COMPUTING MULTI-LEG SCATTERING AMPLITUDES USING LIGHT-CONE ACTIONS

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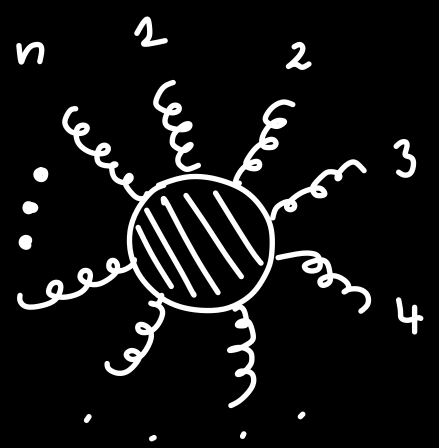
INTRODUCTION

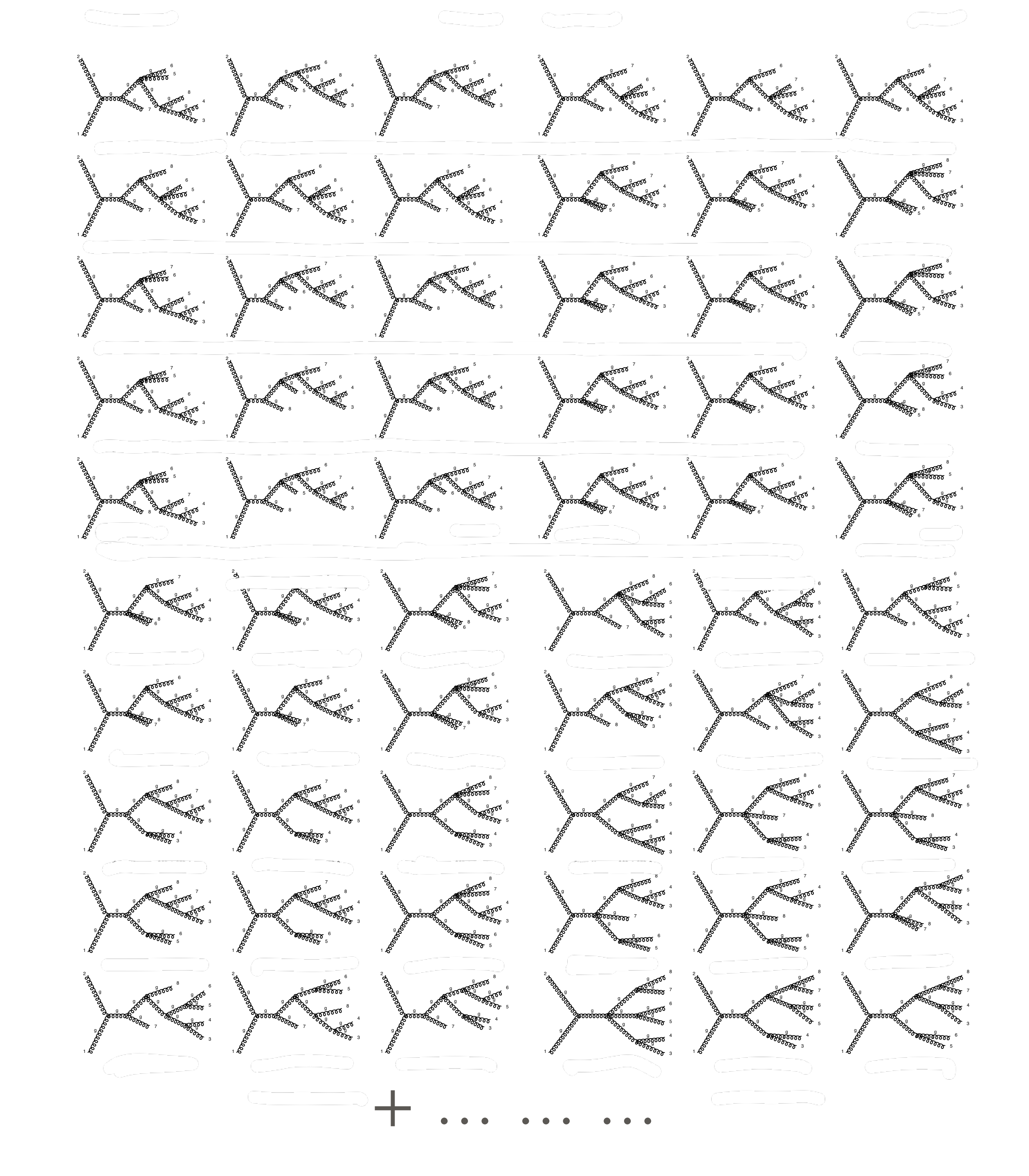
n-gluon scattering amplitudes



# legs	4	5	6	7	8
# diagrams	4	25	220	2485	34300

Many diagrams, but when expressed using the helicity spinors results are suspiciously simple…

Example: tree 8-point amplitude



Color decomposition

$$\mathcal{M}^{a_1,\ldots,a_n} = \sum_{\substack{\text{non-cyclic}\\ \text{permutations}}} \mathrm{Tr}(t^{a_1}\ldots t^{a_n})\, \mathcal{A}(1,\ldots,n)$$

$$\mathrm{Tr}(t^a t^b) = \delta_{ab}$$

$$[t^a, t^b] = i\sqrt{2} f^{abc} t^c$$

color ordered amplitudes

# legs	4	5	6	7	8
# diagrams	4	25	220	2485	34300
# planar diagrams	3	10	38	154	654

Spinor helicity method

SL(2,C) representation for Lorentz group.

$$k_{\alpha\dot\alpha} = (k_\mu \sigma^\mu)_{\alpha\dot\alpha} = \lambda_\alpha \tilde\lambda_{\dot\alpha} \quad \rightarrow \quad \begin{pmatrix} -k^0 + k^3 & k^1 - ik^2 \\ k^1 + ik^2 & -k^0 - k^3 \end{pmatrix}$$

for $k^2 = 0$.

$$v_+ = \begin{pmatrix} \lambda_\alpha \\ 0 \\ 0 \end{pmatrix}, \quad v_- = \begin{pmatrix} 0 \\ 0 \\ \tilde\lambda_{\dot\alpha} \end{pmatrix}$$

where $v_\pm$ are solution to Dirac equation:

$$\not{k}\, v_\pm(k) = 0$$

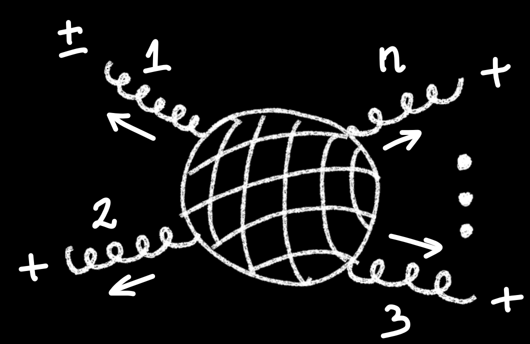
spinor products:

$$\langle ij \rangle = \lambda_i^\alpha \lambda_{j\alpha} \qquad [ij] = \tilde\lambda_i^{\dot\alpha} \tilde\lambda_{j\dot\alpha}$$

for on-shell momenta k_i, k_j.

Tree-level results for any number of gluons

The simplest (color-ordered) helicity amplitudes:

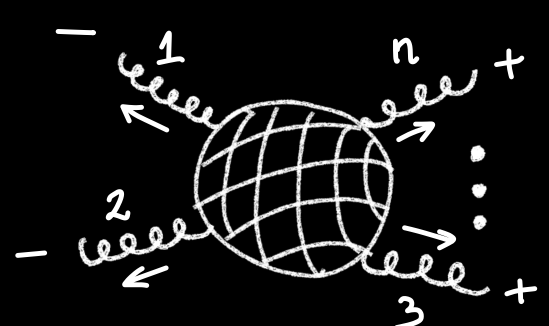


helicity

$$\mathscr{A}(1^+, 2^+, \ldots, n^+) = 0$$

$$\mathscr{A}(1^-, 2^+, \ldots, n^+) = 0$$

Maximally Helicity Violating (MHV) amplitudes:



$$\mathscr{A}(1^-, 2^-, 3^+, \ldots, n^+) = g^{n-2} \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle (n-1) n \rangle \langle n 1 \rangle}$$

[S.J. Parke, T.R Taylor, 1986]

Simplicity of the MHV amplitudes triggered incredible developments in theory over last 20 years…

INTRODUCTION

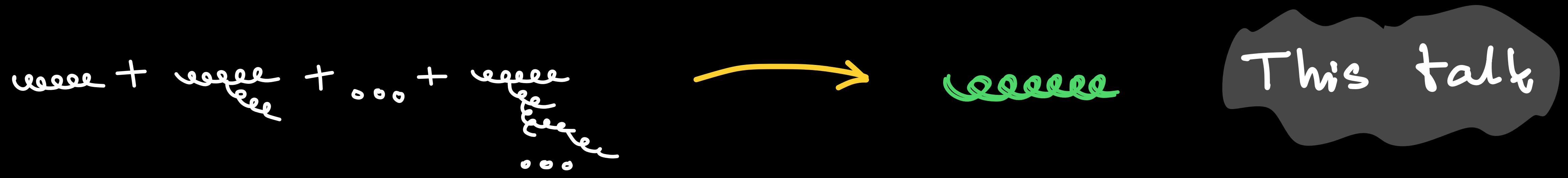
Two approaches to understand amplitudes:

- Geometry & on-shell methods

~~Fields~~ Analytic S-Matrix

  - The story starts with Witten (2004) and his twistor space approach that provided geometric understanding of the MHV amplitudes.
  - Soon after, on-shell recursion relations have been discovered (Britto,Cachazo,Feng,Witten, 2004).
  - Amplitudes in N=4 SYM as volumes of positive Grassmanians (Arkani-Hamed, Trnka, 2013).

