Constraining neutrino models predictions

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Based on arXiv:2310.20681, submitted to PPNP

Phenomenology of Lepton Masses and Mixing with Discrete Flavor Symmetries

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and

arXiv:24xx.xxxxx
work in progress

XXX Cracow EPIPHANY Conference on Precision Physics at High Energy Colliders, Kraków, January 11, 2024

Neutrino mixing, 3ν.

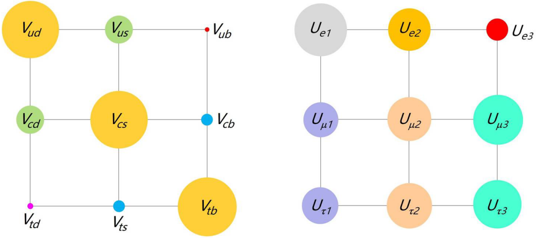
Neutrino flavor and mass eigenstates are related by

$$|\nu_\alpha\rangle = \mathrm{U}_{\alpha i}\,|\nu_i\rangle$$

Pontecorvo–Maki–Nakagawa–Sakata parametrization of mixing matrix

$$\mathrm{U}_{\mathrm{PMNS}} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{s3} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{bmatrix} \begin{bmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, $\delta \equiv \delta_{\mathrm{CP}}$.



(a) CKM quark flavor mixing (b) PMNS lepton flavor mixing

Taken from arXiv:2210.11922, Figure 2.

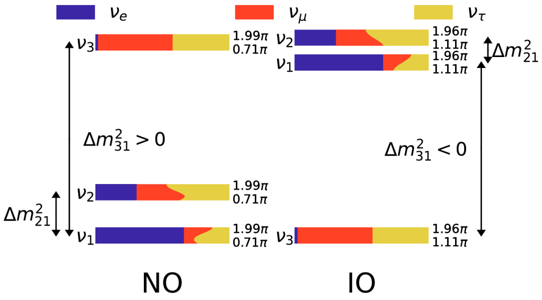
Mass ordering, $m_0 = m_{lightest}$.

Normal mass ordering (NO)

$$m_1 = m_0,$$
$$m_2 = \sqrt{m_0^2 + \Delta m_{21}^2},$$
$$m_3 = \sqrt{m_0^2 + \Delta m_{31}^2},$$

Inverted mass ordering (IO)

$$m_1 = \sqrt{m_0^2 - \Delta m_{21}^2 - \Delta m_{32}^2},$$
$$m_2 = \sqrt{m_0^2 - \Delta m_{32}^2},$$
$$m_3 = m_0,$$



Taken from https://globalfit.astroparticles.es, updated arXiv:1806.11051, Figure 1

Oscillation data, with SK.

