p_b} = \frac{p_b^0 - p_2'^0}{p_b^0} + \mathcal{O}\left(\frac{M^2}{s}\right). \quad (5)$$

Here the energies of the incoming and outgoing protons, respectively, are

$$p_a^0 = p_b^0 = \frac{\sqrt{s}}{2}, \quad (6)$$

$$p_{1,2}'^0 = \frac{1}{2\sqrt{s}}(s + m_p^2 - s_{2,1}), \quad (7)$$

and we set $M^2 = \max(m_p^2, |t_1|, |t_2|, k_\perp^2)$. Alternatively, the proton relative energy-loss parameters can be expressed by the kinematical variables of the photon,

$$\xi_1 = \frac{k_\perp}{\sqrt{s}} \exp(y) + \mathcal{O}\left(\frac{M^2}{s}\right), \quad \xi_2 = \frac{k_\perp}{\sqrt{s}} \exp(-y) + \mathcal{O}\left(\frac{M^2}{s}\right). \quad (8)$$

Theoretical formalism, $pp \rightarrow pp\gamma$

The cross section for the photon yield can be calculated as follows

$$d\sigma(pp \rightarrow pp\gamma) = \frac{1}{2\sqrt{s(s-4m_p^2)}} \frac{d^3k}{(2\pi)^3 2k^0} \int \frac{d^3p'_1}{(2\pi)^3 2p'_1{}^0} \frac{d^3p'_2}{(2\pi)^3 2p'_2{}^0} \\ \times (2\pi)^4 \delta^{(4)}(p'_1 + p'_2 + k - p_a - p_b) \frac{1}{4} \sum_{\mathbf{p} \text{ spins}} \mathcal{M}_\mu \mathcal{M}_\nu^* (-g^{\mu\nu}); \quad (9)$$

see Eqs. (2.33)–(2.35) of [Lebiedowicz, Nachtmann, Szczurek 2022](#).

\mathcal{M}_μ is the radiative amplitude.

Our standard photon-bremsstrahlung amplitude, $\mathcal{M}_\mu^{\text{standard}}$, treated in the **tensor-pomeron approach**.

The amplitudes (a), (b), (d), and (e), corresponding to photon emission from the external protons, are determined by the off-shell pp elastic scattering amplitude. **The contact terms, (c) and (f), are needed in order to satisfy gauge-invariance constraints.**

Theoretical formalism, $pp \rightarrow pp\gamma$

In the following, we shall compare our standard results to two soft-photon approximations, SPA1 and SPA2, as defined in Sec. III of [Lebiedowicz, Nachtmann, Szczurek 2022](#). In both SPAs we keep only the [pole terms](#) $\propto \omega^{-1}$.

We consider only the pomeron-exchange contribution for the radiative amplitudes, the leading term at high energies.

In SPA1, the radiative amplitude has the form

$$\mathcal{M}_\mu \rightarrow \mathcal{M}_{\mu, \text{SPA1}} = e\mathcal{M}^{(\text{on shell})pp}(s, t) \left[-\frac{p_{a\mu}}{(p_a \cdot k)} + \frac{p_{1\mu}}{(p_1 \cdot k)} - \frac{p_{b\mu}}{(p_b \cdot k)} + \frac{p_{2\mu}}{(p_2 \cdot k)} \right], \quad (10)$$

where $\mathcal{M}^{(\text{on shell})pp}(s, t)$ is the amplitude for on-shell pp -scattering

$$\begin{aligned} p(p_a, \lambda_a) + p(p_b, \lambda_b) &\rightarrow p(p_1, \lambda_1) + p(p_2, \lambda_2), \\ p_a + p_b &= p_1 + p_2; \end{aligned} \quad (11)$$

see (2.19) and (3.1) of [Lebiedowicz, Nachtmann, Szczurek 2022](#).