- Field transformations



Apply field redefinitions in the path integral to get a new theory, perhaps better suited to study amplitudes.

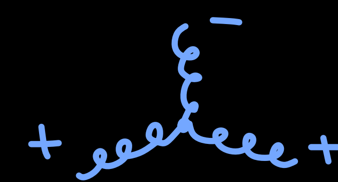
Removing unphysical degrees of freedom

[J. Scherk, J.H. Schwarz, 1975]

Set the light cone gauge $A^+ = 0$ and integrate out A^- :

$$\hat{A} \equiv t^a A_a(x^+, \mathbf{x})$$

$$\mathbf{x} \equiv (x^-, x^\bullet, x^\star)$$

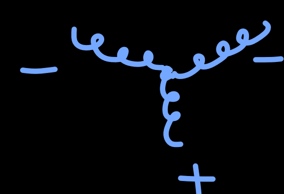


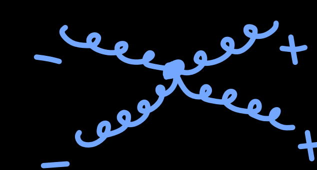
$$S^{(\mathrm{LC})}_{\mathrm{Y-M}}\left[A^\bullet, A^\star\right] = \int dx^+ \int d^3\mathbf{x} \Big\{ -\mathrm{Tr}\,\hat{A}^\bullet \Box \hat{A}^\star - 2ig\,\mathrm{Tr}\,\partial_-^{-1}\partial_\bullet \hat{A}^\bullet \left[\partial_- \hat{A}^\star, \hat{A}^\bullet\right] - 2ig\,\mathrm{Tr}\,\partial_-^{-1}\partial_\star \hat{A}^\star \left[\partial_- \hat{A}^\bullet, \hat{A}^\star\right] - 2g^2\,\mathrm{Tr}\left[\partial_- \hat{A}^\bullet, \hat{A}^\star\right] \partial_-^{-2} \left[\partial_- \hat{A}^\star, \hat{A}^\bullet\right] \Big\}$$

+ −

helicity field

includes instantaneous interactions





double-null coordinates

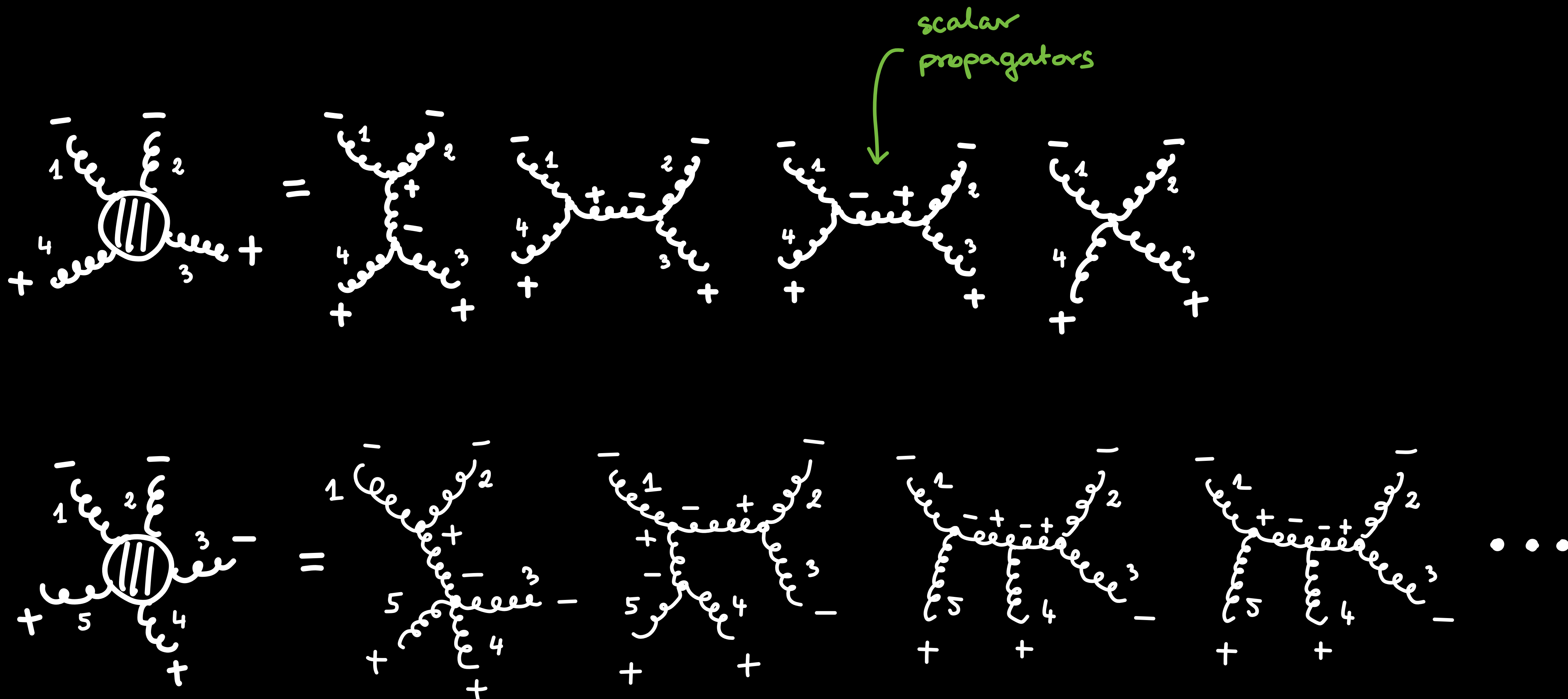
$$v^+ = v \cdot \eta = \frac{1}{\sqrt{2}}(v^0 + v^3) \qquad v^\bullet = v \cdot \varepsilon_\perp^+ = \frac{1}{\sqrt{2}}(v^1 + iv^2)$$

$$v^- = v \cdot \tilde{\eta} = \frac{1}{\sqrt{2}}(v^0 - v^3) \qquad v^\star = v \cdot \varepsilon_\perp^- = \frac{1}{\sqrt{2}}(v^1 - iv^2)$$

$$\eta = \frac{1}{\sqrt{2}}(1,0,0,-1) \qquad \varepsilon_\perp^+ = \frac{-1}{\sqrt{2}}(0,1,+i,0)$$

$$\tilde{\eta} = \frac{1}{\sqrt{2}}(1,0,0,1) \qquad \varepsilon_\perp^- = \frac{-1}{\sqrt{2}}(0,1,-i,0)$$

Scattering amplitudes

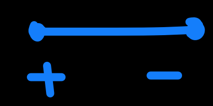


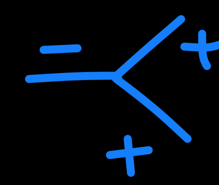
Number of diagrams is of the same order as in the ordinary method.

Self Dual (SD) Yang-Mills theory

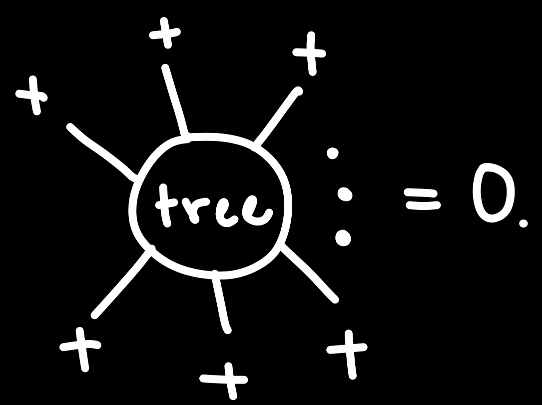
One of the simplest 4D theory with nontrivial amplitudes.

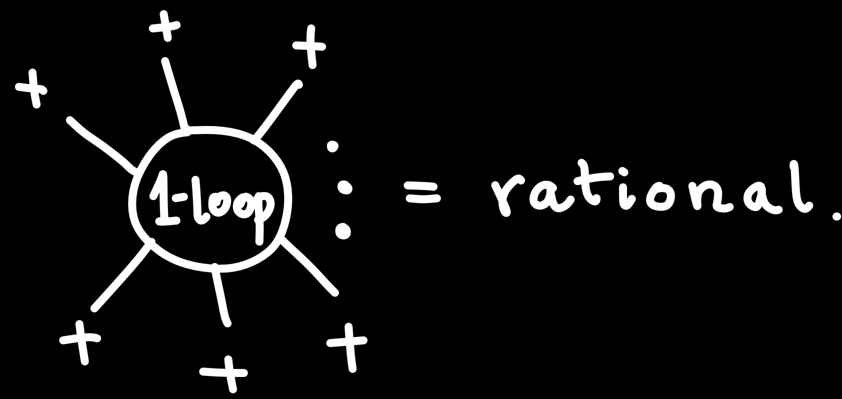
Truncation of the Yang-Mills theory to a single triple gluon vertex gives the self-dual theory $F = \star F$





$$S_{\rm SD}^{(\rm LC)}\left[A^{\bullet}, A^{\star}\right] = \int dx^{+} \int d^{3}\mathbf{x} \left\{ -\operatorname{Tr}\hat{A}^{\bullet}\Box\hat{A}^{\star} - 2ig\operatorname{Tr}\partial_{-}^{-1}\partial_{\bullet}\hat{A}^{\bullet}\left[\partial_{-}\hat{A}^{\star}, \hat{A}^{\bullet}\right] \right\}$$





Classical SD theory is integrable (has infinite set of conserved currents)

- tree amplitudes must vanish
- loop amplitudes are effect of a quantum anomaly of classical symmetries

[W.A. Bardeen, 1996]

