

[2311.15892]

Precise calculations for decays of Higgs bosons in extended Higgs sectors

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In collaboration with

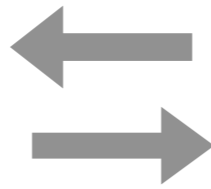
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The shape of the Higgs sector is unknown

- Extended Higgs sector is interesting possibility for new physics.

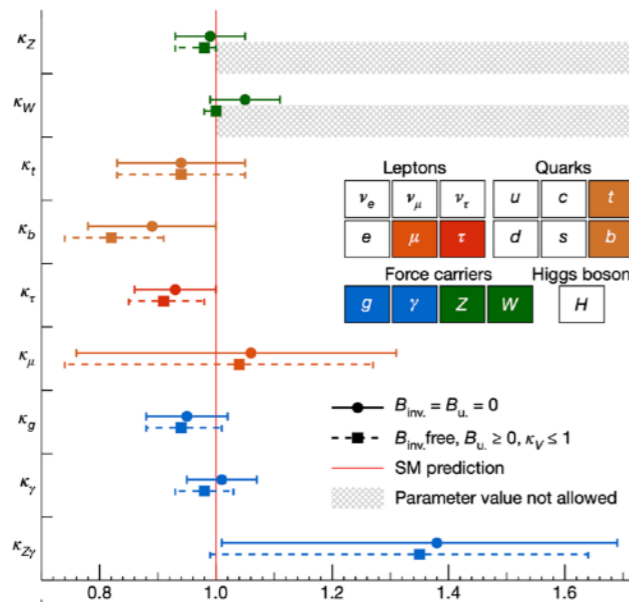
$$\Phi_{\text{SM}} + \Phi' + \dots$$



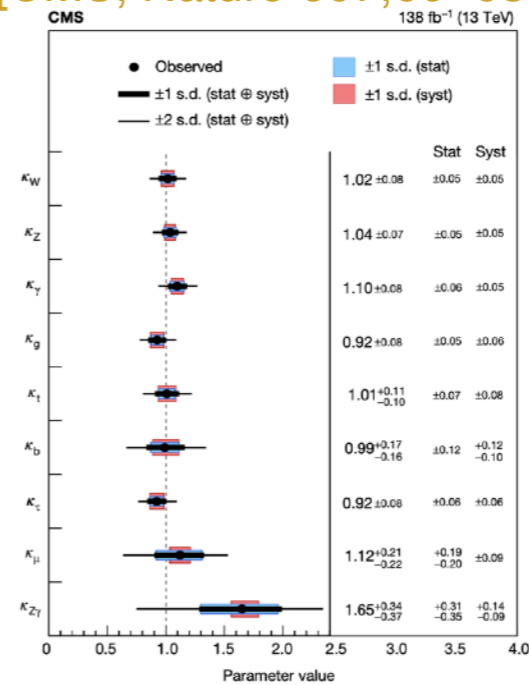
Dark matter, Neutrino masses,
 Baryon asymmetry of the Universe,
 muon-2, W mass, GWs, ...

- It is not completely excluded by the current LHC experiments.

[ATLAS, Nature 607,60–68 (2022)]



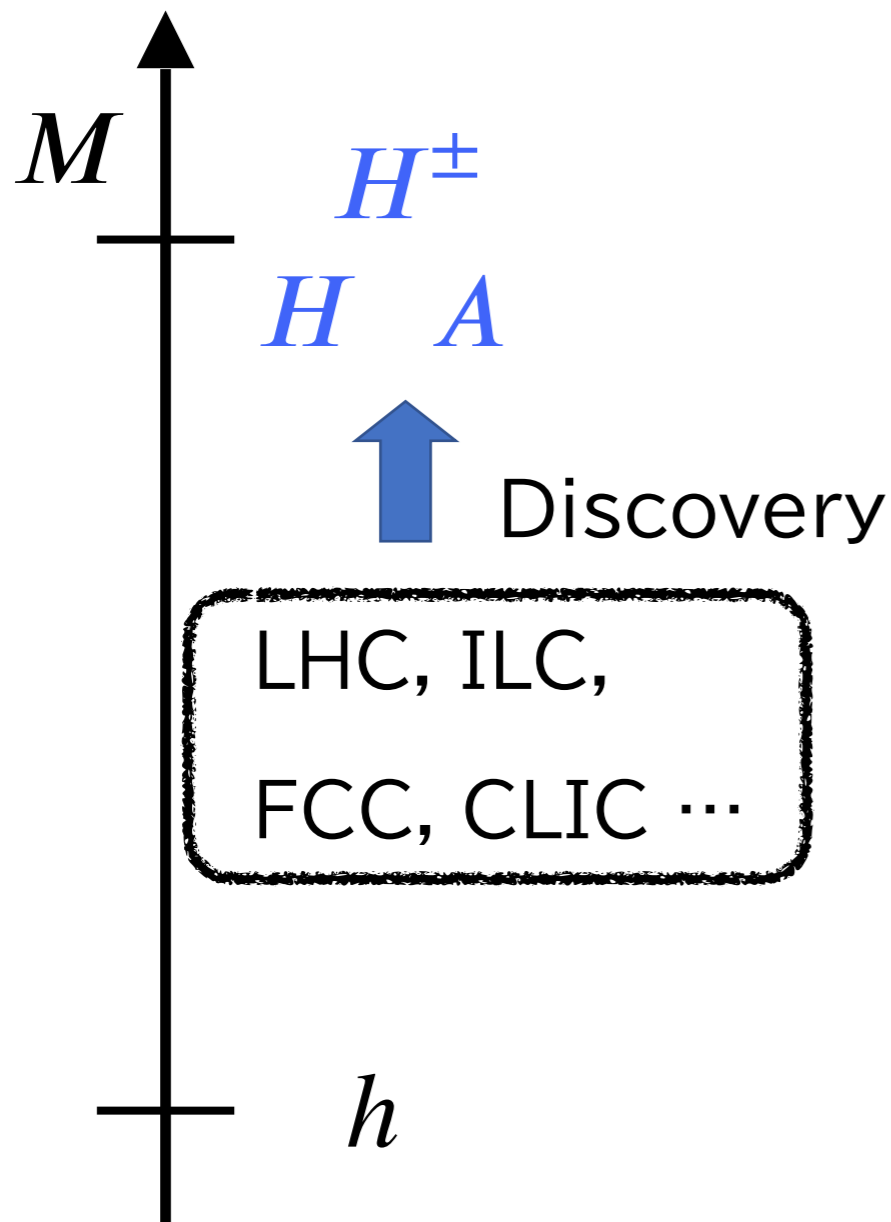
[CMS, Nature 607,60–68 (2022)]



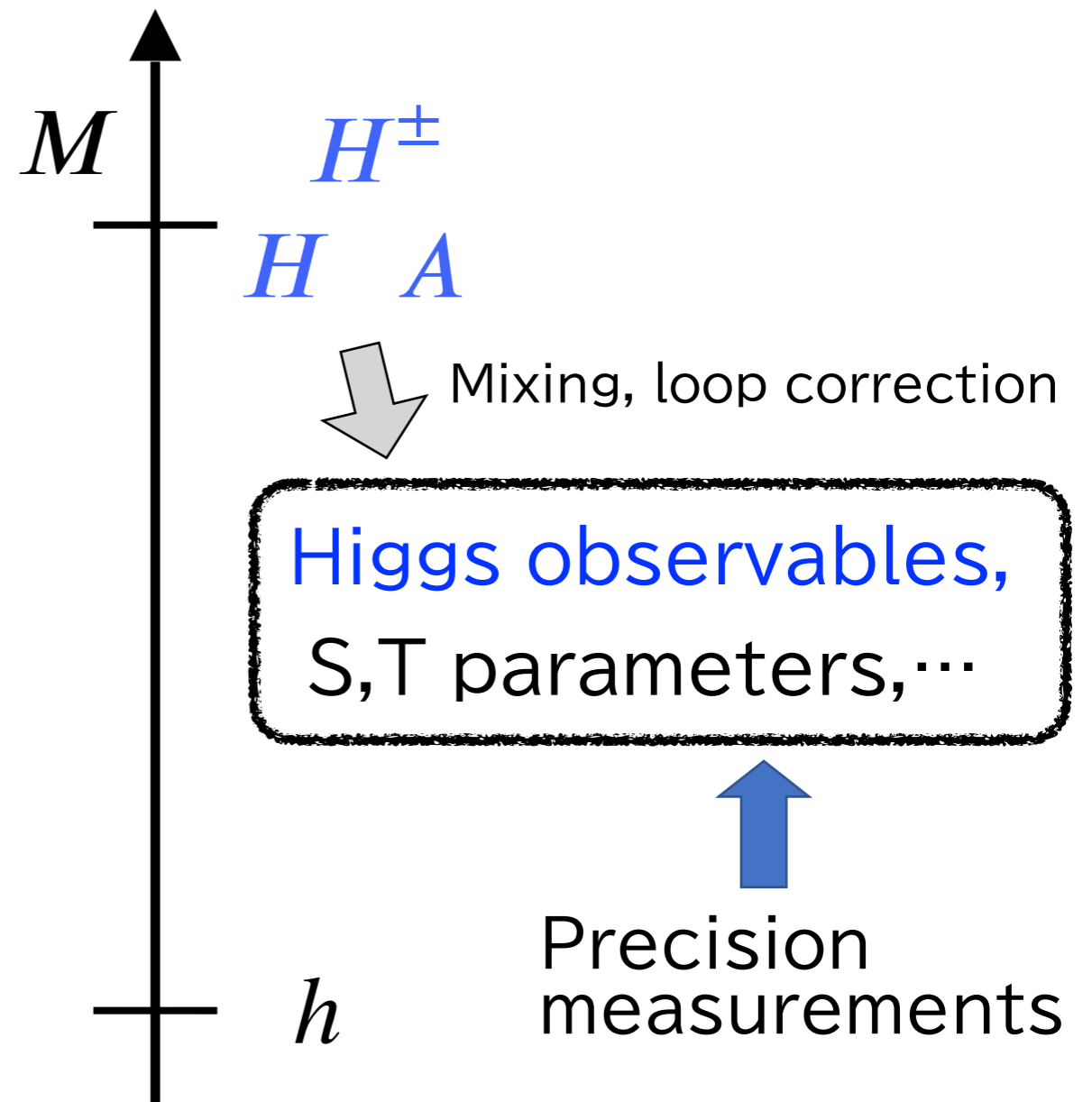
Reconstructing Higgs sector is an important task for finding new physics.

How is the Higgs sector tested?

Direct search



Indirect search



A combination of these two ways can test the Higgs sector.

Contents

- Introduction
- Model
 - 2HDM, motivations.
- Radiative corrections to Higgs decays
 - HCOUP, results for $A \rightarrow Zh$ and $h \rightarrow ZZ^*$
- Summary

Two Higgs doublet models (2HDMs) [1/2]

2HDM is a simple example of the extended Higgs sector. It has several motivations to study.

- Higgs potential

$$V = m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - m_3^2 (\Phi_1^\dagger \Phi_2 + \text{h.c.}) \\ + \frac{1}{2} \lambda_1 |\Phi_1|^4 + \frac{1}{2} \lambda_2 |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{1}{2} \lambda_5 [(\Phi_1^\dagger \Phi_2)^2 + \text{h.c.}].$$

- Softly broken Z_2 symmetry is imposed.

$$\Phi_1 \rightarrow \Phi_1, \quad \Phi_2 \rightarrow -\Phi_2$$

- Component fields and physical states

$$\Phi_i = \begin{bmatrix} w_i^+ \\ \frac{1}{\sqrt{2}}(v_i + h_i + iz_i) \end{bmatrix}, \quad (i = 1, 2),$$

3 of 8 d.o.f are G^0, G^\pm

$$\begin{pmatrix} w_1^\pm \\ w_2^\pm \end{pmatrix} = R(\beta) \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}, \quad \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = R(\beta) \begin{pmatrix} G^0 \\ A \end{pmatrix},$$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R(\alpha) \begin{pmatrix} H \\ h \end{pmatrix}, \quad R(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

h : SM-like Higgs boson

H, A, H^\pm : Heavy Higgs bosons

Two Higgs doublet models (2HDMs) [2/2]

- Model parameters

$$m_{1-3}, \lambda_{1-5}$$



$$m_{H,A,H^\pm}, M^2, \sin(\beta - \alpha), \tan \beta, \\ v (= 246\text{GeV}), m_h (= 125\text{GeV})$$

$$M^2 = \frac{m_3^2}{\cos \beta \sin \beta}$$

- Yukawa sector

- Due to Z_2 symmetry, either Φ_1 or Φ_2 couple with each matter field.
 - Flavor-changing neutral current (FCNC) is forbidden at the tree level.
- There are 4 types of interactions.

$$-\mathcal{L}_Y = Y_u \bar{Q}_L i\sigma_2 \Phi_u^* u_R + Y_d \bar{Q}_L \Phi_d d_R + Y_e \bar{L}_L \Phi_e e_R + \text{h.c.}$$

	Φ_u	Φ_d	Φ_e	
Type I	Φ_2	Φ_2	Φ_2	Light H^\pm scenario
Type II	Φ_2	Φ_1	Φ_1	MSSM
Type X	Φ_2	Φ_2	Φ_1	Radiative seesaw models
Type Y	Φ_2	Φ_1	Φ_2	

Decoupling limit and alignment limit

Alignment limit: $\alpha \rightarrow \beta - \pi/2$

- All scalars are diagonalized by β : $\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = R(\beta) \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \ni h$
 $\ni H, A, H^\pm$
- Higgs boson couplings coincide with the SM:

$$\begin{cases} \kappa_V = \sin(\beta - \alpha) \rightarrow 1 \\ \kappa_f = \sin(\beta - \alpha) + \zeta_f \cos(\beta - \alpha) \rightarrow 1 \end{cases}$$

Decoupling limit: $M^2 \rightarrow \infty$

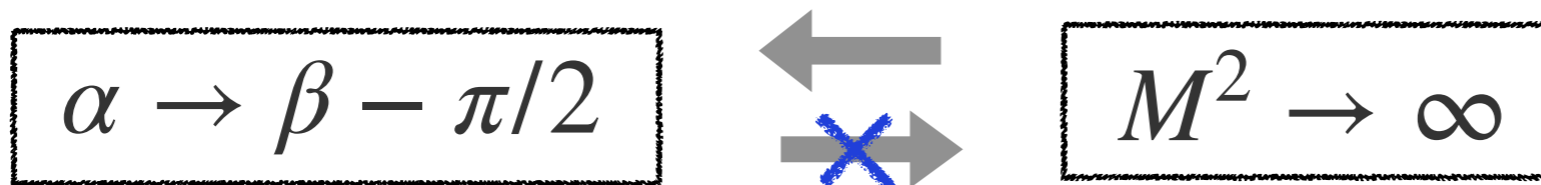
- Heavy Higgs masses is governed by M^2 : $m_\Phi^2 \sim M^2 + f(\lambda_i)v^2$
- All effects of heavy Higgs is suppressed by $1/M^2$.

The relation between α and M

$$\tan 2(\beta - \alpha) = \frac{\sum_i c_i \lambda_i v^2}{\sum_i c_i \lambda_i v^2 + M^2} \xrightarrow{M^2 \rightarrow \infty} 0$$

- The decoupling limit leads to the alignment limit (if λ_i perturbative).