Theoretical formalism, $pp \rightarrow pp\gamma$

The inclusive photon cross section for the SPA1 case is

$$\begin{aligned}
 d\sigma(pp \rightarrow pp\gamma)_{\text{SPA1}} &= \frac{d^3 k}{(2\pi)^3 2k^0} \int d^3 p_1 d^3 p_2 e^2 \frac{d\sigma(pp \rightarrow pp)}{d^3 p_1 d^3 p_2} \\
 &\times \left[-\frac{p_{a\mu}}{(p_a \cdot k)} + \frac{p_{1\mu}}{(p_1 \cdot k)} - \frac{p_{b\mu}}{(p_b \cdot k)} + \frac{p_{2\mu}}{(p_2 \cdot k)} \right] \\
 &\times \left[-\frac{p_{a\nu}}{(p_a \cdot k)} + \frac{p_{1\nu}}{(p_1 \cdot k)} - \frac{p_{b\nu}}{(p_b \cdot k)} + \frac{p_{2\nu}}{(p_2 \cdot k)} \right] (-g^{\mu\nu}). \quad (12)
 \end{aligned}$$

Here

$$\begin{aligned}
 \frac{d\sigma(pp \rightarrow pp)}{d^3 p_1 d^3 p_2} &= \frac{1}{2\sqrt{s(s-4m_p^2)}} \frac{1}{(2\pi)^3 2p_1^0 (2\pi)^3 2p_2^0} \\
 &\times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_a - p_b) \frac{1}{4} \sum_{p \text{ spins}} |\mathcal{M}^{\text{(on shell)}}{}_{pp}(s, t)|^2, \quad (13)
 \end{aligned}$$

where we neglect the photon momentum k in the energy-momentum conserving $\delta^{(4)}(\cdot)$ function.

For SPA1 results we impose restrictions on the proton's relative energy loss variables ξ_i by using (8) neglecting terms of $\mathcal{O}(M^2/s)$.

Theoretical formalism, $pp \rightarrow pp\gamma$

In the SPA2 case, we keep the exact energy-momentum relation (2). Here we calculate the photon yield using (9) replacing the radiative amplitude as follows

$$\mathcal{M}_\mu \rightarrow \mathcal{M}_{\mu, \text{SPA2}} = \mathcal{M}_{\mathbb{P}, \mu}^{(a+b+c)1} + \mathcal{M}_{\mathbb{P}, \mu}^{(d+e+f)1}. \quad (14)$$

The explicit expressions of these terms are given by (3.4), (B4), and (B15) of [Lebiedowicz, Nachtmann, Szczurek 2022](#).

Theoretical formalism, $pp \rightarrow pp\gamma\gamma$

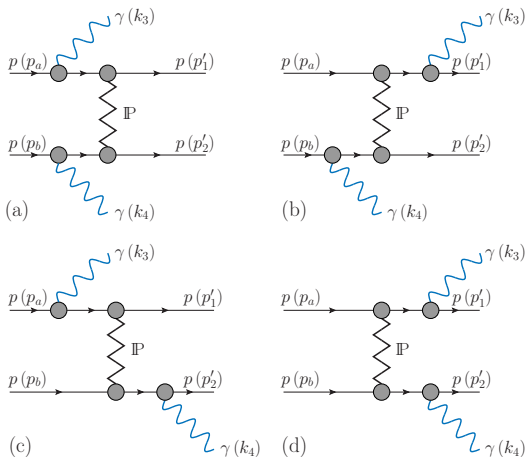


Figure: Diffractive two-photon bremsstrahlung for the reaction $pp \rightarrow pp\gamma\gamma$ (15) with exchange of the pomeron \mathbb{P} . These four diagrams (a – d) contribute to the SPA1 amplitude (17).

Theoretical formalism, $pp \rightarrow pp\gamma\gamma$

Here we consider the reaction

$$p(p_a) + p(p_b) \rightarrow p(p'_1) + p(p'_2) + \gamma(k_3) + \gamma(k_4). \quad (15)$$

We shall study this reaction under specific conditions. We shall require that one photon is emitted at forward and one at backward rapidities, $8.5 < y_3 < 9$ and $-9 < y_4 < -8.5$, respectively, and that $0.02 < \xi_{1,2} < 0.1$.

