A MULTI-CLASS QUANTUM KERNEL-BASED CLASSIFIER

(Published in Advanced Quantum Technologies, https://onlinelibrary.wiley.com/doi/full/10.1002/qute.202300249)

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QUANTUM TECHNIQUES IN MACHINE LEARNING 2023

QTML

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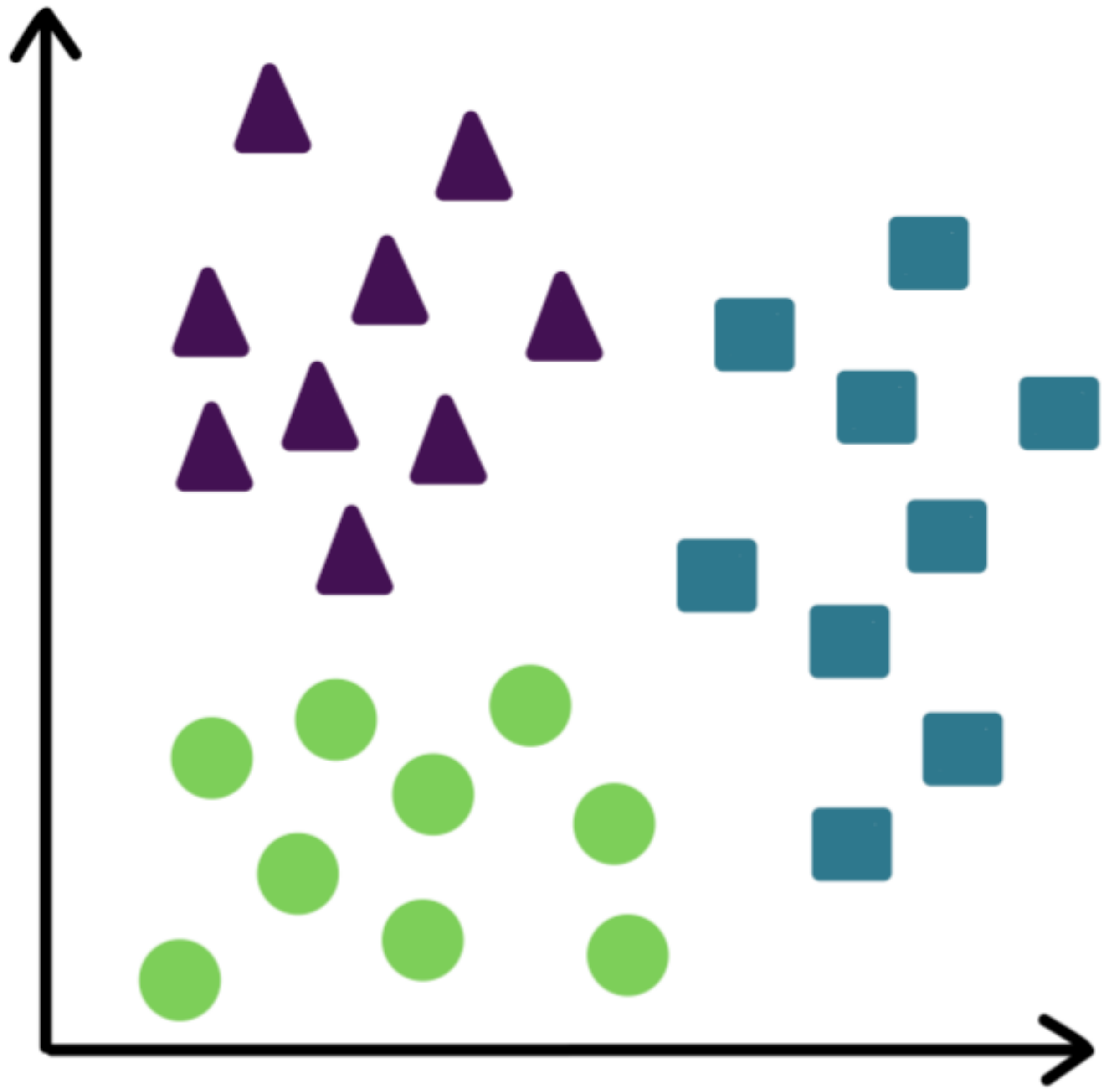
MULTI-CLASS CLASSIFICATION

PROBLEM STATEMENT

An L-class classification problem can be defined as follows: given a dataset

$$D = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_m, y_m)\}$$

consisting of pairs $(\mathbf{x}_i, y_i)$ that contain the data $\mathbf{x}_i \in \mathbb{R}^N$ and their respective labels $y_i \in \{1, \ldots L-1\}$, determine the label $y_i \in \{1, \ldots L-1\}$ corresponding to some new, unseen datum $\tilde{\mathbf{x}}$.



QUANTUM KERNELS

WHAT IS A QUANTUM KERNEL?

Given two classical datapoints, a quantum kernel can be estimated as the squared state overlap between quantum states encoding these two datapoints.