- The opposite is not true : $0 \stackrel{\text{Alignment limit}}{=} \sum_i c_i \lambda_i v^2$ This doesn't depend on M .



In 2HDM, alignment without decoupling scenario is possible.

counterexample: MSSM

$\alpha \rightarrow \beta - \pi/2$

\longleftrightarrow

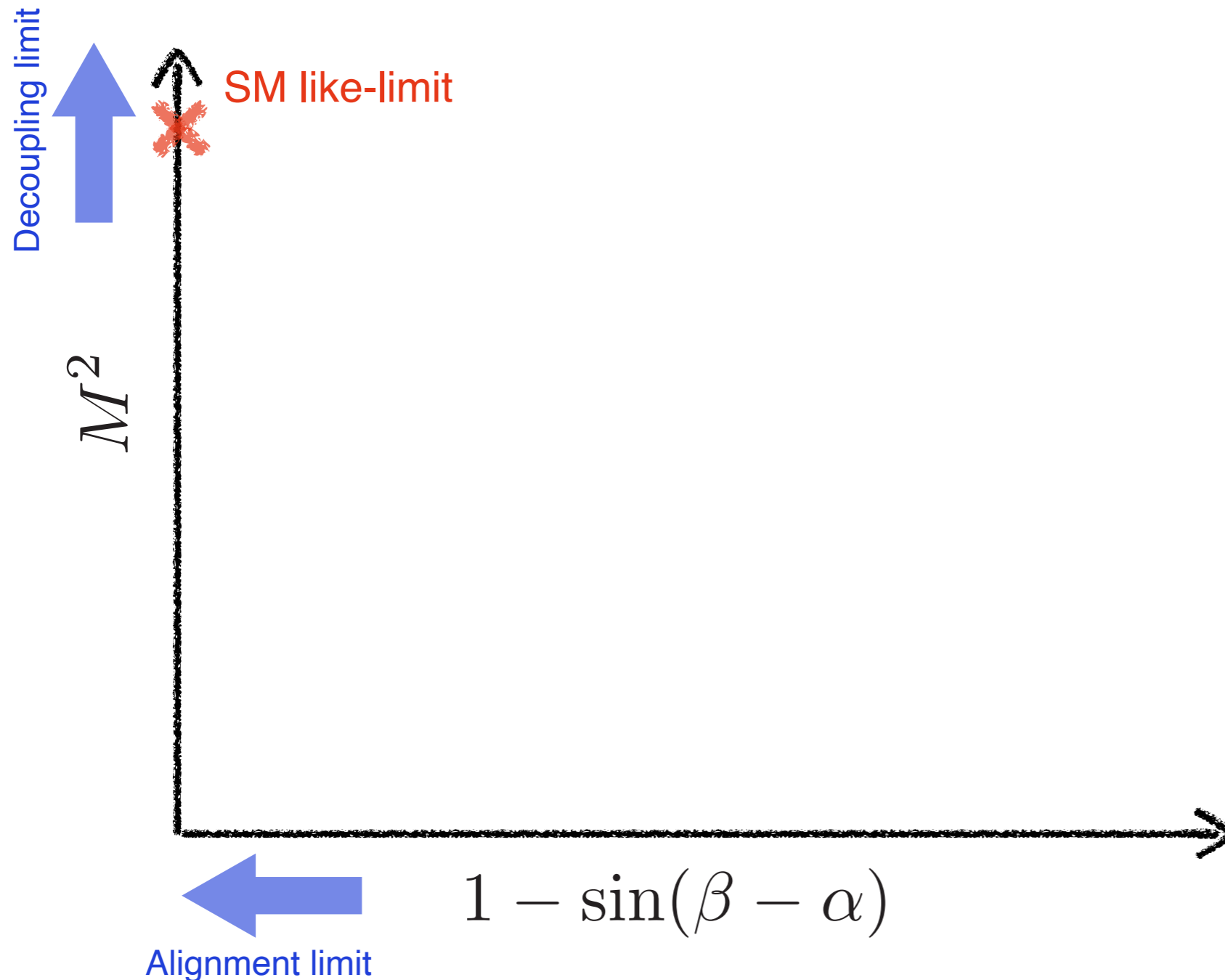
$m_A^2 \rightarrow \infty$

$$\frac{\tan 2\alpha}{\tan 2\beta} = \frac{m_H^2/m_A^2 + m_h^2/m_A^2}{1 + m_Z^2/m_A^2}$$

$$m_H^2 = m_A^2 + m_Z^2(1 - \cos^2 2\beta)$$

Distinct scenarios [1/3]

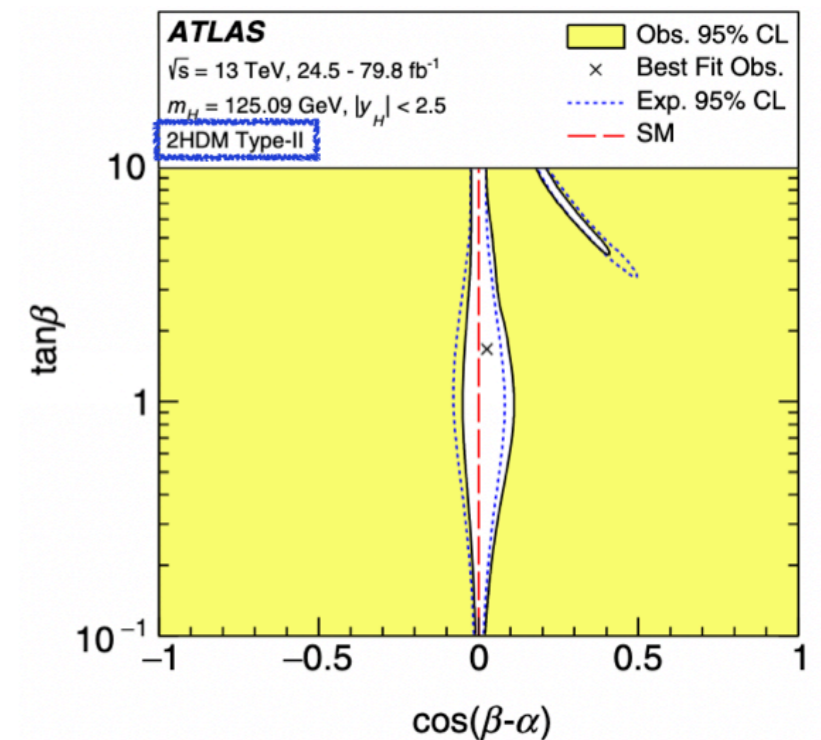
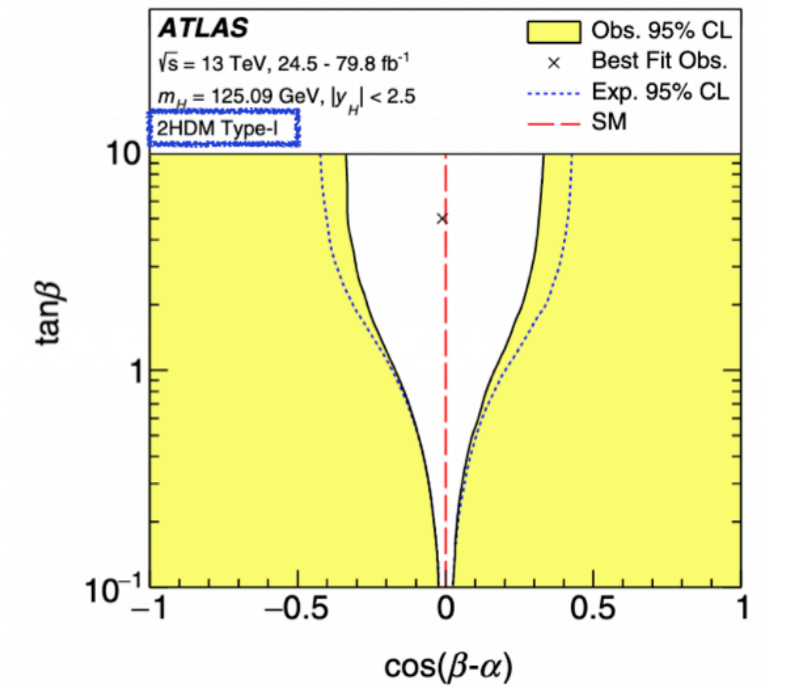
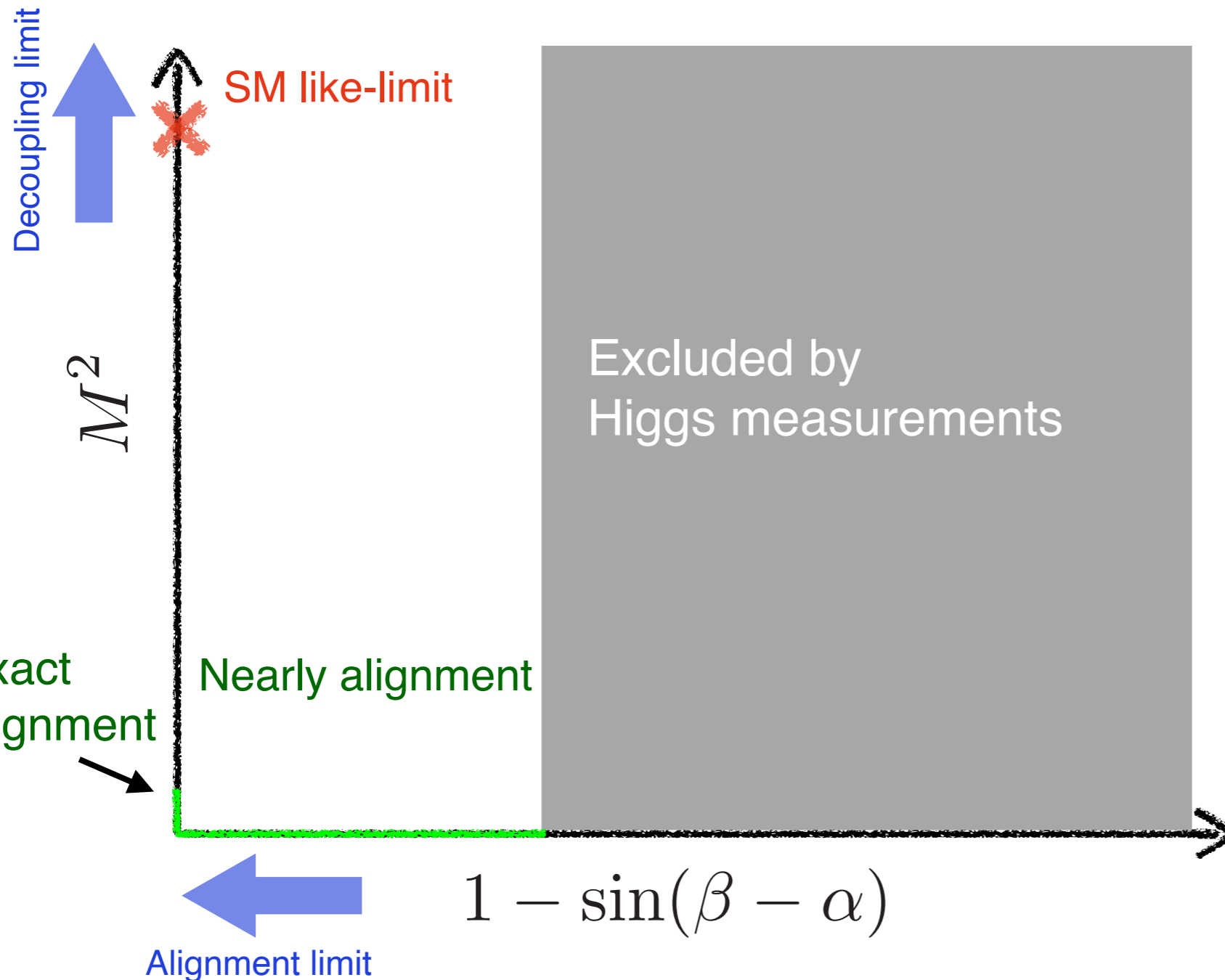
We can describe the model parameter space by the decoupling parameter and the alignment parameter.



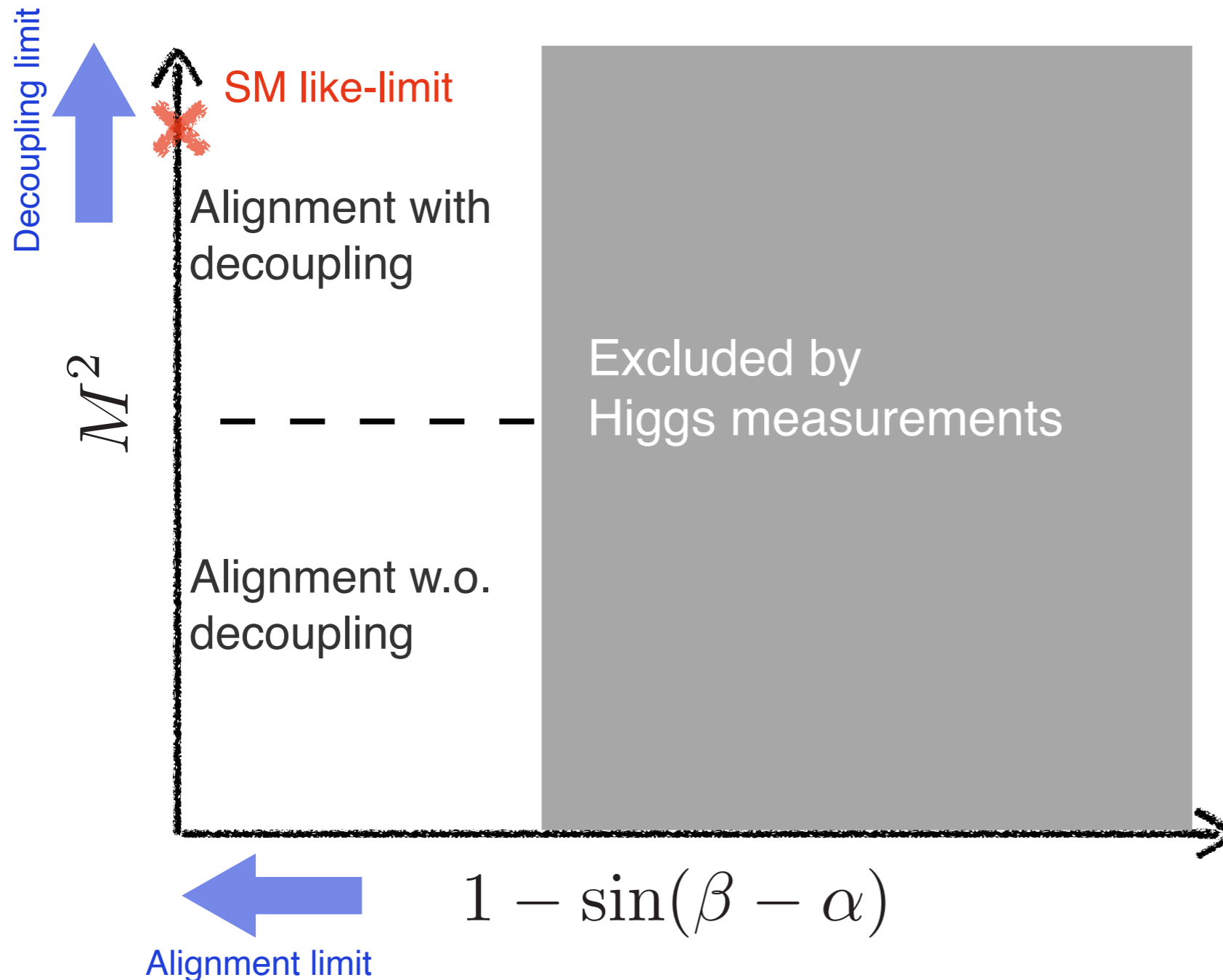
Distinct scenarios [2/3]

Exact alignment or nearly alignment scenarios are favored.

