Diego Garcia-Martin[1,2]@ QTML23

The landscape of QAOA Max-Cut Lie algebras:

Diego Garcia-Martin

- I am not Diego.
- If you came to see Diego, you are right to be pissed off. I apologize.

1. T y
2. C Laboratory

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gello im diego garxia martin ans today i want to talk about qaoa maxcut.

no, ictually im not siego diego is this awesome and very good looking guy here depicted.

Martin Larocca[1,2]@ QTML22

The landscape of QAOA Max-Cut Lie algebras

if you go ***deep, dont forget your initialization!***

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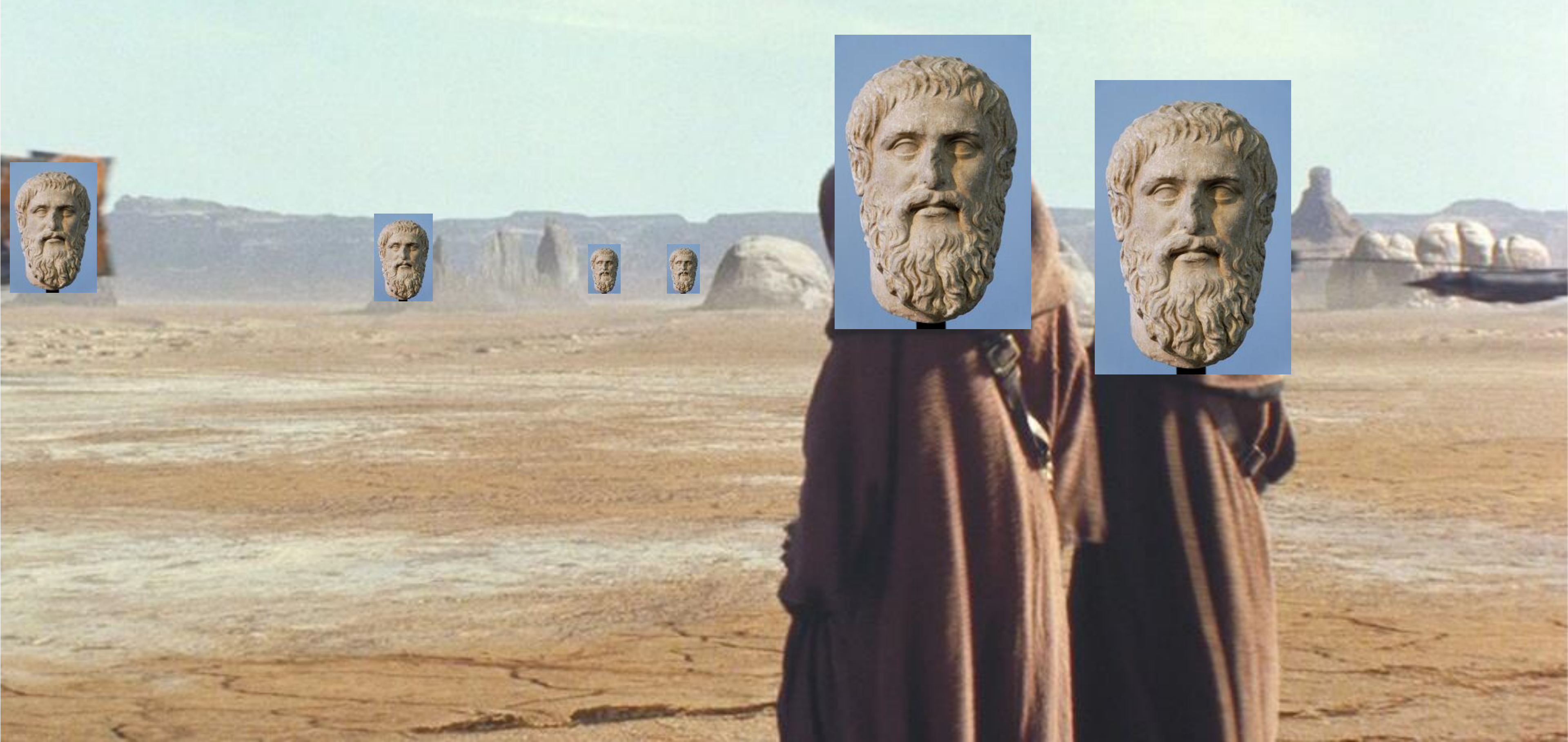
what is this talk about?