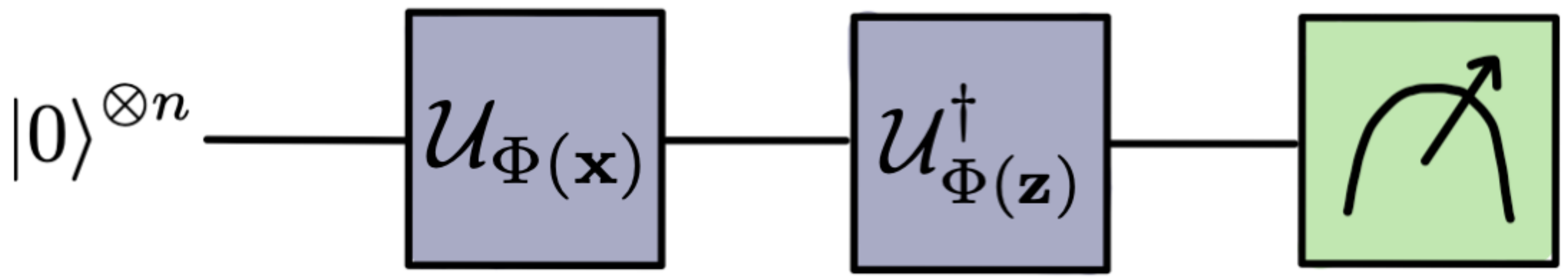
If $U_{\Phi}(\mathbf{x})$ defines some unitary operation that encodes the classical datum $\mathbf{x}$ into an n-qubit quantum state as $|\Phi(\mathbf{x})\rangle = U_{\Phi}(\mathbf{x})|0\rangle^{\otimes n}$ then the kernel of two classical datapoints $\mathbf{x}$ and $\mathbf{z}$ is

$$k(\mathbf{x}, \mathbf{z}) = |\langle\Phi(\mathbf{z})|\Phi(\mathbf{x})\rangle|^2$$