[ATLAS collaboration, PRD 101, 012002 (2020)]



Distinct scenarios [3/3]



Alignment with decoupling

- It is difficult to test.

Alignment w.o. decoupling

- Deviations in the HCs.
- Strong 1st EWPT
- Gravitational wave

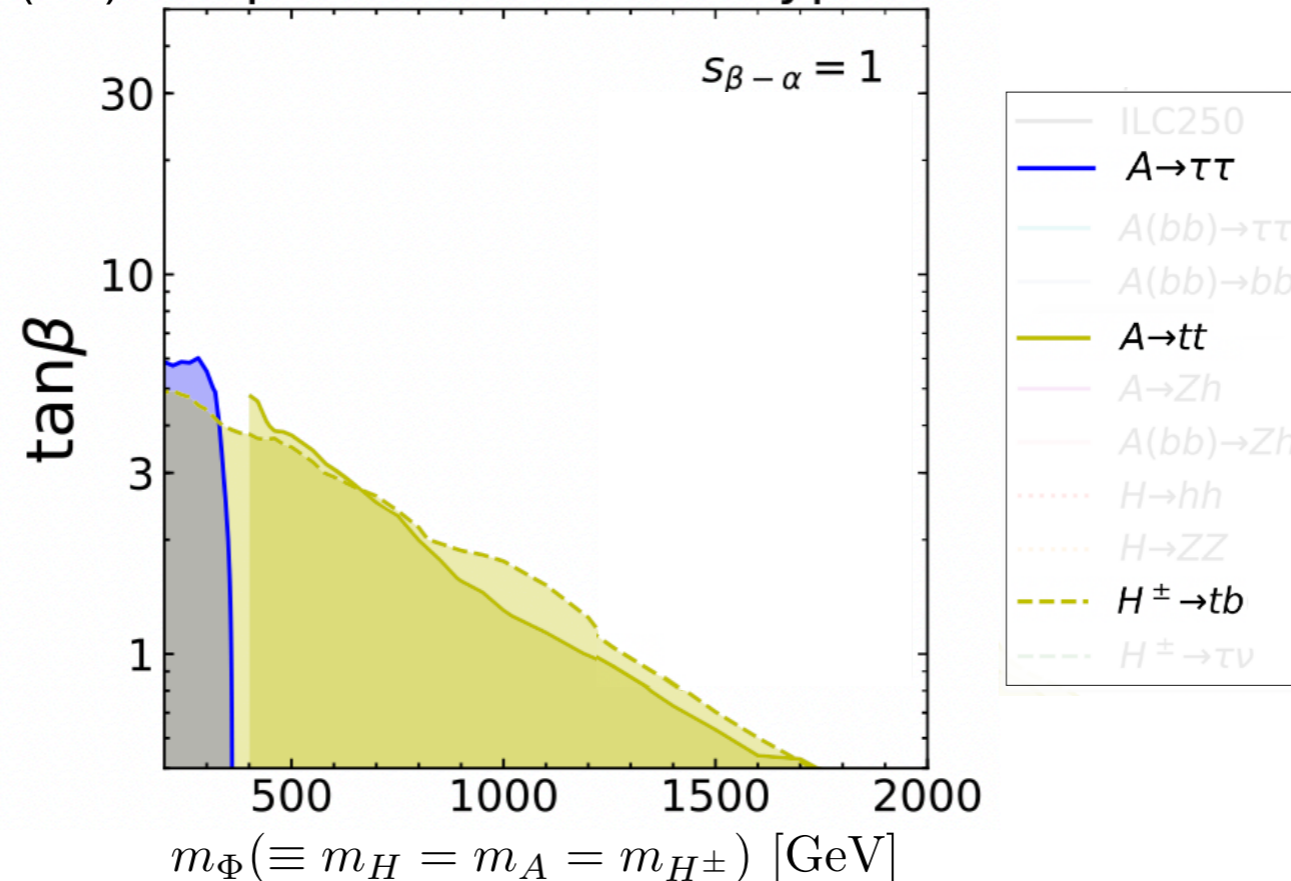
Intriguing scenario. The testability should be studied.

Synergy between direct and indirect searches[1/2]

Exact alignment scenario: $\sin(\beta - \alpha) = 1$

[M. Aiko, S. Kanemura, M. Kikuchi, K. Mawatari, KS, K. Yagyu, NPB 966 (2021) 115375]

(Ex.): Expected exclusion; Type-I at HL-LHC



$$\Gamma(A \rightarrow f\bar{f}) \propto \frac{m_A}{\tan^2 \beta}$$

$$\Gamma(H^\pm \rightarrow t\bar{b}) \propto \frac{m_{H^\pm}}{\tan^2 \beta}$$

Direct searches : Lower bounds for m_Φ and $\tan \beta$ are given.

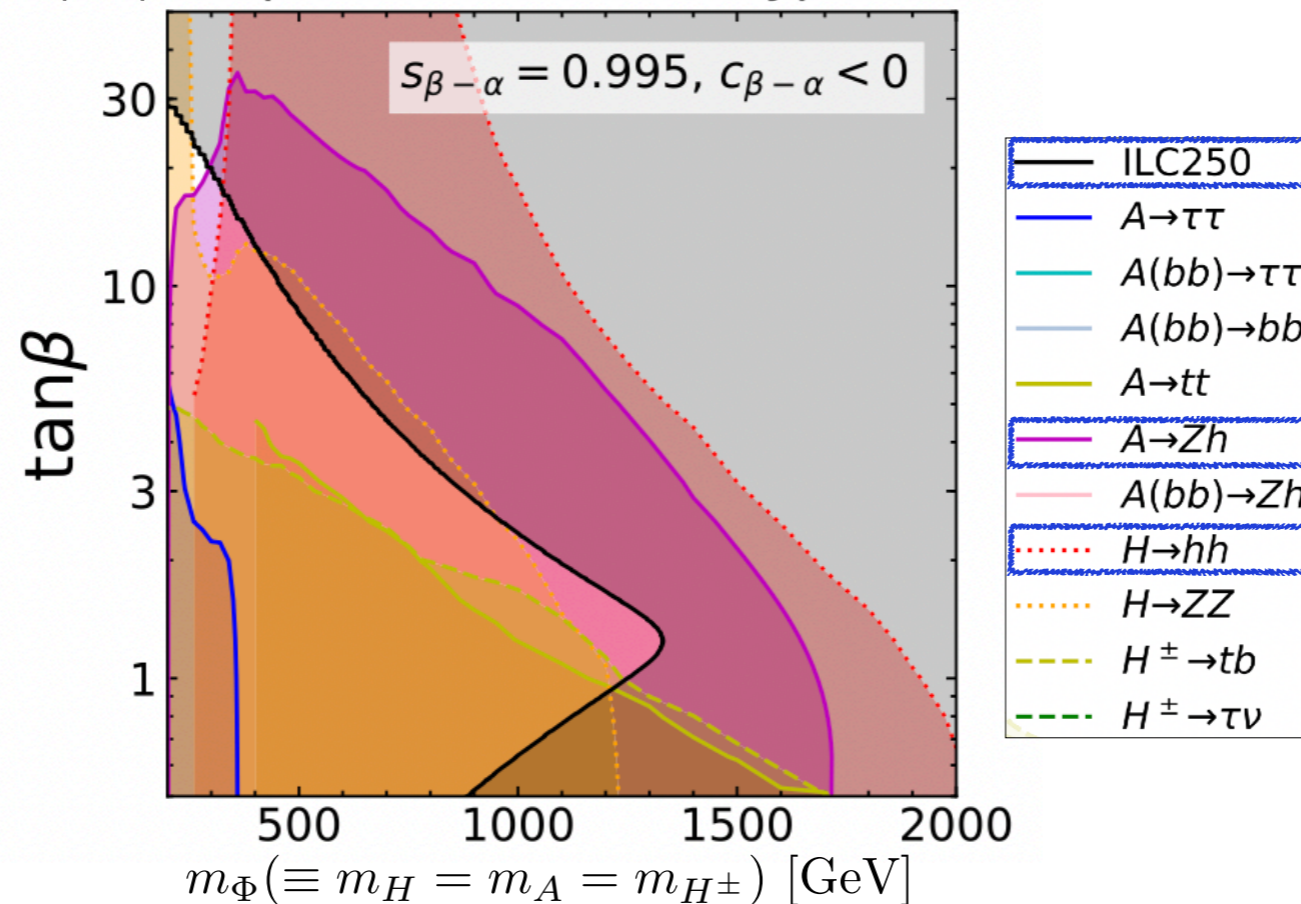
Indirect searches : No sensitivity since Higgs couplings do not deviate.

Synergy between direct and indirect searches [2/2]

Near alignment scenario: $\sin(\beta - \alpha) = 0.995$ ($\cos(\beta - \alpha) \simeq -0.1$)

[M. Aiko, S. Kanemura, M. Kikuchi, K. Mawatari, KS, K. Yagyu, NPB 966 (2021) 115375]

(Ex.): Expected exclusion; Type-I at HL-LHC



$$\kappa_Z = \sin(\beta - \alpha)$$

$$\Gamma(A \rightarrow Zh) \propto \cos(\beta - \alpha)^2 \frac{m_A^3}{16\pi v^2}$$

$$\Gamma(H \rightarrow hh) \sim \cos(\beta - \alpha)^2 \frac{m_H^3}{16\pi v^2}$$

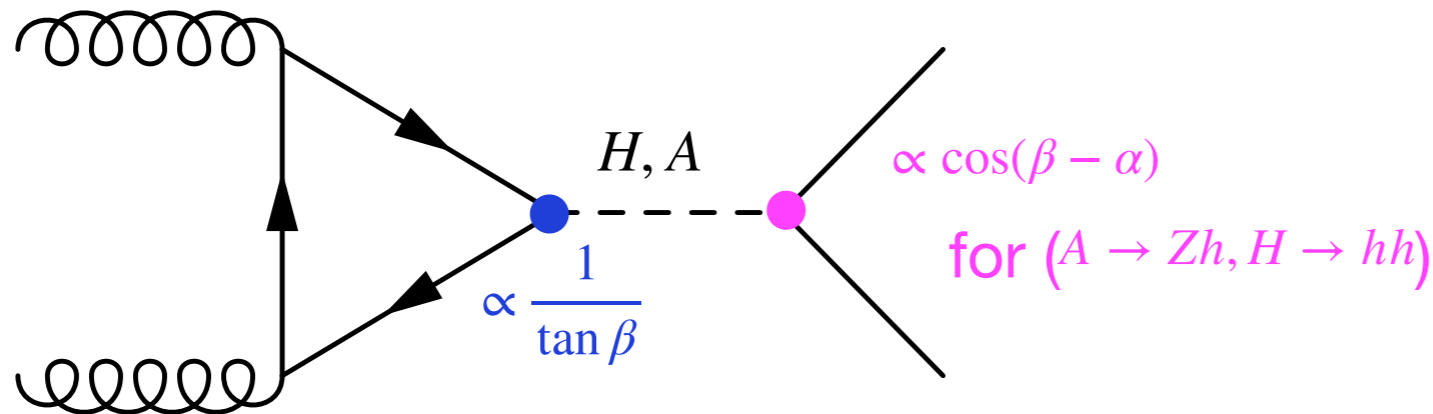
Direct searches: $A \rightarrow Zh$ and $H \rightarrow hh$ give wider sensitivity regions for $(m_\Phi, \tan\beta)$ plane.

Indirect searches: If a deviation in hZZ founds, the upper bounds for m_Φ are given.

→ Most parameter space can be surveyed by the combination of scalar-to-scalar decays and precision measurements of the Higgs coupling.

Importance of NLO corrections to heavy Higgs

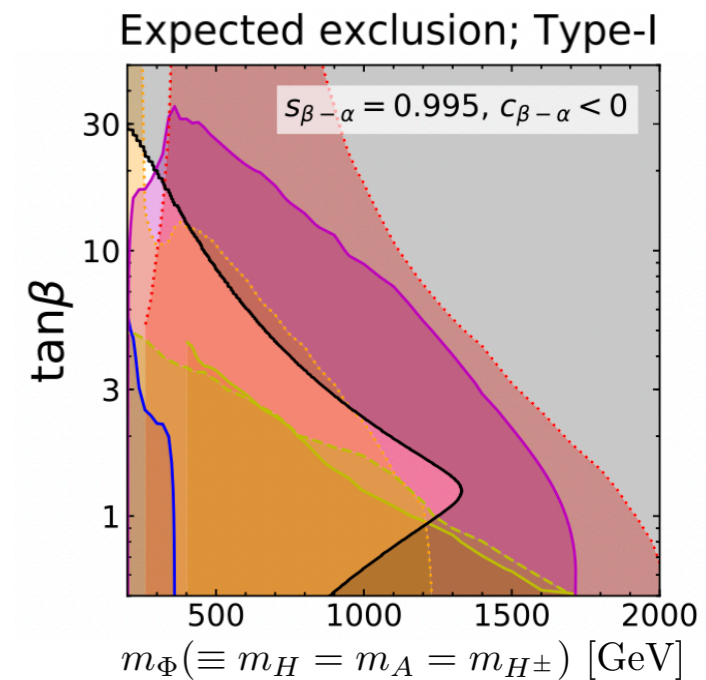
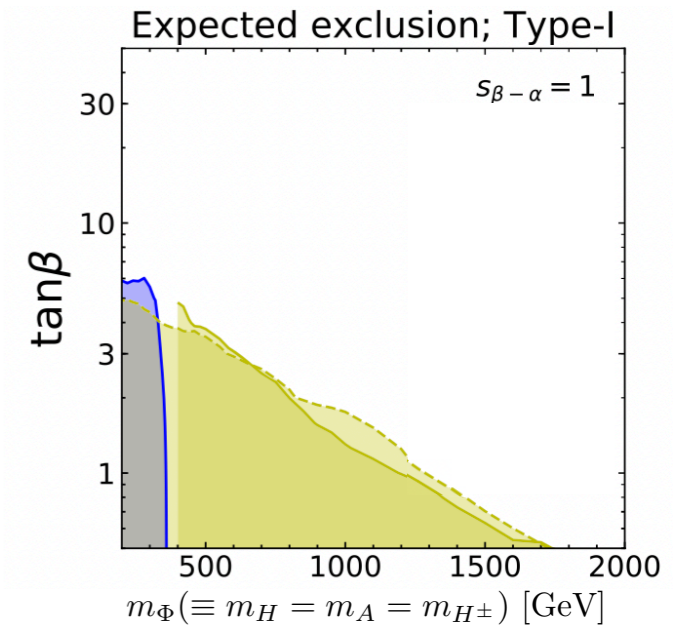
Sensitivity regions by direct searches are drastically changed by $\sin(\beta - \alpha)$, especially for BRs.



→ The loop effect on heavy Higgs decays can be significant.

