Oblique corrections, when $m_W \neq \cos\theta_W m_Z$

(based on 2305.14050)

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Oblique parameters

- Electroweak sector in the SM is highly predictive:
  - given 3 measured input values e.g. (G_F, α, m_Z), SM predicts all the rest of the electroweak observables (Z decays, $\sin\theta_W^{\text{effective}}$, ρ_*, LR, FB assymetries....)
- Consider Beyond Standard model (BSM), with these assumptions
  - Only $SU(2) \times U(1)$ gauge fields (only the SM-like)
  - Tree level relation $m_W = \cos\theta_W m_Z$ holds
- **Oblique** parameters, $S, T, U \in \mathbb{R}$, paremeterize the difference between the SM prediction for the observable O_{SM} and the BSM prediction O_{BSM}[Peskin 1992]:

$$O_{BSM} = O_{SM}\left(1 + a_1 S + a_2 T + a_3 U\right) ,$$

where coefficients $a_i \in \mathbb{R}$ are calculated for each observable.

SM at tree-level

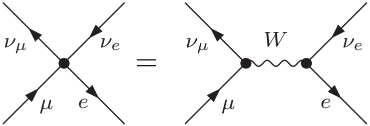
- Weinberg angle

$$s \equiv \sin\theta_W\,, \quad c \equiv \cos\theta_W$$

$$A_\mu = cB_\mu + sW^3_\mu\,, \quad Z_\mu = cW^3_\mu - sB_\mu$$

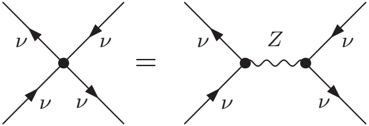
- We will label all **tree-level** parameters with **hats**.
- Fermi constant can be measured from $\mu \to e\nu\nu$ decay:

$$\widehat{G}_{F(charged)} = \frac{\sqrt{2}\widehat{e}^2}{8\widehat{s}^2\,\widehat{m}^2_W} \quad \Leftrightarrow$$



- Analogously (in principle), $\nu\nu \to \nu\nu$:

$$\widehat{G}_{F(neutral)} = \frac{\sqrt{2}\widehat{e}^2}{8\widehat{s}^2\widehat{c}^2\,\widehat{m}^2_Z} \quad \Leftrightarrow$$



ρ ir ρ_*

- Veltmann ρ parameter at tree-level:

$$\widehat{\rho} = \frac{\widehat{G}_{F(neutral)}}{\widehat{G}_{F(charged)}} = \frac{\widehat{m}_W^2}{\widehat{c}^2\,\widehat{m}_Z^2}$$

- SM scalar potential has **custodial** $SU(2)$ symmetry, which implies $\widehat{\rho} = 1$.
- Veltmann ρ gets corrections at higher orders:

$$\rho \equiv \frac{m_W^2}{c^2 m_Z^2} = \widehat{\rho}\,(1 + \text{loops})\,,$$

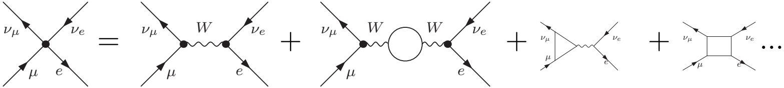
- Define "rho-star" parameter

$$\rho_* \equiv \frac{G_{F(neutral)}}{G_{F(charged)}} = \widehat{\rho}\,(1 + (\text{other})\ \text{loops})$$

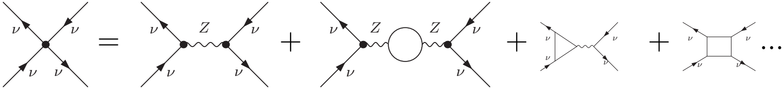
- In general $\rho \neq \rho_*$.
- ρ definition depends on which 3 EW observables one takes as an input.
- Both equations are **predictions**, when $\widehat{\rho} = 1$.

- At one-loop Fermi constant gets both "**oblique**" (i.e. via gauge boson propagator only) corrections, and "direct" (via triangle an box diagrams):

$$G_{F(charged)} = \widehat{G}_{F(charged)} \left(1 - \frac{\Pi_{WW}(0)}{m_W^2} + \triangle + \square\right)$$



$$G_{F(neutral)} = \widehat{G}_{F(neutral)} \left(1 - \frac{\Pi_{ZZ}(0)}{m_Z^2} + \triangle + \square\right),$$



$\triangle + \square$ are triangle and box diagrams (direct corrections). They will be neglected in the end.