For the calculation of the radiative amplitudes we use SPA1. Here in the $2 \rightarrow 4$ kinematics for SPA1 we define

$$\begin{aligned} \xi_1 &= \frac{k_{\perp 3}}{\sqrt{s}} \exp(y_3) + \frac{k_{\perp 4}}{\sqrt{s}} \exp(y_4), \\ \xi_2 &= \frac{k_{\perp 3}}{\sqrt{s}} \exp(-y_3) + \frac{k_{\perp 4}}{\sqrt{s}} \exp(-y_4). \end{aligned} \quad (16)$$

Theoretical formalism, $pp \rightarrow pp\gamma\gamma$

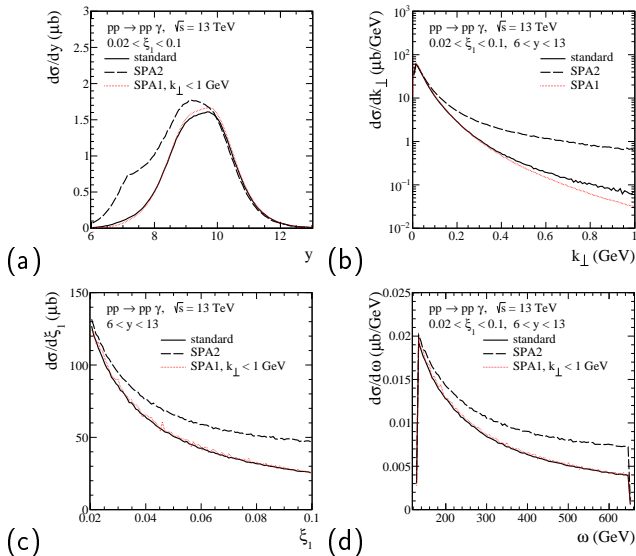
Then, the two-photon bremsstrahlung amplitude has the form

$$\mathcal{M}_{\mu\nu, \text{SPA1}} = e^2 \mathcal{M}^{(\text{on shell}) pp}(s, t) \left[-\frac{p_{a\mu}}{(p_a \cdot k_3)} + \frac{p_{1\mu}}{(p_1 \cdot k_3)} \right] \left[-\frac{p_{b\nu}}{(p_b \cdot k_4)} + \frac{p_{2\nu}}{(p_2 \cdot k_4)} \right], \quad (1)$$

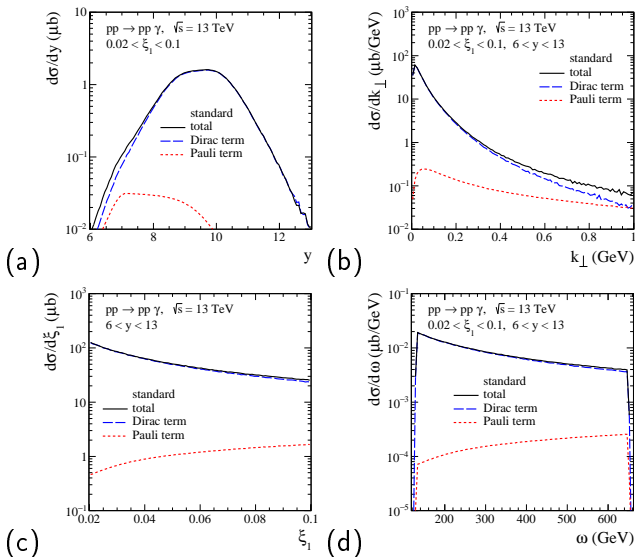
and the inclusive two-photon cross section is

$$\begin{aligned} d\sigma(pp \rightarrow pp\gamma\gamma)_{\text{SPA1}} &= \frac{d^3 k_3}{(2\pi)^3 2k_3^0} \frac{d^3 k_4}{(2\pi)^3 2k_4^0} \int d^3 p_1 d^3 p_2 e^4 \frac{d\sigma(pp \rightarrow pp)}{d^3 p_1 d^3 p_2} \\ &\times \left[-\frac{p_{a\mu}}{(p_a \cdot k_3)} + \frac{p_{1\mu}}{(p_1 \cdot k_3)} \right] \left[-\frac{p_{b\nu}}{(p_b \cdot k_4)} + \frac{p_{2\nu}}{(p_2 \cdot k_4)} \right] \\ &\times \left[-\frac{p_{a\alpha}}{(p_a \cdot k_3)} + \frac{p_{1\alpha}}{(p_1 \cdot k_3)} \right] \left[-\frac{p_{b\beta}}{(p_b \cdot k_4)} + \frac{p_{2\beta}}{(p_2 \cdot k_4)} \right] \\ &\times (-g^{\mu\alpha})(-g^{\nu\beta}). \end{aligned} \quad (18)$$