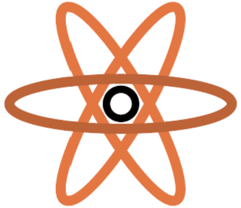
Barren Platos



Deep (= poly depth)
QAOA ansatz

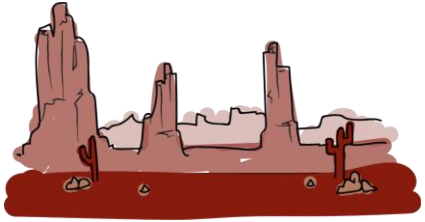
+

random init



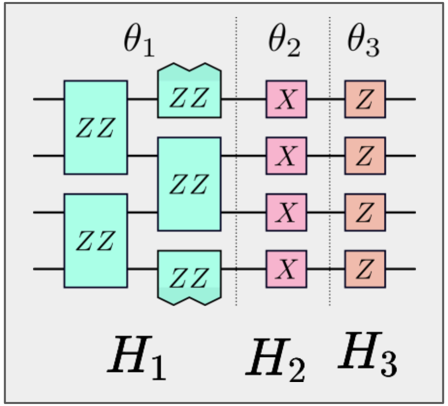
=

barren
plateau



Variational Quantum Algorithms

$$|\psi\rangle$$



$$C_\theta = \langle\psi_\theta|O|\psi_\theta\rangle$$

$$U_\theta = \prod_{m=1}^{M} e^{iH_m\theta_m}$$

Update parameters to minimize

The **Barren Plateau** problem: $\mathrm{Var}_\theta[C_\theta] \in \mathcal{O}(\frac{1}{\exp(n)})$

Expectation values **concentrate (over landscape):** if I sample points with vhp I will not be able to see a difference from the mean → cannot determine descent direction with poly shots

why do BPs happen?

Dynamical Lie algebra **(DLA) [*]**

'The DNA of PQCs'

$$\mathfrak{g} = \mathrm{span}_{\mathbb{R}} \langle \{iH_j\} \rangle_{\mathrm{Lie}} \subseteq \mathfrak{u}(d)$$

Intuition: the DLA constitutes the fingerprint of a PQC since it characterizes its (potential) expressiveness

$$\forall \theta,\, U(\theta) \in e^{\mathfrak{g}} \subseteq \mathbb{U}(d)$$

Conjecture ML21': Var ~ 1/dimg (more expressive = more concentrating)

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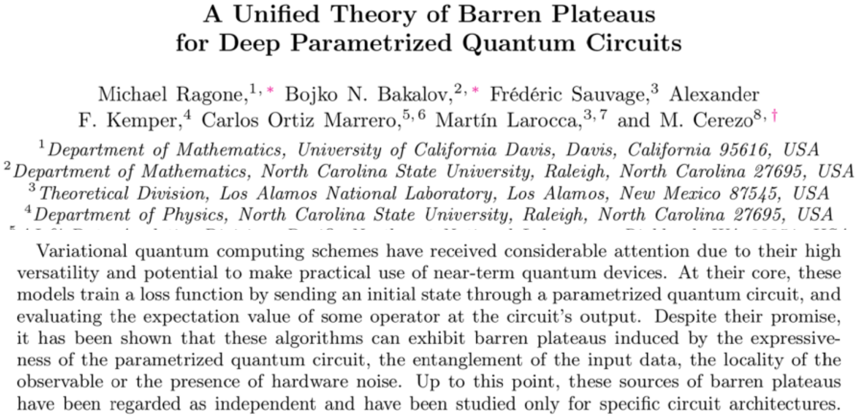
[*] ML21': **ML** et al., Diagnosing barren plateaus with tools from quantum optimal control, Quantum 6, 824 (2022).

