Vilnius University

Relations between basis sets of fields in the renormalization procedure

arXiv: 2307.01642

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Problem

basis independence $\Longleftarrow$ tree-level tension loops $\Longrightarrow$ basis dependence

$\Downarrow$ basis-invariants; **no** mixing angles

(e.g. generic 2HDM written without $\tan\beta$)

?

$\Downarrow$ Mixing angle counterterm

Practicalities

+ model/process dependence

often gauge-dependent

often singular

for degenerate masses; $m_i = m_j$

Goals / Results

1) Remove the tension!
   ↳ Go towards basis-independence

2) Propose a consistent set of counterterms, on which schemes may be built
   ↳ MIXING MATRIX COUNTERTERM = 0
   (angle)

Contents

- Trivial mixing matrix renormalization:
  - „Philosophical" arguments:
    - Comparing arbitrary bases
  - „Practical" arguments:
    - Gauge-(in-)dependence
    - (non-) singular $m_j \to m_i$ limit
- Conclusions

The setup I

Consider n bare fields $\vec{\phi}_0 = \begin{pmatrix} \phi_1^0 \\ \vdots \\ \phi_n^0 \end{pmatrix}$ related to $\vec{h}_0 = R_0 \vec{\phi}_0 = \begin{pmatrix} h_1^0 \\ \vdots \\ h_n^0 \end{pmatrix}$

arbitrary $n \times n$ rotation matrix

$R_0^T R_0 = I$

The bare kinetic term

mass² matrix

$$\mathcal{K} = \vec{\phi}_0^T (p^2 - M_0^2) \vec{\phi}_0$$

$$= \vec{h}_0^T (p^2 - R_0^T M_0^2 R_0) \vec{h}_0$$

$$= \vec{h}_0^T (p^2 - \tilde{M}_0^2) \vec{h}_0$$

momentum²

Bases

1

2.1

2.2

$$\tilde{M}_0^2 = R_0^T M_0^2 R_0$$

Full Lagrangian not needed to discuss R!

The setup II

(blindly) Introducing the counterterms

$$\vec{\Phi}_0 = (I + \delta Z)\vec{\Phi} \; ; \; \vec{h}_0 = (I + \delta\tilde{Z})\vec{h}$$

$$M_0^2 = M^2 + \delta M^2 \; ; \; \tilde{M}_0^2 = \tilde{M}^2 + \delta\tilde{M}^2$$

Importantly: $R_0 = R + \delta R$

with $R^T R = I$ and $\delta(R_0^T R_0) = 0 \Rightarrow \boxed{R^T \delta R = -\delta R^T R}$

The setup III

Kinetic term at 1-loop: Bases

$$K = \vec{\Phi}^T \left(p^2 - M^2 + \delta Z^T (p^2 - M^2) + (p^2 - M^2)\delta Z - \delta M^2\right) \vec{\Phi} \qquad 1$$

$$= \vec{h}^T \left(p^2 - R^T M^2 R + \delta \tilde{Z}^T (p^2 - R^T M^2 R) + (p^2 - R^T M^2 R)\delta \tilde{Z} - \delta R^T M^2 R - R^T M^2 \delta R - R^T \delta M^2 R\right) \vec{h} \qquad 2.1$$

$$= \vec{h}^T \left(p^2 - \tilde{M}^2 + \delta \tilde{Z}^T (p^2 - \tilde{M}^2) - (p^2 - \tilde{M}^2)\delta \tilde{Z} - \delta \tilde{M}^2\right) \vec{h} \qquad 2.2$$

The setup III

Kinetic term at 1-loop : Bases

$$K = \vec{\Phi}^T\left(p^2 - M^2 + \delta Z^T (p^2 - M^2) + (p^2 - M^2)\delta Z - \delta M^2\right)\vec{\Phi} \qquad 1$$

$$= \vec{h}^T\left(p^2 - \tilde{M}^2 + \delta\tilde{Z}^T(p^2 - \tilde{M}^2) + (p^2 - \tilde{M}^2)\delta\tilde{Z} + \delta R^T R\tilde{M}^2 - \tilde{M}^2 R^T\delta R - \underline{R^T\delta M^2 R}\right)\vec{h} \qquad 2.1$$

$$\boxed{\tilde{M}^2 = R^T M^2 R}$$

$$= \vec{h}^T\left(p^2 - \tilde{M}^2 + \delta\tilde{Z}^T(p^2 - \tilde{M}^2) - (p^2 - \tilde{M}^2)\delta\tilde{Z} - \delta\tilde{M}^2\right)\vec{h} \qquad 2.2$$