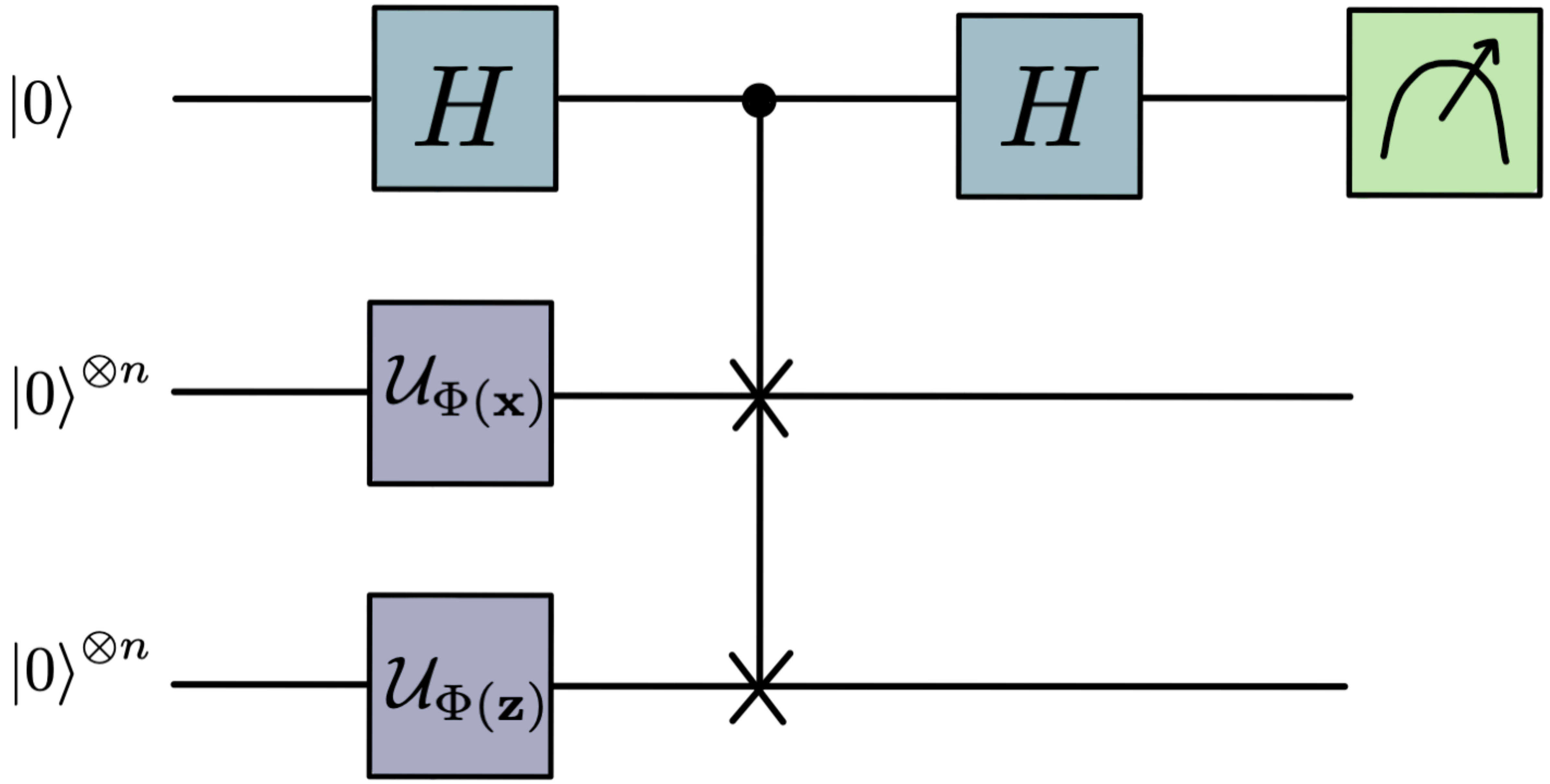
ESTIMATING QUANTUM KERNELS

ROUTINES FOR EVALUATING QUANTUM KERNELS

The **Inversion Test**



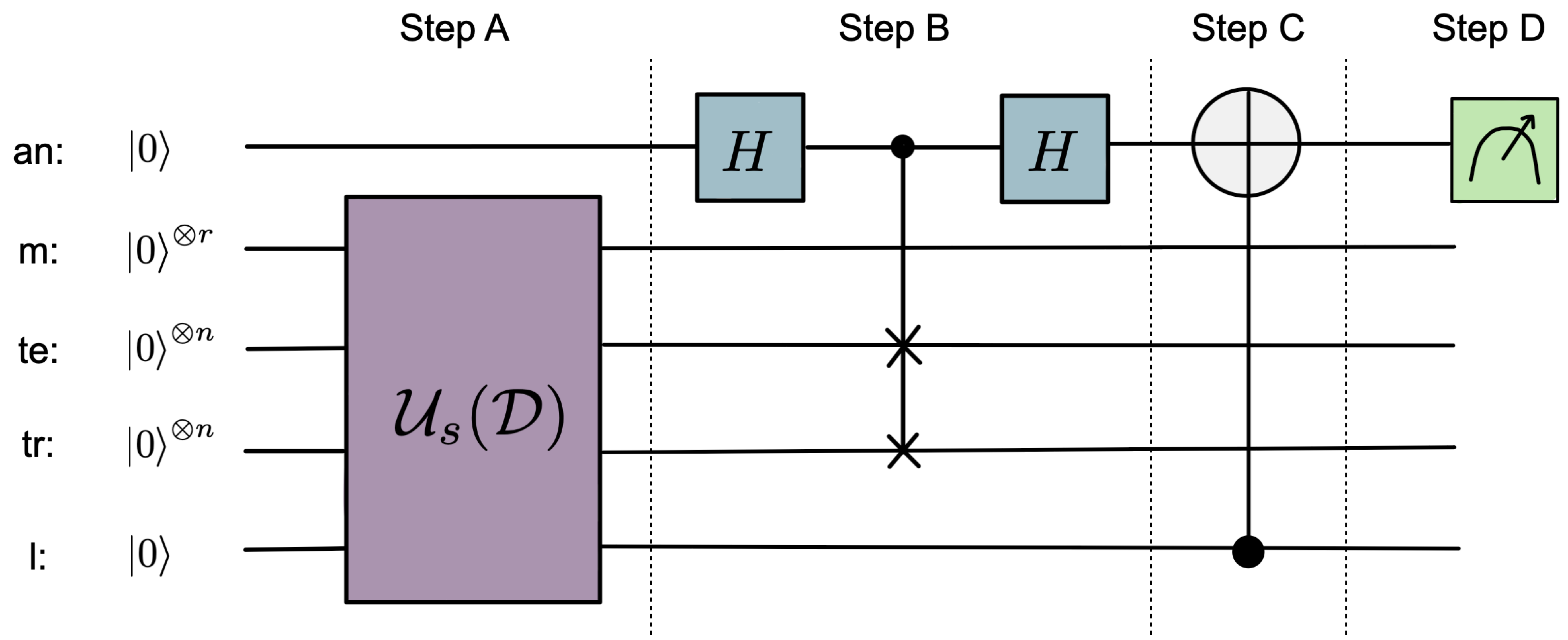
The **SWAP-Test**



THE BINARY SWAP-TEST CLASSIFIER

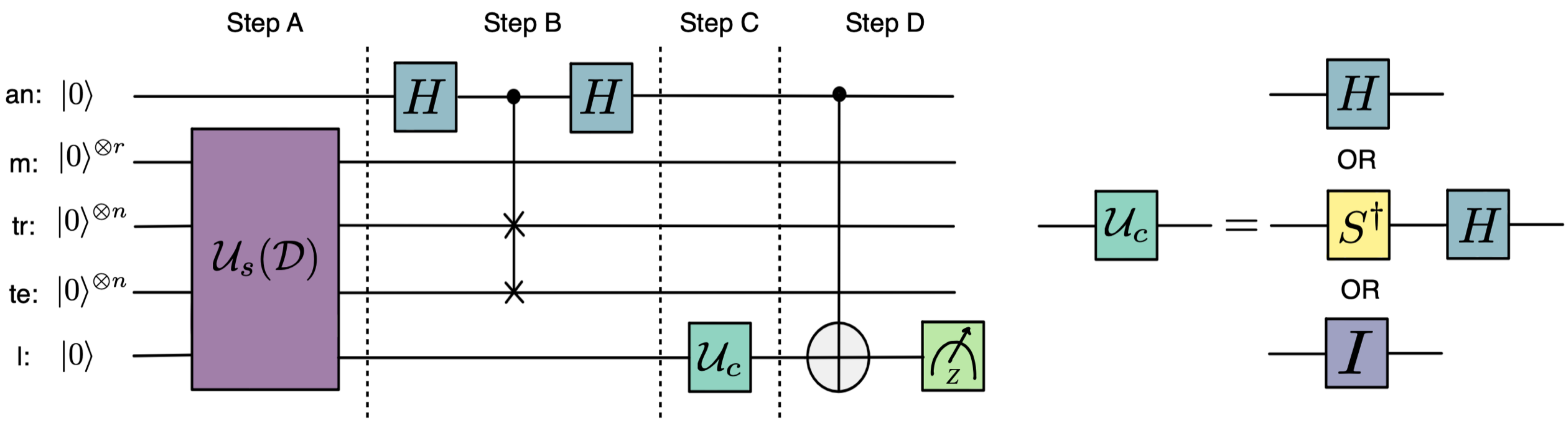
The binary SWAP-Test classifier is a quantum kernel method that allows us to estimate a **weighted power sum of kernel values** between a test datum and all the training data

$$\langle \sigma_z \rangle = \sum_{m=1}^{M} (-1)^{y_m} \, w_m \, k(\tilde{\mathbf{x}}, \mathbf{x}_m)^d \longrightarrow \tilde{y} = \frac{1}{2}\left(1 - \mathrm{sgn}\langle \sigma_z \rangle\right)$$



THE MULTI-CLASS SWAP-TEST CLASSIFIER

The Multi-Class SWAP-Test Classifier is inspired by the binary SWAP-Test classifier [1,2].