Parameter	Ordering	NuFIT 5.2 (2022)		de Salas et al. (2021)		Capozzi et al. (2021)	
		bf$\pm 1\sigma$	3σ range	bf$\pm 1\sigma$	3σ range	bf$\pm 1\sigma$	3σ range
$\sin^2\theta_{12}/10^{-1}$	NO, IO	$3.03^{+0.12}_{-0.12}$	$2.70 - 3.41$	$3.18^{+0.16}_{-0.16}$	$2.71 - 3.69$	$3.03^{+0.13}_{-0.13}$	$2.63 - 3.45$
$\sin^2\theta_{23}/10^{-1}$	NO	$4.51^{+0.19}_{-0.16}$	$4.08 - 6.03$	$5.74^{+0.14}_{-0.14}$	$4.34 - 6.10$	$4.55^{+0.18}_{-0.15}$	$4.16 - 5.99$
θ_{23} octant	IO	$5.69^{+0.16}_{-0.21}$	$4.12 - 6.13$	$5.78^{+0.10}_{-0.17}$	$4.33 - 6.08$	$5.69^{+0.12}_{-0.21}$	$4.17 - 6.06$
$\sin^2\theta_{13}/10^{-2}$	NO	$2.225^{+0.056}_{-0.059}$	$2.052 - 2.398$	$2.200^{+0.069}_{-0.062}$	$2.000 - 2.405$	$2.23^{+0.07}_{-0.06}$	$2.04 - 2.44$
$\neq 0$	IO	$2.223^{+0.058}_{-0.058}$	$2.048 - 2.416$	$2.225^{+0.064}_{-0.070}$	$2.018 - 2.424$	$2.23^{+0.06}_{-0.06}$	$2.03 - 2.45$
δ_{CP}/π	NO	$1.29^{+0.20}_{-0.14}$	$0.80 - 1.94$	$1.08^{+0.13}_{-0.12}$	$0.71 - 1.99$	$1.24^{+0.18}_{-0.13}$	$0.77 - 1.97$
can be 0?	IO	$1.53^{+0.12}_{-0.16}$	$1.08 - 1.91$	$1.58^{+0.15}_{-0.16}$	$1.11 - 1.96$	$1.52^{+0.15}_{-0.11}$	$1.07 - 1.90$
$\Delta m^2_{21}/10^{-5}\text{eV}^2$	NO, IO	$7.41^{+0.21}_{-0.20}$	$6.82 - 8.03$	$7.50^{+0.22}_{-0.20}$	$6.94 - 8.14$	$7.36^{+0.16}_{-0.15}$	$6.93 - 7.93$
$\lvert\Delta m^2_{atm}\rvert/10^{-3}\text{eV}^2$	NO	$2.507^{+0.026}_{-0.027}$	$2.427 - 2.590$	$2.55^{+0.02}_{-0.03}$	$2.47 - 2.63$	$2.485^{+0.023}_{-0.031}$	$2.401 - 2.565$
	IO	$2.486^{+0.028}_{-0.025}$	$2.406 - 2.570$	$2.45^{+0.02}_{-0.03}$	$2.37 - 2.53$	$2.455^{+0.030}_{-0.025}$	$2.376 - 2.541$
$\Delta\chi^2$	IO - NO	6.4		6.4		6.5	

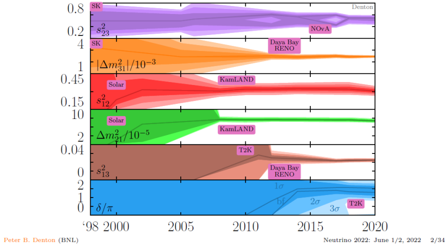


Figure taken form Peter B. Denton talk, link here.

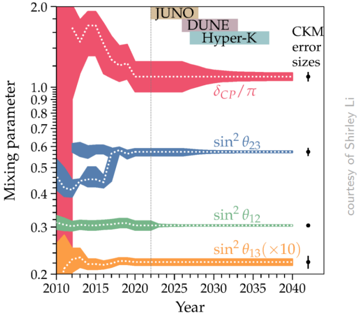


Figure taken form Biswajit Karmakar talk, link here.
See also arXiv:2012.12893, Figure 1 and arXiv:2204.08668, Figure 2.1

BM, TB, GR, HG, $\theta_{13} = 0$ (early 2010s).

$$\mathrm{U_{PMNS}} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{s3} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{bmatrix}$$

$$\theta_{13} = 0^\circ \qquad \Downarrow \qquad \theta_{23} = 45^\circ$$

$$\mathrm{U_0} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\Downarrow$$

$\theta_{12} = 45^\circ$, $s_{12} = 1/\sqrt{2}$ — Bimaximal Mixing

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$\theta_{12} = 35,26^\circ$, $s_{12} = 1/\sqrt{3}$ — Tribimaximal Mixing

$$\begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$\theta_{12} = 31,7^\circ$ — Golden Ratio Mixing

$$\begin{bmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$\theta_{12} = 30^\circ$, $s_{12} = 1/2$ — Hexagonal Mixing

$$\begin{bmatrix} \sqrt{\frac{3}{4}} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Golden Ratio Mixing: $\mathrm{tg}\,\theta_{12} = 1/\varphi$, $\varphi = (1+\sqrt{5})/2$ being the golden ratio.

Based on Biswajit Karmakar talk, link here.

$\theta_{13} \neq 0$, Daya Bay, RENO (2012).