Arguments for having

$$\delta R = 0$$

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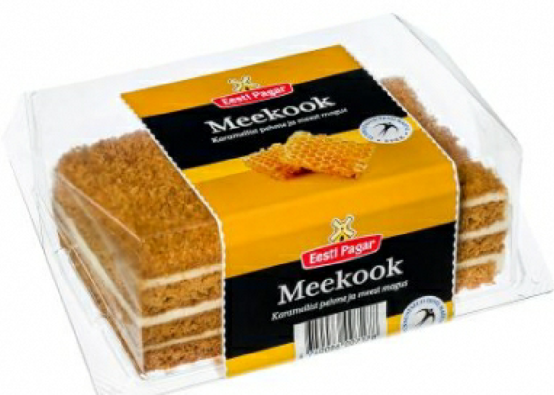

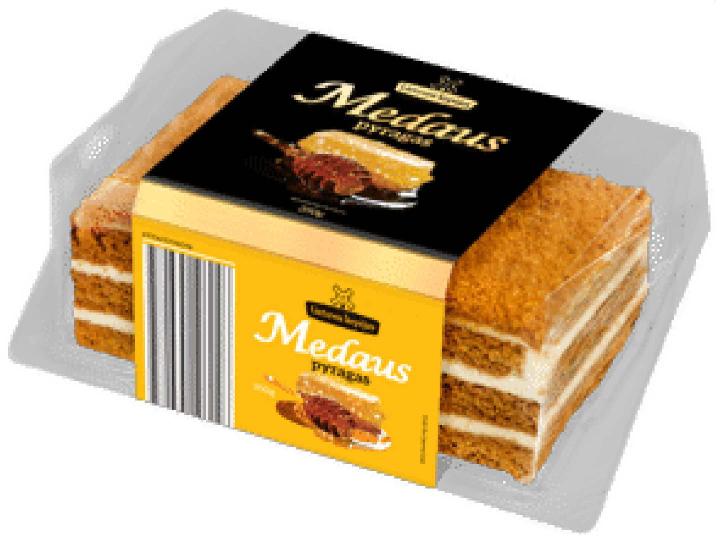

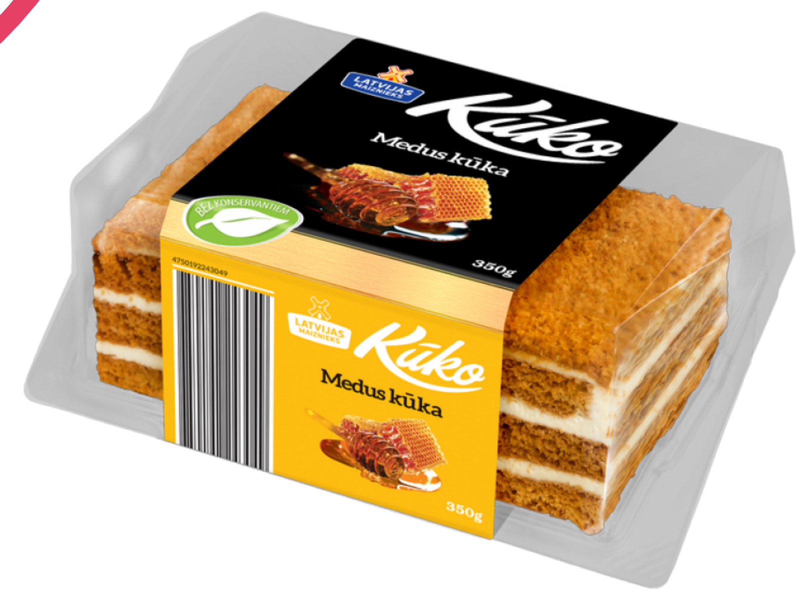

Basis-independence I

$$\mathcal{K} = \begin{matrix}\vec{h}^T \\ \vec{\phi}^T\end{matrix}\left(p^2 - \tilde{m}^2 + \delta\tilde{Z}^T(p^2-\tilde{m}^2) + (p^2-\tilde{m}^2)\delta\tilde{Z} - \delta\tilde{m}^2\right)\begin{matrix}\vec{h} \\ \vec{\phi}\end{matrix}$$ Bases 1 and 2.2