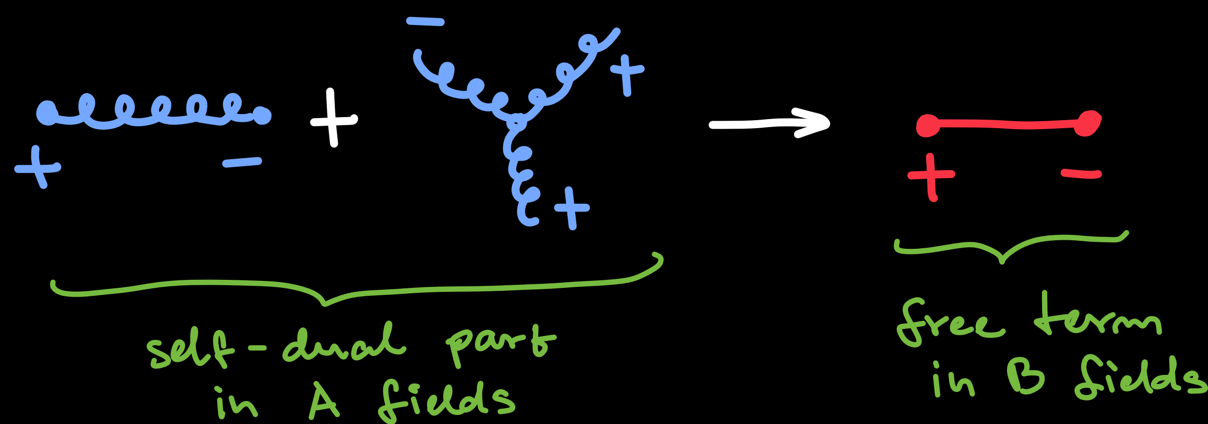
[P. Chattopadhyay, K. Krasnov, 2022, 2023]

[R. Monteiro, R. Stark-Muchao, S. Wikeley, 2023]

Field transformation: MHV action

Apply the canonical field transformation (at equal LC time) $\left\{A^\bullet, A^\star\right\} \rightarrow \left\{B^\bullet, B^\star\right\}$ [P. Mansfield, 2006]

such that:



and

$$\partial\_A^\star_a\left(x^+;\mathbf{x}\right) = \int d^3\mathbf{y}\, \frac{\delta B^\bullet_c\left(x^+;\mathbf{y}\right)}{\delta A^\bullet_a\left(x^+;\mathbf{x}\right)}\, \partial\_B^\star_c\left(x^+;\mathbf{y}\right)$$

equal LC time

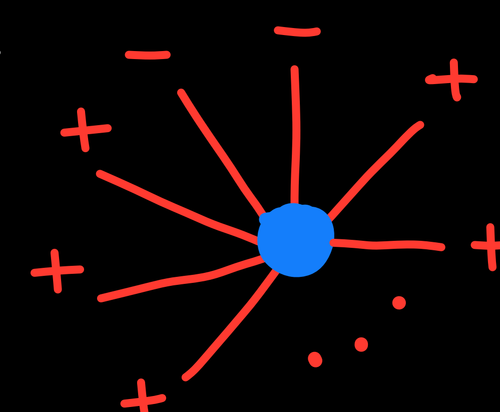
the solutions:

$$\widetilde{A}^\bullet_a[B^\bullet](x^+;\mathbf{P}) = \sum_{n=1}^{\infty} \int d^3\mathbf{p}_1 \ldots d^3\mathbf{p}_n\, \widetilde{\Psi}_n^{ab_1\ldots b_n}(\mathbf{P};\mathbf{p}_1,\ldots,\mathbf{p}_n) \prod_{i=1}^{n} \widetilde{B}^\bullet_{b_i}(x^+;\mathbf{p}_i)$$

$$\widetilde{A}^\star_a[B^\bullet,B^\star](x^+;\mathbf{P}) = \sum_{n=1}^{\infty} \int d^3\mathbf{p}_1 \ldots d^3\mathbf{p}_n\, \widetilde{\Omega}_n^{ab_1b_2\cdots b_n}(\mathbf{P};\mathbf{p_1},\mathbf{p_2},\ldots,\mathbf{p_n}) \widetilde{B}^\star_{b_1}(x^+;\mathbf{p}_1) \prod_{i=2}^{n} \widetilde{B}^\bullet_{b_i}(x^+;\mathbf{p}_i)\,,$$

$$S^{(\mathrm{LC})}_{\mathrm{Y-M}}\left[B^\bullet,B^\star\right] = \int dx^+ \left(-\int d^3\mathbf{x}\,\mathrm{Tr}\,\hat{B}^\bullet\,\square\,\hat{B}^\star + \mathcal{L}^{(\mathrm{LC})}_{--+} + \ldots + \mathcal{L}^{(\mathrm{LC})}_{--+\ldots+} + \ldots\right)$$

MHV vertices

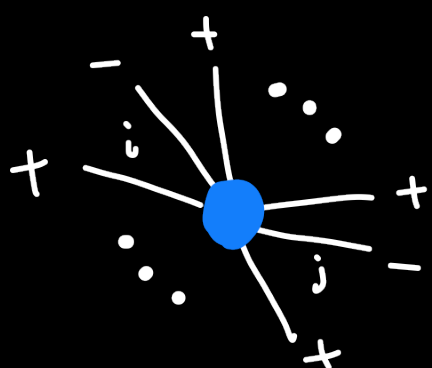


Cachazo-Svrcek-Witten (CSW) method

MHV vertices

The Maximally Helicity Violating (MHV) amplitudes can be treated as interaction vertices.

[F. Cachazo, P. Svrcek, E. Witten, 2004]



$$= g^{n-2} \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n1 \rangle}$$

↑ off-shell spinor products

spinor products (on-shell)

$$\langle ij \rangle = \epsilon_{\alpha\beta} \lambda_i^\alpha \lambda_j^\beta$$

← + helicity spinor

$$[ij] = \epsilon_{\dot\alpha\dot\beta} \tilde\lambda_i^{\dot\alpha} \tilde\lambda_j^{\dot\beta}$$

← − helicity spinor

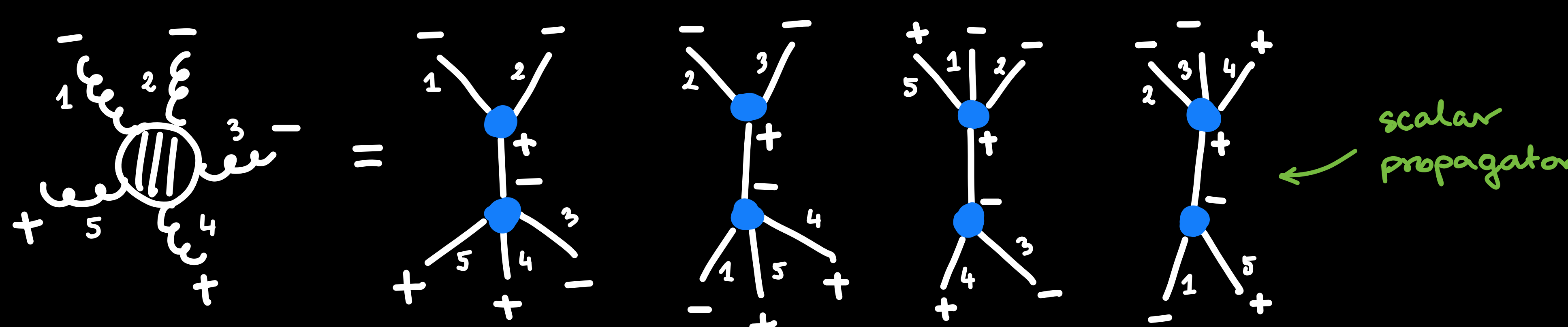
off-shell continuations

$$\lambda^*_{i\,\alpha} = (k_i)_{\alpha\dot\alpha} \tilde\eta_j^{\dot\alpha} = (k_i^\mu \sigma_\mu)_{\alpha\dot\alpha} \tilde\eta_j^{\dot\alpha}$$

↑ $(\frac{1}{2},0)\times(0,\frac{1}{2})$ representation of k^μ, $k^2 \neq 0$

↑ auxiliary spinor

example: tree-level NMHV



(Incomplete) List of papers on the subject

CSW method

- F. Cachazo, P. Svrcek, E. Witten, JHEP 09 (2004) 006
- F. Cachazo, P. Svrcek, E. Witten, JHEP 10 (2004) 074
- G. Georgiou, V. Khoze, JHEP 05 (2004)
- G. Georgiou, E.W.N. Glover, V.V. Khoze, JHEP 07 (2004)
- J.-B. Wu, C.-J. Zhu, JHEP 07 (2004) 032
- L.J. Dixon, E.W.N. Glover, V.V. Khoze, JHEP 12 (2004) 015
- K. Risager, JHEP 12 (2005) 003
- A. Brandhuber, B. Spence, G. Travaglini, JHEP 01 (2006) 142
- M. Kiermaier, S.G. Naculich, JHEP 05 (2009) 072
- T. Adamo, L. Mason, Phys.Rev.D 86 (2012) 065019

Lagrangian formulation of CSW

- P. Mansfield, JHEP 03 (2006) 037
- J.H. Ettle, T.R. Morris, JHEP 08 (2006) 003
- A. Gorsky, A. Rosly, JHEP 01 (2006) 101
- J.H. Ettle, T.R. Morris, Z. Xiao, JHEP 08 (2008) 103
- T.R. Morris, Z. Xiao, JHEP 12 (2008) 028
- H. Feng, Y.-T. Huang, JHEP 04 (2009) 047
- C.-H. Fu, JHEP 04 (2010) 044
- S. Buchta, S. Weinzierl, JHEP 09 (2010) 071
- P.K, A. Stasto, JHEP 09 (2017)
- H. Kakkad, P.K, A. Stasto, Phys.Rev.D 102 (2020) 9