BM, TB, GR, HG disfavored by non-zero θ_{13}.

$$\mathrm{U_{PMNS}} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{s3} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{bmatrix}$$

$\theta_{13} \neq 0^\circ$ $\Downarrow$ $\theta_{23} = 45^\circ$

$$\mathrm{U_0} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$\Downarrow$

$\theta_{12} = 45^\circ$, $s_{12} = 1/\sqrt{2}$ $\quad$ $\theta_{12} = 35,26^\circ$, $s_{12} = 1/\sqrt{3}$ $\quad$ $\theta_{12} = 31,7^\circ$ $\quad$ $\theta_{12} = 30^\circ$, $s_{12} = 1/2$

Bimaximal Mixing

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Tribimaximal Mixing

$$\begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Golden Ratio Mixing

$$\begin{bmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Hexagonal Mixing

$$\begin{bmatrix} \sqrt{\frac{3}{4}} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Golden Ratio Mixing: tg $\theta_{12} = 1/\varphi$, $\varphi = (1 + \sqrt{5})/2$ being the golden ratio.

Based on Biswajit Karmakar talk, link here.

Non-zero θ_{13}: Successors of tribimaximal mixing, TM_1, TM_2.

$$\mathrm{U_{TBM}} = \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \qquad \mathrm{U_{PMNS}} \simeq \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0.15 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix},$$

$$\mathrm{U_{TM_1}} = \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} & \frac{s_\theta}{\sqrt{3}} e^{-i\gamma} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s_\theta}{\sqrt{2}} e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s}{\sqrt{2}} e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{bmatrix},$$

$$\mathrm{U_{TM_2}} = \begin{bmatrix} \frac{2c_\theta}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2s_\theta}{\sqrt{6}} e^{-i\gamma} \\ -\frac{c_\theta}{\sqrt{6}} + \frac{s}{\sqrt{2}} e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{c_\theta}{\sqrt{6}} + \frac{s}{\sqrt{2}} e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{bmatrix},$$

$$|\mathrm{U_{TM_1}}| = \begin{bmatrix} \frac{2}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \end{bmatrix}, \qquad |\mathrm{U_{TM_2}}| = \begin{bmatrix} * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \end{bmatrix}.$$

Based on Biswajit Karmakar talk, link here.

TM_1 oscillation parameters predictions.

$$
\mathrm{U_{PMNS}} \quad = \quad \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{s3} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{bmatrix},
$$

$$
\mathrm{U_{TM_1}} \quad = \quad \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} & \frac{s_\theta}{\sqrt{3}}e^{-i\gamma} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s_\theta}{\sqrt{2}}e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}}e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s}{\sqrt{2}}e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}}e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{bmatrix}.
$$

Comparing the corresponding elements of the first column of $\mathrm{U_{PMNS}}$ and $\mathrm{U_{TM_1}}$.

$$
|\mathrm{U_{e1}}|^2 = \mathrm{c}_{12}^2\mathrm{c}_{13}^2 = 2/3 \quad : \quad s_{12}^2 = \frac{1-3s_{13}^2}{3-3s_{13}^2},
$$

$$
|\mathrm{U}_{\mu 1}|^2 = |\mathrm{U}_{\tau 1}|^2 = 1/6 \quad : \quad \cos\delta_{\mathrm{CP}} = \frac{(1-5s_{13}^2)(2s_{23}^2-1)}{4s_{13}s_{23}\sqrt{2(1-3s_{13}^2)(1-s_{23}^2)}}.
$$

Based on arXiv:1212.3247

TM_2 oscillation parameters predictions.

$$\mathrm{U_{PMNS}} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{s3} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{bmatrix},$$