Both have the same form!

$$\mathcal{K} = \vec{h}^T\left(p^2-\tilde{m}^2 + \delta\tilde{Z}^T(p^2-\tilde{m}^2) + (p^2-\tilde{m}^2)\delta\tilde{Z} - R^T\delta m^2 R + \boxed{R^T\delta R\,\tilde{m}^2 - \tilde{m}^2 R^T\delta R}\right)\vec{h}$$ Basis 2.1

Basis 2.1 with $\delta R = 0$ looks the same and implies

$$\boxed{\delta\tilde{Z} = R^T\delta Z R, \quad \delta\tilde{m}^2 = R^T\delta m^2 R, \quad \vec{\phi} = R\vec{h}, \quad R_0 = R}$$

i.e. with $\delta R = 0$ the same form of the Lagrangian and the same transformation rules

<u>in all bases!</u> ✓

Otherwise, if $\delta R \neq 0$, $\delta\tilde{Z}^A = R^T\delta Z^A R - R^T\delta R$ ✗

Basis-independence II

Rearrange in Basis 2.1 (with $\partial R \neq 0$)

anti-commutator

$$\mathcal{K} = \vec{h}^T \Big(p^2 - \tilde{M}^2 + \big\{ p^2 - \tilde{M}^2 , \delta \tilde{Z}^S \text{ symmetric} \big\} - R^T \delta M^2 R$$

commutator

$$- \big[\tilde{M}^2 , \underbrace{R^T \delta R + \delta \tilde{Z}^A}_{\text{DEGENERATE!}} \text{ anti-symmetric} \big] \Big) \vec{h}$$

Basis-independence II

Rearrange in Basis 2.1 (with $\partial R \neq 0$)

anti-commutator

$$\mathcal{K} = \vec{h}^T\Big(p^2 - \tilde{m}^2 + \{p^2 - \tilde{m}^2, \delta\tilde{Z}^{S}_{\text{symmetric}}\} - R^T\delta M^2 R$$

commutator

$$- [\tilde{m}^2, \underbrace{R^T\delta R + \delta\tilde{Z}^{A}}_{\text{DEGENERATE!}}{}_{\text{anti-symmetric}}]\Big)\vec{h}$$

- Can fix $R^T\partial R + \partial\tilde{Z}^A$, but not ∂R or $\partial\tilde{Z}^A$ separately!
- $\partial\tilde{Z}^A$ can „absorb" $R^T\partial R$!
- $\partial\tilde{Z}^A$ is not physical $\Longrightarrow$ so is $R^T\partial R$!

$\partial R = 0$

Basis-independence III

Rotate back from basis 2.1 by $\vec{h}' = R^T \vec{h}$, then

$$\mathcal{K} = \vec{h}'\Big(\underbrace{p^2 - M^2 + \{p^2 - M^2, \delta Z^S\} - \delta M^2}_{\text{Basis 1, no } R!} \underbrace{- [M^2, \delta R R^T + R \delta \tilde{Z}^A R^T]}_{\text{Different counter terms!}}\Big)\vec{h}'$$

Basis-independence !!!

Rotate back from basis 2.1 by $\vec{h}' = R^T \vec{h}$, then

$$\mathcal{K} = \vec{h}'\Big(\underbrace{p^2 - M^2 + \{p^2 - M^2, \delta Z^s\} - \delta M^2}_{\text{Basis 1, no } R!} - \underbrace{[M^2, \delta R R^T + R \delta \tilde{Z}^A R^T]}_{\text{Different counterterms!}}\Big)\vec{h}'$$

- Two-point function as in Basis 1, but renormalized differently unless $\delta Z^A = R\delta\tilde{Z}^A R^T + \delta R R^T$!
- Rotating with R^T removes R from the Lagrangian, but keeps δR!