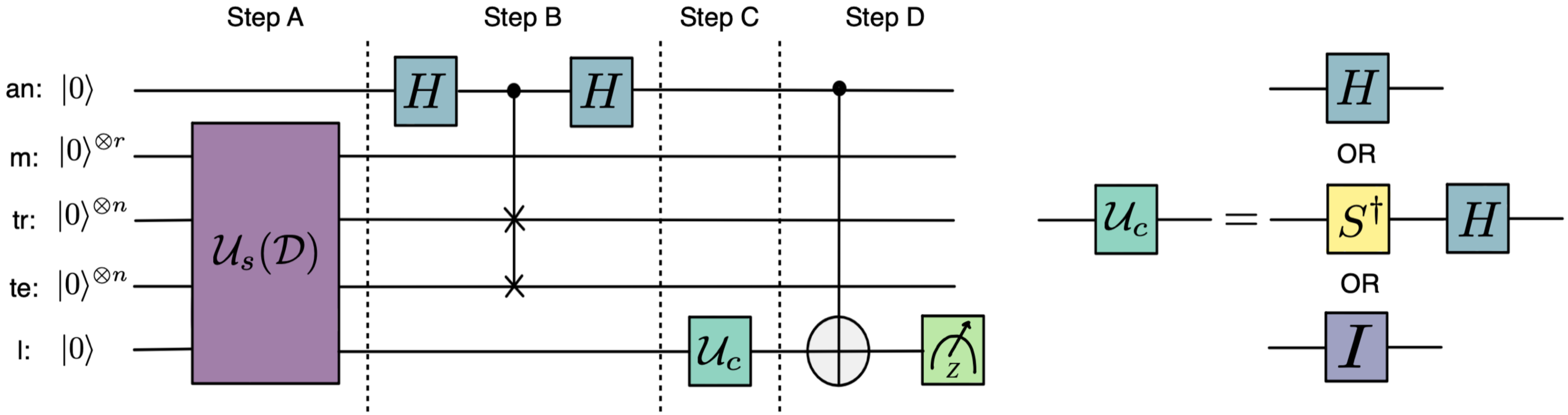
[1] Blank C, Park DK, Rhee JK, Petruccione F. Quantum classifier with tailored quantum kernel. npj Quantum Information. 2020 May 15;6(1):41.

[2] Park DK, Blank C, Petruccione F. Robust quantum classifier with minimal overhead. In2021 International Joint Conference on Neural Networks (IJCNN) 2021 Jul 18 (pp. 1-7). IEEE.

THE MULTI-CLASS SWAP-TEST CLASSIFIER

Given an L-class classification problem, the classifier is realised by first preparing a quantum state encoding the test datum $\tilde{\mathbf{x}}$, the training data $\{\mathbf{x}_m\}_{m=1}^{M}$ and their respective labels $\{y_m\}_{m=1}^{M}$ in the following format:

$$|\Psi\rangle = \sum_{m=1}^{M} \sqrt{w_m}\,|0\rangle\,|m\rangle\,|\mathbf{x}_m\rangle\,|\tilde{\mathbf{x}}\rangle\,|y_m\rangle$$



THE MULTI-CLASS SWAP-TEST CLASSIFIER

STORING OF LABELS

Each label is mapped to a unique label state:

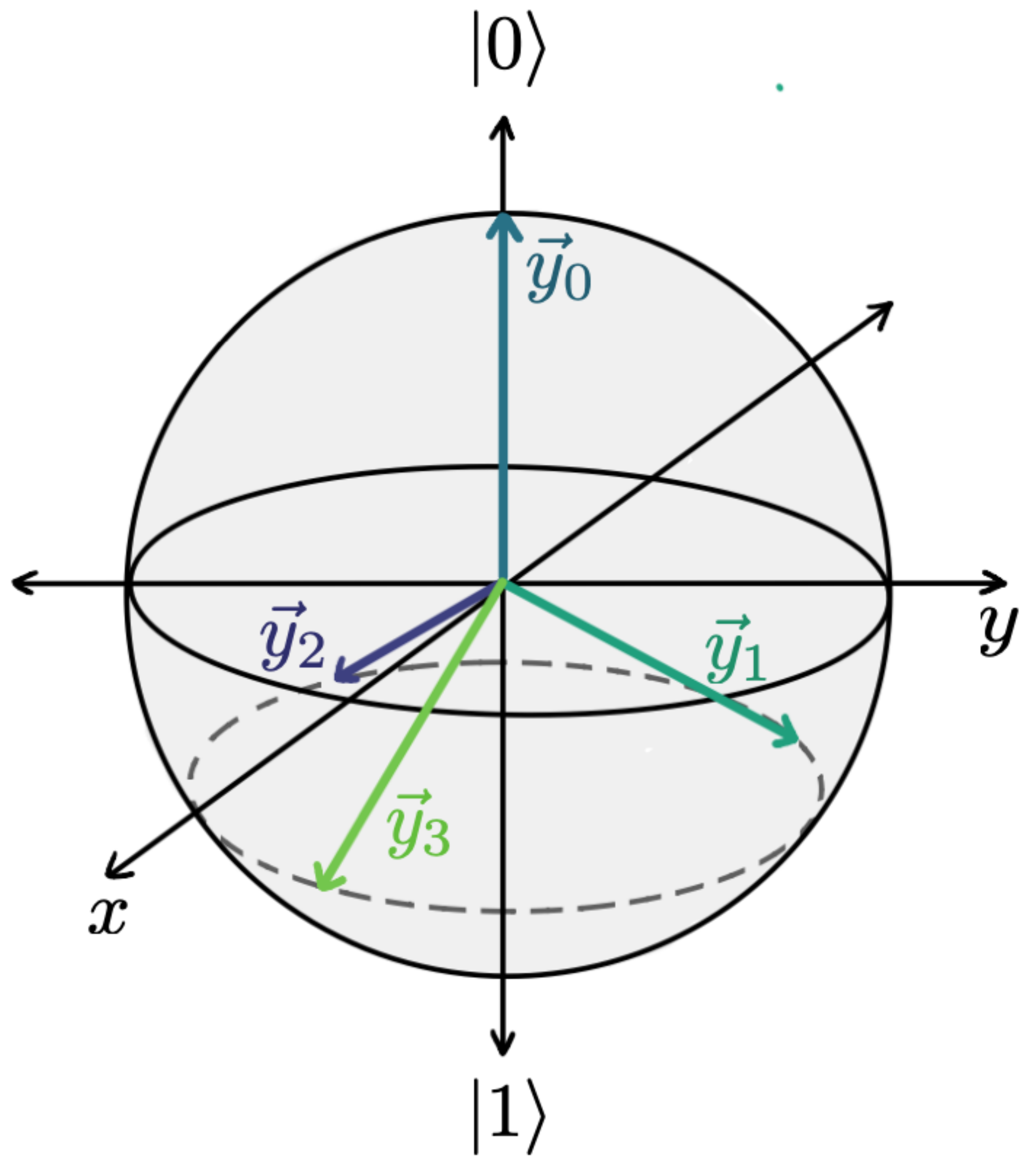
$$y_i \rightarrow |y_i\rangle$$

Each label state

$$|y_i\rangle = \cos\left(\frac{\theta_{y_i}}{2}\right)|0\rangle + e^{i\phi_{y_i}} \sin\left(\frac{\theta_{y_i}}{2}\right)|1\rangle$$

with $0 \le \theta_{y_i} \le \pi$ and $0 \le \phi_{y_i} \le 2\pi$ can be represented as a Bloch vector:

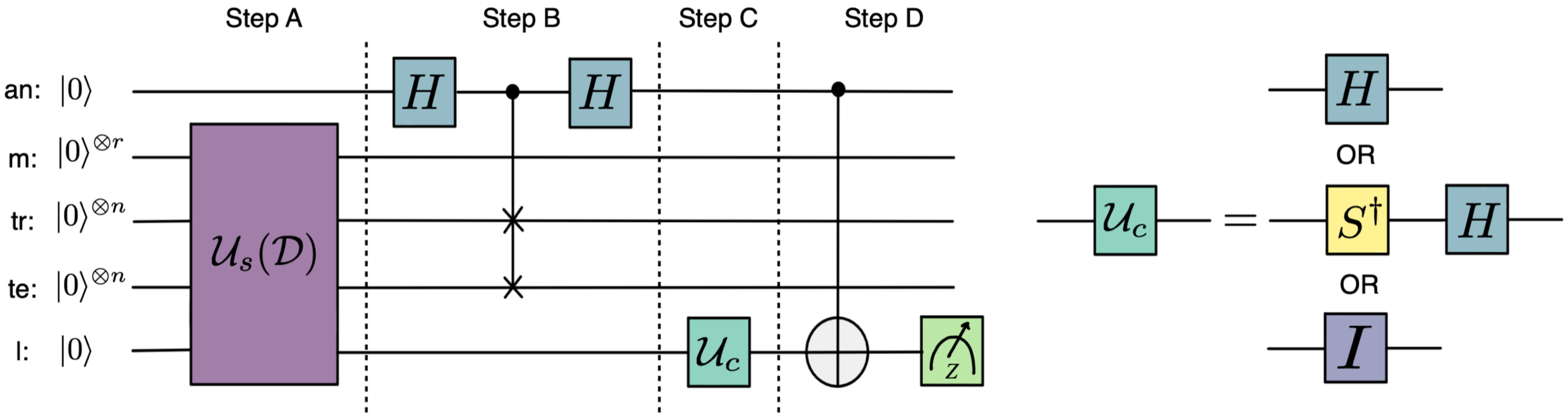
$$\mathbf{y}_i = \begin{pmatrix} \cos\phi_{y_i}\sin\theta_{y_i} \\ \sin\phi_{y_i}\sin\theta_{y_i} \\ \cos\theta_{y_i} \end{pmatrix}$$



THE MULTI-CLASS SWAP-TEST CLASSIFIER

MEASUREMENT

A modified SWAP-Test, involving a state reconstruction of the qubit storing the label states, is then performed on the prepared state.