ρ_* corrections

- Use the corrected G_F expressions:

$$\rho_* = \frac{G_{F(neutral)}}{G_{F(charged)}} = \frac{\widehat{G}_{F(neutral)}\left(1 - \frac{\Pi_{ZZ}(0)}{m_Z^2} + \triangle + \square\right)}{\widehat{G}_{F(charged)}\left(1 - \frac{\Pi_{WW}(0)}{m_W^2} + \triangle + \square\right)}$$

$$= \widehat{\rho}\left(1 + \frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2} + O(2\text{ loops}) + \triangle + \square\right)$$

- In SM $\widehat{\rho} = 1$, then $(\rho_*)_{SM}$ is calculated as **prediction** from (G_F, α, m_Z) and **finite** loop corrections
- When $\widehat{\rho}$ is a free parameter, ρ_* is **not** predicted from (G_F, α, m_Z), since ρ_* depends on $\widehat{\rho}$
  $\Rightarrow$loop corrections are **not** finite
  $\Rightarrow$one needs additional input parameter to fix $\widehat{\rho}$ (i.e. renormalize ρ parameter).

$$\rho_* = \widehat{\rho}\left(1 + \frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2} + \triangle + \Box\right) \tag{1}$$

- Consider that we have two predictions: SM prediction $(\rho_*)_{SM}$ and BSM prediction $(\rho_*)_{BSM}$.
- If BSM has custodial symmetry, $\widehat{\rho} = 1$, both $(\rho_*)_{SM}$ and $(\rho_*)_{BSM}$ can be calculated from (1).
- Divide one from the other, neglect $\triangle + \Box$ (BSM contribution is small [Kennedy&Lynn1989, Peskin1992]) and $O(2 \text{ loops})$:

$$\frac{(\rho_*)_{BSM}}{(\rho_*)_{SM}} = \underbrace{\frac{\widehat{\rho}_{BSM}}{\widehat{\rho}_{SM}}}_{=1}\left(1 + \frac{\Pi_{WW}^{new}(0)}{m_W^2} - \frac{\Pi_{ZZ}^{new}(0)}{m_Z^2}\right) \equiv 1 + \alpha T,$$

in which $\Pi^{new} = \Pi^{BSM} - \Pi^{SM}$, i.e. **only** BSM contribution to self-energies

- T is **oblique** parameter.

- Most often used definitions for oblique parameters ($\widetilde{\Pi}\left(m^2\right) \equiv \frac{\Pi\left(m^2\right)-\Pi(0)}{m^2}$):

$$S = \frac{4s^2c^2}{\alpha}\left[\widetilde{\Pi}_{ZZ}\left(m_Z^2\right) + \frac{s^2-c^2}{sc}\Pi'_{ZA}(0) - \Pi'_{AA}(0)\right],$$

$$T = \frac{1}{\alpha}\left[\frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2}\right],$$

$$U = \frac{4s^2}{\alpha}\left[\widetilde{\Pi}_{WW}\left(m_W^2\right) - c^2\widetilde{\Pi}_{ZZ}\left(m_Z^2\right) - 2sc\Pi'_{ZA}(0) - s^2\Pi'_{AA}(0)\right].$$

ρ and θ_W definitions

- In the same way we can derive oblique corrections for $\rho = \frac{m_W^2}{c^2 m_Z^2}$.
- When $\widehat{\rho} = 1$, we choose input: $\left(G_{F(charged)}, \alpha = \frac{e^2}{4\pi}, m_W\right)$
- Then "observed" c^2 is expressed in terms of input by promoting tree-level expression at all loops:

$$\widehat{G}_{F(charged)} = \frac{\sqrt{2}\widehat{e}^2}{8\widehat{s}^2\,\widehat{m}_W^2} \longrightarrow 1 - c^2 \equiv \frac{\pi\alpha}{\sqrt{2}G_{F(charged)}m_W^2}$$