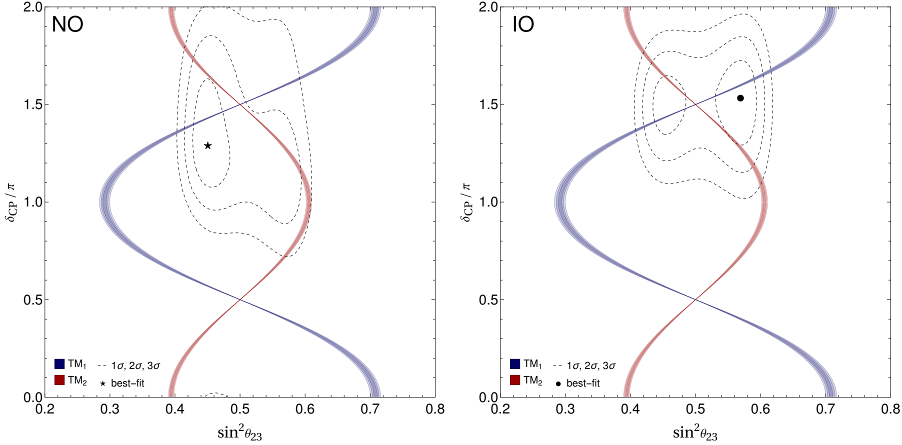
$$\mathrm{U_{TM_2}} = \begin{bmatrix} \frac{2c_\theta}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2s_\theta}{\sqrt{6}}e^{-i\gamma} \\ -\frac{c_\theta}{\sqrt{6}} + \frac{s}{\sqrt{2}}e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_\theta}{\sqrt{3}}e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{c_\theta}{\sqrt{6}} + \frac{s}{\sqrt{2}}e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_\theta}{\sqrt{3}}e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{bmatrix}.$$

Comparing the corresponding elements of the second column of $\mathrm{U_{PMNS}}$ and $\mathrm{U_{TM_2}}$.

$$|\mathrm{U_{e2}}|^2 = \mathrm{s}_{12}^2\mathrm{c}_{13}^2 = 1/3 \quad : \quad s_{12}^2 = \frac{1}{3 - 3s_{13}^2},$$

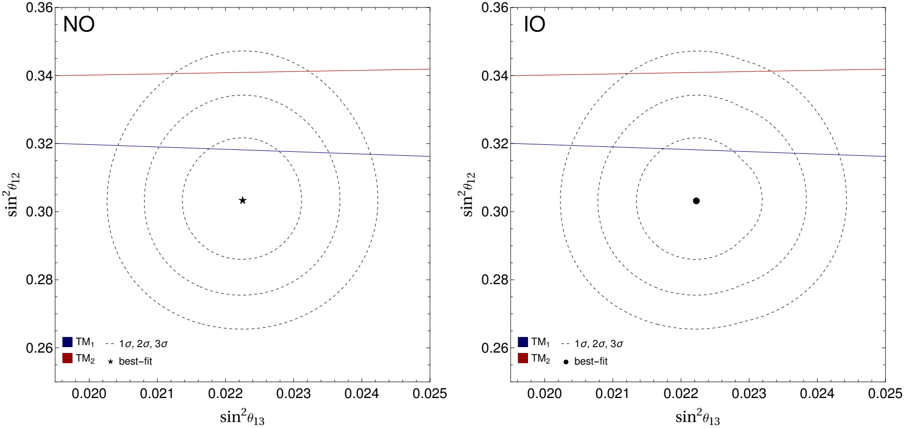
$$|\mathrm{U}_{\mu 2}|^2 = |\mathrm{U}_{\tau 2}|^2 = 1/3 \quad : \quad \cos\delta_{\mathrm{CP}} = -\frac{(2 - 4s_{13}^2)(2s_{23}^2 - 1)}{4s_{13}s_{23}\sqrt{(2 - 3s_{13}^2)(1 - s_{23}^2)}}.$$

TM_1 and TM_2, δ_{CP} *vs.* $\sin^2\theta_{23}$.



$$\mathrm{TM}_1: \quad \cos\delta_{\mathrm{CP}} = \frac{(1-5s_{13}^2)(2s_{23}^2-1)}{4s_{13}s_{23}\sqrt{2(1-3s_{13}^2)(1-s_{23}^2)}}, \qquad \mathrm{TM}_2: \quad \cos\delta_{\mathrm{CP}} = -\frac{(2-4s_{13}^2)(2s_{23}^2-1)}{4s_{13}s_{23}\sqrt{(2-3s_{13}^2)(1-s_{23}^2)}}.$$

TM_1 and TM_2, $\sin^2\theta_{12}$ *vs.* $\sin^2\theta_{13}$.