Lagrangian formulation of CSW at loop level

- A. Brandhuber, B. Spence, G. Travaglini, JHEP 02 (2007) 088
- A. Brandhuber, B. Spence, G. Travaglini, K. Zoubos, JHEP 07 (2007) 002
- J.H. Ettle, C.-H. Fu, J.P. Fudger, P. Mansfield, T.R. Morris, JHEP 05 (2007) 011
- R. Boels, C. Schwinn, JHEP 07 (2008) 007
- C.-H. Fu, J.P. Fudger, P. Mansfield, T.R. Morris, Z. Xiao, JHEP 06 (2009) 035
- H. Elvang, D.Z. Freedman, M. Kiermaier, JHEP 06 (2012) 015
- H. Kakkad, P.K, A. Stasto, JHEP 11 (2022) 132

Field transformations as Wilson lines

Inverse solutions to field transformations

[PK, A. Stasto, 2017]
[H. Kakkad, PK, A. Stasto, 2020]

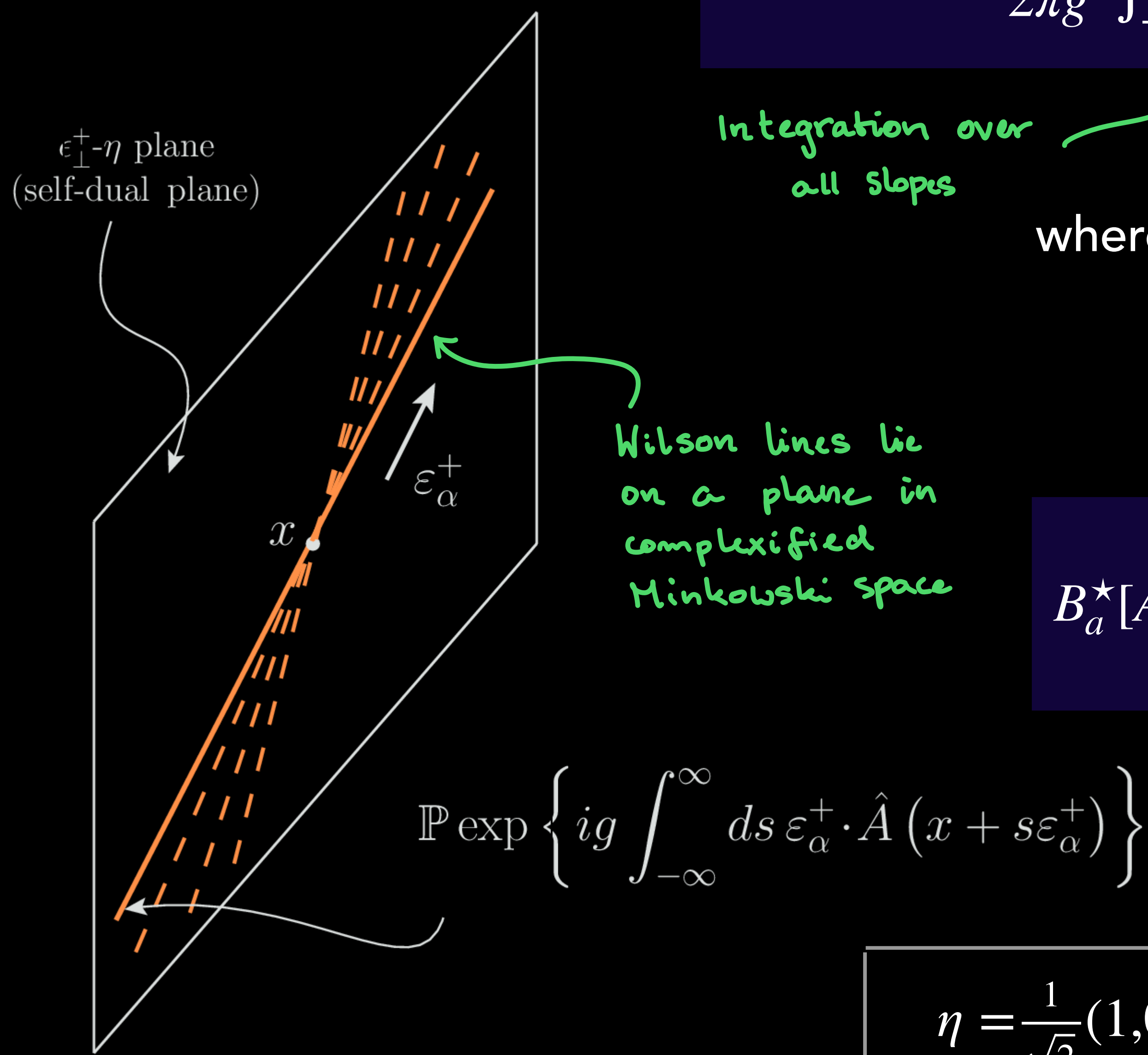
Solving the field transformations for B fields, we get "Wilson lines":

$$B^{\bullet}_{a}[A^{\bullet}](x) = \frac{1}{2\pi g}\int_{-\infty}^{\infty} d\alpha \, \mathrm{Tr}\left\{ t^{a}\partial_{-}\, \mathbb{P}\exp\left[ig\int_{-\infty}^{\infty} ds \; \underbrace{\varepsilon^{+}_{\alpha}\cdot\hat{A}}_{\hat{A}^{\bullet}}\left(x+s\varepsilon^{+}_{\alpha}\right)\right]\right\}$$

Integration over all slopes

where $\varepsilon^{\pm\,\mu}_{\alpha} = \varepsilon^{\pm\,\mu}_{\perp} - \alpha\eta^{\mu}$ "slope" of the Wilson line. "polarization vector"

$$x \equiv (x^{+}, x^{-}, x^{\bullet}, x^{\star})$$



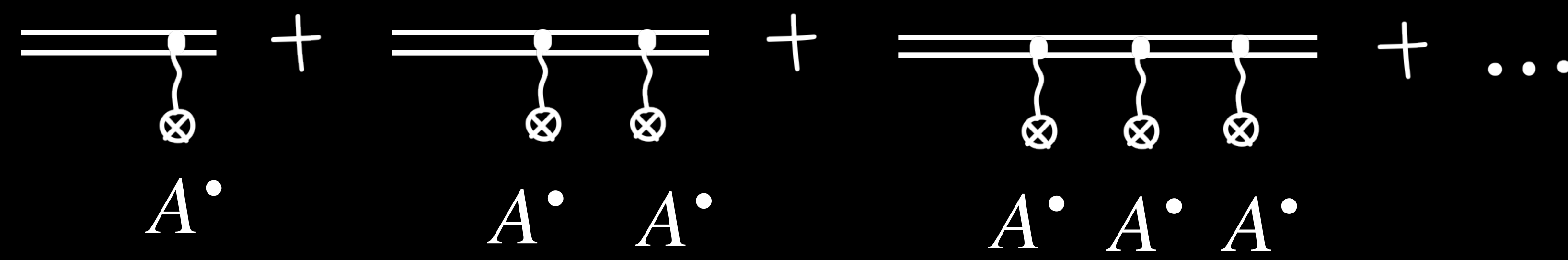
$$B^{\star}_{a}[A^{\bullet}, A^{\star}](x) = \int d^{3}\mathbf{y}\, \left[\frac{\partial^{2}_{-}(y)}{\partial^{2}_{-}(x)}\,\frac{\delta B^{\bullet}_{a}[A^{\bullet}](x^{+};\mathbf{x})}{\delta A^{\bullet}_{c}(x^{+};\mathbf{y})}\right] A^{\star}_{c}(x^{+};\mathbf{y})$$

$$\eta = \frac{1}{\sqrt{2}}(1,0,0,-1) \qquad \varepsilon^{+}_{\perp} = \frac{-1}{\sqrt{2}}(0,1,+i,0) \qquad \varepsilon^{-}_{\perp} = \frac{-1}{\sqrt{2}}(0,1,-i,0)$$

Collective degrees of freedom

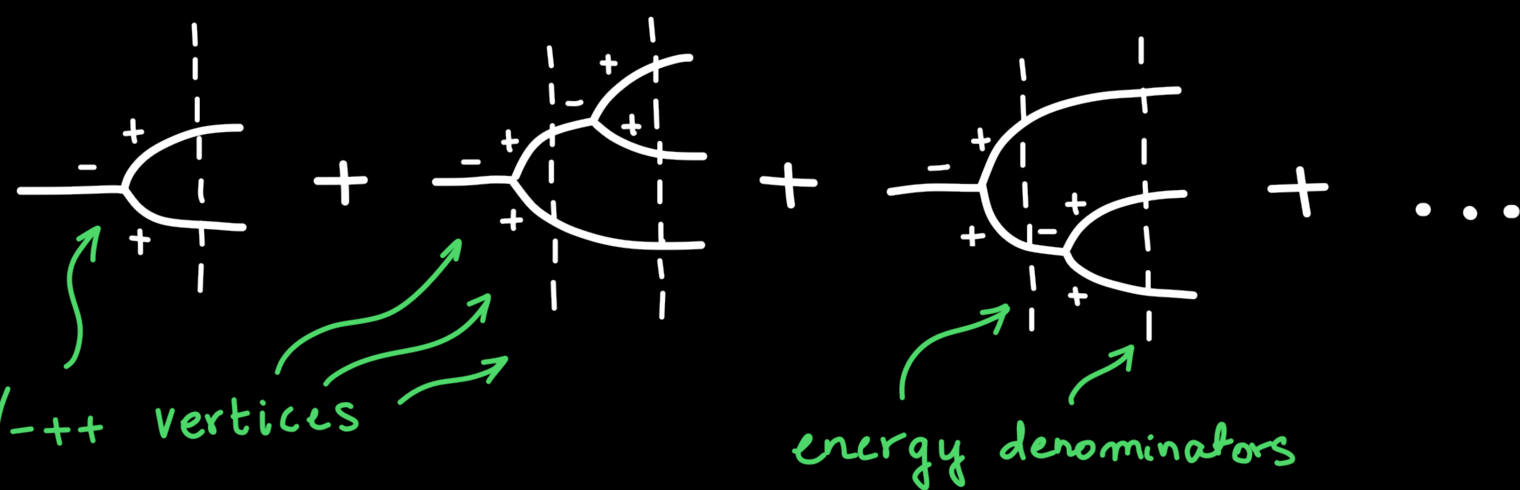
"Wilson line" in momentum space:

$$\widetilde{B}^{\bullet}_{a}[A^{\bullet}](p) =$$



Exactly corresponds to:

$$\widetilde{B}^{\bullet}_{a}[A^{\bullet}](p) =$$



Wilson line along polarization vector resums $(-++)$ triple-gluon interactions!