Results, single photon emission



Results, single photon emission



Results, single photon emission

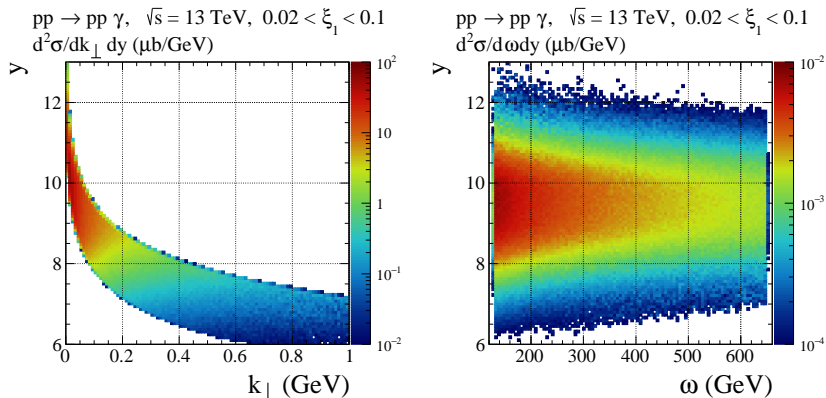


Figure: The two-dimensional distributions in (k_{\perp}, y) and in (ω, y) for the $pp \rightarrow pp\gamma$ reaction for our standard bremsstrahlung model calculated for $\sqrt{s} = 13 \text{ TeV}$, $6 < y < 13$, and $0.02 < \xi_1 < 0.1$.

Results, single photon emission

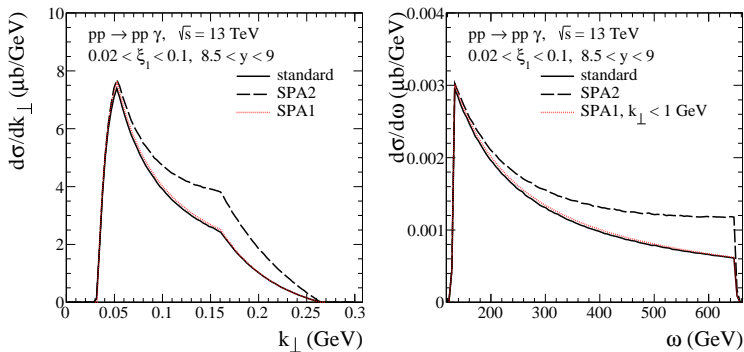


Figure: The differential distributions in the transverse momentum of the photon (left) and in the energy of the photon (right) for the $pp \rightarrow pp\gamma$ reaction. The calculations were done for $\sqrt{s} = 13$ TeV, $8.5 < y < 9$, and $0.02 < \xi_1 < 0.1$. The meaning of the lines is the same as in Fig. 3.

Results, single photon emission

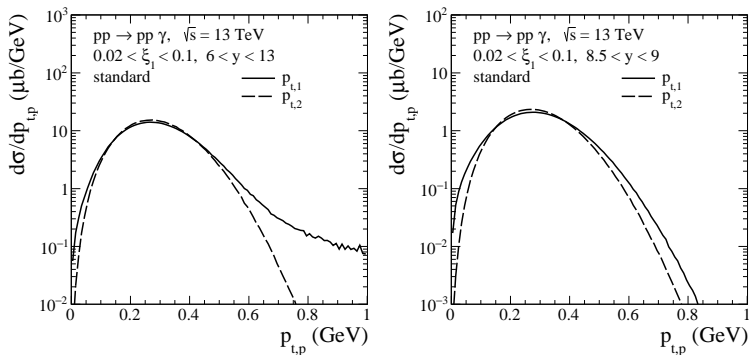


Figure: The distributions in the absolute values of the transverse momenta of the outgoing protons for the $pp \rightarrow pp\gamma$ reaction calculated for $\sqrt{s} = 13 \text{ TeV}$, $0.02 < \xi_1 < 0.1$, $6 < y < 13$ (left panel), and $8.5 < y < 9$ (right panel). The solid (dashed) line, denoted as $p_{t,1}$ ($p_{t,2}$), corresponds to the proton $p(p'_1)$ ($p(p'_2)$).