RESOLVED RECENTLY (SEPT23)!

just the typical QIP deadline situation… **two proofs** for the **conjecture** came out simultanously!



A Unified Theory of Barren Plateaus for Deep Parametrized Quantum Circuits

Michael Ragone,[1,*] Bojko N. Bakalov,[2,*] Frédéric Sauvage,[3] Alexander F. Kemper,[4] Carlos Ortiz Marrero,[5,6] Martín Larocca,[3,7] and M. Cerezo[8,†]

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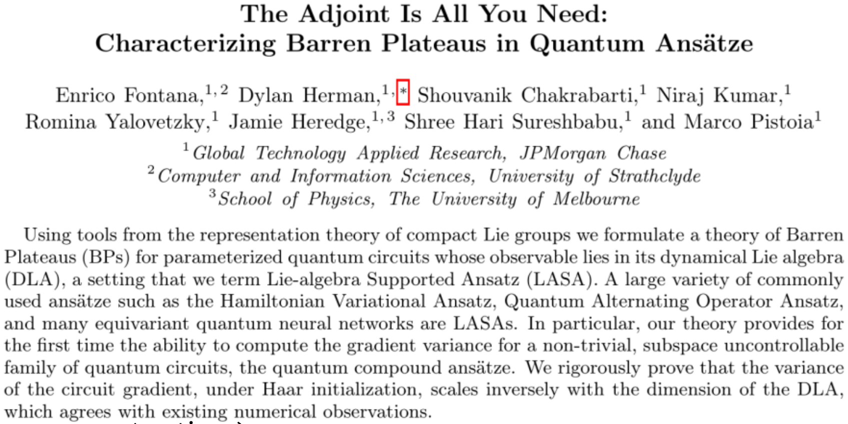
Variational quantum computing schemes have received considerable attention due to their high versatility and potential to make practical use of near-term quantum devices. At their core, these models train a loss function by sending an initial state through a parametrized quantum circuit, and evaluating the expectation value of some operator at the circuit's output. Despite their promise, it has been shown that these algorithms can exhibit barren plateaus induced by the expressiveness of the parametrized quantum circuit, the entanglement of the input data, the locality of the observable or the presence of hardware noise. Up to this point, these sources of barren plateaus have been regarded as independent and have been studied only for specific circuit architectures.



The Adjoint Is All You Need: Characterizing Barren Plateaus in Quantum Ansätze

Enrico Fontana,[1,2] Dylan Herman,[1,*] Shouvanik Chakrabarti,[1] Niraj Kumar,[1] Romina Yalovetzky,[1] Jamie Heredge,[1,3] Shree Hari Sureshbabu,[1] and Marco Pistoia[1]

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[3]*School of Physics, The University of Melbourne*

Using tools from the representation theory of compact Lie groups we formulate a theory of Barren Plateaus (BPs) for parameterized quantum circuits whose observable lies in its dynamical Lie algebra (DLA), a setting that we term Lie-algebra Supported Ansatz (LASA). A large variety of commonly used ansätze such as the Hamiltonian Variational Ansatz, Quantum Alternating Operator Ansatz, and many equivariant quantum neural networks are LASAs. In particular, our theory provides for the first time the ability to compute the gradient variance for a non-trivial, subspace uncontrollable family of quantum circuits, the quantum compound ansätze. We rigorously prove that the variance of the circuit gradient, under Haar initialization, scales inversely with the dimension of the DLA, which agrees with existing numerical observations.