[PK, A. Stasto, 2017]

Can we do the same with the $(++-)$ interactions?

New fields $Z^\bullet, Z^\star$

[H. Kakkad, PK, A. Stasto, 2021]

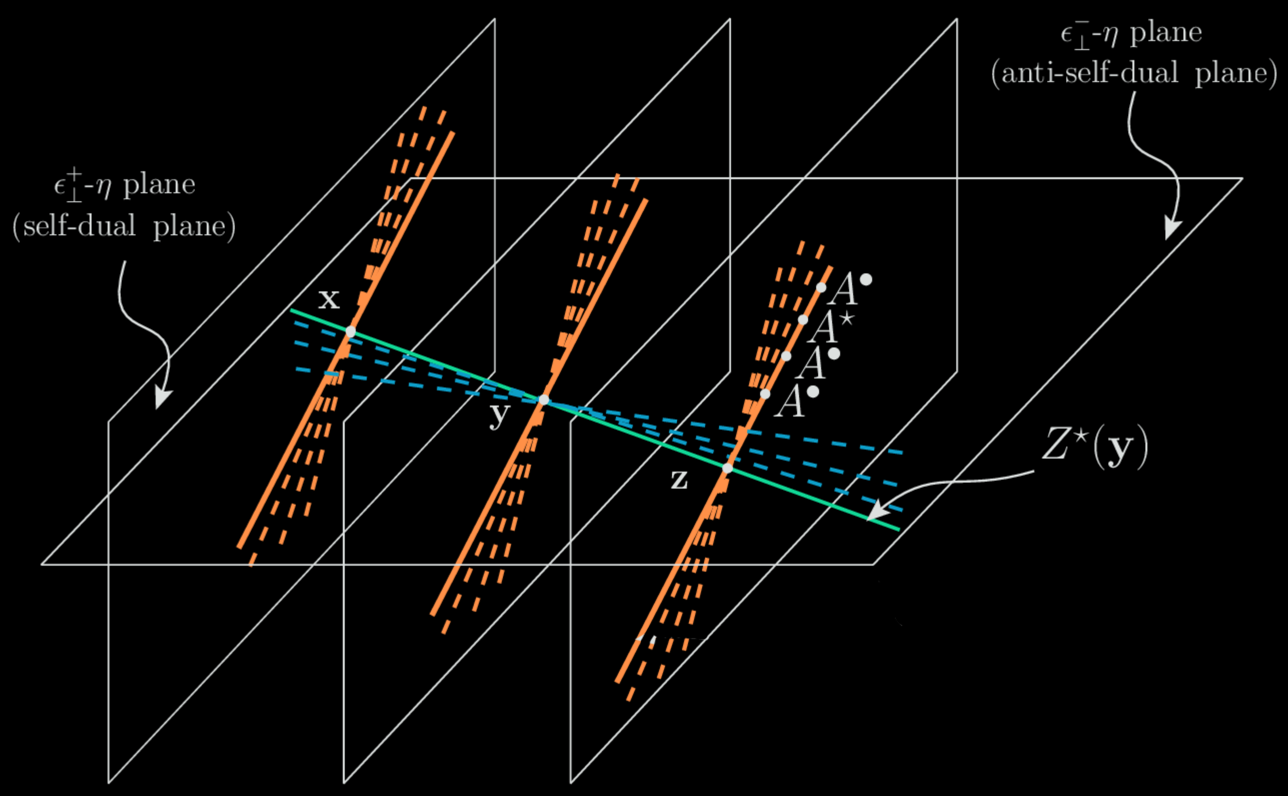
Introduce a canonical transformation $\{A^\bullet, A^\star\} \to \{Z^\bullet, Z^\star\}$ given by the generating functional:

$$\mathcal{G}[A^\bullet, Z^\star](x^+) = -\int d^3\mathbf{x}\ \mathrm{Tr}\,\hat{\mathcal{W}}^{-1}_{(-)}[Z](x)\ \partial_-\hat{\mathcal{W}}_{(+)}[A](x)$$

where

$$\mathcal{W}^a_{(\pm)}[K](x) = \frac{1}{2\pi g}\int_{-\infty}^{\infty} d\alpha\, \mathrm{Tr}\left\{t^a\partial_-\,\mathbb{P}\exp\left[ig\int_{-\infty}^{\infty} ds\ \varepsilon_\alpha^\pm \cdot \hat{K}\left(x + s\varepsilon_\alpha^\pm\right)\right]\right\}$$

Wilson line on self-dual or anti-self-dual plane



Relations between A and Z fields:

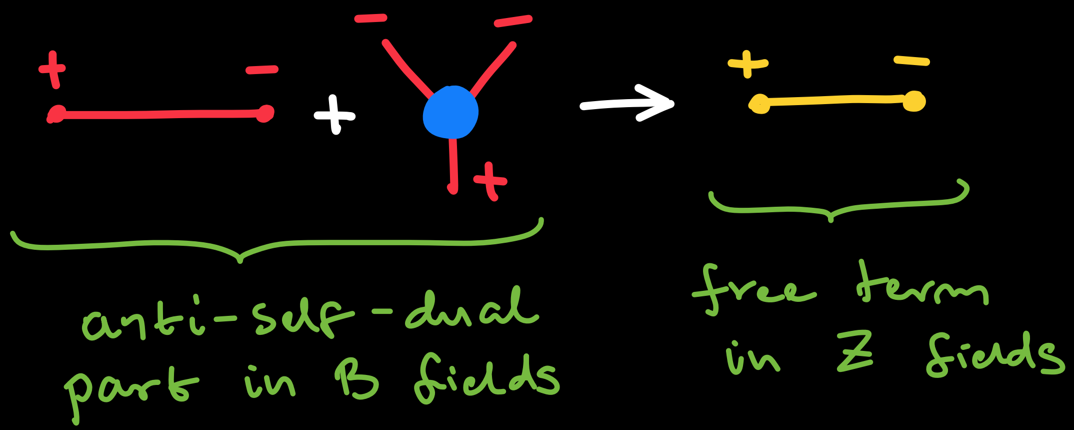
$$\partial_- A_a^\star(x^+, \mathbf{y}) = \frac{\delta\,\mathcal{G}[A^\bullet, Z^\star](x^+)}{\delta A_a^\bullet\,(x^+, \mathbf{y})}$$

$$\partial_- Z_a^\bullet(x^+, \mathbf{y}) = -\frac{\delta\,\mathcal{G}[A^\bullet, Z^\star](x^+)}{\delta Z_a^\star\,(x^+, \mathbf{y})}$$

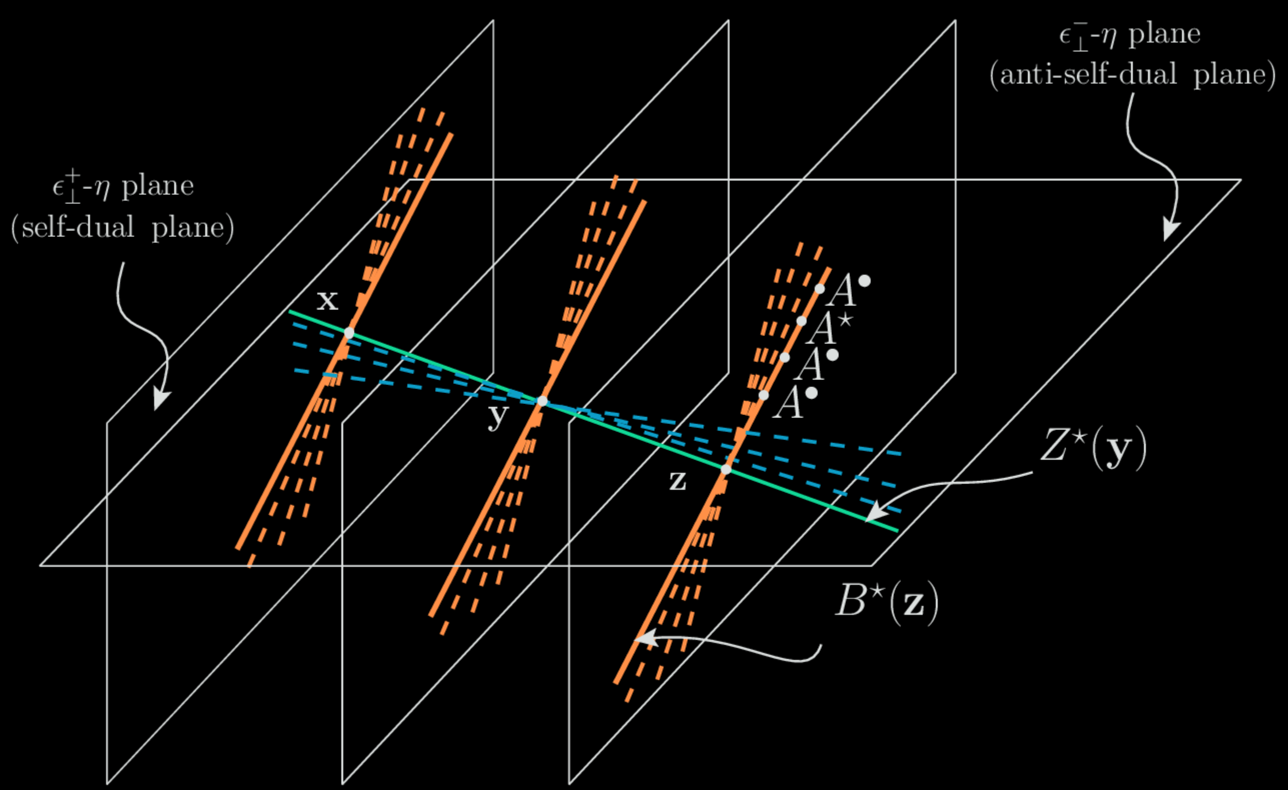
Canonical transformation of the B fields in the MHV action