- We can also choose $\left(G_{F(charged)}, \alpha, m_Z\right)$ [Peskin1992], then (using $\widehat{m}_W = \widehat{c}\widehat{m}_Z$!) we define:

$$\widehat{G}_{F(charged)} = \frac{\sqrt{2}\widehat{e}^2}{8\widehat{s}^2\widehat{c}^2\,\widehat{m}_Z^2} \longrightarrow \bar{c}^2\left(1 - \bar{c}^2\right) \equiv \frac{\pi\alpha}{\sqrt{2}G_{F(charged)}m_Z^2}$$

- c and $\bar{c}$ differs at one-loop by definition and thus ρ is different from $\bar{\rho}$!

ρ and $\bar{\rho}$

- Let us calculate $\rho = \frac{m_W^2}{c^2 m_Z^2}$ (T^{full}, means all oblique corrections of a model).

$$\rho = \frac{m_W^2}{c^2 m_Z^2} = \frac{\widehat{m}_W^2 + \delta m_W^2}{(\widehat{c}^2 + \delta c^2)\left(\widehat{m}_Z + \delta m_Z^2\right)} \quad = \widehat{\rho}\left(1 + \alpha T^{\text{full}} - \alpha K^{\text{full}} + \triangle + \square\right)$$

$$\bar{\rho} = \frac{m_W^2}{c^2 m_Z^2} = \frac{\widehat{m}_W^2 + \delta m_W^2}{(\widehat{c}^2 + \delta \bar{c}^2)\left(\widehat{m}_Z + \delta m_Z^2\right)} \quad = \widehat{\rho}\left(1 + \frac{\bar{c}^2}{\bar{c}^2 - \bar{s}^2}\left[\alpha T^{\text{full}} - \alpha K^{\text{full}}\right] + \ldots\right)$$

$$K \equiv \frac{1}{2c^2} S + \frac{s^2 - c^2}{4s^2 c^2} U$$

- SM vs. BSM predictions (when $\widehat{\rho} = 1$):

$$\rho_{BSM} = \rho_{SM}\left(1 + \alpha T - \alpha K\right), \qquad \text{when input is } \left(G_{F(charged)}, \alpha, m_W\right)$$

$$\bar{\rho}_{BSM} = \bar{\rho}_{SM}\left(1 + \frac{\bar{c}^2}{\bar{c}^2 - \bar{s}^2}\left[\alpha T - \alpha K\right]\right), \qquad \text{when input is } \left(G_{F(charged)}, \alpha, m_Z\right)$$

when $\widehat{\rho} \neq 1$

- Consider we have a **base model** BM with a more complex scalar potential giving unfixed $\widehat{\rho} \neq 1$, and some **beyond base model** BBM (also with $\widehat{\rho} \neq 1$).
- Then
$$\widehat{G}_{F(charged)} = \frac{\sqrt{2}\widehat{e}^2}{8\widehat{s}^2\widehat{m}_W^2} \neq \frac{\sqrt{2}\widehat{e}^2}{8\widehat{s}^2\widehat{c}^2\widehat{m}_Z^2} = \widehat{G}_{F(neutral)}$$
and so (Peskin used, when $\widehat{\rho} = 1$) $\bar{\rho}$, $\bar{c}$ definitions cannot be used!
- Equation
$$\rho = \widehat{\rho}\left(1 + \alpha T^{\text{full}} - \alpha K^{\text{full}} + \triangle + \square\right)$$
not a prediction neither in BM nor BBM, when $\widehat{\rho} \neq 1 \Rightarrow$ need additional input.
- input $\left(G_{F(charged)}, \alpha, m_W, m_Z\right)$ gives by definition:

$$\frac{m_W^2}{c^2 m_Z^2} = \frac{m_W^2}{\left[1 - \frac{\pi\alpha}{\sqrt{2}G_{F(charged)}m_W^2}\right] m_Z^2} = \rho_{BM} = \rho_{BBM} \quad \Rightarrow \quad \frac{\widehat{\rho}_{BBM}}{\widehat{\rho}_{BM}} = (1 - \alpha T + \alpha K)$$

BM vs. BBM

- Previously we derived

$$\frac{(\rho_*)_{BSM}}{(\rho_*)_{SM}} = \frac{\widehat{\rho}_{BSM}}{\widehat{\rho}_{SM}}(1+\alpha T) = (1+\alpha T)$$

since $\widehat{\rho}_{BSM} = \widehat{\rho}_{SM} = 1$.

