Time-series quantum reservoir computing with weak and projective measurements

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ICFO

npj Quantum Inf. **9**, 16 (2023)

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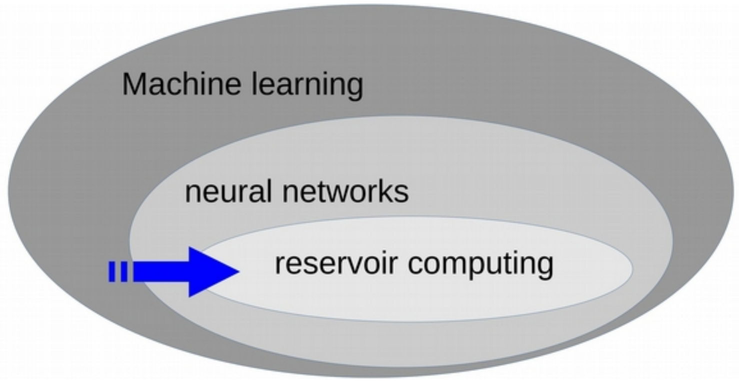
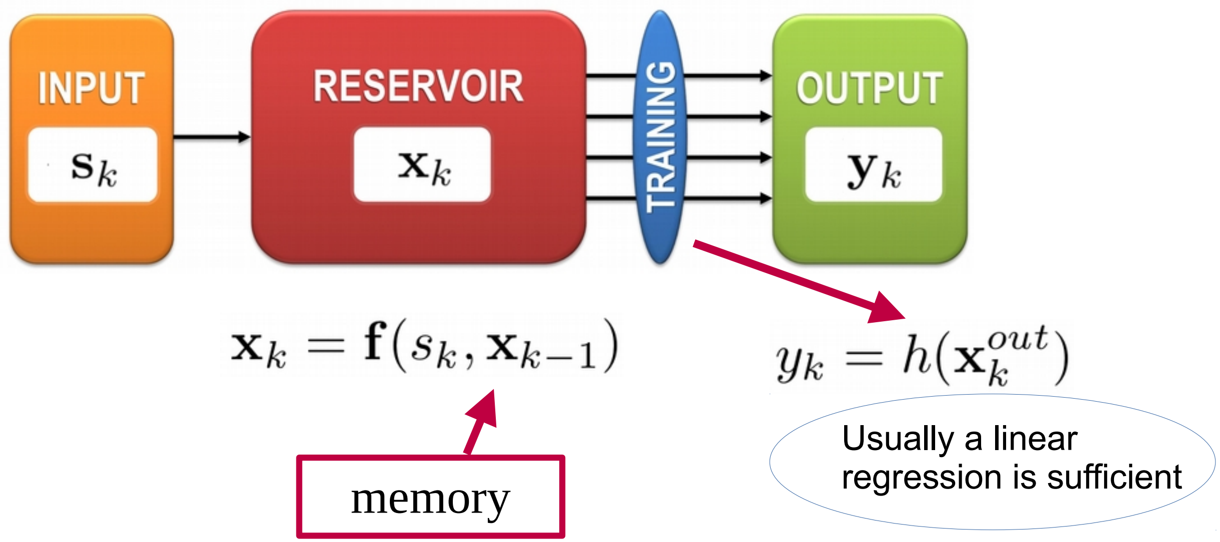
November 2023

- **Supervised Machine Learning** Technique for Temporal Series Processing



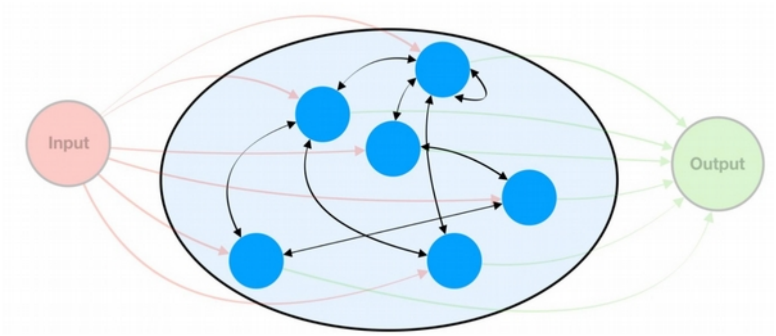


- Reservoir Computing → **Time Series Processing**: Weather Forecasting, speech recognition

