A Multi-Class Quantum Kernel-Based Classifier

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An L-class classification problem can be defined as follows: given a dataset

$$
D = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_m, y_m)\}\
$$

 \mathbf{x}_i consisting of pairs (\mathbf{x}_i, y_i) that contain the data $\mathbf{x}_i \in \mathbb{R}^N$ and their respective labels $y_i \in \{1,...L-1\}$, determine the label $y_i \in \{1,...L-1\}$ corresponding to some new, unseen datum $\tilde{\mathbf{x}}$.

Multi-class classification Problem Statement

Given two classical datapoints, a quantum kernel can be estimated as the squared state overlap between quantum states encoding these two datapoints.

datapoints **x** and **z** is n -qubit quantum state as $|\Phi(\mathbf{x})\rangle = U_\Phi(\mathbf{x})|0\rangle^{\otimes n}$

$$
k(\mathbf{x}, \mathbf{z}) = |\langle \Phi(\mathbf{z}) | \Phi(\mathbf{x}) \rangle|^2
$$

If $U_{\Phi}(\mathbf{x})$ defines some unitary operation that encodes the classical datum \mathbf{x} into an -qubit quantum state as $\ket{\Phi(\mathbf{x})} = U_{\Phi}(\mathbf{x}) \ket{0}^{\otimes n}$ then the kernel of two classical

Quantum Kernels What is a quantum kernel?

Estimating quantum kernels Routines for evaluating quantum kernels

The Inversion Test **Inversion** Test

The binary SWAP-Test classifier is a quantum kernel method that allows us to estimate a **weighted power sum of kernel values** between a test datum and all the training data

The binary swap-test classifier

$$
\langle \sigma_z \rangle = \sum_{m=1}^{M} (-1)^{y_m} w_m k(\tilde{\mathbf{x}}, \mathbf{x}_m)^d \longrightarrow \tilde{y} = \frac{1}{2} (1 - \text{sgn} \langle \sigma_z \rangle)
$$

\n
$$
\begin{array}{ccc}\n\text{Step A} & \text{Step B} & \text{Step C} & \text{Step D} \\
\text{an:} & |0\rangle & \text{Step A} & \text{Step C} & \text{Step D} \\
\text{in:} & |0\rangle^{\otimes r} & \text{Step D} & \text{Step D} \\
\text{in:} & |0\rangle^{\otimes n} & \text{Step D} & \text{Step D} \\
\vdots & |0\rangle & \text{Step D} & \text{Step D} \\
\end{array}
$$

The multi-class swap-test classifier consisting of training data *{*x*i}^M ⁱ*=1 and their respective labels *{yi}^M ⁱ*=1, the goal of supervised classification is to develop a model for classifying unlabelled data. The algorithms for developing these models are called classifiers. This section describes the steps that constitute the multi-class SWAP-Test classifier. These steps are also outlined

[1] Blank C, Park DK, Rhee JK, Petruccione F. Quantum classifier with tailored quantum kernel. npj Quantum Information. 2020 May
15:6(1):41 15;6(1):41.

The second register is the second register is the second register in the index register with minimal overhead In
The training loint Conference on Neural in the training register with minimal overhead In2021 International J Networks (IJCNN) 2021 Jul 18 (pp. 1-7). IEEE. [2] Park DK, Blank C, Petruccione F. Robust quantum classifier with minimal overhead. In2021 International Joint Conference on Neural

The Multi-Class SWAP-Test Classifier is inspired by the binary SWAP-Test classifier [1,2].

The multi-class swap-test classifier **2.1 Classification with the Multi-Class SWAP-Test Classifier** Classification is a fundamental problem in machine learning. Given a dataset

following format: The algorithms for developing for developing α

a b) α and α and encoding the test datum $\tilde{\mathbf{x}}$, the training data $\{\mathbf{x}_m\}_{m=1}^M$ and their respective labels $\{y_m\}_{m=1}^M$ in the Given an L-class classification problem, the classifier is realised by first preparing a quantum state

 $\overline{w_m}$ $|0\rangle$ $|m\rangle$ $|\mathbf{x}_m\rangle$ $|\tilde{\mathbf{x}}\rangle$ $|y_m\rangle$