Results, single photon emission

Now we consider the azimuthal angles ϕ_i of the transverse momenta of the protons $p(p'_i)$ and the photon $\gamma(k)$

$$\begin{aligned} \mathbf{p}'_{i\perp} &= p_{t,i} e^{i\phi_i}, \quad (i = 1, 2), \\ \mathbf{k}_\perp &= k_\perp e^{i\phi_3}, \\ 0 \leq \phi_i &< 2\pi, \quad (i = 1, 2, 3). \end{aligned} \quad (19)$$

Here the azimuth $\phi = 0$ corresponds to some fixed transverse direction in the LHC system which is also the c.m. system for our reactions.

Transverse-momentum conservation requires

$$\mathbf{p}'_{1\perp} + \mathbf{p}'_{2\perp} + \mathbf{k}_\perp = 0. \quad (20)$$

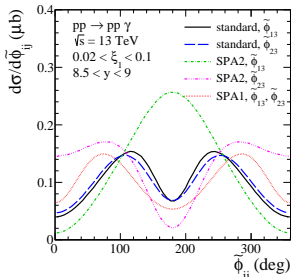
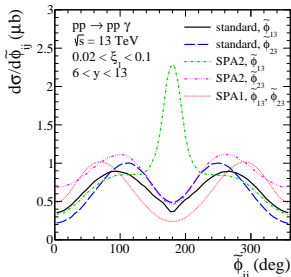
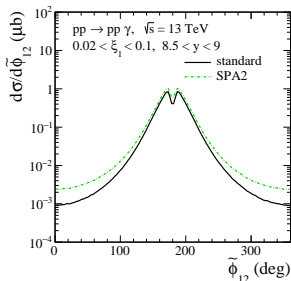
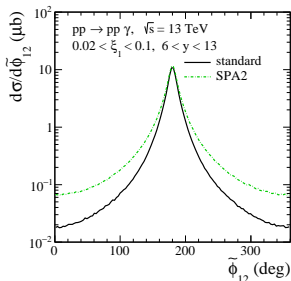
Therefore, a measurement of $\mathbf{p}'_{1\perp}$ and \mathbf{k}_\perp determines also $\mathbf{p}'_{2\perp}$.

Figure 8 shows the distributions in $\tilde{\phi}_{ij}$ defined as

$$\tilde{\phi}_{ij} = \phi_i - \phi_j \pmod{2\pi}, \quad (21)$$

where we require

Results, single photon emission



Results, single photon emission, background

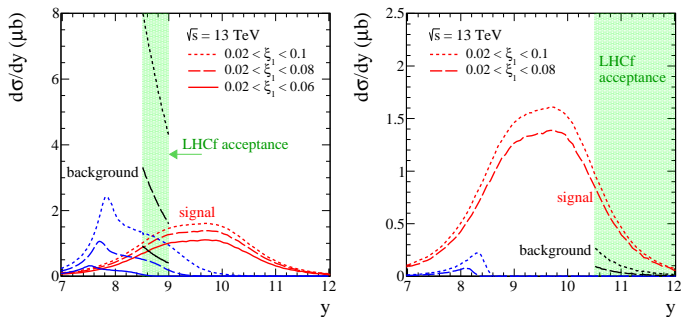
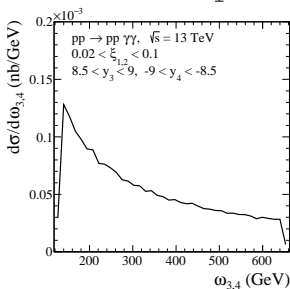
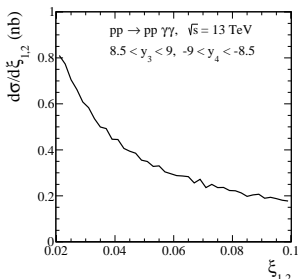
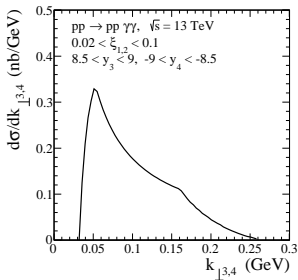
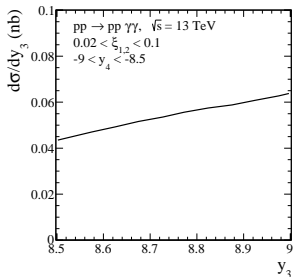