Conjecture ML21': Var ~ 1/dimg (more expressive = more concentrating)

new result:

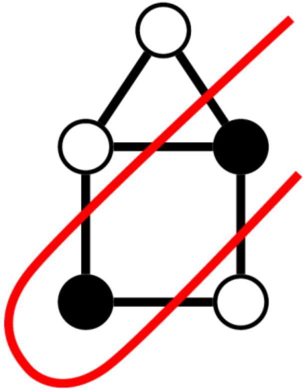
$$\mathrm{Var}_{\boldsymbol{\theta}}[\ell_{\boldsymbol{\theta}}(\rho,O)] = \underset{\text{expressiveness}}{\frac{1}{\dim(\mathfrak{g})}}\,\overset{\text{locality}}{\mathrm{Tr}\left[O_{\mathfrak{g}}^{2}\right]}\,\underset{\text{entanglement}}{\mathrm{Tr}\left[\rho_{\mathfrak{g}}^{2}\right]}$$

"new physics": (besides the conjectured dependence on *expressiveness*) the exact expression provides unforeseen connection between DLA and two other reported *causes* of BP: *entanglement* and *locality*.

it is official: symmetry = less concentration = less QPU time for a given task

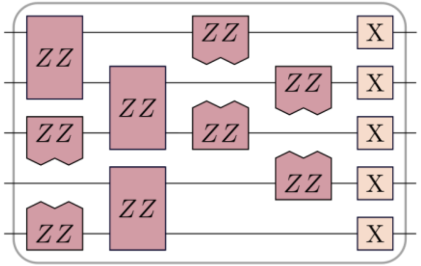
Remark: the theory assumes O in g. Luckily, this is the case of QAOA (and other popular VQAs).
For a theory beyond this constraint see:
Diaz et al., Showcasing a Barren Plateau Theory Beyond the Dynamical Lie Algebra, *arXiv:2310.11505* (2023).

MAXCUT and QAOA



QAOA: variational quantum algorithm with

$$|\psi\rangle = |+\rangle^{\otimes n}$$

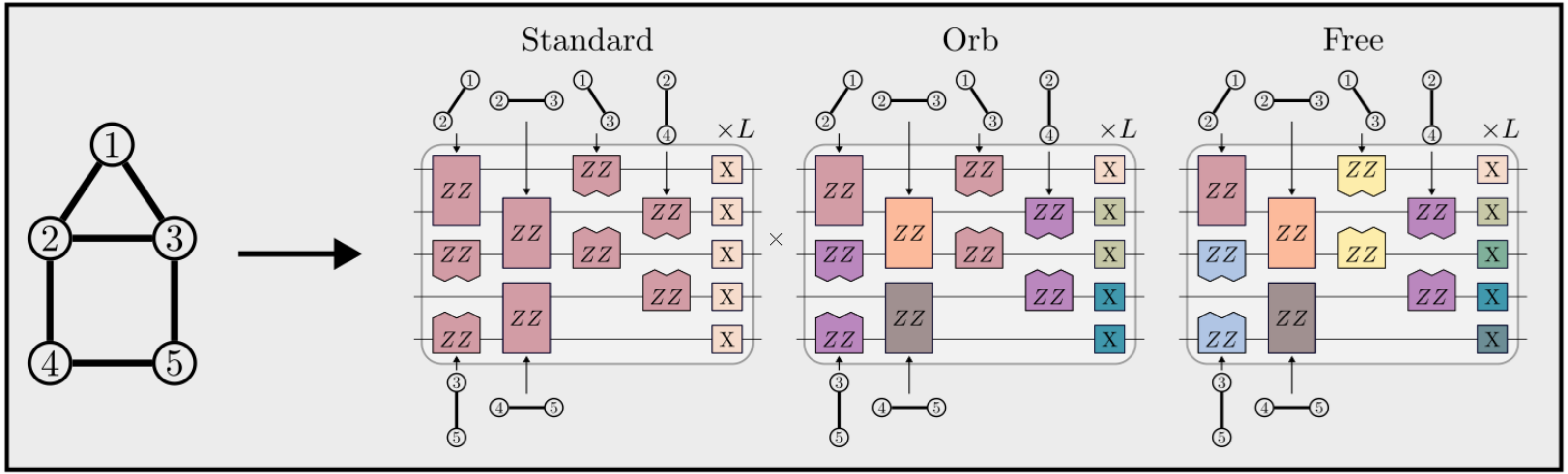


and

$$O = -\frac{1}{2}(|E| - H_P), \quad \text{where } H_P = \sum_{\{i,\})\in E} Z_i Z_j$$

$$C(\boldsymbol{\gamma}, \boldsymbol{\beta}) = \langle \psi(\boldsymbol{\gamma}, \boldsymbol{\beta}) | H_p | \psi(\boldsymbol{\gamma}, \boldsymbol{\beta}) \rangle$$