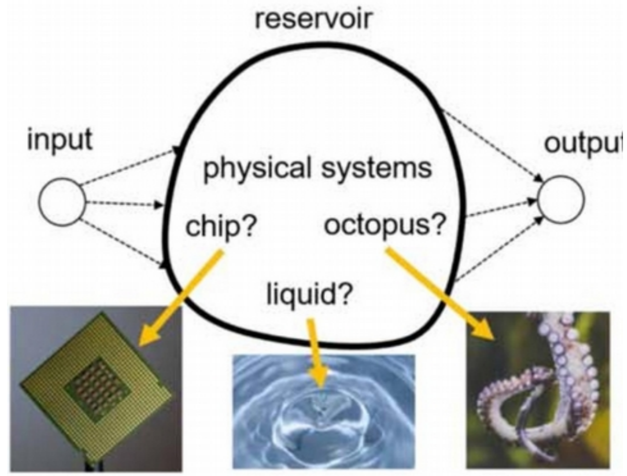
- Successful perfomance by exploiting a **high dimensional system**, internal **memory** and **nonlinearity**.

- Classical Reservoirs

Echo State Networks, H. Jaeger (2001).

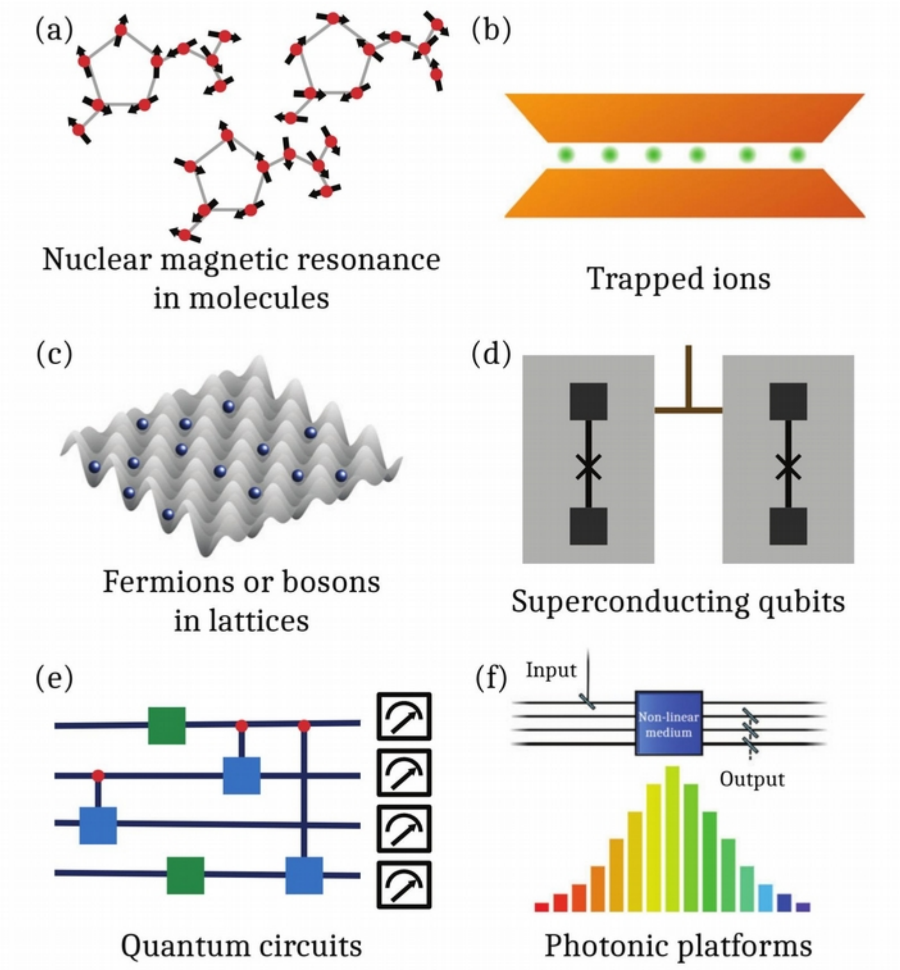


Physical Systems



From K. Nakajima Jpn. J. Appl. Phys. **59** 060501(2020)