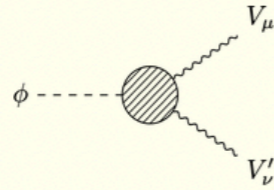
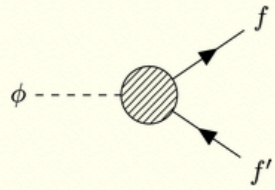
$$\Gamma_{S \rightarrow SV}^{\text{NLO}} = \tilde{\Gamma}^{\text{LO}} (c_{\beta-\alpha}^2 + c_{\beta-\alpha} \Delta^{\text{NLO}})$$

$$\Delta^{\text{NLO}} > c_{\beta-\alpha} ?$$

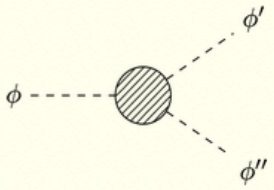
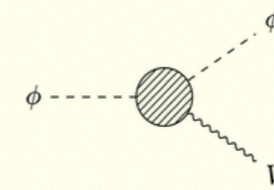


Our interest : impact of NLO corrections on heavy Higgs boson decays.

Correlation between decays of heavy Higgs and $h(125)$.



H-COUP



- Fortran program to evaluate the NLO EW corrections and (N)NLO QCD corrections to various Higgs decays.
- Outputs (EW correction: On-shell scheme, QCD correction:MS scheme)
 - h : On-shell 2-body decays (e.g., $h \rightarrow \bar{f}f$)
 - Off-shell 3-body decays (e.g., $h \rightarrow ZZ^* \rightarrow Z\bar{f}f$)
 - H, A, H^\pm : On-shell 2-body decays (e.g., $A \rightarrow \bar{f}f, Zh$)
- Model
 - Two Higgs doublet models (Type I, Type II, Type X, Type Y)
 - Inert doublet model
 - Higgs singlet model (without global symmetry)

Other public tools

- **2HDECAY, sHDECAY, NHDECAY** [M. Krause, M. Mühlleitner, M. Spira, CPC 246 (2020) 106852], [M. Krause, M. Mühlleitner, 1904.02103] [F. Egle, M. Mühlleitner, R. Santos, J. Viana, JHEP 11 (2023) 116]
 - 2-body decays of all Higgs boson with NLO EW and the state of the art QCD corrections.
 - Model: THDMs (2HDECAY), the Singlet extension of the SM with Z_2 (sHDECAY), THDMs + real singlet (NHDECAY)
- **PROPHECY4F** [L. Altenkamp, S. Dittmaier, H. Rzehak, JHEP 1803 (2018) 110]
 - CP-even Higgs decays into 4fermions with NLO EW and QCD corrections.
 - Model: THDMs, the Singlet extension of the SM with Z_2 .
- **Flexibledecay** [P. Athron, A. Büchner, et. al., Comput.Phys.Commun. 283, (2023) 108584]
 - 2-body decays of neutral Higgs bosons with SM QCD/EW corrections in HSM, Type II 2HDM, CMSSM, MRMSSM.
 - BSM effect is at LO.

Numerical analysis

h	H	A	H^\pm
$h \rightarrow ff$	$H \rightarrow ff$	$A \rightarrow ff$	$H^\pm \rightarrow ff'$
$h \rightarrow ZZ^*$	$H \rightarrow ZZ, WW$	$A \rightarrow ZZ, WW$	$H^\pm \rightarrow Wh$
$h \rightarrow WW^*$	$H \rightarrow hh$	$A \rightarrow Zh$	$H^\pm \rightarrow WH, WA$
$h \rightarrow gg$	$H \rightarrow ZA, WH^\pm$	$A \rightarrow HZ, WH^\pm$	$H^\pm \rightarrow W\gamma, WZ$
$h \rightarrow \gamma\gamma, Z\gamma$	$H \rightarrow gg$	$A \rightarrow gg$	
	$H \rightarrow \gamma\gamma, Z\gamma$	$A \rightarrow \gamma\gamma, Z\gamma$	

- We chose $A \rightarrow Zh$ and $h \rightarrow ZZ^*$ to illustrate the NLO corrections.

$A \rightarrow Zh$: Suppression of $\Gamma_{A \rightarrow Zh}^{\text{LO}}$ $h \rightarrow ZZ^*$: precise measurement in future colliders (e.g. 0.76% @ILC)

[1710.07621]

- Benchmark scenarios for alignment w.o. decoupling ($|c_{\beta-\alpha}| < 0.1$)

(I) $m_A = m_{H^\pm} = 300\text{GeV}$ in Type I 2HDM

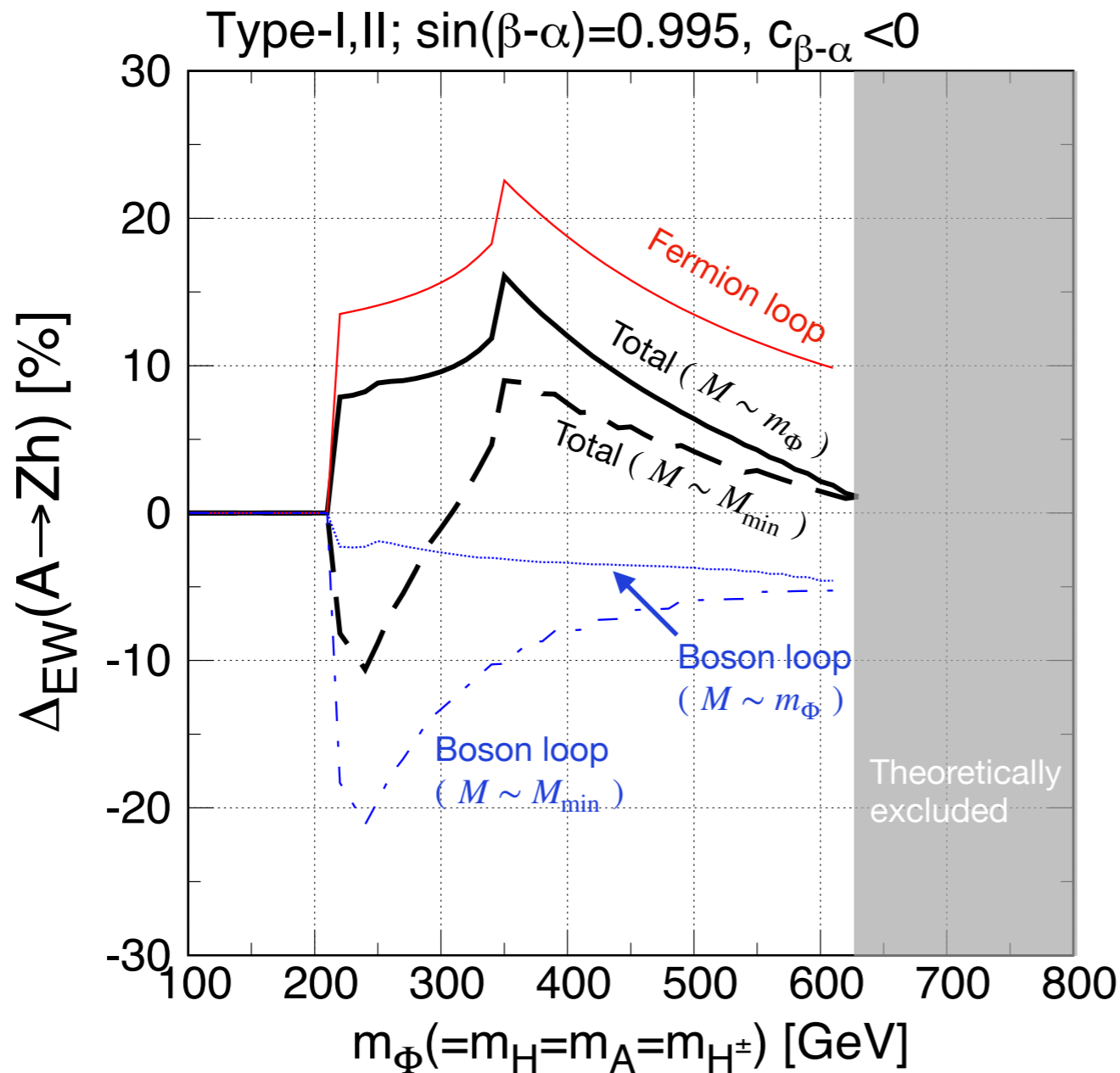
The scenario can be explored by HL-LHC.

(II) $m_A = m_{H^\pm} = 800\text{GeV}$ in Type I,II 2HDMs

A realistic scenario for Type II due to $B \rightarrow X_s \gamma$ ($m_{H^\pm} \gtrsim 600\text{GeV}$).

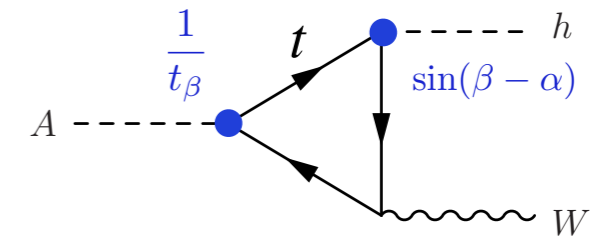
Non-decoupling effects in $\Gamma_{A \rightarrow Zh}$

[M. Aiko, S. Kanemura, KS]



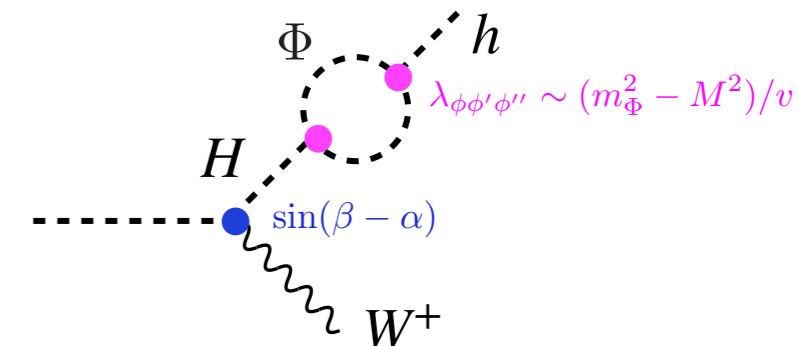
Typical graph :

fermion loop Suppression by t_β, m_Φ^2



$$\mathcal{M}_{A \rightarrow Zh}^F \sim -\frac{1}{16\pi^2} \frac{s_{\beta-\alpha}}{t_\beta} \frac{m_t^4}{v^2 m_\Phi^2} \quad (m_t \ll m_\Phi)$$

Boson loop Nondecoupling effects



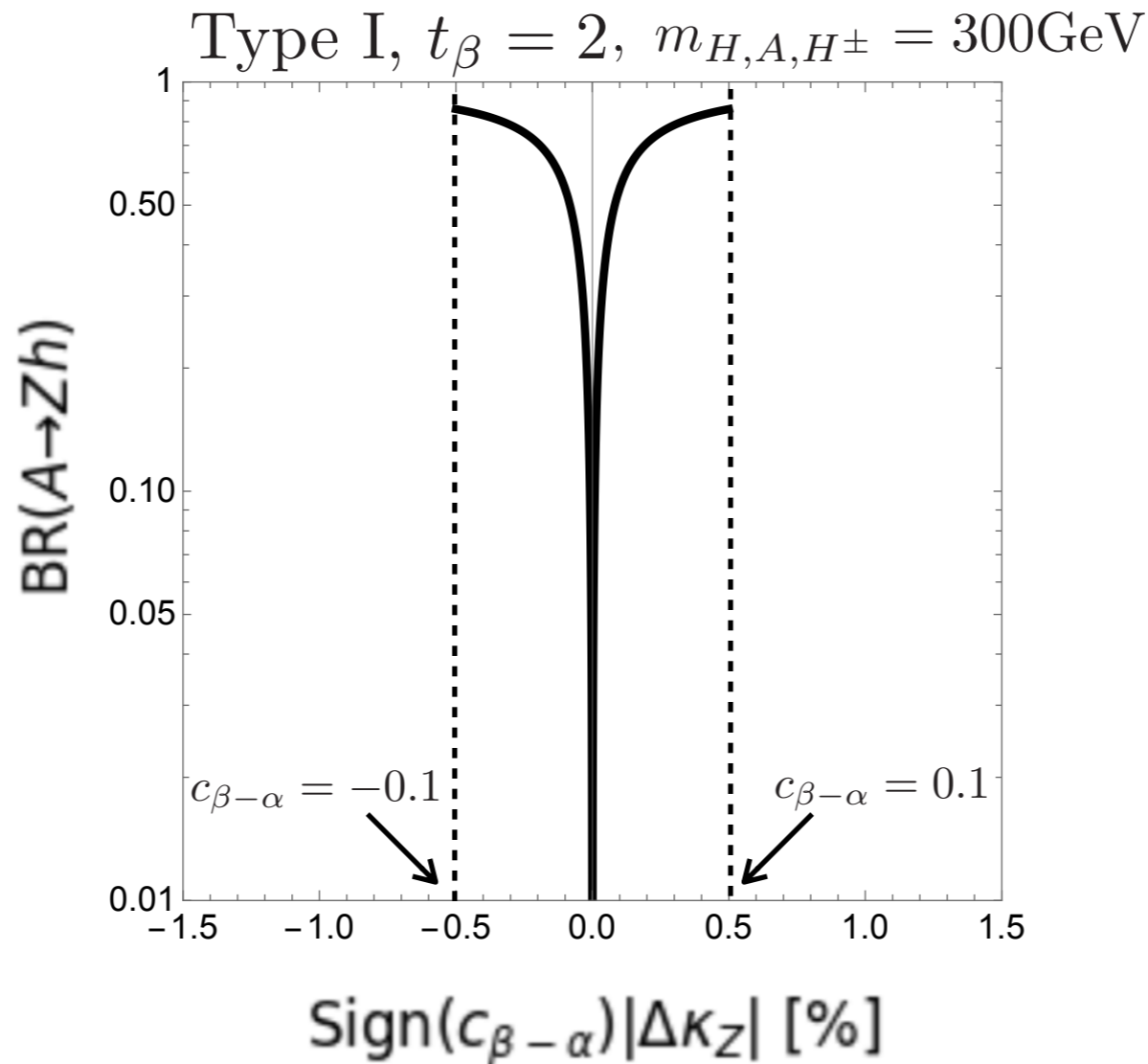
$$\mathcal{M}_{A \rightarrow Zh}^B \sim \begin{cases} \frac{1}{16\pi^2} s_{\beta-\alpha} \frac{m_\Phi^2}{v^2} & (M \sim v) \\ \frac{1}{16\pi^2} s_{\beta-\alpha} \frac{m_h^4}{v^2 m_\Phi^2} & (M \gg v) \end{cases}$$

- Some diagrams are not suppressed by $c_{\beta-\alpha}$.
- Fermion loop and Boson loop are destructive.

→ Total corrections reach ~15%.