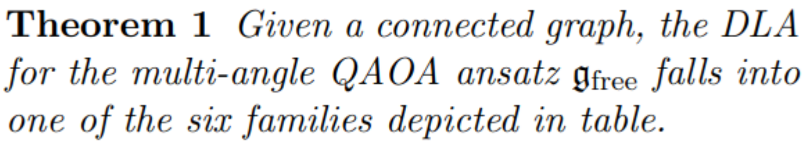
MAXCUT and QAOA: various ansatzes



Sn, symmetric group. Its action on graphs generates new (isomorphic) graphs.
Automorphism group (graph): graph-dependent subgroup of Sn that leaves a given graph invariant.

Aut(complete graph) = Sn
Aut(random graph n>>) = trivial → in this regime (the relevant for maxcut) orb = free!

FREE/MA ANSATZ: Dynamical Lie Algebra



Theorem 1 *Given a connected graph, the DLA for the multi-angle QAOA ansatz $\mathfrak{g}_{\text{free}}$ falls into one of the six families depicted in table.*

We can **completely characterize** and classify the **MA-QAOA DLAs** for ***any*** **graph.**

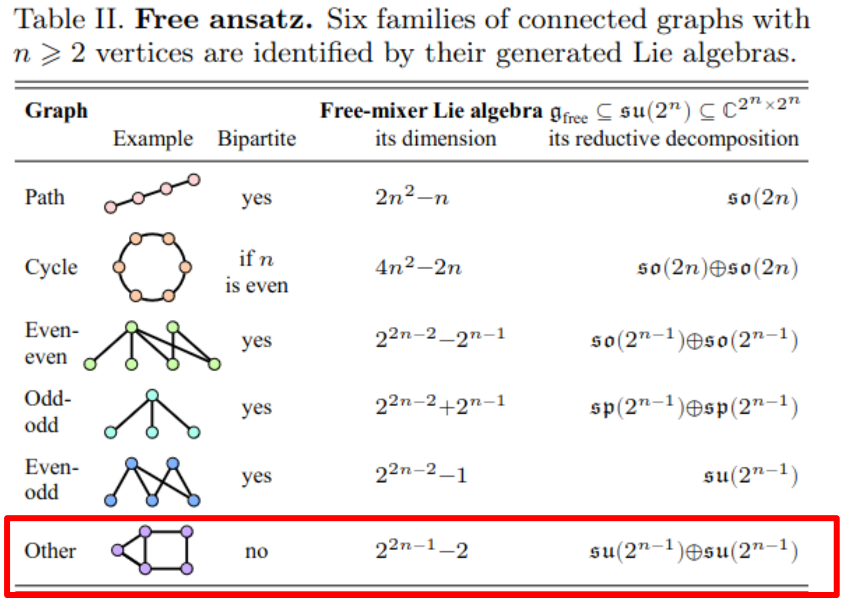


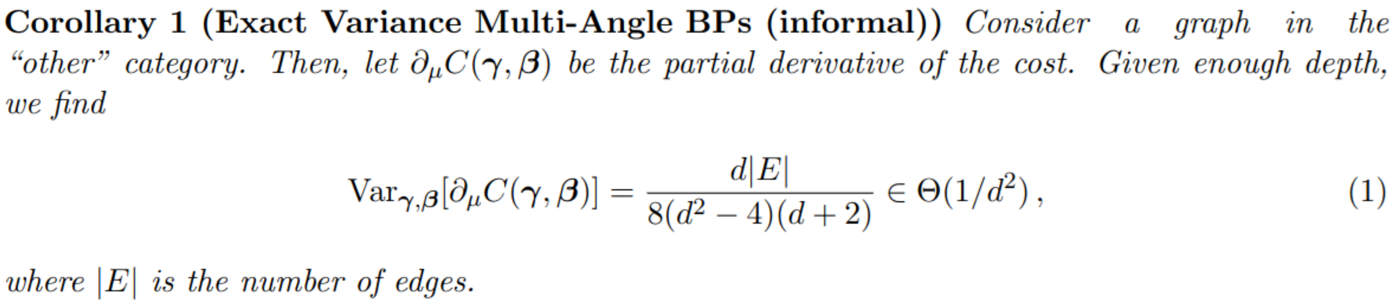
Table II. **Free ansatz.** Six families of connected graphs with $n \geqslant 2$ vertices are identified by their generated Lie algebras.