Each label is mapped to a unique label state: $y_i \rightarrow |y_i\rangle$

Each label state

The multi-class swap-test classifier Storing of labels

$$
|y_i\rangle = \cos\left(\frac{\theta_{y_i}}{2}\right)|0\rangle + e^{i\phi_{y_i}}\sin\left(\frac{\theta_{y_i}}{2}\right)|1\rangle
$$

with $0\leq\theta_{y_i}\leq\pi$ and $0\leq\phi_{y_i}\leq2\pi$ can be represented as a Bloch vector: $y_i =$ cos*ϕyi* sin*θyi* sin*ϕyi* sin*θyi* $cos\theta_{y_i}$

A modified SWAP-Test, involving a state reconstruction of the qubit storing the label states, is then performed on the prepared state.

The multi-class swap-test classifier *^D* ⁼ *{*(x*i, ^yi*)*}^M ⁱ*=1 [⊂] ^R*^N* [×] *{yi}^L ⁱ*=1*,* (1) develop a model for classification when the algorithms for developing th *ⁱ*=1 and their respective labels *{yi}^M*

This effectively yields a linear combination of label vectors,

kernel values between the test data and all the training data with that label.

y*pred* =

$$
= \sum_{i=1}^{L} \alpha_i \mathbf{y_i}
$$

The contribution of each label vector $\alpha_i = \sum w_m k(\tilde{\mathbf{x}}, \mathbf{x}_m)$ is a weighted sum of $m|y_m = i$

The multi-class swap-test classifier Measurement

The multi-class swap-test classifier At first, the significance of y*pred* may not seem clear. However, we can use the fact that the fidelities that result from the modified SWAP-Test represent a valid kernel *k*(x˜*,* x*m*) = *|*〈x˜*|*x*m*〉*|* ². Then, if α*ⁱ* = " *^m|*y*m*=*ⁱ* ^w*mk*(x˜*,* ^x*m*) the

The predicted vector is then used in the following assignment function: *L* α*i* $\overline{}$ $\overline{\mathbf{v}}$ sinφ*yⁱ* sinθ*yⁱ* **a** 2991 TUILEE TUILLIUIL.

$$
\mathbf{X}_{y_i} \{ \mathbf{y_i} \cdot \mathbf{y}_{pred} \}
$$

which is evaluated classically. α*ⁱ* increases the overlap between the y*pred* and y*ⁱ* and indicates a high similarity between the test datum and the

i=1

Robustness to noise

We also consider the effect of a single qubit **depolarising** channel acting only on the label qubit right before the required measurement.

Our analysis shows that the predicted vector obtained is **only scaled** by a factor of $(1-p)$.

This has **no effect** on the outcome of the classification.

Through variance analysis, we show that the number of label states that can be accurately distinguished on a single qubit **grows linearly** with the number of repetitions of the required measurements.

No. of Label States $= O(R)$

Number of classes

The effectiveness of the multi-class SWAP-Test classifier is demonstrated by applying Table 2. The accuracies obtained by the multi-class SWAP-Test class SWAP-Test can be much contained by the multi-

it to a number of different classification problems.

Effectiveness which would arise from amplitude encoding. In other cases, we use the kernel *k*(x*,* z) = ² which would arise from angle encoding. Once the predicted vector is constructed, it is the assignment function given in the assignment function given in equation (14) to the assignment function \mathbf{v} COLASSIFY THE RESULTS OF THESE EXPERIMENTS OF THE RESULTS OF THE RESULTS OF THE RESULTS CAN BE SEEN IN TABLE 2. IT be seen that the accuracies are high, with accuracies are high, with a seen to 90% being than or equal to 90% being the accuracies greater than or equal to 90% being than or equal to 90% being than or equal to 90% being th Analytical results

The effectiveness of the multi-class SWAP-Test classifier is demonstrated by applying it to a number of different classification problems.

EFFECTIVENESS depolarising noise in the circuits. Here, *R^y* is a Y-rotation with ! = 2*p*). NUMERICAL RESULTS. This is in Table 1999.

(1 *p*).

THANK YOU!