THE MULTI-CLASS SWAP-TEST CLASSIFIER

MEASUREMENT

This effectively yields a linear combination of label vectors,

$$\mathbf{y}_{pred} = \sum_{i=1}^{L} \alpha_i \mathbf{y_i}$$

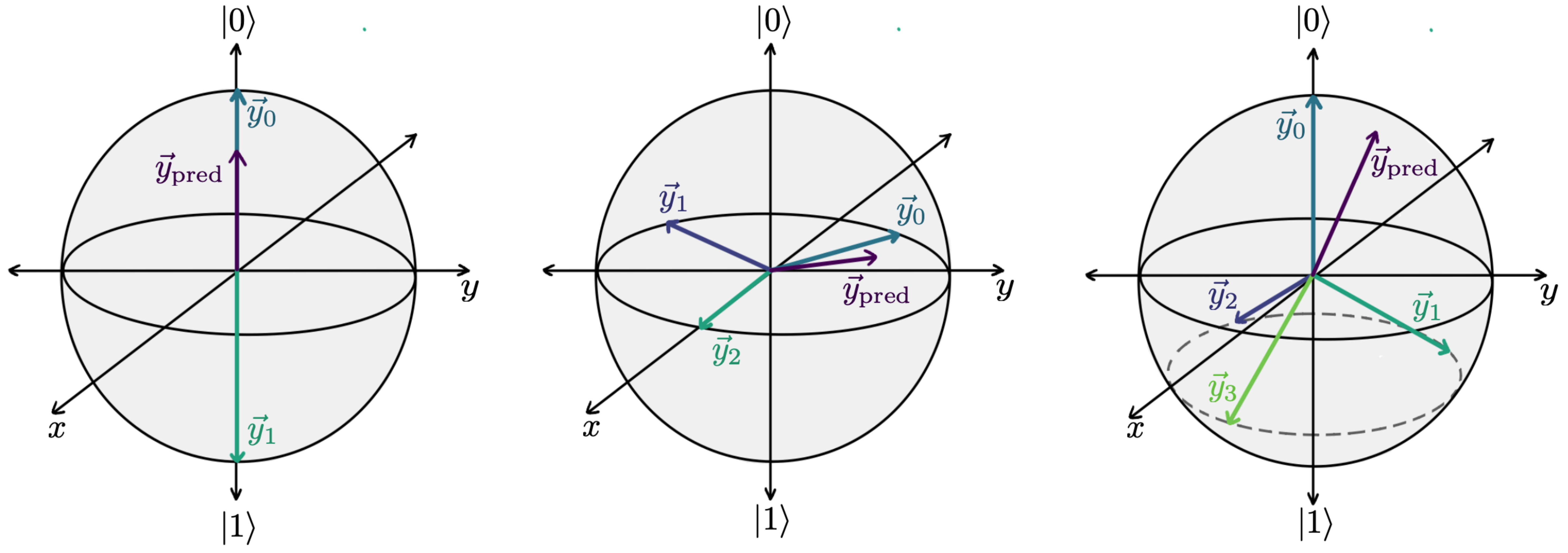
The contribution of each label vector $\alpha_i = \sum_{m|\mathrm{y}_m = i} \mathrm{w}_m k(\tilde{\mathbf{x}}, \mathbf{x}_m)$ is a weighted sum of kernel values between the test data and all the training data with that label.

THE MULTI-CLASS SWAP-TEST CLASSIFIER

The predicted vector is then used in the following assignment function:

$$\tilde{y} = \max_{y_i}\{\mathbf{y_i} \cdot \mathbf{y}_{pred}\}$$

which is evaluated classically.