Figure: The LHCf acceptance regions are marked by the green shaded areas. In the left panel, the results within the LHCf acceptance correspond to the acceptance region $8.5 < y < 9$, while for the right panel to $y > 10.5$. For the background contribution we show the distributions of both photons from the decay of π^0 . The distributions of the **first (measured) photon** correspond to the black lines, while the distributions of the **second photon** correspond to the blue lines. The dashed, long dashed, and full lines correspond to the three ξ_1

Results, emission of two photons



Results, emission of two photons

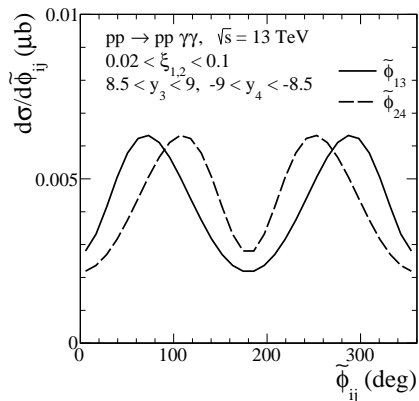


Figure: Azimuthal correlations for $pp \rightarrow pp\gamma\gamma$. Shown are results for SPA1.

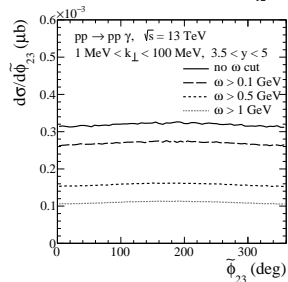
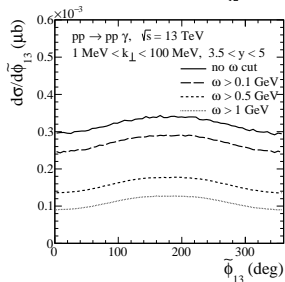
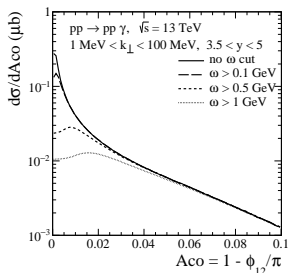
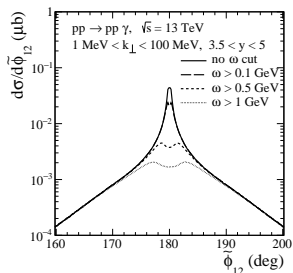
Results, emission of two photons

The fact of seemingly different distributions for forward and backward emissions is due to the way how the azimuthal angles are defined in (19) and (21) which leads to the symmetry relation

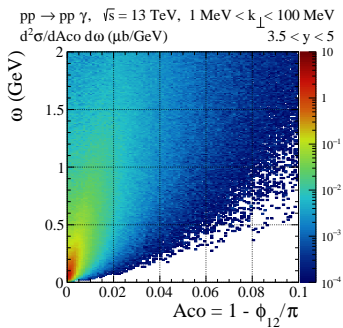
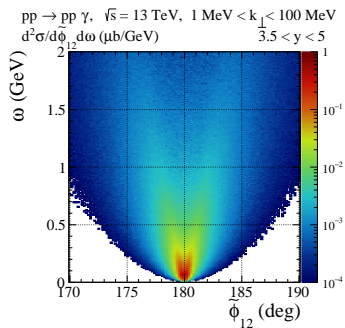
$$\frac{d\sigma^{\text{backward}}}{d\phi}(\phi) = \begin{cases} \frac{d\sigma^{\text{forward}}}{d\phi}(\phi + \pi) & \text{for } 0 \leq \phi < \pi, \\ \frac{d\sigma^{\text{forward}}}{d\phi}(\phi - \pi) & \text{for } \pi \leq \phi < 2\pi. \end{cases} \quad (23)$$

This explains the observed differences between the solid and dashed lines.