Scenario I: BR(A → Zh) vs Δκ_Z at LO

[M. Aiko, S. Kanemura, KS]



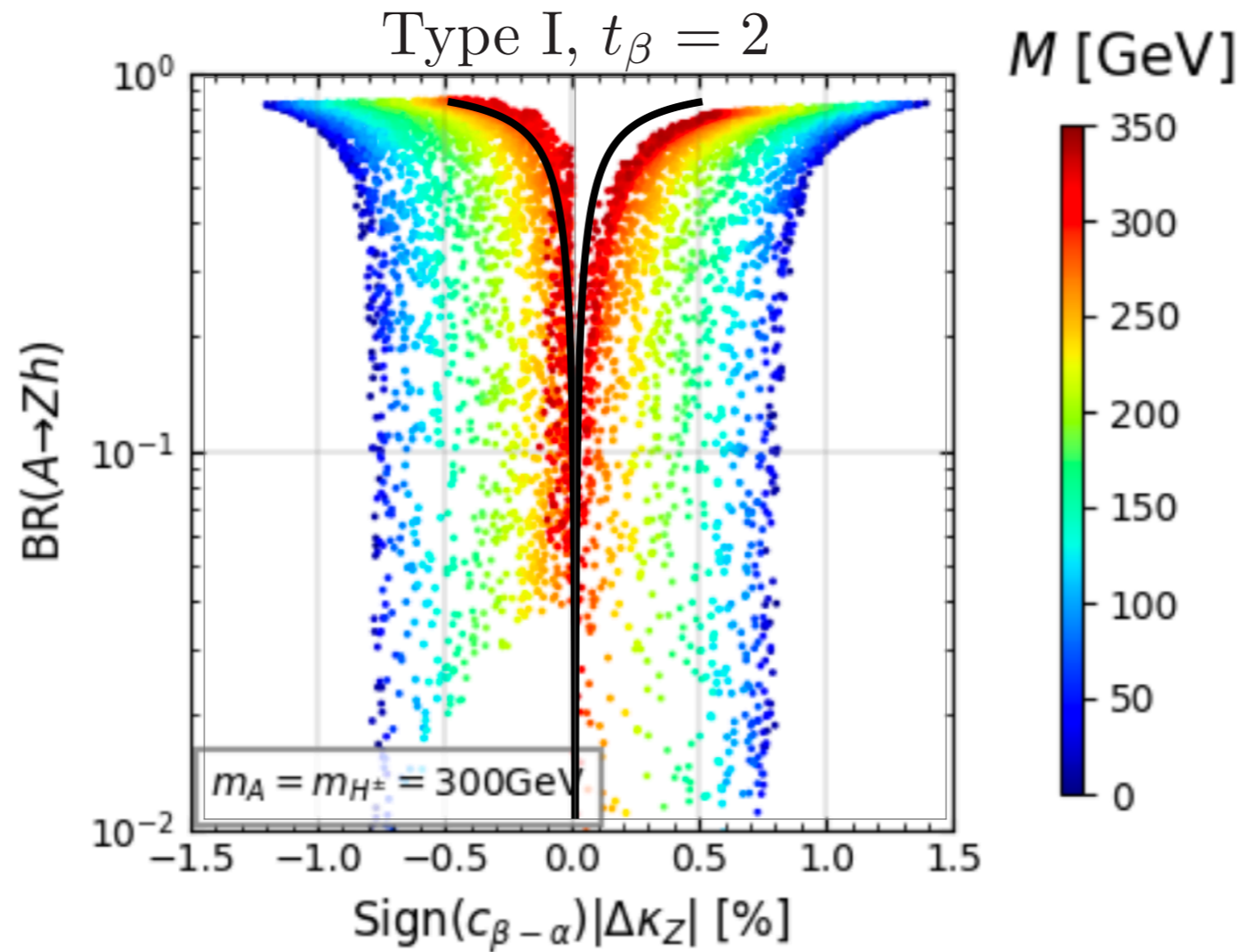
$$\Delta\kappa_Z = \sqrt{\frac{\Gamma_{h \rightarrow ZZ^*}^{2\text{HDM}}}{\Gamma_{h \rightarrow ZZ^*}^{\text{SM}}}} - 1$$

- $\Delta\kappa_Z$ and $\text{BR}(A \rightarrow Zh)$ are governed by $c_{\beta-\alpha}^2$. Positive correlation.

$$\Delta\kappa_Z = s_{\beta-\alpha} - 1 \simeq -c_{\beta-\alpha}^2/2, \quad \text{BR}(A \rightarrow Zh) \simeq c_{\beta-\alpha}^2 \frac{m_A^3}{16\pi\Gamma_{\text{tot}}^A v^2}$$

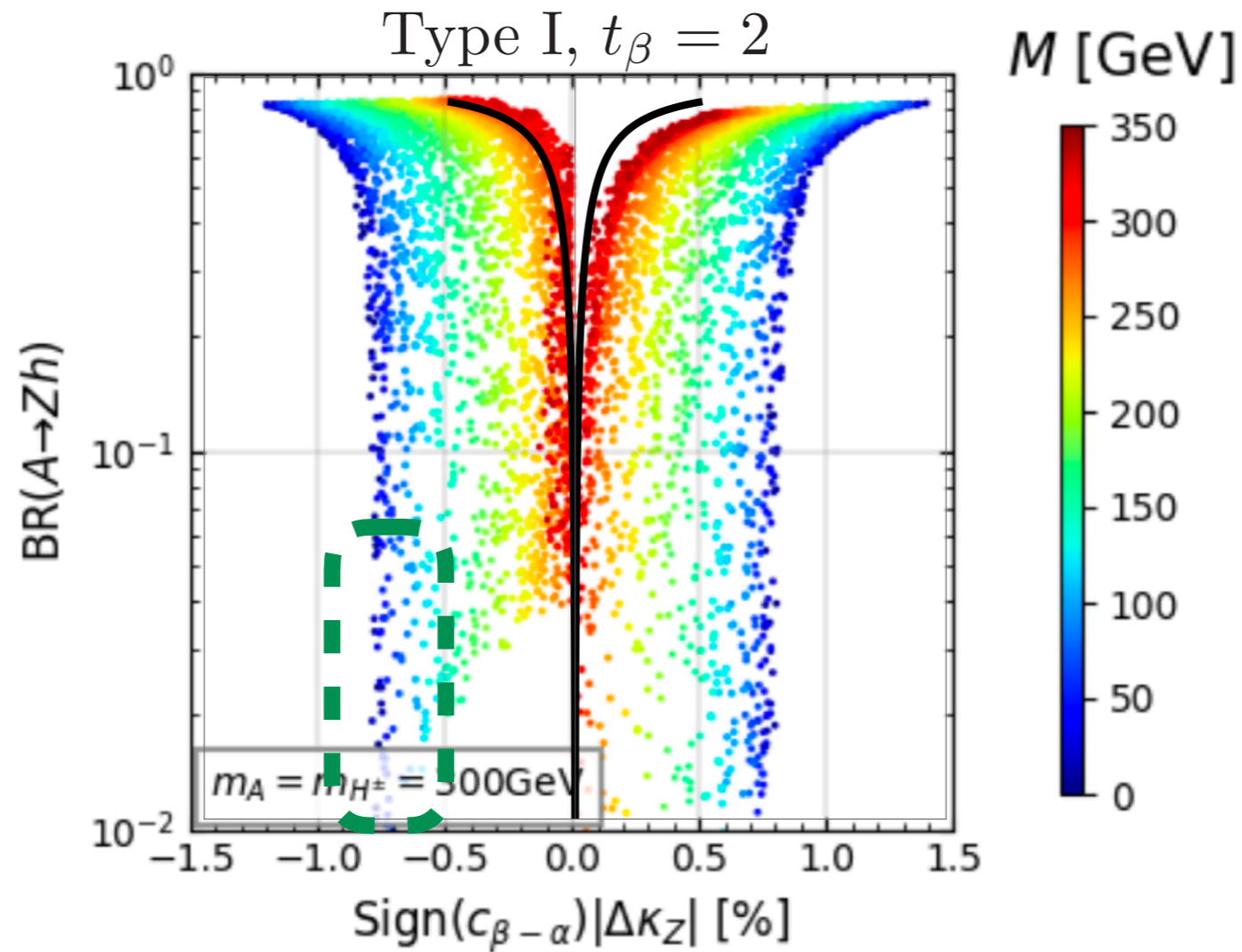
- The maximum of $\Delta\kappa_Z$ is 0.5% at LO.

Scenario I: BR(A → Zh) vs Δκ_Z at NLO [M. Aiko, S. Kanemura, KS]



$$\Delta\kappa_Z = \sqrt{\frac{\Gamma_{h \rightarrow ZZ^*}^{2\text{HDM}}}{\Gamma_{h \rightarrow ZZ^*}^{\text{SM}}} - 1}$$

Scenario I: BR(A → Zh) vs Δκ_ν at NLO [M. Aiko, S. Kanemura, KS]

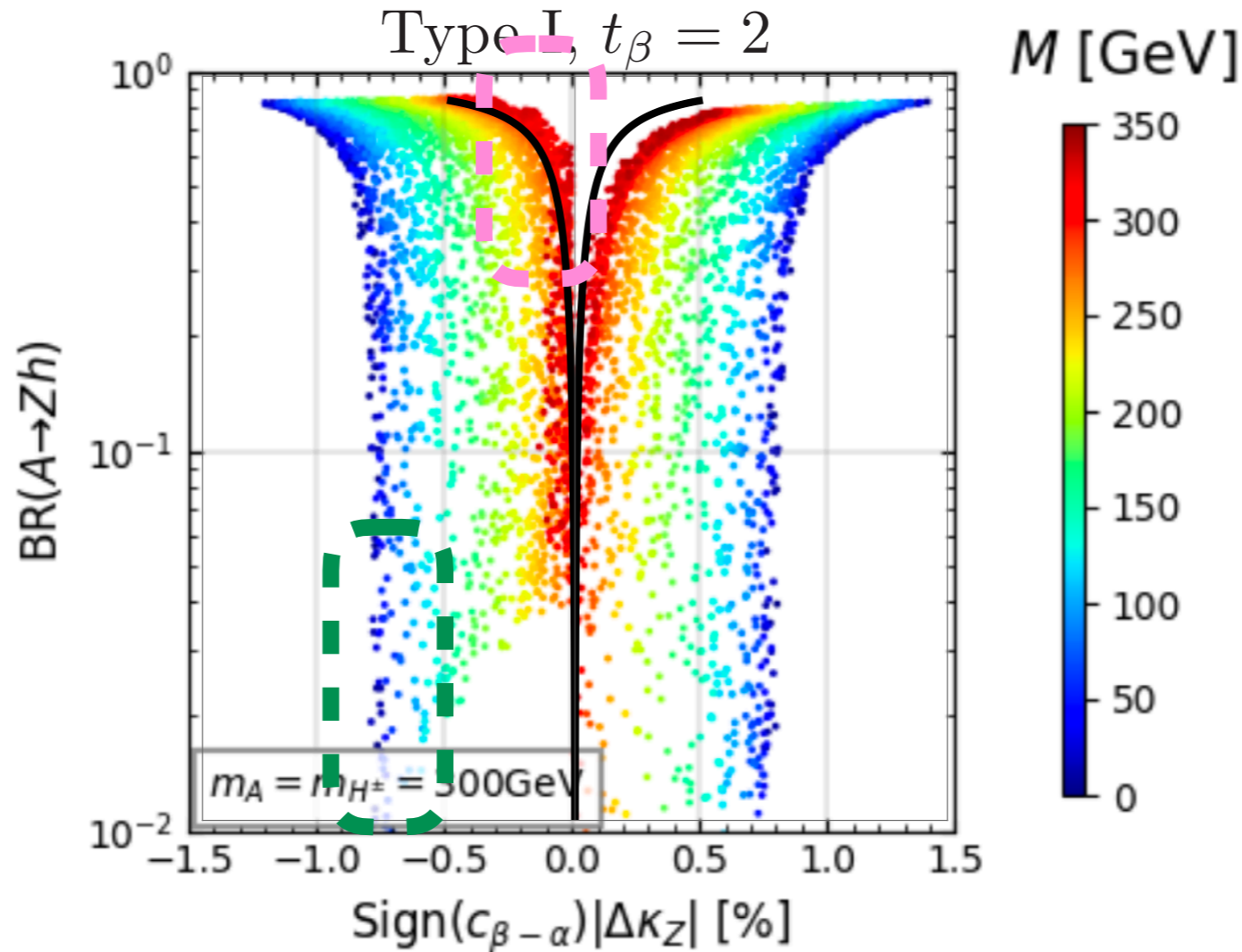


$$\Delta\kappa_Z = \sqrt{\frac{\Gamma_{h \rightarrow ZZ^*}^{\text{2HDM}}}{\Gamma_{h \rightarrow ZZ^*}^{\text{SM}}} - 1}$$

$M^2 \simeq 0, c_{\beta-\alpha} \simeq 0$: Nondecoupling effect of H, A, H^\pm enhances $\Delta\kappa_Z \rightarrow \Delta\kappa_Z \neq 0$ but BR ~ 1%

Scenario I: BR(A → Zh) vs Δκ_Z at NLO

[M. Aiko, S. Kanemura, KS]

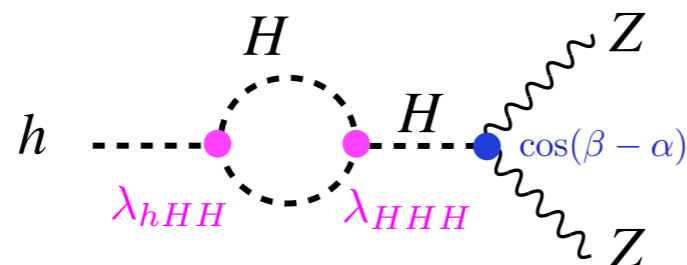


$$\Delta\kappa_Z = \sqrt{\frac{\Gamma_{h \rightarrow ZZ^*}^{2\text{HDM}}}{\Gamma_{h \rightarrow ZZ^*}^{\text{SM}}} - 1}$$

$M^2 \simeq 0, c_{\beta-\alpha} \simeq 0$: Nondecoupling effect of H,A,H[±] enhances Δκ_Z → Δκ_Z ≠ 0 but BR~1%

$M^2 \simeq m_A^2, |c_{\beta-\alpha}| \sim 0.1$: Nondecoupling effect of H can affect → Δκ_Z ~ 0 but BR~O(10)%