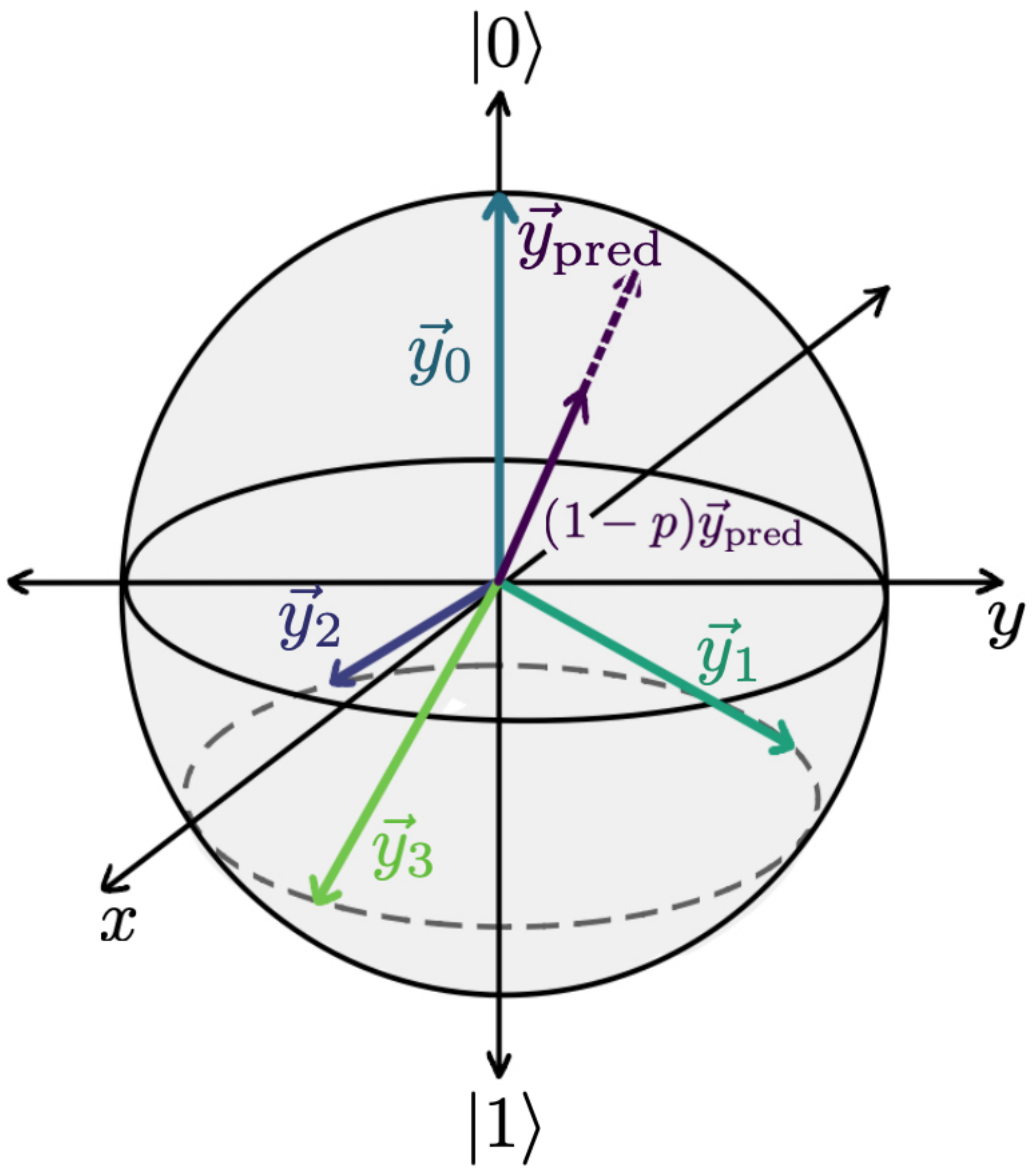


ROBUSTNESS TO NOISE

We also consider the effect of a single qubit **depolarising** channel acting only on the label qubit right before the required measurement.

Our analysis shows that the predicted vector obtained is **only scaled** by a factor of $(1-p)$.

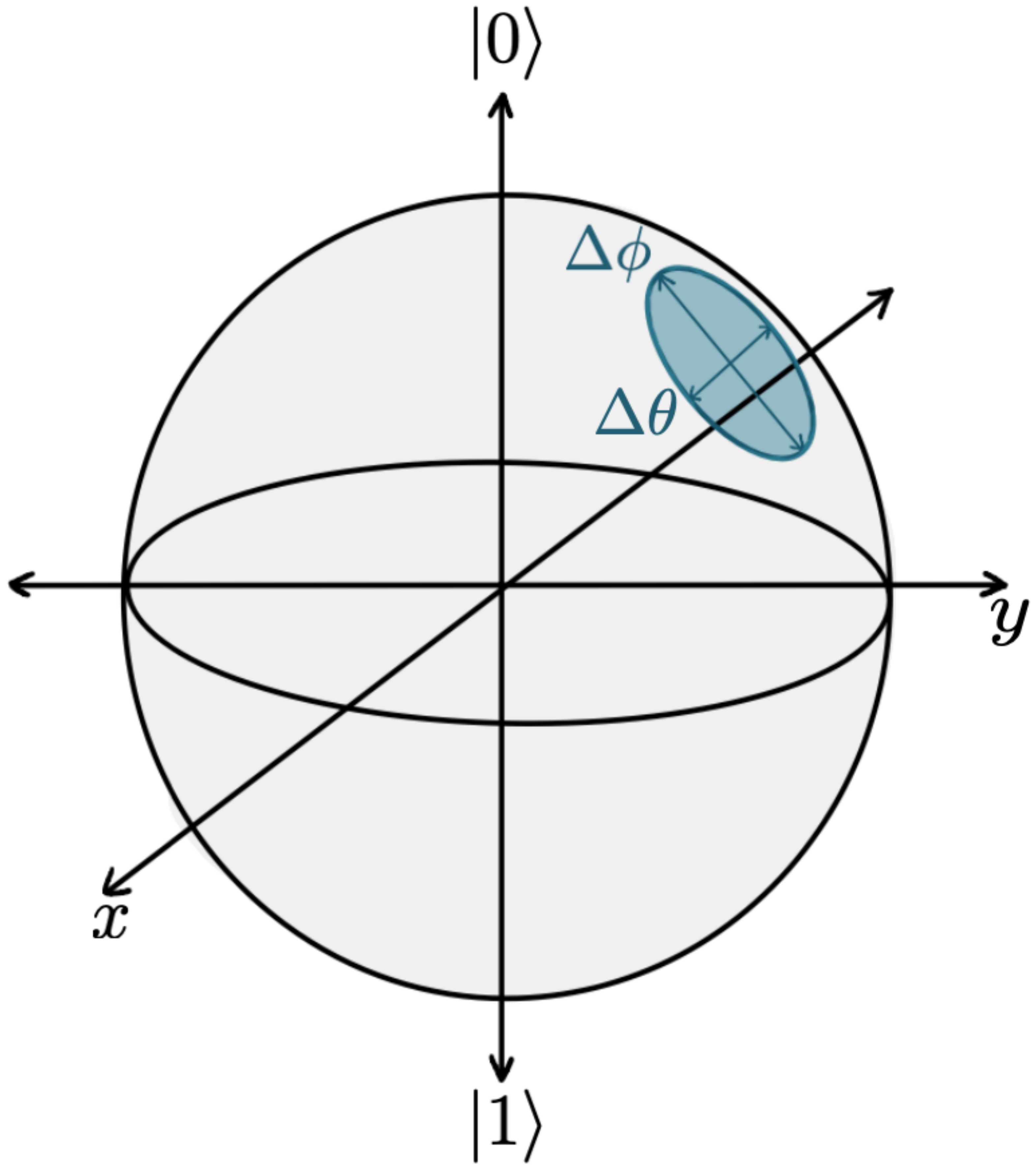
This has **no effect** on the outcome of the classification.