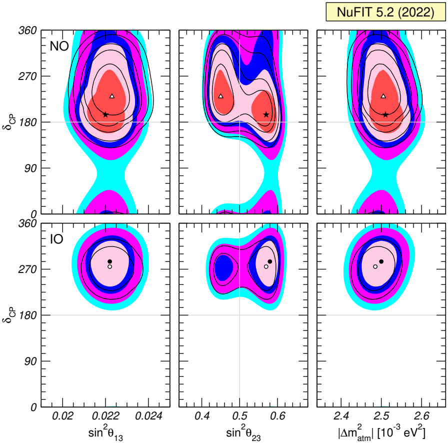


$$\mathrm{TM}_1: \quad s_{12}^2 = \frac{1-3s_{13}^2}{3-3s_{13}^2}, \qquad \mathrm{TM}_2: \quad s_{12}^2 = \frac{1}{3-3s_{13}^2}.$$

NuFIT 5.2 (2022).

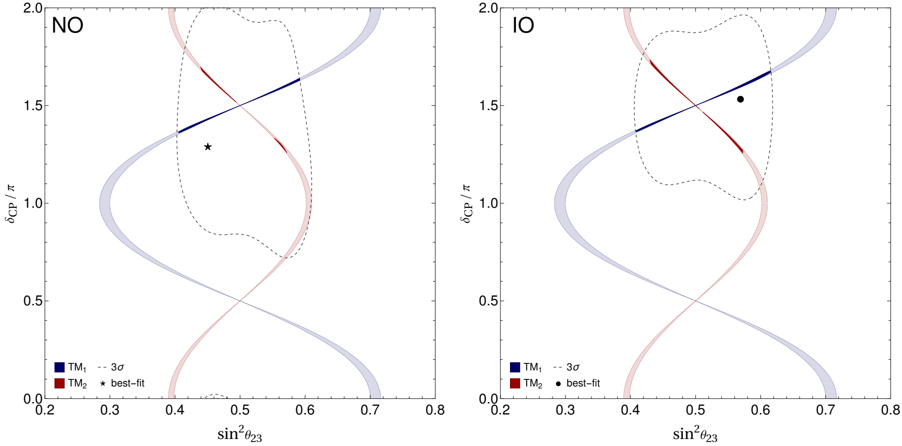
NuFIT 5.2 (2022)

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
	$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
	$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.406 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
	$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
	$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00059}$	$0.02029 \to 0.02391$	$0.02219^{+0.00060}_{-0.00057}$	$0.02047 \to 0.02396$
	$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.12}$	$8.19 \to 8.89$	$8.57^{+0.12}_{-0.11}$	$8.23 \to 8.90$
	$\delta_{CP}/^\circ$	197^{+42}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.511^{+0.028}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.025}$	$-2.581 \to -2.408$
		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.4$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.012}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
	$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
	$\sin^2\theta_{23}$	$0.451^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.569^{+0.016}_{-0.021}$	$0.412 \to 0.613$
	$\theta_{23}/^\circ$	$42.2^{+1.1}_{-0.9}$	$39.7 \to 51.0$	$49.0^{+1.0}_{-1.2}$	$39.9 \to 51.5$
	$\sin^2\theta_{13}$	$0.02225^{+0.00056}_{-0.00059}$	$0.02052 \to 0.02398$	$0.02223^{+0.00058}_{-0.00058}$	$0.02048 \to 0.02416$
	$\theta_{13}/^\circ$	$8.58^{+0.11}_{-0.11}$	$8.23 \to 8.91$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.94$
	$\delta_{CP}/^\circ$	232^{+36}_{-26}	$144 \to 350$	276^{+22}_{-29}	$194 \to 344$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.507^{+0.026}_{-0.027}$	$+2.427 \to +2.590$	$-2.486^{+0.025}_{-0.028}$	$-2.570 \to -2.406$



Taken from http://www.nu-fit.org, updated arXiv:2007.14792, Table 3.