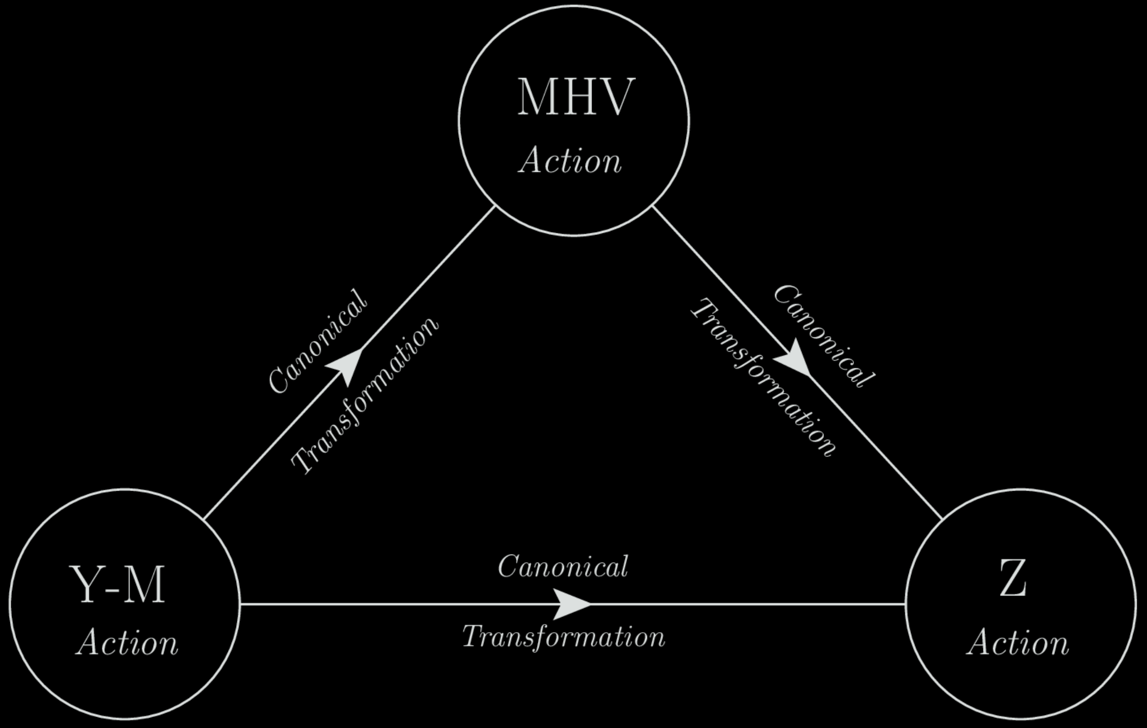
[H. Kakkad, PK, A. Stasto, 2021]

It turns out, that the new fields can be introduced also from the MHV action :



$$\Rightarrow \begin{cases} Z^{\star}_a[B^{\star}](x) = \mathscr{W}^a_{(-)}[B](x) \\ Z^{\bullet}_a[B^{\bullet},B^{\star}](x) = \int d^3\mathbf{y} \left[\frac{\partial^2_-(y)}{\partial^2_-(x)} \frac{\delta Z^{\star}_a[B^{\star}](x^+;\mathbf{x})}{\delta B^{\star}_c(x^+;\mathbf{y})} \right] B^{\bullet}_c(x^+;\mathbf{y}) \end{cases}$$





Z-field action

[H. Kakkad, PK, A. Stasto, 2021]

Solving the field transformation relations we get the following action:

$$S_{\mathrm{Y-M}}^{(\mathrm{LC})}\left[Z^{\bullet},Z^{\star}\right] = \int dx^{+} \Big\{ -\int d^{3}\mathbf{x}\, \mathrm{Tr}\, \hat{Z}^{\bullet} \Box \hat{Z}^{\star}$$

$$+\mathscr{L}^{(\mathrm{LC})}_{--++} + \mathscr{L}^{(\mathrm{LC})}_{--+++} + \mathscr{L}^{(\mathrm{LC})}_{--++++} + \dots$$

$$+\mathscr{L}^{(\mathrm{LC})}_{---++} + \mathscr{L}^{(\mathrm{LC})}_{---+++} + \mathscr{L}^{(\mathrm{LC})}_{---++++} + \dots$$

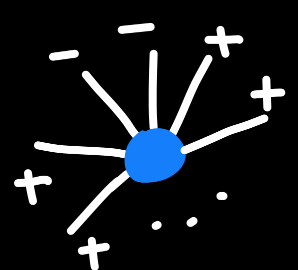
$$\vdots$$

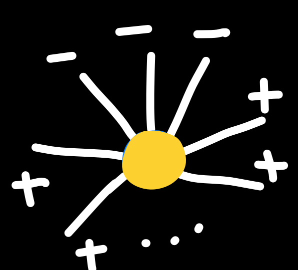
$$+\mathscr{L}^{(\mathrm{LC})}_{---\dots-++} + \mathscr{L}^{(\mathrm{LC})}_{---\dots-+++} + \mathscr{L}^{(\mathrm{LC})}_{---\dots-++++} + \dots \Big\}$$

MHV vertices →

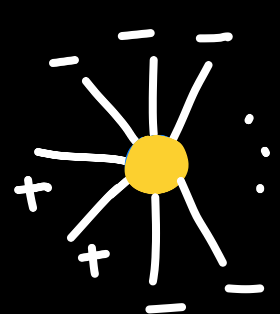
NMHV →

etc...





$\overline{\text{MHV}}$ vertices

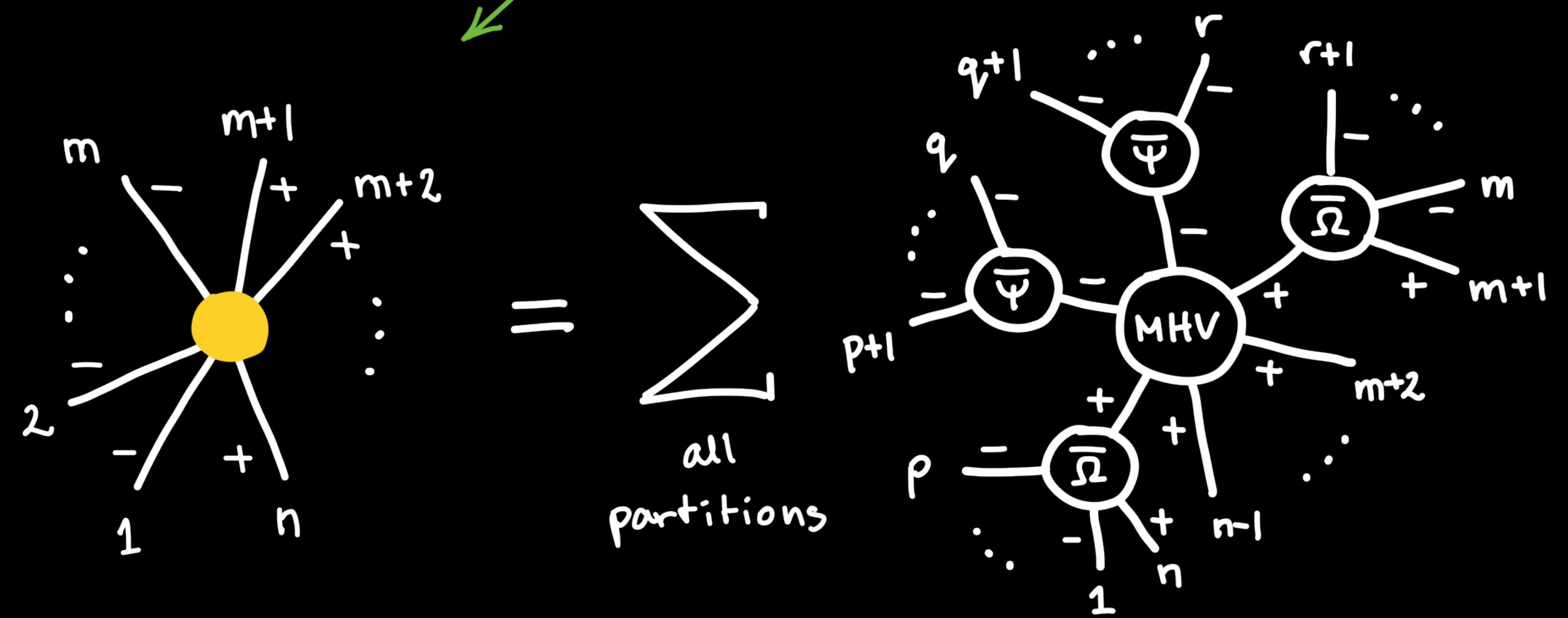


No triple gluon vertices!