Dominant diagrams :
for h → ZZ*

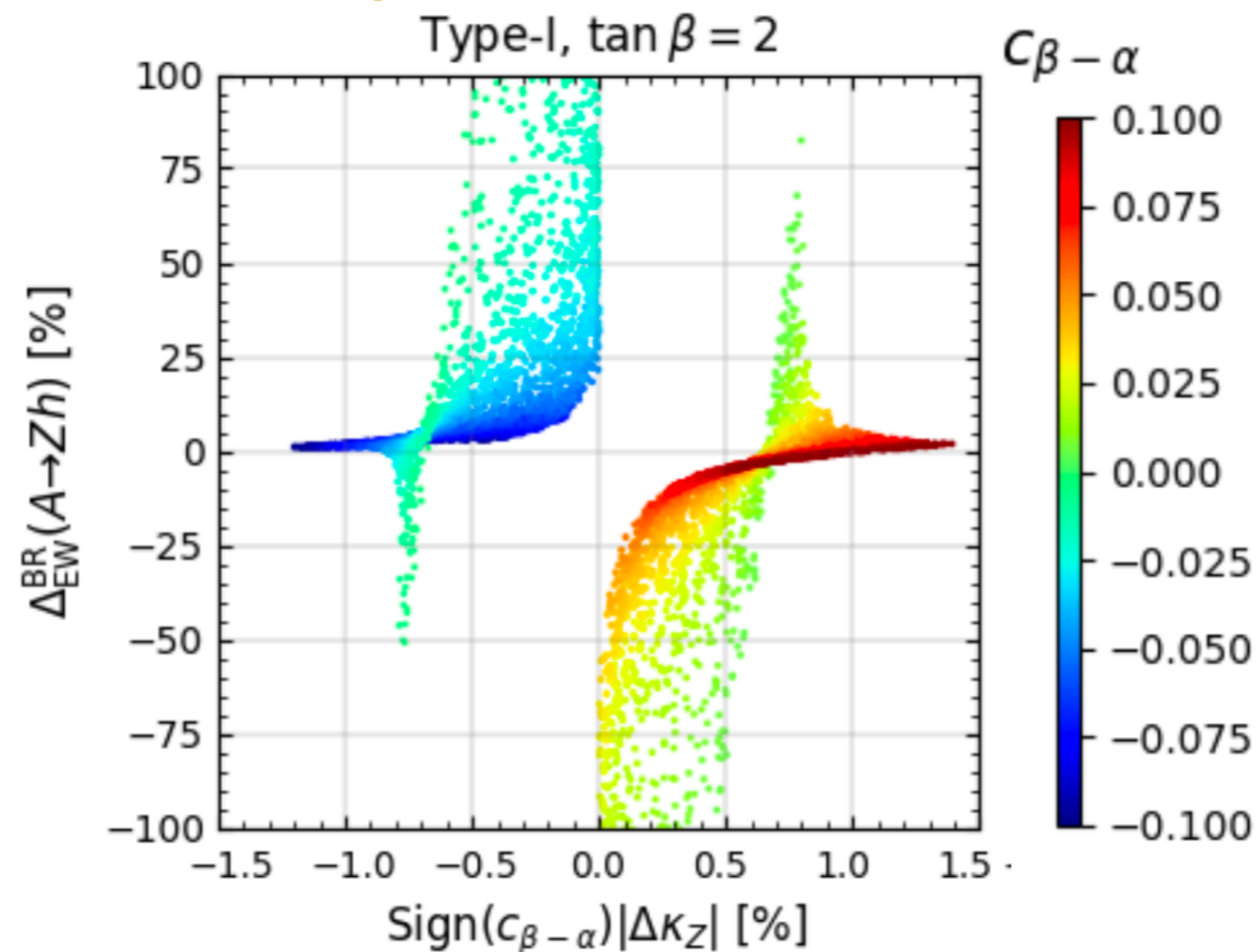


$$\sim \frac{1}{16\pi^2} (m_H^2 - M^2)^2 c_{\beta-\alpha}$$

This compensates tree level contributions in Δκ_Z.

Scenario I: NLO corrections for BR(A → Zh)

[M. Aiko, S. Kanemura, KS]



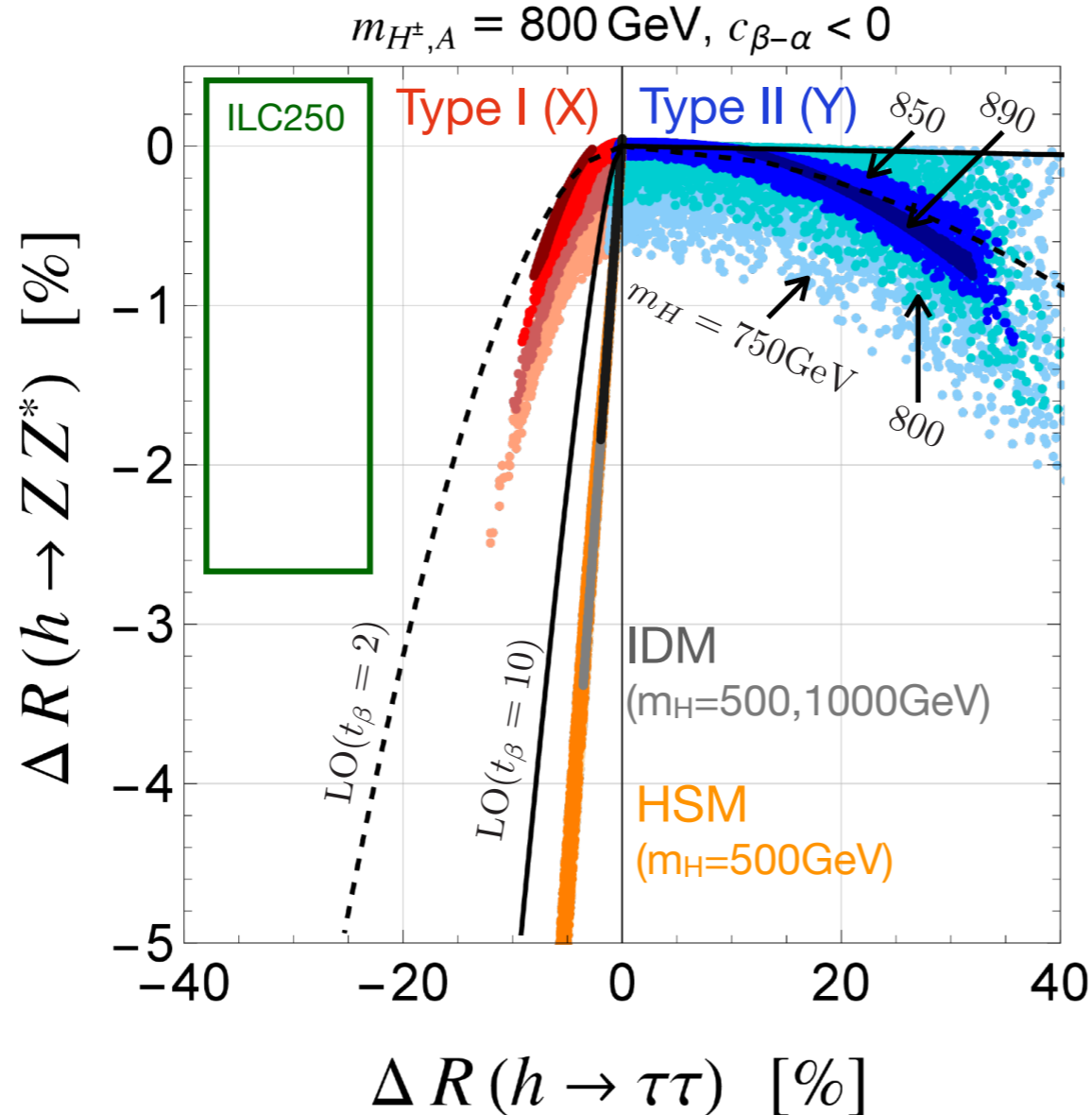
$|c_{\beta-\alpha}| \sim 0.1$: Δ^{BR} is close to 0%

$$\Delta_{\text{EW}}^{\text{BR}} = \frac{1 + \Delta_{A \rightarrow Zh}^{\text{EW}}}{1 + \underbrace{\Delta_{\text{tot}}^{\text{EW}}}_{\sim \Delta_{A \rightarrow Zh}^{\text{EW}}}} - 1 \simeq 0$$

$|\Delta\kappa_Z| \lesssim 0.5\%$: Δ^{BR} can exceed 100%

$$|\mathcal{M}(A \rightarrow Zh)|^2 = C_{AZh} \left(\underbrace{\frac{g_Z^2}{4} c_{\beta-\alpha}^2}_{\text{Tree}} + \underbrace{g_Z c_{\beta-\alpha} \text{Re}\Gamma_{AZh}^{\text{loop}} + |\Gamma_{AZh}^{\text{loop}}|^2}_{\text{1-loop}} \right)$$

Scenario II: Correlation in $h(125)$ decays



$$\Delta R_Z = \frac{\Gamma_{h \rightarrow ZZ^*}^{2\text{HDM}}}{\Gamma_{h \rightarrow ZZ^*}^{\text{SM}}} - 1$$

- HSM, IDM: $\Delta R_{h \rightarrow \tau\tau} \simeq \Delta R_{h \rightarrow ZZ}$, 2HDMs: $|\Delta R_{h \rightarrow \tau\tau}| \gg \Delta R_{h \rightarrow ZZ}$
 - The models can be distinguished from the difference in the gradients
- If $m_A - m_H \simeq 50 \text{ GeV}$, the loop effect of A, H^\pm make $\Delta R_{h \rightarrow ZZ^*}$ small.

Summary

- The tree-level contributions to scalar-to-scalar decays ($A \rightarrow Zh$ and $H \rightarrow hh$) are suppressed by the scalar mixing $c_{\beta-\alpha}$.
- We evaluated the NLO corrections by using H-COUP.
- We investigated the impact of NLO corrections to $A \rightarrow Zh$ and $h \rightarrow ZZ^*$.
- We found that the NLO corrections to $A \rightarrow Zh$ dominate if $|\left(\Gamma_{h \rightarrow ZZ^*} / \Gamma_{h \rightarrow ZZ^*}^{\text{SM}}\right)^{1/2} - 1| < 0.5\%$.
- Also, the correlation between $A \rightarrow Zh$ and $h \rightarrow ZZ^*$ can be different from the tree-level result.

Back up

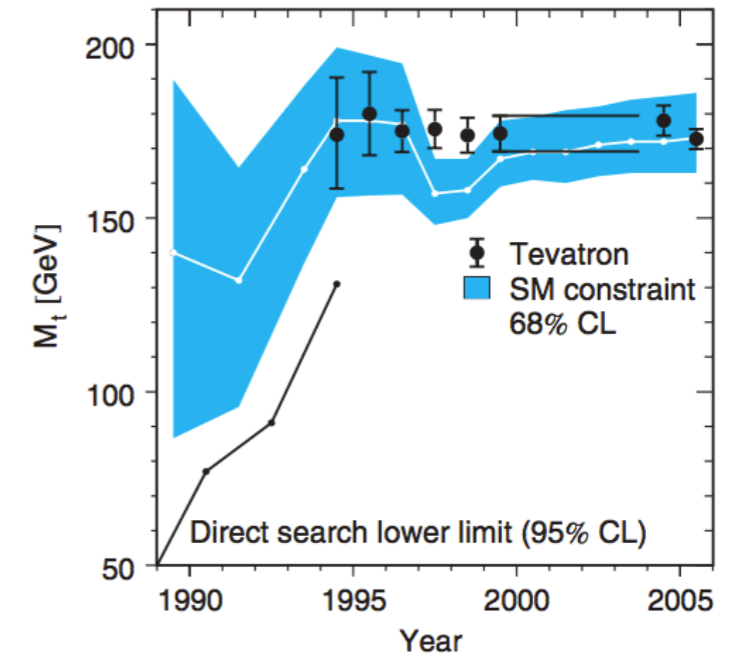
Impact of the precise measurements

Ex.) S, T parameter

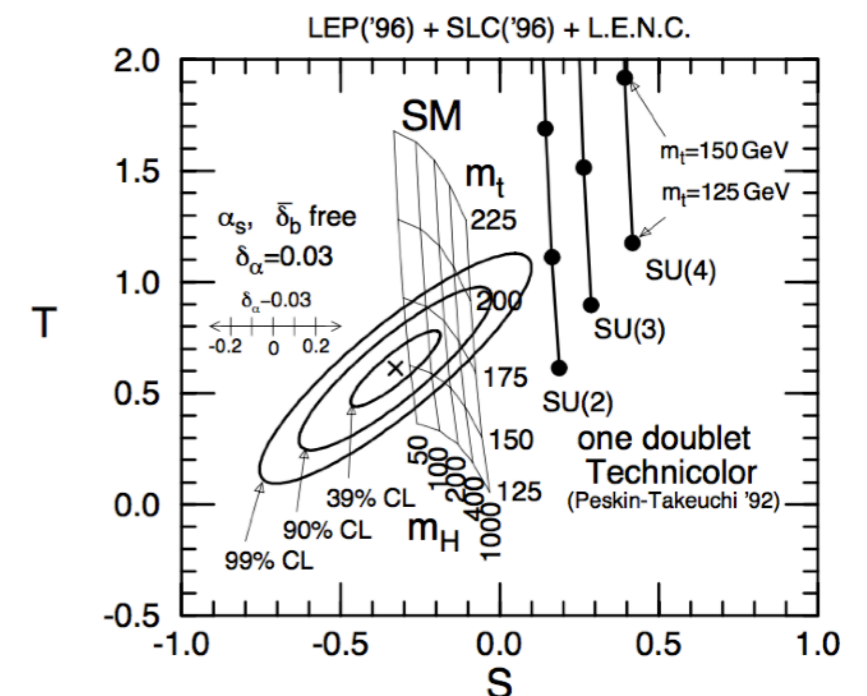
Top mass had been severely restricted before the discovery.

$$\alpha_{\text{EM}} T \simeq \frac{3G_F}{8\sqrt{2}\pi^2} (m_t^2 - m_Z^2 s_W^2 \log \frac{m_h^2}{m_Z^2})$$

Non-decoupling effect



[Physics Reports 427(2006)257]



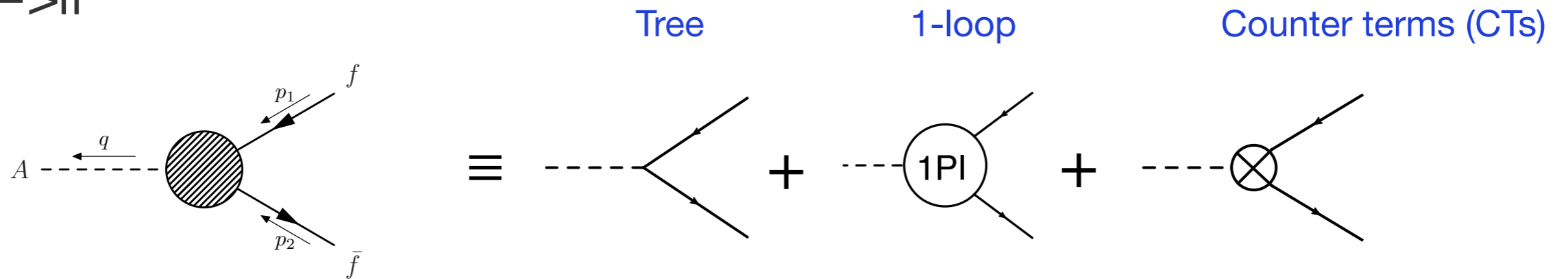
[Hagiwara, Haidt Matsumoto]

Same things can be applied to the Higgs physics.

Renormalization

Details of the calculations of NLO EW corrections

Ex). $A \rightarrow f\bar{f}$



Renormalization scheme : on-shell scheme

$$\left. \begin{array}{l} \delta m_\phi, \delta Z_{\phi_1\phi_2} : \\ \delta\alpha, \delta\beta : \end{array} \right\} \text{On-shell} \quad \leftarrow \text{The CTs are renormalized by } \hat{\Pi}_{ij}(p^2)$$

δM^2 : $\overline{\text{MS}}$ scheme

Another choice: $\hat{\Gamma}_{\Phi \rightarrow SS} = 0$

- Limit for parameter space by kinematics
- Numerical instability [M. Krause, M. Muhlleitner, R. Santos, H. Ziesche]

$\delta T_h, \delta T_H$: standard tadpole scheme, alternative tadpole scheme

Renormalization of tadpoles

- Standard tadpole scheme (STS) [W.F.L. Hollik, Fortschr. Phys. 38 (1990) 165.]