NUMBER OF CLASSES

Through variance analysis, we show that the number of label states that can be accurately distinguished on a single qubit **grows linearly** with the number of repetitions of the required measurements.

$$\text{No. of Label States} = O(R)$$



EFFECTIVENESS

ANALYTICAL RESULTS

The effectiveness of the multi-class SWAP-Test classifier is demonstrated by applying it to a number of different classification problems.

Dataset	# Classes	# Features	# Points	Encoding	Accuracy (%)
XOR	2	2	100	Amplitude	100
	4	3	200	Amplitude	99
	8	4	400	Amplitude	99
Iris	3	4	150	Angle	95
Wine	3	13	144	Angle	92
Digits	10	64	1740	Angle	92
Letter Recognition	12	16	8808	Angle	90

EFFECTIVENESS

NUMERICAL RESULTS

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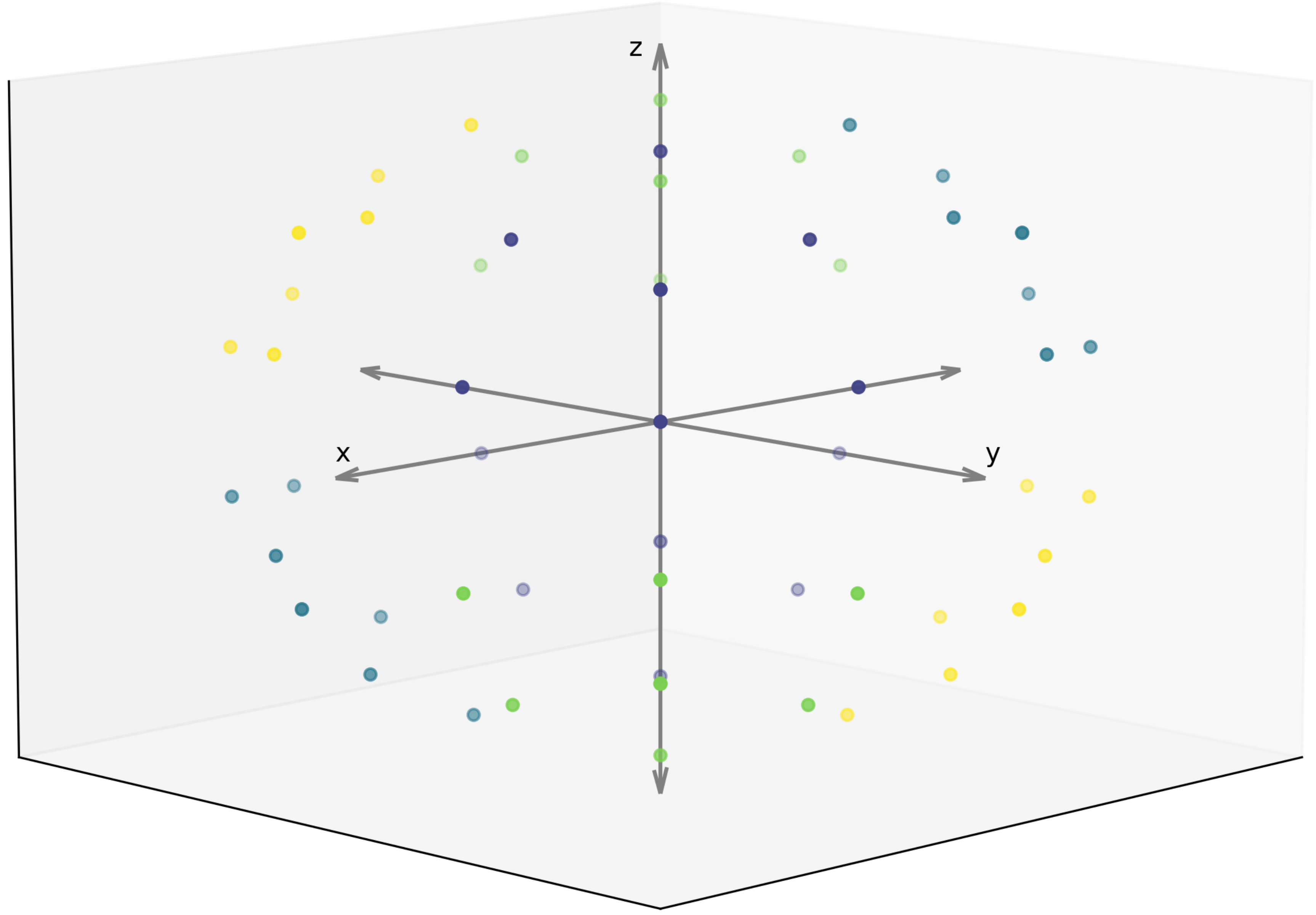

Depolarisation Rate (p)	Accuracy (%)	Av. Norm of Predicted Vector
0	100	0.1356
0.02	100	0.1331
0.04	100	0.1302
0.06	100	0.1275
0.08	100	0.1248
0.1	100	0.1223

THANK YOU!