- Quantum Reservoirs

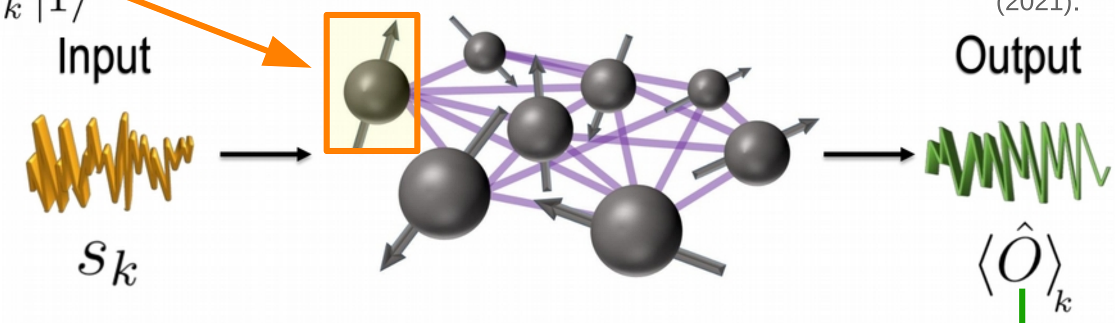


P. M. *et al.*, Adv. Quantum Technol. **4**, 2100027 (2021).

Quantum Reservoir Computing

P. M. *et al.*, Adv. Quantum Technol. **4**, 2100027 (2021).

$$|\psi_k\rangle = \sqrt{1-s_k}\,|0\rangle + \sqrt{s_k}\,|1\rangle$$



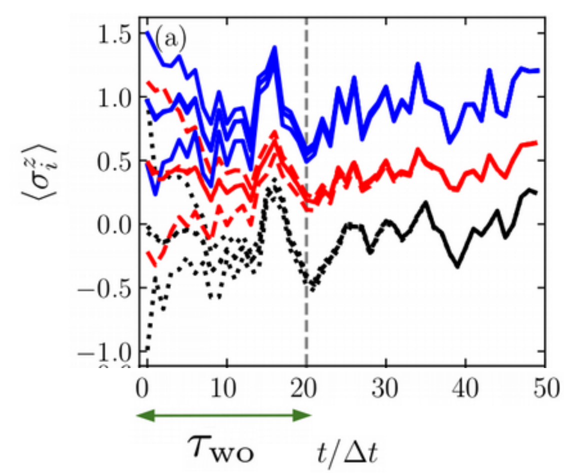
- Supervised Machine Learning with Simple Training: $\tilde{y}_k = \sum_{m=1}^{M} w_m \langle\hat{O}\rangle_{k,(m)} + w_{M+1}$
- System of qubits. Transverse-field Ising Model:

K. Fuji and K. Nakajima, Phys. Rev. Applied **8**, 024030 (2017).
R. Martínez-Peña *et al.*, Phys. Rev. Lett. **127**, 100502 (2021).

$$\hat{H} = \frac{1}{2}\sum_{i=0}^{N-1} h\hat{\sigma}_i^z + \sum_{i<j}^{N-1} J_{ij}\hat{\sigma}_i^x\hat{\sigma}_j^x$$

Fading memory

Trajectories converge after the influence of different initial conditions vanishes:



Large state space (Hilbert space) and entanglement

Dimension = 2^N

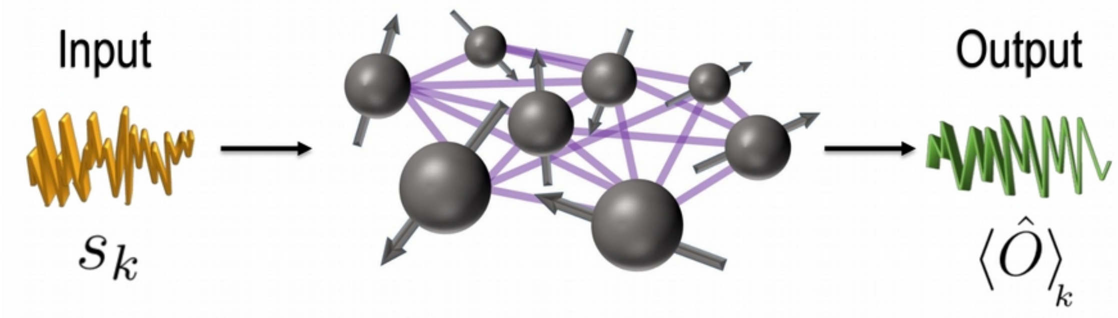
Nonlinearity

$$\langle\hat{O}\rangle^{\infty}_{\rho'_k} = \mathrm{Tr}(\hat{O}\rho_k'^{(0)}) + s_k\mathrm{Tr}(\hat{O}\rho_k'^{(1)}) + r_k\mathrm{Tr}(\hat{O}\rho_k'^{(\mathrm{nl})})$$

P. M. *et al.*, J. Phys. Complex. **2**, 045008 (2021).

$$r_k \equiv \sqrt{s_k(1-s_k)}$$

Quantum Reservoir Computing



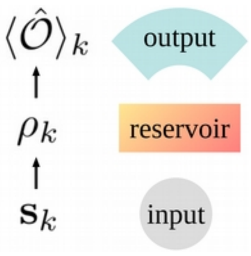
- [Statistical uncertainty] **Expectation values** of observables require **ensemble measurements** on ideally infinite but in experiments N_{meas} copies.

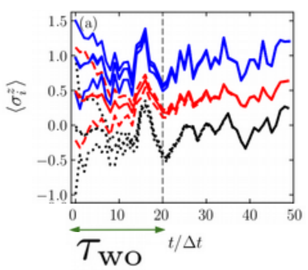
- [Back-action] **Measurements** on the **quantum reservoir "perturb"** its **state.**

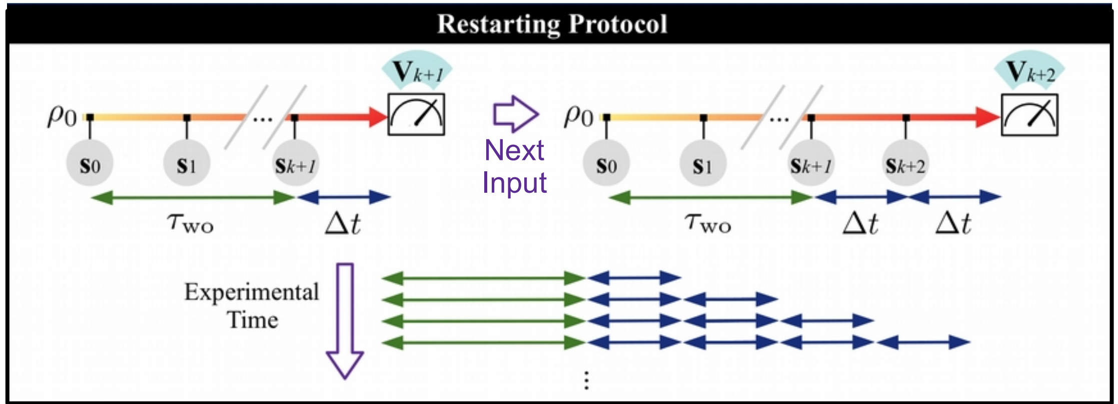
> - Can we use a **quantum system** for **time-series processing** while it is **continuously measured**?
>
> Our Proposal: **Online Measurement Protocol** (OLP) with **Weak Measurements**

Measurement Protocols

- We propose and analyze different **measurement strategies**:







Naive approach

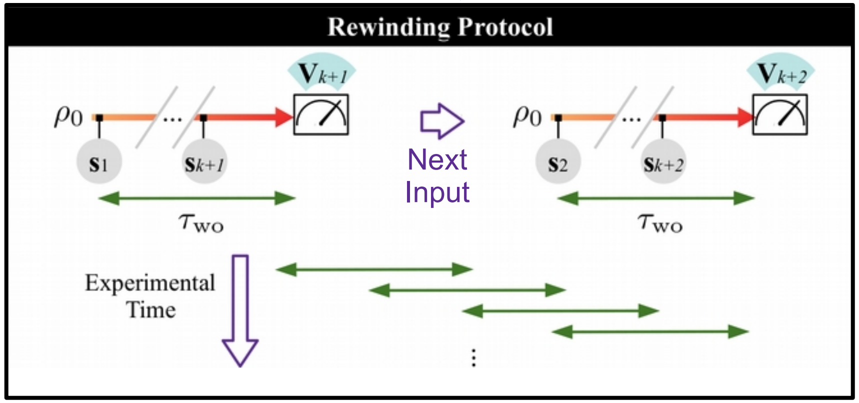
External Memory necessary for input series reinjection

Time consumption

$$t_{\mathrm{exp}}^{\mathrm{RSP}} \sim N_t^2$$

Fading Memory

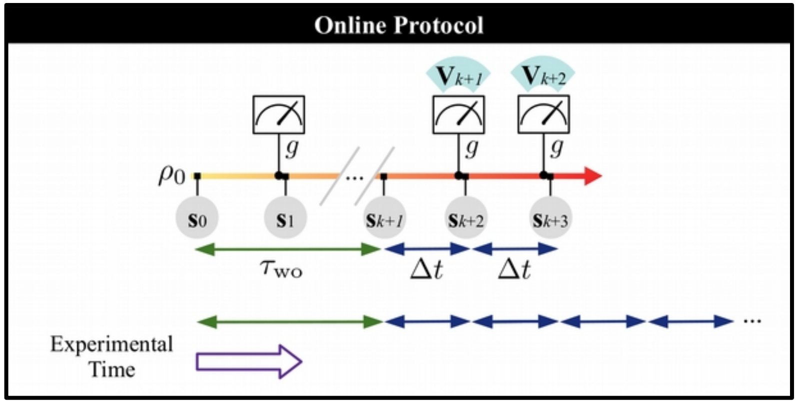
Measurement Strength



Reduced External Memory

Less Experimental Time

$$t_{\mathrm{exp}}^{\mathrm{RWP}} \sim \tau_{\mathrm{WO}} N_t$$



Measurements enter into the Dynamics of the System

No External Memory

Less Experimental Time

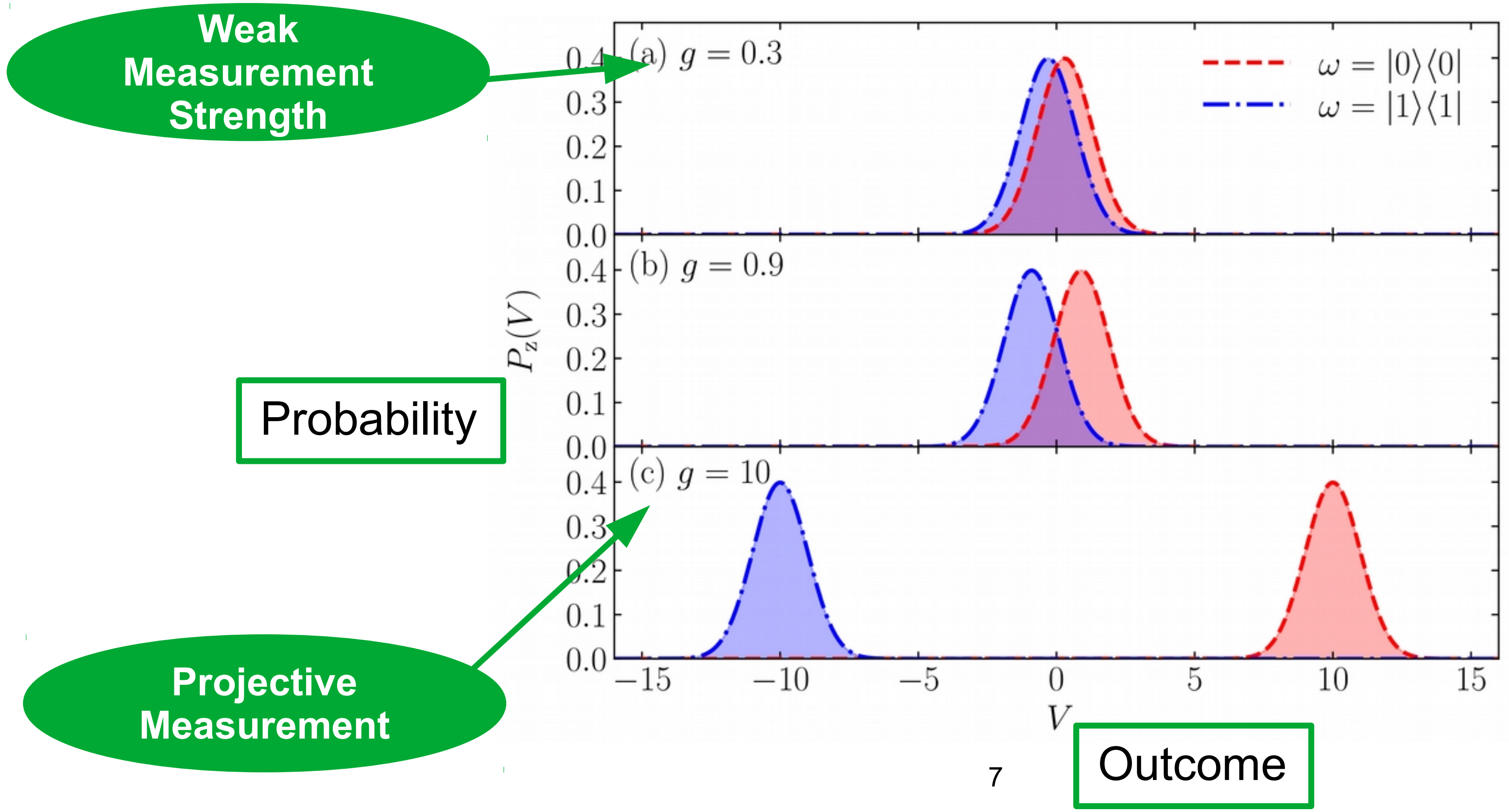
$$t_{\mathrm{exp}}^{\mathrm{OLP}} \sim N_t$$

- Implementation: **Indirect measurements** on a qubit through a coupled probe:

Qubit + probe → Joint unitary evolution (coupling) → Measurement on the probe

$$\hat{\Omega}_V^z = \frac{1}{\sqrt[4]{2\pi}}\left(e^{-\frac{(V-g)^2}{4}}|0\rangle\langle 0| + e^{-\frac{(V+g)^2}{4}}|1\rangle\langle 1|\right)$$

$$P_z(V) = \mathrm{Tr}(\hat{\Omega}_V^{z\dagger}\hat{\Omega}_V^z\omega) = \omega_{00}\frac{1}{\sqrt{2\pi}}e^{-\frac{(V-g)^2}{2}} + (1-\omega_{00})\frac{1}{\sqrt{2\pi}}e^{-\frac{(V+g)^2}{2}}$$