Master formula for a vertex

[H. Kakkad, PK, A. Stasto, 2021]

$$\mathcal{L}^{(\mathrm{LC})}_{\underbrace{-\,\cdots\,-}_{m}\,\underbrace{+\,\cdots\,+}_{n-m}} = \int d^3\mathbf{y}_1\ldots d^3\mathbf{y}_n\, \mathcal{U}^{b_1\ldots b_n}_{-\ldots-+\ldots+}\left(\mathbf{y}_1,\cdots\mathbf{y}_n\right)\prod_{i=1}^{m} Z^{\star}_{b_i}(x^+;\mathbf{y}_i)\prod_{j=1}^{n-m} Z_{b_j}(x^+;\mathbf{y}_j)$$



Inverse Wilson line kernels

$$\overline{\widetilde{\Psi}}{}_{n}^{\,a\{b_1\cdots b_n\}}(\mathbf{P};\{\mathbf{p}_1,\ldots,\mathbf{p}_n\}) = -\,(-g)^{n-1}\,\frac{\widetilde{v}_{(1\cdots n)1}}{\widetilde{v}_{1(1\cdots n)}}\,\frac{\delta^3(\mathbf{p}_1+\cdots+\mathbf{p}_n-\mathbf{P})\ \mathrm{Tr}(t^{a}t^{b_1}\cdots t^{b_n})}{\widetilde{v}_{21}\widetilde{v}_{32}\cdots\widetilde{v}_{n(n-1)}}$$

$$\overline{\widetilde{\Omega}}{}_{n}^{\,ab_1\{b_2\cdots b_n\}}(\mathbf{P};\mathbf{p}_1,\{\mathbf{p}_2,\ldots,\mathbf{p}_n\}) = n\left(\frac{p_1^+}{p_{1\cdots n}^+}\right)^2\overline{\widetilde{\Psi}}{}_{n}^{\,ab_1\cdots b_n}(\mathbf{P};\mathbf{p}_1,\ldots,\mathbf{p}_n)\,.$$

$$\widetilde{v}_{ij} = p_i^+\left(\frac{p_j^\star}{p_j^+}-\frac{p_i^\star}{p_i^+}\right) = -\,(\varepsilon_i^-\cdot p_j)\ \sim [ij]$$

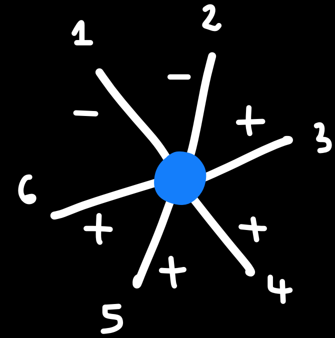
Example of amplitude calculation (1)

6-point amplitudes

[H. Kakkad, PK, A. Stasto, 2021]

$$\mathscr{A}(1^{\pm},2^{+},3^{+},4^{+},5^{+},6^{+}) = 0$$

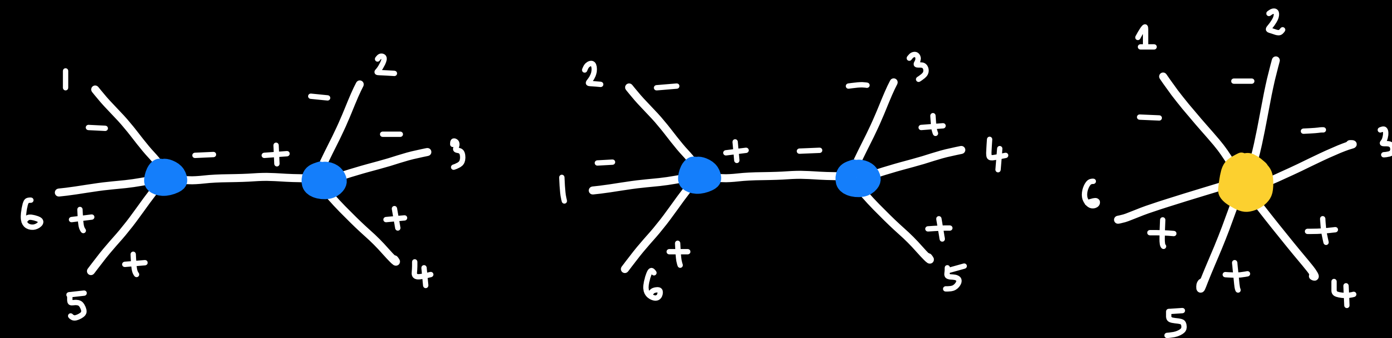
$$\mathscr{A}(1^{-},2^{-},3^{+},4^{+},5^{+},6^{+}) =$$



$$= g^4 \left(\frac{p_1^+}{p_2^+}\right)^2 \frac{\widetilde{u}_{21}^4}{\widetilde{u}_{16}\widetilde{u}_{65}\widetilde{u}_{54}\widetilde{u}_{43}\widetilde{u}_{32}\widetilde{u}_{21}}$$

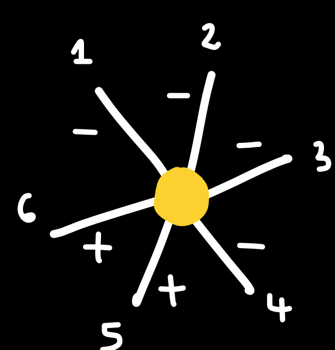
MHV

$$\mathscr{A}(1^{-},2^{-},3^{-},4^{+},5^{+},6^{+}) =$$



NMHV

$$\mathscr{A}(1^{-},2^{-},3^{-},4^{-},5^{+},6^{+}) =$$



$$= g^4 \left(\frac{p_5^+}{p_6^+}\right)^2 \frac{\widetilde{v}_{65}^4}{\widetilde{v}_{16}\widetilde{v}_{65}\widetilde{v}_{54}\widetilde{v}_{43}\widetilde{v}_{32}\widetilde{v}_{21}}$$

$\overline{\text{MHV}}$

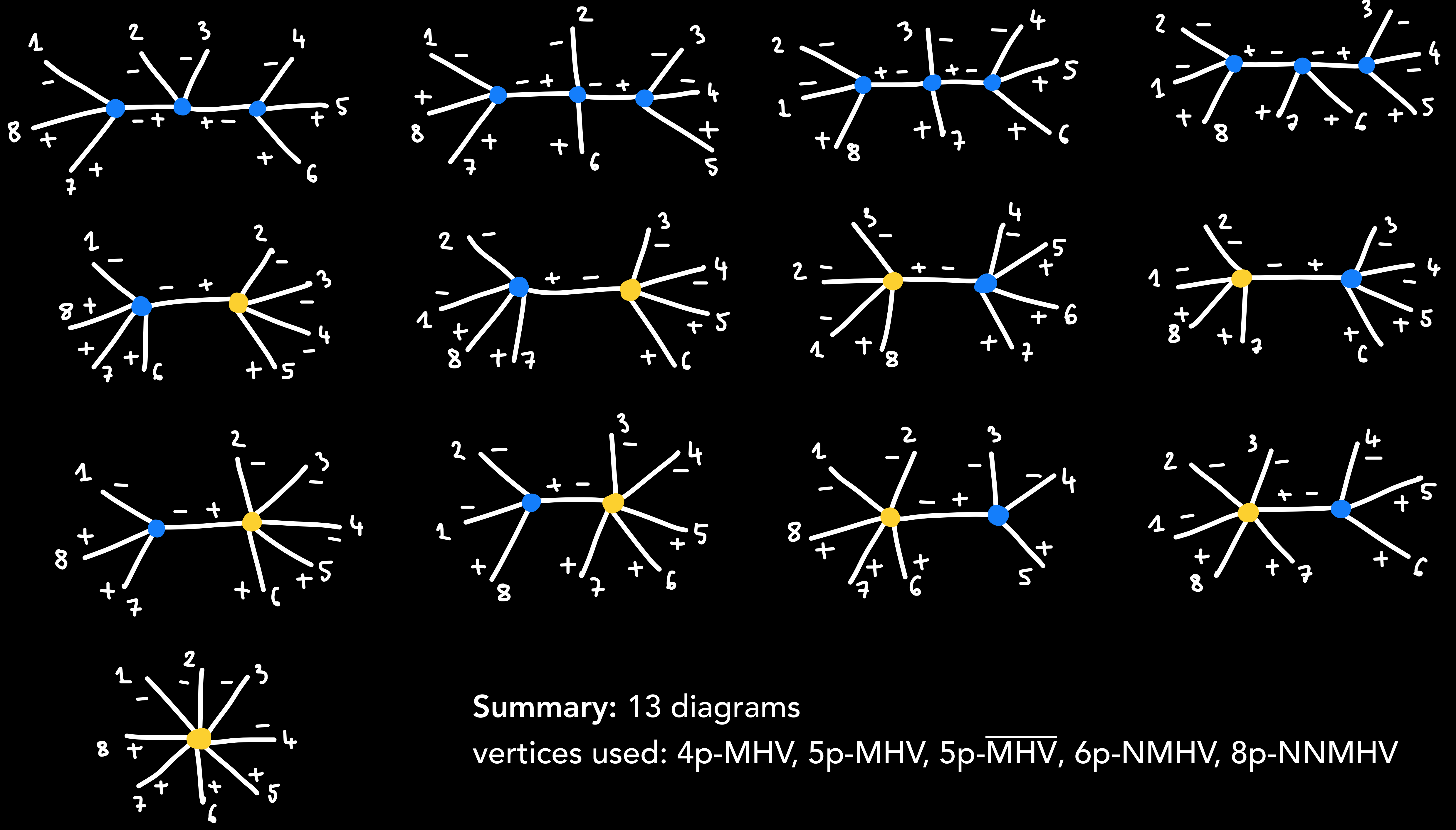
$$\widetilde{v}_{ij} = -\left(\varepsilon_i^- \cdot p_j\right) = p_i^+ \left(\frac{p_j^\star}{p_j^+} - \frac{p_i^\star}{p_i^+}\right) \sim [ij]$$

$$\widetilde{u}_{ij} = -\left(\varepsilon_i^+ \cdot p_j\right) = p_i^+ \left(\frac{p_j^\bullet}{p_j^+} - \frac{p_i^\bullet}{p_i^+}\right) \sim \langle ij \rangle$$

Example of amplitude calculation (2)

8-point NNMHV amplitude $\mathscr{A}(1^-,2^-,3^-,4^-,5^+,6^+,7^+,8^+)$

[H. Kakkad, PK, A. Stasto, 2021]



Summary: 13 diagrams

vertices used: 4p-MHV, 5p-MHV, 5p-$\overline{\text{MHV}}$, 6p-NMHV, 8p-NNMHV

Dellanoy numbers

It can be demonstrated, that the number of diagrams for an amplitude with $p+2$ “minus” helicity legs and $q+2$ “plus” follows the so-called Dellanoy series:

[H. Kakkad, PhD thesis, 2024]

$$\mathscr{D}(p,q) = \sum_{i=0}^{\min(p,q)} 2^i \binom{p}{i}\binom{q}{i}$$

Dellanoy numbers have geometric interpretation. → Hiren's talk

For example, for 9 gluon amplitude, the series predicts at most just 25 diagrams.