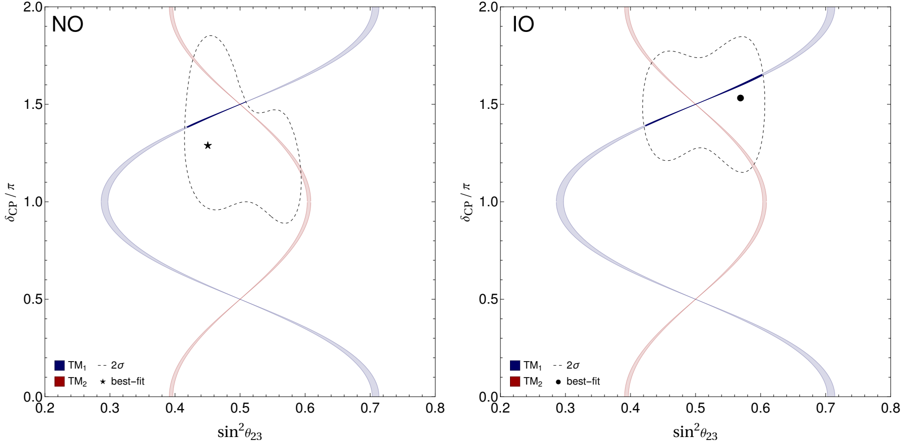
TM_1 and TM_2, δ_{CP} *vs.* $\sin^2\theta_{23}$, constrained, 3σ.



$$\mathrm{TM}_1: \quad \cos\delta_{\mathrm{CP}} = \frac{(1-5s_{13}^2)(2s_{23}^2-1)}{4s_{13}s_{23}\sqrt{2(1-3s_{13}^2)(1-s_{23}^2)}}, \qquad \mathrm{TM}_2: \quad \cos\delta_{\mathrm{CP}} = -\frac{(2-4s_{13}^2)(2s_{23}^2-1)}{4s_{13}s_{23}\sqrt{(2-3s_{13}^2)(1-s_{23}^2)}}.$$

Darker shaded regions = constrained with correlations.

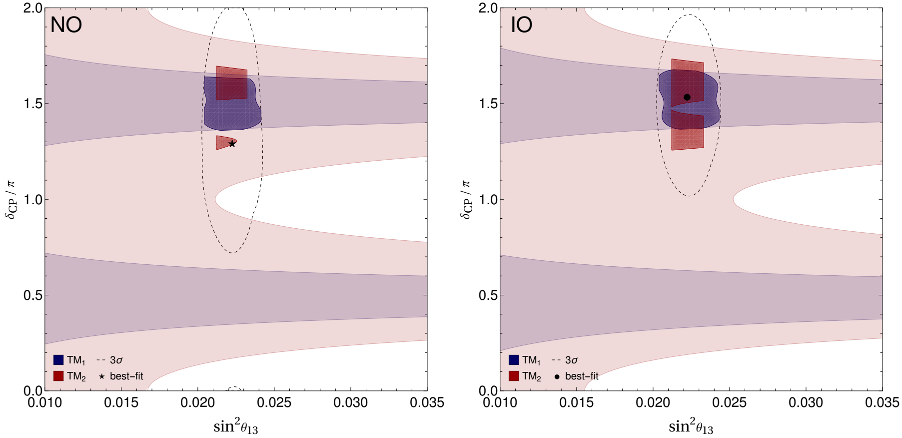
TM_1 and TM_2, $\delta_{\rm CP}$ *vs.* $\sin^2\theta_{23}$, constrained, 2σ.



$$\mathrm{TM}_1: \quad \cos\delta_{\rm CP} = \frac{(1-5s_{13}^2)(2s_{23}^2-1)}{4s_{13}s_{23}\sqrt{2(1-3s_{13}^2)(1-s_{23}^2)}}, \qquad \mathrm{TM}_2: \quad \cos\delta_{\rm CP} = -\frac{(2-4s_{13}^2)(2s_{23}^2-1)}{4s_{13}s_{23}\sqrt{(2-3s_{13}^2)(1-s_{23}^2)}}.$$

Darker shaded regions = constrained with correlations.