- Using the first equation, and $BSM \to BBM$ ir $BM \to SM$, plug $\frac{\widehat{\rho}_{BBM}}{\widehat{\rho}_{BM}} = (1 - \alpha T + \alpha K)$ we get:

$$\frac{(\rho_*)_{BBM}}{(\rho_*)_{BM}} = (1+\alpha K)$$

Result: when $\widehat{\rho} \neq 1$, we use as input $\left(G_{F(charged)}, \alpha, m_W, m_Z\right)$ in both BM and BBM, then we can use same expressions as SM/BSM [Peskin1992, Maksymyk1994] with a substitution:

$$T \to K = \frac{1}{2c^2}S + \frac{s^2 - c^2}{4s^2c^2}U$$

Problems for SM vs. BBM

- One needs different number of input parameters (4 and 3).
- Since $m_W^{SM} \neq m_W^{BBM}$ gauge sector does not cancel in the same way, i.e.:

  $$\Box^{BBM} + \triangle^{BBM} - \Box^{SM} - \triangle^{SM} = \text{Gauge dependent} \Leftrightarrow \Pi^{BBM} - \Pi^{SM} = \text{Gauge dependent}$$

  $\Rightarrow$ oblique parameters are gauge dependent
  tested for S, U parameters in triplet extensions of SM.
- checks of [Kennedy&Lynn1989] which shows $\Box + \triangle$ being negligible does not apply.

Summary

- When $\widehat{\rho} = 1$ in both SM and BSM, well known expressions exist with input $\left(G_{F(charged)}, \alpha, m_Z\right)$[Peskin 1992]:

$$O_{BSM} = O_{SM}\left(1 + a_1 S + a_2 T + a_3 U\right)$$

- When $\widehat{\rho} \neq 1$ in two models(BM and BBM), we can compare their predictions with the same equations with a substutution $T \to K$ and input parameters $\left(G_{F(charged)}, \alpha, m_Z, m_W\right)$

$$O_{BBM} = O_{BM}\left(1 + a_1 S + a_2 \left[\frac{1}{2c^2} S + \frac{s^2 - c^2}{4s^2 c^2} U\right] + a_3 U\right),$$

- Not yet clear: how to compare SM($\widehat{\rho} = 1$) with BBM($\widehat{\rho} \neq 1$):
$\Rightarrow$I would be careful, interpeting studies of ($\widehat{\rho} \neq 1$) models with oblique parameters (as they use formalism, which is derived for $\widehat{\rho} = 1$ only)...

Thank you!

Thanks

All parameters

- STUVWX parameters

$$S=\frac{4s^2c^2}{\alpha}\left[\widetilde{\Pi}_{ZZ}\left(m_Z^2\right)+\frac{s^2-c^2}{sc}\Pi'_{ZA}\left(0\right)-\Pi'_{AA}\left(0\right)\right]$$

$$T=\frac{1}{\alpha}\left[\frac{\Pi_{WW}\left(0\right)}{m_W^2}-\frac{\Pi_{ZZ}\left(0\right)}{m_Z^2}\right]$$

$$U=\frac{4s^2}{\alpha}\left[\widetilde{\Pi}_{WW}\left(m_W^2\right)-c^2\widetilde{\Pi}_{ZZ}\left(m_Z^2\right)-2sc\Pi'_{ZA}\left(0\right)-s^2\Pi'_{AA}\left(0\right)\right]$$

$$V=\frac{1}{\alpha}\left[\Pi'_{ZZ}\left(m_Z^2\right)-\widetilde{\Pi}_{ZZ}\left(m_Z^2\right)\right]$$

$$W=\frac{1}{\alpha}\left[\Pi'_{WW}\left(m_W^2\right)-\widetilde{\Pi}_{WW}\left(m_W^2\right)\right],$$

$$X=\frac{1}{\alpha}\left[\Pi'_{ZA}\left(0\right)-\widetilde{\Pi}_{ZA}\left(m_Z^2\right)\right]$$