⇒ tested explicitly by computing 9-leg amplitude !

[M. Kulig, Engineer degree thesis, 2024]

LOOP AMPLITUDES

Problems with transformed actions

All-plus-helicity amplitude at one loop

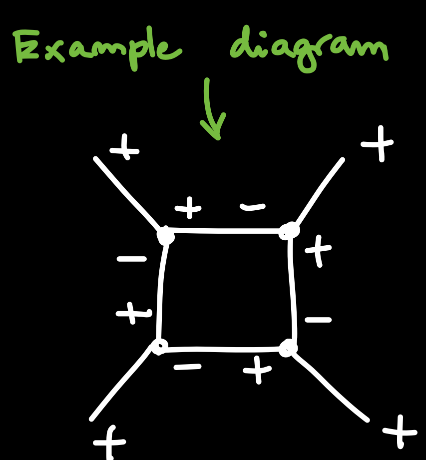
Amplitude with all same helicity gluons is non-zero at one loop and is a rational function:

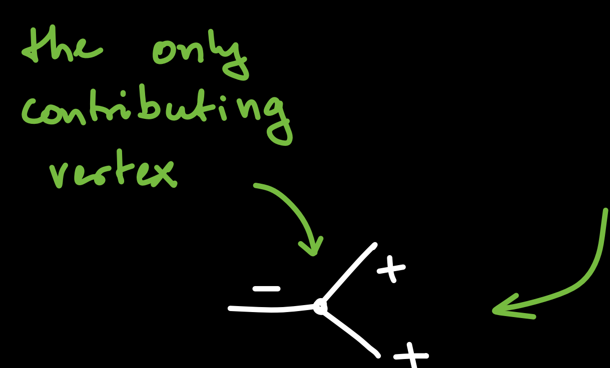
$$\mathscr{A}^{(1)}(1^+,2^+,3^+,\ldots,n^+) = g^n \sum_{q\le i<j<k<l\le n} \frac{\langle ij\rangle[jk]\langle kl\rangle[li]}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

[Z. Bern, G. Chalmers, L. Dixon, D.A. Kosover, 1993]
[G. Mahlon, 1994]

Problems with MHV action (or Z-field action) at quantum level

It is not possible to reconstruct $(\pm + + \ldots +)$ amplitudes in the MHV theory or the Z-field action.





The self-dual vertex $(- + +)$ was removed in both MHV action and in the Z-field action.

One also cannot get the rational parts of other (singular) amplitudes...

[J. Bedford, A. Brandhuber, B.J. Spence, G. Travaglini, 2005]

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See Hiren's talk…

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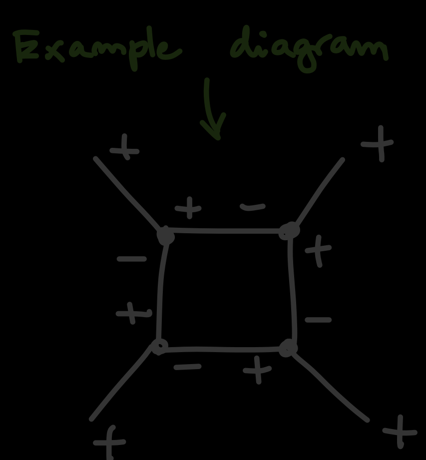

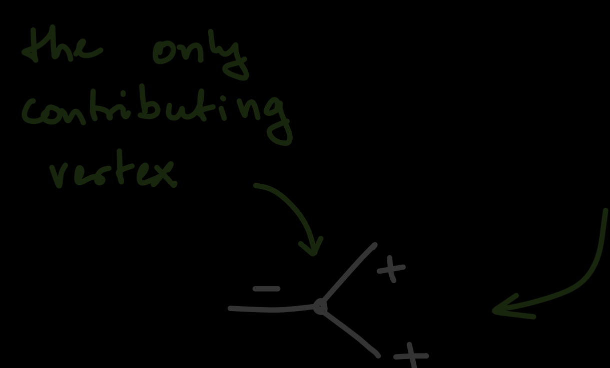

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CONCLUSIONS

- We developed a new theory, classically equivalent to the Yang-Mills theory on the Light Cone.
- Theory is local in the light-cone time.
- Dramatically fewer diagrams when computing tree level scattering amplitudes.
- The number of diagrams follows Dellanoy number series (see Hiren's talk).
- Collective degrees of freedom — a nontrivial slope-integrated Wilson line-type functionals extending in complexified Minkowski space.
- Encoded geometry of scattering amplitudes?
- The structure at quantum level gets very complicated (unlike the classical level), probably because of the quantum anomalies in the self-dual and anti-self dual sectors of the Yang-Mills.

BACKUP

Quantum Z-field theory action (1)

One loop effective action

Generating functional for Y-M theory: $\mathscr{Z}_{\mathrm{Y-M}}[J] = \int [dA^{\bullet}][dA^{\star}]\, e^{i\left\{S[A^{\bullet},A^{\star}] + \int d^4x\, \mathrm{Tr}(\hat{J}_{\bullet}\hat{A}^{\bullet} + \hat{J}_{\star}\hat{A}^{\star})\right\}}$

Apply the field transformation $A \to Z$: $\mathscr{Z}[J] = \int [dZ^{\bullet}][dZ^{\star}]\, e^{i\left\{S[Z^{\bullet},Z^{\star}] + \int d^4x\, \mathrm{Tr}(\hat{J}_{\bullet}\hat{A}^{\bullet}[Z] + \hat{J}_{\star}\hat{A}^{\star}[Z])\right\}}$

(warning: transform also the current terms!)

Integrate quadratic fluctuations around the classical solution:

$$\mathscr{Z}[J] \approx \exp\left\{ iS\left[Z_c^{\bullet}[J], Z_c^{\star}[J]\right] - i\frac{1}{2}\mathrm{Tr}\ln\left(\mathbb{M}_Z[J] + \mathbb{M}_{\mathrm{src}}[J]\right) \right\}$$

Z-field theory vertices

$$\left(\mathbb{M}_Z[J]\right)_{kl} = \left(\frac{\delta^2 S[Z]}{\delta Z^k \delta Z^l}\right), \quad \left(\mathbb{M}_{\mathrm{src}}[J]\right)_{kl} = \left(J_m \frac{\delta^2 A^m[Z]}{\delta Z^k \delta Z^l}\right)$$

missing contributions

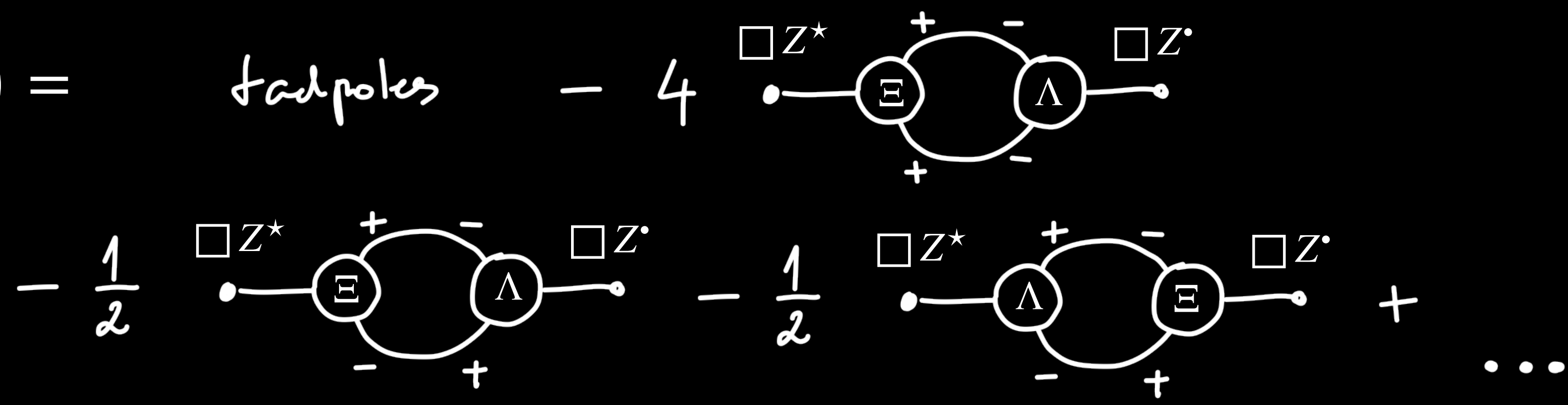
where $k, l, m, \ldots = \{\{\bullet, \star\}, x^{\mu}, a\}$ are collective indices.

Legendre transform to one-loop effective action: $\mathscr{Z}[J] \to \Gamma[\phi] = S[\phi] - \frac{1}{2}\mathrm{Tr}\ln\left(\mathbb{M}_Z + \mathbb{M}_{\mathrm{src}}\right)_{Z_c[J]=\phi}$

Further steps

- The log can be "computed" to the desired number of legs, but it's complicated as there are multiple nested sums (I used `FORM` …)

$$\mathrm{Tr}\ln\left(\mathbb{M}_Z + \mathbb{M}_{\mathrm{src}}\right) =$$



- We can prove that the one loop effective action is "quantum complete" (the rational contributions are not missing).

- Diagrams can be calculated in 4D world-sheet regularization scheme Chakrabarti-Qiu-Thorn (CQT).

[for some examples see H. Kakkad thesis, ArXiv:2308.07695]