$\Longrightarrow$ $\boxed{\text{Cannot form the bare parameter } R_0!}$

Basis-independence IV

Restoring the bare Lagrangian in $\vec{h}'$ basis by inverting $\vec{h}'_0 = (I + \delta Z)\vec{h}'$

$$\mathcal{K}' = \vec{h}'_0 \left(p^2 - M_0^2 - [M^2, \delta R R^T + R \delta \tilde{Z}^A R^T - \delta Z^A]\right) \vec{h}'_0 \neq \mathcal{K}$$

The bare Lagrangian changes if $\delta R \neq 0$!

(unless $\delta Z^A = R \delta \tilde{Z}^A R^T + \delta R R^T$, not ensured automatically)

Basis-independence V

Start in Basis 1 and rotate to Basis 2.1

$$\{\delta Z, \delta M^2, \delta \ldots\} \xRightarrow{R} \{\delta \tilde{Z}, \delta \tilde{M}^2, \delta R, \delta \tilde{\ldots}\}$$

But, can also

start in Basis 2.2 and rotate to Basis 1

$$\{\delta \tilde{Z}, \delta \tilde{M}^2, \delta \tilde{\ldots}\} \xRightarrow{R^T} \{\delta Z, \delta M^2, \delta R, \delta \ldots\}$$

δR can „select" any basis! Best to have:

$$\boxed{\{\delta Z, \delta M^2, \delta \ldots\} \overset{R^T}{\underset{R}{\Longleftrightarrow}} \{\delta \tilde{Z}, \delta \tilde{M}^2, \delta \tilde{\ldots}\}, \text{ i.e. } \delta R = 0}$$

Practicalities: gauge dependence

Gauge dependence via Nielsen Identities

$$\partial_\xi \Pi^0 = \Lambda^T (p^2 - \tilde{m}^2) + (p^2 - \tilde{m}^2)\Lambda$$

from BRST

describes gauge-dependence

$\Lambda = \Lambda^S + \Lambda^A$

$(\Lambda^S)^T = \Lambda^S$, $(\Lambda^A)^T = -\Lambda^A$

ξ: gauge param.

Π^0: bare self-energy

No Λ for the mass

$$\partial_\xi \Pi = \{\partial_\xi \delta\tilde{Z}^S + \Lambda^S, p^2 - \tilde{m}^2\} - \boxed{[\tilde{M}^2, R^T \partial_\xi \delta R + \partial_\xi \delta\tilde{Z}^A + \Lambda^A]} - R^T \partial_\xi \delta m^2 R$$

Π: Renormalized self-energy

natural gauge-dependence!

- δR can only be made gauge-independent by hand!
- More consistent (simple) to set $\boxed{\delta R = 0}$

Practicalities: $m_j \to m_i$ limit 1

Take Basis 2.1 with $\tilde{M}^2 = \mathrm{diag.}(m_1^2, m_2^2, \dots, m_n^2)$

$R^T \delta M^2 R$ is not diagonal in general!

The renormalized self-energy is then

$$\Pi_{ij} = \Pi^0_{ij} + \delta\tilde{Z}^S_{ij}(p^2 - m_i^2) + (p^2 - m_j^2)\delta\tilde{Z}^S_{ij}$$
$$- (m_i^2 - m_j^2)\left((R^T \delta R)_{ij} + \delta\tilde{Z}^A_{ij}\right) - (R^T \delta M^2 R)_{ij}$$

Practicalities: $m_j \to m_i$ limit 1

Take Basis 2.1 with $\tilde{M}^2 = \mathrm{diag}(m_1^2, m_2^2, \dots, m_n^2)$

$R^T \delta M^2 R$ is not diagonal in general!

The renormalized self-energy is then

$$\Pi_{ij}\Big|^{\cancel{p^2=m^2_{i/j}}}_{UV} = \Pi^0_{ij}\Big|^{\cancel{p^2=m^2_{i/j}}}_{UV} + \cancel{\delta\tilde{Z}^s_{ij}(p^2-m_i^2) + (p^2-m_j^2)\delta\tilde{Z}^s_{ij}}$$
$$- (m_i^2-m_j^2)\left((R^T\delta R)_{ij} + \delta\tilde{Z}^A_{ij}\right) - \cancel{(R^T\delta M^2 R)_{ij}}$$

- UV parts; no mass ct.; off-diag ($i \neq j$); no $p^2 - m_{i,j}$ terms

↳ very common!