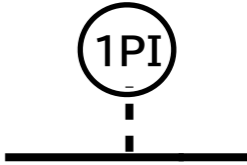
$$\left[\begin{array}{l} t_i^B = t_i^R + \delta t_i \quad (i = h, H) \\ \hat{\Gamma}_i = t_R + \delta t_i + \Gamma_i^{1PI} \end{array} \right] \xrightarrow{(t_i^R = 0, \hat{\Gamma}_i = 0)} \delta t_i = -\Gamma_i^{1PI}$$

- Alternative tadpole scheme (ATS) [J. Fleischer and F. Jegerlehner, PRD23, 2001 (1981)]

$$\left[\begin{array}{l} \Phi_m \rightarrow \Phi_m + \Delta v_m \quad (m = 1, 2) \\ \hat{\Gamma}_i = t^B + f(\Delta v_m) + \Gamma_i^{1PI} \end{array} \right] \xrightarrow{(t_i^B = 0, \hat{\Gamma}_i = 0)} \Delta v_m = \sum_i R_{mi} \Gamma_i^{1PI} / m_i^2$$

- Difference between STS and ATS

- While in STS tadpole affects only scalar self-energy, in ATS all self-energy has tadpole contributions.

$$\hat{\Pi}_{ij}^{\text{ATS}} = \hat{\Pi}_{ij} + \text{tadpole diagram}$$


- This makes self-energy gauge-independent at on-shell mass.

Gauge invariant CTs can be obtained in ATS.

Gauge dependence in mixing angles

- In renormalization of mixing angle, there is a technical issue, namely, gauge dependence appears. [Yamada, PRD64(2001)036008]

- We can check gauge dependence from Nielsen identify:

$$\partial_\xi \Pi_{ij} = (2p^2 - m_i^2 - m_j^2) \tilde{\Pi}_{ij}$$

$$i, j = h, H, A, H^\pm$$

$\tilde{\Pi}_{ij}$: function of loop functions

- $i = j = h$: $\delta m_h^2 = \Pi_{hh}^{1PI}(m_h^2)$

$$\partial_\xi \Pi_{hh}(p^2) = 0 \quad \text{at } p^2 = m_h^2 \quad \Rightarrow \quad \delta m_h^2 \text{ is gauge-independent.}$$

- $i = h, j = H$: $\delta\alpha = \{\Pi_{hH}^{1PI}(m_h^2) + \Pi_{hH}^{1PI}(m_H^2)\} / (m_H^2 - m_h^2)$

$$\partial_\xi \Pi_{Hh} \neq 0 \quad \text{at } p^2 = m_H^2 = m_h^2, \quad \Rightarrow \quad \text{Gauge dependence for } \delta\alpha$$

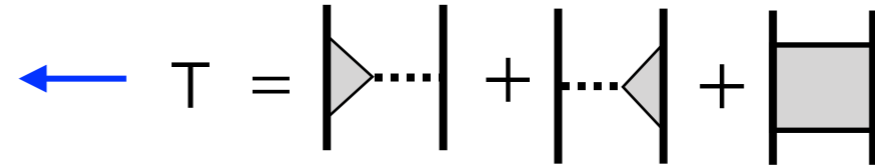
- Though this, the decay amplitudes are also gauge-dependent.

$$\begin{aligned} \frac{\partial \mathcal{M}_{A \rightarrow Zh}}{\partial \xi} &= \frac{\partial}{\partial \xi} \left(\mathcal{M}_{A \rightarrow Zh}^{\text{tree}} + \mathcal{M}_{A \rightarrow Zh}^{1PI} + \delta \mathcal{M}_{A \rightarrow Zh} \right) \\ &= \frac{\partial}{\partial \xi} \left(\underbrace{\mathcal{M}_{A \rightarrow Zh}^{1PI}}_{=0} + f(\delta Z_i) + g(\delta\alpha, \delta\beta) + h(\delta m_i) \right) = \frac{\partial}{\partial \xi} g(\delta\alpha, \delta\beta) \neq 0 \end{aligned}$$

Gauge independent renormalization of mixing angles

- In order to remove the gauge dependence in $\delta\alpha$, $\delta\beta$, we utilize pinch technique.

Basic idea: $\Pi_{Hh} \rightarrow \Pi_{Hh} + \Pi_{Hh}^{\text{Pinch}}$



$\rightarrow \partial_\xi \delta\alpha = 0, \partial_\xi \delta\beta = 0$

This should arise from the full NLO amp.

e.g., $gg \rightarrow A/H \rightarrow \bar{f}f$

$$\mathcal{M}_{gg \rightarrow h/H \rightarrow \bar{f}f}^{\text{NLO}} \ni \mathcal{M}_{gg \rightarrow h/H \rightarrow \bar{f}f}^{\text{SE-like}}$$

- Another scheme for mixing angles in gauge invariant way

- p_* scheme : $\hat{\Pi}_{Hh}(p^2 = [m_H^2 + m_h^2]/2) = 0$

[Espinosa, Yamada, PRD67(2003) 036003]

- On-shell conditions with S matrix (THDM+ ν_{Ri} ($i=1,2$), $y_{\nu i} \rightarrow 0$)

[Denner, Dittmaier, Lang, JHEP 1811(2018)104]

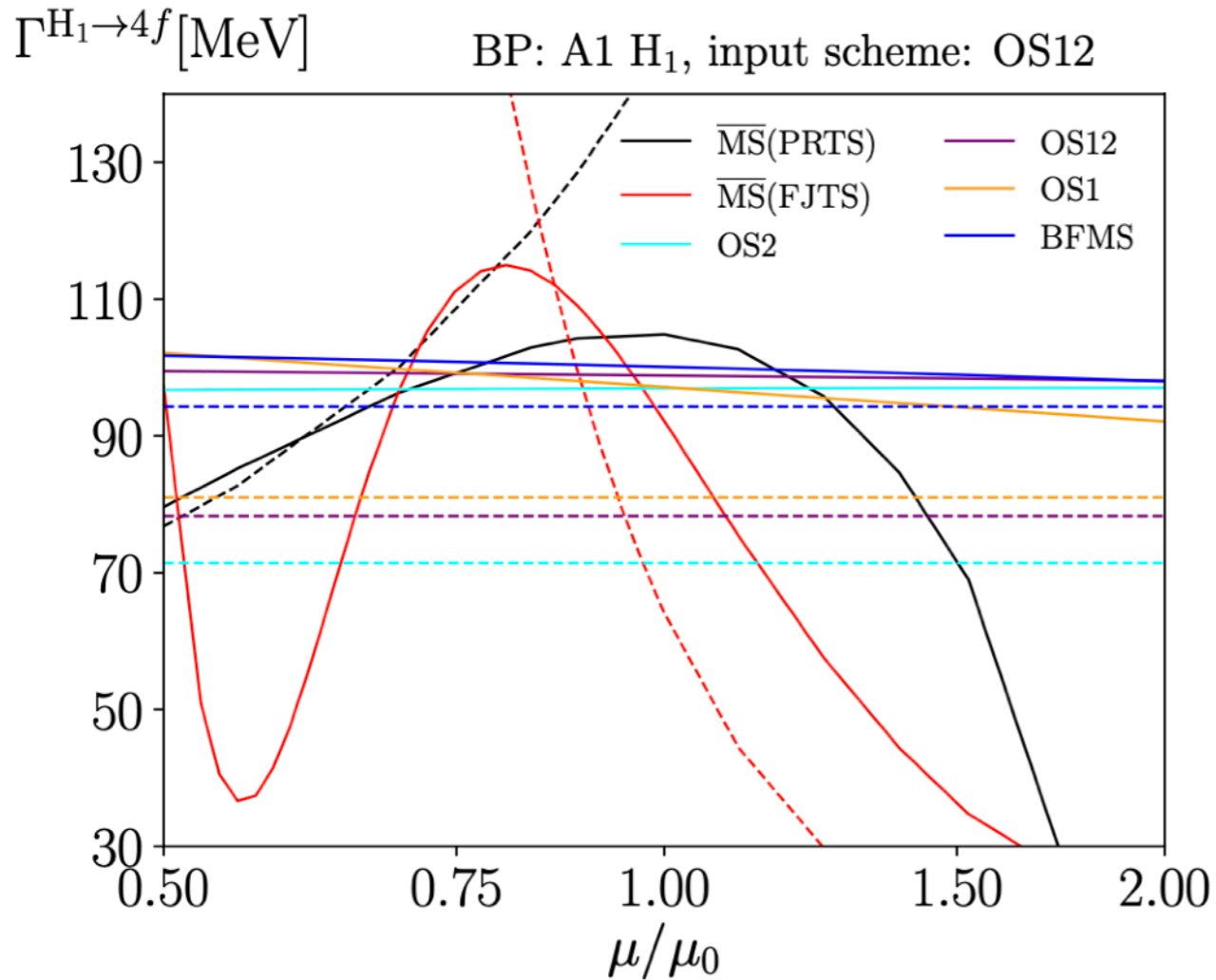
$$\frac{\mathcal{M}_{H \rightarrow \nu_{R1} \nu_{R1}}^{\text{loop}}}{\mathcal{M}_{h \rightarrow \nu_{R1} \nu_{R1}}^{\text{loop}}} = \frac{\mathcal{M}_{H \rightarrow \nu_{R1} \nu_{R1}}^{\text{tree}}}{\mathcal{M}_{h \rightarrow \nu_{R1} \nu_{R1}}^{\text{tree}}} = \frac{c_\alpha}{s_\alpha}$$

$h(H) \rightarrow \nu_{Ri} \nu_{Ri} = y_{\nu i} c_\alpha (s_\alpha)$

Scheme difference in counterterms of mixing angles

[A. Denner, S. Dittmaier, J.N. Lang, 1808.03466]

BP: A1 $M_{H_2} = 125\text{GeV}$, $M_{H_1} = 300\text{GeV}$, $M_{A,H^\pm} = 460\text{GeV}$,
 $\lambda_5 = -1.9$, $t_\beta = 2$, $c_{\beta-\alpha} = 0.1$, $\mu_0 = (m_{H_2} + m_{H_1} + M_A + 2M_{H^\pm})/5$



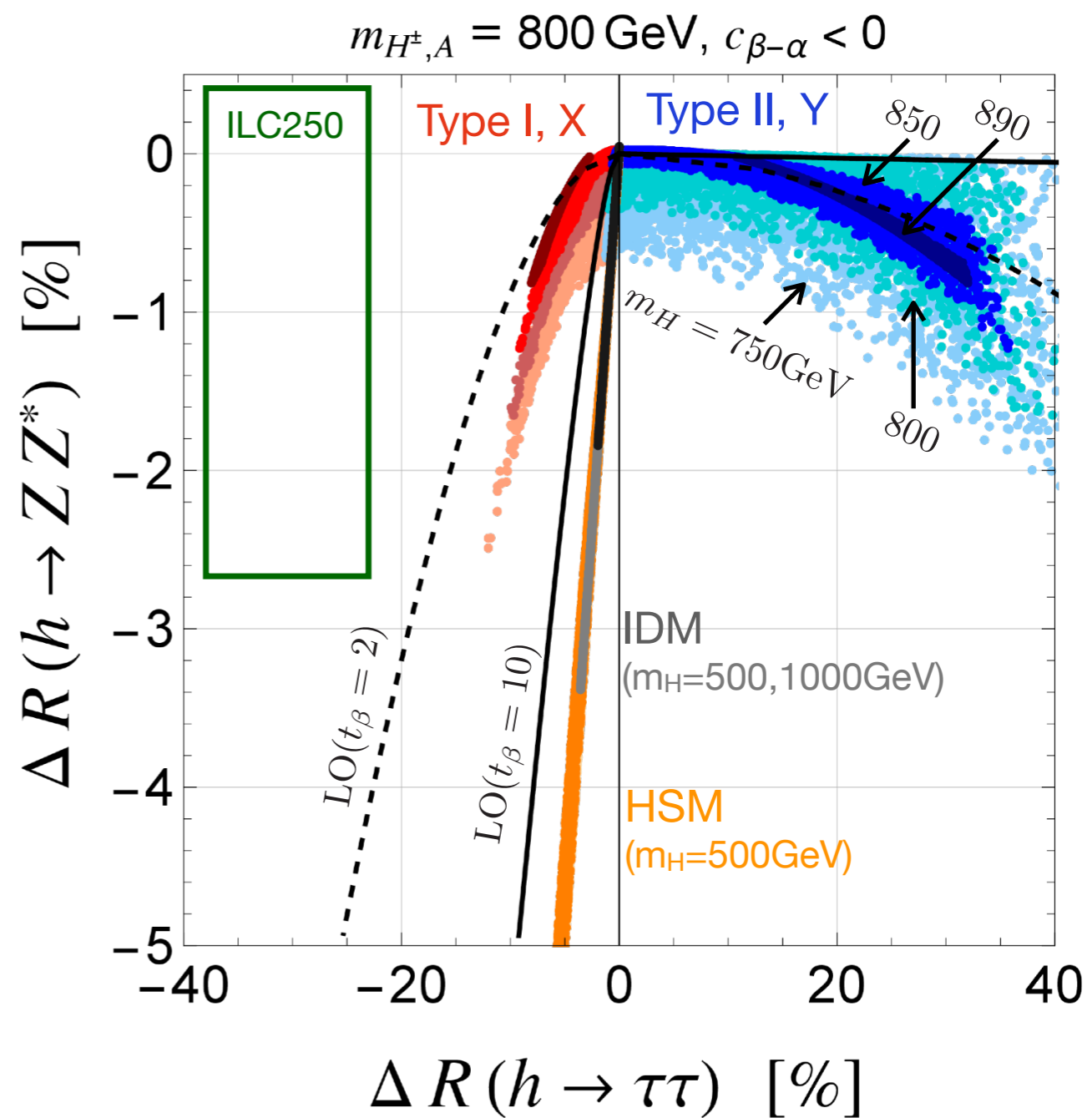
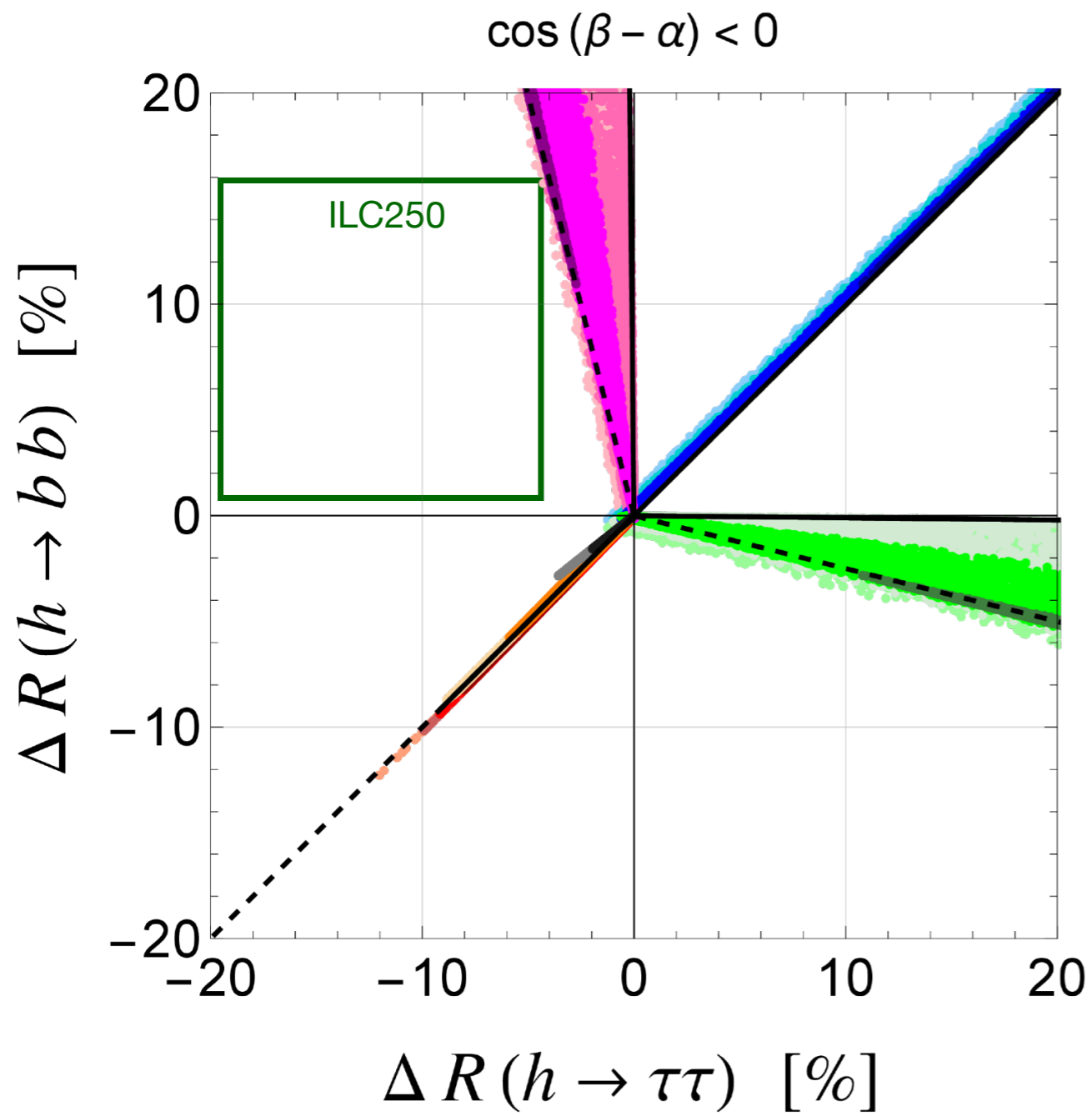
Scheme	A1	
	LO	NLO
$\overline{\text{MS}}(\text{PRTS})$	$147.102(4)_{-47.8\%}^{+100\%}$	$104.86(2)_{-24.1\%}^{-100\%}$
$\overline{\text{MS}}(\text{FJTS})$	$64.096(2)_{>+100\%}^{-86.9\%}$	$92.17(1)_{+5.6\%}^{-81.4\%}$
OS1	$80.992(2)$	$97.145(7)_{+5.1\%}^{-5.2\%}$
OS2	$71.429(2)$	$96.95(1)_{-0.2\%}^{+0.1\%}$
OS12	$78.304(2)$	$98.812(8)_{+0.7\%}^{-0.8\%}$
BFMS	$94.265(2)$	$100.117(5)_{+1.6\%}^{-2.2\%}$