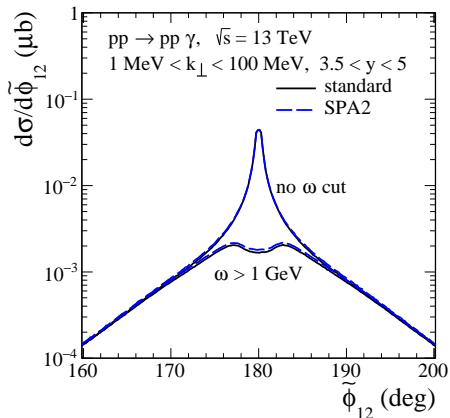
ALICE3, new results



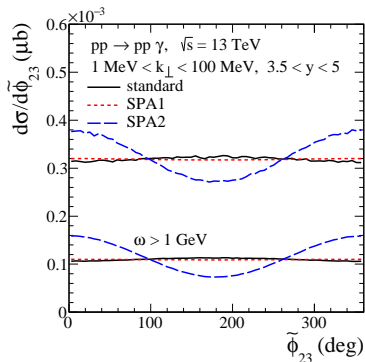
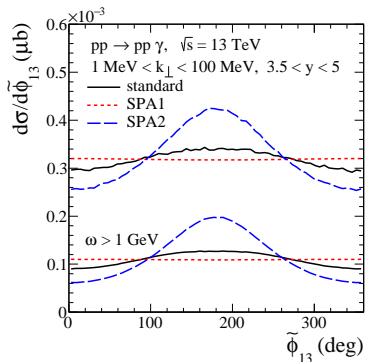
ALICE3, new results



ALICE3, new results



ALICE3, new results



interesting to measure

Conclusions

- We have studied **single- and double-photon bremsstrahlung** at very-forward and backward photon rapidities in proton-proton collisions at high c.m. energies. To calculate the amplitudes of the reactions $pp \rightarrow pp\gamma$ and $pp \rightarrow pp\gamma\gamma$ the framework of the **tensor-pomeron model** was used.
- We have compared our standard bremsstrahlung results and the results using the approximations SPA1 and SPA2.
- We have studied the **azimuthal angle correlations between outgoing particles**. We observe very interesting correlations between protons and photons. Detailed comparisons of our predictions with experiment in order to distinguish our standard and the approximate approaches measurement of the outgoing protons would be most welcome.
- We have estimated the coincidence cross section for **two-photon bremsstrahlung** in the $pp \rightarrow pp\gamma\gamma$ reaction within the **SPA1 approach**. We have required that the final state protons and photons can be measured by the **ATLAS forward proton spectrometers** (AFP) and **LHCf detectors**, respectively.

Conclusions

- We have also briefly estimated the background contribution due to the $pp \rightarrow pp\pi^0$ diffractive process for single photon bremsstrahlung. We have compared the signal and background contributions in two LHCf acceptance regions: $8.5 < y < 9$ and $y > 10.5$. One can increase the signal-to-background ratio to about 1 for the first acceptance region. For the second acceptance region the ratio is bigger than 3.5.
- We conclude that there is a chance to measure single photon bremsstrahlung with the present experimental configuration discussed here.
- The single photon bremsstrahlung mechanism should be identifiable by the measurement of proton and photon on one side and by checking the exclusivity condition (no particles in the main detector) without explicit measurement of the opposite side proton by AFP. Whether this is sufficient requires further studies, since such a measurement will probably include one-side diffractive dissociation, which can be of the order of 20-30 %.
- The cross section for two photons on different sides is rather small but should be measurable. For the two-photon bremsstrahlung a background estimate is much more complicated and goes beyond the scope of this analysis. A study of the background contributions should be done in future.