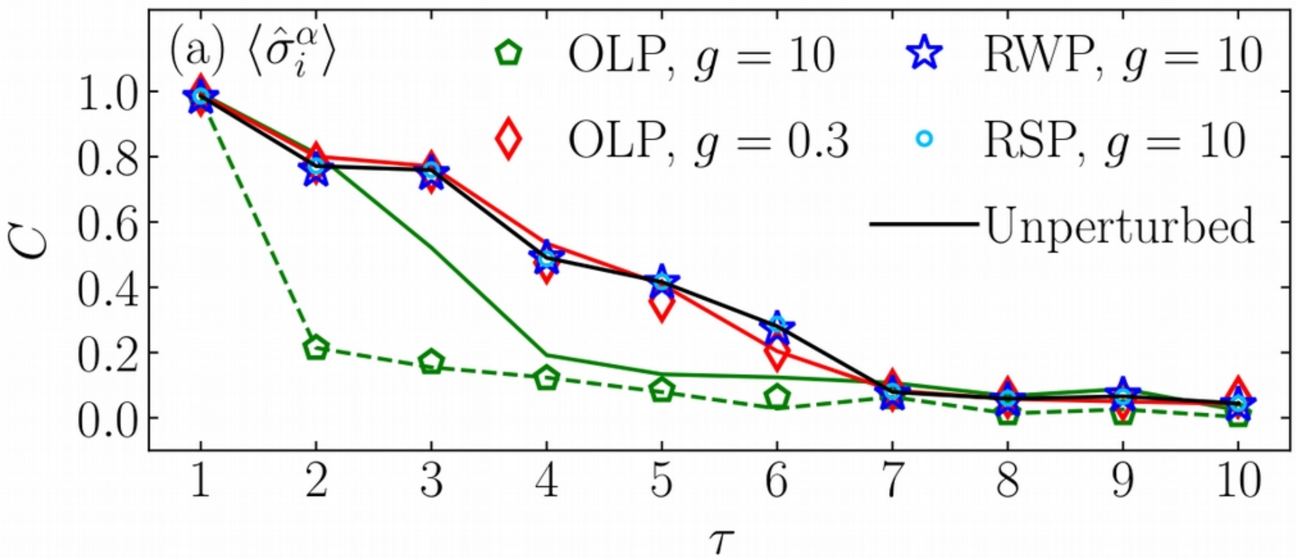


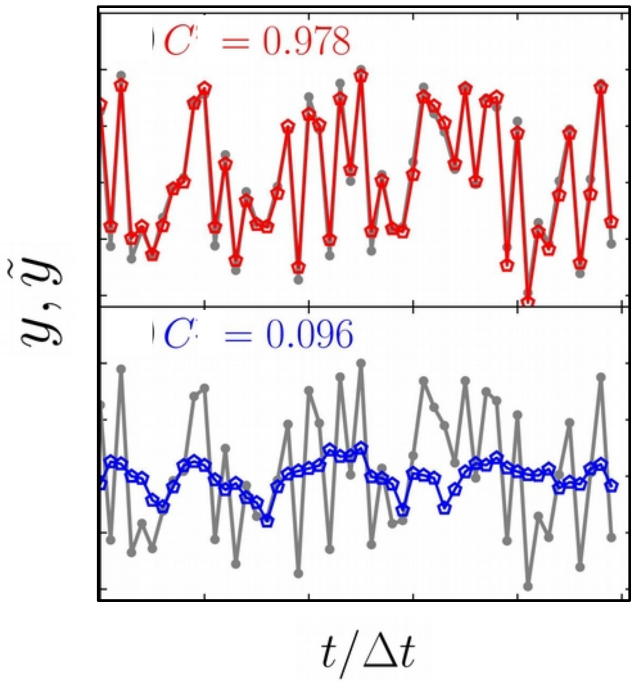
Short-Term Memory Capacity

- **Capacity** of the system to **reproduce recent-past inputs** (delay τ):