Graph			**Free-mixer Lie algebra** $\mathfrak{g}_{\text{free}} \subseteq \mathfrak{su}(2^n) \subseteq \mathbb{C}^{2^n \times 2^n}$	
	Example	Bipartite	its dimension	its reductive decomposition
Path		yes	$2n^2-n$	$\mathfrak{so}(2n)$
Cycle		if n is even	$4n^2-2n$	$\mathfrak{so}(2n)\oplus\mathfrak{so}(2n)$
Even-even		yes	$2^{2n-2}-2^{n-1}$	$\mathfrak{so}(2^{n-1})\oplus\mathfrak{so}(2^{n-1})$
Odd-odd		yes	$2^{2n-2}+2^{n-1}$	$\mathfrak{sp}(2^{n-1})\oplus\mathfrak{sp}(2^{n-1})$
Even-odd		yes	$2^{2n-2}-1$	$\mathfrak{su}(2^{n-1})$
Other		no	$2^{2n-1}-2$	$\mathfrak{su}(2^{n-1})\oplus\mathfrak{su}(2^{n-1})$

relevant case:

$$\mathfrak{g} = \mathfrak{su}(d/2) \oplus \mathfrak{su}(d/2)$$

FREE ANSATZ: Exact Variance



Corollary 1 (Exact Variance Multi-Angle BPs (informal)) *Consider a graph in the "other" category. Then, let* $\partial_\mu C(\boldsymbol{\gamma}, \boldsymbol{\beta})$ *be the partial derivative of the cost. Given enough depth, we find*

$$\mathrm{Var}_{\boldsymbol{\gamma},\boldsymbol{\beta}}[\partial_\mu C(\boldsymbol{\gamma}, \boldsymbol{\beta})] = \frac{d|E|}{8(d^2-4)(d+2)} \in \Theta(1/d^2)\,, \tag{1}$$

where $|E|$ *is the number of edges.*

For all relevant **graphs**, **MA-QAOA exhibits BP**.
Trivially, same holds for **ORB.**

STD ANSATZ:

slightly more nuanced but:

$$\mathfrak{g} \sim \mathfrak{g}_{\text{Free}} = \mathfrak{su}(d/2) \oplus \mathfrak{su}(d/2)$$

consequence: standard QAOA follows same fate… **almost universal DLA** that combined with **deep ansatz** + **random initialization** means a no go.

DISCUSSION:

- **QAOA** doesnt seem to be much of a **“problem-inspired” ansatz** after all. EXACTLY same DLA for different graphs!
  In the deep regime the **graph-dependent information** –except automorphisms– **gets washed out.** We did maxcut but expect same to happen in other combinatorial opt problems.

- **“Loopholes”** and **opportunities**:
  - **shallow regime:** specific graph **topology not washed out**.
    We don't have a theory for BP here but would guess this regime is trainable. It's not obvious to me how powerful this regime is / how truly quantum (at least expectation values). **Work is needed!**

  - **Deep Initializations**: if you are going deep don't forget to bring them. For some reason you are able to initialize in the narrow gorge and use QAOA to do a final tweaking.

Thanks for your attention! Questions?

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Normal Comp.

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FREE ANSATZ: Barren Plateau with a single layer!

When the graph has O(n) edges → even a single layer of MA-QAOA exhibits a BP.