Practicalities: $m_j \to m_i$ limit II

$$\Pi_{ij}\Big|_{UV}^{p^2=m^2_{ij}} = \Pi^0_{ij}\Big|_{UV}^{p^2=m^2_{ij}} - (m_i^2 - m_j^2)\left((R^T\delta R)_{ij} + \delta\tilde{Z}^A_{ij}\right)$$

Has to be UV-finite!

$$\Rightarrow \Pi^0_{ij}\Big|_{UV}^{p^2=m^2_{ij}} = (m_i^2 - m_j^2)\left((R^T\delta R)_{ij} + \delta\tilde{Z}^A_{ij}\right)$$

For $m_j \to m_i$: $\Pi^0_{ij}\Big|_{UV}^{p^2=m^2_{ij}} \neq 0$, but $(m_i^2 - m_j^2) = 0$

$$\Rightarrow (R^T\delta R)_{ij} + \delta\tilde{Z}^A_{ij} \sim \frac{1}{m_i^2 - m_j^2} \xrightarrow{m_j \to m_i} \infty!$$

Practicalities: $m_j \to m_i$ limit III

Mass ct. is the solution

$$\Pi^0_{ij}\Big|_{UV}^{p^2=m^2_{i,j}} = (m_i^2 - m_j^2)\left((R^T\delta R)_{ij} + \delta\tilde{Z}^A_{ij}\right) - (R^T\delta\mathcal{M}^2 R)_{ij}$$

For $m_j \to m_i$: $\Pi^0_{ij}\Big|_{UV}^{p^2=m^2_{i,j}} = -(R^T\delta\mathcal{M}^2 R)_{ij}$!

$$\Longrightarrow \quad (R^T\delta R)_{ij} + \delta\tilde{Z}^A_{ij} \text{ non-singular}$$

δR could only account for $\boxed{(m_i^2 - m_j^2)\times \text{gauge-indep.}}$

$\Longrightarrow$ Simpler and <u>consistent</u> to have $\delta R = 0$

Conclusions

- Some bases are convenient, but still arbitrary
- Rotation matrices are basis-dependent objects
- The counterterm δR should be trivial for basis-independence
- $\delta R = 0$ also avoids practical issues

Implications for RG running I

Usually, running is determined from counterterms, but $R_0 = R \Leftrightarrow \delta R = 0$. So what happens?

In Basis 1 $\quad \partial_{s\text{cale}} M_0^2 = 0 \Rightarrow \partial_s M^2 = -\partial_s \delta M^2$

Change to Basis 2 (.1/2)

$$\partial_s (R \tilde{M}^2 R^T) = -\partial_s (R\, \delta \tilde{M}^2 R^T)$$

algebra ....

$$\partial_s \tilde{M}^2 = -\partial_s \delta \tilde{M}^2 + [\tilde{M}^2 + \delta \tilde{M}^2, R^T \partial_s R]$$

or

$$\boxed{\partial_s \tilde{M}_0^2 = [\tilde{M}_0^2, R^T \partial_s R]}$$

Implications for RG running II

$$\boxed{\partial_s \tilde{M}_0^2 = [\tilde{M}_0^2, R^T \partial_s R]}$$

If $\partial_s R \neq 0$, then $\partial_s \tilde{M}_0^2 \neq 0$! Contradiction!

So $\boxed{\partial_s R = 0}$, but the whole $\tilde{M}^2$ runs!

If $\tilde{M}^2(s_0) = \text{diag.}$, then $\tilde{M}^2(s_1) \neq \text{diag.}$

But at s_1 can rediagonalize with R'.

No „natural" running between arbitrary bases.

Implications for RG running III

$$\partial_s \tilde{M}^2_{ij} = (m_i^2 - m_j^2)(R^T \partial_s R)_{ij} \,, \quad \text{for } i \neq j$$

$$\Rightarrow \quad \partial_s R_{ij} \sim \sum_k R_{ik} \frac{1}{m^2_{i(k)} - m_j^2} \, \partial \tilde{M}^2_{kj}$$

- $(m_i^2 - m_j^2)^{-1}$ is singular and associated with gauge-dependence! (comes from the commutator)
- Keeping $\tilde{M}^2 = \text{diag.}$ selects a family of R, but the choice is not physical.