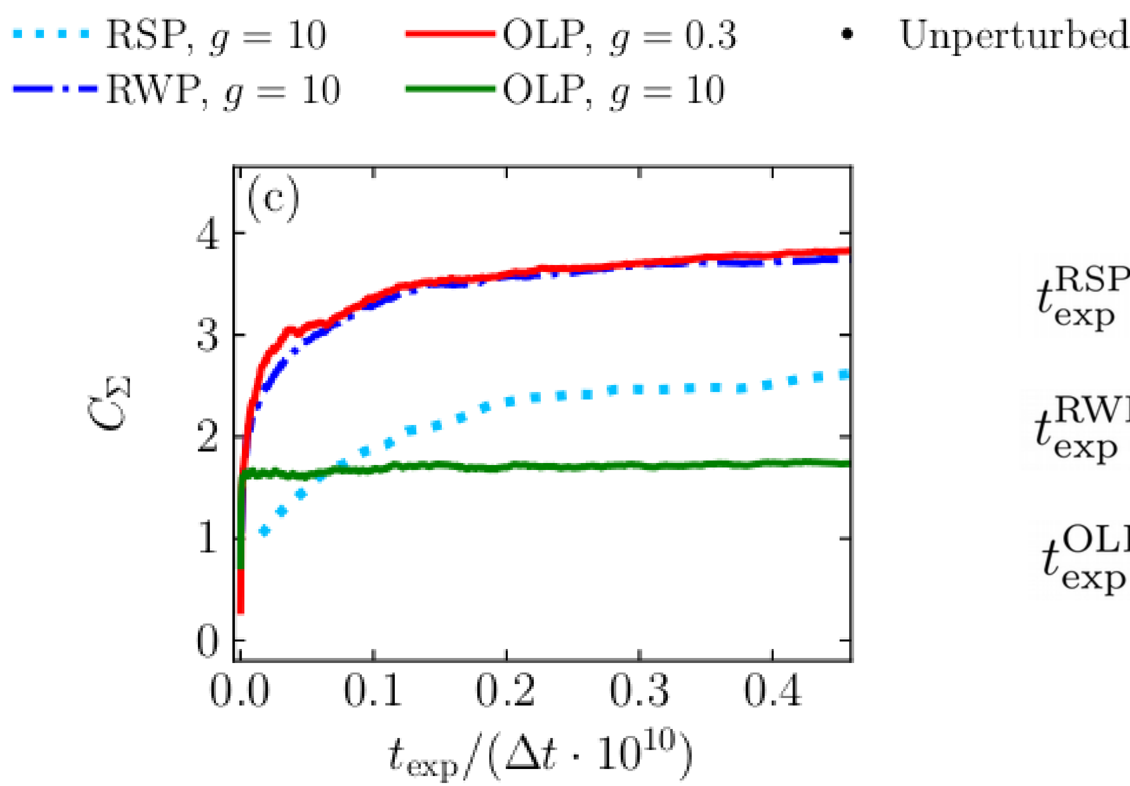
$$C \equiv \frac{\mathrm{cov}^2(\boldsymbol{y}, \tilde{\boldsymbol{y}})}{\mathrm{var}(\boldsymbol{y})\mathrm{var}(\tilde{\boldsymbol{y}})}$$

Target: $y_k = s_{k-\tau}$





- **Back-action reduces** the **STM memory capacity** when the OLP is followed with **projective measurements**.
- We can choose an **optimal** value of the **measurement strength,** g.



Sum capacity:

$$C_\Sigma \equiv \sum_{\tau=1}^{10} C(\tau)$$

$$t_{\mathrm{exp}}^{\mathrm{RSP}} \sim N_t^2$$

$$t_{\mathrm{exp}}^{\mathrm{RWP}} \sim \tau_{\mathrm{wo}} N_t$$

$$t_{\mathrm{exp}}^{\mathrm{OLP}} \sim N_t$$

- Both, the **RWP** and the **OLP** are **competitive** in terms of **experimental resources.**
- The minimum **measurement strength** to be advantageous depends on the **memory of the reservoir**.

$$g \geq \sqrt{\frac{1}{N_{\mathrm{wo}} - 1}}$$

Measurement Strength → g

Fading Memory → N_{wo}

Conclusions

- We find **quantum measurement strategies** for time-series processing that make a clever use of **the fading memory property** and **weak measurements**.
- We take into account and differentiate **statistical effects** from **back-action**.
- We find that **optimum performance** is reachable **beyond ideal assumptions** both in the **RWP with projective measurements** and in the **OLP with weak measurements**.
- In the **OLP,** we show that **online time-series processing without** other **external resources** with **quantum reservoirs** is **possible** and **advantageous** in terms of **experimental implementation**.
- **Back-action** (purely quantum) effects in the dynamics can **increase the performance** in time-series processing tasks.

THANK YOU

for your attention

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