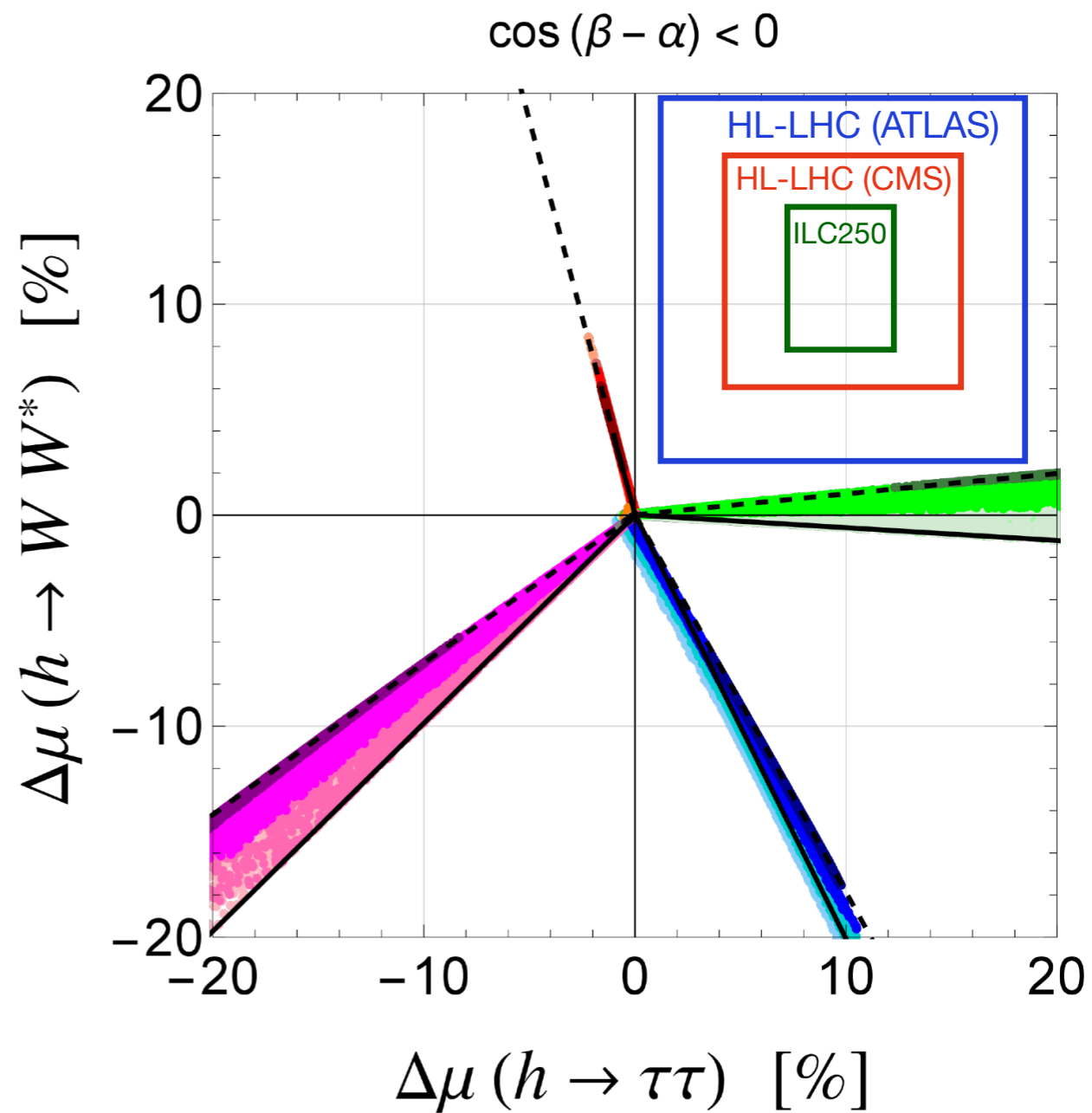
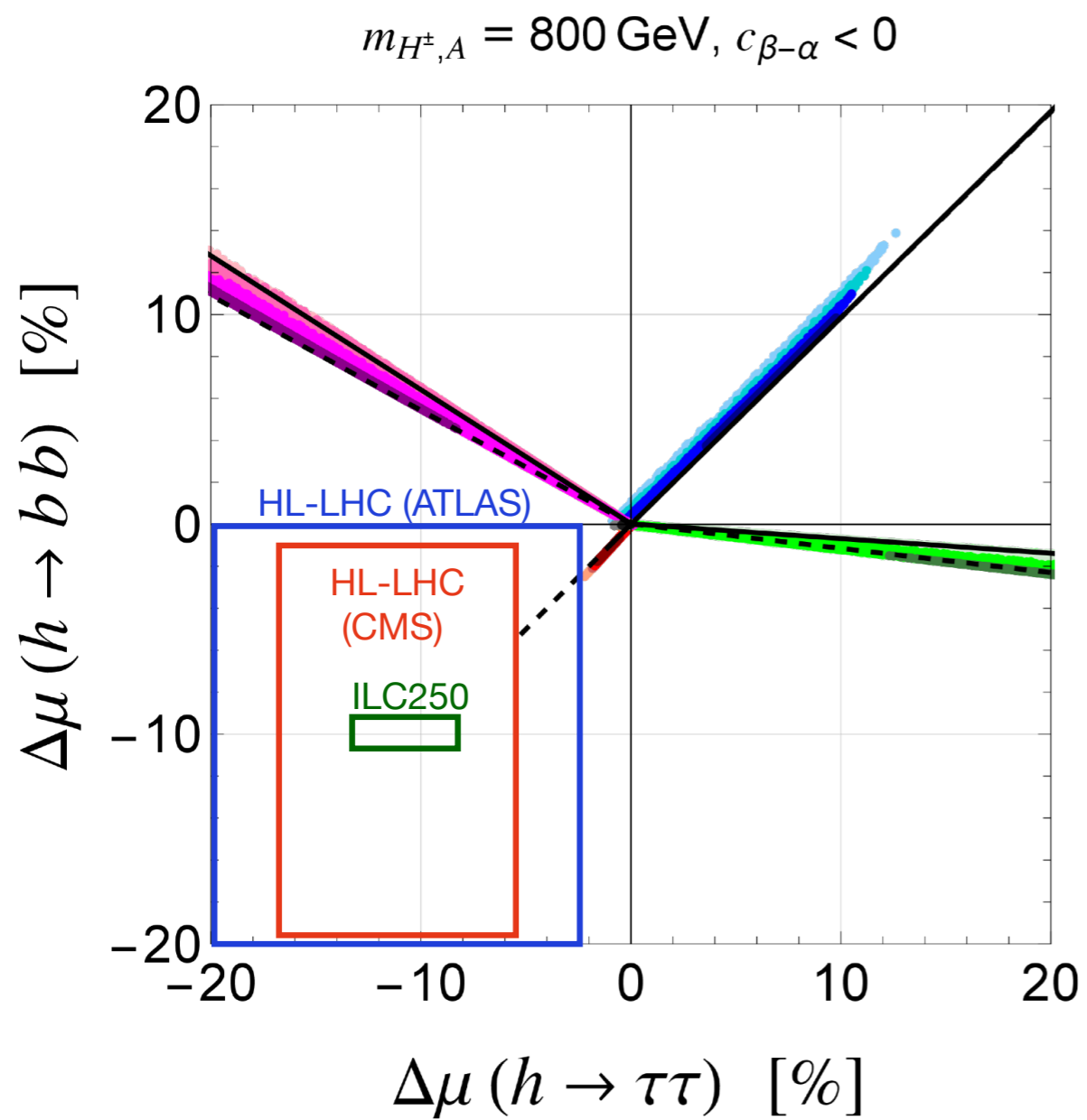
OS1,2,12: On shell with $H, h \rightarrow \nu_{Ri} \bar{\nu}_{Ri}$

BFMS: On-shell with $\hat{\Pi}_{Hh}$ and the PT

Theoretical uncertainty (scheme difference) for on-shell scheme is a few %.

Other plots





[1710.07621]

	ILC250		+ILC500	
	κ fit	EFT fit	κ fit	EFT fit
$g(hbb)$	1.8	1.1	0.60	0.58
$g(hcc)$	2.4	1.9	1.2	1.2
$g(hgg)$	2.2	1.7	0.97	0.95
$g(hWW)$	1.8	0.67	0.40	0.34
$g(h\tau\tau)$	1.9	1.2	0.80	0.74
$g(hZZ)$	0.38	0.68	0.30	0.35
$g(h\gamma\gamma)$	1.1	1.2	1.0	1.0
$g(h\mu\mu)$	5.6	5.6	5.1	5.1
$g(h\gamma Z)$	16	6.6	16	2.6
$g(hbb)/g(hWW)$	0.88	0.86	0.47	0.46
$g(h\tau\tau)/g(hWW)$	1.0	1.0	0.65	0.65
$g(hWW)/g(hZZ)$	1.7	0.07	0.26	0.05
Γ_h	3.9	2.5	1.7	1.6
$BR(h \rightarrow inv)$	0.32	0.32	0.29	0.29
$BR(h \rightarrow other)$	1.6	1.6	1.3	1.2

ATLAS						
3000 fb ⁻¹ relative uncertainty [%]						
		Total	Stat	Exp	SigTh	BkgTh
B ^{γγ}	S1	6.0	1.2	4.7	3.3	1.4
	S2	3.7	1.2	2.9	1.8	0.6
B ^{WW}	S1	5.8	1.0	2.8	4.3	2.6
	S2	4.4	1.0	2.4	3.2	1.6
B ^{ZZ}	S1	5.3	1.6	3.0	3.7	1.7
	S2	3.8	1.6	2.7	1.9	1.0
B ^{bb}	S1	7.6	2.0	2.4	5.0	4.7
	S2	5.0	2.0	1.9	2.8	3.2
B ^{ττ}	S1	6.0	1.7	2.7	4.4	2.4
	S2	4.4	1.7	2.5	2.8	1.7
B ^{μμ}	S1	14.9	12.7	3.2	6.8	0.3
	S2	13.7	12.7	3.2	3.7	0.3
B ^{Zγ}	S1	24.2	20.3	4.5	12.2	0.0
	S2	24.2	20.3	4.5	12.2	0.0

CMS						
3000 fb ⁻¹ relative uncertainty [%]						
		Total	Stat	Exp	SigTh	BkgTh
B ^{γγ}	S1	4.4	1.3	2.6	3.3	0.3
	S2	3.0	1.3	1.7	1.9	0.3
B ^{WW}	S1	4.0	1.0	1.4	3.5	1.0
	S2	2.8	1.0	1.1	2.2	0.9
B ^{ZZ}	S1	5.0	1.6	2.5	3.5	1.9
	S2	3.2	1.6	1.7	2.1	0.7
B ^{bb}	S1	7.0	2.1	2.3	5.2	3.6
	S2	4.7	2.1	1.7	2.4	2.9
B ^{ττ}	S1	3.9	1.6	1.9	2.6	1.5
	S2	2.9	1.6	1.4	1.9	0.6
B ^{μμ}	S1	12.8	9.1	7.6	4.7	0.8
	S2	9.6	9.1	1.7	2.6	0.8

[1902.00134]

	-80% e ⁻ , +30% e ⁺ polarization:		ILC 250 fb ⁻¹					
			250 GeV		350 GeV		500 GeV	
			Zh	νν̄h	Zh	νν̄h	Zh	νν̄h
σ [50–53]			2.0		1.8		4.2	
h → invis. [54, 55]			0.86		1.4		3.4	
h → b̄b [56–59]			1.3	8.1	1.5	1.8	2.5	0.93
h → c̄c [56, 57]			8.3		11	19	18	8.8
h → gg [56, 57]			7.0		8.4	7.7	15	5.8
h → WW [59–61]			4.6		5.6 *	5.7 *	7.7	3.4
h → ττ [63]			3.2		4.0 *	16 *	6.1	9.8
h → ZZ [2]			18		25 *	20 *	35 *	12 *
h → γγ [64]			34 *		39 *	45 *	47	27
h → μμ [65, 66]			72 *		87 *	160 *	120 *	100 *
a [27]			7.6		2.7 *		4.0	
b			2.7		0.69 *		0.70	
ρ(a, b)			-99.17		-95.6 *		-84.8	
+80% e ⁻ , -30% e ⁺ polarization:								

[1708.08912]