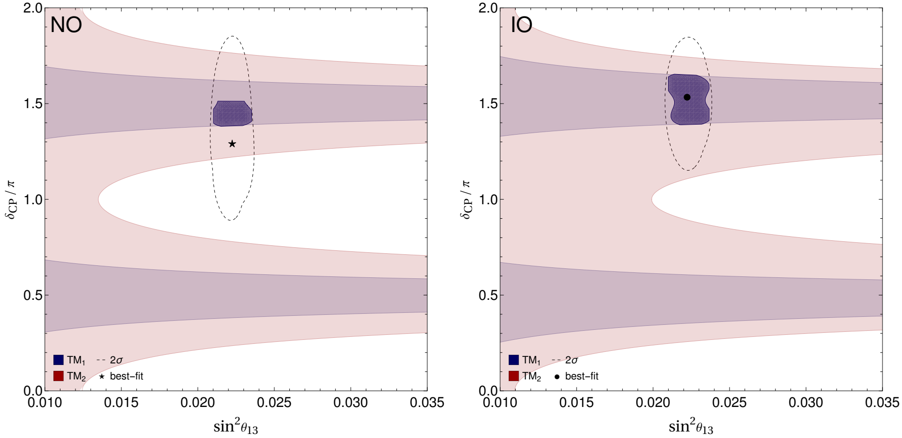
TM_1 and TM_2, δ_{CP} *vs.* $\sin^2\theta_{13}$, constrained, 3σ.



$$\mathrm{TM}_1: \quad \cos\delta_{\mathrm{CP}} = \frac{(1-5s_{13}^2)(2s_{23}^2-1)}{4s_{13}s_{23}\sqrt{2(1-3s_{13}^2)(1-s_{23}^2)}}, \qquad \mathrm{TM}_2: \quad \cos\delta_{\mathrm{CP}} = -\frac{(2-4s_{13}^2)(2s_{23}^2-1)}{4s_{13}s_{23}\sqrt{(2-3s_{13}^2)(1-s_{23}^2)}}.$$

Darker shaded regions = constrained with correlations.

TM_1 and TM_2, δ_{CP} *vs.* $\sin^2\theta_{13}$, constrained, 2σ.



$$\mathrm{TM}_1: \quad \cos\delta_{\mathrm{CP}} = \frac{(1-5s_{13}^2)(2s_{23}^2-1)}{4s_{13}s_{23}\sqrt{2(1-3s_{13}^2)(1-s_{23}^2)}}, \qquad \mathrm{TM}_2: \quad \cos\delta_{\mathrm{CP}} = -\frac{(2-4s_{13}^2)(2s_{23}^2-1)}{4s_{13}s_{23}\sqrt{(2-3s_{13}^2)(1-s_{23}^2)}}.$$

Darker shaded regions = constrained with correlations.

Summary

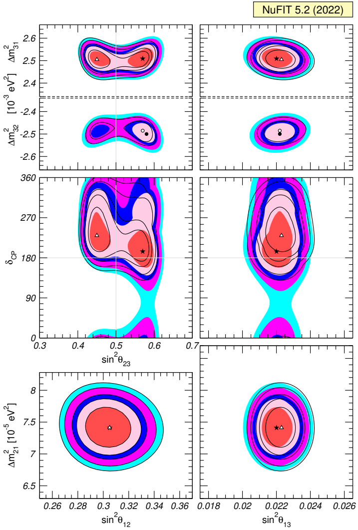
Improvements in precise determination neutrino oscillation parameters triggered construction and tests of discrete symmetry neutrino flavor models.

1. disfavored by oscillation parameters (history)
   - Bimaximal, Tribimaximal, Golden Ratio, Hexagonal Mixings disfavored by non-zero θ_{13};
2. successors of Tribimaximal Mixings (current work)
   - TM_1 and TM_2 predictions are significantly constrained by oscillation parameters correlations;
   - TM_2 is not applicable at 2σ or less;
3. conclusion
   - taking into account correlations between oscillation parameters is a significant step forward in testing and constraining the predictions of discrete symmetry neutrino flavor models;

Thank you
for your attention.

Neutrino oscillation parameters data files.

NuFIT 5.2, link here,



Valencia neutrino global fit, link here.